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Taylor scale and effective magnetic Reynolds number determination from plasma sheet and solar wind magnetic field fluctuations

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[1]   Cluster data from many different intervals in the magnetospheric plasmas sheet and the solar wind are employed to determine the magnetic Taylor microscale from simultaneous multiple point measurements. For this study we define the Taylor scale as the square root of the ratio of the mean square magnetic field (or velocity) fluctuations to the mean square spatial derivatives of their fluctuations. The Taylor scale may be used, in the assumption of a classical Ohmic dissipation function, to estimate effective magnetic Reynolds numbers, as well as other properties of the small scale turbulence. Using solar wind magnetic field data, we have determined a Taylor scale value of 2400 ± 100 km, which is used to obtain an effective magnetic Reynolds number of about 260,000 ± 20,000, and in the plasma sheet we calculated a Taylor scale of 1900 ± 100 km, which allowed us to obtain effective magnetic Reynolds numbers in the range of about 7 to 110. The present determination makes use of a novel extrapolation technique to derive a statistically stable estimate from a range of small scale measurements. These results may be useful in magnetohydrodynamic modeling of the solar wind and the magnetosphere and may provide constraints on kinetic theories of dissipation in space plasmas.

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1.   Introduction

[2]   In the cascade picture of broad band turbulence, energy resides mainly at large scales, but is transferred across scales by nonlinear processes, eventually reaching small scales where dissipation mechanisms of kinetic origin limit the transfer, dissipate the fluid motions, and deposit heat. This general picture is expected in hydrodynamics and in fluid plasma models such as magnetohydrodynamics (MHD) when the associated Reynolds number and magnetic Reynolds number are large compared to unity. This signifies that the nonlinear couplings are very much stronger than the dissipation processes at the large scales and that, therefore, structures having a wide range of spatial scales will be involved in the dynamics. This broad-band character is found in observations of fluctuations of the magnetic field (and other quantities such as velocity and density) in the solar wind and in the plasma sheet. Many studies of turbulence in these systems [*Borovsky et al.*, 1997; *Tu and Marsch*, 1995; *Goldstein et al.*, 1995] analyze the cascade process through spectral analysis or through analysis of

structure functions at various orders. This emphasizes the self-similar range of scale properties that give rise to descriptions such as the famous power law of Kolmogorov theory [*Kolmogorov*, 1941], and its variants [*Kraichnan*, 1965]. The self-similar range is typically defined as extending from an energy-containing scale (correlation scale) down to a Kolmogorov dissipation scale, – thus the two most studied length scales in turbulence studies are simply those that define the long wavelength and short wavelength ends, respectively, of the power law inertial spectral range. However, provided that the spectral distribution of energy is suitably well behaved in both the inertial and dissipative range, the mean square spatial derivatives of the turbulent magnetic or velocity field will be well defined, and in particular will attain definite values relative to the mean energy density. This implies the existence of a third characteristic scale of turbulence, determined by the ratio of mean square fluctuations to a measure of the mean square spatial derivatives of the fluctuations. The corresponding length is the Taylor microscale. This scale has been almost completely ignored in space plasma turbulence prior to this time, mainly due to instrumental limitations. Here, using the accurate magnetic field data at relatively close separations that is afforded by the Cluster mission, we are able to develop a methodology for extracting accurate values of the Taylor scale. We apply this technique to derive measurement of the Taylor scale in both the solar wind and in the plasma sheet. This will permit, under assumption of a standard resistive or Ohmic dissipation function, an evalu-

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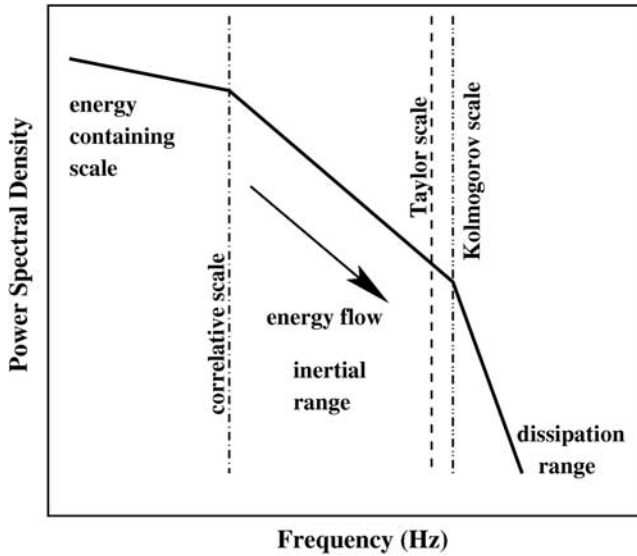
Figure 1. Schematic and power spectral density plot for turbulence solar wind. At the lowest frequencies is the energy containing scale. At the highest frequencies is the dissipation range. The correlative scale separates the energy containing scales and the inertial range and the Kolmogorov scale separates the inertial range and the dissipation range.

ation of an effective magnetic Reynolds number of the turbulence.

[3] Turbulence in the solar wind is thought to originate at least in part in the source regions of the solar wind, where it is the product of coronal dynamical processes. Solar wind turbulence may also be driven by stream interactions, including compressions and shears, which are almost certainly responsible for augmentation of turbulence seen at 1 AU and beyond, as well as the disappearance of Alfvénic correlations with increasing heliocentric distance [*Roberts et al.*, 1987], and the heating that underlies the observed highly nonadiabatic temperature profile that extends from inside 1 AU to beyond 60 AU as observed by Voyager and Pioneer [*Gazis et al.*, 1994; *Richardson et al.*, 1995; *Williams et al.*, 1995; *Smith et al.*, 2001].

[4] The magnetospheric plasma sheet also displays properties associated with turbulence. Electromagnetic energy is stored in the magnetotail lobes and transferred to the plasma sheet and the dominant mechanism for this transport is most likely reconnection. As a consequence, fluctuations in the flow that are produced in the plasma sheet have a very turbulent appearance [*Borovsky et al.*, 1997; *Weygand et al.*, 2005]. Understanding of these fluctuations is important for understanding the physics of this transport of lobe electromagnetic energy and mass to the plasma sheet. However, measurements to ascertain the role of plasma sheet turbulence in energy transport remain incomplete. Some of the key turbulent characteristics that have not yet been thoroughly studied are the scale sizes of the boundaries of the turbulent energy cascade in the inertial range from the driving scale (i.e., the energy containing scales) down to the energy dissipation scales. Assuming a mapping between wave vectors and frequencies, those boundaries are often evident in power spectral density plots as breaks in the

spectral index. A schematic example of a power spectrum with breaks is shown in Figure 1.

[5] Moving toward longer wavelengths beyond the typical inertial range of the power spectrum, one encounters the scales that typically contain most of the energy in a turbulent flow. In some cases the peak of the omnidirectional spectrum [*Batchelor*, 1970] is defined as the "energy containing scale." One can also compute using standard methods a correlation scale and this will frequently be of the same order as the energy-containing scale. Another similar, and in many cases almost equivalent, scale is that which demarcates the lower wave number limit of the inertial range – this can be thought of as a "bendover scale" of the spectrum. For purposes of the present work, we identify these variations of the turbulence outer scale as the correlation scale, which we physically associate with the size of the energetically dominant large scale turbulent eddies [*Weygand et al.*, 2006].

[6] At the opposite end of the inertial range (i.e., small scales) is the Kolmogorov or dissipation scale (see Figure 1). Dissipative processes in hydrodynamics or collisional MHD are important in describing the evolution of eddies at scales smaller than the Kolmogorov scale. The situation is less clear in low collisionality plasmas, where dispersion and dissipation may set in when the inertial range terminates, but also at the same scales new or different modes of plasma motion can appear. Here we will continue to denote the high wave number end of the inertial range as the (reciprocal) dissipation scale. A standard means of identifying the dissipation wave number is to associate it with the breakpoint at the high wave number end of the inertial range above which the spectral index of the power spectral density becomes steeper [*Denskat et al.*, 1983; *Goldstein et al.*, 1994; *Leamon et al.*, 1998].

[7] The magnetic Taylor microscale can be defined as

$$\lambda_T = \sqrt{\langle b^2 \rangle / \left\langle (\nabla \times \mathrm{b})^2 \right\rangle} \qquad (1)$$

where b is the fluctuation of the total magnetic field, which can be written as $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, for a suitably defined averaging procedure $\langle \ldots \rangle$ that defines the mean field $\mathbf{B}_0 = \langle \mathbf{B} \rangle$. It is clear from this definition that λ_T is a length associated with the mean square spatial derivatives of b. An analogous microscale for the turbulent velocity field can be defined and differs from similar definitions in the hydrodynamics turbulence literature by constant factors that are unimportant in the present context [see, e.g., *Batchelor*, 1970].

[8] An equivalent formulation of the Taylor scale in terms of the spectrum is

$$(1/\lambda_T)^2 = \int\limits_0^\infty d^3k \; k^2 E(\mathbf{k}) / \int\limits_0^\infty d^3k \, E(\mathbf{k}) \qquad (2)$$

where the denominator on the right hand side is the total fluctuation energy. This form makes the sensitivity of the Taylor scale to the steepness of the spectrum particularly evident, since $k^2E(k)$ must fall off faster than $1/k$ for the

integral to exist. The Taylor scale can also be defined in terms of the correlation function

$$R(r) = \langle \mathrm{b(x)} \bullet \mathrm{b(x}+r) \rangle \qquad (3)$$

which is the standard trace correlation function (i.e., the sum of the vector component correlations) for the zero mean fluctuation vector $\mathbf{b(x},t)$. In homogeneous turbulence R does not depend on the absolute position $\mathbf{x}$, but only on the separation vector $\mathbf{r}$. In general, for anisotropic turbulence (as expected in the solar wind) $R(\mathbf{r})$ depends on the vector separation $\mathbf{r}$ [*Matthaeus et al.*, 1990; *Milano et al.*, 2005; *Dasso et al.*, 1983], but in isotropic turbulence, or upon averaging over direction, it depends on the magnitude r. For the remainder of the present paper we will assume we are dealing with direction-averaged correlation functions, unless we specify otherwise. It is straightforward to show that the Taylor scale appears explicitly in the small $|\mathbf{r}|$ expansion of the correlation function, assuming isotropy or direction averaging, through the relation,

$$R(r) \approx \langle b^2 \rangle \left(1 - \frac{r^2}{2\lambda_T^2} \right) + \dots \qquad (4)$$

where... represents higher order terms in a power series in r. Consequently, in principle a direct evaluation of the correlation function for small separations r can be employed to determine the Taylor scale. Note that the second order structure function can be written in terms of the same asymptotic relation

$$S(r) = \left\langle |b(x) - b(x+r)|^2 \right\rangle \approx \langle b^2 \rangle (r^2/\lambda_T^2 + \dots) \qquad (5)$$

We can combine equations (4) and (5) to reconstruct the correlation function as $R(r) = \langle b^2 \rangle - S(r)/2$. Now we have a function that relates the magnetic field fluctuations across space with the Taylor scale. Below we will show how to implement the identification of the Taylor scale with multispacecraft data.

[9] Previously we presented preliminary results on computing the correlation scale and Taylor scale in solar wind turbulence using simultaneous two point measurements employing pairs of interplanetary spacecraft observations [*Matthaeus et al.*, 2005]. Here we focus on the methodology for determining the Taylor scale using a stable extrapolation technique and derive an interplanetary magnetic Taylor scale at 1 AU that is more accurately determined, yet is found to agree well with previous estimates *Matthaeus et al.* [2005]. We also apply the new methodology to determine the Taylor scale in plasma sheet turbulence. In both cases, interplanetary and plasma sheet, we employ the results of our Taylor scale estimate and correlation scale determined from previous studies to derive quantitative estimates of the effective magnetic Reynolds number

$$R_{eff} = \left(\frac{\lambda_{CS}}{\lambda_T} \right)^2 \qquad (6)$$

where λ_{CS} is the correlation scale and λ_T is the Taylor scale [*Batchelor*, 1970]. Finally, we compare the present results with previously published estimates based on single space-craft observations of the associated scales, and theoretical estimates of the Reynolds numbers based on collisional resistivity formulations.

2. Instrumentation

[10] For this study the magnetic field measurements taken within the plasma sheet and solar wind were obtained from the Cluster spacecraft. The Cluster mission, supported jointly by the European Space Agency (ESA) and National Aeronautics and Space Administration (NASA), consists of four identical spacecraft, optimally in a tetrahedral configuration, with a perigee of 4 R_E, an apogee of 19.6 R_E, and a spin period of about 4 s. These four spacecraft provide the first three-dimensional measurements of large- and small-scale phenomena in the near-Earth environment [*Escoubet et al.*, 1997]. Each Cluster spacecraft carries 11 instruments. This study uses data from the magnetometer (FGM) [*Balogh et al.*, 1997] and the ion spectrometer (CIS) [*Rème et al.*, 1997]. The Cluster spacecraft orbital plane precesses around the Earth annually. From 2001 to 2004, between July and October the Cluster spacecraft apogees were in the magnetotail and between January and April they were potentially in the solar wind. At apogee the spacecraft were located at the vertices of nearly regular tetrahedrons. In the magnetotail seasons of 2001 and 2004 the tetrahedron's scale was about 1000 km. During the 2002 season the scale was 5000 km (i.e., on the order of the inertial range for turbulence within the plasma sheet). The latter spacing is ideal for examining turbulent eddy scale sizes that are on the order of 5000 km [*Neagu et al.*, 2002; *Weygand et al.*, 2005]. From July to October 2003, Cluster obtained another series of plasma sheet crossings at an inter-spacecraft spacing of about 100 km (i.e., on the order of dissipation range).

[11] Each Cluster spacecraft carries a boom-mounted triaxial fluxgate magnetometer [*Balogh et al.*, 1997]. Magnetic field vectors routinely are available at 22 Hz resolution (nominal mode). Both pre-flight and in-flight calibrations of the two magnetometers have been performed to produce carefully calibrated (and inter-calibrated) magnetic field data. The relative uncertainty in the data after calibration is at most 0.1 nT, an estimate determined by examining the drift in the offset after calibration (K.K. Khurana and H. Schwarzl, private communication, 2004). The digital resolution of the magnetometer is on the order of 8 pT [*Balogh et al.*, 1997].

[12] The CIS instrument [*Rème et al.*, 1997] along with the magnetic field data are essential in identifying periods when Cluster enters the plasma sheet and when it is in the solar wind. CIS also provides fundamental plasma parameters such as density, velocity vectors, the pressure tensor, and heat flux. The uncertainties in most of these quantities are not significant for this study. To help identify the plasma sheet periods we need only to know when the density and ion temperature values significantly increase or decrease so that we can determine when Cluster enters and exits the plasma sheet. Although plasma data are available from only 2 or 3 spacecraft, this is not a problem for identifying the solar wind and plasma sheet regions because the spacecraft
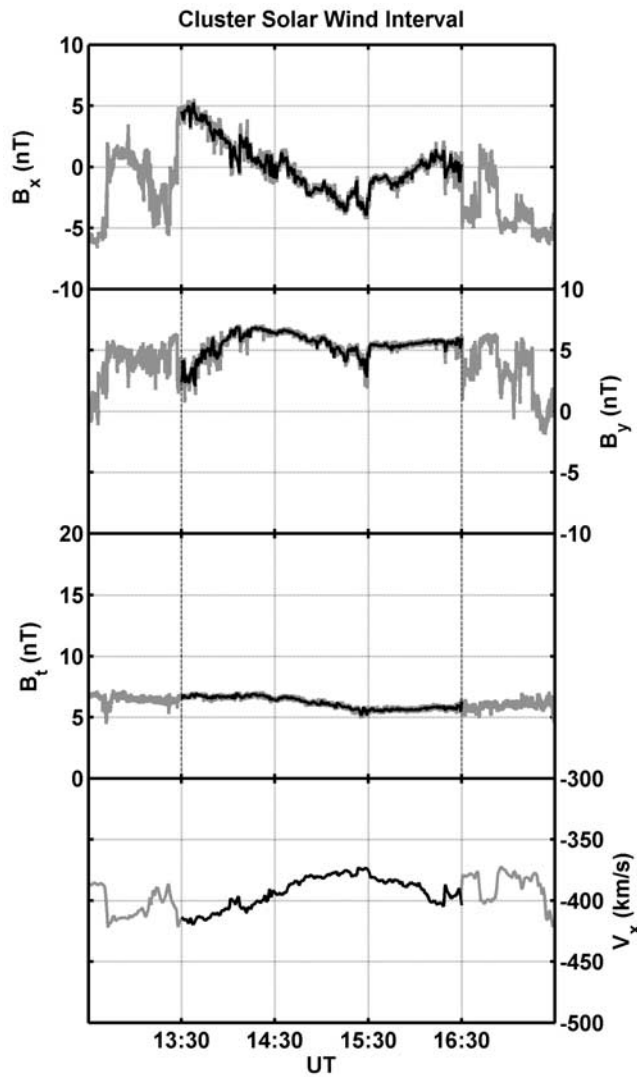
Figure 2. Cluster solar wind observations from 25 February 2003. The black portion of the curve displays the 30-s averaged magnetic field data used in the study. The gray portion of the curve displays 4 s resolution data. The gray data shown before and after the black curve did not meet the criteria for this study.

are close to one another and we conservatively estimate the boundaries of the region of interest.

3. Procedure and Observations

[13] The intervals used in this study are obtained from two distinct regions: the solar wind and the plasma sheet. For each plasma region, data selection criteria are specified. The solar wind data intervals are selected visually from plotted data and bow shock or magnetosheath data are excluded. The solar wind is identified from the magnetic field data, which had typical magnitudes of around 5 to 10 nT, the plasma density, which was of the order of several particles per cm^{-3}, and the solar wind speed, which was of the order of 400 km s^{-1}. We do not use solar wind data that shows sharp rotations in the IMF B_x and B_y components to avoid sector boundary crossings. We also avoid solar wind

shocks and we include intervals only if magnetic field data are continuous for more than an hour. Because the Cluster orbit remained in relatively close proximity to the magnetosphere, even when in the solar wind, we include some measurements in which foreshock waves were present in the solar wind. To reduce the contribution of these waves to our analysis, the solar wind magnetic field measurements are averaged to 30-s resolution, which is approximately the longest period for ion foreshock waves. As an additional check we have examined a subset of solar wind intervals when there are no foreshock waves or shocklets and we obtained very similar results. Figure 2 shows a typical example of Cluster solar wind observations, these being from 25 February 2003.

[14] Data selection for the plasma sheet is similarly restrictive. Entries and exits from the plasma sheet correspond to times when the ion temperature significantly increases or decreases, and within the plasma sheet we require the ion density be greater than 0.1 cm^{-3} and that the magnetic field B_x component have values between -10 and 10 nT. Intervals where the spacecraft appear to enter lobe regions due to magnetotail flapping or other phenomena are eliminated. A minimum of 1 hour's worth of 4 s average magnetic field data are needed. For the plasma sheet data analysis we remove a background magnetic field determined with a cubic fit to the entire data interval. This step is necessary because we are interested in the turbulent magnetic fluctuations and the large scale structure of the magnetotail field influences the cross correlation values. Figure 3 displays magnetic field data from a typical Cluster plasma sheet crossing on 9 September 2002. Not shown in Figure 3 are the velocity vectors. For this interval the speed rapidly fluctuates between 0430 and 0510 UT and reaches values as high as 300 km/s. After 0510 UT the flow speeds drop below 50 km/s and the fluctuation amplitude decreases. We combine many different plasma sheet intervals with a variety of plasma and flow conditions to calculate an average Taylor scale value. Our data set was not large enough to subdivide into statistically usable subsets in different ranges of flow speed.

3.1. Solar Wind Procedure

[15] For each selected data interval we calculate the time-averaged cross correlation of the vector magnetic field for each of the spacecraft pairs (six in total for each interval assuming all four spacecraft record measurements). We then compute the trace correlation, or sum of the vector component correlations. This correlation value is assigned a separation distance, which is the time average of the corresponding spacecraft separation distances for that interval. Each correlation estimate is normalized by the vector variance of the magnetic field fluctuation in the respective interval [*Matthaeus et al.*, 2005]. By collecting normalized correlation values from a large number of suitable solar wind intervals, we find estimates of the trace correlation function as it varies with spatial separation r.

[16] Figure 4 displays the correlation values versus the spacecraft separation in the solar wind. This figure demonstrates that, as the spacecraft separation increases, the correlation decreases, as expected in a turbulent plasma.

[17] In determining a value of the Taylor scale from the data, we must confront several issues: the asymptotic nature
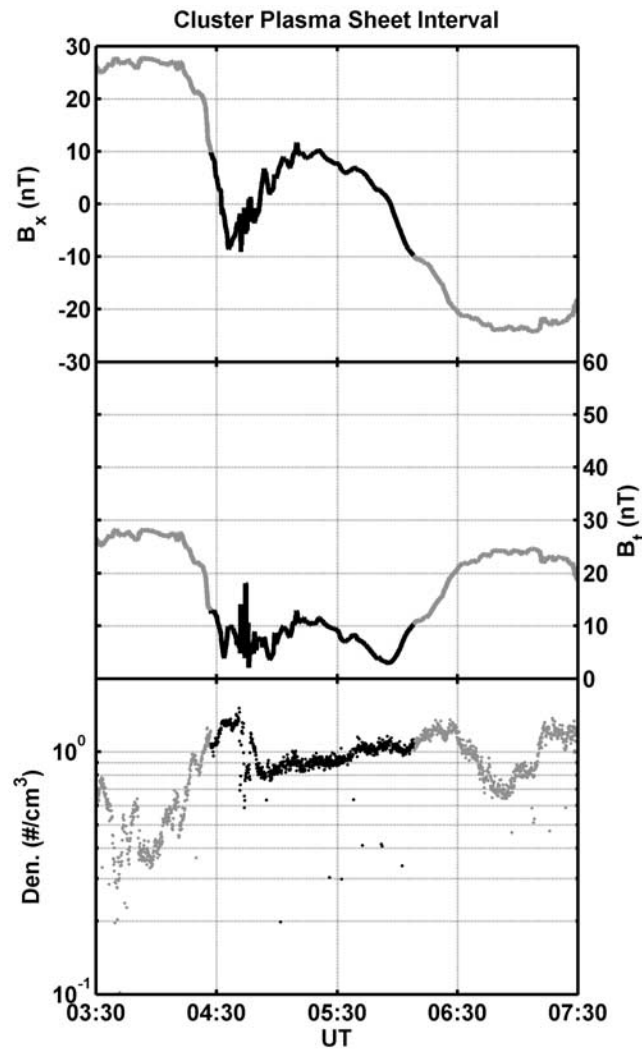
Figure 3. Cluster plasma sheet crossing on 9 September 2002. This figure has the same format as Figure 2.

of equation (4), limitations inherent in the analysis of data from closely separated spacecraft, and the possible departures of plasma structure at small scales from expectations based on viscous hydrodyamics and collisional gasdynamics. We discuss these issues in turn.

[18] First, equation (4) becomes a good approximation only for small values of separation. In fact, we expect that it will become accurate asymptotically only for very small $|\mathbf{r}|$, on the order of the dissipation scale, by which we mean the scale corresponding to the break point in the solar wind spectrum that separates the inertial range from the steeper kinetic-dominated range. The dissipation scale in the solar wind is generally near the ion inertial scale, approximately 500–1000 km at 1 AU [*Leamon et al.*, 1998]. Assuming that the behavior of the second order structure function, $S \sim r^2$, sets in at some scales smaller than the dissipation scale, it is clear that even with the wide range of separations available in the Cluster mission, only a small percentage of possible two spacecraft correlation estimates fall into the range where one might, even in principle, see the asymptotic r^2 dependence. The second order structure function for

this study examines the ensemble average of the square of the fluctuations in the magnetic field as a function of the spacecraft spatial separation. In order to gain statistical weight, and to make optimal use of available data, one would like a method that does not rely on fitting the data with a quadratic equation only in the asymptotic range of separations.

[19] Intervals with small spacecraft spatial separation occur infrequently in the Cluster data and, therefore, estimates obtained from them have low statistical weight. In addition, small separation implies small differences in measured fields. For well calibrated magnetometers on two spacecraft separated by a few hundred kilometers, the associated structure functions, according to our error estimates, remain above the likely noise threshold (which we estimate as that associated with 0.1 nT error). However, the margin is not great and we must allow for possible additional noise in the measured structure function in this regime of small separation.

[20] Finally, from hydrodynamics with a viscous dissipation function we expect that the the $S(r) \sim r^2$ regime will set in at scales smaller than the dissipation scale that marks the end of the inertial range. That is, in hydrodynamics the radius of curvature of the structure function at the origin is larger than the dissipation scale, but this behavior becomes evident only from measurements made at separations smaller than the dissipation scale. Furthermore, in a collisionless plasma, there may be scales that mark the onset of several different types of dissipation, such as resonant absorption, Landau damping, excitation of nonlinear electron effects, reconnection, lower hybrid turbulence, and so on. Consequently there is no a priori way to estimate when the asymptotic regime is entered. In particular, our ensemble of correlation estimates is likely to contain intervals of diverse nature. For example, anisotropic dissipation mechanisms are characteristic of space plasmas. Some affect fluctuations observed for spacecraft separations along the local mean magnetic field (resonance) whereas others can affect fluctuations observed at oblique angles (e.g., Landau damping). However, in the present ensemble, in order to maximize the number of correlation estimates, we have gathered all the intervals together without regard to mean field direction.

[21] For the above reasons, we assume that the very short separation regime sets in at a scale controlled by the spectral break near 1000 km that has been identified elsewhere [*Leamon et al.*, 1998]. By using extrapolation techniques (see below) we derive a stable answer based upon this assumption. In future work we will examine a larger ensemble of measurements in order to identify anisotropy effects and other plasma effects that will give insight into multiple scales relevant to the asymptotic behavior of the structure function at small distances.

[22] The implication of the above discussion is that we assume that the experimental determination of the value of the Taylor scale in a magnetized plasma confronts only those difficulties that arise in hydrodynamics. This is already a subtle issue. We seek a method that recognizes that the quadratic behavior sets in at some small $|r|$, which we can estimate but which is not known exactly. Our estimate for the transition scale is the dissipation scale at distances of the order of the ion gyroradius, so we focus on
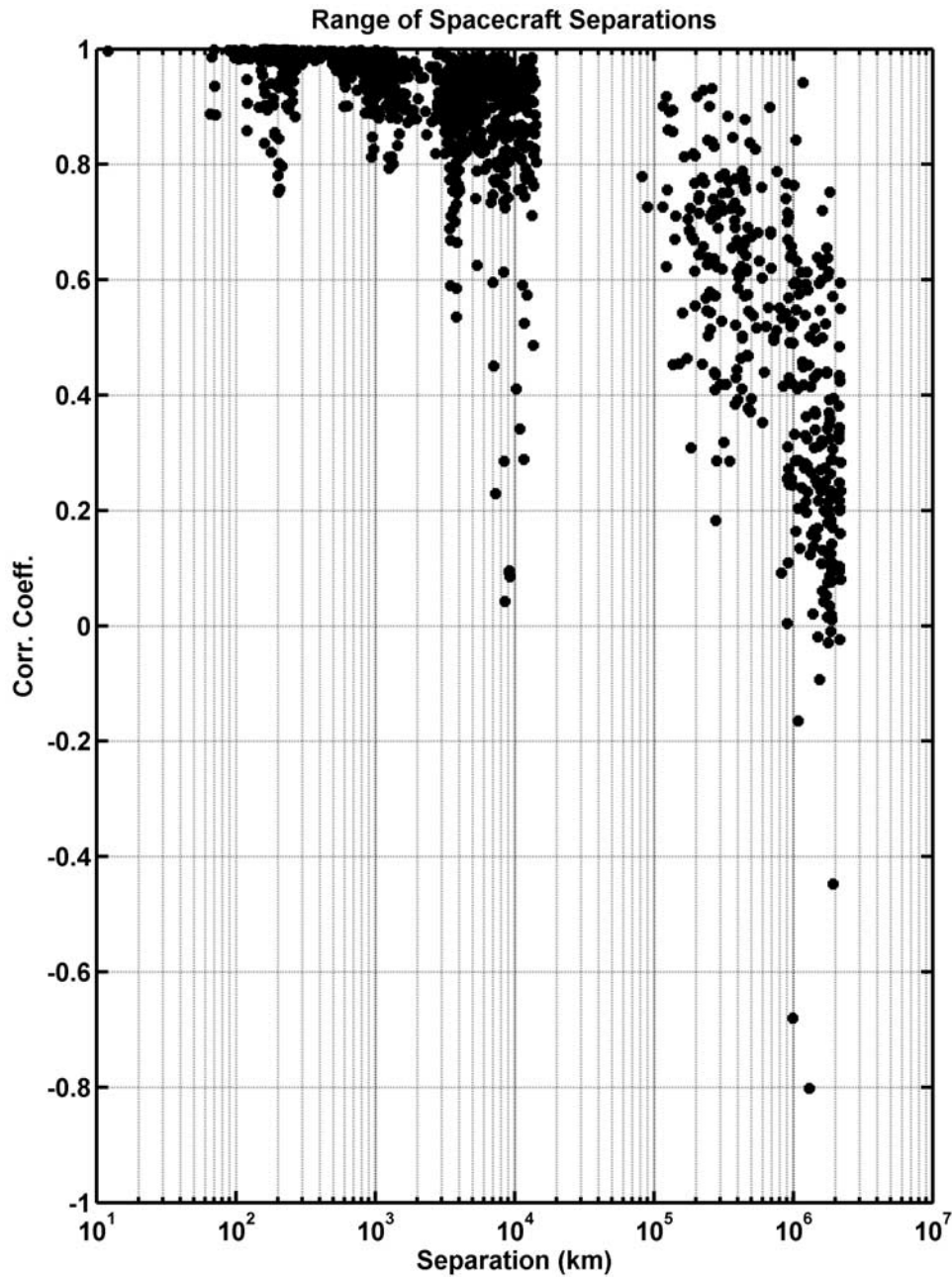
Figure 4. Solar wind correlation values versus Cluster spacecraft separation. This plot shows the decrease in the correlation coefficient for the magnetic field vectors with increasing spacecraft separation.

fits to the radius of curvature at approximately that range of separations. However, it has been found that the maximum separation distance in the data influences the inferred Taylor scale value. The smaller the maximum separation, the smaller the inferred Taylor scale in hydrodynamic studies [*Belmabrouk and Michard*, 1998; *Belmabrouk*, 2000].

[23] In order to overcome the problem of an inferred Taylor scale that changes with separation distance, we apply a method that we call Richardson extrapolation after similar methods in the numerical analysis literature [*Press et al.*, 1999]. This method consists of two parts, which will be discussed in more detail in the next two paragraphs. In the first part we determine an array of tentative estimates of the Taylor scale for a range of spacecraft separations; each of

these is a parabolic fit of the form of equation (4). In the second part we use the first array of Taylor scales to estimate a second set of improved Taylor scale values, which are obtained by extrapolation of the first set of fits to zero separation.

[24] To determine the first array of Taylor scale values we start with N values of the estimated correlation, each of which is associated with a nominal spacecraft separation. We order these from smallest to largest separations. Using the first three smallest spacecraft separations we fit those data points with equation (4) to get a single Taylor scale value. This is the first element of our array of estimates. The next Taylor scale value in the array is then calculated by adding the next larger spacecraft separation data point to the
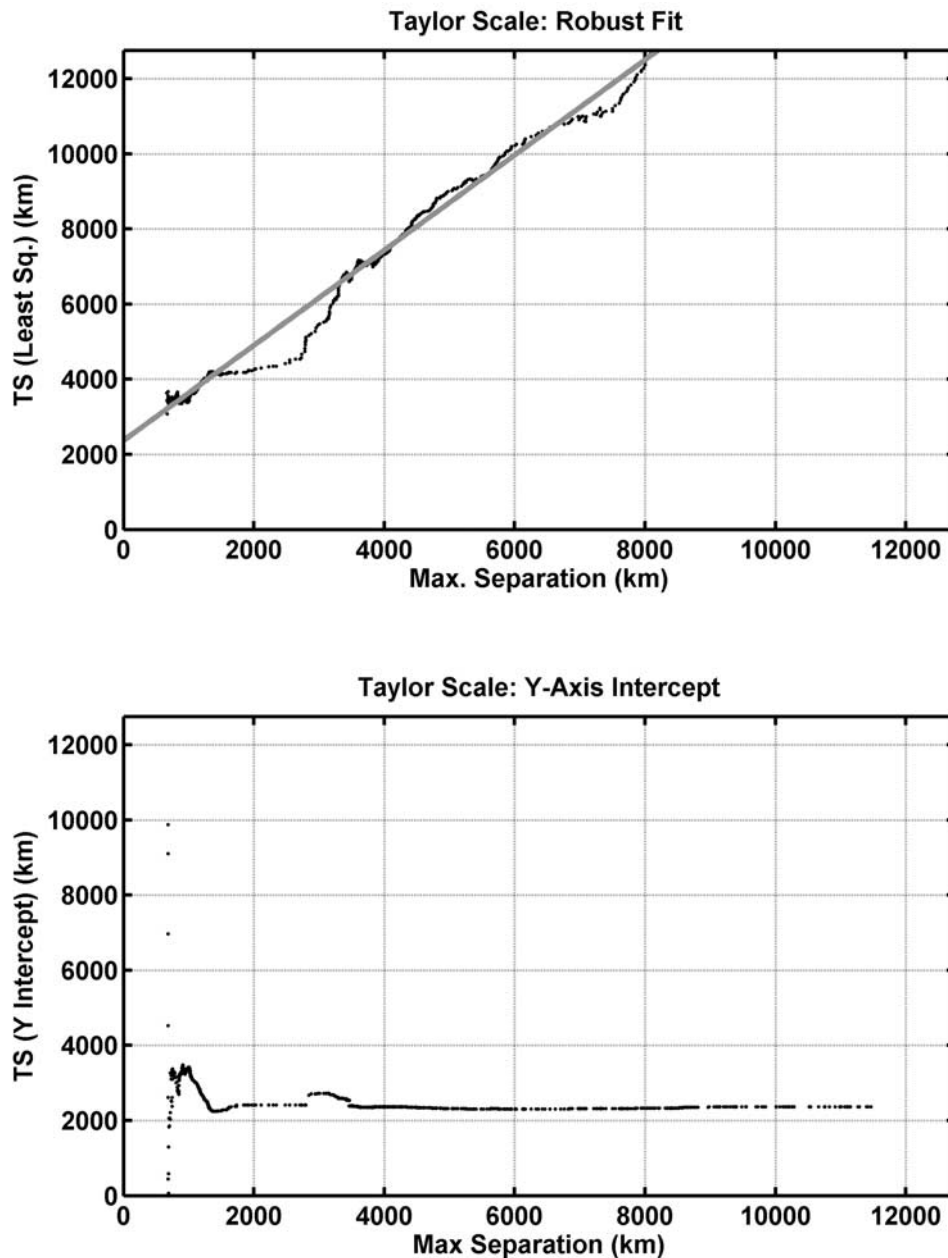
Figure 5. Solar wind Taylor scale determination. The top panel shows the steady increase in the value of the Taylor scale as larger and larger spacecraft separations are included in the robust fit. The bottom panel shows the Taylor scale values determined from a linear fits to the data points in the top panel.

fitting procedure. We continue this process until we have N-2 Taylor scale values calculated from N-2 fits to the correlation versus spacecraft separation. The top panel of Figure 5 is a plot of the N-2 Taylor scale values determined from a robust fit of equation (4) to an increasing number of points for each fit. The top panel of Figure 5 also shows that as we increase the number of points in the robust fit, the calculated value for the Taylor scale increases and no stable Taylor scale value can be found from this step alone. This is because additional points, while providing increased statistical weight, also move the fit away from the asymptotic regime of zero separation (y intercept).

[25] In the next step of the Richardson extrapolation we use the first array of Taylor scale values to extrapolate the estimates back to zero spacecraft separation, where the expansion in equation (3) takes on physical significance. To estimate the radius of curvature at zero spacecraft separation we extrapolate the value from a linear fit to data points in the top panel of Figure 5. However, just like the first stage of this extrapolation, it is unclear how many points to include in the linear fit. We will see, however, that the second array of Taylor scale values appears to plateau to a stable value.

[26] Specifically, in the second stage we begin by using a linear fit to the first two data points of the top panel of Figure 5 to determine a new Taylor scale value at the y intercept. The next estimate is obtained by adding the next larger spacecraft separation data point, carrying out a new linear fit, and again extrapolating back to zero separation.
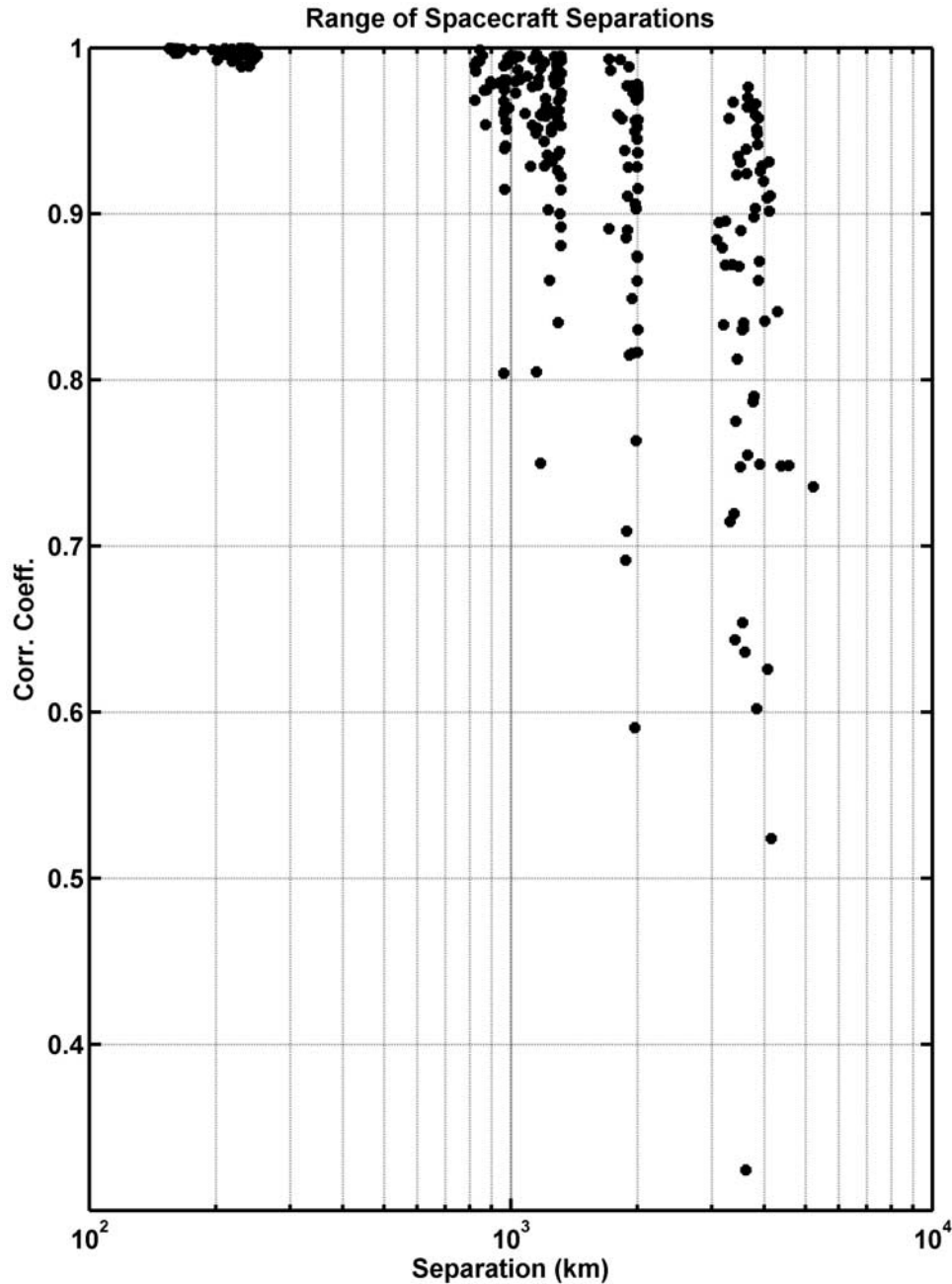
Figure 6. Plasma sheet correlation values versus Cluster spacecraft separation. This figure has the same format as Figure 5.

We continue this process until we have N-3 Taylor scale estimates in the second array. The bottom panel of Figure 5 is a plot of the *y* intercepts, i.e., Taylor scale values, obtained in this way, versus the maximum spacecraft separations used in the linear fit. This figure shows that as additional points are added to the linear fitting procedure, the estimated value for the Taylor scale attains a stable value of about 2400 ± 100 km for spacecraft separations between 2000 and 15,000 km, where the uncertainty is the standard deviation.

3.2. Plasma Sheet Observations

[27] In this section we apply the procedure outlined in section 3.1 to the plasma sheet data. Figure 6 displays the correlation versus spacecraft separation for the Cluster observations in the plasma sheet. The format of this plot is the same as Figure 4 and the same relative decrease in the correlation values with increasing separation is seen. Next we apply the Richardson extrapolation method and obtain the two plots shown in Figure 7. The distribution of the points in Figures 7a and 7b is similar to Figures 5a and 5b and the stable value of the Taylor scale is found to be 1900 ± 100 km.

4. Analysis

[28] Previous studies have determined the correlation scales in the solar wind [*Matthaeus et al.*, 2005] and the
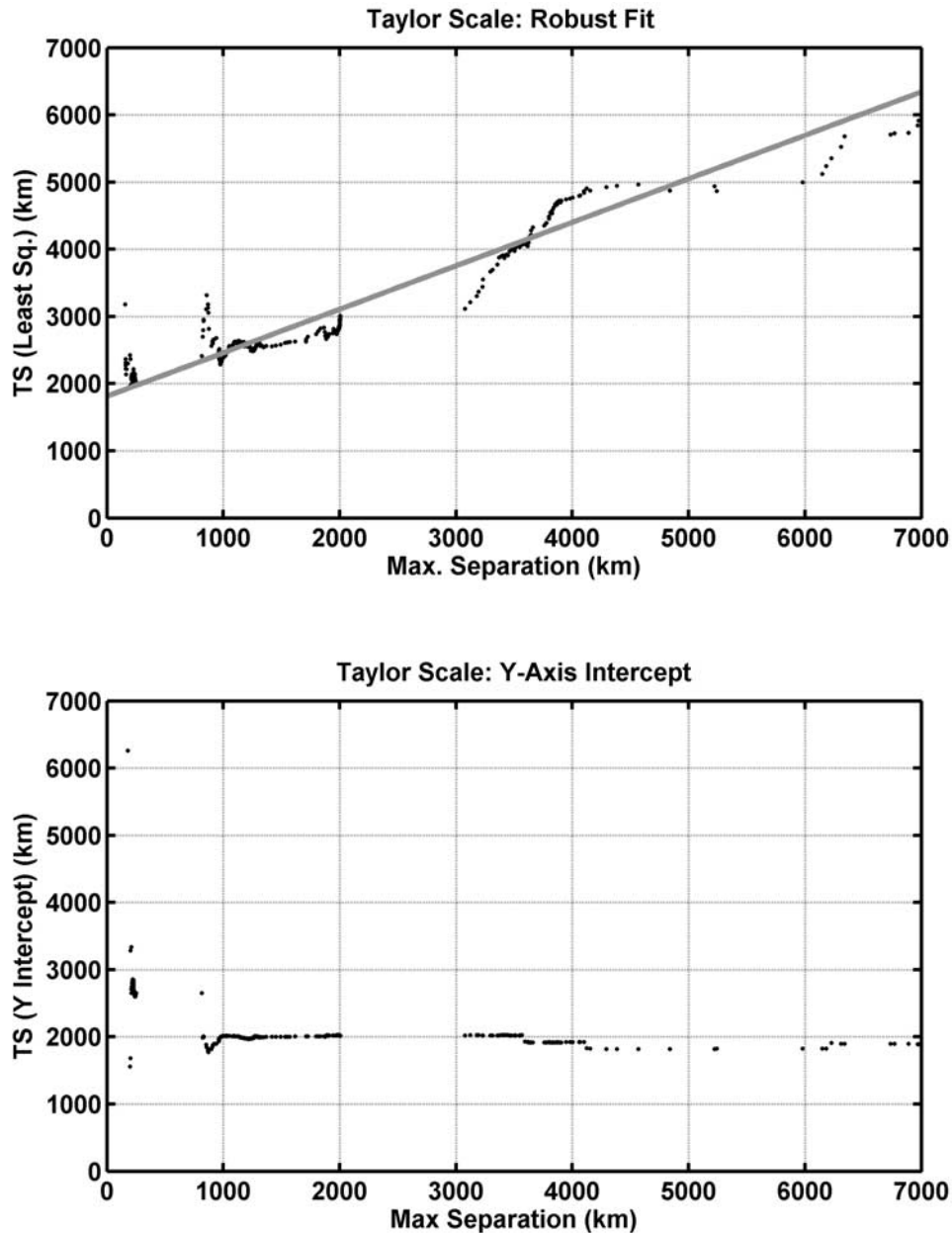
Figure 7. Plasma sheet Taylor scale determination. These panels have the same format as Figure 4.

plasma sheet [*Borovsky et al.*, 1997; *Neagu et al.*, 2002; *Weygand et al.*, 2005]. With the Taylor scale values for the solar wind and the plasma sheet and those previously determined the correlation scales, we can determine the effective magnetic Reynolds number.

4.1. Solar Wind Effective Magnetic Reynolds Number

[29] *Matthaeus et al.* [2005] calculated a correlation scale value of about $1.2 \cdot 10^6$ km for the solar wind near 1 AU from an exponential fit to the correlations determined from ACE-Wind pairs and Cluster spacecraft pairs versus spacecraft separations. With our Taylor scale value and the *Matthaeus et al.* [2005] correlation scale, equation (6) gives a Reynolds number of $260,000 \pm 20,000$. The uncertainty is taken from the uncertainty in establishing the Taylor scale.

4.2. Plasma Sheet Effective Magnetic Reynolds Number

[30] Recent studies have determined correlation scale values within the plasma sheet from magnetic field data. Values as low as 3800 km have been reported by *Neagu et al.* [2002], a value of about 10,000 km was given in *Borovsky et al.* [1997], and a range of values from about 5000 to 20,000 km was determined by *Weygand et al.* [2005]. Using the highest and the lowest of these, we calculate effective magnetic Reynolds numbers between 7 ± 1 and 111 ± 12.

5. Discussion

[31] As far as we are aware two other studies have estimated the value of the Taylor scale in the solar wind.

Matthaeus and Goldstein [1982] used relatively coarse time resolution Voyager 1 and 2 magnetic field data to arrive at a Taylor scale value of about $3.2 \cdot 10^5$ km at 1 AU. This was a single spacecraft determination employing the frozen-in flow approximation to convert time lags to spatial lags. This value is most likely incorrect because the time resolution of the data was most likely insufficient. More recently, *Matthaeus et al.* [2005] employed basically the same technique as that used here to study the correlation versus spacecraft separation. However, in that earlier report, we used Cluster data only from April, 2003 and mid-January to early February 2004 and used a simple fit to extract the Taylor scale. The present determination improves on this value by using additional Cluster data from the solar wind seasons in the years 2001 to 2005, providing a larger range of spacecraft separations. The present analysis uses 1200 2-spacecraft data intervals with spacecraft separations ranging from about 100 km to just over 10,000 km. This range covers scales starting at distances smaller than the thermal ion gyroradius (about 100 km), which is generally thought of as the dissipation scale boundary, and extends to many times the ion gyroradius, which is well within the inertial region. The greatest improvement in the present study, however, is methodological, in that we use Richardson extrapolation to obtain a stable Taylor scale value. *Matthaeus et al.* [1990] fit equation (4) directly to the available data to obtain a Taylor scale value of about 2500 ± 700 km, a value that agrees with the present estimate of 2400 ± 100 km within the uncertainty of the estimates.

[32] With the Taylor scale determined from the solar wind and the correlation scale given from *Matthaeus et al.* [1990] we calculate an effective magnetic Reynolds number of $260,000 \pm 20,000$ using equation (6), which is again similar to the value of 230,000 given in *Matthaeus et al.* [1990]. As far as we are aware these are the only published estimates of the effective magnetic Reynolds number of the solar wind.

[33] Using the Richardson extrapolation technique for the plasma sheet, we have found the Taylor scale to be 1900 ± 100 km and the Reynolds number to be 7 to 110. We cannot compare our results to earlier studies because these quantities have not previously been determined. However, we believe that the values we have obtained make physical sense. In hydrodynamic fluid turbulence it is believed that the Taylor scale is several times larger than the dissipation scale. In the plasma sheet, dissipation is thought to occur on the scale of the ion gyroradius. The ion gyroradius in the center of the plasma sheet is approximately 400 km for mean ion energies of about 5 keV [*Borovsky et al.*, 1997], which is about 5 times smaller than our inferred Taylor scale.

[34] These values that we have inferred for the effective magnetic Reynolds number of the plasma sheet are considerably smaller than the magnetic Reynolds number of about 10^{13} reported by *Borovsky et al.* [1997] and *Borovsky and Funsten* [2003], and the value of 1600 calculated by *Vörös et al.* [2006] from plasma sheet flow data. However, a direct comparison of these different Reynolds numbers is not meaningful. The magnetic Reynolds numbers calculated by *Borovsky et al.* [1997] and *Borovsky and Funsten* [2003] were determined by measuring the root mean square flow speed and the correlation scale and using the Spitzer conductivity. Both papers assume that the plasma sheet turbulence is fully developed, but *Weygand et al.* [2005,

2006], *Vörös et al.* [2006], and *Lui* [1998, 2001] suggest that the plasma sheet turbulence at the inertial scale and at the kinetic scale is most likely not fully developed. Similarly the flow Reynolds number given in *Vörös et al.* [2006] cannot be directly compared with the effective magnetic Reynolds number of this study because *Vörös et al.* [2004] limited their data set to "bursty bulk flows" with plasma flow speeds of the order of about 300 km s^{-1}.

[35] The value that we find for the effective magnetic Reynolds number suggests that the plasma sheet plasma is fairly viscous and does not readily response to driving. Although it is possible that we have not determined the true effective magnetic Reynolds number, rough calculations suggest that our value is reasonable. We believe that correlation scales as large as about 20,000 km [*Weygand et al.*, 2005] are reasonable because the correlation scale represents the size of the largest eddies that can develop in the plasma sheet. The smallest dimension of the plasma sheet is in the z direction. According to *Thompson et al.* [2005] the thickness of the plasma sheet can be as large as 50,000 km under extremely quiet conditions, but is typically around 25,000 km. Thus a correlation scale of the order of tens of thousands of km is reasonable. The measured Taylor scale is several times larger than the dissipation scale, which is on the order of the ion gyroradius, about 400 km in the central part of the plasma sheet [*Borovsky et al.*, 1997]. If we were to use about twice the ion gyroradius as the Taylor scale and 20,000 km as the correlation scale, then with equation (6) we would obtain an effective magnetic Reynolds number of about 600, which differs by less than an order of magnitude from our maximum value of 110.

[36] In order to obtain the plasma sheet Taylor scale and Reynolds number, we have combined many plasma sheet intervals. These intervals include both low and high speed flows, which range from typically tens of km/s to hundreds of km/s. A distribution demonstrating the full range of possible plasma sheet flows can be found in *Borovsky et al.* [1997]. We have assumed that combining all these intervals will produce an average Taylor scale and Reynolds number in the plasma sheet. *Vörös et al.* [2004, 2005] and *Weygand et al.* [2005] have shown that the Taylor scale, the correlative scale, the dissipative scale, and the Reynolds number are velocity dependent. Continuing acquisition of plasma sheet data will allow us in the future to subdivide the data into different flow regimes in order to examine the velocity dependence of these characteristic quantities.

[37] As a final means of verifying our values for the Taylor scale and effective magnetic Reynolds number we can determine the Kolmogorov dissipation scale value from our measurements and compare the result to the expected value for the Kolmogorov scale, which we expect to be on the order of the ion gyroradius. From *Batchelor* [1970], the Kolmogorov scale is calculated from $\lambda_{KS} = \lambda_T (R_{\text{eff}})^{-1/4}$. Using the Taylor scales and effective magnetic Reynolds numbers we obtained from the solar wind and the plasma sheet we obtained a Kolmogorov scale of about 100 km and 700 km, respectively. These values are within a factor of 2 of the ion gyroradius values given for the solar wind in *Kivelson and Russell* [1995], which is 80 km, and for the plasma sheet in *Borovsky et al.* [1997], which is 400 km.

[38] The effective magnetic Reynolds number is an important parameter in computational MHD models. We

compared our range of effective magnetic Reynolds numbers with the Reynolds numbers obtained from coupled magnetosphere-ionosphere, three-dimensional global MHD models of the terrestrial magnetosphere and its interaction with the solar wind [*Raeder et al.*, 1995, 1998, 2001; *El-Alaoui*, 2001]. Part of this simulation code is an ionospheric model that includes three sources of ionospheric conductance for closure of field-aligned currents. The model solves the ideal MHD equations for the magnetosphere and a potential equation for the ionosphere. Numerical effects, such as diffusion, viscosity, and resistivity are necessarily introduced by the numerical methods, which are discussed in detail by *Raeder et al.* [1995, 1998, 2001] and *El-Alaoui* [2001]. The MHD simulation has previously been used successfully to model the magnetotail's particle sources [*Ashour-Abdalla et al.*, 1997, 2000], substorm dynamics, localized reconnection [*Ashour-Abdalla et al.*, 1999, 2002], and the global dynamics of magnetic storms [*Berchem et al.*, 2001]. From their MHD model, Reynolds numbers are calculated from the equation $R = \mu_o v L \sigma$ where v is the plasma flow speed, L is the correlation scale length, and σ is the conductivity. In the plasma sheet the Reynolds numbers vary from about 50 to 1000 [El-Alaoui, Private communication, 2007] and depend on the value used for the correlation scale, flow speed, and conductivity. The lower end of the range of values is closest to our effective magnetic Reynolds numbers. The higher values obtained from their model were commonly obtained in high speed flow regions.

6. Summary and Conclusions

[39] For this study we calculated Taylor scale values of 2400 ± 100 km for the solar wind and 1900 ± 100 km for the plasma sheet from spatial correlation coefficients versus spacecraft separations derived from magnetic field data. As far as we are aware, this is the first study to use two point spacecraft measurement to obtain Taylor scale values in the plasma sheet. The value we obtained for the solar wind is similar to that reported in previous work, although the methodology is improved in the present study, and the Taylor scale value is within the expected range of plausible values. With the Taylor scale values and previously determined correlation scale lengths, we calculated effective magnetic Reynolds numbers. Our effective magnetic Reynolds number for the solar wind (260,000 ± 20,000) is close to the effective magnetic Reynolds number from *Matthaeus et al.* [2005]. The range of effective magnetic Reynolds numbers between 7 ± 1 and 110 ± 12 obtained for the plasma sheet is considerably smaller than the high speed flow Reynolds number calculated by *Vörös et al.* [2006], but our value is determined from magnetic field measurements rather than from flow measurements in high speed flow events. We believe that the small effective magnetic Reynolds number obtained for the plasma sheet suggests that the plasma is viscous and does not readily respond to driving. When compared to a widely used MHD model [*Raeder et al.*, 1995, 1998, 2001; *El-Alaoui*, 2001] our values were similar to those derived for the model plasma sheet.

[40] We close with a reminder that our methodology for determining the Taylor microscale recognizes the well known difficulties encountered in the analogous determination in hydrodynamics. We wish to determine an asymptotic behavior of the structure function at small separations, but any statistically significant sampling of our data covers a finite and nonzero range of separation distances. We address this difficulty with an extrapolation method that looks at the trend in the fits as the separation distance shrinks, for a sequence of fits that employs varying amounts of data. Using this approach, we do identify a stable range of estimated Taylor scale values and it is these values that we report. However, the medium in this case is a plasma, with the likelihood of distinctive dissipative and dispersive effects setting in at scales smaller than the ion inertial scale. Therefore as we move past the end of the power law inertial range into the smaller scales to establish the Taylor scale value associated with mean square spatial derivatives of the magnetic field, we need to recognize that there may be a much greater range of possible behavior in this plasma "dissipation range" than there is in hydrodynamics with a simple viscosity. Future work needs to focus on this, and to understand the distinctive dynamical plasma effects, including anisotropy and variation of other plasma parameters, that are influencing extremely small scale correlations. Further studies of this type will be crucial in understanding dissipation and transport at the small scales in the solar wind, plasma sheet, and other space plasmas.

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