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Polytropic universe

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Exact solution of cosmological field equations by assuming polytropic relation between pressure and density has been obtained

1. INTRODUCTION

A well known line element derived from the assumption of spatial isotropy is given by eq. (1) of the next section (Tolman 1934). The exact form of the function $g(t)$ is not always needed and a number of qualitative conclusions has often been drawn without exact form of $g(t)$ It remains of interest however if exact solution can be obtained. In this paper cosmological models isotropic and homogeneous in space have been considered and it has been shown how exact solution in the form of quadrature is possible corresponding to a polytropic relation between pressure (p) and density (ρ) Main interest in this paper lies in the solution of a differential equation corresponding to cosmological field equations.

2. EQUATION USING POLYTROPIC RELATION

The line element in spatially isotropic and homogeneous space can be written as

$$ds^2 = -\frac{e^{g(t)}}{\left(1+\frac{r^2}{4R_0^2}\right)^2}(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2)dt^2 \qquad \ldots (1)$$

where R_0^2 is a constant which can be positive, negative or infinite.

We assume that the material filling the model can be treated as perfect fluid so that in co-moving coordinate, the components of energy momentum tensor are

$$T_1^1 = T_2^2 = T_3^3 = -p \quad \text{and} \quad T_4^4 = \rho.$$

Field equations without cosmological constant immediately yield the following equations (Tolman 1934)

$$8\pi p = -\frac{1}{R_0^2}e^{-g(t)} - \ddot{g} - \tfrac{3}{4}\dot{g}^2 \qquad \ldots (2)$$

$$8\pi\rho = \frac{3}{R_0^2}e^{-g(t)} + \tfrac{3}{4}\dot{g}^2 \qquad \ldots (3)$$

where dots denote differentiation with respect to time.

We now assume a polytropic relation between pressure and density in the fluid so that

$$p = K\rho^{1+\frac{1}{n}}$$

where k and n are constants

Substituting the values of p and ρ from eqs (2) and (4) in eq. (4), we get

$$\frac{e^{-g(t)}}{R_0^2}+\ddot{g}+\tfrac{3}{4}\dot{g}^2 = -\frac{K}{(8\pi)^{1/n}}\left(\frac{3}{R_0^2}e^{-g(t)}+\tfrac{3}{4}\dot{g}^2\right)^{(1+1/n)} \quad \dots \quad (5)$$

i.e.,

$$\frac{2}{\dot{g}}\frac{d}{dt}\left(\frac{1}{R_0^2}e^{-g(t)}+\tfrac{1}{4}\dot{g}^2\right) = -\left[\frac{3^{1+1/n}K}{(8\pi)^{1/n}}\left(\frac{1}{R_0^2}e^{-g(t)}+\tfrac{1}{4}\dot{g}^2\right)^{(1+1/n)}\right.$$

$$\left.+3\left(\frac{1}{R_0^2}e^{-g(t)}+\tfrac{1}{4}\dot{g}^2\right)\right] \quad \dots \quad (6)$$

As the right hand side of eq (6) is a linear combination of the quantities in bracket of left hand side, the integration can be easily performed given $\dot{g}$ in terms of g. Hence g as a function of t is given by

$$t = \frac{1}{2A^{3/4}3^{n/2}}\int \frac{e^{g/2}dg}{\left[\dfrac{e^{g}}{\left(e^{3g/2n}-\dfrac{3^{1+1/n}K}{(8\pi)^{1/n}}A^{3/2n}\right)^{n}}-\dfrac{1}{A^{3/2}R_0^2 3^n}\right]^{\frac{1}{2}}}.$$

where A is a constant.

We do not propose to study this further in this paper

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References

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