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Radiation due to a fluctuating acoustic ring source in motion*

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The problem of radiation due to a fluctuating acoustic ring-source moving with uniform velocity in an unbounded medium has been analysed The analytical expressions for acoustic power emitted and the frequency spectrum obtained, show that the well known results of a uniformly convected monopole are rediscovered

1 Introduction

The problem of radiation from various types of moving acoustic sources is of great interest in the theory of Jet-Noise (Lighthill 1952, 1954) In general the effect of source motion results in augmentation of the total acoustic power emitted by the source, besides the well known Doppler-effect An elegant treatment of the problem of uniformly moving acoustic monopole has been given by Morse & Ingrad (1968), while the same problem has been discussed by Mani (1972) when the monopole is uniformly convected at the axis of a jet flow (the velocities of jet-flow and monopole being equal)

In this communication we have analysed the problem of radiation due to a fluctuating acoustic ring-source moving with a uniform velocity in an unbounded medium at rest. The results obtained show that the results of a uniformly convected monopole for output power and frequency spectrum are rediscovered, both for subsonic ($M < 1$) and supersonic ($M > 1$) speeds

2 Formulation and Solution of the Problem

Let us calculate the output power and frequency spectrum due a fluctuating acoustic ring-source moving with uniform velocity V in z-direction in unbounded medium. The wave equation governing the propagation of acoustic pressure field by the moving acoustic ring-source in cylindrical coordinates is

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2}-\nabla^2 p=\frac{\partial}{\partial t}\left[g(t)\,\frac{\delta(\gamma-\gamma_0)}{\gamma}\,\delta(z-Vt)\right] \qquad \ldots \quad (2.1)$$

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with

$$\nabla^2 = \frac{1}{\gamma}\frac{\partial}{\partial\gamma}\gamma\left(\frac{\partial}{\partial\gamma}\right) + \frac{\partial^2}{\partial z^2},$$

c_0 is the propagation velocity of acoustic pressure wave, and $q(t)\frac{\delta(\gamma-\gamma_0)}{\gamma}\delta(z-Vt)$ is the ring-source distribution density. Now using the Dirac-delta function property

$$\delta(z-Vt) = \delta\left(V\left(t-\frac{z}{V}\right)\right) = \frac{1}{V}\delta\left(t-\frac{z}{V}\right) \quad .. \quad (2.2)$$

the Fourier transform of right hand side of equation (2.1) gives

$$(2\pi)^{-1}\int_{-\infty}^{\infty}\exp(i\omega t)\frac{\partial}{\partial t}[q(t)\delta(z-Vt)]dt = -\frac{i\omega}{2\pi V}q\left(\frac{z}{V}\right)\exp(i\omega(z/V)) \quad .. \quad (2.3)$$

Similarly defining the Fourier transform of the acoustic field as

$$\tilde{p}(\gamma, z, \omega) = (2\pi)^{-1}\int_{-\infty}^{\infty}p(\gamma, z, t)\exp(i\omega t)dt. \quad .. \quad (2.4)$$

We have eq. (2.1), vide eqs. (2.3) and (2.4), as

$$\nabla^2\tilde{p}+\left(\frac{\omega}{c_0}\right)^2\tilde{p} = \frac{i\omega}{2\pi V}q\left(\frac{z}{V}\right)\exp(i\omega(z/V))\frac{\delta(\gamma-\gamma_0)}{\gamma} \quad . \quad (2.5)$$

Assuming the sine harmonic time dependence of the ring-source as

$$q(t) = q_0\sin\omega_0 t = \left(\frac{q_0}{2i}\right)(\exp(i\omega_0 t)-\exp(-i\omega_0 t)) \quad ... \quad (2.6)$$

and considering the contribution of $\exp(i\omega_0 t)$ to acoustic field denoted by $\tilde{p}_+$, eq. (2.5) becomes

$$\nabla^2\tilde{p}_+ + \left(\frac{\omega}{c_0}\right)^2\tilde{p}_+ = \frac{q_0\omega}{4\pi V}\exp(i(\omega+\omega_0)(z/V))\frac{\delta(\gamma-\gamma_0)}{\gamma}. \quad ... \quad (2.7)$$

The solution of eq. (2.7) will obviously be of the form

$$\tilde{p}_+ = f(\gamma)\exp(i(\omega+\omega_0)(z/V)) \quad ... \quad (2.8)$$

which will modify eq. (2.7) to Helmoholtz equation in two dimensions

$$\left[\frac{1}{\gamma}\frac{\partial}{\partial\gamma}\left(\gamma\frac{\partial}{\partial\gamma}\right)+\left(\frac{\omega}{c_0}\right)^2-\left(\frac{\omega+\omega_0}{V}\right)^2\right]f(\gamma) = \frac{q_0\omega}{4\pi V}\frac{\delta(\gamma-\gamma_0)}{\gamma} \quad ... \quad (2.9)$$

whose solution is

$$f(\gamma) = \frac{i\omega q_0}{16\pi V} H_0^{(1)}(k+R)$$

so that the complete solution of eq. (2.7) is

$$\tilde{p}_+ = \frac{iq_0\omega}{16\pi V} H_0^{(1)}(k+R)\exp(i(\omega+\omega_0)(z/V)) \quad \ldots \quad (2.10)$$

with

$$R = \gamma - \gamma_0$$

$$k_+^2 = \left[\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{\omega+\omega_0}{V}\right)^2 \right] \quad \ldots \quad (2.11)$$

and $H_0^{(1)}$ is the Hankel function of first kind

Similarly one can find the contribution of $\exp(-i\omega_0 t)$ term, denoted by $\tilde{p}_-$ to the acoustic field and is given by

$$\tilde{p}_- = \frac{-iq_0\omega}{16\pi V} H_0^{(1)}(k-R)\exp(i(\omega-\omega_0)(z/V)) \quad \ldots \quad (2.12)$$

with

$$k_-^2 = \left[\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{\omega-\omega_0}{V}\right)^2 \right] \quad (2.13)$$

Thus the total acoustic pressure field is the inverse Fourier transform of the sum of eqs. (2.10) and (2.12) and is given by

$$p(\gamma, z, t) = \int_{-\infty}^{\infty} (\tilde{p}_+ + \tilde{p}_-)\exp(it\omega))d\omega$$

$$= \frac{iq_0}{16\pi V} \int_{\infty -}^{\infty} [\omega H_0^{(1)}(k+R)\exp(i(\omega+\omega_0)(z/V))$$

$$-\omega H_0^{(1)}(k-R)\exp(i(\omega-\omega_0)(z/V))]\exp(-i\omega t)d\omega \quad \ldots \quad (2.14)$$

Also as the acoustic particle velocity is defined as

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial \gamma}$$

hence the radial velocity of the acoustic particles from eq. (2.14) is

$$u_\gamma(\gamma, z, t) = \frac{q_0}{16 u V \rho} \int [k_+ H_1^{(1)}(k_+ R)\exp(i(\omega+\omega_0)(z/V))$$

$$-k_- H_1^{(1)}(k_- R)\exp(i(\omega-\omega_0)(z/V))]\exp(-i\omega t)d\omega \quad .. \quad (2.15)$$

where use has been made of the relation,

$$\frac{d H_0^{(1)}(z)}{dz} = - H_1^{(1)}(z)$$

Before calculating the output power and frequency spectrum of the ring-source, let us discuss the frequency dependence of

3 Discussion of Frequency Dependence

From equation (2.11) we have

$$k_+^2 = \left(\frac{\omega}{c_0}\right)^2 - \left(\frac{\omega+\omega_0}{V}\right)^2$$

$$= \frac{M^2-1}{V^2} \cdot \left(\omega + \frac{\omega_0}{M+1}\right)\left(\omega - \frac{\omega_0}{M-1}\right) \quad \ldots \quad (3.1)$$

with

$$M = (V/c_0)$$

Eq. (3.1) shows that the real values of the propagation constant k_+ for subsonic ($M > 1$) motion of the ring source, are confined to the frequency range

$$-\frac{\omega_0}{M+1} < \omega < \frac{\omega_0}{M-1} . \quad .. \quad (3.2)$$

Similarly for supersonic motion ($M > 1$) of the ring-source the real values of the propagation constant k_+ lie in the frequency ranges

$$-\infty < \omega < -\frac{\omega_0}{1-M}, \quad -\frac{\omega_0}{1+M} < \omega < \infty \quad \ldots \quad (3.3)$$

The corresponding frequency ranges for real values of propagation constant k_- are obtained by replacing ω_0 by $-\omega_0$ in eqs. (3.2) and (3.3). The reason for not considering the negative values of the propagation constants k_+^2 and k_-^2 is that k_+ and k_- will be imaginary in character and as the Hankel function for these values will not contribute to the acoustic field in the far field approximation

4. The Emitted Power and Frequency Spectrum of the Ring Source

The acoustic power emitted by the ring source in far field approximation ($\gamma > \gamma_0$, $R \simeq \gamma$) is calculated by integrating the radial acoustic energy flux, over the surface of an infinitely long cylinder enclosing the line of motion of motion of the ring source over one period ($2\pi/\omega_0$) of the sources as

$$P = \int 2\pi\gamma\,\overline{p u_\gamma}\, dz \quad .. \quad (4\ 1)$$

where γ is the radius of the cylinder and p and u_γ are given by eqs (2 14) and (2 15) respectively

Now following Morse & Ingrad (1968), the average acoustic power output and frequency spectrum of the ring-source in subsonic motion ($M < 1$) for the frequency range

$$-\frac{\omega_0}{M+1} < \omega < \frac{\omega_0}{1-M}\,.$$

are given by

$$P = \frac{q_0^2\Lambda_0^2}{8\pi\rho c_0}\;\frac{1}{(1-M^2)^2} \quad \ldots \quad (4.2)$$

and

$$I(\omega) = \frac{q_0^2}{16\pi\rho c_0}\;\frac{\omega}{M} \quad \ldots \quad (4\ 3)$$

Similarly for supersonic motion ($M > 1$) of the ring-source and for the frequency range given by eq. (3 3), the average acoustic power emitted is

$$P = \frac{q_0^2\omega_0^2}{8\pi\rho c_0}\;\frac{1}{(1-M^2)^2} \quad . \quad (4.4)$$

5 Discussion

Eqs. (4 2) and (4 4) show that the average acoustic power emitted by the ring source is increased by a factor of $(1-M^2)^{-2}$ and the increase is same for both kinds of ring-source motion. However, the frequency spectrum given by eq. (4 3) is different for subsonic and supersonic motions, as the frequency ranges given by eqs. (3.2) and (3 3) are different for the two cases of interest

Eqs (4 2) through (4 4) are the same as those obtained for a uniformly convected monopole acoustic source radiation (Morse & Ingrad 1968) This similarity of the results was to be expected because a ring source can be through of as integrated effect of monopole sources arranged on a ring, whose strength will be equal to the total strength of all the monopole sources

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