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The first-forbidden 1.98 MeV beta decay in rhenium-188

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Nuclear matrix elements governing the 1^- (1·98 MeV beta)2^+ transition in ^{188}Re decay are extracted from the various experimental observables on the transition using the Buhring's formalism in the notation of Simms. A cancellation of vector matrix elements in Y (combination of rank one matrix elements) and the domination of B_{ij} matrix element is observed. The CVC ratio shows deviation from Fujita's estimate, thus indicating the significance of the off-diagonal elements in Coulomb Hamiltonian.

1 INTRODUCTION

Studies on the beta decays of rhenium isotopes is of importance as these isotopes lie just outside the deformed region. In the present work, the 1·98 MeV non-unique first-forbidden beta transition in ^{188}Re decay is carefully analysed.

The decay scheme of ^{188}Re is well established and a partial decay scheme (Yamazaki 1969) of the same is shown in Fig 1. The 1·98 MeV (end point

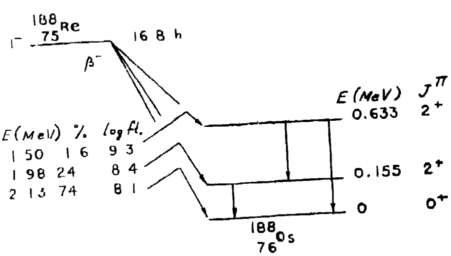

Fig 1 Partial decay scheme of Re-188

energy, W_0 4·84 in natural units) non-unique first forbidden beta transition in ^{188}Re is characterised by a high log ft value 8·6. The shape (Johns *et al* 1956, Bashandy *et al* 1963, Andre *et al* 1968, Trudel *et al* 1970, Vanderwerf *et al* 1969, Nielsen & Nielsen 1958) and beta-gamma directional correlation (Trudel *et al* 1970, Wyley *et al* 1963, Gronacs *et al* 1965) of this non-unique first forbidden transition have been extensively studied. Many authors report a non-statistical shape and a large beta-gamma anisotropy. However, the measurements of

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Bashandy & El-Nesr (1963) and Trudel *et al* (1970) indicate energy independent shape Further, to facilitate a determination of nuclear matrix elements (hereafter referred to as NMEs) experimental data on beta-gamma circular polarisation (Gygax & Hess 1966) longitudinal polarisation (Kaminker *et al* 1965) and nuclear orientation (Brewer & Shirley 1970) is also available.

Earlier attempts to determine matrix elements commence with the computations of Wyly *et al* (1963) who used the approximate formulae of Kotani (1959) Andre & Liaud (1968) also analysed the experimental observables to obtain NMEs but in this data on polarisation was not used. Manthuruthil *et al*

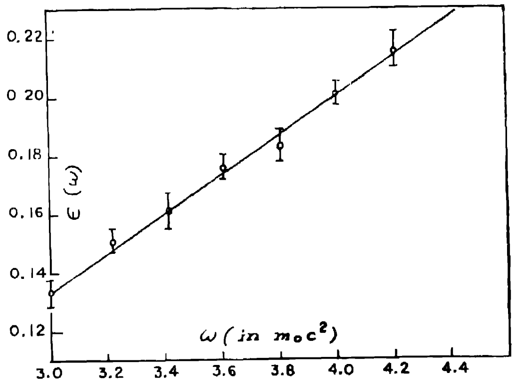

Fig 2 The directional correlation data used in the present analysis

(1971), who used all the experimental observables for the determination of NME parameters, employed the formulae of Morita & Morita (1958). Meanwhile, Bogdan *et al* (1968) theoretically computed the NME parameters using both Nilsson and Woods-Saxon wave functions and predicted the various experimental observables. But the agreement between experiment and theory is rather unsatisfactory, in particular regarding the angular correlation and the log ft value. Boherens & Bogdan (1970) recalculated the same using the ideas (i) the radial dependence of the electron radial wave function (Buhring 1963, 1965) and (ii) deformed part of the coulomb potential as pointed out by Damgaard & Winther (1966) Still one sees a disagreement between the experiment and theory in the case of the log ft value

The present study of the 1·98 MeV beta transition essentially concerned a re-analysis of the various experimental observables for the determination of NMEs This is done using more rigorous formulae of Buhring (1963, 1965) as modified by Simms (1965) The results thus obtained are compared with those that follow the analysis of Manthuruthil (1971) and also the model dependent NMEs due to Bogdan (1968, 1970).

2 Matrix Element Analysis and Results

The present beta transition involves unit spin change and is governed by four zeroth order matrix elements belonging to tensor ranks 1 and 2, besides

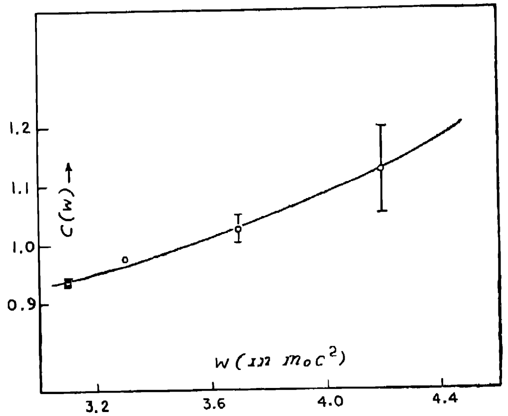

Fig. 3 The shape data used in the present analysis

the corresponding higher order matrix elements arising out of the finite size of the nucleus. The matrix element parameters Y, x, u and z are given by

$$z = -\frac{1}{\eta} C_A \int \frac{iB_{ij}}{R}, \qquad D'y = \frac{1}{\eta} C_V \int \bar{\alpha}$$

$$x = \frac{1}{\eta} C_V \int \frac{i\bar{r}}{R}, \qquad u = \frac{1}{\eta} C_A \int \frac{\sigma \times r}{R}$$

and

$$DY = D'y_0 - D(x+u)(1+d\lambda)/(1+a)$$

The parameters D, a and d are defined in the paper by Simms (1965) and are $D = 0{\cdot}306$, $a \approx -0{\cdot}130$ and $d \approx -0{\cdot}262$ (natural units are used) for the present case. R is the nuclear radius ($R = 0{\cdot}0179$ in natural units). D' distinguishes the relativistic matrix elements from the non-relativistic ones and $D' = |D|$. $y_0 = (y+ay')/(1+a)$. y' is the higher order matrix element parameter that arises because of the finite nuclear size effects, corresponding to y. λ is the ratio of higher order matrix element x' to the first order matrix elemtnt x and is given by

$$\lambda = x'/x = \int \frac{i\bar{r}}{R}(r/R)^2 / \int i\frac{\bar{r}}{R}.$$

The theoretical vector matrix element ratio, Λ_{CVC}, of Damgaard & Winther (1966) is given by (for β^- decay)

$$\int \bar{\alpha} / \int \frac{i\bar{r}}{R} = \Lambda_{CVC} = \Lambda^0_{CVC} + \tfrac{1}{2}\alpha z(0{\cdot}6-\lambda)$$

where Λ^0_{CVC} is the vector matrix element ratio due to Fujita (1962) given by

$$\Lambda^0_{CVC} = 2{\cdot}4\xi R + (W_0 - 2{\cdot}5)R.$$

The 1^-(1·98 MeV beta)2^+ decay of ^{188}Re was analysed using the Buhring (1963, 1965) formulae as modified by Simms (1965), with the following experimental data imposed : (i) The angular correlation due to Trudel *et al* (1970), (ii) the shape factor due to Vanderworf (1969), (iii) the beta-gamma circular polarisation correlation due to Gygax & Hess (1966), (iv) the nuclear orientation due to Brewer & Shirley (1970), and (v) the longitudinal polarisation due to Kaminker *et al* (1965). The usual search method was adopted for the evaluation of the matrix elements and the analysis was carried out on the IBM 1130 computer. That value of λ for which the experimental Λ_{CVC} equals the theoretical value, is taken as the value of λ. The standard matrix element η is calculated as detailed by Simms (1965). Finally the computations yielded satisfactory solutions in the ranges given below :

$$Z_0 = 1$$
$$0{\cdot}094 \leqslant Y \leqslant 0{\cdot}109$$
$$-0{\cdot}02 \leqslant u_0 \leqslant +0{\cdot}02$$
$$0{\cdot}26 \leqslant x_0 \leqslant 0{\cdot}31$$
$$0{\cdot}128 \leqslant \eta \leqslant 0{\cdot}134$$
$$2{\cdot}27 \leqslant \lambda \leqslant 2{\cdot}37$$

The NME parameters x_0, u_0, etc. contain the higher order ones x', u', etc. They are related as follows :

$$x_0 = \frac{x+(4/5)ax'}{1+(4/5)a}$$

$$u_0 = \frac{u+(4/5)au'}{1+(4/5)a}$$

$$z_0 = \frac{z+(4/5)az'}{1+(4/5)a}$$

To separate out x and x' and u and u', the following relationship is used :

$$x'/x = \lambda \approx u'/u.$$

From the above one observes that the solutions for the NME parameters confine to close limits due to the wealth of the experimental data available on the transition. From the ranges given above three typical sets (*A*, *B* and *C*) are chosen and shown in table 1

Table 1 Typical matrix element parameter sets

Set	z_0	x	u	$D'y_0$	Y	λ	η
A	1.00	0.341	0.017	0.064	0.104	2.35	0.129
B	1.00	0.352	0.007	0.067	0.099	2.34	0.131
C	1.00	0.364	0.109	0.068	0.099	2.36	0.130

The angular correlation and shape $C(W)$ calculated by these solutions as a function of beta energy are given in table 2. Table 3 contains the calculated values $P_\gamma(\theta)$, the circular polarisation parameter, U_2, the orientation parameter and P_γ, longitudinal polarisation parameter along with the corresponding experimental values.

Table 2. Shape and angular correlation parameter predicted by the NME

Energy W (in m_0c^2)	Shape $C(W)$			Correlation coefficient $\epsilon(W)$		
	A	B	C	A	B	C
3.010	0.964	0.970	0.974	0.132	0.135	0.133
3.220	0.969	0.972	0.975	0.147	0.151	0.148
3.420	0.978	0.979	0.980	0.160	0.165	0.163
3.610	1.000	1.000	1.000	0.176	0.182	0.180
3.810	1.025	1.024	1.023	0.192	0.197	0.196
4.010	1.026	1.022	1.019	0.198	0.205	0.203
4.210	1.049	1.044	1.040	0.210	0.217	0.215

Table 3. Theoretical prediction for typical sets

Description	$P_\gamma(135°)$	$P_\gamma(180°)$		$P_\gamma/(p/W)$
Set A	0.21	−0.32	0.299	0.976
Set B	−0.21	−0.32	0.307	0.979
Set C	0.21	−0.32	0.307	0.974
Experimental	0.21±0.08*	−0.17±0.06*	0.266±0.014**	1.005±0.016***

* Experimental value of F. Gygax & R. Hess (1966).

** Experimental value of W. D. Brewer & D. A. Shirley (1970).

*** Experimental value of D. M. Kaminket *et al* (1965).

The absolute values of the zeroth order matrix elements, that cause the non-unique first forbidden 1^-(1·98 MeV beta)2^+ transition in ^{188}Re, due to different authors are summarised in table 4. For comparison, the values from the present work are shown corresponding to set A only as the magnitudes of the values of the other two sets are not much different. From table 4 it can be seen that the values of Manthuruthil *et al* (1971) who follow the formalism of Morita & Morita (1968) disagree with the present values where the formalism of Buhring (1963, 1965) is used The disagreement is essentially in the values of the vector matrix elements $|\int i\bar{r}/R|$ and $|\int \bar{\alpha}|$. This brings significant difference on the estimation of Λ_{CVC} and hence λ Figures 4 and 5 show $\epsilon'(W) = \epsilon(W)/(p^2/W)$ and $C(W)$

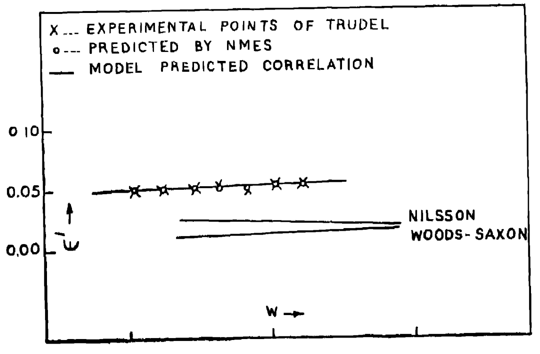

Fig. 4. Reduced correlation coefficient Vs. energy.

Table 4. Absolute values of nuclear matrix elements

$\lvert\int B_{ij}/R\rvert$	$\lvert\int i\bar{r}/R\rvert$	$\lvert\int \bar{\sigma}\times\bar{r}/R\rvert$	$\lvert\int\bar{\alpha}\rvert$	Authors Reference
0.700	0.010	0.390	0.003	Woods-Saxon, Bogdan *et al* (1968)
0.353	0 010	0.116	0.005	Nilsson, Bogdan *et al* (1968)
0.632	0.001	0.176	0 008	Beherns *et al* (1970)
0 180±0.050	0.003±0.003	0.008±0.004	0 002±0.001	Manthuruthil *et al* (1971)
0.107	0.044	0.002	0.013	Present work.

functions respectively due to Woods-Saxon and Nilsson wave function for the

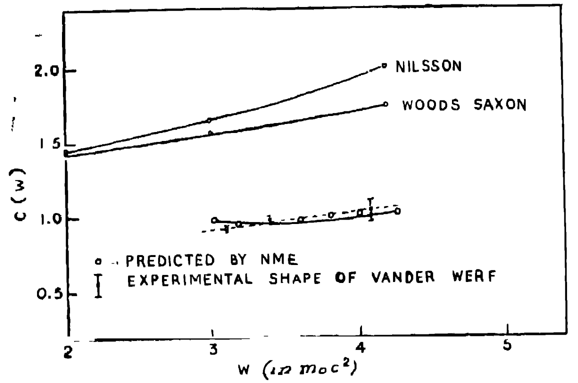

Fig. 5. Shape factors Vs. energy

present beta decaying states. Figures 6 and 7 represent the same due to Nilsson wave functions including the correction due to the higher order effects. These prediction were made in the works of Bogdan *et al* (1968) and Boherns & Bogdan

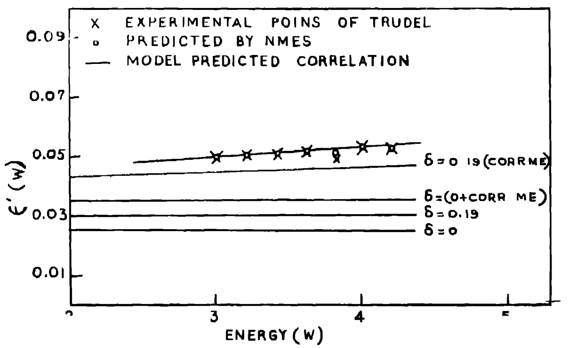

Fig. 6. Reduced correlation coefficient Vs energy.

(1970). The corresponding experimental and the calculated values for set *A* matrix elements are also superposed in these figures. The deformation parameters used in each case are indicated against each curve.

The values of $\frac{1}{\xi R} \frac{\int \alpha}{\int i\bar{r}/R}$ due to different authors are summarised in table 5, the expected value of which is 2·4 according to Fujita. The present analysis

Table 5. Vector matrix element ratio summary

Description	$\frac{1}{\xi R} \frac{\int \alpha}{\int ir/R}$
Fujita	2.4
Woods-Saxon	1.450
Nilsson	2.032
Manthuruthil	2.368 ± 2.663
Present work	0.92 ± 0.12

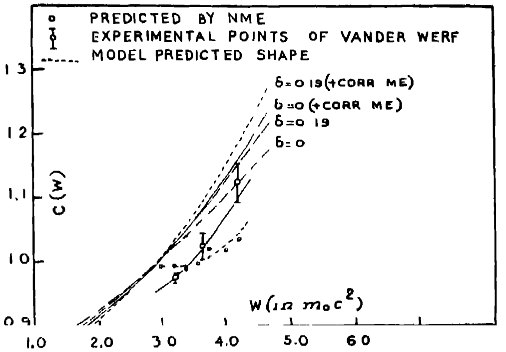

Fig 7. Shape factor Vs. energy.

was also carried out using the statistical shape of Trude *et al* (1970). But no satisfactory solutions for NME parameters consistent with other experimental observables could be predicted.

3. Discussion

The lower excitations of ^{188}Re and ^{188}Os nuclei can be viewed as due to pure rotations (Alago *et al* 1955). However, the ratio of the ft values of the beta transitions proceeding from the ground states is $3{\cdot}2 \pm 0{\cdot}6$, while the predicted value is 2 for pure rotational states. The experimentally determined $B(E2)$ and $B(M1)$ values for the low-lying γ-rays in ^{188}Os also differ from the pure rotational picture. Thus departures from the pure rotational band region of lowlying states in ^{188}Os are noted. Bodgon *et al* describe these states in terms of Nilsson model and have obtained the NME parameters.

From Figs 4 and 5 the disagreement between the model predicted values and the, experiment can be clearly seen. However, Behorens & Bogdon (1970) recalculated the same by introducing the higher order terms due to the finite nuclear charge distribution Buhring (1963, 1965) and coulomb Hamiltonian (Damgaard & Winther 1966) This substantially improves the accuracy in calculations as evidenced from the agreement seen between experimental and theory in Figs. 6 and 7 But still one finds considerable difference between the predicted log ft value (= 7·4) and the experimental value (= 8·4). From table 4 it is seen but for the value of $\int \frac{\bar{\sigma}\times\bar{r}}{R}$ all the other absolute matrix elements of Behorens & Bogdan and the present values are nearly of the same order of magnitude. In the work of Behorens & Bogdan (1970), from the small values of $\int \bar{r}$, a large deviation from Fujita's estimate for CVC ratio is predicted. The values of η of this analysis ranging from 2·27 to 2·37 substantiate this prediction. Now the disagreement between the results of Manthuruthil *et al* and the present analysis regarding, $\int \frac{\bar{i}}{R} \int \bar{\alpha}$ and Λ_{CVC} is evident, inasmuch as, the formalisms adopted are different and the present expressions are considered to be more accurate The values of λ further indicate the importance of the off-diagonal elements in the coulomb Hamiltonian H, which were neglected in arriving at the relationship of Fujita. From Table 4 one finds that all the first forbidden matrix elements are attenuated to a high degree when compared to the allowed matrix elements and the B_{ij} matrix element dominates the three vector matrix elements. This explains the energy dependence of the several experimental observables and the failure of ξ-approximation No K-selection rule hindrance of the vector matrix elements for the present 1^-—2^+ beta transition takes place, inasmuch as, the connecting states have K-quantum numbers 1 and 0 thus obeying the K-selection rule. The values of DY are too small as can be seen from Table 2 which suggest a cancellation of rank 1 matrix elements. Thus the domination of B_{ij} matrix element and the cancellation of rank one matrix elements cause a deviation from ξ-approximation for the 1·98 MeV non-unique first forbidden beta decay in ^{188}Re.

From the present analysis the following conclusions may be drawn .

1) The theoretical computations of Beherens & Bogdan appear to be more accurate.

2) A deviation from Fujita's estimate for the CVC ratio is observed which stresses the importance of the non-diagonal matrix elements in the coulomb Hamiltonian.

3) For the failure of ξ-approximation, the domination of B_{ij} matrix element and the cancellation of vectoı matrix elements are ascribed.

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