

Galois and universal universal coverings of linear categories and fibre products

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Abstract

Let k be a commutative ring. We study the behaviour of coverings of k -categories through fibre products and find a criterion for a covering to be Galois or universal.

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1 Introduction

In this paper we study Galois coverings of k -categories and we look for conditions that ensure the existence of a universal covering. With this purpose in mind we consider the fibre product of two Galois coverings. Differences with the usual algebraic topology setting appear, since the fibre product of coverings of k -categories does not provide in general a covering through the projection functor. Nevertheless, the fibre product of a Galois covering with a fully faithful functor is a Galois covering.

The theory of Galois coverings is deeply related to the fundamental group and gradings of k -linear categories. In [6] we introduce the intrinsic fundamental group $\pi_1(\mathcal{B})$ attached to a k -linear category \mathcal{B} as the automorphism group of the fibre functor which sends any Galois covering F to its fibre $F^{-1}(b)$ over a chosen object b . This fundamental group takes into account the linear structure of the category \mathcal{B} . It is intrinsic in the sense that it is not attached to a presentation of the category by generators and relations. However, the fundamental groups previously constructed by R. Martínez-Villa and J.A. de la Peña in [11] and by K. Bongartz and P. Gabriel [3, 8] associated to a presentation of the category by a quiver and relations and depending on it [1, 4] are quotients of the intrinsic π_1 we introduce. In case a universal covering $U : \mathcal{U} \rightarrow \mathcal{B}$ exists, the fundamental group that we have defined is isomorphic to the automorphism group $\text{Aut}(U)$.

The concept of Galois covering is also related to gradings and to the smash product construction. In [7] we make an explicit comparison between Galois coverings and smash coverings of a k -category \mathcal{B} . More precisely, we provide an

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equivalence between the category of Galois coverings of \mathcal{B} and its full subcategory whose objects are the smash product coverings. We consider the fundamental group that has been defined in [6] using Galois coverings and show that we can restrict to smash coverings when computing the fundamental group $\pi_1(\mathcal{B})$. From [5] we get that each connected grading of \mathcal{B} provides a Galois covering through the smash product, and hence the fundamental group is derived from all the groups grading the linear category in a connected way. This allows the computation of the fundamental groups of matrix algebras, triangular algebras, group algebras and diagonal algebras, see [7]. In particular this provides a confirmation of the fact that the fundamental group of an algebra takes into account the matrix structure, in other words it is not a Morita invariant: matrices do not admit a universal covering since there exist at least two connected components of the category of connected gradings.

Our main results here are: a characterization of Galois coverings, since F is Galois if and only if the fibre product of F with itself is a trivial covering; and a characterization of universal coverings, since F is universal if and only if the fibre product of F with any Galois covering provides a trivial covering. We finish this article with an application.

2 Preliminaries

In this section we recall some definitions and results from [6] that will be useful for this article.

Let k be a commutative ring. A k -category is a small category \mathcal{B} such that each morphism set ${}_y\mathcal{B}_x$ from an object $x \in \mathcal{B}_0$ to an object $y \in \mathcal{B}_0$ is a k -module, the composition of morphisms is k -bilinear and the identity at each object is central in its endomorphism ring. In particular each endomorphism set is a k -algebra, and ${}_y\mathcal{B}_x$ is a ${}_y\mathcal{B}_y - {}_x\mathcal{B}_x$ -bimodule.

Each k -algebra A provides a single object k -category \mathcal{B}_A with endomorphism ring A . Of course the structure of A can be described more precisely by choosing a finite set E of orthogonal idempotents of A , such that $\sum_{e \in E} e = 1$, the k -category $\mathcal{B}_{A,E}$ has set of objects E and morphisms from e to f the k -module fAe . Note that $\mathcal{B}_{A,\{1\}} = \mathcal{B}_A$.

Definition 2.1. *The star $\text{St}_b\mathcal{B}$ of a k -category \mathcal{B} at an object b is the direct sum of all the morphisms with source or target b :*

$$\text{St}_b\mathcal{B} = \left(\bigoplus_{y \in \mathcal{B}_0} {}_y\mathcal{B}_b \right) \oplus \left(\bigoplus_{y \in \mathcal{B}_0} {}_b\mathcal{B}_y \right).$$

Note that this k -module counts twice the endomorphism algebra at b .

Definition 2.2. *Let \mathcal{C} and \mathcal{B} be k -categories. A k -functor $F : \mathcal{C} \rightarrow \mathcal{B}$ is a covering of \mathcal{B} if it is surjective on objects and if F induces k -isomorphisms between the corresponding stars. More precisely, for each $b \in \mathcal{B}_0$ and each x in the non-empty fibre $F^{-1}(b)$, the map*

$$F_b^x : \text{St}_x\mathcal{C} \longrightarrow \text{St}_b\mathcal{B}.$$

induced by F is a k -isomorphism.

Remark 2.3. Each star is the direct sum of the source star $\text{St}_b^- \mathcal{B} = \bigoplus_{y \in \mathcal{B}_0} {}_y \mathcal{B}_b$ and the target star $\text{St}_b^+ \mathcal{B} = \bigoplus_{y \in \mathcal{B}_0} {}_b \mathcal{B}_y$. Since the St^- and St^+ components are stable under F , the condition of the definition is equivalent to the requirement that the corresponding target and source stars are isomorphic through F .

Moreover this splitting goes further: the restriction of F to $\bigoplus_{y \in F^{-1}(c)} {}_y \mathcal{C}_x$ is k -isomorphic to the corresponding k -module ${}_c \mathcal{B}_b$. The same holds with respect to the target star and morphisms starting at all objects in a single fibre.

Remark 2.4. The previous facts show that Definition 2.2 coincides with the one given by K. Bongartz and P. Gabriel in [3].

Definition 2.5. Given k -categories $\mathcal{B}, \mathcal{C}, \mathcal{D}$, the set $\text{Mor}(F, G)$ from a covering $F : \mathcal{C} \rightarrow \mathcal{B}$ to a covering $G : \mathcal{D} \rightarrow \mathcal{B}$ is the set of pairs of k -linear functors (H, J) where $H : \mathcal{C} \rightarrow \mathcal{D}$, $J : \mathcal{B} \rightarrow \mathcal{B}$ such that $GH = JF$, with J an isomorphism.

We will consider within the group of automorphisms of a covering $F : \mathcal{C} \rightarrow \mathcal{B}$, the subgroup $\text{Aut}_1(F)$ of invertible endofunctors H of \mathcal{C} such that $FH = F$.

We next recall the definition of a connected k -category. In fact, given a covering, we shall be interested in each connected component.

We use the following notation: given a morphism f , its source object is denoted $s(f)$ and $t(f)$ is its target object. We will also make use of walks. For this purpose we consider the set of formal pairs (f, ϵ) as "morphisms with sign", where f is a morphism in \mathcal{B} and $\epsilon \in \{-1, 1\}$. We extend source and target maps to this set as follows:

$$s(f, 1) = s(f), s(f, -1) = t(f), t(f, 1) = t(f), t(f, -1) = s(f).$$

Definition 2.6. Let \mathcal{B} be a k -category. A non-zero walk in \mathcal{B} is a sequence of non-zero morphisms with signs $(f_n, \epsilon_n) \dots (f_1, \epsilon_1)$ such that $s(f_{i+1}, \epsilon_{i+1}) = t(f_i, \epsilon_i)$. We say that this walk goes from $s(f_1, \epsilon_1)$ to $t(f_n, \epsilon_n)$.

A k -category \mathcal{B} is connected if any two objects b and c of \mathcal{B} can be joined by a non-zero walk.

Proposition 2.7. [6, Proposition 2.8] Let $F : \mathcal{C} \rightarrow \mathcal{B}$ be a covering of k -categories. If \mathcal{C} is connected, then \mathcal{B} is connected.

The following two statements correspond respectively to [6, Proposition 2.9] and [6, Corollary 2.10].

Proposition 2.8. [9] Let $F : \mathcal{C} \rightarrow \mathcal{B}$ and $G : \mathcal{D} \rightarrow \mathcal{B}$ be coverings of k -linear categories. Assume \mathcal{C} is connected. Two morphisms $(H_1, J), (H_2, J)$ from F to G such that H_1 and H_2 coincide on some object are equal.

Corollary 2.9. Let $F : \mathcal{C} \rightarrow \mathcal{B}$ be a connected covering of a k -linear category \mathcal{B} . The group $\text{Aut}_1(F) = \{(H, 1) : F \rightarrow F \mid H \text{ an isomorphism of } \mathcal{C}\}$ acts freely on each fibre.

Next we recall the definition of a Galois covering:

Definition 2.10. A covering $F : \mathcal{C} \rightarrow \mathcal{B}$ of k -categories is a Galois covering if \mathcal{C} is connected and if $\text{Aut}_1(F)$ acts transitively on some fibre.

Of course, the condition "on some fibre" actually means that the automorphism group acts transitively at every fibre whenever it acts transitively on a particular one, as it is proven in [6, 9, 10].

Proposition 2.11. *Let $F : \mathcal{C} \rightarrow \mathcal{B}$ be a Galois covering. Then $\text{Aut}_1(F)$ acts transitively on each fibre.*

The following structure theorem gives an explicit description of Galois coverings.

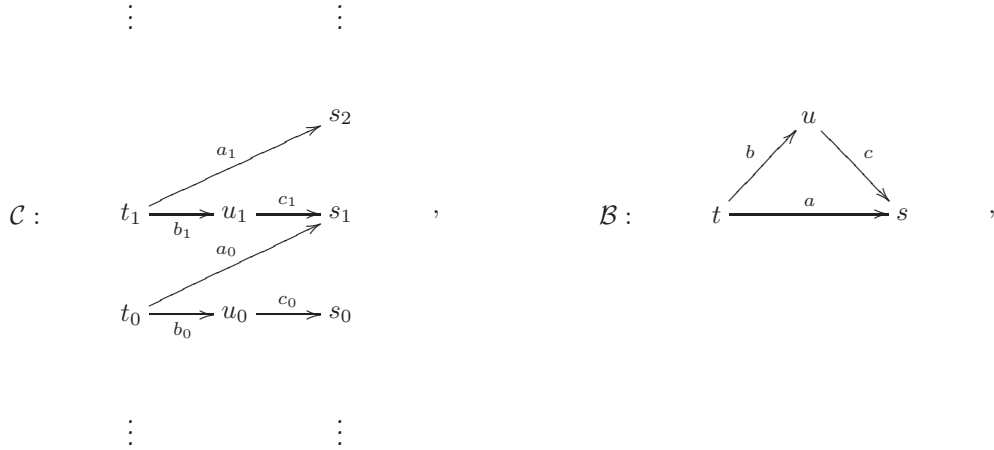
Theorem 2.12. [6, Theorem 3.7]. *Let $F : \mathcal{C} \rightarrow \mathcal{B}$ be a Galois covering. Then there exists a unique isomorphism of categories $F' : \mathcal{C}/\text{Aut}_1(F) \rightarrow \mathcal{B}$ such that $F'P = F$, where $P : \mathcal{C} \rightarrow \mathcal{C}/\text{Aut}_1(F)$ is the Galois covering given by the categorical quotient.*

Examples of coverings that are not Galois can be found in [6, 9, 10].

Definition 2.13. *A universal covering $U : \mathcal{U} \rightarrow \mathcal{B}$ is a Galois covering of \mathcal{B} such that for any Galois covering $F : \mathcal{C} \rightarrow \mathcal{B}$, and for any $u \in \mathcal{U}_0, c \in \mathcal{C}_0$ with $U(u) = F(c)$, there exists a unique morphism $(H, 1)$ from U to F such that $H(u) = c$.*

In case of existence, a universal covering is unique up to isomorphisms of Galois coverings. In general universal coverings do not exist, as the following Example shows.

Example 2.14. *Let k be a field and consider the k -categories*



and $F_1, F_2 : \mathcal{C} \rightarrow \mathcal{B}$ Galois coverings of \mathcal{B} given by $F_1(a_i) = a, F_2(a_i) = a + cb, F_1(b_i) = F_2(b_i) = b$ and $F_1(c_i) = F_2(c_i) = c$.

Since \mathcal{C} is simply connected, it admits no proper Galois covering, see [2, 11], and there is no morphism between F_1 and F_2 . Hence \mathcal{B} admits no universal covering.

3 Fibre product of coverings

The fibre product of coverings that we define below will be useful in order to provide a criterion for a covering to be Galois and to be universal.

Definition 3.1. *Let $F : \mathcal{C} \rightarrow \mathcal{B}$ and $G : \mathcal{D} \rightarrow \mathcal{B}$ be k -functors of k -categories. The fibre product $\mathcal{C} \times_{\mathcal{B}} \mathcal{D}$ is the k -category defined as follows: objects are pairs $(x, y) \in \mathcal{C}_0 \times \mathcal{D}_0$ such that $F(x) = G(y)$; the set of morphisms from (x, y) to (x', y') is the k -submodule of ${}_{x'}\mathcal{C}_x \oplus {}_{y'}\mathcal{D}_y$ given by pairs of morphisms (φ, ψ) verifying $F\varphi = G\psi$. Composition of morphisms is defined componentwise.*

Remark 3.2. The fibre product as defined above is in fact the categorical fibre product in the category of k -linear categories. In particular, the following diagram is commutative

$$\begin{array}{ccc} \mathcal{C} \times_{\mathcal{B}} \mathcal{D} & \xrightarrow{\text{pr}_{\mathcal{C}}} & \mathcal{C} \\ \text{pr}_{\mathcal{D}} \downarrow & & \downarrow F \\ \mathcal{D} & \xrightarrow{G} & \mathcal{B} \end{array}$$

Theorem 3.3. With the same notations as above, assume that F is a covering and that G is fully faithful. Then $\text{pr}_{\mathcal{D}}$ is a covering of \mathcal{D} .

Proof. Let d be an object of \mathcal{D} . The $\text{pr}_{\mathcal{D}}$ -fibre of d is the set of pairs (c, d) such that $Fc = Gd$, that is, the set of pairs (c, d) such that c is in the F -fibre of Gd .

In order to prove that $(\text{pr}_{\mathcal{D}})_d^{(c,d)}$ is an isomorphism between stars, we first show that it is surjective. Let $f \in {}_y\mathcal{D}_d$, consider $Gf \in \text{St}_{Gd}\mathcal{B}$ and let $\sum g_i$ be the unique sum of morphisms in \mathcal{C} obtained through the isomorphism of stars between $\text{St}_c\mathcal{C}$ and $\text{St}_{Fc}\mathcal{B} = \text{St}_{Gd}\mathcal{B}$. Of course each g_i starts at the common object c and it ends at objects in the F -fibre of Gy . We have that $\sum Fg_i = Gf$. Since Fg_i is a morphism from Gd to Gy in \mathcal{B} and G is full, there is one morphism h_i from d to y in \mathcal{D} such that $Gh_i = Fg_i$. This provides morphisms (h_i, g_i) of the fibre product. Moreover G is faithful, $G\sum h_i = \sum Fg_i = Gf$ hence $\sum h_i = f$ and $\sum (g_i, h_i)$ is the required morphism in the star $\text{St}_{(c,d)}\mathcal{C} \times_{\mathcal{B}} \mathcal{D}$ lying above f .

In order to prove that $(\text{pr}_{\mathcal{D}})_d^{(c,d)}$ is a monomorphism, let $\sum (g_i, h_i) \in \text{St}_{(c,d)}\mathcal{C} \times_{\mathcal{B}} \mathcal{D}$ be such that

$$0 = (\text{pr}_{\mathcal{D}})_d^{(c,d)} \sum (g_i, h_i) = \sum h_i \in \bigoplus_i {}_{y_i}\mathcal{D}_d.$$

Fix i_0 in the set of indices of the last direct sum and consider the subset of indices defined by $J = \{i : y_i = y_{i_0}\}$. Now $0 = \sum_J h_i \in {}_{y_{i_0}}\mathcal{D}_d$ and $F(\sum_J g_i) = \sum_J Gh_i = 0$. Since $F_{F(c)}^c : \text{St}_c\mathcal{C} \rightarrow \text{St}_{Fc}\mathcal{B}$ is an isomorphism, we have that $\sum_J g_i = 0$. Observe that each g_i lies in a different direct summand of $\text{St}_c\mathcal{C}$. Hence $g_i = 0$ for all $i \in J$, $0 = Fg_i = Gh_i$ and we have that $h_i = 0$ for all $i \in J$ because G is faithful. Finally note that the subsets $J = J(i_0)$ are a partition of the set of indices considered. \square

We provide an example of a fibre product of coverings such that its projections are not coverings.

Example 3.4. Consider coverings F_1 and F_2 as in Example 2.14. Its fibre product $\mathcal{C} \times_{\mathcal{B}} \mathcal{C}$ has set of objects isomorphic to the set $\mathcal{C}_0 \times \mathbb{Z}$ by the bijection sending (x_i, x_{i+k}) to (x_i, k) for $x = s, t, u$. Now the vector spaces of morphisms from (t_i, t_j) to (s_{i+1}, s_{j+1}) are zero. So, even if both projections are surjective on objects, the respective maps between stars are not surjective.

Next we prove several results concerning fibre products of Galois coverings. The first one gives a method to produce Galois coverings in an iterative way. After recalling the definition of a trivial Galois covering we give a criterion useful to check if a given covering is trivial.

Proposition 3.5. Let $F : \mathcal{C} \rightarrow \mathcal{B}$ be a Galois covering of k -linear categories. The fibre product of F by itself is a covering of \mathcal{C} , through the projection functor $\text{pr}_{\mathcal{C}}$.

Proof. It is clear that pr_C is surjective on objects since $\text{pr}_C(c, c) = c$. We must prove that

$$(\text{pr}_C)_c^{(c, c')} : \text{St}_{(c, c')}C \times_{\mathcal{B}} C \rightarrow \text{St}_c C$$

is an isomorphism, where $F(c) = F(c')$. Using that F is Galois we deduce that there exists $g \in \text{Aut}(F)$ such that $gc = c'$. The surjectivity on morphisms is also immediate since $(f, gf) \in \text{St}_{(c, c')}C \times_{\mathcal{B}} C$ and $\text{pr}_C(f, gf) = f$. The proof of the injectivity is the same as in the proof of Theorem 3.3. \square

Definition 3.6. A trivial covering of a k -category \mathcal{B} is a covering which is isomorphic to the one given by the product $\mathcal{B} \times E$ of \mathcal{B} by a set E , with objects $\mathcal{B}_0 \times E$ and where the morphisms are ${}_{(y, e)}(\mathcal{B} \times E)_{(x, e)} = {}_y\mathcal{B}_x$ while ${}_{(y, f)}(\mathcal{B} \times E)_{(x, e)} = 0$ if $e \neq f$. The covering functor is the projection functor to the first factor.

Lemma 3.7. Let $F : \mathcal{C} \rightarrow \mathcal{B}$ be a covering of k -categories, where \mathcal{B} is connected. Let S be a section of F , namely a k -functor $\mathcal{B} \rightarrow \mathcal{C}$ such that $FS = 1_{\mathcal{B}}$. Then the subcategory $S\mathcal{B}$ is full and it is a connected component of \mathcal{C} .

Proof. Let f be a non-zero morphism of \mathcal{C} having one extreme object in $S\mathcal{B}$. Let $x \in S\mathcal{B}_0$ and let $f \in {}_y\mathcal{C}_x$. Let $b \in \mathcal{B}_0$ be such that $x = Sb$. In order to prove that y is also in the image of S , consider $\text{St}_x\mathcal{C}$ which is isomorphic through F to $\text{St}_b\mathcal{B}$. Since $FS = Id$, we have that $S : \text{St}_b\mathcal{B} \rightarrow \text{St}_x\mathcal{C}$ is the inverse of F at the star level. Hence $f = SFf$ and $y = SFy$. We have shown that a morphism between objects in the image of S is in the image of S , which shows that $S\mathcal{B}$ is full in \mathcal{C} . Since $S\mathcal{B}$ is isomorphic to \mathcal{B} , the category $S\mathcal{B}$ is connected. \square

Proposition 3.8. Let $F : \mathcal{C} \rightarrow \mathcal{B}$ be a covering of k -categories, where \mathcal{B} is connected. The covering is trivial if and only if for each object $x \in \mathcal{C}_0$ there exists a k -linear functor $S : \mathcal{B} \rightarrow \mathcal{C}$ such that FS is the identity functor of \mathcal{B} and $SFx = x$.

Proof. Assume that for each $x \in \mathcal{C}_0$ there exists a k -linear functor $S : \mathcal{B} \rightarrow \mathcal{C}$ such that FS is the identity functor of \mathcal{B} and $SFx = x$. Let Σ be the set of all these sections of F and consider the functor $\alpha : \mathcal{B} \times \Sigma \rightarrow \mathcal{C}$ given by $\alpha(b, S) = Sb$ and $\alpha({}_{(c, S)}f_{(b, S)}) = Sf$. This functor is clearly faithful and dense. It is also full by the previous lemma. The other implication is immediate. \square

We already know after Proposition 3.5 that when F is a Galois covering the fibre product of F with itself is a covering. In fact, we will now characterize Galois coverings using fibre products.

Theorem 3.9. A connected covering $F : \mathcal{C} \rightarrow \mathcal{B}$ is Galois if and only if the fibre product of F with itself is a trivial covering of \mathcal{C} with respect to the projection functor pr_C to the first factor.

Proof. In case F is Galois, let $x \in \mathcal{C}_0$ and let (x, x') be in the fibre of x in the fibre product, namely $Fx = Fx'$. By transitivity and freeness of the action, there exists a unique $g \in \text{Aut}(F)$ such that $gx = x'$. Let $S : \mathcal{C} \rightarrow \mathcal{C} \times_{\mathcal{B}} \mathcal{C}$ be the functor defined as follows: $Sy = (y, gy)$ on objects and $Sf = (f, gf)$ on morphisms. This functor verifies the hypotheses of the previous proposition, and hence F is trivial.

Conversely let x and x' be in the same F -fibre, in order to define an automorphism carrying x to x' , let S be the section of the functor of pr_1 through (x, x') obtained using Proposition 3.8. The second component of S will provide the required automorphism g , more precisely g is defined by $Sy = (y, gy)$ and $Sf = (f, gf)$. Clearly $Fg = F$ by definition of the fibre product. Note also that this proves the existence of g , we already know that there exists at most one endomorphism g of F such that $gx = x'$. In order to prove that g is invertible, note that for x' and x'' in the same fibre, there exists an endomorphism h of F such that $x' = hx''$. If $x'' = x$, we have $ghx = x$, then $gh = 1$. \square

We already know that since a universal covering U of \mathcal{B} is Galois, the fibre product $\mathcal{U} \times_{\mathcal{B}} \mathcal{U}$ is trivial. In fact, we shall see that this is the case for $\mathcal{U} \times_{\mathcal{B}} \mathcal{C}$, where $\mathcal{C} \rightarrow \mathcal{B}$ is any Galois covering of \mathcal{B} , and moreover, this property characterizes universal coverings.

Theorem 3.10. *A connected covering $U : \mathcal{U} \rightarrow \mathcal{B}$ is universal if and only if the fibre product of U with any Galois covering $F : \mathcal{C} \rightarrow \mathcal{B}$ provides a trivial covering of \mathcal{U} .*

Proof. In case U is universal, let (u, x) be an object in $U \times_{\mathcal{B}} \mathcal{C}$, that is $Uu = Fx$. We have to prove that $\text{pr}_{\mathcal{U}}$ is a covering and that it is trivial. It is clear that $\text{pr}_{\mathcal{U}}$ is surjective on objects. To see that

$$(\text{pr}_{\mathcal{U}})_u^{(u,x)} : \text{St}_{(u,x)} \mathcal{U} \times_{\mathcal{B}} \mathcal{C} \rightarrow \text{St}_u \mathcal{U}$$

is an isomorphism consider the k -linear functor $S : \mathcal{U} \rightarrow \mathcal{U} \times_{\mathcal{B}} \mathcal{C}$ defined as follows: since U is universal there exists a unique functor G such that $FG = U$ and $Gx = u$; take $S = \text{Id}_{\mathcal{U}} \times G$. Now this functor S induces an inverse of $(\text{pr}_{\mathcal{U}})_u^{(u,x)}$. It is also clear that S is a section of $\text{pr}_{\mathcal{U}}$ through (u, x) .

Conversely, let $u \in \mathcal{U}_0$ and $x \in \mathcal{C}_0$ such that $Uu = Fx$. In order to define G such that $FG = U$ and $Gx = u$, let S be a section of $\text{pr}_{\mathcal{U}}$ through (u, x) , and define $G = \text{pr}_{\mathcal{C}} S$. We have $FG = F\text{pr}_{\mathcal{C}} S = U\text{pr}_{\mathcal{U}} S = U$. \square

As an application we use this criterion in order to prove that the coverings in Example 2.14 are not universal.

Example 3.11. *Consider the Galois coverings $F_1, F_2 : \mathcal{C} \rightarrow \mathcal{B}$ as in Example 2.14. We have already verified in Example 3.4 the projections from the fibre product $\mathcal{C} \times_{\mathcal{B}} \mathcal{C}$ to both coordinates are not coverings. Hence, neither F_1 nor F_2 is a universal covering.*

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