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2D simulation of wave-particle coupling inspired by walking droplets

Onofrio R Battaglia and Claudio Fazio

Dipartimento di Fisica e Chimica, University of Palermo, Italy

E-mail: onofriorosario.battaglia@unipa.it

Abstract

In recent years, a fluid dynamics phenomenon has been observed that shows interesting analogies with several quantum mechanical ones. Under specific experimental conditions, a liquid droplet released on a vibrating liquid persists in jumping, forming a localized wave-particle, and its behaviour resembles that of a de Broglie wave-particle. In this paper we discuss a simplified model for this phenomenon and the results of numerical fluid dynamics simulations implemented on the basis of the model. In spite of the relevant simplifying assumptions of our approach, we observe that a wave-droplet coupling is obtained and the droplet travels at nearly constant velocity, as it is observed in experiments. This suggests that the model describes the basic features of the phenomenon well, and that the simulation could be used to introduce undergraduate students to the study of quantum mechanics.

Keywords: physics education, fluid dynamics, modelling, simulation, walking droplet

(Some figures may appear in colour only in the online journal)

1. Introduction

An interesting fluid dynamics phenomenon has recently been observed by Y. Couder and co-workers [1]. They have shown that in an appropriate range of parameters, an oil droplet deposited on a vertically vibrating bath of the same liquid can bounce indefinitely and generate a wave in the liquid, which is strongly coupled with the droplet. The vibrating tank acts as an energy reservoir and pump, as shown in figure 1.

This phenomenon has raised some interest, as the occurrence of a persistently jumping solid body on a vibrating rigid surface is expected, but a similar occurrence when the body is a liquid droplet falling from a small height on a vibrating viscous liquid is less obvious. Moreover, with regard to the specific values of the physical parameters characterising the
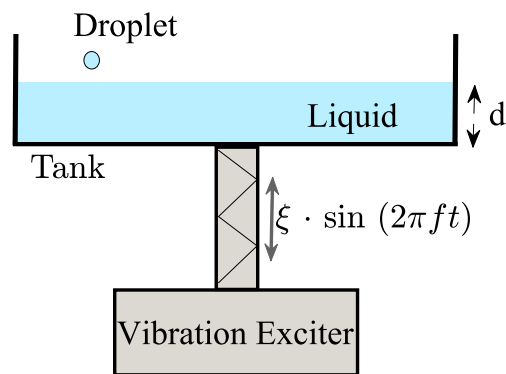
Figure 1. Diagram of the 'walking droplet' experimental apparatus.

phenomenon, both the wave and the droplet move at the same constant horizontal velocity. For this reason, this phenomenon has been called 'the walking droplet'.

In order to observe this peculiar behaviour, it is fundamental that the amplitude of the liquid vibration be very close to (actually slightly below) the Faraday instability threshold [1]. The phenomenon of the Faraday instability is well known. To explain this, let us consider a viscous liquid, subjected to vertical oscillations of a given amplitude. At low values of vibration amplitude, the fluid remains quiescent, but above the Faraday instability threshold, whose value depends on the frequency of the oscillations, which in turn depends on the surface tension, the viscosity and density of the liquid, the layer becomes unstable to a field of Faraday waves [2]. The frequencies of the resulting waves are quantized, as they are integer multiples of half the driving frequency [3]. Although higher harmonics may be excited for the silicone oils used in the walker system, the most unstable waves are subharmonic, with half the frequency of the vibrational force imposed [3].

The need to be slightly below the Faraday instability threshold is due to the fact that, in this condition, vertical oscillation of the liquid bath preconditions it for a monochromatic wave field that can be generated by particle impact [3]. Only in proximity of the droplet– liquid collision zone are waves produced.

The walking droplet phenomenon is currently being studied by several researchers, partly because it highlights features previously thought to be peculiar to the quantum realm [3–6]. This suggests it might be a good idea to discuss some peculiar characteristics of this phenomenon to introduce undergraduate students to some basic concepts of quantum mechanics, such as the de Broglie idea of wave-particle coupling that occurs on atomic and subatomic scales.

Setting up a 'walking droplet' experiment and considering the related problems and configurations does not need to be expensive or technically complicated, at least if accurate control of parameters is not required. Harris *et al* [7] presented an interesting example of a low-cost experimental apparatus that could be useful as a high school physics classroom demonstration, especially in a school laboratory. However, precision experiments require careful levelling and control of the vibrational forces [3]. Any variation from level non-uniform vibration, or spurious resonance, results in a spatial dependence of the Faraday threshold. The walker dynamics and statistics are highly sensitive to all the system parameters: drop size, fluid properties, and forcing. Consequently, repeatable experiments require the simultaneous control and documentation of each.

In this case, a computer-based simulation can help the teacher and/or the students to overcome the difficulties of the experiment and still satisfactorily study the phenomenon. For example, a simulation describing a single walking droplet could be used to introduce the students to the discrete energy levels of a quantum harmonic oscillator, or two interacting walking droplets could introduce them to the idea of electron-proton interaction in the hydrogen atom. Moreover, simulations with different boundary conditions can reproduce interesting quantum-like effects, like single-particle diffraction, tunnelling, quantised orbits and orbital level splitting [5, 6]. However, these quantum analogies still need to be analyzed in-depth (some as those regarding the double-slit experiments even have been recently contested [8, 9]) and they will not be the focus of this paper. Instead, it will describe a model and some simulation results showing the basic characteristics of the walking droplet phenomenon and also deals with its pedagogical presentation in high schools and undergraduate physics courses.

In the last decade, many research papers [10–13] have discussed theoretical models with the aim of studying the walking droplet phenomenon using computer-based simulations. They solve the Navier–Stokes equations for the liquid bath [12], including the effect of the droplet on the liquid by an ad-hoc pressure or treating the droplet as a rigid sphere [13], by using numerical methods. These methods can be very efficient, allowing results to be obtained in a short time. For instance, the method discussed by Durey and Milewski [13] is apparently 50 times faster than the experiment itself. However, to date, all these approaches have only allowed unbounded fluid domains to be considered [13] and are therefore unable to model the interaction with submerged obstacles, such as slits and barriers, when phenomena like diffraction, interference patterns and tunnelling are being studied. Two other theoretical models, very different to the one shown in this paper, have recently taken into account bounded fluid domain. The model developed by Faria [14] captures some important features of the walker-topography interactions not with boundaries but with regions where the wave speeds changes. Another 1D model developed by Nachbin *et al* [15] explores the dynamics and statistics of tunnelling and walker motion in a confined domain.

Our simulation uses Lagrangian numerical fluid dynamics methods, as these methods are more suitable than Eulerian ones when it is necessary to deal with waves deforming a liquid surface [16]. We use a smoothed particles hydrodynamics (SPH) numerical method[1] [17, 18] to simulate the motion of the liquid in the tank. The droplet is simulated by a deformable sphere made up of a small group of points that interact each other and with the discrete particles that make up the liquid by ad-hoc forces, as described below. In order not to have to deal an excessive number of particles to simulate, we do not currently introduce other particles representing air into the simulation, and to mimic the droplet-air interaction we use some simple ad-hoc physical models, as described below. The simulation starts by setting up a small container of viscous liquid, which extends along the X-axis and contains a 'slice' of liquid that vibrates along the Z-axis (vertical axis), close to the Faraday resonance frequency. A 'droplet' is released along Z on the surface of the liquid, and, with some simplifying assumptions, the dynamics of the droplet and the liquid bath can be visualised and studied.

The characteristics of our model, as we will show below, allow bounded fluid domains to be dealt with quite easily, like for instance the walls of the tank which contains the liquid. In our opinion, the possibility of obtaining results that are in good agreement with the experimental ones, by using a simulation with small number of particles, is also an added value of our approach, as it can be implemented without special computational resources.

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[1] SPH is a method for obtaining approximate numerical solutions to the equations of fluid dynamics by replacing the fluid with a set of particles [17].

2. The physical model

In order to model the walking droplet phenomenon the following relevant aspects must be taken into account:

- the oscillation amplitude of the liquid, which must be close to the Faraday instability threshold;
- the viscosity of the liquid needed to stabilize the dynamics;
- the presence of an 'air cushion', which produces a bouncing force;
- the surface tension of the liquid and of the droplet, to enhance the bouncing force caused by the air.

Specific values of the vibration frequency and amplitude, and of the droplet size, produce a coupling of the droplet and the wave, which move synchronously. In our approach, the droplet is made up of a small group of 'points', held together by an appropriate small range force, which has an effect similar to surface tension. It interacts with the particles of the liquid with an ad-hoc force, as it is described below. A liquid such as oil is considered incompressible for many fluid dynamics problems but, in this study, we consider it slightly compressible because better suited to SPH numerical method [17], and we model its pressure by using the Tait equation[17], $P = \frac{\rho_0 c_0^2}{7}[\left(\frac{\rho}{\rho_o}\right)^7 - 1]$, where the sound speed c_0 is chosen 10 times greater than the maximum speed that a fluid particle can assume. In this way the speed of sound is large enough for the density fluctuation to be negligible [17]. So, in our case $c_0 = 10\sqrt{2ad}$, where d is the height of the liquid 'slice' and a is the typical acceleration of the SPH particles.

The kinematic viscosity of the liquid is simulated on the basis of the values of an 'artificial viscosity' [17], typically used in basic SPH algorithms. Artificial viscosity is also used to avoid instabilities in the simulation and only during compression of volumes. The walls of the container are built by using a stratum of fixed particles.

2.1. The droplet surface tension

The SPH method models the surface tension of a liquid by means of appropriate mathematical interpolation criteria for the boundaries [19, 20]. Obviously, this approach requires a sufficient number of interpolating points to accurately calculate the curvature of the surface of the liquid and all the related derivatives.

In the phenomenon that we want to study, the interaction between the liquid and the droplet is certainly mediated by the air cushion above the liquid, but we decided to avoid the complexities of the two fluid treatment (liquid-air) and adopted a simpler, ad-hoc strategy. The liquid in the container has been treated with standard SPH methods, while the droplet has been considered as a set of points, whose mutual interactions provide the surface tension and give to the droplet an internal dynamics. The interaction between the droplet and the liquid is modelled by using another kind of force acting between each droplet point and the SPH particles of the liquid.

Figure 2 shows a diagram of the interactions between each pair of points in the droplet and between the droplet and the particles in the liquid. It is worth noting that the SPH particles are partially overlapped, as the SPH numerical method requires in order to work correctly[2] [17].

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[2] The equations of motion of these particles in the liquid are determined by the continuum equations of fluid dynamics with interpolation from the particles, which, therefore, have to be partially overlapping.
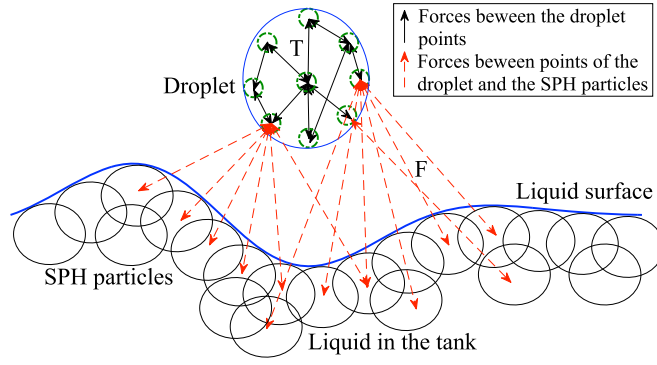
Figure 2. Schematic view of the interactions between the droplet points and between the droplet points and the SPH particles in the liquid bath. The interactions between the SPH particles are not shown.

In our model, an elastic-like force, $\bar{T}_{i,j}$, is exerted between each pair of droplet points, i and j. It is given by the following expression:

$$\bar{T}_{i,j} = -\frac{T_0}{n_{drop}}\left(\frac{r_{i,j} - \delta_{drop}}{\delta_{drop}}\right)\frac{\bar{r}_{i,j}}{r_{i,j}} \tag{1}$$

where δ_{drop} is the initial distance between the droplet points, $r_{i,j}$ is the distance between two droplet points, i and j, at the generic time t, and n_{drop} is the total number of points that make up the droplet. This value actually sets out the droplet size. T_0 is the intensity factor given by $T_0 = 0.1c_0^2\rho_0 2s$, where c_0 is the speed of sound in the liquid, ρ_0 is the density of the liquid, and s is the scaling size of the repulsive droplet–liquid force, which is described below. This force allows droplet deformations and produces a fictitious tension, whose value is not compared with the real one in this study.

2.2. The 'air force'

The interaction between the droplet points and the SPH particles of the liquid in the container is described by a repulsive force, which, as discussed above, should have the role of the air pressure. Therefore, it should increase as the droplet–liquid distance decreases. We tested a variety of functions and found that the one that gives the best simulation results is the following:

$$\bar{F}_{i,j} = F_0 e^{-\left(\frac{\bar{r}_{i,j}}{s}\right)^2}\left(\frac{\bar{r}_{i,j}}{r_{i,j}}\right) \tag{2}$$

with $F_0 = \frac{c_0^2\rho_0 2s}{n_{drop}}$. The scaling size of the repulsive droplet–liquid force, s, can be freely chosen, but it has to be of the order of magnitude of the initial SPH particle separation δ. F_0 is divided by n_{drop} so that the force depends only on the size of the droplet and is independent from the number of points that make up the droplet. This force allows us to avoid the two-phase fluid (air-liquid) treatment and it is exerted between each point in the droplet and each nearby liquid SPH particle. It goes to zero when the distance $r_{i,j}$ is much larger than the liquid particle separation $s \cong \delta$, and it is switched off once and for all if the droplet interacts with a number of liquid particles for a time interval greater than the stability time [21, 22]. This last

condition is needed to simulate the coalescence effect of the droplet. It merges into the liquid and the simulation stops.

It is worth noting that, in a range of the parameter space in which the presence of air can be neglected, the droplet–liquid interface behaves like a linear spring during droplet impact, as shown in [10]. Although this kind of force has the big advantage of an easy mathematical form, it could not be the best choice when one wants to simulate the walking droplet dynamics in a wide range of the space parameter.

3. Governing equation

The fluid governing equations, written in a Lagrangian framework are the mass conservation law:

$$\frac{d\rho}{dt} = -\rho \bar{\nabla} \cdot \bar{u} \tag{3}$$

and Newton's second law:

$$\frac{d\bar{u}}{dt} = -\frac{1}{\rho}\bar{\nabla}P + \bar{g}V(t) + \bar{a} \tag{4}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{u} \cdot \bar{\nabla}$ is the total time derivative operator, $\bar{u}$ is the velocity of a fluid element, P is the pressure in the fluid, ρ is the density, $\bar{a}$ is the local acceleration (i.e. the acceleration of a fluid elements at their positions) due to the interaction between the droplet and the fluid elements and $\bar{g}$ is the gravitational acceleration.

Since we choose to work in the frame of reference of the container, the body and the inertial forces act on both the liquid and the droplet. Therefore, the gravitational acceleration is multiplied by the factor $V(t) = 1 - \xi \sin(2\pi f t)$, where ξ is a dimensionless parameter whose value is equal to the ratio between the maximum value of the acceleration of the container and the gravitational acceleration and f is the vibration frequency. The maximum amplitude of the vibration is $Z_{\max} = \frac{\xi}{4\pi^2 f^2}$, obtained by integrating $V(t)$ over time and assuming that the initial position and velocity of the container are 0.

Each droplet point moves according to the equation:

$$\frac{d\bar{u}_i}{dt} = \bar{g}V_i - \frac{\bar{R}_i}{m_i} \tag{5}$$

where m_i is the mass of the point i and

$$\bar{R}_i = \sum_{j=1}^{n_{drop}} \bar{T}_{i,j} + \sum_{l=1}^{n_{SPH}} \bar{F}_{i,l}. \tag{6}$$

Here i refers to any droplet's point and n_{SPH} is the number of liquid particles interacting with the droplet. $\bar{T}_{i,j}$ and $\bar{F}_{i,l}$ are given by equations (1) and (2), respectively. We compute the motion of each point that makes up the droplet and not the motion of the droplet center of mass. $\bar{R}_i$ is the total force on the droplet point i. This force mimics the behaviour of air and the tension forces. To conserve the momentum, the SPH liquid particles interacting with the droplet points are subjected to opposite air forces, so we also add to each k SPH particle the reaction force $-\sum_{i=1}^{n_{drop}} \bar{F}_{i,k}$. Consequently, the $\bar{a}$ term of equation (4) for the k-th SPH particle is given by $\bar{a}_k = \frac{\sum_{i=1}^{n_{drop}} \bar{F}_{i,k}}{m_k}$.

The computational cost for the droplet–liquid interaction is proportional to $n_{drop} \cdot n_{SPH}$, where $n_{SPH} = \frac{L_X L_Z}{\delta^2}$, and L_X and L_Z are the dimensions of the container. To reduce the
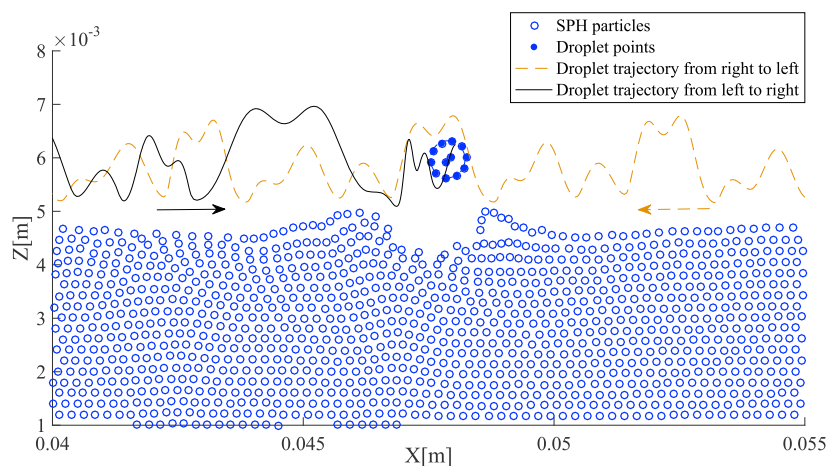
Figure 3. Points that make up the droplet (blue full circles) and the SPH particles (blue empty circles) of liquid bath. The dashed orange and the black lines represent the droplet trajectory above the liquid surface during its back and forth motion in the container. The horizontal axis represents a portion of the container.

computational cost, the interaction between points and SPH particles of the liquid in the container can be limited to the liquid surface SPH particles, which is nearly $\frac{L_X}{\delta}$.

4. Simulation results

We implemented our model by using a Fortran code and performed several simulations for different parameter values, such as the container sizes, the number of droplet points and so on. The particles were set up in equilateral triangles with sides of equal length to the initial distance of the particles, forming a regular array. Typically, the number of neighbours around each particle ranges from 12 to 30, so the criteria for good calculation of the derivatives are well satisfied. The resulting total number of particles ranges from a few thousands to 20,000.

The number of droplet points sets out the size of the drop and the degree of information we want to have on its internal dynamics. In our simulation tests it ranges from 4 to 40. These two values indicatively define an interval within which the compromise between having circular droplets (the minimum value) and having simulation times that are not too long (the maximum value) is taken into account. We tested our simulation for different vibration frequencies, but we will only discuss the $f = 66.7$ Hz case here, which is more suitable for a comparison with experimental data coming from the literature. The dimensions of the container that we simulate are $L_X = 0.1$ m, and $L_Z = 0.005$ m. $\delta = 0.00024$ m, $s = 0.00025$ m and kinematic viscosity $\nu = 4.15 \cdot 10^{-6}$ m^2/s. The reference sound speed is 20 m/s and the time duration of the simulations is about 10 s.

Figure 3 shows the SPH particles (empty blue circles)[3] and the droplet points (full blue circles). The figure refers to a case with vibration amplitude $\xi = 4.75$ and a drop made of 12 points. In this case the droplet 'walks' at a constant velocity and the black lines and the dashed orange lines represent the droplet trajectory above the surface of the liquid during its back and forth motion in the container.

[3] In this case the high number of SPH particles do not allow them to represent overlapping.
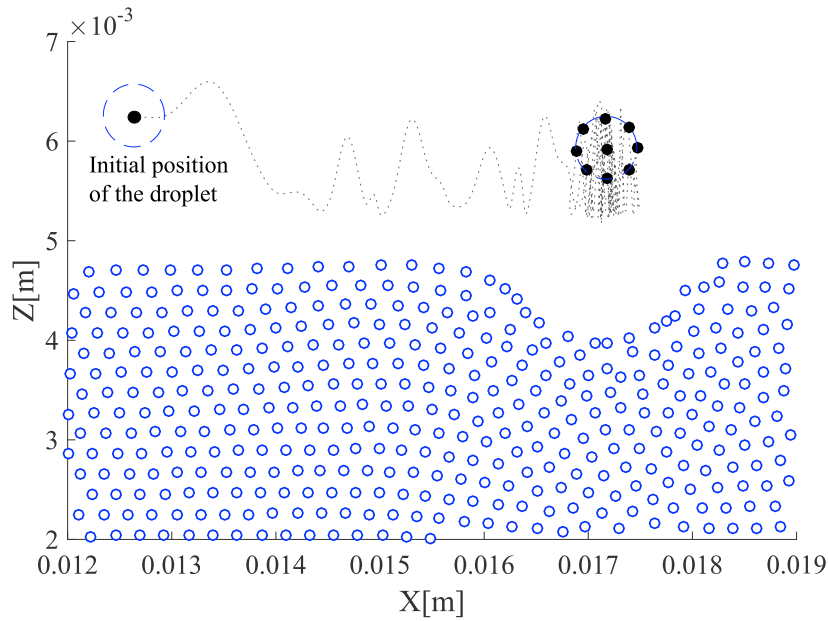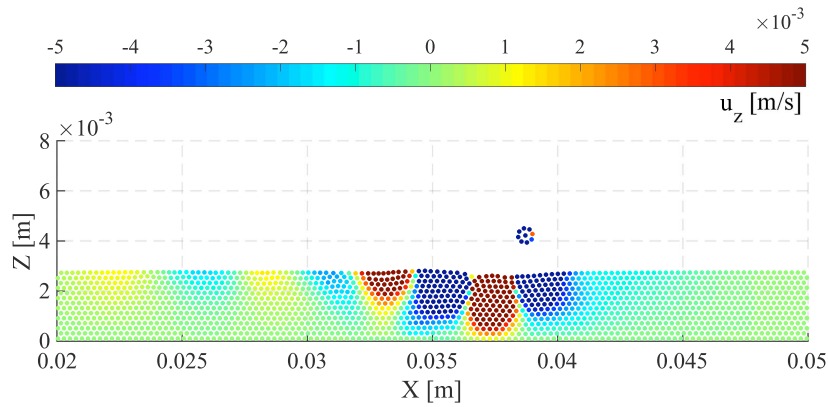
Figure 4. Points that make up the droplet (black full circles) and SPH particles (blue empty circles) of liquid bath are shown. The grey dotted line represents the droplet trajectory above the surface of the liquid. The horizontal axis represents a portion of the container.

Figure 4 shows the simulation results when ξ is equal to 3. The drop (made of nine points) does not move along the container, but after a short movement along X, it persists in oscillating in the same position.

Figure 5 shows the simulation result obtained $\xi = 3.5$ and a droplet made of nine points. The velocities of the droplet points and of the particles in the liquid bath are represented by using different colours. These highlight the presence of waves in the liquid bath, showing a velocity field whose values are significantly different from zero only close to the droplet.

Figure 6 shows the u_X and the scaled u_Z velocity component of the droplet versus time for the same conditions as the previous one. It refers to a view in a short temporal window of the data (0.495–0.7 s). The graph grid is set at 0.015 s intervals. The horizontal velocity is nearly constant; the spikes are due to the 'impact' of the droplet with the liquid surface and denote the synchronisation effect between the droplet and the wave in the liquid bath [12]. Double period bouncing is clearly represented. Its trend is very similar to the experimental one obtained by Milewski [12]. This result is relevant, as it can occur only in a restricted, fine-tuned range of the parameter values [12].

Figure 7 shows the motion states of the droplet for various values of ξ and n_{drop} (i.e. the 'droplet size'). All the other simulation parameters are equal to the ones previously used. The symbols (blue points, red stars, etc) allow us to identify zones on the graph in which the droplet motion states are different. In the 'Walking' state (W-graph zone identified by red stars) the droplet continuously moves ('walks') along the X-axis and bounces above the liquid. In the 'Walking-NotWalking' state (WNW-violet triangles) the droplet still bounces above the liquid, but alternates between walking at a constant speed and time intervals in which it does not walk. The larger ξ is, the smaller is the time interval in which the droplet does not walk. In the 'Disturbed walking' state (DW-black diamonds) the Faraday instability

Figure 5. Points that make up the droplet and SPH particles of liquid bath are shown. u_z velocities component are reported by using different colours.
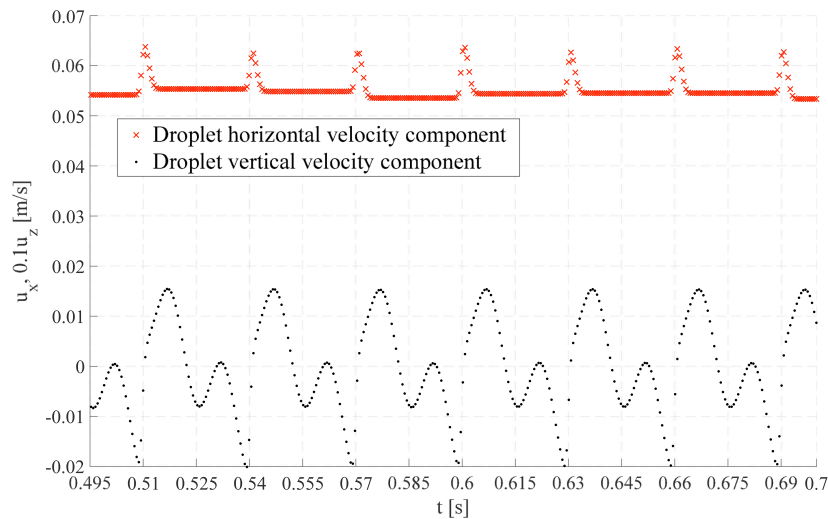

Figure 6. The droplet u_X (red crosses) and u_Z (black dots, values divided by a factor of 10) velocity components versus time.

threshold has been exceeded. The droplet still bounces and walks, but its horizontal motion is disturbed. In the 'Delayed-Walking' state (W*-green triangles) the droplet starts walking after a short time interval. In the 'Bouncing' state (B-blue crosses) the droplet only bounces above the liquid and does not move horizontally. Finally, in the 'Coalescing' state (C-yellow squares) the droplet coalesces. For instance, looking at figure 7 it is clear that a droplet made of 12 particles subjected to $\xi = 4.5$ vibration amplitude definitely walks, while the same droplet simply bounces when $\xi = 3.5$. It should also be pointed out that the trend of the amplitude values ξ for which the droplet coalesces is similar to the experimental one [23].

The behaviour of the droplet also depends on other parameters, like the kinematic viscosity, ν, of the liquid. Figure 8 shows the results of a simulation for a fixed n_{drop} value
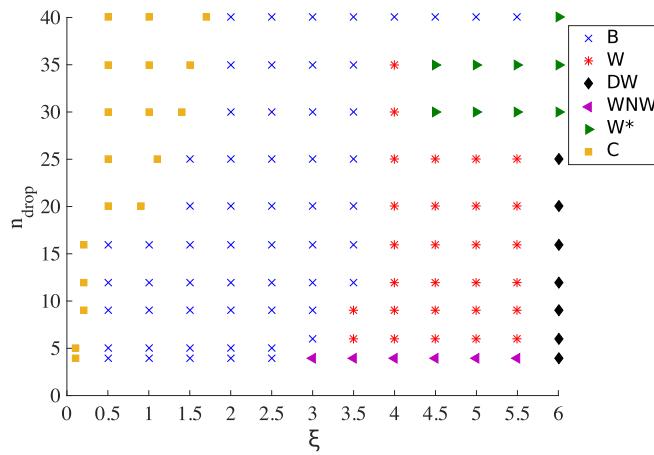
Figure 7. Behaviour of the droplet for $f = 66.7$ Hz and kinematic viscosity of the liquid, $\nu = 4.15 \cdot 10^{-6}$ m^2/s at different values of n_{drop} and ξ. Each point in the graph represents a state of the droplet obtained for different simulations parameters. In the key B stands for the 'Bouncing' state, W for 'Walking', DW for 'Disturbed Walking', WNW for 'Walking-NotWalking', 'W*' for 'Delayed-Walking' and finally C for 'Coalescing'.
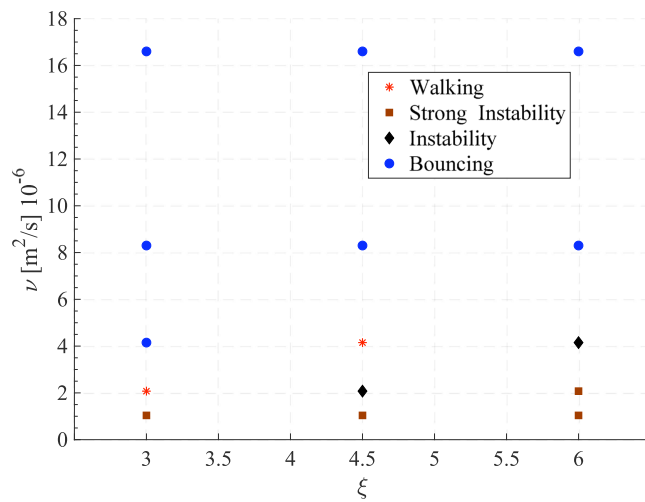
Figure 8. Behaviour of the droplet for $f = 66.7$ Hz and $n_{drop} = 12$ at different values of kinematic viscosity ν and vibration amplitude ξ. Each point in the graph represents a state of the droplet obtained with different simulations parameters.

and different ν and ξ. For example, when ξ is 4.5, the droplet simply bounces above the liquid for ν greater than $4.15 \cdot 10^{-6}$ m^2/s, while at $\nu = 2.1 \cdot 10^{-6}$ m^2/s the walking of the droplet is disturbed because the Faraday instability threshold has been exceeded. At $\nu = 1.0 \cdot 10^{-6}$ m^2/s the liquid's instability is so high that bouncing and walking by the droplet along the X-axis are both prevented. In this case, after a few cycles the simulation stops.

5. Conclusions

In this paper we have presented a very simple model of the walking droplet phenomenon. We have shown that with a numerical simulation based on this model and an SPH Lagrangian code it is possible to reproduce the basic aspects of this phenomenon. For instance, when the wave-droplet coupling is well established, the droplet has a horizontal velocity whose average is constant as it is shown in the experimental results.

We have studied the droplet behaviour in relation to some fundamental quantities, such as the vibration amplitude, the liquid viscosity and the droplet size, qualitatively comparing it with that shown experimentally and founding a good agreement.

High school and undergraduate students can explore the phenomenon by 'playing' with our simulation, adjusting some fundamental quantities by using the simulation parameters. In this way teachers and students can be able to build some graphs like those shown in figures 6 and 7 by using our simulation as a 'tool' for analysing and discussing the droplet behaviour in several different conditions.

Our study of the phenomenon does not explicitly take into account the presence of air particles between the droplet and the liquid, or the 3D nature of the phenomenon itself. A more realistic reproduction of the experimental results, like the ones obtained by Couder *et al* [4] and Milewski [12], could be obtained by setting up a simulation that takes into account the parameters above considered better. Work is in progress to find a way of carrying out a simulation in which the container is fully extended in the X–Y plane and the interaction between the droplet and the surrounding air is dealt with by taking into account the presence of air particles instead of an ad-hoc air force.

Acknowledgments

ORCID iDs

Onofrio R Battaglia ● https://orcid.org/0000-0002-0250-0514

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