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Threshold energies of electrons and holes for impact ionization in silicon

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Impact ionization in silicon has been experimentally studied by various workers (Mckay 1954, Miller 1957, Chynoweth 1958, 1960, Goetzberger *et al* 1963, Moll & Overstraeten 1963, Lee *et al* 1964.) They have measured ionization coefficient (number of ionizations per unit path length from the measurements of breakdown voltages and pre-breakdown multiplication factors. The experimentally measured ionization coefficients are fitted to the theoretical curves of Wolff (1954), Shockley (1961) or Baraff (1962) by adjusting three parameters (1) the threshold energy of ionization, (2) the mean free path for phonon emission a and (3) the constant energy loss associated with the scattering which is equal to ϵ_R the optical phonon energy in silicon as optical mode of scattering dominates in this material. Little attempt seems to have been made for calculating the threshold energy for impact ionization with the help of band structure of the material. Ahmed & Khokle (1967) have, however, calculated the threshold energy of ionization for electrons using only heavy hole mass employing the graphical method of Frantz (1956).

We in this communication have attempted to calculate the threshold energy of electrons and holes for impact ionization by an analytical approach described by Shekhar & Sharma (1974) for gallium arsenide

Silicon has six valleys in its conduction band, each at a distance of $0{\cdot}86 \times 2\pi/a$ from the center of Brillouin zone in $<100>$ directions. Density of states mass of electrons is $0{\cdot}33\ m_0$. The valence band is degenerate having three branches, two of which have coincident tops at k (000) and the third the split off branch also has its top at k (000) but 0·04 ev below the tops of the other two branches. The lattice constant $a = 5{\cdot}43$ Å and the energy gap $E_g = 1{\cdot}107$ eV at 300°K. The masses of heavy light and split off holes are $0{\cdot}5\ m_0$, $0{\cdot}15\ m_0$ and $0{\cdot}23\ m_0$ respectively.

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The values of threshold of ionization of electrons and holes for various possible impact processes are given in tables 1 and 2 below. Values of threshold of ionization more than 4 ev are not reported as they are not meaningful.

Table 1. Ionization thresholds for electrons in silicon

Positions of Electrons			hole mass m_p/m_0	Ionization energy in ev for		
Primary electron	Primary electron after impact	Secondary electron after impact		normal process	Umklapp Process $G = -4\pi/a$	Umklapp Process $G = +4\pi/a$
(100)	(100)	(100)	.50	2.15	—	3.18
			.15	2.15	—	3.75
			.23	2.39	—	3.62
(100)	(100)	(100)	.50	2.15	3.18	—
			.15	2.45	3.75	—
			.23	2.39	3.62	—
(100)	(100)	(100)	.50	—		1.44
			.15	—		1.55
			.23			1.55

Table 2. Ionization thresholds for holes in silicon

Position of holes			Position of electron	Ionization energy in ev for		
Primary hole before impact	Primary hole after impact	Secondary hole		Normal process	Umklapp Process $G = -4\pi/a$	Umklapp Process $G = +4\pi/a$
1	1	1	Conduction Valley	1.84	—	2.64
1	1	1	-do-	2.01	—	2.98
1	2	2	-do-	2.30		3.55
1	1	3	-do-	1.99	—	2.91
1	3	3	-do-	2.20	—	3.30
1	2	3	-do-	2.25		3.41
2	1	1	-do-	2.45	—	3.64
2	1	2	-do-	2.81	—	—
2	2	2	-do-	3.47	—	—
2	1	3	-do-	2.74	—	—
2	3	3	-do-	3.17	—	—
2	2	3	-do-	3.31	—	—
3	1	1	-do-	2.21	—	3.26
3	1	2	-do-	2.50	—	3.81
3	2	2	-do-	3.02	—	—
3	1	3	-do-	2.89	—	3.69
3	3	3	-do-	2.79	—	—
3	2	3	-do-	2.89	—	—

For the calculation of threshold energies for impact ionization both normal and processes have been considered. The momentum conservation equations for normal and Umklapp processes respectively are

$$\boldsymbol{k}_1 = \boldsymbol{k}'_1 + \boldsymbol{k}'_2 + \boldsymbol{k}_h \quad \ldots \quad (1a)$$

$$\boldsymbol{k}_1 = \boldsymbol{k}'_1 + \mathrm{k}'_2 + \boldsymbol{k}_h + \boldsymbol{G} \quad \ldots \quad (1b)$$

where $\boldsymbol{k}_1$ is the wave vector of primary electron before impact, $\boldsymbol{k}_1'$, $\boldsymbol{k}'_2$ and $\boldsymbol{k}_h$ are wave vectors of primary electron secondary electron and secondary hole after the impact $\boldsymbol{G}$ is reciprocal lattice vector. The energy of electron in the conduction band is given by

$$E = \frac{\hbar^2}{2m_e} (\boldsymbol{k} - \boldsymbol{k}_0)^2 \quad \ldots \quad (2)$$

where $\boldsymbol{k}_0$ is the wave vector of the corresponding conduction band minimum Since energy minima in conduction badd lie in equivalent $\langle 100 \rangle$ directions we expect the minimum energy for ionization will be when the motion of the particles involved in the impact process is confined in [100] direction; hence the wave vectors can be considered as scalars

Assuming the energy reference at the bottom of the conduction band, the energy conservation equation for the first process listed in table 1 for the impact ionization by electrons is

$$\frac{\hbar^2}{2m_e}(k_1 - k_0)^2 = \frac{\hbar^2}{2m_e}(k'_1 - k_0)^2 + \frac{\hbar^2}{2m_e}(k'_2 + k_0)^2 + E_g + \frac{\hbar^2 k_h^2}{2m_a} \quad \ldots \quad (3)$$

After eliminating k_h from eq. (3) with the help of eq (1a) and then minimizing it with respect to k'_0 and k'_2, eq (3) becomes

$$k_1^2 \left(\frac{m_h + m_e}{2m_e + m} \right) - 2k_0 k + \left(k_0^2 - \frac{2m_e E_g}{\hbar^2} \right) = 0 \quad \ldots \quad (4)$$

Equation (4) is quadratic in k_0 and is solved numerically to give two values of k_0. Then energy is calculated by using eq. (2) for both values of k_0. The lower value of energy is taken as threshold energy provided that corresponding k_0 is real, and within the first Brilloum zone; corresponding k'_1, k'_2 and k_h are all real and within first Brilloum zone and the wave factors of electrons must correspond to their valley assumed in a particular process Such physical restrictions are applied in the calculations of threshold energy for all the processes listed in tables 1 and 2.

From the table 1 it can be seen that the threshold energy for impact ionization by electrons is lowest when primary electron is in (100) valley and both the electrons are in (100) valley after impact. This is because the Umklapp process reduces the energy requirement for ionization in cases where large momentum transfers are involved in the ionization process. The values of threshold energy for this process range from 1·44 eV to 1·55 eV The lower values of threshold energy for electrons for normal process range from 2·15 eV to 2·45 eV This indicates that when the energy of electrons is increased by applied field, impact ionization by Umklapp process will start before the onset of impact ionization by normal process The impact ionization will increase with the increase of electron energy, consequently with the increase of applied electric field. Thus there will be a range threshold energy for electrons from 1·44 eV to 2·45 eV. These values are in excellent agreement with those determined from experimental data by Lee *et al* (E E 1·5 E with most probable value of 1·6 eV). Moll & Overstraeten (E = 1·8 eV), Mckay & Chynoweth (E = 2·5 eV)

From table 2 we can see that threshold energy of holes is less for normal process than that for Umklapp process The lower values of threshold energy for holes range from 1·84 eV to 3·47 eV These values are also in good agreement with the experimentally determined values of Moll & Overstraeten (E = 2·4 eV). Lee *et al* have fitted their experimentally data of ionization coefficient (number of ionizations per cm) by assuming the same value of threshold energy for electrons and holes (E_i = 1·6 eV), but they have taken different values of mean free path n for phonon scattering by electrons (50 Å $< \lambda_e <$ 70 Å) and by holes (30 Å $< \lambda <$ 45 Å) If the mean free path for holes is assumed the same as for electrons, then Lee *et al* data will give the value of threshold energy of 2·6 eV.

Similarly Chynoweth and Mckay's data are also fitted to Wolff theory by assuming same threshold energy but different mean free path for optical phonon scattering for electrons and holes. If the scattering mean free path is assumed the same, the threshold energy of holes would then be 3·5 eV Our values of threshold energy for impact ionization are thus in excellent agreement with the experimental results. These calculations further show that the impact ionization is not very sharp, because all processes do not start at the same energy and consequently at the same field.

Our values of threshold energy for electrons are higher than those calculated graphically by Ahmed & Khokle (E = 1·18 to 2·1 eV) the difference seems to be due to the different mass used for heavy hole by them (m = 0·78 m_0)

References

Ahmed S. & Khokle W. S. 1967 *J. Phys. Chem. Solids* **28**, 2499.
Baraff G. A. 1962 *Phys. Rev.* **128**, 2507.

Chynoweth A. G. 1958 *Phys Rev.* **109**, 1537
Chynoweth A. G. 1960 *J. Appl. Phys.* **31**, 1161.
Franz W. 1956 *Handbook der Physik* **17**, 185.
Geotzberger A., McDonald B , Hartz R. H. & Scarlett R M. 1963 *J. Appl. Phys.* **34**, 1591.
Lee C. A., Logan R. A., Batdorf R. L., Klemack J. J. & Weighmann W. 1964 *Phys. Rev.* **134**, A651.
McKay K. G. 1954 *Phys. Rev.* **94**, 873.
Eiller S. L. 1957 *Phys Rev* **105**, 1241
Moll J. L. & Overstraeton R. Van 1963 *Solid State Electronics* **6**, 147
Moll J. L., Meyer N I 1961 *Solid State Electron.* **3**, 155
Shekhar Chandra & Sharma S. K. 1974 *Indian J. Pure & Appl. Phys.* **12**, 179.
Shekhar Chandra & Sharma S. K 1974 *Phys Letters* 50.
Shockley W. 1961 *Solid State Electron* **2**, 35.
Wolff P A. 1954 *Phys. Rev.* **95**, 1415

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Comparison of probe and spectroscopic electron temperature measurements in a h.f. hydrogen plasma

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McGregor (1966) has experimentally found that there is an equivalence of the electron temperatures (found by the probe method) and the excitation temperatures (found by the spectroscopic method) for an argon d.c plasma. Ferdinand (1970) has compared the results for the electron temperatures measured by the spectroscopic and the probe methods for the plasma of an H_2–O_2 flame. The author has presently compared electron temperatures in an h.f. plasma of hydrogen gas by the spectroscopic and the double probe methods and the results of T_e have been concurrently checked by calculating T_e using Stoenbeck's formula

For the probe measurements the pyrex glass discharge tube having a length of 51 5 cm and diameter 4 5 cm was used The probe was made of a tungsten wire 0 2 mm in diameter and provided with a capillary cover It was located at the middle of the tube and projected radially inside the tube, having a collecting length of 1 cm terminating at the axis of the tube. The anti-probe was in the form of a grid having a surface area about 150 times as compared to the probe area. The distance between the probe and anti-probe was 3 cm Two external sleeve electrodes of 1 cm wide thin copper foils were used and connected to the Hartley oscillator for exciting the discharge Hydrogen gas was introduced

into the tube by diffusion through a palladium tube. The discharge was studied at five pressures viz. 0.03, 0.07, 0.13, 0 19 and 0.25 torr. and the pressures were measured by the Pirani gauge. The excitation frequency was 18 MHz, and voltage 300 volts. The electron temperatures were calculated from the slope of the curve between $\log\left(\frac{\Sigma I}{i_p}-1\right)$ and V where ΣI is total ionic current to the probe assembly and i, the electronic current to the probe and V, the p d between the probe assembly The results for electron temperatures are reproduced below in the table.

The electron temperatures have been also calculated by using the theoretical formula of Engel & Steenback (1934)

$$\frac{e^x}{\sqrt{x}} = 1\cdot16\times10^7C^2p^2R^2$$

where $x = eV_i/kT_e$, in which e is electronic charge, V_i in onisation potential, k Boltzmann's constant T_e electron temperature and C is a constant whose value for hydrogen is given to be $1\cdot35\times10^{-2}$, R the radius of discharge tube P the pressure in mm Hg

The set up used for spectroscopic determination of the electron temperatures consisted of the discharge tube and a vacuum system the same as used for the probe study, the discharge is studied under the conditions identically the same as those for the probe experiments The central part of the glow column is focussed on the slit of a Hilger C D spectrograph having a dispersion of about 18 A.U./mm at 4600 A U. which is about the mean wave length region of the singlet and tuiplets to be measured The photographs of the spectra are taken on Kodak plates for identification of the lines. The intensity of lines is measured by a Moll self recording microphotometer. The electron temperatures were computed from the curves given by Brasefield (1930) The results are reproduced below :

Table 1

S.No.	Pressure of hydrogen gas in mm. Hg.	T_e, measured by probe method °K	T_e calculated	T_e by spectroscopic method °K
1.	0.03	60,000	56,000	58,000
2.	0.07	57,500	50,000	54,000
3.	0.13	45,000	35,000	36,000
4.	0.19	42,000	30,500	26,000
5.	0.25	41,000	28,000	25,000

It is seen that at higher pressures the spectroscopic values of the electron temperatures do not agree with the probe values This may be due to non-reliability of the values because of increased number of molecular collisions at those pressures which may affect process of excitation and the intensity of lines from which determination of T_e is made by spectroscopic method However we could have a check for the probe value of T_e by an alternative experimental set up

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References

Brasefield, 1930 *Phys Rev.* **35**, 92
Engel V. & Steenbeck, 1934 *Electrische Gasent Ladungen*, **11**, 85.
Ferdinard J. 1970 *Phys. Letters* **31A**, 370.
McGregor W K. & Brewer L E 1966 *Phys. Fluids* **9**, 826.