

# Modelling and Solving the Multi-Day Container Drayage Problem

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**Abstract**—This paper deals with a general Multi-Day Container Drayage Problem (MDCDP) that consists in assigning trucks to container transportation orders during several days. To this aim, a Mixed Integer Linear Programming problem is formulated: the model describes real problems taking into account the orders to be planned for several days, the types of the containers and the rest periods of drivers. In order to address real scenarios, a heuristic algorithm based on the rolling horizon approach is proposed. Some randomly generated MDCDP instances validate the heuristic algorithm and a case study of real dimensions shows the effectiveness of the proposed solution technique.

## I. INTRODUCTION

Container logistics has become of basic importance in the last two decades and in turn the short distance container transportation, both from an initial consignee to a terminal (seaport or railway hub station) and from a terminal to a final receiver performed by trucks [1]. This door-to-door service is usually referred to as *container drayage service* and it is responsible for a significant portion of the total transportation cost [2].

The Container Drayage Problem (CDP) has the following basic elements: a fleet of *trucks* with different characteristics; a set of *truck depots*; a set of *orders*, i.e., requests of moving a container. CDP deals with three-location routes: the starting location where the container is picked up, the intermediate location where the container is loaded or unloaded, the destination location where the container is delivered. Furthermore, many additional conditions must be complied with, concerning the matching of container type with truck characteristics, the service hours regulations for drivers, and the need for them to return empty to the depot at the end of service (which can encompass a day, a week or different periods). The set of the above elements makes the CDP very complex, to the point that it could be impossible to satisfy all the given orders with a limited sized fleet of trucks. This is why in practice decision makers can delay some orders

(possibly paying a penalty) or even refuse some of them (thus giving up the relative income).

Given such elements, the objective in the CDP problem is to determine which truck performs which order while maximizing the number of assigned orders and minimizing a given generalized cost function, which takes into account the number of delayed orders and the total distance travelled by a truck without any container, in order to move from the previous to the following order required locations (these movements involve pure costs for the truck company which are not covered by orders revenues).

Real-life scenarios in container drayage are very complex and are widely studied in the scientific literature (see for instance [1] for a review). However, in order to make the problem tractable, most papers introduce simplifications with respect to real-world scenarios. For example, [3] and [4] consider a single-day planning horizon, imposing that all the trucks return to their own depot at the end of each working day. Moreover, [5] and [6] assume that there are always enough trucks to perform all the container transportation orders. Finally, few papers take into account the restrictions on the drivers daily duty hours, i.e., the service hours regulations [7]. From the evidence of the real case studies [8], [9], modelling and solving real cases to take care of all the particular requirements represent open problems that need new solution techniques and approaches.

This paper deals with general CDPs in order to allow practitioners and decision makers to plan real scenarios. The aim is helping the container trucking company operators in their daily operations of assigning container transportation orders to the available fleet of trucks. To this aim, a Mixed Integer Linear Programming (MILP) formulation for the Multi-Day CDP (MDCDP) is presented: the model describes real problems taking into account the orders to be planned for several days, the types of the containers and the rest periods of drivers. The resulting MILP model is very complex and only instances of small dimensions can be solved in reasonable time. Therefore, in order to address real scenarios, a heuristic algorithm based on the rolling horizon approach [10] is proposed. Moreover, in order to validate the heuristic algorithm, some MDCDP randomly generated instances are solved by both the heuristic algorithm and the MILP model. The results show that the heuristic methodology is able to find good solutions. Finally, a case study of real dimensions shows the effectiveness of the presented solution technique.

The paper is organized as follows. Section II describes the considered problem and presents the MILP model. Section III presents the heuristic algorithm and Section IV discusses

some computational results and a real case study. Finally, Section V draws the conclusions.

## II. PROBLEM STATEMENT

In this section we formalize the MDCDP problem: given a heterogeneous and limited fleet of trucks and a set of container transportation orders, the objective is to optimally assign orders to trucks by maximizing the number of assigned orders and minimizing a generalized cost function, which takes into account the total distance travelled without any container and the number of delayed orders.

The drayage problem is addressed in a deterministic framework, i.e., order and truck parameters are assumed to be known at the beginning of the planning horizon.

*Trucks:* each truck is the association among a driver, a tractor and a trailer, and can carry only certain types of containers. Each truck has its own depot, where it has to return without load by a defined ending time. Truck operations can be performed from a defined starting time and initial position. Moreover, each truck driver should respect a minimum rest period each night.

*Orders:* each container transportation order is characterized by the following issues: *i)* three location types that have to be visited in sequence, i.e., the starting location (type  $A$ ) where the container is picked up, the intermediate location (type  $B$ ) where the container is loaded or unloaded, the destination location (type  $C$ ) where the container is delivered; *ii)* for each location, hard (for location types  $A$  and  $C$ ) or soft (for location type  $B$ ) time windows to be complied with; *iii)* the type of container to be transported.

### A. Time Indexed Formulation

In order to model the system, we present a MILP formulation of the MDCDP based on a Time-Indexed Formulation where the decision variables are the discrete-time instants in which the operations are scheduled. The planning horizon includes  $Q$  days and each day is discretized into  $K$  time periods, each lasting 1 time unit (t.u.). Therefore, the planning horizon starts at time 1 and ends at time  $T = K \cdot Q$ .

1) *Sets:* The following sets are defined:

- $\mathcal{T} = \{t \mid t = 1, 2, \dots, T\}$ : set of *time periods*;
- $\mathcal{G}_i = \{\text{sleep}_{min} + K \cdot (i - 1), \text{sleep}_{min} + K \cdot (i - 1) + 1, \dots, \text{sleep}_{max} + K \cdot (i - 1)\}$ ,  $i \in \{1, 2, \dots, Q - 1\}$ : set of *time periods* during which it is allowed to begin the night rest, where  $\text{sleep}_{min}$  (respectively,  $\text{sleep}_{max}$ ) is the earliest time (latest time) at which the night rest can start every day;
- $\mathcal{R} = \{c \mid c = 1, 2, \dots, R\}$ : set of available trucks;
- $\mathcal{S} = \{v \mid v = 1, 2, \dots, S\}$ : set of orders to be performed during the planning horizon;
- $\mathcal{S}^s = \{v \mid v = S+1, S+2, \dots, S+S^s\}$ : set of dummy orders ( $S^s$  in number) modelling the night rest;
- $\mathcal{S}^* = \mathcal{S} \cup \mathcal{S}^s$ : total set of orders;
- $\mathcal{E} = \{e \mid e = 1, 2, \dots, E\}$ : set of container typologies;
- $\Lambda = \{\lambda \mid \lambda = A, B, C\}$ : set of the location types.

2) *Trucks:* The trucks  $c \in \mathcal{R}$  are characterized by the following parameters:

$r_{ce} \in \{0, 1\}$ :  $r_{ce} = 1$  if  $c$  can carry a container of type  $e \in \mathcal{E}$ ;

$[t_c^{start}, t_c^{end}] \subseteq \mathcal{T}$ : time interval during which truck  $c$  can perform operations;

$d'_{cv}$ : time distance (in t.u.) between the starting position of  $c$  and location of type  $A$  of order  $v \in \mathcal{S}$ ;

$d''_{cv}$ : time distance (in t.u.) between location of type  $C$  of order  $v \in \mathcal{S}$  and the depot of truck  $c$ .

3) *Orders:* Each order  $v \in \mathcal{S}^*$  is characterized by the following parameters:

$s_{ve} \in \{0, 1\}$ :  $s_{ve} = 1$  if the type of container to be transported in order  $v$  is  $e \in \mathcal{E}$ . If  $v \in \mathcal{S}^s$ , then  $s_{ve} = 0$ ;

$[t_v^{\lambda L}, t_v^{\lambda U}] \subseteq \mathcal{T}$ : time window during which operations at location  $\lambda \in \{A, C\}$  for order  $v$  must be performed;

$[t_v^{BL}, t_v^{BU} + \delta_v^B] \subseteq \mathcal{T}$ : time window during which operations at location  $B$  for order  $v$  must be performed, with  $\delta_v^B$  maximum delay admitted at location  $B$ ;

$\tau_v^\lambda$ : if  $\lambda = A$ , then  $\tau_v^A = 0$  for each  $v \in \mathcal{S}^*$ . If  $\lambda = B$ , then  $\tau_v^B$  is the time distance (in t.u.) between location type  $A$  and location type  $B$  of  $v$  for  $v \in \mathcal{S}$ , while  $\tau_v^B = 0$  for  $v \in \mathcal{S}^s$ . If  $\lambda = C$ , then  $\tau_v^C$  is the sum of  $\tau_v^B$  and the time distance (in t.u.) between location type  $B$  and location type  $C$  of  $v$  for  $v \in \mathcal{S}$ , while  $\tau_v^C$  is equal to the length of the night rest for  $v \in \mathcal{S}^s$ ;

$d_{vw}$ : time distance (in t.u.) between location type  $C$  of order  $v$  and location type  $A$  of order  $w \in \mathcal{S}$ , with  $w \neq v$ .

4) *Decision variables:* The following decision variables  $\{0, 1\}$  are introduced:

$x_{cv}(t)$  for  $c \in \mathcal{R}$ ,  $v \in \mathcal{S}^*$ ,  $t \in \mathcal{T}$ :  $x_{cv}(t) = 1$  if  $v$  is assigned to  $c$ , which moves from its current position at time  $t$ ;

$z_{cvw}(t)$  for  $c \in \mathcal{R}$ ,  $v, w \in \mathcal{S}$ ,  $v \neq w$ ,  $t \in \mathcal{T}$ :  $z_{cvw}(t) = 1$  if  $c$  performs order  $w$  at time  $t$  immediately after order  $v$ ;

$p_{cvw}(t)$  for  $c \in \mathcal{R}$ ,  $v \in \mathcal{S}$ ,  $w \in \mathcal{S}^s$ ,  $t \in \mathcal{T}$ :  $p_{cvw}(t) = 1$  if  $c$  performs a night rest  $w$  at time  $t$ , after having started order  $v$ ;

$y_c(t)$  for  $c \in \mathcal{R}$ ,  $t \in \mathcal{T}$ :  $y_c(t) = 1$  if  $c$  begins the night rest at time  $t$ ;

$l_{cv}$  for  $c \in \mathcal{R}$ ,  $v \in \mathcal{S}$ :  $l_{cv} = 1$  if  $c$  performs  $v$  arriving at location type  $B$  during the time window  $(t_v^{BU}, t_v^{BU} + \delta_v^B]$ ;

$z'_{cv}$  for  $c \in \mathcal{R}$ ,  $v \in \mathcal{S}$ :  $z'_{cv} = 1$  if  $c$  performs  $v$  from its starting position;

$z''_{cv}$  for  $c \in \mathcal{R}$ ,  $v \in \mathcal{S}$ :  $z''_{cv} = 1$  if  $c$  performs  $v$  and goes to its ending position.

The objectives of the model are the following: *i)* maximizing the total number of orders performed during the planning horizon; *ii)* minimizing the total distance travelled by all trucks without any load; *iii)* maximizing the number of on time orders.

Hence, the following multi-objective MILP problem is formulated:

$$\max [f_1, -f_2, f_3] \quad (1)$$

where

$$f_1 = \left( \sum_{c \in \mathcal{R}} \sum_{v \in \mathcal{S}} \sum_{t \in \mathcal{T}} x_{cv}(t) \right) / S \quad (2)$$

$$f_2 = \left( \sum_{c \in \mathcal{R}} \sum_{u \in \mathcal{S}} \sum_{v \in \mathcal{S}} \sum_{t \in \mathcal{T}} z_{cuv}(t) \cdot d_{uv} + \sum_{c \in \mathcal{R}} \sum_{v \in \mathcal{S}} z'_{cv} \cdot d'_{cv} + \sum_{c \in \mathcal{R}} \sum_{v \in \mathcal{S}} z''_{cv} \cdot d''_{cv} + \sum_{v \in \mathcal{S}} \tau_v^C \right) / \sum_{v \in \mathcal{S}} \tau_v^C \quad (3)$$

$$f_3 = \sum_{c \in \mathcal{R}} \sum_{v \in \mathcal{S}} l_{cv} / S \quad (4)$$

s. t.:

$$\sum_{c \in \mathcal{R}} \sum_{t \in \mathcal{T}} x_{cv}(t) \leq 1 \quad \forall v \in \mathcal{S}, \quad (5)$$

$$\begin{aligned} & \sum_{t \in \mathcal{T}} t \cdot x_{cw}(t) - (d'_{cv} \cdot z'_{cv} + \sum_{t \in \mathcal{T}} (t + \tau_v^C) \cdot x_{cv}(t) + \\ & \sum_{\substack{u \in \mathcal{S} \\ u \neq v}} \sum_{t \in \mathcal{T}} d_{uv} \cdot z_{cw}(t) + \sum_{u \in \mathcal{S}^s} \sum_{t \in \mathcal{T}} \tau_u^C \cdot p_{cwu}(t)) \geq \\ & \left( \sum_{t \in \mathcal{T}} z_{cww}(t) - 1 \right) \cdot M \quad \forall c \in \mathcal{R}, v \in \mathcal{S}, w \in \mathcal{S}, \quad (6) \end{aligned}$$

$$\sum_{t \in \mathcal{T}} t \cdot x_{cw}(t) - \sum_{t \in \mathcal{T}} t \cdot x_{cv}(t) \geq \left( \sum_{t \in \mathcal{T}} p_{cww}(t) - 1 \right) \cdot M \quad \forall c \in \mathcal{R}, v \in \mathcal{S}, w \in \mathcal{S}^s, \quad (7)$$

$$t_c^{start} \cdot z'_{cw} + \sum_{v \in \mathcal{S}} \sum_{t \in \mathcal{T}} t \cdot z_{cvw}(t) \leq \sum_{t \in \mathcal{T}} t \cdot x_{cw}(t) \leq$$

$$t_c^{end} \cdot z'_{cw} + \sum_{v \in \mathcal{S}} \sum_{t \in \mathcal{T}} t \cdot z_{cvw}(t) \quad \forall c \in \mathcal{R}, w \in \mathcal{S}, \quad (8)$$

$$\sum_{v \in \mathcal{S}} \sum_{t \in \mathcal{T}} t \cdot p_{cww}(t) = \sum_{t \in \mathcal{T}} t \cdot x_{cw}(t) \quad \forall c \in \mathcal{R}, w \in \mathcal{S}^s, \quad (9)$$

$$\sum_{w \in \mathcal{S}^s} \sum_{t \in \mathcal{T}} p_{cww}(t) \leq \sum_{t \in \mathcal{T}} x_{cv}(t) \quad \forall c \in \mathcal{R}, v \in \mathcal{S}, \quad (10)$$

$$\sum_{c \in \mathcal{R}} \sum_{w \in \mathcal{S}} \sum_{\substack{t \in \mathcal{T} \\ w \neq v}} z_{cww}(t) \leq 1 \quad \forall v \in \mathcal{S}, \quad (11)$$

$$\sum_{c \in \mathcal{R}} \sum_{v \in \mathcal{S}} \sum_{\substack{t \in \mathcal{T} \\ v \neq w}} z_{cww}(t) \leq 1 \quad \forall w \in \mathcal{S}, \quad (12)$$

$$\sum_{c \in \mathcal{R}} \sum_{w \in \mathcal{S}^s} \sum_{t \in \mathcal{T}} p_{cww}(t) \leq 1 \quad \forall v \in \mathcal{S}, \quad (13)$$

$$\sum_{\substack{t \in \mathcal{T} \\ t \leq t_c^{start} - 1}} x_{cv}(t) = 0 \quad \forall c \in \mathcal{R}, v \in \mathcal{S}^*, \quad (14)$$

$$\begin{aligned} & d''_{cw} \cdot z''_{cw} + \sum_{v \in \mathcal{S}} \sum_{t \in \mathcal{T}} (t + \tau_w^C + d_{vw}) \cdot z_{cvw}(t) + \\ & \sum_{u \in \mathcal{S}^s} \sum_{\substack{t \in \mathcal{T} \\ t \leq t_w^{CU}}} \tau_u^C \cdot p_{cwu}(t) \leq t_c^{end} \quad \forall c \in \mathcal{R}, w \in \mathcal{S}, \quad (15) \end{aligned}$$

$$\begin{aligned} & t_w^{\lambda L} \cdot \sum_{t \in \mathcal{T}} x_{cw}(t) \leq d'_{cw} \cdot z'_{cw} + \sum_{t \in \mathcal{T}} (t + \tau_w^\lambda) \cdot x_{cw}(t) + \\ & \sum_{v \in \mathcal{S}} \sum_{t \in \mathcal{T}} d_{vw} \cdot z_{cvw}(t) + \sum_{u \in \mathcal{S}^s} \sum_{\substack{t \in \mathcal{T} \\ t \leq t_w^{\lambda U}}} \tau_u^C \cdot p_{cwu}(t) \leq t_w^{\lambda U} \\ & \forall w \in \mathcal{S}, c \in \mathcal{R}, \lambda \in \{A, C\}, \quad (16) \end{aligned}$$

$$t_w^{BL} \cdot \sum_{t \in \mathcal{T}} x_{cw}(t) \leq d'_{cw} \cdot z'_{cw} + \sum_{t \in \mathcal{T}} (t + \tau_w^B) \cdot x_{cw}(t) +$$

$$\sum_{v \in \mathcal{S}} \sum_{t \in \mathcal{T}} d_{vw} \cdot z_{cvw}(t) + \sum_{u \in \mathcal{S}^s} \sum_{\substack{t \in \mathcal{T} \\ t \leq t_w^{BU}}} \tau_u^C \cdot p_{cwu}(t) \leq$$

$$t_w^{BU} + l_{cw} \cdot \delta_w^B \quad \forall w \in \mathcal{S}, c \in \mathcal{R}, \quad (17)$$

$$s_{ve} \cdot \sum_{t \in \mathcal{T}} x_{cv}(t) \leq r_{ce} \quad \forall e \in \mathcal{E}, c \in \mathcal{R}, v \in \mathcal{S}, \quad (18)$$

$$\sum_{t \in \mathcal{G}_i} y_c(t) = 1 \quad \forall c \in \mathcal{R}, i \in \{1, 2, \dots, Q\}, \quad (19)$$

$$\sum_{t \in \mathcal{T} \setminus \mathcal{G}_i} y_c(t) = 0 \quad \forall c \in \mathcal{R}, i \in \{1, 2, \dots, Q\}, \quad (20)$$

$$\begin{aligned} & \sum_{\substack{t \in \mathcal{G}_{i+1} \\ t_c^{start} \leq t \leq t_c^{end}}} t \cdot y_c(t) - \sum_{\substack{t \in \mathcal{G}_i \\ t_c^{start} \leq t \leq t_c^{end}}} t \cdot y_c(t) \leq K \\ & \forall c \in \mathcal{R}, i \in \{1, \dots, Q-2\}, \quad (21) \end{aligned}$$

$$\sum_{\substack{v \in \mathcal{S}^s \\ t \in \mathcal{G}_i}} t \cdot x_{cv}(t) = \sum_{t \in \mathcal{G}_i} t \cdot y_c(t) \quad \forall c \in \mathcal{R}, i \in \{1, \dots, Q\}, \quad (22)$$

$$\sum_{v \in \mathcal{S}} z'_{cv} \leq 1 \quad \forall c \in \mathcal{S}, \quad (23)$$

$$\sum_{v \in \mathcal{S}} z''_{cv} \leq 1 \quad \forall c \in \mathcal{S}, \quad (24)$$

$$x_{cv}(t) \in \{0, 1\} \quad \forall c \in \mathcal{R}, v \in \mathcal{S}^*, t \in \mathcal{T}, \quad (25)$$

$$z_{cvw}(t) \in \{0, 1\} \quad \forall c \in \mathcal{R}, v \in \mathcal{S}, w \in \mathcal{S}, t \in \mathcal{T}, \quad (26)$$

$$p_{cww}(t) \in \{0, 1\} \quad \forall c \in \mathcal{R}, v \in \mathcal{S}, w \in \mathcal{S}^s, t \in \mathcal{T}, \quad (27)$$

$$y_c(t) \in \{0, 1\} \quad \forall c \in \mathcal{R}, t \in \mathcal{T}, \quad (28)$$

$$l_{cv} \in \{0, 1\} \quad \forall c \in \mathcal{R}, v \in \mathcal{S}, \quad (29)$$

$$z'_{cv} \in \{0, 1\} \quad \forall c \in \mathcal{R}, v \in \mathcal{S}, \quad (30)$$

$$z''_{cv} \in \{0, 1\} \quad \forall c \in \mathcal{R}, v \in \mathcal{S}. \quad (31)$$

The three terms of the objective function are normalized as follows:  $f_1$  (2) is the fraction of assigned orders over the total number of orders;  $f_2$  (3) is the fraction of the actual distance travelled by all trucks over the total length of all orders;  $f_3$  (4) is the fraction of on time orders over the total number of orders.

Constraints (5) are the *assignment uniqueness constraints* and ensure that each order is performed at most by one truck.

Constraints (6)-(13) are the *orders sequencing constraints*. In particular, (6) ensure that, if order  $w \in \mathcal{S}$  is performed immediately after order  $v \in \mathcal{S}$ , then order  $w$  can not start before order  $v$  has been completed. On the other hand, since a dummy order  $w \in \mathcal{S}^s$  can interrupt the previous real order  $v$ , constraints (7) guarantee that the starting time of  $w$  is subsequent to the starting time of  $v$ . Note that in constraints (6) and (7)  $M \in \mathbb{N}^+$  is a sufficiently large number. Moreover, constraints (8) (constraints (9)) ensure that order  $w \in \mathcal{S}$  ( $w \in \mathcal{S}^s$ ) can be executed after order  $v \in \mathcal{S}$  by truck  $c \in \mathcal{R}$  if  $w$  is effectively assigned to  $c$ . Constraints (10) impose that order  $w \in \mathcal{S}^s$  can follow a real order  $v \in \mathcal{S}$  on truck  $c \in \mathcal{R}$  if  $v$  is effectively assigned to  $c$ . Finally, constraints (11) and (12) guarantee that each order can have at most one successor and one predecessor, respectively, while constraints (13) impose that a real order can be followed by at most one night rest.

Constraints (14) and (15) are the *truck availability constraints* and impose that each truck  $c \in \mathcal{R}$  can perform its operations only during the defined time interval  $[t_c^{start}, t_c^{end}]$ .

Constraints (16) are the *time constraints* at location types  $A$  and  $C$ , respectively, while constraints (17) are the *time constraints* at location type  $B$ .

Constraints (18) are the *container type constraints*.

Constraints (19)-(22) are the *service hours constraints*, i.e., they guarantee that each driver has a night rest of  $\tau_v^C$  t.u., with  $v \in \mathcal{S}^s$ , every day of the planning horizon (except the last one).

Finally, constraints (23) and (24) are the *depot constraints* and impose that each truck leaves its starting position and returns back to its depot at the end of the planning horizon.

Constraints (25)-(31) are the binary variables definitions.

### III. THE ROLLING HORIZON HEURISTIC

The presented MILP formulation is suitable for instances of small dimensions, but it requires huge computational efforts for large-scale instances. Therefore, in order to address real scenarios, a heuristic algorithm based on the rolling horizon approach [10] and on the weighted sum method for multi-objective optimization [11] is introduced.

In particular, a two-step problem decomposition is applied.

First, the MDCDP is decomposed into  $Q$  single-day Sub-Problems  $SP^q$ ,  $q=1, 2, \dots, Q$ . Each sub-problem  $SP^q$  uses the *information about day  $q-1$*  to update the starting position and time availability for each truck of the fleet, and the *information about day  $q+1$*  to minimize the overall distance travelled without any load.

Second, each single-day problem  $SP^q$  is in turn decomposed into a sequence of interdependent Assignment Problems  $AP^{qh}$ , each dealing with the same type of containers and similar required visit times at location  $B$ .

In addition, a single objective function is obtained as an appropriately weighted sum of the conveniently normalized three terms of the MDCDP objective function.

In order to describe  $AP^{qh}$ , the following notation is introduced:

$I_h \subset \mathcal{T}$ : set of time periods included in the planning horizon of  $AP^{qh}$ ;

$S^{qh} \subset \mathcal{S}$ : set of orders to be assigned by  $AP^{qh}$ ;

$\mathcal{R}^{qh} \subset \mathcal{R}$ : set of available trucks considered in  $AP^{qh}$ .

Problem  $AP^{qh}$  has the following binary decision variables:

$\bar{x}_{cv} \in \{0, 1\}$  for  $c \in \mathcal{R}^{qh}, v \in S^{qh}$ , with  $\bar{x}_{cv} = 1$  if  $v$  is assigned to  $c$ . Obviously, it holds  $\bar{x}_{cv} = \sum_{t \in I_h} x_{cv}(t)$ .

Trucks and orders are characterized by the same parameters defined for the MILP formulation: for each  $AP^{qh}$ , the same symbols used for MILP are retained with the  $qh$  apex added. For instance,  $t_c^{start, qh} \in \mathcal{T}$  is the operations starting time of truck  $c$  for  $AP^{qh}$ .

Moreover, the following additional elements are introduced:

- $\varrho$ , length of the night rest (in t.u.);
- $duty_{max}$ , maximum daily working time (in t.u.) for each truck according to the service hours regulations;
- $duty_c^{qh}$  for  $c \in \mathcal{R}^{qh}$ , residual working time (in t.u.) of truck  $c$  according to the service hours regulations during time intervals  $I_h$ ;
- $m_{cv}^{qh}$  for  $c \in \mathcal{R}^{qh}$  and  $v \in S^{qh}$  considers the information about the future orders: it is used to penalise the assignment of order  $v$  to truck  $c$  if location  $C$  of  $v$  has low demand at day  $q+1$  for the types of containers that can be carried by truck  $c$ . In practice, this element allows avoiding to send a truck where it is not required the day after.

At this point, in order to formalize the  $AP^{qh}$  we define a *generalized cost matrix*  $\mathbf{O} \in \mathbb{R}^{\mathcal{R}^{qh} \times S^{qh}}$  where each element  $o_{cv}$  is a cost that takes into account: i) the distance travelled without any load; ii) the number of the not delayed orders; iii) the time distance between the order location  $C$  and the depot; iv) the information about the future orders.

More formally, the element  $o_{cv}$  of  $\mathbf{O}$  is defined as follows (note that for the sake of simplicity we omit the apex  $qh$  in the elements of  $\mathbf{O}$  and in the weights  $p_i$ ,  $i = 1, 2, 3, 4$ ):

$$o_{c,v} = \begin{cases} p_1 \cdot d'_{cv} + p_2 \cdot l_{cv} + & \text{if the assignment of } v \\ p_3 \cdot d''_{cv} + p_4 \cdot m_{cv}^{qh} & \text{to } c \text{ is feasible} \\ \infty & \text{otherwise.} \end{cases} \quad (32)$$

The feasibility of the assignment of order  $v$  to truck  $c$  is evaluated according to constraints (14) – (18) of the MILP.

**Remark 1.** The weights  $p_i$  for  $i=1, \dots, 4$  can have different values for different  $AP^{qh}$  problems. In particular,  $p_3$  is increased as  $q$  increases in order to foster the return to the depot;  $p_4$  is increased with  $h$  since the information about the future orders are more important for the last orders of the day.

Each assignment problem  $AP^{qh}$  is formulated as follows [12]:

$$\min \sum_{\substack{c \in \mathcal{R}^{qh} \\ v \in S^{qh}}} o_{cv} \cdot \bar{x}_{cv} \quad (33)$$

s.t.

$$\sum_{v \in S^{qh}} \bar{x}_{cv} = 1 \quad \forall c \in \mathcal{R}^{qh}, \quad (34)$$

$$\sum_{c \in \mathcal{R}^{qh}} \bar{x}_{cv} \leq 1 \quad \forall v \in S^{qh}, \quad (35)$$

$$\bar{x}_{cv} \in \{0, 1\} \quad \forall c \in \mathcal{R}^{qh}, v \in S^{qh}. \quad (36)$$

The objective function (33) converts the multi-objective function of the MILP formulation using the defined cost matrix  $\mathbf{O}$ . Constraints (34) impose that each truck performs one order, while constraints (35) ensure that each order is assigned at most to one truck. Finally, (36) are the binary variable definitions.

Given the above premises, the MDCDP is addressed by the Heuristic Algorithm 1. An iterative procedure based on the solution of  $h$  interdependent assignment problems  $AP^{qh}$  is applied to solve each sub-problem  $SP^q$ ,  $q=1, 2, \dots, Q$ . Starting from an initial value of  $h = 1$ , the procedure solves a sequence of assignment problems  $AP^{qh}$  with  $h = 1, 2, \dots$  until all the orders to be performed in the day  $q$  are considered. In particular, in order to address each assignment problem  $AP^{qh}$  three preliminary steps are performed.

First, the trucks parameters are initialized. If  $q=1$  and  $h=1$  then the parameters are set to the same values considered in the MILP formulation, i.e., starting time and position at the beginning of the planning horizon (Step 3). If  $q > 1$  and  $h=1$  then the parameters are set on the basis of the solution of problem  $SP^{q-1}$  (Step 3), i.e., the starting time of a truck for day  $q$  is equal to the time at which it completes the last order of day  $q-1$ , considering also the night rest of length



$\varrho$  t.u., the starting position is the location of type  $C$  of the last order of day  $q - 1$ , while the residual working time is set to its maximum possible value. Finally, if  $q > 1$  and  $h > 1$  then the parameters are set on the basis of the solution of problem  $AP^{q(h-1)}$  (Step 8), similarly to what done for the parameters update between problem  $SP^{q-1}$  and  $SP^q$ , with the exception of the residual working time, decreased according to the duration of the order assigned to the truck by  $AP^{q(h-1)}$ .

Second, the subset of orders for  $AP^{qh}$  are determined (Step 4).

Third, the generalized cost matrix  $\mathbf{O}$  is constructed (Step 6).

The assignment process for the sub-problem  $SP^q$  is completed if all the orders of day  $q$  are analysed (Step 5). The Heuristic Algorithm 1 ends when all the  $Q$  days of the planning horizon of the MDCDP are considered (Step 10).

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#### Algorithm 1 Heuristic Algorithm

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- 1: **Set**  $q=1$ .
  - 2: **Set**  $h=1$ .
  - 3: **Initialize trucks parameters.** **Set**  $duty_c^{qh} = duty_{max}$ . **If**  $q=1$  **then Set**  $t_c^{start,qh} = t_c^{start}$ ,  $d_{cv}^{qh} = d_{cv}$  **else Set**  $t_c^{start,qh}$  and  $d_{cv}^{qh}$  on the basis of the solution of problem  $SP^{(q-1)}$  and considering the night rest ( $\varrho$  t.u.). **If** information about orders for day  $q+1$  is available **then** initialize  $m_{cv}^{qh}$ .
  - 4: **Determine the new set of orders to be assigned.** Determine  $S^{qh}$  on the basis of the type of container to be transported and the time requested at location  $B$ . Determine  $\mathcal{R}^{qh}$  accordingly.
  - 5: **Check if the assignment process for  $q$  is completed.** **If**  $S^{qh} = \emptyset$  **then Go to** 10.
  - 6: **Determine the generalized cost matrix.** **Set** the weights  $p_i$ ,  $i=1, 2, 3, 4$ , on the basis of the actual values of  $h$  and  $q$  (see Remark 1) and determine  $\mathbf{O}$ .
  - 7: **Solve the assignment problem**  $AP^{qh}$ .
  - 8: **Update the trucks parameters.** **Set**  $t_c^{start,qh}$ ,  $d_{cv}^{qh}$ ,  $duty_c^{qh}$  on the basis of the solution of  $AP^{qh}$ . **If** information about orders for day  $q+1$  is available **then** update  $m_{cv}^{qh}$ .
  - 9: **Set**  $h=h+1$  and **Go to** Step 4.
  - 10: **Check if the assignment process is completed.** **If**  $q < Q$  **then Set**  $q=q+1$  and **Go to** Step 2 **else END**.
- 

## IV. COMPUTATIONAL RESULTS

The multi-objective MILP formulation is implemented by introducing a lexicographic order among the objectives [13]: first, assign as many orders as possible ( $f_1$ ), since in this way revenues for the transportation company increase; then, minimise the distance travelled by trucks from one order to the following one ( $f_2$ ), since it implies costs (driving time, fuel, etc.) with no revenues; finally, maximise the number of orders on time ( $f_3$ ), since delays may sometimes imply penalties.

The MILP formulation is implemented in C++ by using ILOG Concert 2.9 and CPLEX 12.5, and the proposed heuristic is implemented in the MATLAB software environment. The two algorithms run on a 3.3 GHz Intel i7 980X with 24 GB of memory. The CPLEX code uses three threads and the CPU time limit is 3600 seconds.

### A. Random Data Set

The MILP formulation and the Heuristic Algorithm 1 are tested on 18 randomly generated instances.

In all instances,  $Q=5$  days are considered, with  $K=96$  time periods per day. In particular, for each instance, the number  $R$  of trucks and the number  $S$  of orders are fixed. In addition, the remaining parameters are randomly generated as follows: given a set of Italian geographical locations on the street map, location types  $A$ ,  $B$  and  $C$  are randomly selected; the time window for location type  $B$  is randomly generated between 7 a.m. and 5 p.m.; the time windows for location types  $A$  and  $C$  are defined according to the time windows for  $B$ ; the maximum admitted delay at location  $B$  is 1 hour; all the orders have the same type of container. Moreover, for each truck, the following assumptions are considered: the starting time is 4 a.m. of day  $q=1$  and the ending time is 12 p.m. of day  $q=5$ ; the depot location is randomly selected among a given list of geographical locations; all trucks can carry all types of containers. Finally, the length of the night rest is of 9 hours.

Table I shows the results obtained by both the MILP model and the Heuristic Algorithm 1 for the 18 instances. In particular, # identifies the instance,  $R$  is the number of trucks and  $S$  is the number of orders. The values of the objective functions  $f_1$ ,  $f_2$  and  $f_3$  are reported both for the MILP model and Algorithm 1 (remind that  $f_1$  and  $f_3$  have to be maximised and  $f_2$  minimised). The CPU time is shown only for the MILP model since the computation time of the heuristic algorithm is less than 5 seconds in all cases.

In 10 cases the MILP model reaches the time limit without finding the optimal solution: in 7 of them (denoted by an asterisk in the CPU time) a feasible solution is found and the relative figures for  $f_1$ ,  $f_2$  and  $f_3$  are reported. The exact model shows good behaviour with 2 trucks and up to 25 orders. On the contrary, with 3 trucks only 2 out of 10 instances are solved to optimality and for 5 of them a feasible solution is provided. It is worth to point out that, for different instances with the same number of trucks and orders, the MILP model sometimes shows quite different behaviours: for instance, case 3 is solved to optimality in slightly more than 1 minute, while case 4 requires almost 20 minutes. This behaviour is typical of NP-hard problems: in particular, huge symmetry features characterise the considered model.

The Heuristic Algorithm 1 always shows a good behaviour: only in two cases the values of  $f_1$  obtained by the heuristic are worse than the optimum; in one case it is better than the feasible solution provided by the MILP. The values of  $f_2$  determined by Algorithm 1 are generally near the optimum, with a maximum increase of 5%. Summing up, on the basis of the obtained results, the proposed heuristic algorithm appears to be a good tool to manage real-sized cases.

### B. Real Case Study

A real case study is solved using Heuristic Algorithm 1 by considering a planning horizon of  $Q=5$  days,  $K=1440$  time periods (the minute is considered as t.u.),  $R=500$  trucks,

TABLE I  
RESULTS OBTAINED FOR 18 RANDOMIZED DATA SETS

#	R	S	MILP model			Algorithm 1			
			CPU Time [s]	$f_1$	$f_2$	$f_3$	$f_1$	$f_2$	$f_3$
1	2	15	19.88	0.93	1.35	0.93	0.93	1.36	1.00
2	2	15	512.13	0.87	1.50	0.87	0.87	1.57	0.93
3	2	20	67.39	0.85	1.42	0.90	0.80	1.44	1.00
4	2	20	1159.14	0.85	1.37	1.00	0.85	1.43	1.00
5	2	25	2363.96	0.80	1.22	0.96	0.80	1.28	0.96
6	2	25	348.82	0.84	1.38	0.92	0.80	1.36	1.00
7	2	30	3600.00*	0.77	1.27	1.00	0.77	1.26	0.97
8	2	30	3600.00*	0.77	1.29	0.97	0.77	1.24	0.90
9	3	15	3600.00*	1.00	1.46	0.93	1.00	1.48	1.00
10	3	15	957.28	1.00	1.38	1.00	1.00	1.45	1.00
11	3	20	3600.00*	1.00	1.33	0.85	1.00	1.37	1.00
12	3	20	301.616	0.95	1.35	1.00	0.95	1.35	1.00
13	3	25	3600.00*	1.00	1.43	0.96	1.00	1.45	1.00
14	3	25	3600.00*	0.96	1.35	1.00	0.96	1.37	1.00
15	3	30	3600.00*	0.87	1.35	0.80	0.93	1.41	0.97
16	3	30	3600.00	-	-	-	0.90	1.30	0.93
17	3	35	3600.00	-	-	-	0.83	1.32	0.94
18	3	35	3600.00	-	-	-	0.77	1.27	0.94

$S = 4000$  orders and  $E = 6$  container types. Location types  $A$ ,  $B$  and  $C$  are Italian geographical locations in an area of about  $41000 \text{ km}^2$ .

The truck-to-order assignments of a single day are computed in less than 8 minutes. Note that for each sub-problem  $SP^q$  with  $q = 1, \dots, 5$ , it turns out that  $h \leq 3$ , i.e., at most 3 assignment problems  $AP^{qh}$  are solved.

Figure 1 depicts the values of the objective functions (2), (3) and (4) defined in the MILP formulation and obtained by Algorithm 1 for each single-day problem  $SP^q$  with  $q = 1, 2, \dots, 5$ .

The results show that most of the orders are effectively assigned, as highlighted by the high values of  $f_1$  (the fraction of the assigned orders) in all the working days of the week.

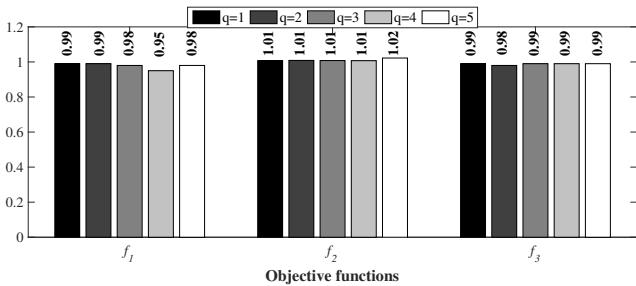


Fig. 1. Objective functions values in each single-day problem  $SP^q$ .

On the other hand, values of  $f_2 > 1$  for  $q = 1$  and  $q = 5$  are due to the requirements of starting from and returning to the depot.

Finally, the values of  $f_3$  (the fraction of on time orders) point out that the possibility of delaying an order is effectively exploited to improve  $f_1$ .

## V. CONCLUSIONS

This paper studies real Multi-Day Container Drayage Problems (MDCDPs) that take into account many complex features that are not considered together in the related literature.

We model the MDCDP as a Mixed Integer Linear Programming problem that turns out to be very complex and can solve in reasonable time only instances of small dimensions. Hence, a heuristic algorithm based on a rolling horizon approach. Some MDCDP randomly generated instances solved by both the heuristic algorithm and the MILP model show that the heuristic iterative procedure is able to find good solutions in short time. Finally, we deal with a case study of real dimensions and the results show the effectiveness of the presented solution technique that allows to obtain good performance indices.

Comparing the considered MDCDP with the drayage problems faced in the literature, we simultaneously deal with a number of different items, such as for instance: multi-day planning horizon with orders extended over two consecutive days and restrictions on the driver work hours ([3] and [4] consider a single-day planning horizon only); limited size of the truck fleet, which may force to refuse or to delay some orders ([5] and [6] assume unlimited truck fleet).

Future research will study the stochastic aspects of the orders and the use of the heuristic algorithm in the real time environments.

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