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Current driven electrostatic ion cyclotron waves in presence of parallel electric field in the magnetosphere

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Abstract : Electrostatic ion cyclotron (EIC) waves driven by the electron currents and ion beams at frequencies near the harmonics of the ion cyclotron have been a common feature of the auroral magnetospheric plasma as observed by satellite. In the generation mechanism of EIC waves, the resonance excitation of the beam-ion-cyclotron modes by the target ions has been considered. The counterstreaming of electrons and ion beams along auroral magnetic field line in the region above parallel electric field has been dealt earlier. In the present paper the growth rate of EIC waves has been derived from linear dispersion relation for low frequency electrostatic waves in an anisotropic warm magnetised plasma consisting of beam ions, target ions and drifting bulk electrons in the region of weak parallel electric field. Findings of the theoretical investigations of growth rate are used to explain the excitation mechanism of EIC waves in the ELF range as observed by satellites in the auroral region of the magnetosphere.

Keywords : EIC wave, parallel electric field, magnetosphere

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1. Introduction

The current driven electrostatic ion cyclotron (EIC) instability has been very important to both space and laboratory plasmas. In most of the studies, the field aligned currents or ion beams have served as the driving mechanism for such instabilities [1]. An evident reason is that the EIC instabilities are among the lowest threshold instabilities which can be excited in a magnetized plasma and therefore arise naturally as a result of internal perturbation in the plasma [2].

In-situ and the ground based observations of EIC waves in naturally occurring plasmas have also been reported by Kintner and Gurnett [3].and Bering [4]. EIC waves observation made on a sounding rocket in the auroral zone show the current-driven EIC waves by Earle and Kelley [5]. The EIC waves may be unstable to current driven instabilities in the auroral magnetosphere at altitude above 1000 km [6]. It is widely recognised that the electric field plays an important role in the dynamics of the plasma in the ionosphere and magnetosphere. The electric field being the driving force for the current and the plasma instabilities providing the resistivity to limit the current and thereby maintain the field.

There has been much discussion concerning possible free energy sources needed to drive the EIC waves [7]. Kintner *et al* [8] considered the possibility that either electron drifts or upstreaming ions (ion beams) were the source of free energy for EIC waves as observed from S3–3 Satellite. Later, Cattell [9] explained that the EIC waves could be driven by a combination of ion beams and field aligned current. In subsequent work, Kaufmann and Kintner [10] considered that the observed EIC waves by S3–3 Satellite are being driven by ion beams rather by cold drifting ions. It has also been observed that EIC waves driven by electron currents and ion beams can be generated at frequencies near the harmonics of the ion cyclotron frequency in an ion beam-plasma system. The generation mechanism is the resonant excitation of the ion beam-cyclotron modes by the relative drift of the target ions [11]. It is assumed that the only resonant particles are responsible for the energy exchange between wave and particles while the non resonant particles supports the oscillatory motions of the waves.

The experimental measurements of parallel electric field using rocket probe was carried out by Mozer and Bruston [12] and they have reported that the field of 20 mV/m is present in the auroral zone. The measurements of parallel electric field also carried out during active aurora by Kelley *et al* [13]. He has shown that a small electric field of the order of 30 mV/m parallel to the magnetic field lines exist whose magnitude may change from time to time. Recently, Block and Faslthammar [14] have reported that the weaker parallel electric fields of the order of a few mV/m to 20 mV/m are more common. The electric field accelerates charged particles which transfer energy to the interacting wave leading to wave amplification.

The efforts to make *in-situ* measurements of the electric field parallel to the geomagnetic field in the magnetosphere [12–15] inspired many researchers to make detailed investigations of wave propagational effects and wave-particle interactions. Role of electric field on the electrostatic wave instability in earth's magnetoplasma has been recently discussed by Juhl and Treumann [16] and Bajaj and Tiwari [17]. In the present paper, we have investigated the effect of parallel electric field through modification in parallel ion beam temperature on the current driven EIC instability. This growth mechanism could be applicable to explain the presence of EIC wave ranging from 10 to 50 Hz (ELF) in the auroral region where counterstreaming electrons and ion beams are present.

2. Theoretical considerations

The magnetospheric plasma has been considered to contain a uniform static magnetic field B_0 and an electric field E_0 parallel to it. In the present paper the following assumptions have been made to solve the dispersion relation.

1. Small amplitude conditions are satisfied so the terms involving the product of time dependent quantities are negligible.
2. One dimensional analysis is applicable in which all quantities vary only with one spatial variable and

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

3. The effective electron collision frequency is assumed to be velocity independent and very weak.
4. All time varying quantities are assumed to be small and to have harmonic dependence of the form exp $i(\omega t - kt)$ where ω and k are the wave angular frequency and propagation constant while z and t denote the spatial and temporal variables.

In the present analysis, we have considered small electric field (few mV/m) and weak collisions. Under these conditions, electrons would not lead to the phenomenon of runaway [16–18] and they are responsible for the wave growth *via* energy exchange under wave - particle interaction processes.

The dispersion relation for low frequency electrostatic waves in a magnetised plasma consisting of beam ions, target ions and drifting bulk electrons is given by Singh *et al* [11],

$$D(\omega,k) = 1 + \frac{1}{k^2\lambda_e^2}\left\{1 + \xi_e Z(\xi_e)\right\} + \frac{1}{k^2\lambda_i^2}\left\{1 + \sum_n \Gamma_n \frac{\omega}{k_\parallel V_i} Z(\xi_i)\right\}$$

$$\frac{}{k^2\lambda_{h_\parallel}^2}\left\{1 + \sum_n \Gamma_n(\xi_{ib})\, Z(\xi_{ib})\right\} = 0, \qquad (1)$$

where $k^2 = k_\perp^2 + k_\parallel^2$, $k_\perp$ and $k_\parallel$ are perpendicular and parallel wave number with respect to magnetic field B_0 and,

$$\xi_e = \frac{\omega + k_\parallel V_{de} + i\nu}{k_\parallel V_e} \qquad \xi_i = \frac{\omega - n\Omega_i}{k_\parallel V_i}, \qquad \xi_{ih} = \frac{\omega - k_\parallel U_b - nA_b\Omega_i}{k_\parallel V_{h_\parallel}}$$

$$\lambda_e = \qquad \lambda_i = \frac{V_i}{\sqrt{2}\,\omega_i}, \qquad \lambda_{h_\parallel} = \frac{V_{h_\parallel}}{\sqrt{2}\,\omega_h},$$

U_b Ion beam velocity,

V_{de} Electron drift velocity in the direction opposite to U_b,

$V_{h_\perp}$ Perpendicular ion beam thermal velocity with respect to B_0,

$V_{h_\parallel}$ Parallel ion beam thermal velocity with respect to B_0,

T_b Perpendicular ion beam temperature,

$T_{b_\parallel}$ Parallel ion beam temperature,

k_B Boltzmann constant,

ν Electron-neutral collision frequency,

V_e Thermal velocity of electrons,

V_i Thermal velocity of target ions,

$\Omega_i = \frac{eB_0}{M}$ Ion cyclotron frequency,

$\omega_{p\alpha} = \frac{ne^-}{\epsilon_0 m_\alpha}^{1/2}$ Plasma frequencies of various plasma component ($\alpha = e, i, ib$),

$A_b = 1 - T_{b_\parallel}/T_{b_\perp}$,

M Mass of ion, and

m_e Mass of electron.

The function Γ_n is defined by

$$\Gamma_n(\mu_i) = I_n(\mu_i)\exp(-\mu_i), \tag{2}$$

where

$$\mu_i = \frac{k_\perp^2 V_\perp^2}{2\Omega_i^2} = 1/2\, k_\perp^2 \rho_i^2,$$

ρ_i ion gyro radius.

Γ_n is modified Bessel function of n-th order.

The effect of parallel electrostatic field E_0 with respect to magnetic field B_0 has been incorporated through the modification of particles thermal velocity [16–18]. Under this effect temperature $T_\parallel$ modifies to complex temperature $T_{\parallel C}$ in the direction of magnetic field and is given by

$$T_{\parallel C} = T_\parallel \quad 1 + \frac{iqE_0}{M k \alpha_\parallel^2} \qquad \alpha_\parallel \qquad \frac{2k_B T_\parallel}{M} \tag{3}$$

where q is the charge of particles and k_B is Boltzmann constant, while i is $\sqrt{-1}$. Replacement of temperature by complex temperature has been discussed by Juhl and Treumann [16]. This approach is valid for smaller electric field where we assume $\left(q\, E_0/M\, k\alpha_\parallel^2\right) << 1$ for linear theory. Introducing the effect of parallel electric field, via complex temperature the dispersion relation (1) modifies to :

$$D(\omega, k) = 1 + \frac{1}{k^2\lambda_e^2}\left\{1 + \xi_e Z(\xi_e)\right\} + \frac{1}{k^2\lambda_i^2}$$

$$\times\left\{1+\sum_n \frac{\Gamma_n\omega}{k_\parallel V_{b_\parallel}} Z(\xi_i)\right\} + \frac{1}{k^2\lambda^2_{b_\parallel}}\left\{1+\sum_n \Gamma_n\right.$$

$$\left.\times\left(\frac{\omega-k_\parallel U_b-nA_b\Omega_i}{k_\parallel V_{b_\parallel}} - i\,\frac{n\Omega_i}{k_\parallel V_{b_\parallel}}\,\frac{T_{b_\parallel}}{T_{b_\perp}}\,\frac{qE_0}{M\,k\,\alpha^2_{\parallel_b}}\right)Z(\xi_{ib})\right\} = 0. \qquad (4)$$

In the absence of electric field ($E_0 = 0$), we can recover the standard dispersion relation for EIC waves in the auroral magnetospheric plasma [11].

Using plasma dispersion function [19]

$$Z(\xi_i) = i\sqrt{\pi},\ \exp\ +\left(-\xi_i^2\right) - \frac{1}{\xi_i} \qquad (5)$$

and separating the real and imaginary part of the dispersion relation (4), we get

$$\operatorname{Re} D(\omega,k) = 1 + \frac{1}{k^2\lambda_i^2} - \sum_n \frac{\Gamma_n}{k^2\lambda_i^2}\left[\frac{\alpha_{\parallel i}}{\xi_i}\right]$$

$$+\frac{1}{k^2\,\lambda^2_{b_\parallel}}\left[1+\sum_n \Gamma_n\left[\frac{n\Omega_i}{k_\parallel\, V_{b_\parallel}}\ \frac{q\,E_0\,T_{b\parallel}}{M\,k\,\alpha^2_{\parallel b}\,T_{b_\perp}}\right.\right.$$

$$\left.\left.\times\sqrt{\pi}\ \exp\left(-\xi^2_{ib}\right) - 1 - \frac{n\Omega_i\left(1-A_b\right)}{k_\parallel V_{b_\parallel}\xi_{ib}}\right]\right], \qquad (6)$$

$$\operatorname{Im} D(\omega,k) = \frac{\xi_e}{k^2\lambda_e^2}\,\frac{\sqrt{\pi}}{\sum_n\Gamma_n}\ \exp\ +\left(-\xi_e^2+\frac{V_e^2}{k_\parallel^2V_e^2}\right)+\sum_n\frac{\Gamma_n}{k^2\lambda_i^2}\alpha_{\parallel i}$$

$$\times\sqrt{\pi}\ \exp\left(-\xi_i^2\right)+\sqrt{\pi}\left(\frac{\sum_n\Gamma_n}{k_2\lambda^2_{b_\parallel}}\right)\exp\left(-\xi^2_{ib}\right)\left\{\xi_{ib}+\frac{n\Omega_i\left(1-A_b\right)}{k_\parallel V_{b_\parallel}}\right.$$

$$\left.+\frac{1}{\xi_{ib}}\,\frac{n\Omega_i}{k_\parallel\,V_{b_\parallel}}\,\frac{T_{b_\parallel}}{T_{b_\perp}}\,\frac{qE_0}{M\,k\,\alpha^2_{\parallel_b}}\right\}, \qquad (7)$$

where

$$\alpha_{\parallel_b} = V_{b_\parallel} = \sqrt{\frac{2k_BT_{b_\parallel}}{M}},$$

$$\alpha_{\parallel_i} = V_i = \sqrt{\frac{2k_BT_i}{M}}$$

Using the basic definition of growth rate γ as

$$\gamma = \frac{-\operatorname{Im} D(\omega,k)}{\partial/\partial\omega\ \operatorname{Re} D(\omega,k)} \qquad (8)$$

and substituting eqs. (6) and (7) into eq. (8) the expression of the growth rate for fundamental EIC mode ($n = 1$) in presence of electric field is given by

$$-\left(\frac{\lambda_{b_\parallel}}{\lambda_e}\right)^2 \frac{\sqrt{\pi}}{\sum_n \Gamma_n}\left(\frac{x+X}{Y}\right) \exp\left[-\left(\frac{x+X}{Y}\right)^2+\frac{1}{\xi_{ve}^2}\right]-\left(\frac{\lambda_{b_\parallel}}{\lambda_i}\right)^2 \sqrt{\pi}\ \exp\left[-\left(\frac{x-1}{Z}\right)^2\right]\times\left(\frac{x}{z}\right)$$

$$\frac{\gamma}{\Omega_i} = \frac{-\sqrt{\pi}\exp\ -\left(\frac{x-y-1}{z}\right)^2\left(\frac{x-y-1}{z}+\frac{1-A_b}{z}\right)-\frac{1}{(x-y-1)}\frac{T_{b_\parallel}}{T_{b_\perp}}\tilde{k}}{\left(\frac{\lambda_{b_\parallel}}{\lambda_i}\right)^2\frac{1}{(x-1)^2}-\frac{1-A_b}{(x-y-1)^2}-2\sqrt{\pi}\,\tilde{k}\,\frac{T_{b_\parallel}}{T_{b_\perp}}\left(\frac{x-y-1}{z}\right)\frac{1}{z^2}}$$

$$\times\exp\left[-\left(\frac{x-y-1}{z}\right)^2\right]+\frac{2\sqrt{\pi}}{k^2\lambda_e^2}\frac{\xi_e}{\xi_{ve}}-\exp\left\{-\left(\xi_e^2-\frac{1}{\xi_{ve}^2}\right)\right\} \tag{9}$$

where

$X = \frac{k_\parallel V_{de}}{\Omega_e}$, $x = \frac{\omega}{\Omega_i}$;

$Y = \frac{k_\parallel V_e}{\Omega_e}$; $y = \frac{k_\parallel U_b}{\Omega_i}$; $\tilde{k} = \frac{qE_0}{M k \alpha_{\parallel_b}^2}$;

$Z = \frac{k_\parallel V_i}{\Omega_i} = K_\parallel$; $z = \frac{k_\parallel V_{b_\parallel}}{\Omega_i}$; $\xi_{ve} = \frac{k_\parallel V_e}{v_e}$.

Using magnetospheric plasma parameters, the growth/damping are computed from eq. (9) and effect of electric field on the waves generation is discussed in detail.

3. Results and discussion

In the present work, the theoretical assumptions for deriving the growth rate of EIC waves are almost same as occurring in the case of magnetospheric region of interest. In this region, the wave growth is not significantly affected by weak collisions [20,21], where as the present study limits to choose weak electric field strength due to linear approach. The electric field generates the drift between oppositely charged particles, which affects the drifted bi-Maxwellian distribution, but at smaller electric fields the effect is not appreciable. We have chosen magnetospheric plasma parameter to compute the growth rate from eq. (9). The effect of electric field is being discussed with reference to temperature anisotropy of the beam, and electron drift. The calculations have been performed for long wave length (*i.e.* low frequency), for fundamental mode ($n = 1$) of EIC waves.

There are various possibilities of wave-particle interactions which could excite waves in magnetised (auroral) plasma consisting of drifting bulk electrons, target ions, and beam ions. The generation of EIC waves due to electron-ion beam interaction is being estimated in the presence of target ion and weak electric field. This problem is important for understanding the auroral physics, where the counterstreaming of electrons (upwards currents) and ion beams along auroral magnetic field lines are common in parallel electric fields region. Therefore the EIC wave growth can be directly affected by presence of electric field in the earth's magnetoplasma region. Satellite observations have been clearly established that such a counterstreaming is a common feature of auroral plasma [9]. The range of

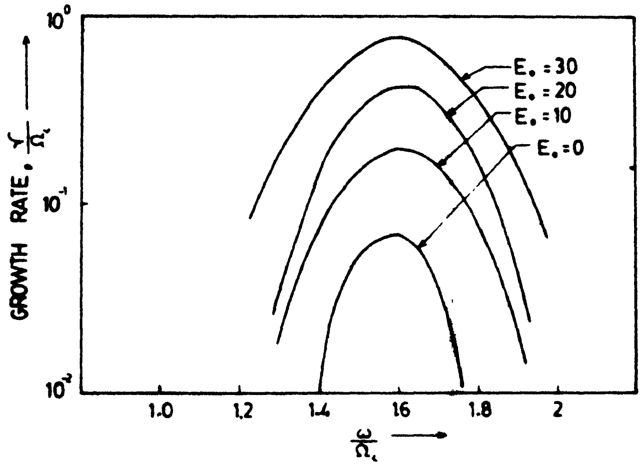

Figure 1. Variation of normalized growth-rate of EIC wave with ω/Ω_i for different values of E_0 and $T_{b_\parallel}/T_{b_\perp} = 0.1$.

excitation of ion cyclotron frequency is around 10 to 50 Hz, which lies in the extremely low frequency (ELF) range. This is also observed by S3–3 satellite in the auroral region [22].

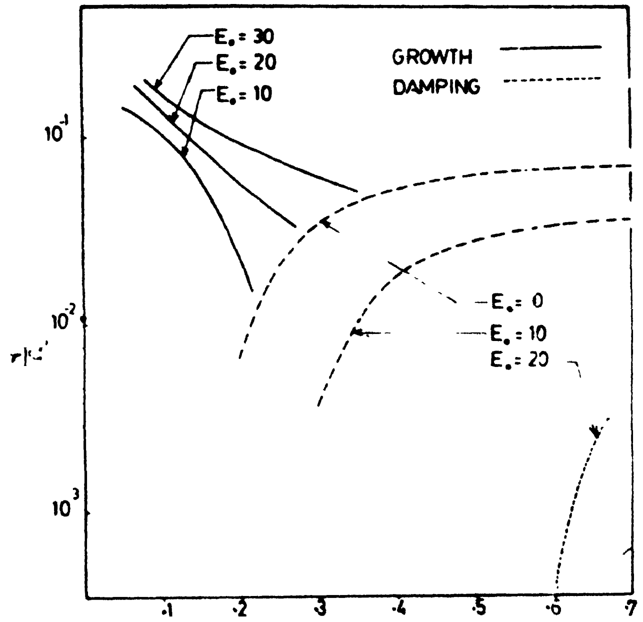

Figure 2. Variation of normalised growth-rate of EIC wave with $T_{b_\parallel}/T_{b_\perp}$ for different values of E_0.

In Figure 1 the normalised growth rate has been plotted as a function of ω/Ω_i for the magnetospheric plasma parameters used by Singh *et al* [11], *i.e.*, $T_{b_\parallel}/T_b = 0.1$, $n_b \sim n_t =$

12n_e, V_{de} is varied from 0 to 43 V_t, $X = 40$, $Y = 20$, $Z = 10$, $y = 2$, $z = 1$. As the magnitude of electric field increases the resonant growth rate also increases and extends to broad band generation of EIC waves while lower value of electric field excites the narrow band emissions. Thus the electric field E_0 enhances the range of frequency from narrow band to broad band for the generation of EIC waves. Such type of waves with different frequency ranges [8,22] have been observed in the earth's magnetosphere by different satellites. There is no cutoff in the entire range of frequency instead of this we observe resonance at $\omega/\Omega_i \sim 1.6$. We are considering the resonant interaction in the analysis ($\omega = kV_{th}$), where maximum growth rate takes place at $\omega/\Omega_i \sim 1.6$. Effect of temperature anisotropy of beam has been shown in Figure 2. EIC wave growth rate increases as temperature anisotropy increases while for its lower values damping is prominant [23]. Presence of electric field restricts the damping to occur at higher values of temperature anisotropy.

Figure 3 reveals the variation of growth rate over a wide range of $k_{\parallel}$ (*i.e.* $\Omega_i/U_b < k_{\parallel} < \Omega_i/U_b$) for high temperature anisotropy ($T_{b_\parallel}/T_{b_\perp} = 0.1$). The parallel wave number has approximately the resonance condition $k_{\parallel} = \Omega_i/U_b$. EIC waves are excited in the above range. The effect of electric field enhances the growth rate at $k_{\parallel} < \Omega_i/U_b$ while at higher values $k_{\parallel} > \Omega_i/U_b$ it almost gets saturated as shown in figure.

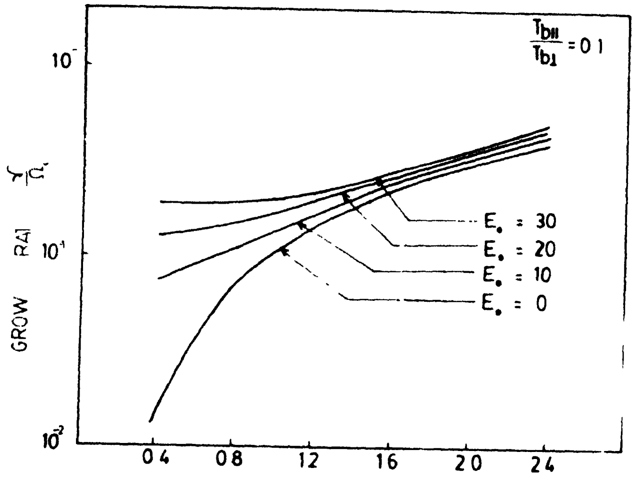

Figure 3. Variation of normalized growth-rate of EIC wave with k for different values of E_0 and $T_{b_\parallel}/T_{b_\perp} = 0.1$.

Figure 4 depicts the growth rate of EIC waves which are significantly modified by the presence of drifting electrons. Electron drift can lead to current driven EIC wave instabilities in collisional [24,25] and collisonless [1] plasmas. Entire range of parallel wave number can be divided into growth and damping regions from where one can estimate the threshold condition. Since drift velocity of electrons exceeds the target ions, therefore, in energy exchange process, the wave is amplified for higher values of $K_{\parallel}$ and damping occurs at its lower values. Kindel and Kennel [6] have pointed out that the electron drift is also one of the source of free energy in the magnetosphere. Generally in this case $K_{\parallel}$ lies in the range of 0.2 to 0.6. In the absence of electric field E_0, the growth rate increases as the drift velocity of

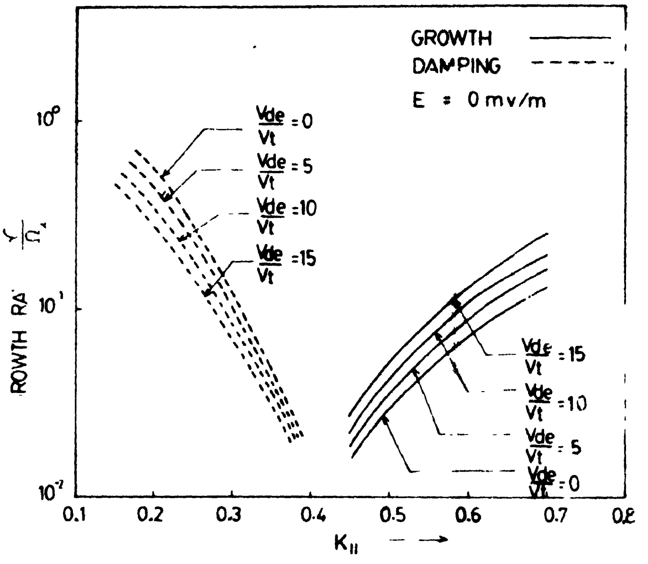

Figure 4. Variation of normalized growth-rate of EIC wave with k for different values of V_{de}/V_t when $E_0 = 0$ mV/m.

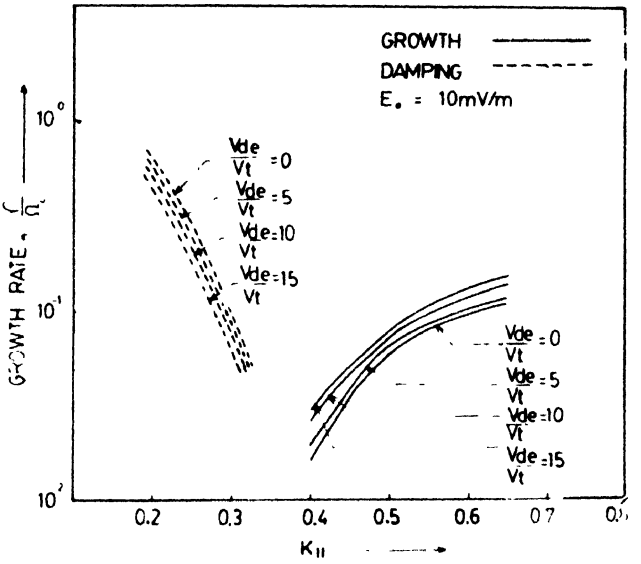

Figure 5. Variation of normalized growth-rate of EIC wave with $k_{\parallel}$ for different values of V_{de}/V_t when $E_0 = 10$ mV/m

electron increases for higher value of $K_{\parallel}$ while damping occurs at its lower values as shown in Figure 5. The same variation of growth rate with respect to $K_{\parallel}$ is observed in presence of electric field E_0 = 10 mV/m as shown in Figure 6. Electric field once again decides the threshold condition for the occurrence of instability in k-space.

4. Conclusion

In the present communication, we have investigated the excitation of low frequency EIC waves in magnetised (auroral) plasmas which consist of ion beams, drifting bulk electrons and background ions. Excitation of EIC waves has been explained on the basis of linear wave-particle resonant interaction. We may summarize our results with respect to interaction between : electron-target ion, electron-beam ion and target ion beam ions. In general the waves are excited by an energetic electrons through resonance interaction with beam ion and the back ground target ions support the instability. A band of EIC waves can be excited when an ion beam permeates a current carrying plasma with electron drifting in the opposite to that of the ion beam. In presence of parallel electric field, a significant enhancement of the growth rate of EIC waves occurs. Anisotropic ion beam may also give rise to a substantial growth rate of EIC waves at its higher values. We have studied the resonant growth rate at fundamental mode of ion cyclotron frequency and shown that there is a significant enhancement of the growth rate of EIC waves in presence of electric field. Mozer *et al* [26] have predicted the observation of current driven electrostatic ion cyclotron waves which have been observed by S3–3 and Viking satellite [27]. Further, electric field plays an important role in the generation of EIC waves and acceleration of auroral particles in the magnetosphere. This may be confirmed from ISEE–1 satellite data [28]. Since the computation of analytical growth rate is being done by choosing magnetospheric plasma parameters, so these theoretical findings can be used for the study of EIC waves generation and its excitation in the magnetosphere.

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