IMPLEMENTATION AND VERIFICATION OF A GEOSYNTHETIC-SOIL INTERFACE CONSTITUTIVE MODEL IN THE GEOGRID ELEMENT OF FLAC^{3D}

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Abstract

Due to the complexity of geosynthetic-soil interactions, the simple interface constitutive models embedded in the geosynthetic elements of general computing software cannot satisfy the requirements for a numerical simulation of different geosynthetic-soil interface behaviours. Based on the direct shear test results of a composite geomembrane (CGM) and polyurethane (PUR) mixed crushed stones interface, a nonlinear elastic, perfectly plastic model was used to describe the interface behaviours. The method of incorporating an interface constitutive model into the Geogrid element of a fast Lagrange analysis of continua in three dimensions (FLAC^{3D}) procedure was presented in detail through a user-defined program in the FISH environment. Then the incorporated model of the Geogrid element was used to simulate the direct tests of the CGM-PUR mixed crushed stones interface. The results of the numerical tests confirmed the validity and reliability of the incorporated model. The method and program flowchart for implementing the nonlinear elastic, perfectly plastic interface constitutive model into the Geogrid element can provide a reference for users who want to simulate other geosynthetic-soil interface behaviours with FLAC^{3D}.

1 INTRODUCTION

Geosynthetics are increasingly used with soils as a composite structure due to their numerous advantages. This is relevant to many geotechnical engineering situations, such as the geomembrane barrier of an earth-rock-fill dam and waste landfills, geotextile- or geogrid-reinforced structures. A deeper understanding of the geosynthetics-soil interaction is of paramount importance for a stability assessment and deformation analysis of composite structures. The key point of solving the geosynthetic-soil interaction problem is to accurately describe the shear stress-displacement relationship of the interface. Several researchers, such as Byrne [1]; Jenevein et al. [2]; Esterhuizen et al. [3]; Kim [4]; Zhang et al. [5]; and Bacas et al. [6] have proposed different constitutive models to describe the geosynthetic-soil interface behaviours based on the results of laboratory tests and field investigations. The interface constitutive models were also incorporated into numerical programs to simulate the geosynthetics-soil interactions involved in the different structures [7, 8, 9, 10]. Several numerical computing software packages, such as PLAXIS, ABAQUS and FLAC/FLAC^{3D}, can provide structural elements to simulate the mechanical behaviours of flexible geosynthetics. But these structural elements only provide a single and simple generalized interface model for users to simulate the different geosynthetic-soil interactions. This seriously limits the application of the software in solving problems that contain different geosynthetic-soil interfaces.

Due to its distinct advantage of solving the large strain problems of rock and soil structures, the finite-difference code FLAC/FLAC^{3D} has been used to solve geotechnical engineering problems containing geosynthetics by many researchers, such as Jones and Dixon [11]; Fowmes et al. [12,13,14]; Wu. et al. [15]; and Wu and Shu. [16]. The Geogrid element in FLAC and FLAC^{3D} can only simulate the shear stress vs. shear displacement relationship of a geosynthetic-soil interface according to the linear elastic, perfectly plastic model. But it is not able to simulate the geosynthetic-soil interfaces characterized by nonlinear or strain-softening behaviours.

Studies on the improvement and further development of Geogrid elements in the FLAC and FLAC^{3D} software have rarely been reported. Jones and Dixon [7]; Fowmes et al. [9]; Wu et al. [17] have incorporated the results of interface direct tests into the FLAC and FLAC^{3D} program to simulate the strain-softening behaviours of a geosynthetic-soil interface using the embedded FISH language, respectively. Their simulations were achieved by incorporating the geosynthetic-soil interface constitutive model into the Interface element, but not the Geogrid element of the FLAC or FLAC^{3D}. Although the interface element can simulate the geosynthetic-soil interface behaviours, it cannot simulate the mechanical behaviour of the geosynthetics itself. Only the shear stress distribution at the interface can be solved using their incorporated models. And they cannot be used to calculate the stress and strain of the geosynthetics when considering the geosynthetic-soil interaction. So a further improvement needs to be performed to accomplish a numerical simulation of the geosynthetic-soil interaction using the Geogrid element in FLAC^{3D}.

In this paper, the implementation of the direct shear testing results of the CGM-PUR mixed crushed stones interface was taken as an example to present the method of incorporating a new geosynthetic-soil interface constitutive model into the Geogrid element of the FLAC^{3D}. Firstly, a nonlinear elastic, perfectly plastic model for the interface was used to describe the direct shear test results for the CGM-PUR mixed crushed stones interface. Then the interface model was incorporated into the Geogrid elements of the FLAC^{3D} through a user-defined program in the FISH environment. Finally, the direct shear tests on the CGM-PUR mixed crushed stones interface were simulated by an improved Geogrid element to verify the incorporated geosynthetic-soil interface model.

2 ORIGINAL INTERFACE CONSTITUTIVE MODEL OF THE GEOGRID ELEMENT IN FLAC^{3D}

The Geogrid element in FLAC^{3D} is a type of plane-stress element that can resist a membrane load but not a bending load [18]. The Geogrid elements can be used to model the flexible geosynthetics whose shear interactions with the soil are important, such as the geomembrane, the geotextiles and the geogrids.

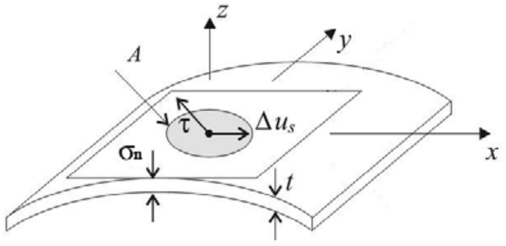

Figure 1. Mechanical behaviour in the shear direction of the Geogrid-soil interface.

The Geogrid element is embedded in the interior of the grid zones (soil) in FLAC^{3D}. As shown in Fig. 1, the interaction between Geogrid and the zone element is defined by the mechanical behaviours of the Geogrid-soil interface. The interface behaviour is represented numerically at each Geogrid node by a rigid attachment in the normal direction and a spring-slider in the tangent plane to the Geogrid surface.

In the normal direction the Geogrid element is slaved to the soil-grid motion. In order to be different from the common soil-structure interface, the normal discontinuous deformation such as penetration or separation on the geosynthetic-soil interface is not considered. And the effective normal stress is assumed to be acting equally on both sides of the Geogrid surface. The velocity and displacement normal to the Geogrid surface are transferred directly to the nodes of the soil zone. The node exerts no normal force on the soil-grid if all the Georgids that share the node are co-planar; however, if they are not co-planar, then a proportion of their membrane force will act in the normal direction[18].

In the tangent plane of the Geogrid surface, a shear-directed frictional interaction occurs between the Geogrid and the soil grid. The relative displacement between the Geogrid and the soil grid is the source of the frictional shear stress on the interface. In computing the relative displacement at the Geogrid-soil interface, an interpolation scheme is used that is based on the displacement field in the zone to which the node is linked. The interpolation scheme uses weighting factors that are determined by the distance to each of the zone grid points [18].

The shear stress that is exerted on the node of the Geogrid during the calculation time step $t+\Delta t$ can be expressed as follows:

$$\tau^{(t+\Delta t)} = \tau^{(t)} + \Delta\tau \qquad (1)$$

where $\tau^{(t+\Delta t)}$ is the shear stress exerted on the node of the Geogrid during the calculation time step $t+\Delta t$; $\tau^{(t)}$ is the shear stress exerted on the node of the Geogrid during the calculation time step t, and $\Delta\tau$ is the incremental shear stress between t and $t+\Delta t$.

The incremental shear stress $\Delta\tau$ is determined by the Geogrid-soil interface constitutive model, i.e., the shear stress-shear displacement relationship:

$$\tau = f(u) \qquad (2)$$

where τ is the shear stress of the Geogrid-soil interface, and u is the shear displacement of Geogrid-soil interface.

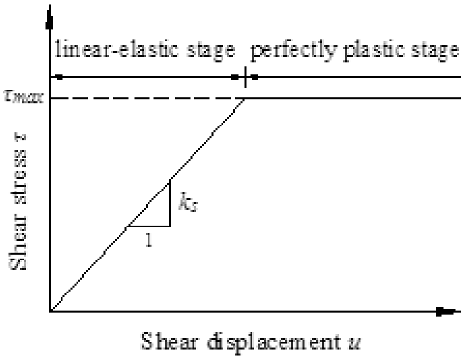

Figure 2. Elastic, perfectly plastic model of the Geogrid-soil interface.

The original interface constitutive model of the Geogrid element in FLAC^{3D} is shown in Fig.2. The relationship between the shear stress and the shear displacement is defined by the linear elastic, perfectly plastic model. In the linear elastic stage, the shear stress and shear displacement relationship can be expressed as:

$$\tau = f(u) = k_s u \qquad (3)$$

where k_s is the shear stiffness (constant) of the Geogrid-soil interface.

After the shear stress reaches a peak shear strength of the interface, plastic failure occurs with an increase of the shear displacement. The interface shear strength of the Geogrid element in FLAC^{3D} is defined by the Mohr-Coulomb failure criterion:

$$\tau = \tau_{\max} = c + \sigma_n \tan\phi \qquad (4)$$

where $\tau_{\max}$ is the shear strength of the Geogrid-soil interface; c is the cohesion of the Geogrid-soil interface; ϕ is the friction angle of the Geogrid-soil interface, and σ_n is the normal stress of the Geogrid-soil interface.

3 DIRECT SHEAR TEST OF THE CGM-PUR MIXED CRUSHED STONES INTERFACE

The direct shear test results of the CGM- PUR mixed crushed stones interface will be taken as an example to illustrate the method of incorporating a new geosynthetic-soil interface constitutive model into the Geogrid elements of FLAC^{3D}. And the direct shear tests of the CGM-PUR mixed crushed stones interface will be introduced in a simple way.

3.1 Test materials

The CGM commonly used as a surface barrier for rock-fill dams in China was chosen for the tests. A photograph of a CGM is shown in Fig. 3. It consists of a 0.8-mm-thick HDPE geomembrane laminated to 400 g/m^2 PET needle punched nonwoven geotextile on both sides. Its ultimate tensile strength (ASTM D4595, 2005) in the machine direction and the cross-machine direction are 75.9kN/m and 58.3kN/m, respectively.

As shown in Fig. 4, the PUR mixed crushed stones is a new elastic porous material that is casted using polyure-

Figure 3. Composite geomembrane.

Figure 4. PUR mixed crushed stones.

thane adhesive mixed crushed stones. Due to its higher bending strength, excellent resilience and permeability, it can be used as the cushion layer of CGM in the surface barrier of a high rockfill dam on a thick riverbed alluvial deposit [19]. The basic properties of the crushed stones are given in Table 1.

Table 1. Basic properties of the crushed stones for the tests.

Size range (mm)	d_{50} (mm)	C_u	C_c	ρ_d (g/cm^3)	Φ (°)
5-20	10	4.2	2.4	1.725	48

3.2 Test apparatus and procedure

A large-scale direct shear apparatus was used for the tests. The device comprises a 300-mm square-top box and a 300 mm × 350 mm lower box. The maximum shear displacement can reach 50 mm with no loss in the area of the shear plane. A rigid iron block was filled in the lower box. The tests were performed according to the procedure in ASTM D5321-08 [20]. The CGM specimen was glued to the rough top face of the lower block with adhesive. The left end of the CGM was fixed on the side of the lower box with a steel bar and screws. Then the 300 mm × 300 mm pre-casted PUR mixed crushed stones specimen was placed in the top box. The tests were carried out in dry conditions. And the bottom surface of the PUR mixed crushed stones keeps a good contact state with the top surface of the CGM. The tests were performed on each interface at constant normal stresses of 25, 50, 75 and 100kPa, respectively. The rate of shearing was kept at 1.0 mm/s. Each test was conducted until the shear displacement reached 20 mm.

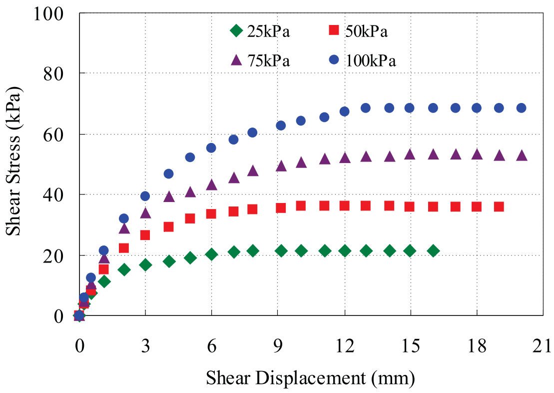

Figure 5. Shear stress vs. shear displacement of the CGM-PUR mixed crushed stones interface.

3.3 Test results

The shear behaviours of the CGM-PUR mixed crushed stones interface are shown in Fig. 5. The interface shear stress vs. shear displacement curves under different normal stresses show obviously nonlinear characteristics at the beginning of the shearing. When the shear stress reaches the peak strength, plastic failure occurs with an increase of the shear displacement. The CGM-PUR mixed crushed stones interface shows the failure mode of the elastic, perfectly plastic and sliding along the interface.

The peak shear stress versus normal stress for the CGM-PUR mixed crushed stones interface is shown in Fig. 6.

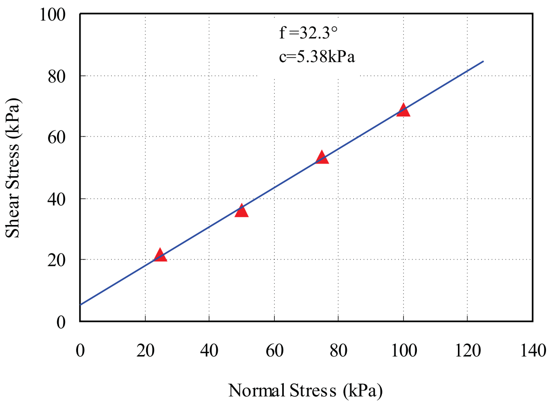

Figure 6. Shear stress vs. normal stress of the CGM-PUR mixed crushed stones interface.

It is clear that the peak shear stresses increase with the increasing of the normal stress. The shear strength of the interface can be expressed as a function of the normal stress using the Mohr-Coulomb criterion. The interface shear strength parameters φ and c obtained by fitting a straight line through the plots of the peak shear stress vs. the normal stress are 32.3° and 5.38kPa, respectively.

4 CONSTITUTIVE MODEL OF THE CGM-PUR MIXED CRUSHED STONES INTERFACE

4.1 Nonlinear elastic, perfectly plastic interface model

Based on the results of the direct shear tests, a nonlinear elastic, perfectly plastic interface constitutive model (Fig. 7) that combines the nonlinear hyperbolic model

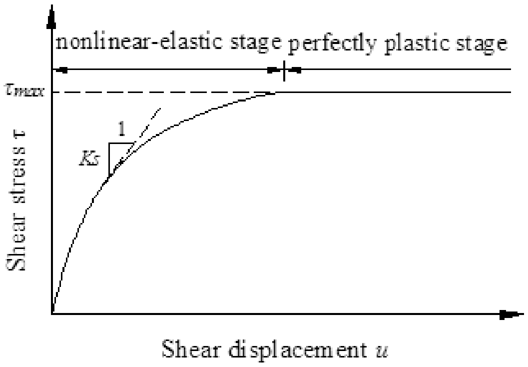

Figure 7. Nonlinear elastic, perfectly plastic model of the interface.

[21] with the Mohr-Coulomb plastic failure envelope can be used to describe the mechanical behaviour of the CGM-PUR mixed crushed stones interface. As shown in Fig.7, the complete shear stress, shear displacement response of the nonlinear elastic, perfectly plastic interface constitutive model can be divided into two sections: the pre-peak nonlinear elastic stage and the post-peak perfectly plastic failure stage.

4.1.1 Nonlinear-elastic stage

Before the shear stress reaches the peak strength, the relationship between the interface shear stress and the shear displacement can typically be modelled by a hyperbolic equation proposed by Kondner [22]:

$$\tau = \frac{u}{a+bu} \qquad (5)$$

The parameters a and b can be expressed as:

$$a = \frac{1}{k_1 \gamma_w \left(\frac{\sigma_n}{Pa}\right)^n} \qquad (6)$$

$$b = \frac{R_f}{\sigma_n \tan\phi + c} \qquad (7)$$

where γ_w is the unit weight of water, σ_n is the normal effective stress of the interface; Pa is the atmospheric pressure; τ is the shear stress; c is the cohesion of the interface; ϕ is the friction angle of the interface; k_1 , n and R_f are nonlinear parameters that can be derived from the interface direct shear tests.

The shear stiffness of the interface k_s can be represented by a tangent modulus of the shear stress vs. shear displacement curve[23]:

$$k_s = \frac{\partial \tau}{\partial u} \qquad (8)$$

By combining the four equations above, the shear stiffness of the interface k_s can be expressed as follows:

$$k_s = K_1 \gamma_w \left(\frac{\sigma_n}{Pa}\right)^n \left[1 - R_f \frac{\tau}{c + \sigma_n \tan\phi}\right]^2 \qquad (9)$$

4.1.2 Perfectly plastic failure stage

When the shear stress reaches the peak shear strength of the CGM-PUR mixed crushed stones interface, plastic failure occurs with a further increase of the shear displacement. The shear-strength envelope in the post-peak stage is the same as that of the original interface's shear-strength failure criterion (Eq. (4)) of the Geogrid element in FLAC^{3D}.

4.2 Parameters of the CGM-PUR mixed crushed stones interface model

The fitting curves of the interface shear stress vs. the shear displacement from the test results using the nonlinear elastic, perfectly plastic interface model are shown in Fig. 8. The fitting parameters for the interface model are given in Table 2. It is clear that the fitting curves using the model show good agreement with the test results under different normal loads.

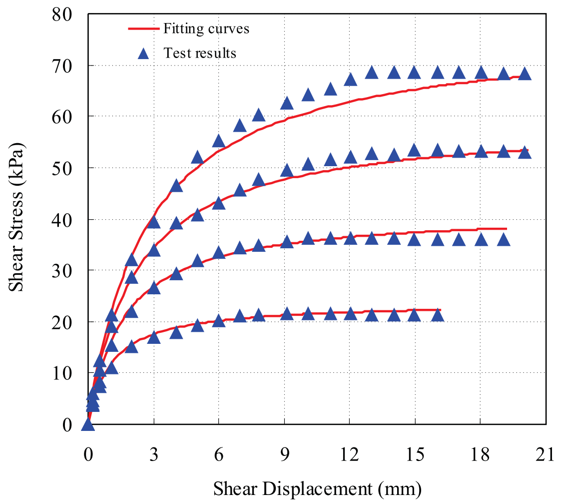

Figure 8. Fitted curves of shear stress vs. shear displacement for the CGM-PUR mixed crushed stones interface.

Table 2. Fitting parameters of the interface model.

K	n	R_f	c(Pa)	φ (°)
2871	0.185	0.893	5380	32.3

5 IMPLEMENTATION OF THE CGM-PUR MIXED CRUSHED STONES INTERFACE MODEL

In order to simulate the mechanical behaviours of the CGM-PUR mixed crushed stones interface, the nonlinear elastic, perfectly plastic interface model was incorporated into the Geogrid element of FLAC^{3D} by applying the user-defined FISH program.

By comparing the original interface constitutive model of the Geogrid element in FLAC^{3D} (Fig. 2) with the nonlinear elastic, perfectly plastic model for the CGM-PUR mixed crushed stones interface (Fig. 7). It is clear that the main difference is the pre-peak stage of the two models. So the implementation works mainly focus on replacing the linear-elastic stage (Eq. (3)) of the shear stress vs. displacement curve by the nonlinear-elastic stage (Eq. (8) and Eq. (9)) with the user-defined FISH program. The general method and detailed procedure for incorporating the interface model into the Geogrid element of FLAC^{3D} are described in this section.

5.1 Procedure for incorporating the program

A detailed program flowchart for the implementation of the CGM-PUR mixed crushed stones interface model is shown in Fig. 9.

Firstly, a loop-control function is used to define the total number of calculation steps at the beginning of the main program of FLAC^{3D}.

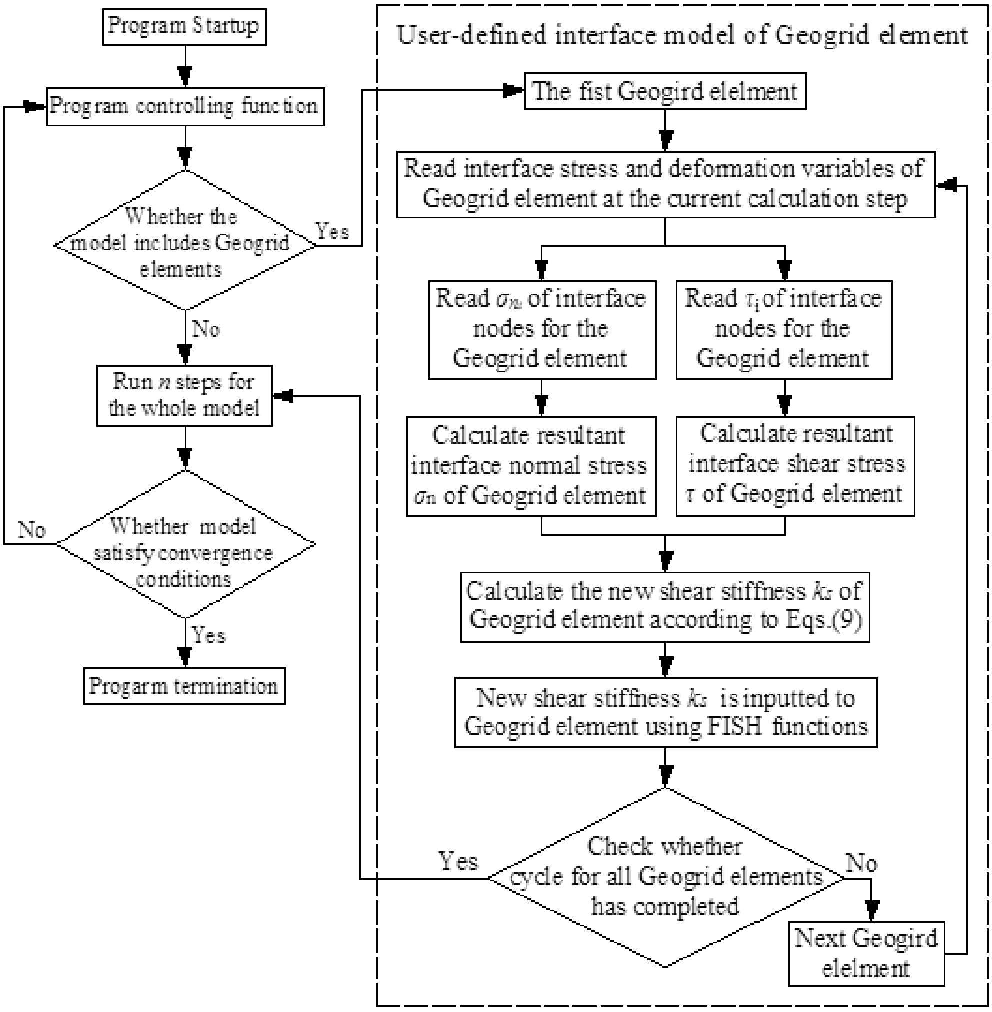

Figure 9. Program flowchart for the implementation of the nonlinear elastic, perfectly plastic interface model in the Geogrid element of FLAC^{3D}.

Secondly, a case statement of Fish is used to check whether the numerical model contains the Geogrid element. If there are Geogrid elements included in the model, the program module that defines the user-defined interface model will be called. Through the running of the user-defined interface model, the parameters and the theoretical formulae of the shear stress vs. shear displacement relationship of all the Geogrid elements will be updated. Then, the main program will be activated to run *n* steps. In the event that there is no Geogrid element that can be detected by the case statement, the main program will be directly activated to run *n* steps without calling the program module of the user-defined interface model.

Thirdly, the main program of FLAC^{3D} will check whether all the elements in the model satisfy the convergence conditions. If the convergence conditions are satisfied, the main program runs to completion. In the case that the convergence conditions are not satisfied, the main program will return to the loop-control function and a new cycle will be carried out by repeating the steps above until all the elements reach an equilibrium state.

The detailed program for the user-defined interface model of the Geogrid element (right part of Fig. 9) is described as follows.

At every calculation step, the program first reads the normal effective stress and the shear stress of the three interface nodes for every Geogrid element. The resultant interface normal stress and shear stress of the Geogrid element are calculated according to the node variables. Then the stiffness of the interface is calculated according to the resultant Geogrid element variables by Eq. (9). And the shear stiffness is inputted into the Geogrid element using Fish functions. Then the case statement of the Fish is used to check whether the cycle of all the Geogrid elements has completed. If it has been completed, the main program returns to the step of running *n* steps. In the other case the cycle will be carried out for the next Geogrid element. In this way, the user-defined interface model is continuously carried out until the cycle for all the Geogrid elements has completed.

5.2 Several points need attention

Several key points needed attention during the programming of the implementation of the CGM-PUR mixed crushed stones interface model and are presented here:

(1) When defining the total number of calculation steps, a large enough but appropriate integer should be set to ensure that all the elements can reach an equilibrium state within the number of steps.

(2) At every calculation step, it is should be checked to see whether the interface normal effective stress σ_n read from the last step of every Geogrid element is positive. At the beginning of every cycle, the stress and displacement of the elements are still in an unbalanced state and the normal effective stress read from the last step may be a negative number or zero. This may result in an error of the main program. In this case, a small positive number should be inputted as the initial value of σ_n.

(3) The interface shear stress and the shear displacement are not variables of the Geogrid element, but variables of the nodes of the Geogrid element. Since the parameters updating and inputting for the Geogrid element is based on the shear stress and shear displacement of the element, the nodal shear stress and shear displacement obtained from the last step must be transformed into element variables to calculate the new interface parameters.

(4) In the perfectly plastic stage of the interface model, some plastic sliding along the interface occurs. Then the interface elements begin to yield, and the shear stress will be corrected according to the original yield criterion (Eq. (4)) of FLAC^{3D}. So the updating of the shear stiffness and the shear strength parameters in plastic stage is not required in the program module of the user-defined interface model.

(5) When the parameters of all the Geogrid elements have been updated through the program module of the user-defined interface model, the main program will be activated to run *n* steps. An integer that is equal to or larger than *1* should be set to *n*. If *n* is too large, a shorter calculation time and a lower calculation accuracy of the model may be the result. So the users should adjust *n* according to the limitation of the calculation time and the requirement of the calculation accuracy.

6 VERIFICATION OF THE INCORPORATED INTERFACE MODEL

In order to verify the correctness of the incorporated interface model of the Geogrid element, a numerical example is used to model the interface direct shear tests between the CGM and the cushion material casted by the PUR mixed crushed stones. As shown in Fig. 10, the numerical model of the test is composed of two parts. The upper part is a shear box with a cushion material in it; the lower is a rigid block where the geomembrane is glued to the top surface. In order to keep a constant contacting area during shearing, the area of the lower box is larger than that of the upper. The incorporated nonlinear elastic, perfectly plastic interface model of the

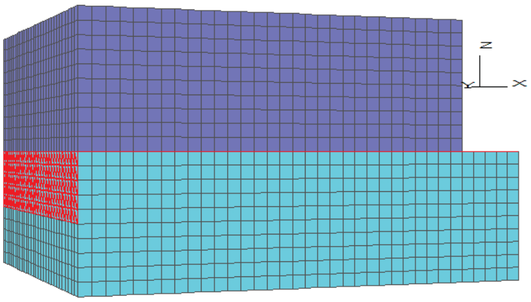

Figure 10. Numerical model of direct shear test for the CGM-PUR mixed crushed stones interface.

Geogrid element in last section is used to simulate the CGM-PUR mixed crushed stones interface behaviours.

In order to compare the numerical results with the results of the theoretical model and the test results, a linear elastic model was employed for the cushion material in the upper box as well as the rigid block in the lower box. Gravitational forces were not considered during the numerical experiments. The parameters in Table 2 resulting from the direct shear tests were used for the imbedded interface model. According to the typical procedures for the direct shear test, a constant normal pressure was applied on the top surface of the cushion material. Then, the displacements and velocities of all elements were reset to zero. A fixed shear velocity of 1×10^{-5} m/s was applied to all the elements of the lower part to simulate the actual shearing rate of 1 mm/min. This led to a displacement on the interface between the upper cushion and the lower CGM. Four numerical direct-shear tests were simulated with constant normal pressures of 25kPa, 50kPa, 75kPa, and 100kPa, respectively.

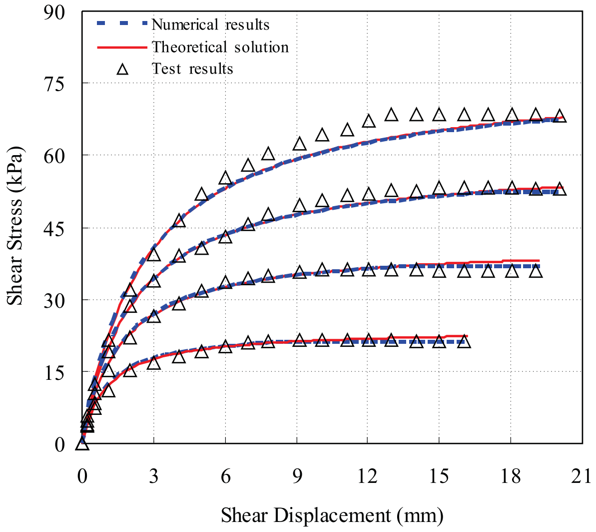

Figure 11. Relationship for shear stress vs. shear displacement of the Geogrid-Zone interface.

The numerical results of the average interface shear stress vs. the shear displacement curves and the comparison with the test results and the theoretical solution are shown in Fig. 11. It is obvious that the numerical results are very close to the theoretical solutions calculated by Eq. (5)-(7) using the same parameters. And both the numerical and theoretical solutions also show good agreements with the shear direct test results. Fig. 11 illustrates that the incorporated interface model in the Geogrid element of the FLAC^{3D} procedure is capable of modelling the nonlinear elastic, perfectly plastic behaviour of the interface between the CGM and the PUR mixed crushed stones.

7 CONCLUSIONS

(1) The Geogrid element in FLAC^{3D} can only model geosynthetic-soil interactions that accord with the linear elastic, perfectly plastic model. That restricts the application range of FLAC^{3D} in solving problems involving different geosynthetic-soil interfaces. The implementation of the direct shear testing results on the CGM-PUR mixed crushed stones interface was taken as an example to present the method of incorporating a new geosynthetic--soil interface constitutive model into the Geogrid elements of FLAC^{3D}.

(2) By fitting the direct shear test data, the nonlinear elastic, perfectly plastic interface model can be used to describe the mechanical behaviours of the CGM--PUR mixed crushed stones interface. The constitutive model of the CGM-PUR mixed crushed stones interface was incorporated into the Geogrid element of the FLAC^{3D} procedure by the user-defined FISH program. The method and program flowchart of the implementation of geosynthetic-soil interface model into the Geogrid element of FLAC^{3D} was presented in detail.

(3) A numerical simulation of the direct shear test of a CGM-PUR mixed crushed stones interface was performed to verify the correctness of the incorporated interface model. The numerical results represent good agreement with the theoretical solution and the test results. The improved Geogrid element FLAC^{3D} can be used for the nonlinear and plastic characteristics of the geosynthetic-soil interface behaviour.

(4) The method and basic procedures of the implementation of a geosynthetic-soil interface constitutive model into the Geogrid element in FLAC^{3D} described in this paper can offer a reference for the

incorporation of other geosynthetic-soil interface constitutive models into FLAC^{3D} using the FISH programming platform.

(5) It must be emphasized that the incorporated interface model in the Geogrid elements of FLAC^{3D} of this paper is mainly used to simulate the monotonical shear behaviours in the tangential direction of the geosynthetic-soil interface. The simulation of the cyclic shear and normal dilatant behaviours of the geosynthetic-soil interface may be a future study on the basis of this work.

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