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Formation of protonium in the collision of antiproton with hydrogen atom

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Abstract : We carry out the quantum mechanical calculations of the total cross sections for the formation of protonium, for the first time within the intermediate energy range (50–250 keV) using the Born approximation. Two notable characteristics are considered. Firstly, unlike the case of electron capture by protons, here the differential cross sections increase slowly with the scattering angle. Secondly, for a heavy particle collision, usually the major contributions comes from below the angular region of the order of one miliradian (m rad.). But in the present case, contributions come from the entire angular region (0° – 180°) which is usually true for the light particle scattering.

Keywords : Protonium, Born approximation, probability amplitude, capture cross sections

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1. Introduction

We are investigating the formation of protonium in antiproton-hydrogen atom collision. We are interested in problems connected with rearrangement collisions which give rise to the formations of 'Protonium' as a result of disintregation of hydrogen atom with the collisions of anti-protons and the electron is liberated from the system

$$\bar{p} + H = p\bar{p} + e^{-}. \tag{1}$$

We have a dynamical system composed of the antiproton and proton giving rise to symmetric quantum states. The theoretical work on the capture of negative particles of heavy mass (negative mesotron) was discussed for the first time by Fermi and Teller [1]. They found out

⁺On leave from Physics Department, B. C. College, Assansol, Dist. Burdwan, West Bengal, India the time for overall process of capture and energy loss of the antiparticles as projectiles. The moderation of negative μ^- and π^- mesons as well as negatively charged particle of mass 1000 m (τ^-) and 1837 m ($\bar{p}$) in hydrogen had been done by Wightman [2] before the experimental discovery by Segré and Wiegand [3], in high energy proton-proton collision using the Bevatron at California. Here m is the electron mass. This experiment took a quarter of a century after Dirac's prediction of an anti-particle for proton. Since an anti-proton can be created only in a pair with a proton (in proton-proton collisions), we need at least the energy equivalent to the mass of two protons which by Einstein's relation $E = mc^2$, amounts to 2×938 or 1876 MeV. After the proton-proton collisions we shall have four particles, the two original protons plus a newly created proton-antiproton pair. In the experiment of Segre and Weigand [3], each of the four particles emerged from the collisions, with a kinetic energy amounting to about one thousand MeV. Thus the generation of an antiproton by this method took 2000 MeV (creation of proton-anti-proton pair) plus 4000 MeV (the kinetic energy of the four emerging particles). In recent years, some works both theoretical and experimental, have been started on the formation of a protonium, —a bound state of proton and an antiproton —by the collision of an antiproton with a hydrogen atom. The CERN with its low energy antiproton ring (LEAR) facilities is conducting experiments, specially in the field of excitation and ionisation in antiproton atom collisions. Bracci *et al* [4] discussed the formation of protonium in low energy collisions of antiprotons with protons, neutral and negatively ionised hydrogen atoms. They used a purely classical method. Cohen [5] used the classical trajectory Monte-Carlo (CTMC) approach to calculate the protonium formation cross section in antiproton-hydrogen atom collisions at very low energy. We use atomic units throughout our calculation.

2. Theory

The transition matrix for the formation of 'protonium' from anti-proton and hydrogen atom collision may be written as

$$T_{if} = \langle \psi_f | V | \psi_i \rangle, \tag{2}$$

where, (cf. Figure 1)

$$\psi_i = e^{i\boldsymbol{k}_i \cdot \boldsymbol{\rho}} \, \Phi_i(\boldsymbol{r}), \tag{3a}$$

$$\psi_f = e^{i\boldsymbol{k}_f \cdot \boldsymbol{r}'} \, \Phi_f(\boldsymbol{R}), \tag{3b}$$

$$V = \frac{1}{x} - \frac{1}{r} = \frac{1}{|\boldsymbol{r} - \boldsymbol{R}|} - \frac{1}{r}, \tag{3c}$$

with

$$\Phi_i(\boldsymbol{r}) = N_i \, e^{-\lambda_i r}, \qquad \Phi_f(\boldsymbol{R}) = N_f \, e^{-\lambda_f R}$$

$$\lambda_i = 1, \qquad \lambda_f = M/2 = 918, \tag{3d}$$

$$N_i = 1/\sqrt{\pi}, \qquad N_f = \frac{\cdot}{\sqrt{\pi}}(918)^{3/2}.$$

Now the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{v'}{v}\right)|f(\theta,\Phi)|^2, \tag{4}$$

where v' = final velocity of protonium,

v = initial velocity of antiproton.

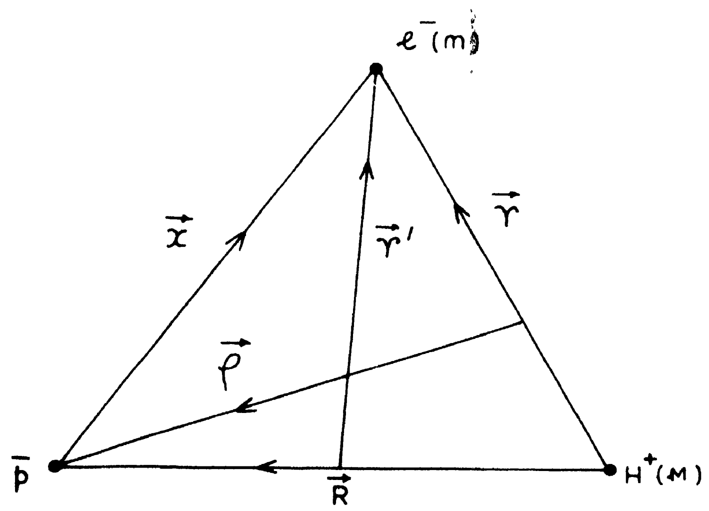

Figure 1. Capture phenomenon in antiproton-hydrogen atom collisions.

The scattering amplitude is given by

$$f(\theta,\Phi) = \frac{\mu_f N_i N_f^*}{2\pi}\left[I_1 - I_2\right], \tag{5}$$

where $\mu_f = M/2$, M is the proton mass,

$$I_1 = \int e^{-\lambda_i r}\, e^{-\lambda_f R}\, e^{i\boldsymbol{B}\cdot\boldsymbol{R}}\, e^{-i\boldsymbol{C}\cdot\boldsymbol{r}} \left(\frac{1}{\boldsymbol{r}-\boldsymbol{R}}\right) d\boldsymbol{r}\, d\boldsymbol{R}, \tag{6}$$

$$I_2 = \int e^{-\lambda_i r}\, e^{-\lambda_f R}\, e^{i\boldsymbol{B}\cdot\boldsymbol{R}}\, e^{-i\boldsymbol{C}\cdot\boldsymbol{r}}\, \frac{1}{r}\, d\boldsymbol{r}\, d\boldsymbol{R}, \tag{7}$$

with

$$\boldsymbol{B} = \boldsymbol{k}_i + \frac{1}{2}\boldsymbol{k}_f, \qquad \boldsymbol{C} = \frac{m}{M+m}\boldsymbol{k}_i + \boldsymbol{k}_f. \tag{8}$$

The space integration in (6) and (7) are then evaluated analytically and we arrive at

$$I_1 = 8\pi^2 \frac{\partial}{\partial\lambda_i}\cdot\frac{\partial}{\partial\lambda_f} L\left(\lambda_i, \lambda_f, B, C\right) \tag{9}$$

and

$$I_2 = \frac{4\pi}{\left(\lambda_i^2 + C^2\right)}\cdot\frac{8\pi\lambda_f}{\left(\lambda_f^2 + B^2\right)^2}, \tag{10}$$

where $L\left(\lambda_i, \lambda_f, B, C\right)$ represents the Lewis integral [6] and is given by

$$L = \frac{1}{\sqrt{\beta^2 - \alpha\gamma}} \log \frac{\beta + \sqrt{\beta^2 - \alpha\gamma}}{\beta - \sqrt{\beta^2 - \alpha\gamma}}, \tag{11}$$

$$\beta = \lambda_i\left(B^2 + \lambda_f^2\right) + \lambda_f\left(C^2 + \lambda_i^2\right), \tag{12}$$

$$\alpha\gamma = \left[(C-B)^2 + \left(\lambda_i + \lambda_f\right)^2\right]\left[\left(B^2 + \lambda_f^2\right)\left(C^2 + \lambda_i^2\right)\right]. \tag{13}$$

3. Results and discussions

Protonium is an electromagnetically bound system of p and $\bar{p}$. It is useful in the study of nuclear forces as discussed by Sapiro [7] and Erickson [8]. Protonium is also important in the study of the colour force between the quarks as shown by Veneziano [9]. Thus, the dominance of interest of 'protonium' formation ranges from atomic physics to nuclear physics and particle physics. Bracci *et al* [4] obtained expression for the cross sections of protonium formation. The energy *versus* cross section curve is nearly represented by a straight trajectory. According to them, the highest population is achieved in low n-states so that a large fraction of atoms immediately annihilate. The traditional way of forming protonium is by stopping antiprotons in high density hydrogen. According to Cohen [5] the atoms are formed very efficiently in highly excited states, but because of large l-mixing rates in the dense fluid rather quickly reach an s, p or d state and annihilate. The protonium formation cross sections obtained by Cohen is of the order of 10^{-16} cm^2 for the energy range 0.05 (a.u) –0.7 (a.u).

In Table 1; we present our results for the total cross sections of 'protonium' formation in antiproton-hydrogen atom collision for the energy range 50 – 250 keV. Throughout this energy, range our cross sections are approximately of the order of 10^{-24} cm^2. This is in agreement with the investigation by Fermi and Teller [1] in which the mesotron was found to be captured in the Bohr orbit with a radius of the order of 10^{-12} cm. We also plot our total cross sections in Figure 2. Here we note that for very low energy region, Cohen [5] obtained cross sections of atomic dimension whereas our results are of the nuclear dimension.

However, variations of the total cross sections as a function of energy in these two investigations are similar in nature.

Table 1. Anti-protons energy *vs* total cross sections for the formation of protonium. A(–ob) represents $A \times 10^{-b}$.

Nos.	Energy (KeV)	Total cross section $\sigma_{p\bar{p}}\left(a_0^2\right)$
1	50	2.877428 (–05)
2	75	8.5821684 (–06)
3	100	3.4032712 (–06)
4	125	1.6079654 (–06)
5	150	8.5575670 (–07)
6	175	4.9646508 (–07)
7	200	3.0750067 (–07)
8.	225	2.0049731 (–07)
9.	250	1.3625326 (–07)

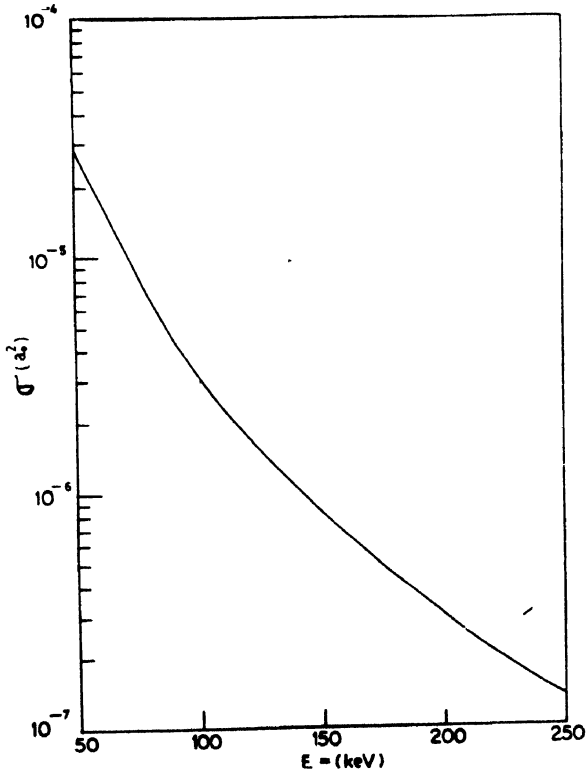

Figure 2. Total protonium formation cross sections in units of a_0^2 as a function of incident antiproton energy in keV. A(–B) represents. $A \times 10^{-B}$.

We have also studied the differential cross sections for the protonium formation in antiproton-hydrogen atom collision at energies 50 keV, 100 keV and 200 keV. These results are presented in Figures 3–5 respectively. Two interesting characteristics are noted in the

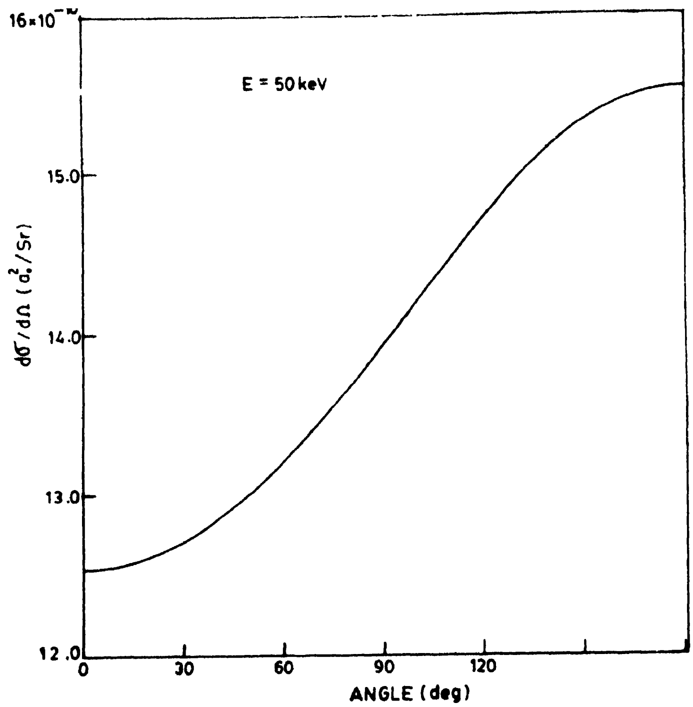

Figure 3. Differential cross section in units of a_0^2/Sr at an incident antiproton energy of 50 keV.

differential cross section at all the three energies considered. Firstly, the differential cross section at each energy is found to be slowly increasing with the increase of the angle which the protonium atom makes with incident direction of the antiproton. This may be attributed to the fact that the electrons liberated in this process will occupy more space in the forward direction and hence more protonium atoms are expected to be available in the backward directions. Secondly, in heavy particle collision, usually the major contribution comes from below the angular region of one mili-radian. But in the present case, contributions from the entire angular region (0°–180°) are important. This behaviour is usually observable in light particle scattering. All the three curves (Figures 3–5) representing angular distributions of the protonium formation clearly show two valleys in each of them —one between 0°–60° and the other between 120°–180°. In between the two valleys, nearly a straight trajectory is obtained.

In order to check the variation of the differential cross sections at each energy, we calculate

$$\gamma' = (d\sigma/d\Omega)_{\theta s}/(d\sigma/d\Omega)_{\theta_l},$$

where

$$\theta_s = 4.24° \text{ and } \theta_l = 175.76°.$$

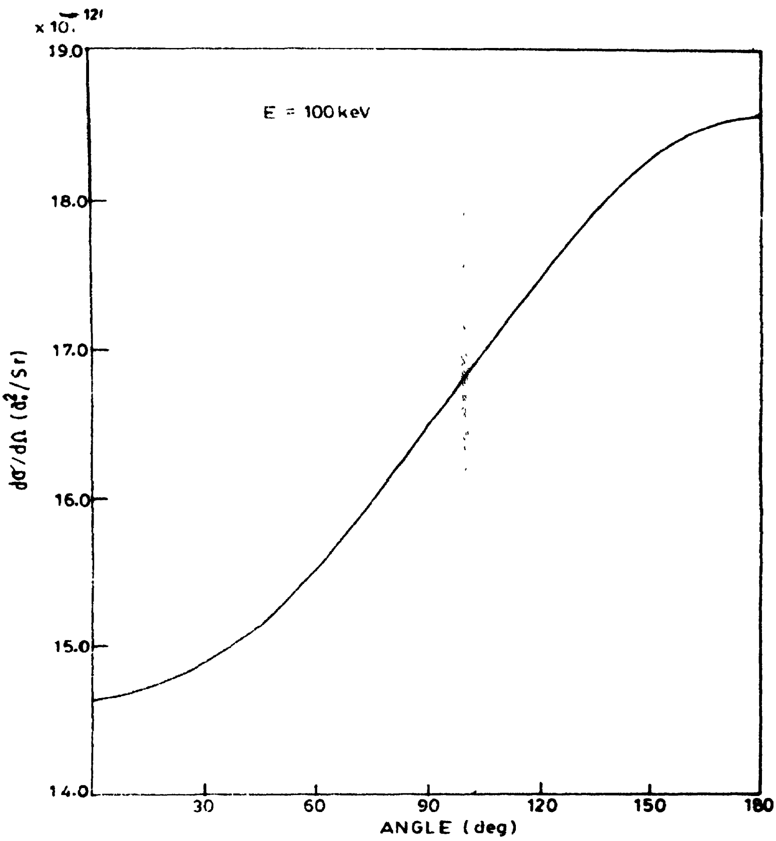

Figure 4. Same as in Figure 3 at 100 keV.

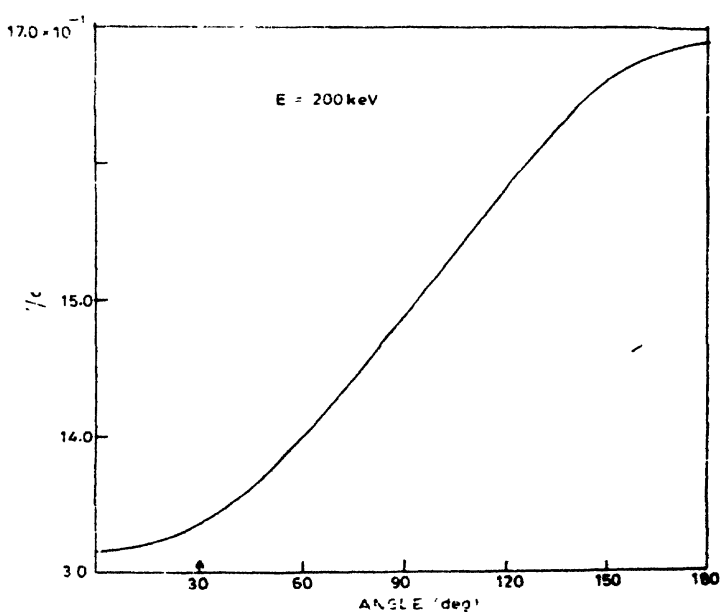

Figure 5. Same as in Figure 3 at 200 keV.

These results are tabulated in Table 2. γ' is found to be decreasing slowly with increasing energy. Here θ_s and θ_l corresponds to the first and last points of integration for a set of 32 Gauss-Legendre quadrature points.

Table 2. Antiproton energy and γ', the ratio of initial and final angular distributions.

Nos.	Energy (keV)	$\gamma' = \left(\frac{d\sigma}{d\Omega}\right)_{\theta_s} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\theta_l}$
1	50	0.8064
2	75	0.7961
3	100	0.7900
4	125	0.7863
5	150	0.7837
6	175	0.7815
7	200	0.7802
8	225	0.7789
9	250	0.7779

4. Conclusion

We have applied the first Born approximation to calculate the protonium formation cross section in antiproton-hydrogen atom collision for the intermediate energy range. Two interesting characteristics are found in the differential cross sections : (i) the differential cross section increases with the scattering angle, (ii) the entire angular range (0°–180°) is important for the calculation of cross sections. The total cross sections obtained are of the order of nuclear dimension. Unfortunately, no theoretical or experimental results are available for a direct comparison. However, experiments [10,11] are being done with antiprotons as projectiles but for ionization problems.

The fundamental constituents of matter come in matched pairs. The symmetrical pairing of proton and antiproton is required in order to unite the two great theories of twentieth century physics (a) Theory of Relativity, and (b) Quantum Mechanics –the formations of protonium and annihilation of it will usher in new area of research.

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