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Fractional Order Modeling and Control of a Smart Beam

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Abstract— **Smart beams are one of the most frequently used means of studying vibrations in airplane wings. Their mathematical models have been so far solely based on classical approaches that ultimately involve integer order transfer functions. In this paper, a different approach towards modeling such smart beams is considered, an approach that is based on fractional calculus. In this way, a fractional order model of the smart beam is obtained, which is able to better capture the dynamics of the system. Based on this novel fractional order model, a fractional order PD$^{\mu}$ controller is then tuned according to a set of three design constraints. This design leads to a closed loop system that exhibits a much smaller resonant peak compared to the uncompensated smart beam system. Experimental results are provided, considering both passive and active control responses of the smart beam, showing that a significant improvement of the closed loop behavior is obtained using the designed controller.**

I. Introduction

Vibration is one of the most studied domains in research, since it occurs in numerous applications and can have disastrous effects if not properly accounted for. In the airplane wings, vibrations can lead to increased oscillations that could cause even the system failure. Such problems need to be actively dealt with.

The most common way to study vibrations in airplane wings, at a research level, is through some dedicated cantilever beams that simulate on a simplified level the dynamics of airplane wings. These cantilever beams are usually made out of aluminum and are equipped with sensors, actuators and real-time devices that implement a dedicated control algorithm [1-3].

To design a dedicated control algorithm, the mathematical model of the smart beam is required. Quite often, this is obtained based on the finite element approach [4-7] leading to integer order differential equations that could later be put into transfer function or state space form. However, for an accurate representation of the complete dynamics of the beam, the sizes of the finite element model matrices or the resulting transfer functions are usually too big [7]. Simplified models could be considered [8,9] instead, but they could lack precision in approximating the beam dynamics. In this paper, a different approach to modeling is considered, one that is based on the viscoelastic nature of the cantilever beam. It has been already demonstrated that viscoelasticity and related phenomena can be better described using fractional order models [10, 11]. As a consequence, a fractional order model is determined in this paper. The model has its basis in a previously determined integer order model [12], which is then altered by considering the damping in the smart beam to be represented by a fractional order differentiator. Such an approach has been considered before, in the case of mass-spring-damper element [13,14], but never for the case of a smart beam system. Once the structure of the model is selected, the computation of the parameters is performed by minimizing the difference between the model and a set of experimental data. The final results show that the fractional order model provides for an increased accuracy in the approximation of the beam dynamics compared to the integer order model.

Once a model has been developed, the control algorithm is designed based on that model. Several control algorithms have been proposed for vibration attenuation in smart beams, ranging from simple classical strategies [6,7,9,15,16] to advanced algorithms [5,6,8,17-19]. Fractional order PID controllers have also been proposed as adequate solutions for vibration attenuation [12,20,21], however all of these strategies are designed based on an integer order model of the smart beam. The choice of fractional order controllers is based on their ability to enhance the closed loop performance, as well as to increase the closed loop robustness, as compared to the classical integer order controllers [22,23]. In this paper, a fractional order PD controller is designed based on the fractional order model of the beam. From the control effects point of view, usage of an integrator is futile since the beam is in its equilibrium position, while the derivative effect enhances the overall stability of the closed loop. if not properly tuned to account for modeling errors and uncertainties, an integrative effect can easily destabilize the closed loop system. The purpose of the controller is to reduce the settling time of the transient response, and to diminish the amplitude of the oscillation, hence a PD type controller is fit to honor the design specifications. The design considers three performance specifications and leads to a robust controller that can actively diminish the effect of disturbances upon the dynamics of the beam. The designed controller is implemented and tested experimentally on a smart beam built within the Technical University of Cluj-Napoca.

The novelty of the approach consists in the development of a fractional order of the beam based on optimization techniques on the experimental frequency response, the design of a fractional order PD controller based on the fractional model as well as the testing and validation of the results through experiments on the actual system.

The paper is organized as follows. Section II briefly details the case study considered in this paper, while Section III describes the fractional order model of this beam. The design of the fractional order controller, along with

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experimental closed loop tests, is given in Section IV. The last section contains some concluding remarks.

II. Case Study: The Custom Built Smart Beam

The experimental unit considered in this paper has been custom built at the Technical University of Cluj-Napoca, Romania, to serve as a means to simulate the airplane wing as a cantilever beam. To more accurately represent the airplane wing, the beam is made of aluminum, having one fixed end and the other one left free to vibrate and oscillate. The beam dimensions are as follows: 250mm x 20mm x 1mm.

To measure the displacement and to ultimately attenuate the vibrations caused by several possible disturbances, sensors and actuators have been glued near the fixed end of the beam. The sensors used are strain gauge type, 120 ohm Omega Prewired KFG-5-120-C1-11L1M2R, while for the actuators, the P-878 DuraAct Power Patch Transducers were chosen. This piezoelectric (PZT) patches can be used both as sensors and actuators, but for the purpose of this custom built smart beam, they are employed only as actuators, having been positioned in pairs, one pair on each side of the beam. Each patch has 27 mm length, 9.5 mm width and 0.5 mm thickness. The nominal operating voltages are between -20 and 120 V, while the power generation is possible up to the milliwatt range. For control purposes, the control signal is limited to [-20,+20]V. A block diagram of the experimental unit is given in Fig. 2 including the programmable automation controller (PAC) and a dedicated system with chassis (E501.00) comprising a power amplifier (E503.00) and the signal processing module (E509.X3). The PAC embedded system includes a real time controller (NI 9014), a chassis with FPGA (NI 9103), two extension modules with analog input lines (NI 9230) for reading data from the sensors on the beam and analog output lines (NI 9263) for sending the control signal to the PZT patches used as actuators.

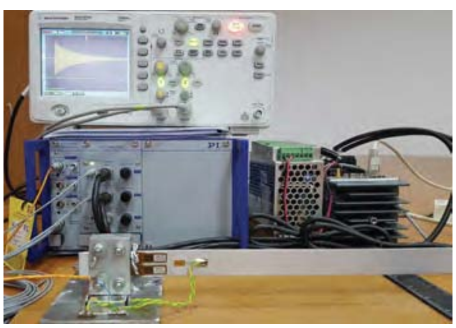


Fig. 1. The custom built smart beam

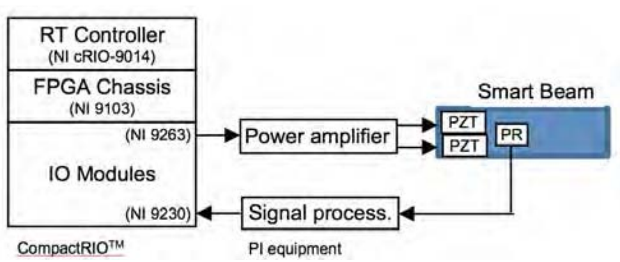


Fig. 2. Block diagram of the experimental stand

III. Fractional Order Modeling of the Smart Beam

An integer order model of this beam has been obtained before based on experimental data [12]:

$$H(s) = \frac{1}{0.0128s^2+0.0156s+104.94} \quad (1)$$

with a resonant frequency of 14.43Hz.

However, because of the viscoelastic nature of the beam, a fractional order model could be more suitable in representing the dynamics of the oscillations. This is based on the assumption that, the second order model in (1) is a mathematical representation of a smart beam as a mass-spring-damper system [4,8]. The second assumption is based on previous researches that link the damping in a mass-spring-damper system to fractional calculus and fractional order derivatives [13,14]. Also, a fractional order model has an increased degree of freedom when adjusting the slope of the magnitude plot of the frequency response and would ensure a better approximation of the frequency response. As a consequence, a fractional order model for the smart beam is proposed as follows:

$$H_{FO}(s) = \frac{1}{a_1 s^2+a_2 s^{\alpha}+a_3} \quad (2)$$

The key issue is now to determine the parameters of this model, a_1, a_2, a_3 and the fractional order α. For this, the complex representation of the fractional order model in (2) is computed as:

$$H_{FO}(j\omega) = \frac{1}{-a_1\omega^2+a_2\omega^{\alpha}cos\frac{\alpha\pi}{2}+a_3+ja_2\omega^{\alpha}sin\frac{\alpha\pi}{2}} \quad (3)$$

where ω is the frequency and $\mathcal{R}(\omega) = -a_1\omega^2 + a_2\omega^{\alpha}cos\frac{\alpha\pi}{2} + a_3$ and $\Im(\omega) = a_2\omega^{\alpha}sin\frac{\alpha\pi}{2}$ are the real and imaginary parts of the denominator in (3). The modulus and phase of (3) can then be determined as:

$$|H_{FO}(j\omega)| = \frac{1}{\sqrt{\mathcal{R}(\omega)^2+\Im(\omega)^2}} \quad (4)$$

$$\angle H_{FO}(j\omega) = atan\frac{\Im(\omega)}{\mathcal{R}(\omega)} \quad (5)$$

Next, a sinusoidal signal of different frequencies ω=[13, 13.97, 14.15, 14.34, 14.52, 14.71, 14.9, 15.08, 15.27, 15.46, 15.83, 16, 16.95, 18.07]Hz and amplitude of 1V is supplied as the input to the smart beam and data regarding its oscillations is collected. Based on these experimental data, the modulus and phase of the smart beam, M_{ω} and φ_{ω} at a specific frequency ω, are determined as follows:

$$M_{\omega}(j\omega) = \frac{A_o}{A_i} \quad (6)$$

$$\varphi_{\omega}(j\omega) = \omega\tau \quad (7)$$

where A_o and A_i are the amplitudes of the output and input signals and τ is the time shift between the input and output signals [24]. Then, an optimization routine (Matlab *fmincon*) is used to determine the parameters of (2) by minimizing the main cost function:

$$F = |H_{FO}(j\omega_r)| - M_{\omega_r}(j\omega_r) \quad (8)$$

where $\omega_r = 14.52$Hz is a frequency close to the resonant frequency of the smart beam. In the optimization, the following constraints are also used:

$$C_{modulus} = |H_{FO}(j\omega)| - M_{\omega}(j\omega) \quad (9)$$

$$C_{phase} = \angle H_{FO}(j\omega) - \varphi_{\omega}(j\omega) \quad (10)$$

where $\omega \neq \omega_r$ are the frequencies used for the experimental tests. The initial values of the optimization routine for a_1, a_2, a_3 are selected to be those resulting from (1), such as: a_1=0.0128, a_2=0.0156 a_3= 104.94 and the fractional order α=0.5.

The final parameters, as computed using the optimization routine, lead to the following transfer function of the fractional order model of the smart beam:

$$H_{FO}(s) = \frac{1}{0.0131s^2+0.027s^{0.89}+107.72} \quad (11)$$

Figure 3 shows the magnitude Bode diagram of the fractional order model compared to the experimental data computed using (6). The figure shows that the fractional order model provides for a good approximation near the resonant frequency.

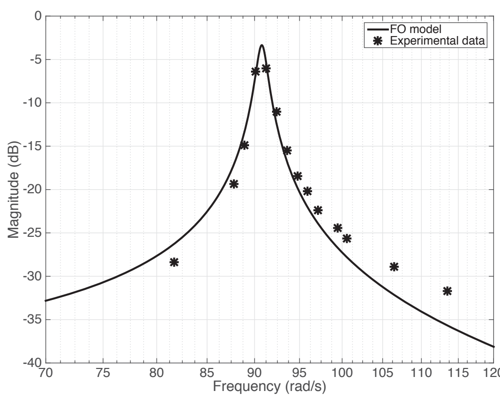


Fig. 3. Magnitude Bode diagram of the fractional order model compared to the experimental data

To compare the integer order (IO) and fractional order (FO) mathematical models, in terms of approximation accuracy, the following performance index is used:

$$J = \sum_{i=1}^{n} \left(y_{exp}(i) - y_s(i)\right)^2 \quad (12)$$

where $y_{exp}(i)$ is the set of experimental data, $y_s(i)$ is the set of simulated data and i=1,n is the number of comparison points. The computed values for the performance index are included in Table 1. In all cases, the input signal had an amplitude of 1V and frequency as indicated in Table 1.

TABLE I. COMPARATIVE PERFORMANCE RESULTS

	Performance index J as in (12)	
Frequency	**FO model**	**IO model**
13	1.73	2.13
13.97	6.70	10.77
14.15	13.87	27.14
14.34	19.35	40.53
14.52	5.18	14.74
14.57	3.76	5.96
14.71	17.99	21.03
14.9	9.19	10.27
15.27	4.66	4.96
16	1.92	1.97
Swept sine [12,16]Hz	6.57	10.3

The comparative simulation results considering the swept sine signal of frequency range [12,16]Hz applied at the input of the smart beam are given in Fig. 4, while Fig. 5 shows the comparison between the two models considering a sine input signal of frequency 14.52Hz. The simulation and experimental data in this case is portrayed for the steady state behavior of the smart beam. The results show that the FO model is able to better capture the dynamics and steady state behavior of the smart beam. An analysis of the results included in Table 1 also demonstrates a better accuracy for the FO model compared to the IO model, with as much as 65% better fit near the resonant frequency.

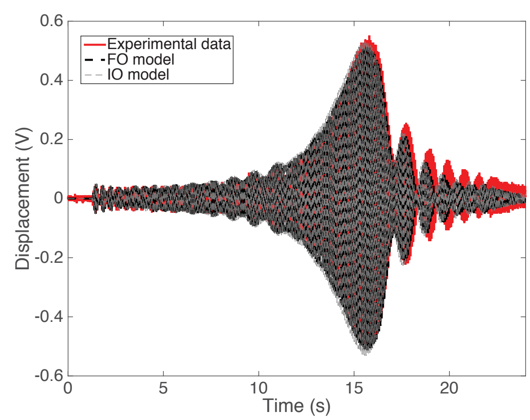


Fig. 4. Fractional order model compared to integer order model based on a swept sine of amplitude 1V and frequency [12,16]Hz applied at the input of the smart beam

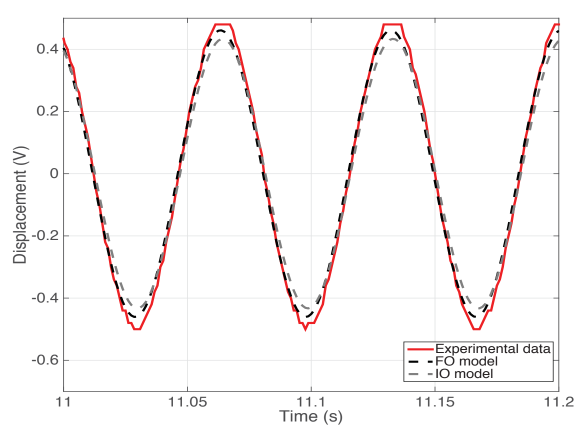


Fig. 5. Fractional order model compared to integer order model based on a sine of amplitude 1V and frequency 14.52Hz applied at the input of the smart beam: zoomed response for steady state

IV. Fractional Order PD Controller Design for Smart Beam

The transfer function of a fractional order PD$^{\mu}$ controller , proposed as the solution for vibration mitigation in the smart beam, is given by:

$$H_{FO-PD}(s)=k_p\left(1+k_d s^{\mu}\right) \tag{13}$$

with k_p and k_d - the proportional and the derivative gains, while $\mu \in (0,1)$ is the fractional order of differentiation. A frequency domain representation of this controller is:

$$H_{FO-PD}(j\omega)=k_p\left(1+k_d\left(j\omega\right)^{\mu}\right) \tag{14}$$

The following relation holds:

$$\left(j\omega\right)^{\mu}=\omega^{\mu}\left(cos\left(\frac{\pi\mu}{2}\right)+j\,sin\left(\frac{\pi\mu}{2}\right)\right) \tag{15}$$

Replacing now (15) into (14), leads to the following complex form of the fractional order controller:

$$H_{FO-PD}(j\omega)=k_p\left[1+k_d\omega^{\mu}\left(cos\left(\frac{\pi\mu}{2}\right)+j\,sin\left(\frac{\pi\mu}{2}\right)\right)\right] \tag{16}$$

To tune the parameters of the fractional order controller in (13), three design constraints are used [12,22,23]: a specific gain crossover frequency and phase margin for the open loop system, as well as the iso-damping property. These three design constraints are mathematically described as:

$$\angle\left(H_{FO-PD}(j\omega_{cg})\right)=-\pi+\varphi_m-\angle\left(H_{FO}(j\omega_{cg})\right) \tag{17}$$

$$\left|\left(H_{FO-PD}(j\omega_{cg})\right)\right|=\frac{1}{\left|\left(H_{FO}(j\omega_{cg})\right)\right|} \tag{18}$$

$$\left.\frac{d\left(\angle H_{FO-PD}(\omega)\right)}{d\omega}\right|_{\omega=\omega_{cg}}=-\left.\frac{d\left(\angle H_{FO}(\omega)\right)}{d\omega}\right|_{\omega=\omega_{cg}} \tag{19}$$

where ω_{cg} is the gain crossover frequency and φ_m is the phase margin.

In (17)-(19), the right hand side of all equations can be easily computed based on the fractional order model of the process previously determined as in (11), while the left-hand side can be determined based on (16). The simplest way of solving the system of equations (17)-(19) and to determine the tuning parameters of the FO-PD controller consists in a graphical approach that has been detailed on numerous occasions [22,23]. For the smart beam, the design constraints are specified to be ω_{cg}=105 rad/s and φ_m=60^{o}. Replacing these values as well as (16), into (17)-(19) leads to the following system of equations that needs to be solved:

$$\frac{k_d\cdot 105^{\mu}\,sin\left(\frac{\pi\mu}{2}\right)}{1+k_d\cdot 105^{\mu}\,cos\left(\frac{\pi\mu}{2}\right)}=tg\left(-\pi+\frac{\pi}{3}-\left(-\frac{177.37\pi}{180}\right)\right) \tag{20}$$

$$k_p=\frac{1}{0.0274}\frac{1}{\sqrt{1+2k_d\cdot 105^{\mu}\,cos\frac{\pi\mu}{2}+k_d^2\cdot 105^{2\mu}}} \tag{21}$$

$$\frac{\mu k_d\cdot 105^{\mu-1}\,sin\frac{\pi\mu}{2}}{1+2k_d\cdot 105^{\mu}\,cos\frac{\pi\mu}{2}+k_d^2\cdot 105^{2\mu}}=-\left(-0.0031\right) \tag{22}$$

Based on (20) and (22), the values of k_d as a function of μ are determined and plotted as indicated in Fig. 6. According to these plots, the intersection point, as the unique solution of (20) and (22), gives the final values for k_d=0.45 and μ=0.9. With these values, the proportional gain is determined using (21): k_p=15. Then, the final transfer function of the fractional order PD controller is:

$$H_{FO-PD}(s)=15\left(1+0.45s^{0.9}\right) \tag{23}$$

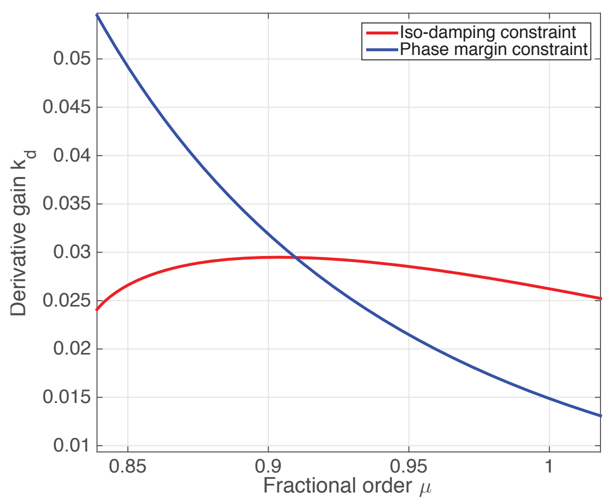


Fig. 6. Computation of the derivative gain k_d and the fractional order μ based on a graphical approach

To implement this controller, a novel approach is considered [12, 25] based on an indirect discretization of the fractional order transfer function in (23). First a rational continuous-time approximation of (23) is obtained using the well-known Oustaloup Recursive Approximation method [26], valid in a frequency domain defined between a lower, ω_L, and an upper bound, ω_H:

$$G(s)=K_c\frac{\prod_{j=1}^{n}\left(s-z_j^c\right)}{\prod_{i=1}^{n}\left(s-p_i^c\right)} \tag{24}$$

where K_c is the gain, z_j^c are the zeros, j=1,2,...,n and p_i^c are the poles, i=1,2,…,n. Then, to compute the discrete-time poles and zeros, a new operator is used [1,9]:

$$z = \frac{1+\alpha+\alpha s T_s}{1+\alpha-sT_s} \tag{25}$$

with α϶[0÷1] – a weighting parameter. For α=0, the Euler discretization rule is obtained, while α=1 leads to the Tustin discretization rule. The effect of this weighting parameter has been detailed before [12,25]. The discrete-time transfer function that approximates (24) is written as:

$$G(z) = K_d \frac{\prod_{j=1}^{n}\left(z-z_j^d\right)}{\prod_{i=1}^{n}\left(z-p_i^d\right)} \tag{26}$$

with K_d the corresponding discrete-time transfer function gain and z_j^d and p_i^d are the zeros and the poles, respectively. Finally, the gain of the discrete-time transfer function that approximates (23) is computed based on the equivalency of the continuous-time and discrete-time transfer functions from (24) and (26) in steady state (s=0 and z=1):

$$K_d = K_c \frac{\prod_{j=1}^{n}\left(-z_j^c\right)\prod_{i=1}^{n}\left(1-p_i^d\right)}{\prod_{i=1}^{n}\left(-p_i^c\right)\prod_{j=1}^{n}\left(1-z_j^d\right)} \tag{27}$$

To approximate the fractional-order PD controller in (23), n=5, ω_L= 0.0628 and ω_H=628 rad/s are chosen for the Oustaloup Recursive Approximation in (24). Then, for the discrete-time approximation, a sampling period T_s=0.005 s and α=0.2 are used. The discrete-time controller is then implemented on the real time control device mentioned in Section II. Figure 7 shows the frequency response of the open loop system, demonstrating that the imposed design contraints are indeed met.

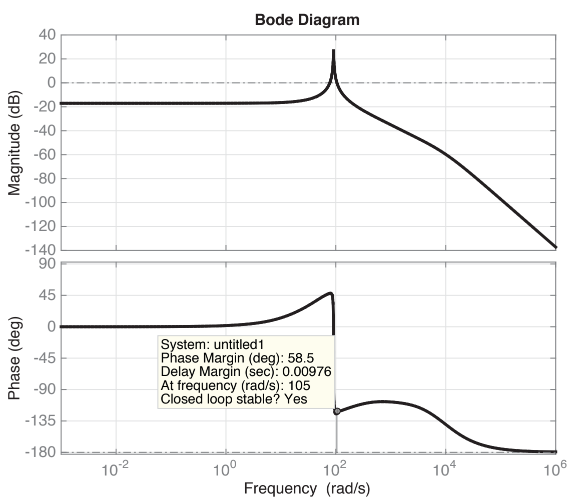


Fig. 7. Frequency response of the open loop system

Figure 8 shows the frequency response of the fractional order model describing the process dynamics. Also, the frequency response of the closed loop is given and clearly shows that the resonant peak has been considerably reduced in comparison to the initial peak of the uncompensated system. This suggests an improved behavior of the closed loop system in comparison to the passive smart beam, without any active controller.

To test the ability of the designed fractional order controller in rejecting disturbances, a sinusoidal signal, of amplitude 4V and resonant frequency, acting on the free end of the smart beam has been considered. The experimental results are included in Fig. 9a) for the smart beam free end displacement, while Fig. 9b) shows the control signal. A quick analysis of the experimental results in Fig. 9a) shows that there is indeed an 88% improvement in terms of the amplitude of oscillation in the active case, compared to the passive one. Different sinusoidal disturbance signals were also considered with similar results in terms vibration attenuation.

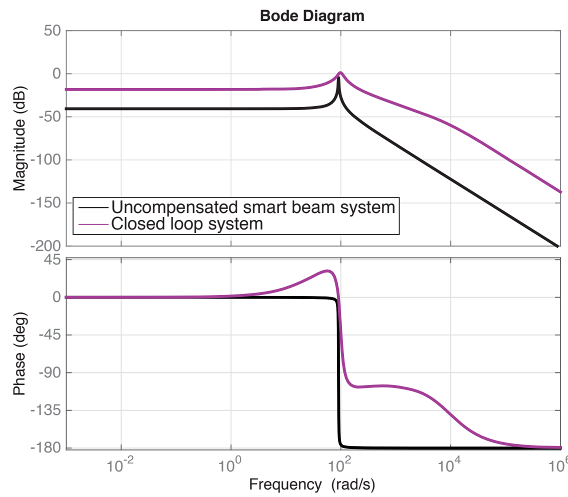
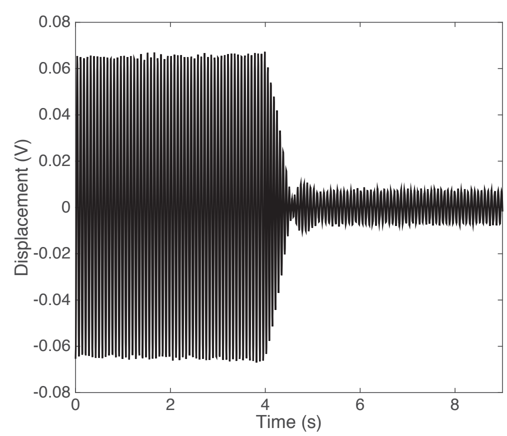
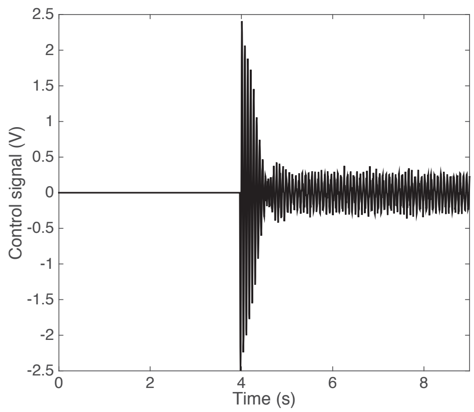


Fig. 8. Frequency response of the uncompensated smart beam system and the closed loop system with the proposed fractional order controller



a)



b)

Fig. 9. a) Displacement of the smart beam when subjected to a sinusoidal disturbance signal of frequency 14.43 Hz b) Control signal

V. CONCLUSION

Smart beams have been quite often used as a means to study vibration in airplane wings and the effect that different types of control algorithms have upon the mitigation of various disturbances. So far, smart beam have been modeled solely using integer order transfer functions. However, because of the viscolesticity involved in the dynamics of such beams, a different approach towards modeling was sought. This different approach is based on the trending principles of fractional calculus and has lead to a fractional order model of the smart beam.

Based on this novel fractional order model, a fractional order PD^{μ} controller is then tuned according to a set of three design constraints that refer to the phase margin and gain crossover frequency of the open loop system, as well as the robustness of the closed loop system to gain variations. As it has been shown, the frequency response of the obtained closed loop system exhibits a much smaller resonant peak compared to the uncompensated smart beam system. This has lead to improved disturbance rejection when the designed fractional order controller was active in comparison to the passive situation.

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