Indian J. Phys. **50**, 929–941 (1976)

Study of fields associated with spin-I particles carrying electric and magnetic charges

Om Parkash and B S Rajput

Department of Physics, Kurukshetra University, Kurukshetra.

(*Received* 18 *February* 1976, *revised* 15 *April* 1976)

It has been shown that electric and magnetic fields due to spin-I particles carrying electric and magnetic charges are symmetrical. In order to avoid the arbitrary string variables in the solution of field equations, two four-potentials are introduced and it has been shown that their components are proportional to electric and magnetic currents source densities respectively.

1. Introduction

Using the methods given by Lomont & Moses (1967), we could derive the reductions of wavefunctions transforming as scalar field (Rajput 1969d; Parkash & Rajput 1975), antisymmetric tensor field (Rajput 1969a, 1969b) and three vector field (1969c), for non-zero as well as for zero mass system, to the irreducible representations of proper, orthochronous, inhomogeneous Lorentz group in terms of Foldy-Shirokov (1956, 1958) and Lomont-Moss (1964) realizations These reductions have been used for the reduction, second quantization and interactions (Rajput 1970, 1971b, 1969c; Parkash & Rajput 1974) of electromagnetic fields for zero and non-zero mass systems. In the reductions for non-zero mass system it has been seen that in the presence of an electric charge source the longitudinal component of electric fields is non-zero while that of magnetic field vanishes. This lack of symmetry though agrees with the experimental observation leading to the conclusion of non-existence of free magnetic charge in nature, is somewhat disturbing because nothing in classical physics forbids their existence. To overcome this lack of symmetry, Dirac (1931, 1948) put forward the idea of magnetic charge as the natural generalization of electricity. He showed that quantum mechanics demands the existence of free magnetic poles, having the pole strength $e/2\alpha$, where α is fine-structure constant. The similar result was deduced independently by Saha (1937, 1948) from very simple considerations of classical electrodynamics taking a point electric charge and a magnetic pole at two points and computing the angular momentum of the system about the line joining these points Schwinger (1966, 1968) formulated the field theory of magnetic charges carried by spin-1/2 particles and Zwanziger (1968, 1971) could generalize this theory for these particles carrying both electric and magnetic charges. Deo & Singh (1973) extended Zwznziger's work -for spin-1/2 fields to the case of electric and magnetic charges carried by spin-zero fields. In the present work, assuming the generalized charge of a spin-1 massive particle as a complex quantity with electric and magnetic charges as its real and imaginary parts we have undertaken the study of reduction of electromagnetic field in the presence of electric charge source density to the symmetric fields of particles carrying both the electric and magnetic charges.

We have carried out the reduction of generalized electromagnetic fields to the irreducible representations of proper, orthochronous, inhomogeneous Lorentz group in linear momentum basis for non-zero mass system. Reduced expansions for electric and magnetic fields are resolved in to longitudinal and transverse parts and it has been shown that both the longitudinal fields are non-vanishing and symmetrical. It has also been shown that the magnetic charge has its contribution in producing electric fields as electric charge (in motion) contributes to the magnetic field. Two vector potentials and two scalar potentials are introduced for the description of transverse and longitudinal electric and magnetic fields and it has been shown that electric and magnetic charge source densities are proportional to fundamental electric and magnetic charges.

Maxwell's field equations are modified in the presence of electric and magnetic current and charge source densities to the generalized field equations in terms of the generalized electromagnetic wave-function and electromagnetic field tensor separately. In order to avoid the arbitrary string variables in the solution of these field equations two four-potentials have been introduced and it has been shown that their components are proportional to fundamental electric and magnetic charges. Reduced expansion of generalized four current density has been reduced and it has been shown that it satisfies the continuity condition ensuring its conservation. Imposition of the generalized field equations on the reduced expansions of electromagnetic fields has led to the proportionality of electric and magnetic current and charge source densities. A suitable Lagrangian density which yields the required field equations has been given.

2. Reduction of Fields of Particles Carrying Electric and Magnetic Charges (Linear Momentum Basis)

Reduction of electric and magnetic fields in the presence of electric and magnetic charges to the irreducible representations of proper, orthochronous, inhomogeneous Lorentz group can not be done in the usual manner in which the reduction of ordinary electromagnetic fields (in the absence of monopoles) is carried out (Rajput 1970, 1971; Parkash & Rajput 1974b) because in this case we have restriction ($\boldsymbol{H} \neq$ cur $\boldsymbol{A}$). In order to carry out this reduction let us start with the results derived by Epstein (1967a) for electric and magnetic fields in the presence of the particles carrying both the charges, (electric and magnetic).

Substituting in these results our reduced expansions of scalar (Rajput 1969e) and vector fields (Rajput 1967c) for non-zero mass system, we get

$$\boldsymbol{E}(x) = \frac{1}{|q|4\pi^{3/2}} \Big[\int d\boldsymbol{p} \Big[ef(\boldsymbol{p}) - \frac{e\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{\omega(\omega+m)} - \frac{g\{\boldsymbol{p}xf(\boldsymbol{p})\}}{\omega} \Big] \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}$$

$$-\int d\boldsymbol{p} \Big[eh^*(\boldsymbol{p}) - \frac{e\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{(\omega+m)\omega} - \frac{g\{\boldsymbol{p}xh^*(\boldsymbol{p})\}}{\omega} \Big] \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} \Big]$$

and

$$\boldsymbol{H}(x) = \frac{1}{|q|4\pi^{3/2}} \Big[\int d\boldsymbol{p} \Big[gf(\boldsymbol{p}) - \frac{g\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{(\omega+m)\omega} + \frac{e\{\boldsymbol{p}xf(\boldsymbol{p})\}}{\omega} \Big] \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)$$

$$-\int d\boldsymbol{p} \Big[eh^*(\boldsymbol{p}) - \frac{e\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{\omega(\omega+m)} + \frac{g\{\boldsymbol{p}xh^*(\boldsymbol{p})\}}{\omega} \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} \Big] \quad \ldots (2)$$

where $f(\boldsymbol{p}) = f(m, \boldsymbol{p})$ is the representation of wave function of particle of mass m and spin-1 in the basis characterized by the Hilbert space on which the generators of inhomogeneous Lorentz group operate; $h(\boldsymbol{p}) = f^*(m, -\boldsymbol{p})$; x is the space-time four-component position vector, $c = \hbar = 1$, and $\omega(p) = (m^2+p^2)^{\frac{1}{2}} = \omega$. The generalized charge q has been considered as a complex quantity with electric and magnetic charges e and g as its real and imaginary parts;

$$q = e - ig. \quad \ldots (3)$$

This consideration has been shown (Parkash & Rajput 1973, 1974a) to have certain advantages over that given by Zwanziger (1968a), regarding the derivation of electric and magnetic coupling parameters. These equations may also be written in the following forms .

$$\boldsymbol{E}(x) = \boldsymbol{E}^L(x) + \mathrm{E}^T(x), \quad \ldots (4)$$

$$\boldsymbol{H}(x) = \boldsymbol{H}^L(x) + \boldsymbol{H}^T(x). \quad \ldots (5)$$

where

$$\boldsymbol{E}^L(x) = \frac{iem}{|q|4\pi^{3/2}} \Big[\int \frac{d\boldsymbol{p}}{\omega} \frac{\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{p^2} \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}$$

$$-\int \frac{d\boldsymbol{p}}{\omega} \cdot \frac{\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{p^2} \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} \Big], \quad \ldots (6a)$$

$$\mathrm{H}^L(x) = \frac{igm}{|q|4\pi^{2/2}} \Big[\int \frac{d\boldsymbol{p}}{\omega} \frac{\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{p^2} \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}$$

$$-\int \frac{d\boldsymbol{p}}{\omega} \cdot \frac{\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{p^2} \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} \quad \ldots (6b)$$

and

$$E^T(x) = \frac{i}{|q|4\pi^{3/2}}\left[\int d\boldsymbol{p}\left[\; ef(\boldsymbol{p}) - \frac{e\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{p^2} - \frac{g\{\boldsymbol{p}\times f(\boldsymbol{p})\}}{\omega}\right]\exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\right.$$

$$\left. -\int d\boldsymbol{p}\left[eh^*(\boldsymbol{p}) - \frac{e\boldsymbol{p}\{\boldsymbol{p}\,h^*(\boldsymbol{p})\}}{p^2} - \frac{g\{\boldsymbol{p}\times h^*(\boldsymbol{p})\}}{\omega}\right]\exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\right], \quad \ldots \text{(7a)}$$

$$H^T(x) = \frac{i}{|q|4\pi^{3/2}}\left[\int d\boldsymbol{p}\left[gf(\boldsymbol{p}) - \frac{g\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{p^2} + \frac{e\{\boldsymbol{p}\times f(\boldsymbol{p})\}}{\omega}\right]\exp\{i(\boldsymbol{p}\;\boldsymbol{x}-\omega t)\}\right.$$

$$\left. -\int d\boldsymbol{p}\left[gh^*(\boldsymbol{p}) - \frac{g\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{p^2} + \frac{e\{\boldsymbol{p}\times h^*(\boldsymbol{p})\}}{\omega}\right]\exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\right] \quad \ldots \text{(7b)}$$

It may be readily seen that

$$\text{div}\; E^T = \text{div}\; H^T = 0$$

and

$$\text{curl}\; E^L = \text{curl}\; H^L = 0 \quad \ldots \text{(8)}$$

The divergenceless parts E^T and H^L of generalized electric and magnetic fields are interpreted as their transverse parts in the conventional manner while non-curl parts E^L and H^L are treated as longitudinal parts. It is obvious from the reduced expressions (6a) and (6b) that both the longitudinal fields are non vanishing and symmetrical in the presence of electric and magnetic charges while in the absence of electric or magnetic charge the corresponding longitudinal field vanishes. For the ordinary fields (i.e., for $g = 0$) the reduced expansions (Parkash & Rajput 1973; Rajput 1971) of electric and magnetic fields are not symmetrical. Moreover the longitudinal parts E^L and H^L are proportional to e and g respectively. This supports the assumption made by Schwinger (1966a) and Zwanziger (1968b) in developing the quantum field theory.

From the transverse fields E^T and H^T given by eqs. (7a) and (7b), we can derive the reduced expansions for the transverse magnetic and electric potentials B^T and A^T respectively, by using following relations :

$$A^T(x) = \nabla x \int D(\boldsymbol{x}-\boldsymbol{x}')H^T(x')d\boldsymbol{x}' \quad \ldots \text{(9)}$$

$$B^T(x) = -\nabla x \int D(\boldsymbol{x}-\boldsymbol{x}')E^T(x')d\boldsymbol{x}' \quad \ldots \text{(10)}$$

where

$$D(\boldsymbol{x}-\boldsymbol{x}') = \frac{1}{4\pi|\boldsymbol{x}-\boldsymbol{x}'|}.$$

We thus get

$$A^T(x) = \frac{1}{|q|4\pi^{3/2}}\Big[\int d\boldsymbol{p}\Big[\frac{gf(\boldsymbol{p})}{\omega} - \frac{g\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{p^2\omega} + \frac{e\{\boldsymbol{p}x f(\boldsymbol{p})\}}{p^2}\Big] \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}$$

$$+\int d\boldsymbol{p}\Big[\frac{eh^*(\boldsymbol{p})}{\omega} - \frac{e\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{p^2\omega} - \frac{g\{\boldsymbol{p}xh^*(\boldsymbol{p})\}}{p^2}\Big] \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\Big] \quad \ldots \quad (11)$$

and

$$B^T(x) = \frac{1}{|q|4\pi^{3/2}}\Big[\int dp\Big[\frac{gf(p)}{\omega} - \frac{gp\{p.f(p)\}}{p^2\omega} + \frac{e\{p\times f(p)\}}{p^2}\Big] \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}$$

$$+\int d\boldsymbol{p}\Big[\frac{gh^*(\boldsymbol{p})}{\omega} - \frac{g\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{p^2\omega} + \frac{e\{\boldsymbol{p}\times h^*(\boldsymbol{p})\}}{p^2}\Big] \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\Big] \quad \ldots \quad (12)$$

from which it is obvious that

$$E^T = -\nabla x B^T \quad \ldots \quad (13a)$$

$$H^T = \nabla x A^T \quad \ldots \quad (13b)$$

Therefore in order to describe the transverse electromagnetic fields in the presence of electric and magnetic charges, the two transverse vector potentials are required. The same result was derived semiclassically by Cabibbo & Ferrari (1962)

Longitudinal parts E^L and H^L given by eqns (6a) and (6b) may also be written as follows :

$$E^L = -\Delta\phi_e \quad \ldots \quad (14a)$$

and

$$H^L = -\Delta\phi_g \quad \ldots \quad (14b)$$

where

$$\phi_e = -\frac{em}{|q|4\pi^{3/2}}\Big[\int \frac{d\boldsymbol{p}}{\omega p^2}\{\boldsymbol{p}.f(\boldsymbol{p})\} \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}$$

$$+\int \frac{d\boldsymbol{p}}{\omega p^2}\{\boldsymbol{p}.h^*(\boldsymbol{p})\} \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\Big] \quad \ldots \quad (15a)$$

and

$$\phi_g = -\frac{gm}{|q|4\pi^{3/2}}\Big[\int \frac{d\boldsymbol{p}}{\omega p^2}\{\boldsymbol{p}.f(\boldsymbol{p})\} \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}$$

$$+\int \frac{d\boldsymbol{p}}{\omega p^2}\{\boldsymbol{p}.h^*(\boldsymbol{p})\} \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\Big]. \quad \ldots \quad (15b)$$

These scalar fields may be interpretted as electric and magnetic scalar potentials, respectively. In terms of these potentials one can introduce in the obvious manner, the electric and magnetic charge source densities $j_0(x)$ and $k_0(x)$ as follows :—

$$\phi_e = \int D(\boldsymbol{x}-\boldsymbol{x}')j_0(\boldsymbol{x}')d\boldsymbol{x}' \qquad \ldots \quad (16a)$$

and

$$\phi_g = \int D(\boldsymbol{x}-\boldsymbol{x}')k_0(\boldsymbol{x}')d\boldsymbol{x}'. \qquad \ldots \quad (16b)$$

Comparing these equations with eqs. (15a) and (15b), we get

$$j_0(x) = \frac{em}{|q|4\pi^{3/2}} \left[\int \frac{d\boldsymbol{p}}{\omega} \{\boldsymbol{p}\, f(\boldsymbol{p})\} \exp\{i(\boldsymbol{p}\, \boldsymbol{x}-\omega t)\} - \int \frac{d\boldsymbol{p}}{\omega} \{\boldsymbol{p}.h^*(\boldsymbol{p})\} \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} \right] \qquad \ldots \quad (17a)$$

and

$$k_0(x) = \frac{gm}{|q|4\pi^{3/2}} \left[\int \frac{d\boldsymbol{p}}{\omega} \{\boldsymbol{p}.f(\boldsymbol{p})\} \exp\{i(\boldsymbol{p}\, \boldsymbol{x}-\omega t)\} - \int \frac{d\boldsymbol{p}}{\omega} \{\boldsymbol{p}\, h^*(\boldsymbol{p}) \exp \{-i(\boldsymbol{p}\, \boldsymbol{x}-\omega t)\} \right] \qquad \ldots \quad (17b)$$

which are the reduced expansions of electric and magnetic charge source densities. It is obvious from these expansions that

$$\frac{j_0}{k_0} = \frac{\phi_e}{\phi_g} = \frac{e}{g} \qquad \ldots \quad (18)$$

Using the expansions (1) and (2), we may derive the following reduced expansion of the wave function, $\psi = \boldsymbol{E}-i\boldsymbol{H}$, which transforms as electromagnetic fields

$$\psi(x) = \frac{iq}{|q|4\pi^{3/2}} \left[\int d\boldsymbol{p} \left[f(\boldsymbol{p}) - \frac{\boldsymbol{p}\,\{\boldsymbol{p}.\, f(\boldsymbol{p})\}}{\omega(\omega+m)} - \frac{\boldsymbol{p}\, x f(\boldsymbol{p})}{\omega} \right] \exp\{i(\boldsymbol{p}\, \boldsymbol{x}-\omega t)\} - \int d\boldsymbol{p} \left[h^*(\boldsymbol{p}) - \frac{\boldsymbol{p}\{\boldsymbol{p}.\, h^*(\boldsymbol{p})\}}{\omega(\omega+m)} - i\,\frac{\boldsymbol{p}\times h^*(\boldsymbol{p})}{\omega} \right] \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} \right] \qquad \ldots \quad (19)$$

It may also be resolved into longitudinal and transverse parts in the following manner :

$$\psi(x) = \psi^L(x) + \psi^T(x), \qquad \ldots \quad (20)$$

and the longitudinal part may be written as

$$\psi^L(x) = -\Delta\phi_g \qquad \ldots \quad (21)$$

where

$$\phi_g = \phi_e - i\phi_g \qquad \ldots \quad (22)$$

which is the generalized scalar potential. Generalized charge source density $J_0(x)$ may be introduced in terms of ϕ_g in the manner described by eqs. (16) Thus we get

$$J_0(x) = -\frac{qm}{|q|4\pi^{3/2}}\left[\int \frac{d\boldsymbol{p}}{\omega}\{\boldsymbol{p}.f(\boldsymbol{p})\}\exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\right.$$

$$\left. +\int \frac{d\boldsymbol{p}}{\omega}\{\boldsymbol{p}\, h^*(\boldsymbol{p})\}\exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\right] \qquad \ldots \quad (23)$$

which may be written as

$$J_0(x) = j_0(x) - ik_0(x), \qquad \ldots \quad (24)$$

where $j_0(x)$ and $k_0(x)$ are given by eqs. (17). Transverse part ψ^T in eq. (20) may be written as

$$\psi^T(x) = -i\nabla x \boldsymbol{V}^T, \qquad \ldots \quad (25)$$

where

$$\boldsymbol{V}(x) = \boldsymbol{A}^T(x) - i\boldsymbol{B}^T(x),$$

for $\boldsymbol{A}^T$ and B^T given by eqs (9) and (10), and therefore, $\boldsymbol{V}^T(x)$ may be interpretted as the generalized transverse vector potential. Obviously, $\boldsymbol{V}^T(x)$ may also be written as

$$\boldsymbol{V}^T(x) = i\nabla x \int D(\boldsymbol{x}-\boldsymbol{x}')\psi^T(x-x')d\boldsymbol{x}' \qquad \ldots \quad (26)$$

Let us put

$$\boldsymbol{A}^T(x) = \boldsymbol{V}^{T1}(x), \quad \boldsymbol{B}^T(x) = \boldsymbol{V}^{T2}(x)$$

and similarly,

$$\boldsymbol{E}^T(x) = \psi^{T1}(x), \quad \boldsymbol{H}^T(x) = \psi^{T2}(x)$$

Then the relations (9), (10) and (26) can be written as a single equation given as

$$V^T = \epsilon^{\alpha\beta}\epsilon_{ijk}\nabla_j \int D(\boldsymbol{x}-\boldsymbol{x}')\psi_k^{T\beta}(x')d\boldsymbol{x}' \qquad \ldots \quad (27)$$

where ϵ_{ijk} is the usual Levicivita three-index symbol and $\epsilon^{\alpha\beta}$ is the antisymmetric symbol with $\epsilon^{12} = 1$.

So far only the transverse vector potentials have been introduced—Total vector potential may be introduced in the following way (Schwinger 1966a)

$$\boldsymbol{A}(x) = \boldsymbol{A}^T(x) + \int a(\boldsymbol{x}-\boldsymbol{x}')k_0(\boldsymbol{x}')d\boldsymbol{x}' \qquad \ldots \quad (28a)$$

$$\boldsymbol{B}(x) = \boldsymbol{B}^T(x) - \int a(\boldsymbol{x}-\boldsymbol{x}')j_0(\boldsymbol{x}')d\boldsymbol{x}' \qquad \ldots \quad (28b)$$

where $a(\boldsymbol{x}-\boldsymbol{x}')$ is a numerical function defined by the following equations :

$$\text{curl}\, a(\boldsymbol{x}-\boldsymbol{x}') = -\nabla D(\boldsymbol{x}-\boldsymbol{x}')+h(\boldsymbol{x}-\boldsymbol{x}') \quad \dots \quad (29)$$

and

$$\nabla . a(\boldsymbol{x}-\boldsymbol{x}') = 0 \quad \dots \quad (30)$$

In eq. (29) $h(\boldsymbol{x}-\boldsymbol{x}')$ is non-zero only along a singular line called a Dirac string and satisfies the following condition

$$\nabla . h(\boldsymbol{x}-\boldsymbol{x}') = -\delta(\boldsymbol{x}-\boldsymbol{x}'). \quad \dots \quad (31)$$

These equations (28a) and (28b) for total potentials are chosen in accordance with the similar relations for the total fields $\boldsymbol{E}(x)$ and $\boldsymbol{H}(x)$, which can be derived in the following form by combining equations (4), (5) and (41a) and (14b) and (16a) and (16b) .

$$\boldsymbol{E}(x) = \boldsymbol{E}^T(x)-\nabla \int D(\boldsymbol{x}-\boldsymbol{x}')j_0(x')d\boldsymbol{x}' \quad \dots \quad (32)$$

$$\boldsymbol{H}(x) = \boldsymbol{H}^T(x)-\nabla \int D(\boldsymbol{x}-\boldsymbol{x}')k_0(x')d\boldsymbol{x}'. \quad \dots \quad (33)$$

Negative sign before the integral in eq. (28b) is chosen keeping in view the space-reflection characteristic of $\boldsymbol{B}^T(x)$. Taking curl on both sides of eqn (28a) and using eqs. (13a) and (29), we get,

$$\Delta x \boldsymbol{A}(x) - \boldsymbol{H}(x) + \int h(\boldsymbol{x}-\boldsymbol{x}')k_0(x')d\boldsymbol{x}' \quad \dots \quad (34)$$

taking divergence of this equation and using eqn (31) we get

$$\text{div}\, \boldsymbol{H}(x) = k_0(x) \quad \dots \quad (35)$$

which is the generalized Maxwell's equation for divergence of magnetic field. This equation obviously maintains the symmetry in field equations for generalized electromagnetic fields. If string is not crosssed, $h(\boldsymbol{x}-\boldsymbol{x}') = 0$ and therefore, we get,

$$\boldsymbol{H}(x) = \text{curl}\, \boldsymbol{A}(x) \qquad \text{(Locality condition).}$$

In the similar manner eqn. (2.28b) yields

$$\text{div}\, \boldsymbol{E}(x) = j_0(x) \quad \dots \quad (36)$$

and also for not crossing the string

$$\boldsymbol{E}(x) = -\nabla x \boldsymbol{B}(x)$$

If we take the divergence of equations (28a) and (28b), we get

$$\nabla . A(x) = \nabla \boldsymbol{B}(x) = 0. \quad \dots \quad (37)$$

Let us now define longitudinal potentials $\boldsymbol{A}^L(x)$ and $\boldsymbol{B}^L(x)$ such that

$$A(x) = \boldsymbol{A}^T(x)+\boldsymbol{A}^L(x) \quad \dots \quad (38a)$$

and

$$\boldsymbol{B}(x) = \boldsymbol{B}^T(x)+\boldsymbol{B}^L(x), \quad \dots \quad (38b)$$

then equations (34) and (37) yield

$$\nabla x \boldsymbol{A}^L = -\nabla\phi_g + \int h(\boldsymbol{x}-\boldsymbol{x}')k_0(x')d\boldsymbol{x}' \quad . \quad (39)$$

and

$$\nabla \boldsymbol{A}^L = 0.$$

Which for some choice of string may be solved to get $\boldsymbol{A}^L$ in terms of string variables (ϕ_g is given by eq. (15b)) Similarly, for the potential $\boldsymbol{B}^L$ we have

$$\nabla x \boldsymbol{B}^L = \nabla\phi_e - \int h(\boldsymbol{x}-\boldsymbol{x}')j_0(x')d\boldsymbol{x}'$$

and

$$\nabla.\boldsymbol{B}^L = 0 \quad .. \quad (40)$$

which may yield $\boldsymbol{B}^L$.

In order to avoid the solutions in terms of arbitrary string variables, let us introduce four-potentials $A = \{A_\mu\}$ and $B = \{B_\mu\}$ such that $A = (\boldsymbol{A}, i\phi_e)$ and $B = (\boldsymbol{B}, i\Phi_g)$ In terms of spatial and temporal parts of these vectors, electric and magnetic fields may be written as

$$E(x) = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\Phi_e - \Delta\times\boldsymbol{B}$$

$$H(x) = -\frac{\partial B}{\partial t} - \nabla\phi_g + \nabla\times\boldsymbol{A} \quad \ldots \quad (41)$$

Comparing these equations with eqs. (1) and (2), we may readily derive the following reduced expansions for the components of these vectors .

$$\boldsymbol{A}(x) = \frac{e}{|q|4\pi^{3/2}}\left[\int\frac{d\boldsymbol{p}}{\omega} f(p)\exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\right.$$

$$\left. + \int\frac{d\boldsymbol{p}}{\omega} h^*(p)\exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\right] \quad \ldots \quad (42a)$$

$$\Phi_e = \frac{e}{|q|4\pi^{3/2}}\left[\int\frac{d\boldsymbol{p}}{\omega(\omega+m)}\{\boldsymbol{p}\, f(\boldsymbol{p})\}\exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\right.$$

$$\left. + \frac{d\boldsymbol{p}}{\omega(\omega+m)}\{\boldsymbol{p}\, h^*(\boldsymbol{x})\}\exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\}\right] \quad \ldots \quad (42b)$$

$$\boldsymbol{B}(x) = \frac{g}{|q|4\pi^{3/2}}\left[\int\frac{d\boldsymbol{p}}{\omega} f(\boldsymbol{p})\exp\{i(\boldsymbol{p}\,\boldsymbol{x}-\omega t)\}\right.$$

$$\left. + \int\frac{d\boldsymbol{p}}{\omega} h^*(\boldsymbol{p})\exp\{-i(\boldsymbol{p}\,\boldsymbol{x}-\omega t)\}\right] \quad \ldots \quad (42c)$$

and

$$\Phi_g(x) = \frac{g}{|g|\,4\pi^{3/2}}\left[\int \frac{d\boldsymbol{p}}{\omega(\omega+m)}\{\boldsymbol{p}\, f(\boldsymbol{p})\}\exp\{i(\boldsymbol{p}\,\boldsymbol{x}-\omega t)\}\right.$$

$$\left.+\int \frac{d\boldsymbol{p}}{\omega(\omega+m)}\{\boldsymbol{p}\, h^*(\boldsymbol{p})\}\exp\{-i(\boldsymbol{p}\,\boldsymbol{x}-\omega t)\}\right] \quad \ldots \quad (42d)$$

It is obvious from these reduced expansions that

$$\frac{A_\mu}{B_\mu} = \frac{e}{g} \quad \ldots \quad (43)$$

which when combined with eqn. (18) yields

$$\frac{A_\mu}{B_\mu} = \frac{j_0}{k_0} = \frac{e}{g} \quad \ldots \quad (44)$$

3. Generalized Field Equation

In the presence of electric and magnetic charge sources two Maxwell's equations of electromagnetic fields have already been derived as eqns (35) and (36). The other two are given by

$$\Delta x\boldsymbol{H} = \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{j} \quad \ldots \quad (45)$$

and

$$\Delta x\boldsymbol{E} = \frac{\partial \boldsymbol{H}}{\partial t} - \boldsymbol{k}, \quad \ldots \quad (46)$$

where $\boldsymbol{j}$ and $\boldsymbol{k}$ are electric and magnetic current densities. In these equations it is assumed that positive magnetic charge is a source while negative magnetic charge is a sink for magnetic field $\boldsymbol{H}$. This assumption determines the sign of k_0 and $\boldsymbol{k}$ in eqs (35) and (46) and permits one to construct a magnetic dipole as the combination of a positive and a negative monopole so that it will be indistinguishable from that produced by suitable electric current.

In terms of wave function ψ given by eq. (20) these field equations can be written as follows

$$\Delta\, \psi = J_0, \quad \ldots \quad (47)$$

$$\Delta x\psi = -i\frac{\partial \psi}{\partial t} - i\boldsymbol{J} \quad \ldots \quad (48)$$

where J_0 is the generalized charge source density given by eq. (24) and $\boldsymbol{J}$ is generalized current density defined as

$$\boldsymbol{J} = \boldsymbol{j} - i\boldsymbol{k}. \quad \ldots \quad (49)$$

It is now possible to introduce generalized four-current density

$$\{J_\mu\} = (\boldsymbol{J}_0, iJ_0) = \{j_\mu - ik_\mu\},$$

where

$$\{j_\mu\} = (\boldsymbol{j}, iJ_0) \quad \text{and} \quad \{k_\mu\} = (\boldsymbol{k}, ik_0)$$

Substituting the reduced expansions (1) and (2) in fields eqns (35), (36), (45) and (46) we get the following expansions in addition to those given by eqs. (17) :

$$\boldsymbol{j}(x) = \frac{em}{|q|4\pi^{3/2}} \left[\int \frac{d\boldsymbol{p}}{\omega} \left[mf(\boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{\omega+m} \right] \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} + \int \frac{d\boldsymbol{p}}{\omega} \left[mh^*(\boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{\omega+m} \right] \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} \right] \quad \ldots \quad (50)$$

$$\boldsymbol{k}(x) = \frac{gm}{|q|4\pi^{3/2}} \left[\int \frac{d\boldsymbol{p}}{\omega} \left[mf(\boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}.f(\boldsymbol{p})\}}{\omega+m} \right] \exp\{i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} + \int \frac{d\boldsymbol{p}}{\omega} \left[mh^*(\boldsymbol{p}) + \frac{\boldsymbol{p}\{\boldsymbol{p}.h^*(\boldsymbol{p})\}}{\omega+m} \right] \exp\{-i(\boldsymbol{p}.\boldsymbol{x}-\omega t)\} \right] \quad \ldots \quad (51)$$

It is obvious from these reduced expansions of generalized charge and current densities that

$$\frac{j_\mu}{k_\mu} = \frac{e}{g}, \quad \ldots \quad (52)$$

which gives the proportionality of electric charge and current source densities with magnetic charge and current source densities respectively in terms of the ratio of electric and magnetic fundamental charges.

In terms of four-potentials $\{A_\mu\}$ and $\{B_\mu\}$ we may define the electromagnetic tensor as follows :

$$F_{\mu\nu} = (A_{\mu\nu} - A_{\nu\mu}) - \frac{1}{2i}\epsilon_{\mu\nu\gamma\rho}(B_{\rho,\gamma} - B_{\gamma,\rho}), \quad \ldots \quad (53)$$

where $\epsilon_{\mu\nu\gamma\rho}$ is completely antisymmetric Ricci-tensor and

$$A_{\nu,\mu} = \frac{\partial A_\nu}{\partial x_\mu}$$

Then the field equations (35), (36), (45) and (46) may be written in the following compact form

$$F_{\mu\nu,\nu} = j_\mu \quad \ldots \quad (54a)$$

and

$$F^d_{\mu\nu,\nu} = k_\mu, \quad \ldots \quad (54b)$$

where $F_{\mu\nu}^d$ are the components of a dual tensor obtained from $F_{\mu\nu}$ by the dual transformations i.e.,

$$F_{\mu\nu}^d = (\beta_{\mu,\nu}-\beta_{\nu,\mu}) + \frac{1}{2i}\epsilon_{\mu\nu\lambda\rho}(A_{\rho,\lambda}-A_{\lambda,\rho}) \quad \ldots \quad (55)$$

Substituting eqs (53) and (55) in field eqs (54a) and (54b), we get

$$J_\mu = \frac{\partial^2 V_\nu}{\partial x_\nu \partial x_\mu} - \frac{\partial^2 V_\mu}{\partial x_\nu^2}, \quad \ldots \quad (56)$$

where $V_\mu = A_\mu - iB_\mu$, and the components of generalized current four-vector are defined by equations (24) and (49).

Reduced expansions of generalized four-current density J_μ may be readily derived by using the reduced expansions (17), (50) and (51) in eqs (24) and (49). It may then be shown

$$\frac{\partial J_0(x)}{\partial t} + \text{div}\, \boldsymbol{J}(x) = 0 \quad \ldots \quad (57)$$

which is the equation of continuity ensuring the conservation of generalized four-current.

Lagrangian Density

$$L = -1/4[\alpha\{(A_{\nu,\mu}-A_{\mu,\nu})^2+(B_{\nu,\mu}-B_{\mu,\nu})^2\}+2\beta\{(A_{\nu,\mu}-A_{\mu,\nu})(B_{\nu,\mu}-B_{\mu,\nu})\}]$$
$$+(\alpha A_\mu-\beta B_\mu)j_\mu-(\beta A_\mu+\alpha B_\mu)k_\mu \quad \ldots \quad (58)$$

where α and β are real arbitrary parameters, can be shown to yield field eq (46) under the independent variations of A_μ and B_μ. For the variation of potentials A_μ and B_μ respectively, we get

$$\alpha\left(\frac{\partial^2 A_\nu}{\partial x_\nu \partial x_\mu} - \frac{\partial^2 A_\mu}{\partial x_\nu^2}\right) - \frac{\beta}{2}\left(\frac{\partial^2 B_\nu}{\partial x_\nu \partial x_\mu} - \frac{\partial^2 B_\mu}{\partial x_\mu^2}\right) = \alpha j_\mu + \beta k_\mu \quad \ldots \quad (59)$$

and

$$\alpha\left(\frac{\partial^2 A_\nu}{\partial x_\nu \partial x_\mu} - \frac{\partial^2 A_\mu}{\partial x_\nu^2}\right) + \frac{\beta}{2}\left(\frac{\partial^2 B_\nu}{\partial x_\nu \partial x_\mu} - \frac{\partial^2 B}{\partial x_\nu^2}\right) = \beta j_\mu + \alpha k_\mu \quad \ldots \quad (60)$$

From these equations the field equations (56) may be readily derived provided that $\alpha^2+\beta^2 \neq 0$

Although the usual Lagrangian for Maxwell system is not invariant under the dual transformations of potential and current, the Lagrangian introduced here is dual invariant. This dual invariance will be discussed in detail in our later paper

References

Cabibbo N & Ferrari E 1962 *Nuovo Cimento* **23**, 1147.

Dirac P A M 1931 *Proc Roy Soc.* **A133** 60

Dirac P A M 1948 *Phys. Rev* **74**, 817

Deo B. B. & Singh L P 1973 *Ind J Phys* **47** 650

Epstein K J 1967a *Phys Rev Letter* **18**, 50

Foldy L L 1956 *Phys Rev* **102**, 568

Lomont J S. & Moses H E 1964 *Journ Math. Phys* **5**, 294

Lomont J. S & Moses H E 1967b *Journ. Math Phys* **8**, 837

Moses H E 1967c *Journ Math Phys.* **8**, 1134

Parkash Om & Rajput B. S. 1973 *Ind. J Phys* **47**, 611

Parkash Om & Rajput B S 1974a *Ind. J Phys* **48**, 152

Parkash Om & Rajput B S 1974b *Ind J Phys* **48**, 215

Parkash Om, Rajput B S & Singh B 1974d *Ind J Phys* **48**, 509.

Parkash Om & Rajput B S 1974c *Ind J Phys* **48** 359

Parkash Om & Rajput B S 1975 *Ind J Pure & Appl Phys* **13**, 471

Rajput B. S 1969a *Ind. J Phys* **43**, 135.

Rajput B. S 1969b *Ind J Phys* **43**, 439

Rajput B S 1969c *Ind J Phys* **43**. 602

Rajput B S 1969d *Ind J Pure & Appl Phys* **7**, 720

Rajput B. S 1969e *Ind J. Pure & Appl Phys* **7**, 823.

Rajput B S 1970a *Nuovo Cimento* **66A**, 517

Rajput B S 1970b *Ind J. Phys* **44**, 569.

Rajput B. S 1971a *Nuovo Cimento Lett.* **2**, 712.

Rajput B S 1971b *Nuovo Cimento* **2B**, 45

Schwinger J 1968 *Phys. Rev* **173**, 1536

Schwinger J. 1966a *Phys Rev.* **144**, 1087

Schwinger J 1966b *Phys Rev* **151**, 1018

Shirokov Iu M 1958 *Sovt Phys JETP.* **6** 919

Saha M. N 1937 *Ind J Phys* **10**. 145

Saha M N 1949 *Phys Rev* **75**, 1968

Zwanziger D 1968a *Phys Rev* **176**, 1480.

Zwanziger D 1968b *Phys Rev* **176**, 1489

Zwanziger D. 1971 *Phys Rev* **3D** 880