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EULERIAN DECORRELATION OF FLUCTUATIONS IN THE INTERPLANETARY MAGNETIC FIELD

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ABSTRACT

A method is devised for estimating the two-time correlation function and the associated Eulerian decorrelation timescale in turbulence. With the assumptions of a single decorrelation time and a frozen-in flow approximation for the single-point analysis, the method compares two-point correlation measurements with single-point correlation measurements at the corresponding spatial lag. This method is applied to interplanetary magnetic field measurements from the *Advanced Composition Explorer* and *Wind* spacecraft. An average Eulerian decorrelation time of 2.9 hr is found. This measures the total rate of distortion of turbulent fluid elements—including sweeping, nonlinear distortion, and wave propagation.

Key words: interplanetary medium – magnetic fields – solar wind – turbulence

Spatial and temporal structure are fundamental in turbulence (Monin & Yaglom 1971, 1975), but are rarely both accessible experimentally in laboratory or terrestrial observations (Hinze 1975; Christensen & Adrian 2002; Gervais et al. 2007; Pal et al. 1998). In space and astrophysical observations, the prospects for measuring both are even more challenging. A fixed position probe in a zero net momentum turbulent medium measures the time decorrelation but cannot easily deduce the wavenumber spectrum. In a rapidly sweeping medium, the same fixed probe estimates the spatial structure through the (Taylor 1938) frozen-in flow hypothesis, but loses direct access to the zero-mean momentum frame (i.e., Eulerian) temporal decorrelation. For the sweeping case, the same problem exists even with multiple probes. Here, we show how, using multiple probes, a simple, but still approximate, extension of the Taylor hypothesis permits determination of the Eulerian decorrelation time. The method is applied to magnetic field data in the solar wind.

Studies of turbulence frequently use wavenumber spectra, or two-point, single-time correlation functions as measures of the distribution of energy in spatial structures of varying size (Batchelor 1970; Monin & Yaglom 1971, 1975; Tu & Marsch 1995). Equally fundamental is the single-point Eulerian correlation (Tennekes 1975; Chen & Kraichnan 1989), which quantifies temporal structure. In cases such as the solar wind (Jokipii 1973; Montgomery et al. 1980) and laboratory wind tunnels (Monin & Yaglom 1971, 1975), fluctuations are swept rapidly past the detectors. Hence, to a good approximation, correlation functions can be obtained using a single detector (single spacecraft) by assuming the frozen-in flow property. However, unless the detector resides in the plasma frame, the Eulerian correlation is not directly accessible. Here, by employing multispacecraft correlation methods, we estimate the single-point, two-time Eulerian correlation function using solar wind magnetic field data and parameterizing the measurement using an exponential form. We find an Eulerian decorrelation timescale of approximately 2.9 hr for the solar wind near 1 AU. The temporal decorrelation of magnetic field fluctuations in interplanetary space is important in fundamental turbulence theory (e.g., Kraichnan 1965). It is also relevant to particle scattering (Schlickeiser & Achatz 1993; Bieber et al. 1994; Shalchi et al. 2006) and predictability for space weather (Ridley 2000; Weimer et al. 2003).

The quantity of interest is the two-time single-point correlation function of a zero-mean fluctuating vector field $\mathbf{b}(\mathbf{x}, t)$; that is,

$$F(\tau) = \langle \mathbf{b}(\mathbf{x}, t) \cdot \mathbf{b}(\mathbf{x}, t + \tau)\rangle \quad (1)$$

$$= \int d\omega \hat{F}(\omega) \exp i\omega\tau. \quad (2)$$

We assume a statistical description that is homogeneous in space and stationary in time.[4] It is reasonable to assume that in most circumstances of interest the magnetic fluctuations become uncorrelated ($F \to 0$) at widely separated times ($\tau \to \infty$). Here, we discuss how to estimate the timescale of this decay from measurements in a moving frame. This method may also be useful in plasma and fluid experiments.

To place the Eulerian function in a broader context, we note that the general second-order two-point, two-time correlation function and its full spacetime Fourier decomposition may be written as

$$R_{\alpha\beta}(\mathbf{r}, \tau) \equiv \langle b_\alpha(\mathbf{x}, t) b_\beta(\mathbf{x} + \mathbf{r}, t + \tau)\rangle$$
$$= \int d^3k \ d\omega \ S_{\alpha\beta}(\mathbf{k}, \omega) e^{i\mathbf{k}\cdot\mathbf{r}} e^{i\omega\tau}. \quad (3)$$

Upon setting $\tau = 0$ we obtain $R_{\alpha\beta}(\mathbf{r}, 0)$, the two-point single-time correlation function. In the usual way, we transform as $S_{\alpha\beta}(\mathbf{k}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} d^3r \ R_{\alpha\beta}(\mathbf{r}) \exp(-i\mathbf{k} \cdot \mathbf{r})$ to obtain spectral tensor $S_{\alpha\beta}(\mathbf{k}) = \int_{-\infty}^{+\infty} d\omega S_{\alpha\beta}(\mathbf{k}, \omega)$. For three-dimensional isotropic Kolmogorov turbulence, the omnidirectional energy spectrum is just $\mathcal{E}(k) = 4\pi k^2 S_{\alpha\alpha}(\mathbf{k}) \sim k^{-5/3}$ (trace implied). Abbreviating $S(\mathbf{k}) \equiv S_{\alpha\alpha}(\mathbf{k})$, we may factorize the time dependence as

$$F(\tau) = \int d^3k S(\mathbf{k}) \Gamma(\mathbf{k}, \tau). \quad (4)$$

The quantity $\Gamma(\mathbf{k}, \tau)$ fixes the rate of decorrelation in time. Specification of Γ for every wave vector $\mathbf{k}$ fully determines

[4] For weak stationarity and homogeneity, correlation functions, denoted by an ensemble average $\langle \cdots \rangle$, are independent of the origin of the space coordinate $\mathbf{x}$ and of the time t. For a stationary signal, one may equivalently discuss the Fourier transform of F, the Eulerian frequency (ω) spectrum, $\hat{F}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\tau F(\tau) \exp -i\omega\tau$.

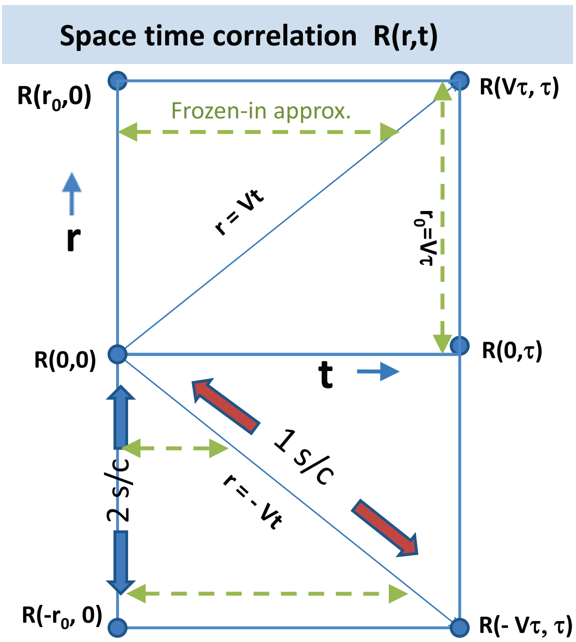

Figure 1. The correlation function $R(r, t)$ depends upon spatial separation r and time separation t. Depicted here is the (r, t) plane in the plasma frame. The ordinary two point correlation function is computed along the vertical axis. The Eulerian two time correlation function is measured along the horizontal axis. For a particular spacecraft separation r_0, the solar wind at speed V travels this distance in time τ, as illustrated. The frozen-in assumption approximates the two-point correlation by values along the diagonal lines, which are spacecraft trajectories with speeds $\pm V$.

the Eulerian function. One sees immediately that $\Gamma(\mathbf{k}, 0) = 1$, while $\Gamma \to 0$ for every $\mathbf{k}$ as $\tau \to \infty$. The dimensionless time correlation Γ appears prominently in closure theories such as the direct interaction approximation (e.g., McComb 1990).

Recall now the G. I. Taylor frozen-in flow approximation (for the interplanetary context see, e.g., Jokipii 1973), which is valid when the fluctuating fields convect past a single detector in a time short compared to the characteristic timescale for their distortion. This allows a time-lagged correlation to be interpreted as a spatial correlation, or specifically, in the zero momentum (unprimed) frame of reference,

$$R(\mathbf{r} = \mathbf{V}\tau, 0) \approx R(\mathbf{r} = \mathbf{V}\tau, \tau). \tag{5}$$

The frozen-in approximation and its relationship to the general spacetime second-order correlations are illustrated in Figure 1, shown in the zero momentum frame of reference. A detector sweeping through the turbulence with speed V samples along the characteristic $r = V\tau$. The plasma and detector (primed) frames are related by $\mathbf{r}' = \mathbf{r} + \mathbf{V}\tau$. Transforming the frozen-in property, Equation (5), into the frame of the detector (primed frame), the same approximation takes the familiar form $R'(\mathbf{r}' = -\mathbf{V}\tau, \tau = 0) \approx R'(0, \tau)$.

It is evident that the frozen-in property is equivalent to the approximation $\Gamma(\mathbf{k}, t) \equiv 1$ for all relevant values of $\mathbf{k}$. If the fluctuations do not distort in transit over the separation distance, then their constituent Fourier amplitudes do not distort. Clearly, $\Gamma(\mathbf{k}, \tau) = 1$ is not tenable for all τ as this is inconsistent with $F \to 0$ as $\tau \to \infty$. However, it is a useful first approximation when the flow is fast enough that the transit times for the structures of interest are shorter than all relevant dynamical timescales.

To refine this picture and enable an estimation of Eulerian time correlations from single detectors in a rapid flow such as

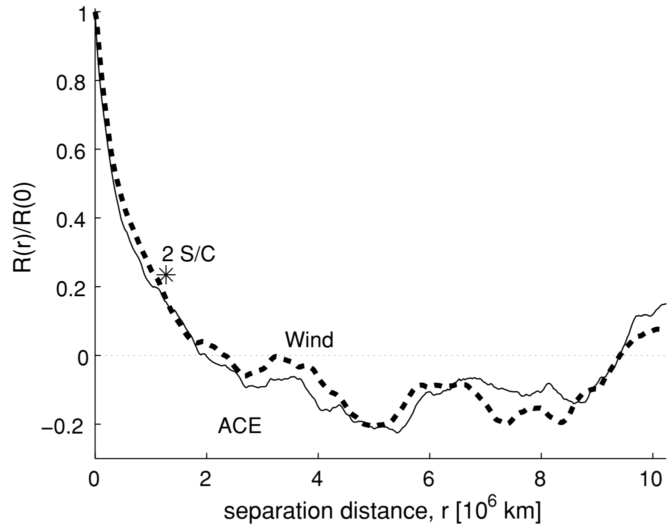

Figure 2. Magnetic correlation functions vs. distance for one 24 hr interval of *ACE* and *Wind* magnetic field data in the solar wind near Earth orbit (1999 October 4, 00–24 UT). Each spacecraft provides a correlation estimate using frozen-in flow (solid and dotted lines). *Wind–ACE* cross correlation provides a single correlation estimate at the spacecraft separation (asterisk). To the extent that these estimates agree, the Taylor hypothesis is exact. Analysis of the small differences in many such observations is used here to estimate the average Eulerian decorrelation rate.

the solar wind, we adopt here a next level improvement beyond the Taylor approximation. Specifically, we make the simplifying assumption that $\Gamma(\mathbf{k}, \tau) = \Gamma(\tau)$ independent of $\mathbf{k}$. Furthermore, we adopt the functional form

$$\Gamma(\tau) = e^{-\tau/\tau_c} \tag{6}$$

with a single Eulerian decorrelation timescale τ_c to be determined by the procedure. Although not a fundamental relation (see, e.g., Chen & Kraichnan 1989; Zhou et al. 2004; Shalchi et al. 2006; Shalchi 2008), the above ansatz leads to a convenient separability of the space and time dependence, and an improvement over the frozen-in flow approximation. In particular, with this simple choice, the single-time two-point correlation and single-point two-time correlation are related by

$$R(r, \tau) = R(r, 0)\Gamma(\tau) = R(r, 0)e^{-\tau/\tau_c}. \tag{7}$$

Note that $F(\tau) = \langle|\mathbf{b}|^2\rangle e^{-\tau/\tau_c}$. This separable form adapts readily to estimation of the decorrelation timescale by combining data from multi-point probes at single times, and from single-point probes at different times, using the assumption of frozen-in flow.

We apply the concepts developed here to data from the solar wind, employing a collection of interplanetary vector magnetic field data sets obtained from the *Wind* and *Advanced Composition Explorer* (*ACE*) spacecraft. The analysis includes 2716 intervals of solar wind observations (from 1998 February to 2008 March) each of which is of 24 hr duration. Interspacecraft separation distances range from 10^5 km (8×10^{-4} AU) to 3×10^6 km (2×10^{-2} AU). The basic methodology is similar to that employed in previous studies (Milano et al. 2004; Dasso et al. 2005; Matthaeus et al. 2005; Weygand et al. 2007). First, we find spacecraft pairs with interspacecraft separations in an appropriate range, generally falling into the expected inertial range of interplanetary turbulence (see Figure 2). The single spacecraft data are used to estimate the two-point single-time

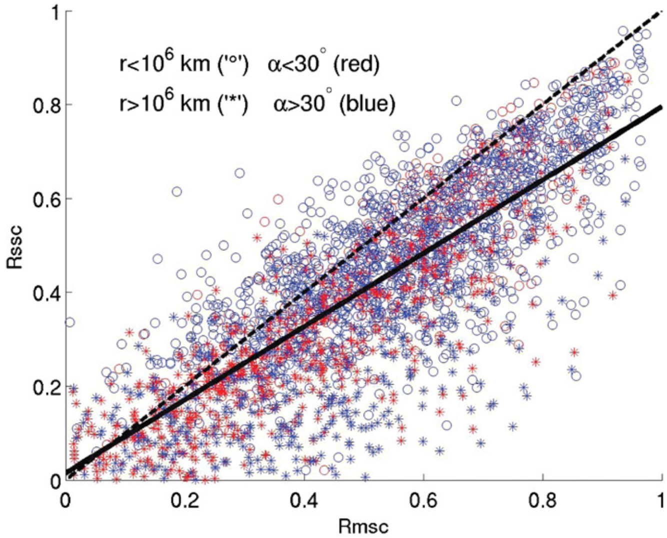

Figure 3. Scatter plot of single-spacecraft correlation (R_{ssc}) vs. two-spacecraft correlation (R_{msc}), at a common spatial lag, where 2584 cases from *ACE* and *Wind* data are used. The line $R_{\mathrm{ssc}} = R_{\mathrm{msc}}$ (dashed) and a linear least-squares line (solid) are shown for reference. Data are sorted according to angle α subtended by spacecraft separation and radial direction.

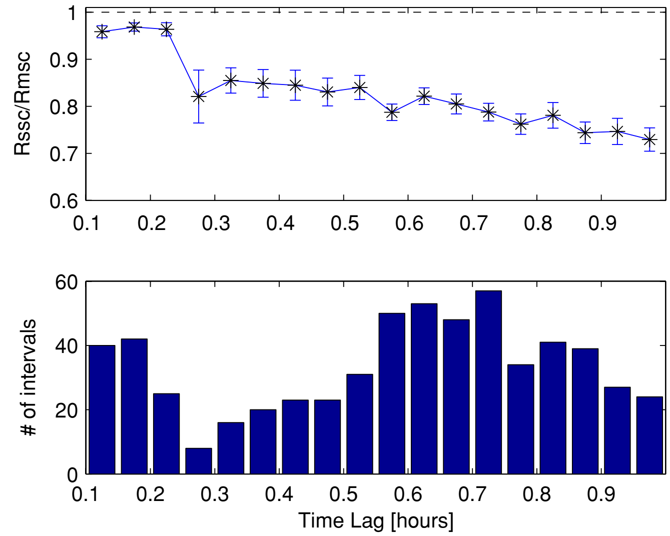

Figure 4. Top: estimated normalized Eulerian correlation $F(\tau)/\langle|\mathbf{b}|^2\rangle$ from ratio of single-spacecraft correlation estimates to two-spacecraft correlation estimates binned by time lags. Symbols are bin averages. Bottom: the number of intervals in each bin. For all intervals spacecraft are within 30° of radial alignment.

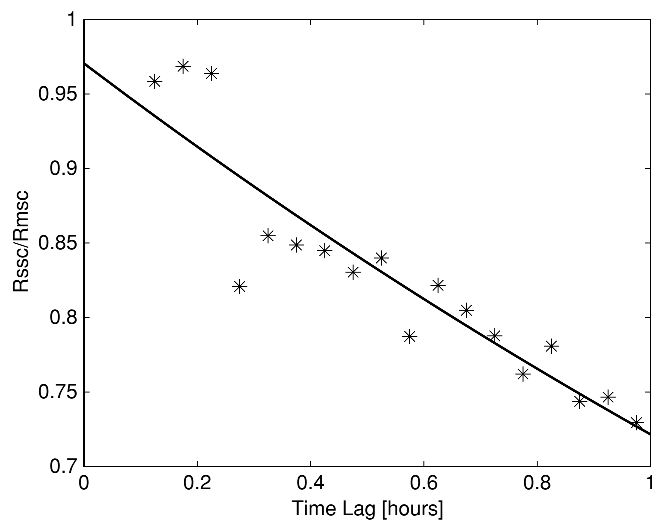

Figure 5. Estimated normalized Eulerian correlation $F(\tau)/\langle|\mathbf{b}|^2\rangle$. An exponential fit is shown with a decorrelation time of 2.9 hr.

correlation using the frozen-in flow approximation. The correlation estimate at the spacecraft separation is also computed directly from the same pair of data sets, using time averaging to obtain the required means, fluctuations, and covariances (Matthaeus et al. 2005). The latter is a true two-point single-time correlation estimate to the extent that spacecraft motion is negligible. In Figure 2, for the particular case shown, the single-spacecraft estimate corresponds to a slightly lower value than the two-spacecraft estimate. Therefore, the Taylor approximation is not quite exact. Our present hypothesis is that the difference between the frozen-in and two-spacecraft correlation values is caused mainly by distortion of eddies during the transit time between the two spacecraft.

After computing single-spacecraft and two-spacecraft correlations for 2584 intervals, we find a considerable statistical spread in the results; see Figure 3. There is a tendency for lower correlation values in the single-spacecraft measurements, as a least-squares fit to the data confirms. The ratios of the single-spacecraft correlation R_{ssc} to the two-spacecraft correlation R_{msc} in each interval were accumulated in binned ranges of (time) separation. Further data refinement included removing outliers, defined to be those intervals where $R_{\mathrm{ssc}}/R_{\mathrm{msc}}$ was outside of the range defined by the mean ± the standard deviation in each bin. We also selected intervals where the interspacecraft separation is closely aligned with the solar wind flow direction. It is under this condition that useful comparisons can be made between R_{msc} and R_{ssc} (see Equation (7)), respectively, a direct estimate of $R(r, 0)$ in the plasma and spacecraft frames, and an estimate of $R(r = Vt, t)$ using frozen-in flow. Since the flow velocity is always close to radial, in practice this condition was approximated by restricting the data set to spacecraft separations subtending an angle relative to the radial direction of $\alpha < 30°$. The restriction to small α also reduces the effects of anisotropy (Dasso et al. 2005). After removing intervals with large α, 601 data intervals remained. From the arguments leading to Equation (7), the ratio of R_{ssc} to R_{msc} can be used to estimate the Eulerian decorrelation time. Figure 4 shows the average of these ratios, the error of the mean, and the number of intervals in each bin. The ratio decreases with increasing spacecraft separation as expected.

The best fit to all the data samples is shown in Figure 5. For this collection of data sets, the associated Eulerian decorrelation time is τ_c =2.9 hr. The rate $1/\tau_c$ is the total rate of decorrelation in a frame moving with the plasma, due to all effects—turbulence, wave propagation, and any possible dissipation or randomization by external forces. The effect of the latter two processes is expected to be small. Dissipation effects are likely negligible since the separations examined are in the inertial range. External plasma driving is probably negligible. One possible source of decorrelation near 1 AU would be wave particle driving due to nonthermal proton beams originating at the terrestrial bow shock (Tsurutani & Rodriguez 1981). However, we screened the data in advance to eliminate intervals strongly perturbed by upstream waves, so we expect the ensemble of events we examined is either undriven or weakly driven by such effects. Another source of solar wind turbulence is long wavelength driving by large-scale velocity shear. Shear is thought to replenish the solar wind MHD scale cascade (Roberts et al. 1992), producing the Kolmogorov picture of driven dissipative turbulence over several decades of wavenumber in the inertial range. The timescale of shear driving is approximately the shear thickness divided by the velocity jump across the shear. This time is roughly the observed passage time of an interaction region ($\sim$10 hr) multiplied by the mean solar wind speed $\sim$400 km s^{-1} and divided by the change in velocity across the shear ($\sim$200 km s^{-1}). Therefore, the shear timescale is $\sim$20 hr, which is considerably longer than the observed decorrelation time. We conclude that the measured temporal decorrelation is not associated with driving but with intrinsic turbulence dynamics. Therefore, $\tau_c = 2.9$ hr is an estimated value of the turbulence Eulerian decorrelation time and, to the best of our knowledge, the first such estimate. We defer a formal error analysis at this time, but we expect that this estimate is good to within an order of magnitude.

The Eulerian timescale enters turbulence theory in several ways. It is associated with predictability, and it estimates the time beyond which the Taylor approximation becomes increasingly invalid. Eddies at the energy-containing scale are processed by the turbulence within this time. Typical fluctuations of inertial range dimension have still faster timescales.

In the simplest analysis, τ_c can be associated with the large-scale eddy turnover time $\tau_{\rm eddy} = \lambda_c/Z$. Here, the correlation scale is $\lambda_c = 0.008\,\mathrm{AU} = 1.2 \times 10^6$ km, computed from multispacecraft measurement (Matthaeus et al. 2005), and $Z \sim 30$ km s^{-1} is the turbulence amplitude around 1 AU. Thus, $\tau_{\rm eddy} \sim 4 \times 10^4$ s or about 10 hr near 1 AU. This is not inconsistent with the measured 2.9 hr decorrelation time, taking into account the potentially substantial contribution (see Equations (2) and (4)) from the small-scale fluctuations, which decorrelate more rapidly than larger scale fluctuations. Through our simplifying ansatz $\Gamma = e^{-\tau/\tau_c}$ we neglect this faster decorrelation. For example, in the inertial range, the local eddy time $\tau_{\rm nl}(k)$ usually estimated as $\tau_{\rm nl}(k) \sim \tau_{\rm eddy}/(k\lambda_c)^{2/3}$, so that $\tau_{\rm nl}(k) \ll \tau_{\rm eddy}$ when $k\lambda_c \gg 1$. In general, the Eulerian correlation function may incorporate various wavenumber-dependent effects (Zhou et al. 2004; Shalchi et al. 2006) including sweeping of the small-scale fluctuations by the large-scale eddies (Tennekes 1975; Chen & Kraichnan 1989, 1997), nonlinear distortion (Sanada & Shanmugasundaram 1991; Bieber et al. 1994), and wave propagation (Schlickeiser & Achatz 1993). Some possible explicit forms of the Eulerian function are given in Shalchi (2008).

One should not confuse the above estimates with the superficially similar timescale $\tau_0 = \lambda_c/V$ (where V is the mean solar wind speed ≈ 400 km s^{-1}). This timescale, $\tau_0 \approx 3 \times 10^3$ s, is the convection time of the correlation scale past the spacecraft by the bulk solar wind and has no fundamental relationship to the Eulerian timescale.

To suggest the utility of the Eulerian timescale, we provide examples of its connection to other quantities of interest in solar wind turbulence. One such example is the coefficient of self diffusion $\mu = \langle X^2(t)\rangle/t \approx 2\langle u^2\rangle\tau_L$. The Lagrangian correlation time τ_L, computed following random displacements $X(t)$ of fluid elements (Monin & Yaglom 1971, 1975; here we have introduced the turbulence energy/mass u^2), is difficult to measure in the solar wind, as it essentially requires the equivalent of tracer particles. However, it is estimated that the Lagrangian and Eulerian decorrelation times might be equal within a factor of about two (e.g., Hesthaven et al. 1995). Using our measured value of τ_c, we find that $\mu \approx 2\times(20\,\mathrm{km\,s^{-1}})^2\times 3\times 10^3\,\mathrm{s} \approx 2.4\times 10^{12}\,\mathrm{cm^2\,s^{-1}}$ for the solar wind. This elementary diffusion coefficient is a simple estimate of kinematic viscosity or turbulent resistivity (in conditions of near equipartition of flow and magnetic energy) and is a first approximation to the β-coefficient that appears in mean field electrodynamics and dynamo theory (Krause & Rädler 1980). We can also formulate a new estimate of the Reynolds number of solar wind turbulence at 1 AU through the association $R \approx u\lambda_c/\mu$, which accordingly yields $R \approx 120{,}000$. This compares favorably with the estimate $R \approx 260{,}000$ (Matthaeus et al. 2005; Weygand et al. 2007) based only on measured length scales. Somewhat larger values were reported by Weygand et al. (2009).

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