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Charged multiplicity distribution and its Bose type nature*

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Bose type distribution of the form $\psi(z) = Az^{m-1}/(e^{az}-1)$ is proposed for the charged particle multiplicity distribution in pp, π^-p and K^-p collisions in the momentum range 50–1500 GeV/c where z is the reduced multiplicity $n_c/<n_c>$. Empirical relations connecting the average charged multiplicity $<n_c>$ and the normalized second moment $<n_c^2>/<n_c>^2$ are used to fix one of the free parameters. Higher moments are calculated from the above distribution and have been compared with the experimental values. Moreover two reaction constants are found viz., $A_1 = (<n_c^2>-3)/(<n_c>^2-3)$ and $A_2 = [<n_c>-(<n_c>-1)^{-1}]/D$ in the momentum range 4–405 GeV/c where D is the dispersion.

1 Introduction

The study of multiplicity distribution at high energy gives the information about the production mechanism and the underlying theory of hadronic interactions. In recent years a large number of data on multiplicity distribution in hadronic interactions have been obtained by different authors (Smith 1971, Boggild *et al*
interactions have been obtained by different authors (Smith 1971 Boggild *et al*
1971 Ammosov *et al* 1972, 1973, Chapman *et al* 1972, Charlton *et al* 1972, Bromberg *et al* 1973, Dao *et al* 1974). Koba, Nielsen & Olesen (1972, to be referred to as KNO) proposed a scaling law which was found to be valid for pp, π^-p and K^-p at high energies. Several authors (Bozoki *et al* 1961, Czyzewski & Rybicki 1972, Slattery 1972, 1973, Ram & Rao & Sarma 1979, Baras & Koba 1973, Roy *et al* 1974, 1975, Bhattacharyya *et al* 1974, 1976) have proposed empirical fits to the charged multiplicity some of which incorporates KNO scaling.

In the present work we have shown that a Bose type distribution gives a good fit to the data at all energies. At high energies the distribution exhibits KNO scaling. The usual constraints of the distribution function $\psi(z)$ determines two among the three parameters. The other free parameter is determined from an empirical relation connecting the normalized second moment and the average charged multiplicity. The relation is obtained from the observed constants $A_1 = (<n_c^2>-3)/(<n_c>^2-3)$ and $A_2 = [<n_c>-(<n_c>-1)^{-1}]/D$

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2 Charged Particle Multiplicity Distribution

The probability for the production of n_c charged prongs at a center of mass energy $s^{\frac{1}{2}}$ is given by

$$P_n(s) = \sigma_n(s) / \sum_n \sigma_n(s) \qquad \ldots \quad (1)$$

where $n = \frac{1}{2}n_c$ and $\sigma_n(s)$ is the partial cross section for n_c charged prongs. The variable n runs over the set of positive integers. We replace the variable n by the reduced multiplicity $z = n/<n>$. Not much error is introduced in replacing the discrete spectrum of z by the continuous spectrum which is the positive half of the real line. This is shown to be valid by Roy *et al* (1975) for the gamma distribution and by Bozoki *et al* (1961) for a modified Gaussian distribution. It can be shown that the Bose type distribution is really a superposition of a large number of gamma distribution. Hence a Bose type continuous distribution can be proposed for the probability distribution (1). We write

$$\psi(z) = <n> P_n(s) = A z^{m-1}/(e^{\alpha z} - 1), \qquad . \quad (2)$$

where A, α and m are three parameter depending on s. The probability distribution $P_n(s)$ satisfies the following relation

$$\sum_{n=1}^{\frac{1}{2}N_{max}} P_n(s) = 1 \qquad . \quad (3)$$

$$\sum_{n=1}^{\frac{1}{2}N_{max}} n P_n(s) = \tfrac{1}{2} < n_c > \qquad . \quad (4)$$

where $<n_c>$ is the experimental average charged particle multiplicity and N_{max} is the maximum value of charged prongs observed. Eq (3) and (4) gives the following constraints on $\psi(z)$

$$\int_0^{z_{max}} \psi(z) dz = \int_0^{z_{max}} z \psi(z) dz = 1 \qquad . \quad (5)$$

where $z_{max} = N_{max}/<n_c>$ and the reduced k-th moment of the distribution is given by

$$c_k = <z^k> = \left[\frac{G(m, a)}{G(m+1, a)}\right]^k \frac{G(m+k, a)}{G(m, a)} \qquad (6)$$

where

$$G(s, a) = \sum_{r=1}^{\infty} \Gamma(s, ra) r^{s} \quad \text{and} \quad a = \alpha z_{max} = N_{max}/<n_c>$$

In the limit of large N_{max}, which is satisfied by the experimental data fairly well we have

$$G(s, a) = G(s, \alpha) = \Gamma(s).\zeta(s) \qquad \text{for } s > 1 \qquad \dots (7)$$

$$\alpha = \frac{G(m+1, a)}{G(m, a)} = m\zeta(m+1)/\zeta(m) \qquad \dots (8)$$

$$c_2 = (1+1/m) \quad \frac{\zeta(m)\zeta(m+2)}{[\zeta(m+1)]^2} \qquad .. (9)$$

and

$$\psi(z) = \frac{m^m}{\Gamma(m)\zeta(m)} \left[\frac{\zeta(m+1)}{\zeta(m)}\right] z^{m-1} \exp\left[\frac{m\zeta(m+1)}{\zeta(m)} - 1\right]^{-1} \qquad \dots (10)$$

where $\zeta(s)$ is the Riemann zeta function Usually the value of a is such that approximation (7) is valid but in the calculation of higher moments this approximation is inadequate The appropriate approximation is

$$G(s, a) \simeq \Gamma(s, a)\, \zeta(s) \qquad . (11)$$

where

$$\Gamma(s, a) = \int_0^a x^{s-1} e^{-x} dx$$

so that we get

$$\alpha = \frac{\Gamma(m+1, a)\zeta(m+1)}{\Gamma(m, a)\,\zeta(m)} \qquad . (12)$$

$$c_k = \left[\frac{\Gamma(m, a)\,\zeta(m)}{\Gamma(m+1, a)\,\zeta(m+1)}\right]^k \frac{\Gamma(m+k, a)\,\zeta(m+k)}{\Gamma(m, a)\,\zeta(m)} \qquad . (13)$$

At asymptotic energies the second reduced moment c_2 becomes equal to 1·25 and from eq (8) we get $m = 4{\cdot}38$ With such a value of m the value of α comes out to be approximately equal to m so that a becomes fairly close to mz_{max} Moreover c_k's in eq (13) are not very much sensitive to the parameter a, and we can replace a in eq. (13) by mz_{max} Thus c_k's become solely a function of the single energy dependent parameter m. The second reduced moment c_2 becomes

$$c_2 = \frac{\Gamma(m, mz_{max})\,\Gamma(m+2, mz_{max})\,\zeta(m)\,\zeta(m+2).}{[\Gamma(m+1, mz_{max})\,\zeta(m+1)]^2} \qquad .. (14)$$

3. New Reaction Constants and Calculation of the Parameter m

Several authors (Malhotra 1963, Czyzewski & Rybicki 1972, Wroblewski 1973, Roy *et al* 1975) have pointed out some regularities in the charged multiplicity data on hadron-hadron collisions Here we propose that the following two quantities maintain a constant value at all energies They are given by

$$A_1 = (\langle n_c^2\rangle - 3)/(\langle n_c\rangle^2 - 3) \qquad \dots (15)$$

$$A_2 = [\langle n_c\rangle - (\langle n_c\rangle - 1)^{-1}]/D. \qquad \dots (16)$$

When A_1 and A_2 are calculated from experimental charged multiplicity distribution in pp, π^-p and K^-p collisions and it is seen that the values of A_1 and A_2 are evenly distributed about their mean values which are 1·25 and 2·00 respectively (this is shown in Fig. 5) This encourages us to conjecture that A_1 and A_2 are energy independent for pp, π^-p and K^-p collisions and have values 1·25 and 2 respectively. With these values of A_1 and A_2 we get two relation connecting c_2 and $<n_c>$ namely

$$c_2 = 1{\cdot}25 - 0{\cdot}75/<n_c>^2 \qquad \text{if } A_1 = 1{\cdot}25 \tag{17}$$

$$= 1{\cdot}25 - 1/<n_c>(<n_c>-1) \qquad \text{if } A_2 = 2{\cdot}00 \tag{18}$$

Relation (17) and (18) differs only at very low energies where $<n_c> < 4$. To calculate m we can use any one of the relation (17) or (18) In our fitting we have used the relation (17) so that we get

$$\frac{\Gamma(m,\bar{a})\,\Gamma(m+2,\bar{a})\,\zeta(m)\,\zeta(m+2)}{[\Gamma(m+1,\bar{a})\,\zeta(m+1)]^2} = 1{\cdot}25 - 1/<n_c>^2 \tag{19}$$

where $\bar{a} = mz_{max} \simeq m_0 z_{max}$, m_0 being the solution of (7) and (17).

4. Results and Discussion

The values of m calculated from eq. (19) and is presented in Figure 1, where m becomes practically independent of $<n_c>$ for above $<n_c> = 5$. Since $<n_c>$

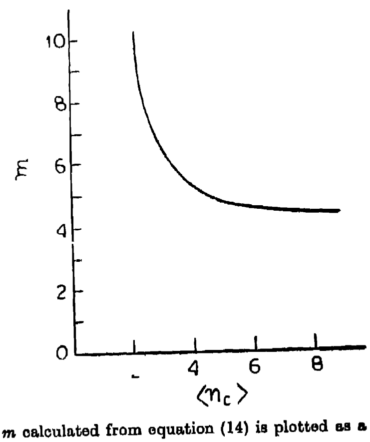


Fig. 1. The value of m calculated from equation (14) is plotted as a function of the average charged multiplicity $<n_c>$.

increases monotonically with energy, hence m becomes practically constant above 50 GeV/c (corresponding to $<n_c> \sim 5$). Above 50 GeV/c the distribution function $\psi(z)$ becomes dependent on s only through the reduced multiplicity. The form of the function becomes independent of energy This is precisely the KNO scaling without assuming it *a priori*

In Figure 2 $\psi(z)$ is plotted as a function of z together with the experimental points for pp, $\pi^- p$ and $K^- p$ collisions The value of m used is 4·6 which is the average value of m between $<n_c> = 5$ (50 GeV/c) and $<n_c> = 13$ (1500 GeV/c)

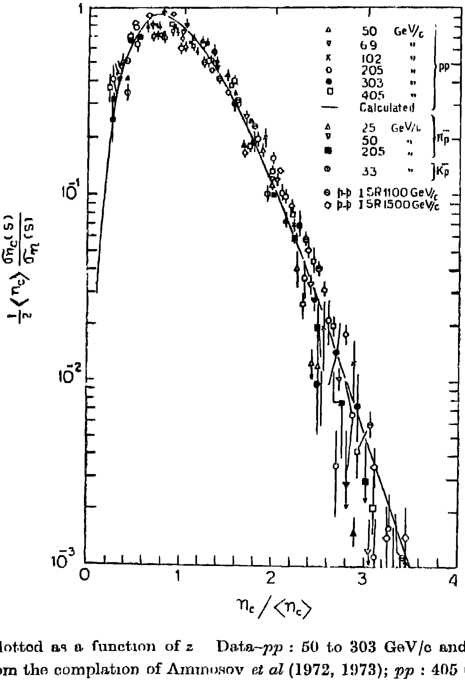


Fig. 2. $\psi(z)$ is plotted as a function of z Data–pp : 50 to 303 GeV/c and $K^- p$: 33 GeV/c taken from the complation of Ammosov *et al* (1972, 1973); pp : 405 GeV/c, Bromberg *et al* (1973), ISR data : Flugge *et al* (1973); $\pi^- p$ 205 GeV/c, Bogert *et al* (1973).

In Fig. 3 the calculated values of c_k(for $k = 2, 3, 4$ and 5) are given at different values of $<n_c>$ for the three reactions along with the experimental values Mueller's (1971) correlation parameter $f_2 = <n_c^2> - <n_c>^2 - <n_c>$ is calculated from the eq (14) and plotted against $<n_c>$ in Figure 4 along with the experimental values

The calculated values of topological cross sections are presented in Figure 5 at different beam momentum along with the experimental data of different authors

Figure 6 shows the plot of A_1 and A_2 against beam momentum for pp, π^-p and K^-p interactions

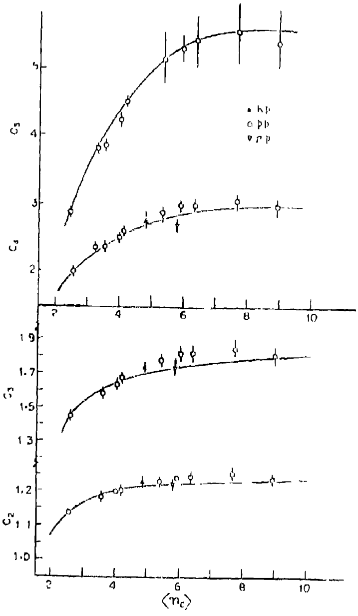


Fig 3. The calculated values of the moments c_k (for $k = 2$, 3, 4 and 5) have been plotted as a function of momentum. Data : Slattery (1973)

5. Conclusion

It is shown that the Bose type distribution gives a fairly good fit to the charged multiplicity distribution for pp, π^-p and K^-p interactions at all energies Recently Suzuki (1974) has shown that a gamma type distribution is obtained if one assumes the formation of clusters in the hadron-hadron collision and using photon statistics. What we have used is really a superposition of gamma distribution and one cannot rule out such a distribution from theoretical stand point.

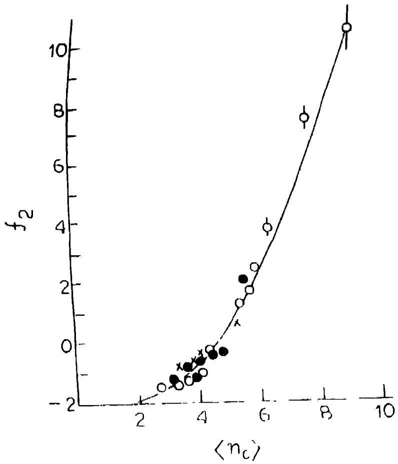
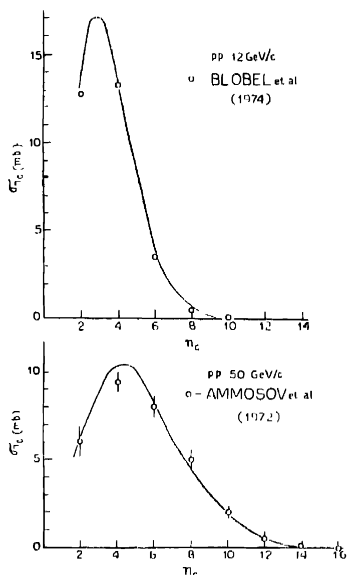


Fig. 4. The calculated values of f_2 for different $<n_c>$. Data : O-pp, ●-π^-p and ×-K^-p. Data taken from the compilation of Ammosov *et al* (1973).



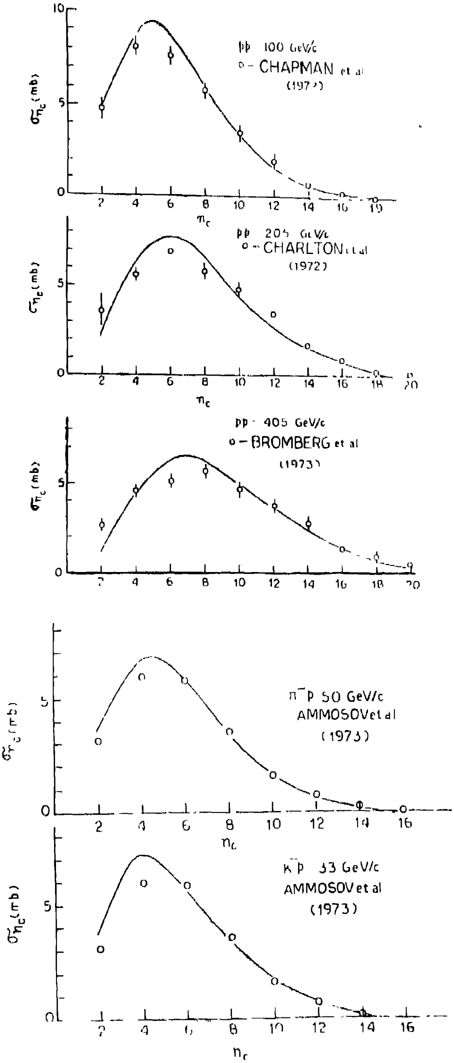


Fig. 5. The calculated values of topological cross sections for pp, π^-p and K^-p collisions at different beam momenta.

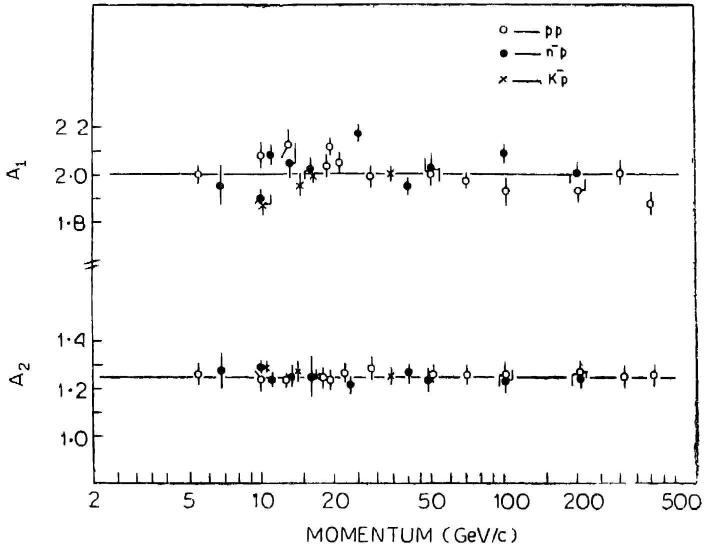


Fig. 6. The new reaction constants A_1 and A_2 have been plotted as a function of beam momentum for pp, π^-p and K^-p reactions. Data : 4–303 GeV/c pp—Ammosov *et al* (1973); 405 GeV/c—Bromberg *et al* (1973).

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