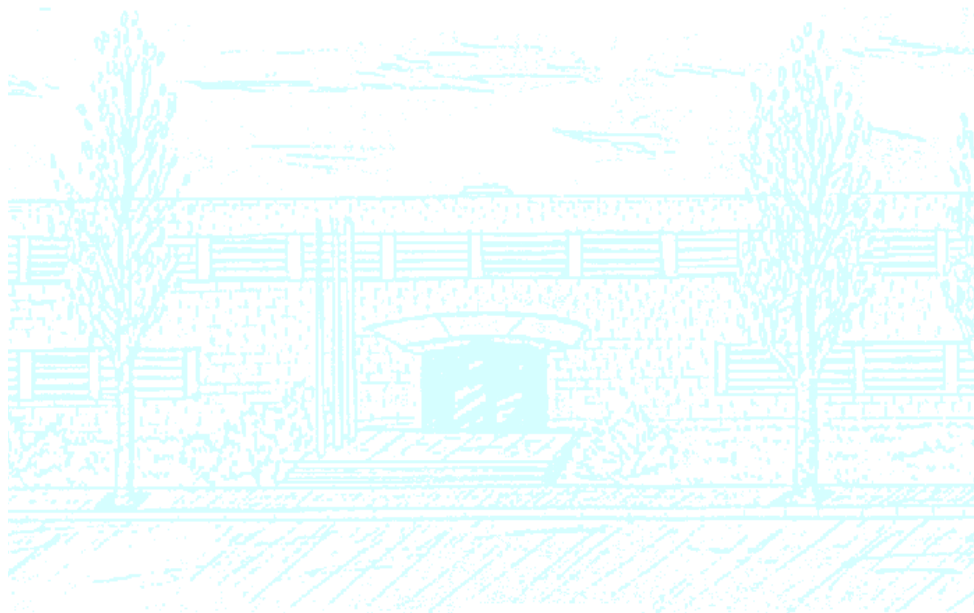# Degree in Mathematics

BACHELOR'S THESIS

Title: Analytical solution of the electromagnetic field of a current carrying conductor in multi-layer media

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Analytical solution of the electromagnetic field of a current carrying conductor in multi-layer media for wireless power transfer applications
Thesis

Submitted in partial fulfillment of the requirements for the degree of Bachelor of Science (Mathematics and Industrial Engineering) at the POLYTECHNIC UNIVERSITY OF CATALONIA

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Vita
Pablo Esteban de la Iglesia
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Educational history

I completed all my bachelor of science in mathematics and industrial engineering in the Polytechnic University of Catalonia, Spain, except my thesis, which I am currently doing at the New York University.

I started working at the NYU Power Lab on September 19th, 2017 and I have been working there for my entire project to finish the thesis.

This thesis is a continuation of Jaimin Jha´s Master of Science thesis, so many calculations and conclusions reached by him are used here.

Wireless power transfer (WPT) is the transmission of electrical power without wires as a physical link. Wireless power transmission technologies use time-varying electric, magnetic, or electromagnetic field. Wireless power transfer is useful to power electrical devices where interconnecting wires are inconvenient, hazardous or impossible.

This thesis uses some of the calculations and conclusions made by Jaimin Jha in his Master of Science´s thesis. And some of those results are used to make calculations and arriving at conclusions.

An example is solved in order to find a general solution for this problem, which is an infinite filament carrying current with parallel plates. The results portrayed are the magnetic vector potential. If this vector is known, the magnetic field can be easily obtained.

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Nomenclature

$\vec{A}_z$: Magnetic vector potential

$\vec{B}$: Magnetic vector

$\vec{J}$: Current density

$\vec{E}$: Electrical field

$\vec{H}$: Magnetic field strength

$\delta_0(x)$: Dirac´s delta

μ: Magnetic permeability

σ: Conductivity of the material

ε: Electrical permittivity

PDE: Partial differential equation

ODE: Ordinary differential equation

1. Introduction

1.1 Wireless power transfer

Wireless power transfer is the transmission of electrical power without any wire or physical contact. Nowadays it has many applications such as magnetic levitated trains or charging many electrical devices.

In this thesis, a 2D model analysis of the magnetic vector potential is made. This model is made by a single carrying current filament, which is infinitely long. Underneath this filament, there is a ferrite material and, below the ferrite material, there is an aluminum shielding, which, is all surrounded by air.

Using Maxwell equations a partial differential equation (PDE) that allows the problem to be solved is obtained. Due to the fact that solving this PDE in a 2D model is very complex, a method called "separation of variables" or "Fourier method" is used. This method is frequently used when solving PDEs. In order to apply this method, all known variables must be decomposed in Fourier series. When the PDE is solved, an algorithm is created. This algorithm is slow, therefore a faster one is needed.

In order to verify the reliability of the algorithm, the algorithm is compared with the results using the finite element method (FEM). The program used to obtain the solution of FEM method is "ANSYS electronics desktop". Both solutions are compared, and if the results are similar, the algorithm is validated.

1.2 The model

The 2D model is made of a ferrite core with a thickness of 4 mm over an aluminum shielding with a thickness of 3.5 mm, and the infinitesimal filament, which has the current, is above the ferrite core. This is all surrounded by air. The current´s frequency is 85,000 Hz and its intensity is 10 A. It must be remembered that the peak value is not 10 A, but $10 \cdot \sqrt{2}$ A.
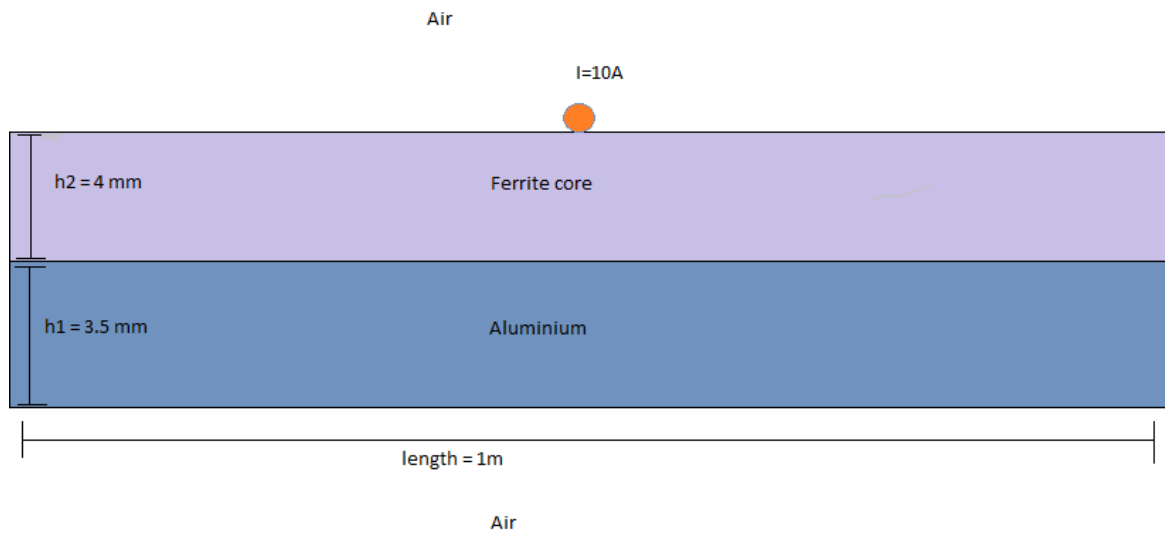
1.3 The analysis

With Maxwell equations a partial differential equation (PDE) that can be solved decomposing in Fourier series is obtained. The following assumptions are made:

1. Magnetic saturation is neglected.

2. On a surface, B_{normal} is continuous (consequence of Gauss law in magnetic field).

3. The magnetic field disappears when the current filament disappears.

2. Maxwell equations

2.1 Introduction

This chapter is taken in its entirely from Jaimin Jha Master of Science thesis. In this part, Maxwell equations are used in order to obtain the equation to solve the problem. These are the equations:

1. Gauss law in electric field. $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon}$
2. Gauss law in magnetic field. $\vec{\nabla} \cdot \vec{B} = 0$
3. Faraday-Lenz law. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
4. Ampere law. $\vec{\nabla} \times \vec{H} = \vec{J}$

Saturation of materials is not considered, so the materials have linear behavior. Thus, the material properties are:

$$\vec{B} = \mu \cdot \vec{H}$$
$$\vec{J} = \sigma \cdot \vec{E}$$

The magnetic flux density can be computed from the magnetic vector potential as:

$$\vec{\nabla} \times \vec{A} = \vec{B}$$

2.2 PDE to solve

First of all, it must be taken into account that the model is 2D, with infinite dimensions on the z-axis. This means the magnetic flux density does not exist in this direction, therefore $B_z = 0$. As a consequence, $A_x = A_y = 0$. The aim is to obtain A_z in order to calculate the magnetic field.

The current in the conductor has a frequency of 85 kHz, so $\omega_t = 2 \cdot \pi \cdot 85000$, as a consequence $\frac{\partial \vec{B}}{\partial t} = j \cdot \omega_t \cdot \vec{B}$.

Now, the equations mentioned before are used.

$$\vec{\nabla} \times \vec{E} \ = \ -\frac{\partial \vec{B}}{\partial t} = \ -j \cdot \omega_t \cdot \vec{B} \text{ (eq 1)}$$
$$\vec{\nabla} \times \vec{H} \ = \vec{J} \text{ (eq 2)}$$
$$\vec{B} = \mu \cdot \vec{H} \text{ (eq 3)}$$
$$\vec{J} = \sigma \cdot \vec{E} \text{ (eq 4)}$$
$$\vec{\nabla} \times \vec{A} = \vec{B} \text{ (eq 5)}$$

From eq 2 and eq 3:

$$\vec{\nabla} \times \vec{H} \ = \vec{\nabla} \times \left(\frac{\vec{B}}{\mu}\right) \ = \vec{J} \text{ (eq 6)}$$

using eq 5,
$$\vec{\nabla} \times \left(\frac{\vec{B}}{\mu}\right) \ = \vec{\nabla} \times \left(\frac{\vec{\nabla} \times \vec{A}}{\mu}\right) \ = \vec{J} \text{ (eq 7)}$$

using eq 4,
$$\vec{\nabla} \times \left(\frac{\vec{\nabla} \times \vec{A}}{\mu}\right) \ = \vec{J} = \sigma \cdot \vec{E} \text{ (eq 8)}$$

from eq 1 and eq 5,
$$\vec{\nabla} \times \vec{E} \ = \ -j \cdot \omega_t \cdot \vec{B} \ = \ -j \cdot \omega_t \cdot (\vec{\nabla} \times \vec{A}),$$
so,
$$\vec{E} = \ -j \cdot \omega_t \cdot \vec{A}. \text{ (eq 9)}$$

This happens because there is no electromotive force or charge distribution, so the gradient $\nabla \phi$ is non-existent. So $\nabla \phi = 0$.

Finally, using eq 8 and eq 9 it is obtained:
$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \ -j \cdot \omega_t \cdot \mu \cdot \sigma \cdot \vec{A} \text{(eq 10)}$$

As mentioned before, we are working with an infinite straight model on the *z*-axis, so the properties are non-variant in the *z* direction. Thus we have:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A})|_z = \frac{\partial}{\partial x}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) = -j \cdot \omega_t \cdot \mu \cdot \sigma \cdot A_z$$

$$\frac{\partial A_x}{\partial z} = \frac{\partial A_y}{\partial z} = 0,$$

And

$$\frac{\partial}{\partial x}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) = -\Delta A_z = -j \cdot \omega_t \cdot \mu \cdot \sigma \cdot A_z.$$

And finally, the PDE is obtained:

$$\boldsymbol{\Delta A_z = j \cdot \omega_t \cdot \mu \cdot \sigma \cdot A_z} \textbf{ (eq 11)}$$

So this PDE must be solved. The boundary conditions are:

1. The magnetic field is 0 at infinity; as a consequence, A_z is a constant.

2. The current in the filament is known.

The solution of this PDE is the objective of this thesis.

In chapter 4, this PDE is solved with a method called "separation of variables". In order to use this method, the known variable (current intensity and frequency in the filament) has to be decomposed in Fourier series. The following chapter provides a brief summary of the Fourier series.

3. Fourier analysis

3.1 Introduction

This section is a short summary of the Fourier series, due to the fact that they are used in order to solve the PDE. This is a summary can be found in any book of real analysis that has a section of the Fourier series. The important result is obtained in point 3.3. This result is a fast way to obtain the Fourier series of a function using complex exponential series.

Some linear algebra´s properties are also considered so as to understand better the Fourier series properties.

3.2 Basic properties

First of all, some terminology that is used is mentioned. It always works in a vectoral space E and F.

1. If $v, w \in E$, the scalar product is $\langle v, w \rangle$.
2. v, w are orthogonal if $\langle v, w \rangle = 0$, and it is represented as $v \perp w$.
3. The norm of a vector v is $||v|| = \sqrt{\langle v, v \rangle}$. $||v|| \geq 0$, and $||v|| = 0 \Leftrightarrow v = 0$.
4. If $E \subset F$, and $v \in F$, the orthogonal projection of v in E is the vector w which satisfies $\min_{w \in E} ||v - w||$.

The orthogonal projection can be calculated when an orthonormal base of E is known. With the orthonormal base w_i, the vector w can be calculated as $w = \sum_{i=1}^{n} a_i w_i$, where n is the dimension of E and $a_i = \langle w_i, v \rangle$.

In this case, the vectoral space F it is the functions when $\int_a^b f^2(x)\, dx$ and $\int_a^b f(x)\, dx$ exists. The scalar product of f and g is $\langle f, g \rangle = \int_a^b f(x) \cdot g(x) \cdot dx$. The orthonormal base of E is

$\alpha \cdot cos(k \cdot \omega \cdot x)$, $\sqrt{\frac{1}{b-a}}$, $\beta \cdot sin(k \cdot \omega \cdot x)$, where $\omega = \frac{2 \cdot \pi}{b-a}$, k is a positive

integer number and $\alpha = \beta = \sqrt{\frac{2}{b-a}}$.

So if it is wanted to project the function f in E, the steps mentioned before must be applied, $w = \sum_{k \geq 1} \alpha_k \cdot \alpha \cdot cos(k \cdot \omega \cdot x) + \sum_{k \geq 1} \beta_k \cdot \beta \cdot sin(k \cdot \omega \cdot x) + c \cdot \sqrt{\frac{1}{b-a}}$.

So it has to be calculated α_k, β_k and c.

$$\alpha_k = \langle f, \alpha \cdot cos(k \cdot \omega) \rangle = \alpha \cdot \int_a^b f(x) \cdot cos(k \cdot \omega \cdot x) \cdot dx.$$

$$\beta_k = \langle f, \beta \cdot sin(k \cdot \omega) \rangle = \beta \cdot \int_a^b f(x) \cdot sin(k \cdot \omega \cdot x) \cdot dx.$$

$$c = \langle f, \sqrt{\frac{1}{b-a}} \rangle = \sqrt{\frac{1}{b-a}} \cdot \int_a^b f \cdot dx.$$

And finally getting:

$$w(x) = \sum_{k \geq 1} \alpha_k \cdot \alpha \cdot cos(k \cdot \omega \cdot x) + \sum_{k \geq 1} \beta_k \cdot \beta \cdot sin(k \cdot \omega \cdot x) + c \cdot \sqrt{\frac{1}{b-a}} =$$

$$= \sum_{k \geq 1} \alpha \cdot \langle f, cos(k \cdot \omega) \rangle \cdot \alpha \cdot cos(k \cdot \omega) +$$

$$+ \sum_{k \geq 1} \beta \cdot \langle f, sin(k \cdot \omega) \rangle \cdot \beta \cdot sin(k \cdot \omega \cdot x) + \sqrt{\frac{1}{b-a}} \cdot \sqrt{\frac{1}{b-a}} \cdot \langle f, 1 \rangle =$$

$$= \sum_{k \geq 1} \frac{2}{b-a} \cdot \langle f, cos(k \cdot \omega) \rangle \cdot cos(k \cdot \omega \cdot x) +$$

$$+ \sum_{k \geq 1} \frac{2}{b-a} \cdot \langle f, sin(k \cdot \omega) \rangle \cdot sin(k \cdot \omega \cdot x) + \frac{1}{b-a} \cdot \langle f, 1 \rangle.$$

Now a formula that lets the orthogonal projection in the base to be calculated. It must be mentioned the Fourier series are calculated with a finite number of terms. The Fourier series is not the best punctual approximation of any function, it is the best approximation of any function on an interval (a, b) with the norm mentioned before, and the

only conditions are that $\int_a^b f^2(x)\,dx$ and $\int_a^b f(x)\,dx$ must exist. Also, the Fourier series are the best approximation in an interval. If it is wanted to extend the Fourier series to a bigger interval, the result achieved is a periodic function, because this result is a sum of periodic functions.

The best punctual approximation of a function is the Taylor series, but the function must be as many times derivable as the number of terms desired. This condition is more restrictive than the Fourier series conditions.

3.3 Exponential Fourier series

At this point the Fourier series using complex numbers is obtained. It must be remembered that $e^{j \cdot x} = cos(x) + j \cdot sin(x)$.

In this case, the base is $e^{j \cdot k \cdot \omega \cdot x}$ where k is an integer number. As seen, this base is not orthonormal, so the formula aforementioned can not be applied. Another formula is used, which is proved later.

$w(x) = \sum_{k=-\infty}^{\infty} \gamma_k \cdot e^{j \cdot k \cdot \omega \cdot x}$. Where $\gamma_k = \frac{1}{b-a} \cdot \int_a^b e^{-j \cdot k \cdot \omega \cdot x} \cdot f(x) \cdot dx$.

As it can be easily proved, $\gamma_k = \frac{1}{b-a} \cdot \int_a^b e^{-j \cdot k \cdot \omega \cdot x} \cdot f(x) \cdot dx =$

$$= \frac{1}{b-a} \cdot \int_a^b (cos(-k \cdot \omega \cdot x) + j \cdot sin(-k \cdot \omega \cdot x)) \cdot f(x) \cdot dx =$$

$\frac{1}{b-a} \cdot (\langle f, cos(k \cdot \omega) \rangle - j \cdot \langle f, sin(k \cdot \omega) \rangle)$, if $k \neq 0$.

If $k = 0$, $\gamma_0 = \frac{1}{b-a} \cdot \int_a^b e^{-j \cdot 0 \cdot \omega \cdot x} \cdot f(x) \cdot dx = \frac{1}{b-a} \cdot \langle f, 1 \rangle$. So:

$w(x) = \sum_{k=-\infty}^{\infty} \gamma_k \cdot e^{j \cdot k \cdot \omega \cdot x} =$

$= \gamma_0 \cdot e^{j \cdot 0 \cdot \omega \cdot x} + \sum_{k \geq 1} (\gamma_k \cdot e^{j \cdot k \cdot \omega \cdot x} + \gamma_{-k} \cdot e^{-j \cdot k \cdot \omega \cdot x}) =$

$$= \gamma_0 + \sum_{k \geq 1} (\gamma_k \cdot e^{j \cdot k \cdot \omega \cdot x} + \gamma_{-k} \cdot e^{-j \cdot k \cdot \omega \cdot x}) = \frac{1}{b-a} \cdot \langle f, 1 \rangle +$$

$$+ \sum_{k \geq 1} \frac{1}{b-a} \cdot (\langle f, cos(k \cdot \omega) \rangle - j \cdot \langle f, sin(k \cdot \omega) \rangle)$$

$$\cdot (cos(k \cdot \omega \cdot x) + j \cdot sin(k \cdot \omega \cdot x)) +$$

$$\sum_{k \geq 1} \frac{1}{b-a} \cdot (\langle f, cos(-k \cdot \omega)\rangle - j \cdot \langle f, sin(-k \cdot \omega)\rangle)$$

$$\cdot ((cos(-k \cdot \omega \cdot x) + j \cdot sin(-k \cdot \omega \cdot x)) =$$

$$= \frac{1}{b-a} \cdot \langle f, 1 \rangle +$$

$$+ \sum_{k \geq 1} \frac{1}{b-a} \cdot (\langle f, cos(k \cdot \omega)\rangle \cdot cos(k \cdot \omega \cdot x) + \langle f, sin(k \cdot \omega)\rangle \cdot sin(k \cdot \omega \cdot x))$$

$$+ \sum_{k \geq 1} \frac{1}{b-a} \cdot j \cdot (\langle f, cos(k \cdot \omega)\rangle \cdot sin(k \cdot \omega \cdot x) - \langle f, sin(k \cdot \omega)\rangle$$

$$\cdot cos(k \cdot \omega \cdot x)) +$$

$$+ \sum_{k \geq 1} \frac{1}{b-a} \cdot (\langle f, cos(k \cdot \omega)\rangle \cdot cos(k \cdot \omega \cdot x) + \langle f, sin(k \cdot \omega)\rangle$$

$$\cdot sin(k \cdot \omega \cdot x)) +$$

$$+ \sum_{k \geq 1} \frac{1}{b-a} \cdot j \cdot (-\langle f, cos(k \cdot \omega)\rangle \cdot sin(k \cdot \omega \cdot x) + \langle f, sin(k \cdot \omega)\rangle$$

$$\cdot cos(k \cdot \omega \cdot x)) =$$

$$= \frac{1}{b-a} \cdot \langle f, 1 \rangle + \sum_{k \geq 1} \frac{2}{b-a} \cdot (\langle f, cos(k \cdot \omega)\rangle \cdot cos(k \cdot \omega \cdot x))$$

$$+ \langle f, sin(k \cdot \omega)\rangle \cdot sin(k \cdot \omega \cdot x))$$

As seen, it is the same expression that is obtained in 3.2, so it implies that the Fourier series with this formula can be obtained. This way to obtain the Fourier series is very useful as seen later.

4. Analytical solution
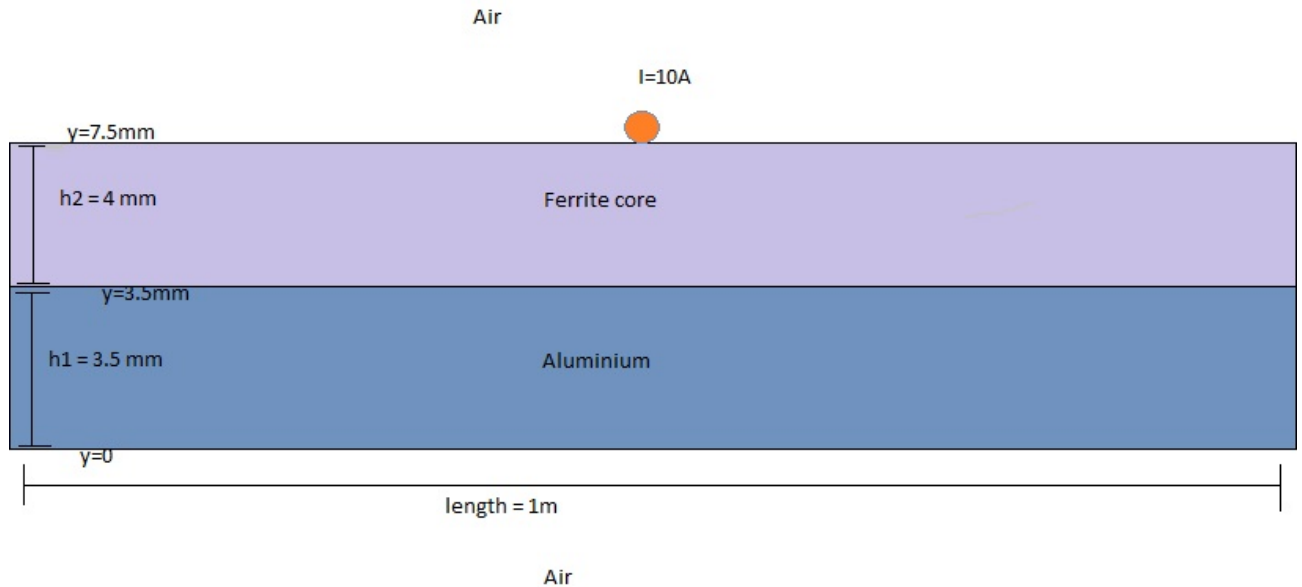
4.1 Introduction

In this chapter, equation 11 obtained. The known conditions are the magnetic vector potential in the infinite, which is a constant, and the current density at $y = 7.5\ mm$.

Now there is a 2D problem. Solving this problem directly is very complex; in fact, it may be impossible. As mentioned before, a method called "substitution of variable" is used.

It is known $\vec{J}(x, 7.5\ mm, t) = 10 \cdot \sqrt{2} \cdot e^{j \cdot \omega_t \cdot t} \cdot \delta_0(x)$. Now and in the future, it is supposed $t = 0$, so if the value is to be obtained in a determined instant in t, the result obtained when $t = 0$ must be multiplied by $e^{j \cdot \omega_t \cdot t}$. This is also true for the others variables.

Now, $\vec{J}(x, 7.5\ mm, 0) = 10 \cdot \sqrt{2} \cdot \delta_0(x)$, if this variable is decomposed in its Fourier series with the formula obtained in 3.3, it is finally obtained $\vec{J}(x, 7.5\ mm) \approx \sum_{k=-n}^{n} \gamma_k \cdot e^{j \cdot k \cdot \omega \cdot x} = \sum_{k=-n}^{n} \frac{10 \cdot \sqrt{2}}{l} \cdot e^{j \cdot k \cdot \omega \cdot x}$.

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Where $\omega = \frac{2 \cdot \pi}{l}$. It must be remembered when a function decomposes in its Fourier series, the function is approximated in an interval with periodic functions. As a consequence, a problem where there is a current filament every l meters is solved. However, if l is chosen large enough, this is not an issue, because there are not interferences between them.

Let $\vec{J}_k(x, 7.5\ mm)$ be $\vec{J}_k(x, 7.5\ mm) = \frac{10 \cdot \sqrt{2}}{l} \cdot e^{j \cdot k \cdot \omega \cdot x}$, so $\vec{J} \approx \sum_{k=-n}^{n} \vec{J}_k$. Now the same PDE is solved, but the initial condition is $\vec{J}_k$ instead of $\vec{J}$. The PDE is transformed into an ordinary differential equation (ODE) that can be solved. Let $A_{z,k}(x, y)$ be the solution of this ODE. The idea of "substitution of variable" is, because the linear behavior of materials is assumed, and $\vec{J} \approx \sum_{k=-n}^{n} \vec{J}_k$, as a consequence, $A_z(x, y) = \sum_{k=-n}^{n} A_{z,k}(x, y)$, which is the desired solution.

4.2 One-dimensional case

This point is fully obtained from Jaimin Jha Master of Science.

The objective here is to solve the PDE when the current density is at $y = 7.5\ mm$ is $\vec{J}_k(x, 7.5\ mm) = \frac{10 \cdot \sqrt{2}}{l} \cdot e^{j \cdot k \cdot \omega \cdot x}$ instead of $\vec{J}(x, 7.5\ mm) = 10 \cdot \sqrt{2} \cdot \delta_0(x)$. Due to the fact that $\vec{J}_k$ is periodic in x-direction, $A_{z,k}(x, y) = A_{z,k}(0, y) \cdot e^{j \cdot \omega \cdot k \cdot x}$. Let $A_{z,k}(y)$ be $A_{z,k}(0, y)$.

$$\frac{\partial^2 A_{z,k}(x, y)}{\partial x^2} = \frac{\partial^2 e^{j \cdot k \cdot \omega \cdot x}}{\partial x^2} \cdot A_{z,k}(y) = (j \cdot \omega \cdot k)^2 \cdot e^{j \cdot \omega \cdot k \cdot x} \cdot A_{z,k}(y) =$$
$$= -(\omega \cdot k)^2 \cdot e^{j \cdot \omega \cdot k \cdot x} \cdot A_{z,k}(y)$$

The PDE that has to be solved is: $\Delta A_z = j \cdot \omega_t \cdot \mu \cdot \sigma \cdot A_z$

$$\Delta A_z = \frac{\partial^2 A_{z,k}(x, y)}{\partial x^2} + \frac{\partial^2 A_{z,k}(x, y)}{\partial y^2} =$$
$$= \frac{\partial^2 A_{z,k}(y) \cdot e^{j \cdot \omega \cdot k \cdot x}}{\partial x^2} + \frac{\partial^2 A_{z,k}(y) \cdot e^{j \cdot \omega \cdot k \cdot x}}{\partial y^2} =$$

$$= -(\omega \cdot k)^2 \cdot e^{j \cdot \omega \cdot k \cdot x} \cdot A_{z,k}(y) + \frac{\partial^2 A_{z,k}(y)}{\partial y^2} \cdot e^{j \cdot \omega \cdot k \cdot x} =$$

$$= j \cdot \omega_t \cdot \mu \cdot \sigma \cdot A_{z,k}(y) \cdot e^{j \cdot \omega \cdot k \cdot x}$$

$e^{j \cdot \omega \cdot k \cdot x} \neq 0 \ \forall x$, So the last result is simplified as:

$$-(\omega \cdot k)^2 \cdot A_{z,k}(y) + \frac{\partial^2 A_{z,k}(y)}{\partial y^2} = j \cdot \omega_t \cdot \mu \cdot \sigma \cdot A_{z,k}(y)$$

as a consequence:

$$\frac{\partial^2 A_{z,k}(y)}{\partial y^2} = ((\omega \cdot k)^2 + j \cdot \omega_t \cdot \mu \cdot \sigma) \cdot A_{z,k}(y)$$

An ODE that can be solved is finally obtained. Now, in order to solve the ODE, it must be considered:

(1) $B \to 0$ when $y \to \pm\infty$, so $A_{z,k} \to cte$ when $y \to \pm\infty$, where cte is a constant which could be different when $y \to \infty$ and $y \to -\infty$.

(2) B_x is never infinite (with the exception of the points near the current sheet), so this means $B_x = \frac{\partial A_{z,k}}{\partial y}$ is not infinite. So, $A_{z,k}$ is continuous on the y direction.

(3) If a current sheet J exists between two surfaces, $H_{x,i} + J = H_{x,i+1}$. Where $H_{x,i} = \frac{1}{\mu_i} \cdot \frac{\partial A_i}{\partial y}$.

Now there are 2 different cases, $k \neq 0$ and $k = 0$. With the assumptions mentioned before, both cases are solved.

1. $k \neq 0$.
$\frac{\partial^2 A_{z,k}(y)}{\partial y^2} = ((\omega \cdot k)^2 + j \cdot \omega_t \cdot \mu \cdot \sigma) \cdot A_{z,k}(y)$. As $\omega \cdot k \neq 0$ always, $((\omega \cdot k)^2 + j \cdot \omega_t \cdot \mu \cdot \sigma) \neq 0$, because its real part is never zero. Let α be; $\alpha = \sqrt{((\omega \cdot k)^2 + j \cdot \omega_t \cdot \mu \cdot \sigma)}$ so the solution of this ODE is:

$$\frac{\partial^2 A_{z,k}(y)}{\partial y^2} = \alpha^2 \cdot A_{z,k}(y), \; A_{z,k}(y) = c_{i,1} \cdot e^{-\alpha \cdot y} + c_{i,2} \cdot e^{\alpha \cdot y},$$ for any

region that is in the model:

Region I: air

$$\lim_{y \to -\infty} B_{x,k} = \lim_{y \to -\infty} \frac{\partial A_{z,k}(y)}{\partial y} = \lim_{y \to -\infty} \left(-\alpha \cdot c_{1,1} \cdot e^{-\alpha \cdot y} + \alpha \cdot c_{1,2} \cdot e^{\alpha \cdot y} \right) = 0$$

$$\Rightarrow c_{1,1} = 0$$

Region IV: air

$$\lim_{y \to \infty} B_{x,k} = \lim_{y \to \infty} \frac{\partial A_{z,k}(y)}{\partial y} = \lim_{y \to \infty} \left(-\alpha \cdot c_{4,1} \cdot e^{-\alpha \cdot y} + \alpha \cdot c_{4,2} \cdot e^{\alpha \cdot y} \right) = 0$$

$$\Rightarrow c_{4,2} = 0$$

It must considered $\sigma_{air} = \sigma_{fe} = 0 \Rightarrow \alpha_{air} = \alpha_{fe} = k \cdot \omega$. And also
$\mu_{air} = \mu_{al} = \mu_0$ and $\mu_{fe} = \mu_{r,fe} \cdot \mu_0$. α_{al} is named α

The previous equations are applied:
In the boundary between "Region I" and "Region II", $y_1 = 0$:
$A_{z,k}$ is continuous in $y = y_1$, so $c_{1,2} \cdot e^{k \cdot \omega \cdot y_1} = c_{2,1} \cdot e^{-\alpha \cdot y_1} + c_{2,2} \cdot e^{\alpha \cdot y_1} \Rightarrow$

$$\Rightarrow c_{1,2} \cdot e^{k \cdot \omega \cdot y_1} - c_{2,1} \cdot e^{-\alpha \cdot y_1} - c_{2,2} \cdot e^{\alpha \cdot y_1} = 0$$

Between both surfaces there is no current, so $H_{x,1} = H_{x,2}$:

$$\frac{1}{\mu_1} \cdot \frac{\partial A_{z,1}}{\partial y} = \frac{1}{\mu_2} \cdot \frac{\partial A_{z,2}}{\partial y} \Rightarrow \frac{1}{\mu 0} \cdot c_{1,2} \cdot k \cdot \omega \cdot e^{k \cdot \omega \cdot y_1}$$

$$= \frac{1}{\mu_0} \left(-\alpha \cdot c_{2,1} \cdot e^{-\alpha \cdot y_1} + \alpha \cdot c_{2,2} \cdot e^{\alpha \cdot y_1} \right) \Rightarrow$$

$$\Rightarrow c_{1,2} \cdot k \cdot \omega \cdot e^{k \cdot \omega \cdot y_1} + c_{2,1} \cdot \alpha \cdot e^{-\alpha \cdot y_1} - c_{2,2} \cdot \alpha \cdot e^{\alpha \cdot y_1} = 0$$

If this process is repeated with all surfaces (only $\vec{J_k} \neq \vec{0}$ between
"Region III" and "Region IV"):

$$c_{2,1} \cdot e^{-\alpha \cdot y_2} + c_{2,2} \cdot e^{\alpha \cdot y_2} - c_{3,1} \cdot e^{-k \cdot \omega \cdot y_2} - c_{3,2} \cdot e^{k \cdot \omega \cdot y_2} = 0$$

$$-c_{2,1} \cdot \mu_{r,fe} \cdot \alpha \cdot e^{-\alpha \cdot y_2} + c_{2,2} \cdot \mu_{r,fe} \cdot \alpha \cdot e^{\alpha \cdot y_2} + c_{3,1} \cdot k \cdot \omega \cdot e^{-k \cdot \omega \cdot y_2} - c_{3,2} \cdot k \cdot \omega \cdot e^{k \cdot \omega \cdot y_2} = 0$$

$$c_{3,1} \cdot e^{-k \cdot \omega \cdot y_3} + c_{3,2} \cdot e^{k \cdot \omega \cdot y_3} - c_{4,1} \cdot e^{-k \cdot \omega \cdot y_3} = 0$$

$$-c_{3,1} \cdot k \cdot \omega \cdot e^{-k \cdot \omega \cdot y_3} + c_{3,2} \cdot k \cdot \omega \cdot e^{k \cdot \omega \cdot y_3} + c_{4,1} \cdot k \cdot \omega \cdot \mu_{r,fe} \cdot e^{-k \cdot \omega \cdot y_3} = \mu_0 \cdot \mu_{r,fe} \cdot J_k$$

This linear equation system where the unknown values are $c_{1,2}$, $c_{2,1}$, $c_{2,2}$, $c_{3,1}$, $c_{3,2}$ and $c_{4,1}$ must be solved.

2. If $k = 0$, there are cases in which $\alpha = 0$, so the solution is different. In Regions I, III and IV, $\alpha = 0$, so:

$\frac{\partial^2 A_{z,0}(y)}{\partial y^2} = \alpha^2 \cdot A_{z,0}(y) = 0$. The solution is: $A_{z,0} = a \cdot y + b$, where a, b are complex constants.

In "Region II" the same equation as obtained in last part is achieved, so:

Region I:
$$A_{z,0} = c_{1,1} \cdot y + c_{1,0}$$

Region II:
$$A_{z,0} = c_{2,1} \cdot e^{-\alpha \cdot y} + c_{2,2} \cdot e^{\alpha \cdot y}$$

Region III:
$$A_{z,0} = c_{3,1} \cdot y + c_{3,0}$$

Region IV:
$$A_{z,0} = c_{4,1} \cdot y + c_{4,0}$$

In "Region I" $\lim\limits_{y \to -\infty} B_{x,0} = \lim\limits_{y \to -\infty} \frac{\partial A_{z,0}}{\partial y} = c_{1,1} = 0$

Analogously, in "Region IV" $\lim\limits_{y \to \infty} B_{x,0} = \lim\limits_{y \to \infty} \frac{\partial A_{z,0}}{\partial y} = c_{4,1} = 0$

If the conditions in the equations mentioned before are applied:
$$c_{1,0} - c_{2,1} \cdot e^{-\alpha \cdot y_1} - c_{2,2} \cdot e^{\alpha \cdot y_1} = 0$$
$$c_{2,1} \cdot \alpha \cdot e^{-\alpha \cdot y_1} - c_{2,2} \cdot \alpha \cdot e^{\alpha \cdot y_1} = 0$$
$$c_{2,1} \cdot e^{-\alpha \cdot y_2} + c_{2,2} \cdot e^{\alpha \cdot y_2} - c_{3,1} \cdot y_2 - c_{3,0} = 0$$
$$\mu_{r,fe} \cdot \left(c_{2,1} \cdot e^{-\alpha \cdot y} + c_{2,2} \cdot e^{\alpha \cdot y} \right) - c_{3,1} = 0$$
$$c_{3,1} \cdot y_3 + c_{3,0} - c_{4,0} = 0$$
$$c_{3,1} = \mu_0 \cdot \mu_{r,fe} \cdot J_0$$

And this linear equations system has to be solved in order to obtain the solution if $k = 0$.

4.3 Solving the PDE

As said before, the PDE is solved by the "substitution of variables" method. The magnetic linear behavior is presumed so if there are many current sources, the magnetic vector potential obtained is the sum of the magnetic vector potential produced by all of them separately.

So, $\vec{J} \approx \sum_{k=-n}^{n} \vec{J}_k$, as a consequence, $A_z(x,y) = \sum_{k=-n}^{n} A_{z,k}(x,y) = \sum_{k=-n}^{n} A_{z,k}(y) \cdot e^{j \cdot k \cdot \omega \cdot x}$. Also, $A_{z,k}(y) = A_{z,-k}(y)$. So:

$$A_z(x,y) = \sum_{k=-n}^{n} A_{z,k}(y) \cdot e^{j \cdot k \cdot \omega \cdot x} =$$

$$= A_{z,0}(y) \cdot e^{j \cdot 0 \cdot \omega \cdot x} + \sum_{k=-n}^{-1} A_{z,k}(y) \cdot e^{j \cdot k \cdot \omega \cdot x} + \sum_{k=1}^{n} A_{z,k}(y) \cdot e^{j \cdot k \cdot \omega \cdot x} =$$

$$= A_{z,0}(y) + \sum_{k=1}^{n} A_{z,-k}(y) \cdot e^{-j \cdot k \cdot \omega \cdot x} + \sum_{k=1}^{n} A_{z,k}(y) \cdot e^{j \cdot k \cdot \omega \cdot x} =$$

$$= A_{z,0}(y) + \sum_{k=1}^{n} A_{z,k}(y) \cdot e^{-j \cdot k \cdot \omega \cdot x} + \sum_{k=1}^{n} A_{z,k}(y) \cdot e^{j \cdot k \cdot \omega \cdot x} =$$

$$= A_{z,0}(y) + \sum_{k=1}^{n} A_{z,k}(y) \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x} \right) =$$

$$= A_{z,0}(y) + \sum_{k=1}^{n} A_{z,k}(y) \cdot 2 \cdot \cos(k \cdot \omega \cdot x)$$

4.4 How to obtain $\vec{B}$ vector

Due to the fact that the model is infinite on the z-axis, $B_z = 0$, so B_x and B_y has to be obtained. It is also known that $\frac{\partial \vec{A}}{\partial z} = \vec{0}$, because properties do not change throughout z-axis. This means $\frac{\partial A_x}{\partial z} = \frac{\partial A_y}{\partial z} = 0$. So $B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\partial A_z}{\partial y} = \frac{\partial}{\partial y} \left(A_{z,0}(y) + \sum_{k=1}^{n} A_{z,k}(y) \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x} \right) \right)$

$$= \frac{\partial}{\partial y}\left(A_{z,0}(y)\right) + \frac{\partial}{\partial y}\left(\sum_{k=1}^{n} A_{z,k}(y) \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x}\right)\right) =$$

$$= \frac{\partial A_{z,0}(y)}{\partial y} + \sum_{k=1}^{n} \frac{\partial A_{z,k}(y)}{\partial y} \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x}\right),$$ where it is known how to

obtain $\frac{\partial A_{z,k}(y)}{\partial y}$ $\forall k$, because its expression is known.

If B_y is wanted to be obtained $B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = -\frac{\partial A_z}{\partial x} =$

$$= -\frac{\partial}{\partial x}\left(A_{z,0}(y) + \sum_{k=1}^{n} A_{z,k}(y) \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x}\right)\right) =$$

$$= -\left(\frac{\partial A_{z,0}(y)}{\partial x} + \frac{\partial}{\partial x}\sum_{k=1}^{n} A_{z,k}(y) \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x}\right)\right) =$$

$$= -\sum_{k=1}^{n} A_{z,k}(y) \cdot \frac{\partial}{\partial x}\left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x}\right) =$$

$$= -\sum_{k=1}^{n} A_{z,k}(y) \cdot j \cdot k \cdot \omega \cdot \left(e^{j \cdot k \cdot \omega \cdot x} - e^{-j \cdot k \cdot \omega \cdot x}\right) =$$

$$= \sum_{k=1}^{n} A_{z,k}(y) \cdot j \cdot k \cdot \omega \cdot \left(e^{-j \cdot k \cdot \omega \cdot x} - e^{j \cdot k \cdot \omega \cdot x}\right) =$$

$$= -\sum_{k=1}^{n} A_{z,k}(y) \cdot j \cdot k \cdot \omega \cdot j \cdot 2 \cdot sin(k \cdot \omega \cdot x) =$$

$$= \sum_{k=1}^{n} A_{z,k}(y) \cdot 2 \cdot k \cdot \omega \cdot sin(k \cdot \omega \cdot x)$$

In this way, an expression that solves the problem is obtained, although this algorithm is a bit slow. In the following chapter, a faster algorithm is obtained.

5. Optimization of the algorithm

5.1 Introduction

This algorithm makes $n \cdot m$ operations, where n is the number of terms of the exponential Fourier series, and m is the number of samples on the $y = cte$ axis.

As this algorithm is slow, the objective in this chapter is to solve the same problem with the same result, but faster than before. The optimization is done using the fast Fourier transform (FFT).

This chapter has calculations done by myself and the results are the same. I have posted the codes in ANNEX I (non-optimized algorithm) and ANNEX II (optimized algorithm), and the results of both codes at the end of this chapter.

5.2 DFT and FFT

The direct Fourier transform (DFT) is an algorithm that calculates:

$Y_k = \sum_{n=0}^{M-1} y_n \cdot e^{-j \cdot 2 \cdot \pi \cdot k \cdot \frac{n}{M}}$ where $k = 0, \dots, M - 1$ and all variables y_n are known.

The FFT is an optimization of the algorithm that calculates a DFT in $O(n \cdot log(n))$. This optimization of the algorithm saves a lot of operations. However, only 2^n samples can be taken if a FFT is wanted to be done with an even number of samples.

5.3 The new algorithm

It is known that on $y = y_{cte}$; $A_z(x, y_{cte}) = A_{z,0}(y_{cte}) + \sum_{k=1}^{n} A_{z,k}(y_{cte}) \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x}\right)$. The function is evaluated in a discrete number of points. These points that are evaluated have the form of $x_m = l \cdot \frac{m}{-1}$, where N is the number of

samples in $\left[\frac{-l}{2},\frac{l}{2}\right]$. It must be taken into account that the function is evaluated in $\left[\frac{-l}{2},\frac{l}{2}\right]$, so it is also assumed $x_{-m} = l \cdot \frac{-m}{N-1} = -x_m$. And $m = 0, \pm 1, \dots, \pm \frac{N-1}{2}$.

To evaluate $A_z(x_m, y_{cte}) = A_z(l \cdot \frac{m}{N-1}, y_{cte}) =$

$= A_{z,0}(y_{cte}) + \sum_{k=1}^{n} A_{z,k}(y_{cte}) \cdot \left(e^{j \cdot k \cdot \omega \cdot l \cdot \frac{m}{N-1}} + e^{-j \cdot k \cdot \omega \cdot l \cdot \frac{m}{N-1}}\right)$. If it is considered $\omega = \frac{2 \cdot \pi}{l}$,

$e^{j \cdot k \cdot \omega \cdot l \cdot \frac{m}{N-1}} + e^{-j \cdot k \cdot \omega \cdot l \cdot \frac{m}{N-1}} = e^{j \cdot k \cdot \frac{2 \cdot \pi}{l} \cdot l \cdot \frac{m}{N-1}} + e^{-j \cdot k \cdot \frac{2 \cdot \pi}{l} \cdot l \cdot \frac{m}{N-1}} = e^{j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}}$.

So, $A_z(x_m, y_{cte}) = A_{z,0}(y_{cte}) + \sum_{k=1}^{n} A_{z,k}(y_{cte}) \cdot \left(e^{j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}}\right)$

It is also known: $e^{j \cdot m \cdot 2 \cdot \pi \cdot \frac{k}{N-1}} = e^{j \cdot m \cdot 2 \cdot \pi \cdot \frac{k}{N-1} - j \cdot m \cdot 2 \cdot \pi} = e^{j \cdot m \cdot 2 \cdot \pi \cdot \left(\frac{k}{N-1} - 1\right)} =$

$= e^{-j \cdot m \cdot 2 \cdot \pi \cdot \left(\frac{N-1-k}{N-1}\right)}$.

$$A_z(x_m, y_{cte}) = A_{z,0}(y_{cte}) + \sum_{k=1}^{n} A_{z,k}(y_{cte}) \cdot \left(e^{j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}}\right) =$$

$$= \sum_{k=0}^{n} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{k=1}^{n} A_{z,k}(y_{cte}) \cdot e^{j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} =$$

$$= \sum_{k=0}^{n} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{k=1}^{n} A_{z,k}(y_{cte}) \cdot e^{-j \cdot m \cdot 2 \cdot \pi \cdot \left(\frac{N-1-k}{N-1}\right)}$$

If $n = N - 2$, in order to apply the DFT,

$$A_z(x_m, y_{cte}) =$$

$$= \sum_{k=0}^{n} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{k=1}^{n} A_{z,k}(y_{cte}) \cdot e^{-j \cdot m \cdot 2 \cdot \pi \cdot \left(\frac{N-1-k}{N-1}\right)} =$$

$$= \sum_{k=0}^{N-2} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{k=1}^{N-2} A_{z,k}(y_{cte}) \cdot e^{-j \cdot m \cdot 2 \cdot \pi \cdot \left(\frac{N-1-k}{N-1}\right)}$$

Now is used in the second summation $t = N - 1 - k$, it results in, $k = N - 1 - t$.

$$A_z(x_m, y_{cte}) =$$

$$= \sum_{k=0}^{N-2} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + + \sum_{k=1}^{N-2} A_{z,k}(y_{cte}) \cdot e^{-j \cdot m \cdot 2 \cdot \pi \cdot \left(\frac{N-1-k}{N-1}\right)} =$$

$$= \sum_{k=0}^{N-2} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{t=1}^{N-2} A_{z,N-1-t}(y_{cte}) \cdot e^{-j \cdot m \cdot 2 \cdot \pi \cdot \frac{t}{N-1}}$$

It has to be remembered that what is trying to be obtained is the function's values where $y = y_{cte}$. Therefore, the algorithm is optimized in order to obtain this result. It also must be remembered that the function has a period l, which implies $X_k = X_{k+N-1}$.

Let X_m be $X_m = A_z(x_k, y_{cte})$. It is known $X_m = A_z(x_m, y_{cte}) =$

$$= \sum_{k=0}^{N-2} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{t=1}^{N-2} A_{z,N-1-t}(y_{cte}) \cdot e^{-j \cdot m \cdot 2 \cdot \pi \cdot \frac{t}{N-1}} =$$

$$= \sum_{k=0}^{N-2} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{t=1}^{N-2} A_{z,N-1-t}(y_{cte}) \cdot e^{-j \cdot m \cdot 2 \cdot \pi \cdot \frac{t}{N-1}}.$$

So:

$$(X_0, \dots, X_{N-2}) = DFT\left(A_{z,0}(y_{cte}), \dots, A_{z,N-2}(y_{cte})\right) +$$

$$+ DFT\left(0, A_{z,N-2}(y_{cte}), A_{z,N-3}(y_{cte}), \dots, A_{z,1}(y_{cte})\right) =$$

$$= DFT\left(A_{z,0}(y_{cte}), A_{z,N-2}(y_{cte}) + A_{z,1}(y_{cte}), \dots, A_{z,1}(y_{cte}) + A_{z,N-2}(y_{cte})\right)$$

where all numbers with the form of $A_{z,k}(y_{cte})$ are known for all values of k.

Finally, if this algorithm has to be more efficient than it was before, a FFT must be used instead of an DFT. This is only possible if $N = 2^n + 1$ with $n = 0,1,2, \dots$, because a DFT of 2^n elements is carried out. At last:

$$(X_0, \dots, X_{N-2}) =$$

$$= FFT\left(A_{z,0}(y_{cte}), A_{z,N-2}(y_{cte}) + A_{z,1}(y_{cte}), \dots, A_{z,1}(y_{cte}) + A_{z,N-2}(y_{cte})\right)$$

However, this has a limitation in that 2^n points where the results are evaluated, and 2^n Fourier series are needed. This fact has 2 different issues. The first issue is that if this function needs to be evaluated at more points, more Fourier series are required. The second

issue, which is also a consequence of the first one, is that matlab's the greatest number is $1.7977 \cdot 10^{308}$. The equation proposed to obtain $A_{z,k}(y)$, has some terms such as $e^{k \cdot \omega \cdot y_2}$, so if k is a large number, problems will arise.

5.4 Avoiding the limitations

As said before, the Fourier series cannot be obtained if k is a huge number. In this chapter this issue is solved. The objective to improve the algorithm in order to decrease the interval that takes samples. Assuming that there are 2^n Fourier series, and $N - 1 = 2^n \cdot 2^{n'} = 2^{n+n'}$ samples are desired, the same notation, $x_m = l \cdot \dfrac{m}{N-1}$, is used as before. As proved before:

$$A_z(x_m, y_{cte}) = \sum_{k=0}^{n} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{k=1}^{n} A_{z,k}(y_{cte}) \cdot e^{j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}}$$

Due to the fact that $N = 2^{n+n'}$, and there are 2^n, Fourier series terms, so $m = 2^n - 1$:

$$A_z(x_m, y_{cte}) = \sum_{k=0}^{m} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{k=1}^{m} A_{z,k}(y_{cte}) \cdot e^{j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} =$$

$$= \sum_{k=0}^{2^n-1} A_{z,k}(y_{cte}) \cdot e^{-j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}} + \sum_{k=1}^{2^n-1} A_{z,k}(y_{cte}) \cdot e^{j \cdot k \cdot 2 \cdot \pi \cdot \frac{m}{N-1}}$$

So:

$$(X_0, \dots, X_{N-2}) = DFT\big(A_{z,0}(y_{cte}), A_{z,1}(y_{cte}), \dots, A_{z,N-2}(y_{cte}), 0, \dots, 0\big) +$$
$$+ DFT\big(0, A_{z,N-2}(y_{cte}), A_{z,N-3}(y_{cte}), \dots, A_{z,1}(y_{cte}), 0, \dots, 0\big) =$$
$$= DFT\big(A_{z,0}(y_{cte}), A_{z,N-2}(y_{cte}) + A_{z,1}(y_{cte}), \dots, A_{z,1}(y_{cte})$$
$$+ A_{z,N-2}(y_{cte}), 0, \dots, 0\big)$$

As a DFT of $2^{n+n'}$ terms is done, the FFT can be used in order to optimize the algorithm. So, as before:

$$(X_0, \dots, X_{N-2}) = FFT\big(A_{z,0}(y_{cte}), A_{z,1}(y_{cte}), \dots, A_{z,N-2}(y_{cte}), 0, 0, \dots, 0\big) +$$
$$+ FFT\big(0, A_{z,N-2}(y_{cte}), A_{z,N-3}(y_{cte}), \dots, A_{z,1}(y_{cte}), 0, 0, \dots, 0\big) =$$

$$= FFT\big(A_{z,0}(y_{cte}), A_{z,N-2}(y_{cte}) + A_{z,1}(y_{cte}), \ldots, A_{z,1}(y_{cte})$$
$$+ A_{z,N-2}(y_{cte}), 0, \ldots, 0\big)$$

5.5 How to obtain $\vec{B}$ field

As seen before, $B_z = 0$, $B_x = \frac{\partial A_{z,0}(y)}{\partial y} + \sum_{k=1}^{n} \frac{\partial A_{z,k}(y)}{\partial y} \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x}\right)$,

$$B_y = \sum_{k=1}^{n} A_{z,k}(y) \cdot j \cdot k \cdot \omega \cdot \left(e^{-j \cdot k \cdot \omega \cdot x} - e^{j \cdot k \cdot \omega \cdot x}\right)$$

Which results in,

$$B_x = \frac{\partial A_{z,0}(y)}{\partial y} + \sum_{k=1}^{n} \frac{\partial A_{z,k}(y)}{\partial y} \cdot \left(e^{j \cdot k \cdot \omega \cdot x} + e^{-j \cdot k \cdot \omega \cdot x}\right) =$$

$$= \frac{\partial A_{z,0}(y)}{\partial y} + \sum_{k=1}^{n} \frac{\partial A_{z,k}(y)}{\partial y} \cdot e^{j \cdot k \cdot \omega \cdot x} + \sum_{k=1}^{n} \frac{\partial A_{z,k}(y)}{\partial y} e^{-j \cdot k \cdot \omega \cdot x}$$

If $n = N - 2$, and there are the sample of N points:

$$B_x = \frac{\partial A_{z,0}(y)}{\partial y} + \sum_{k=1}^{n} \frac{\partial A_{z,k}(y)}{\partial y} \cdot e^{j \cdot k \cdot \omega \cdot x} + \sum_{k=1}^{n} \frac{\partial A_{z,k}(y)}{\partial y} \cdot e^{-j \cdot k \cdot \omega \cdot x} =$$

$$= \frac{\partial A_{z,0}(y)}{\partial y} + \sum_{k=1}^{N-2} \frac{\partial A_{z,k}(y)}{\partial y} \cdot e^{j \cdot k \cdot \omega \cdot x} + \sum_{k=1}^{N-2} \frac{\partial A_{z,k}(y)}{\partial y} \cdot e^{-j \cdot k \cdot \omega \cdot x}$$

So:

$$(BX_0, \ldots, BX_{N-2}) =$$

$$= FFT\left(\frac{\partial A_{z,0}(y_{cte})}{\partial y}, \frac{\partial \left(A_{z,N-2}(y_{cte}) + A_{z,1}(y_{cte})\right)}{\partial y}, \ldots, \frac{\partial \left(A_{z,1}(y_{cte}) + A_{z,N-2}(y_{cte})\right)}{\partial y}\right)$$

Analogously:

$$B_y = \sum_{k=1}^{N-2} A_{z,k}(y) \cdot j \cdot k \cdot \omega \cdot \left(e^{-j \cdot k \cdot \omega \cdot x} - e^{j \cdot k \cdot \omega \cdot x}\right) =$$

$$= \sum_{k=1}^{N-2} A_{z,k}(y) \cdot j \cdot k \cdot \omega \cdot e^{-j \cdot k \cdot \omega \cdot x} - \sum_{k=1}^{N-2} A_{z,k}(y) \cdot j \cdot k \cdot \omega \cdot e^{j \cdot k \cdot \omega \cdot x}$$

So:

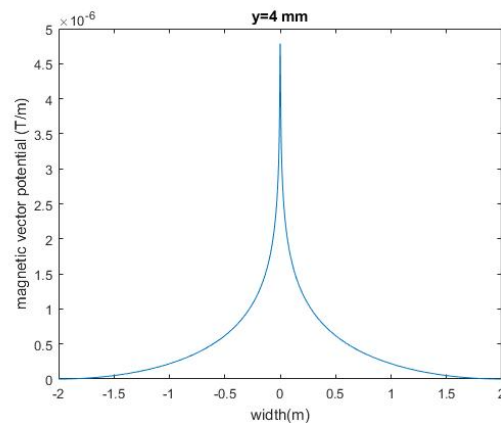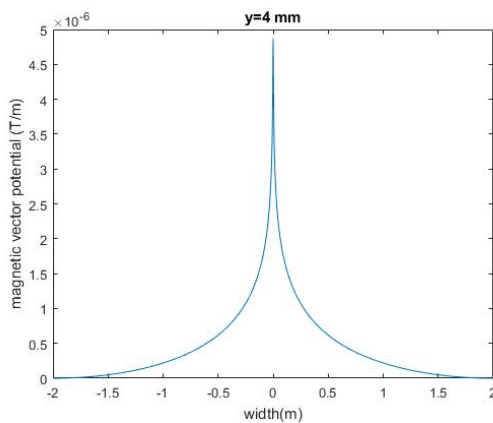$$(BY_0, \dots, BY_{N-2}) = FFT\left(0, j \cdot \omega \cdot \left(A_{z,1}(y_{cte}) - (N-2) \cdot A_{z,N-2}(y_{cte})\right),\right.$$

$$, j \cdot \omega \cdot \left(2 \cdot A_{z,2}(y_{cte}) - (N-3) \cdot A_{z,N-3}(y_{cte})\right), \dots,$$

$$\left. , \dots, j \cdot \omega \cdot \left((N-2) \cdot A_{z,N-2}(y_{cte}) - A_{z,1}(y_{cte})\right), 0,0, \dots, 0\right)$$

Analogously, if $N - 1 = 2^{n'} \cdot 2^n$, it is obtained:

$$(BX_0, \dots, BX_{N-2}) =$$

$$= FFT\left(\frac{\partial A_{z,0}(y_{cte})}{\partial y}, \frac{\partial \left(A_{z,N-2}(y_{cte}) + A_{z,1}(y_{cte})\right)}{\partial y}, \dots, \frac{\partial \left(A_{z,1}(y_{cte}) + A_{z,N-2}(y_{cte})\right)}{\partial y}, 0, \dots, 0\right)$$

$$(BY_0, \dots, BY_{N-2}) = FFT\left(0, j \cdot \omega \cdot \left(A_{z,1}(y_{cte}) - (N-2) \cdot A_{z,N-2}(y_{cte})\right),\right.$$

$$, j \cdot \omega \cdot \left(2 \cdot A_{z,2}(y_{cte}) - (N-3) \cdot A_{z,N-3}(y_{cte})\right), \dots,$$

$$\left. , \dots, j \cdot \omega \cdot \left((N-2) \cdot A_{z,N-2}(y_{cte}) - A_{z,1}(y_{cte})\right), 0,0, \dots, 0\right)$$

5.6 Images of results

The left image shows the result with the non-optimized method, whereas the right image is the result of the optimized method. In order to make this comparison, it uses the same length, the same number of Fourier series and the same points evaluated. The matlab codes of both images are posted in annex I and annex II respectively.

6. Simulation with FEM

6.1 ANSYS electronics desktop

In order to solve an eddy current problem, ANSYS electronics desktop is used in order to simulate the problem.

Three plaques with a length of 1 meter are simulated. The first one is made of aluminum with a width of 3.5 mm, the second is made of ferrite with a width of 0.5 mm as is last one is made of ferrite, but with a width of 3.5 mm. This is done so that the magnetic field at $y = 4\,mm$ can be known. In order to introduce the current, a very small rectangle of copper is made, because a point cannot be made and considered as an object.

6.2 Results

The results of the magnetic field of the solution obtained with the FEM and with the analytical solution are plotted. The magnetic field is plotted, and not magnetic vector potential, because the subtraction may be a constant and that means the magnetic vector is the same, although the magnetic vector potential is not the same.
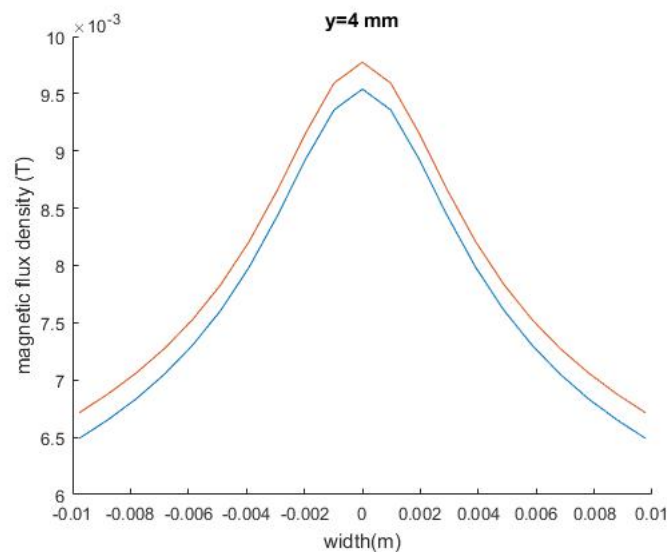
This image shows the differences between the analytical solution (orange line), and the results from the FEM (blue line). In this case an error roughly of 4% at the peak value can be observed.
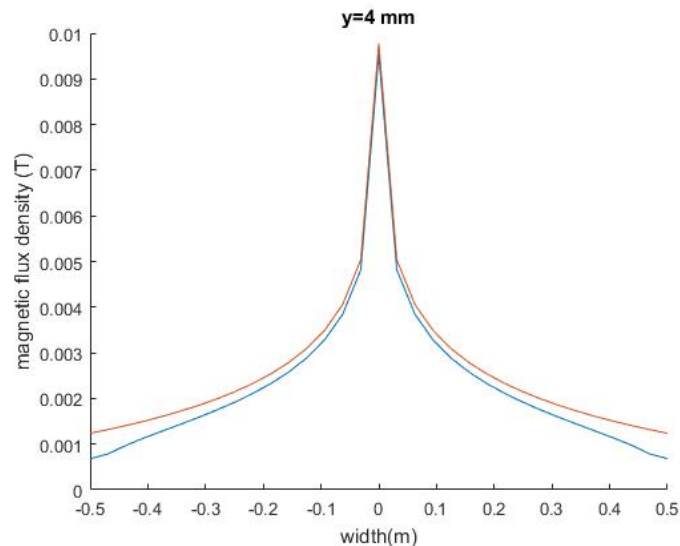
However, on values far from the zero, a greater error is observed.

Near the peak value, the results obtained are more or less the same. The difference between both results maybe because the linear behavior of materials is supposed whereas the FEM program is not, and the FEM program makes an error as a result of meshing operations.

The error along the x-axis is greater on values farther from zero. This fact may be produced by Fourier approximation, because the current density is approximated with Fourier series. Therefore, the approximation is done with periodic functions, so the problem solved is a problem where there is a current filament every distance l. As a consequence, the magnetic field is larger as it is getting closer to another filament current.

7. Conclusions

In the first chapter, the objective of this thesis is defined. In the second chapter, a PDE that solves this problem is obtained. Due to the fact that this is a 2D problem, it is very complex to solve this PDE directly. So, this PDE is solved with a method called "separation of variables" in chapter 4. In order to apply this method, a Fourier analysis must be carried out. With the Fourier analysis, the 2D problem transforms into a 1D problem, so an ODE must be solved instead of a PDE, and then this ODE can be solved. In chapter 5, an optimization of this algorithm is done.

In order to check that the algorithm is correct, the same model is solved using the finite element method, and the results are shown in chapter 6.

With those results it can be concluded the model is validated because there is an error of 4%, which is normal when working with finite elements method.

In the next chapter, the solution is generalized as the model may have more current filaments or more plaques.

8. General solution

8.1 Introduction

In this chapter, the objective is to obtain a solution for the main problem, which is obtaining the magnetic vector potential. But now there may be more current filaments and plaques. The only condition is that the plaques must be parallel.

In this chapter, there are two methods proposed in order to solve this problem.

8.2 First method

The idea of this method is use the magnetic linear behavior of materials. For each current filament that exists, a problem where only there is only current in this filament must be solved. The final solution is the sum of the A_z field obtained for each current filament.

8.3 Second method

This is the general method. For all straight parallel to x-axis where any current filament exists, it must be decomposed in Fourier series. When all Fourier series are obtained, the equation system proposed in chapter 4.2 varies. Guidelines mentioned in that chapter must be considered. When the equation system is set, it can be solved, and finally, it is $A_{z,k}$. Now, if A_z is wanted the steps mentioned in 4.3 and in chapter 5 must be followed.

9. Bibliography

1."Jaimin Jha master of science thesis". From this thesis the information in order to make the chapter 2 (obtain the PDE) and all chapter 4.2 can be found.

2." Introduction to Hilbert´s spaces, operators and specters" by Carlos Fernández González (This book is only in Spanish). All chapter 3.2 is a summary of its chapter 4. The chapter 3.3 is from Wikipedia, but this thesis proves the calculations are right.

3.In chapter 4.3, the PDE $\Delta A_z = j \cdot \omega_t \cdot \mu \cdot \sigma \cdot A_z$ is solved with a method called "Separation of Variables".

4.On chapter 5, the calculations are the writer´s own, but the results are exactly the same as in chapter 4 (obviously with the same number of Fourier series and the same points as samples). In fact, both matlab codes are posted in an annex.

ANNEX (MATLAB CODE)

ANNEX I. NON-EFFICIENT ALGORITHM TO OBTAIN A_Z

```
clc;
close all;

I=10; %total current
l=4; %length
Jm = sqrt(2)*I/l; %current density
f = 85000; %frequency of the current
w=2*pi*f;
mu0=4*pi*1e-7;
ww=2*pi/l;
y=0.004;

%air properties
sigmaair=0;
muair=mu0;

%ferrite properties
mufe = 2400;

%aluminium properties
mual=mu0;
sigmaal=3.77*1e7;

y1=0;
y2=0.0035;
y3=0.0075;

%solving when k=0

aalpha=sqrt(1j*w*mual*sigmaal);

AA=zeros(6,6);

AA(1,1)=1;
AA(1,2)=-1;
AA(1,3)=-1;
AA(2,2)=1;
AA(2,3)=-1;
AA(3,2)=exp(-aalpha*y2);
AA(3,3)=exp(aalpha*y2);
AA(3,4)=-y2;
AA(3,5)=-1;
AA(4,2)=-mufe*aalpha*exp(-aalpha*y2);
AA(4,3)=mufe*aalpha*exp(aalpha*y2);
AA(4,4)=-1;
AA(5,4)=y3;
AA(5,5)=1;
AA(5,6)=-1;
AA(6,4)=1;
```

```matlab
cc=zeros(6,1);

cc(6)=mu0*Jm*mufe;

bb=AA\cc;

%solving when k is not 0

N=2047;

alpha=zeros(1,N);
c12=zeros(1,N);
c21=zeros(1,N);
c22=zeros(1,N);
c31=zeros(1,N);
c32=zeros(1,N);
c41=zeros(1,N);

A=zeros(6,6);
c=zeros(6,1);
c(6)=-mufe*mu0*Jm;

Aair1=zeros(1,N);
Aal=zeros(1,N);
Afe=zeros(1,N);
Aair2=zeros(1,N);

for k=1:N
    alpha(k)=sqrt(k*k*ww*ww+1j*w*mual*sigmaal);
    A(1,1)=1;
    A(1,2)=-1;
    A(1,3)=-1;
    A(2,1)=k*ww;
    A(2,2)=alpha(k);
    A(2,3)=-alpha(k);
    A(3,2)=exp(-alpha(k)*y2);
    A(3,3)=exp(alpha(k)*y2);
    A(3,4)=-exp(-k*ww*y2);
    A(3,5)=-exp(k*y2*ww);
    A(4,2)=-mufe*alpha(k)*exp(-alpha(k)*y2);
    A(4,3)=mufe*alpha(k)*exp(alpha(k)*y2);
    A(4,4)=k*ww*exp(-k*ww*y2);
    A(4,5)=-k*ww*exp(k*ww*y2);
    A(5,4)=exp(-k*y3*ww);
    A(5,5)=exp(k*y3*ww);
    A(5,6)=-exp(-k*y3*ww);
    A(6,4)=exp(-k*y3*ww)*k*ww;
    A(6,5)=-exp(k*ww*y3)*k*ww;
    A(6,6)=-exp(-k*y3*ww)*mufe*k*ww;

    b=A\c;
```

```matlab
    c12(k)=b(1);
    c21(k)=b(2);
    c22(k)=b(3);
    c31(k)=b(4);
    c32(k)=b(5);
    c41(k)=b(6);

    Aair1(k)=b(1)*exp(k*y*ww);
    Aal(k)=b(2)*exp(-alpha(k)*y)+b(3)*exp(alpha(k)*y);
    Afe(k)=b(4)*exp(-k*ww*y)+b(5)*exp(k*ww*y);
    Aair2(k)=b(6)*exp(-k*ww*y)
end

%getting the value of Az,k(x) in terms of his position
%depending on the region where x is

if y<0
    Azf=Aair1;
    Az0=bb(1);
end

if y*(y-y2)<=0
    Azf=Aal;
    Az0=b(2)*exp(-aalpha*y)+b(3)*exp(aalpha*y);
end

if (y-y2)*(y-y3)<0
    Azf=Afe;
    Az0=bb(4)*y+bb(5);
end

if y>=y3
    Azf=Aair2;
    Az0=bb(6);
end

%obtaining the fft as mentioned in chapter 5.2

x=-l/2:l/N:l/2;
n=length(x);
Az=zeros(1,n);
Az=Az+Az0;

for i=1:n
    for k=1:N
        Az(i)=Az(i)+Azf(k)*2*cos(k*ww*x(i));
    end
end

Az=Az-Az(1);

plot(x, abs(Az)),xlabel('width(m)'),ylabel('magnetic vector potential (T/m)'), title('y=4 mm');
```

ANNEX II. EFFICIENT ALGORITHM TO OBTAIN A_Z

```
clc;
close all;

I=10;
l=4; %length
Jm = sqrt(2)*I/l; %current density
f = 85000; %frequency
w=2*pi*f;
mu0=4*pi*1e-7;
ww=2*pi/l;

%air properties

sigmaair=0;
muair=mu0;

N=2^11;

%ferrite properties
mufe = 2400;

%aluminium properties
mual=mu0;
sigmaal=3.77*1e7;
y1=0;
y2=0.0035;
y3=0.0075;

%solving when k=0

aalpha=sqrt(1j*w*mual*sigmaal);

AA=zeros(6,6);
AA(1,1)=1;
AA(1,2)=-1;
AA(1,3)=-1;
AA(2,2)=1;
AA(2,3)=-1;
AA(3,2)=exp(-aalpha*y2);
AA(3,3)=exp(aalpha*y2);
AA(3,4)=-y2;
AA(3,5)=-1;
AA(4,2)=-mufe*aalpha*exp(-aalpha*y2);
AA(4,3)=mufe*aalpha*exp(aalpha*y2);
AA(4,4)=-1;
AA(5,4)=y3;
AA(5,5)=1;
AA(5,6)=-1;
AA(6,4)=1;
```

```matlab
cc=zeros(6,1);

cc(6)=mu0*Jm*mufe;

bb=AA\cc;

alpha=zeros(1,N-1);
c12=zeros(1,N-1);
c21=zeros(1,N-1);
c22=zeros(1,N-1);
c31=zeros(1,N-1);
c32=zeros(1,N-1);
c41=zeros(1,N-1);
Aair1=zeros(1,N-1);
Aal=zeros(1,N-1);
Afe=zeros(1,N-1);
Aair2=zeros(1,N-1);
Azf=zeros(1,N-1);

A=zeros(6,6);
c=zeros(6,1);
c(6)=-mufe*mu0*Jm;

y=0.004;

%solving when k is not 0
for k=1:N-1
    alpha(k)=sqrt(k*k*ww*ww+1j*w*mual*sigmaal);
    A(1,1)=1;
    A(1,2)=-1;
    A(1,3)=-1;
    A(2,1)=k*ww;
    A(2,2)=alpha(k);
    A(2,3)=-alpha(k);
    A(3,2)=exp(-alpha(k)*y2);
    A(3,3)=exp(alpha(k)*y2);
    A(3,4)=-exp(-k*ww*y2);
    A(3,5)=-exp(k*y2*ww);
    A(4,2)=-mufe*alpha(k)*exp(-alpha(k)*y2);
    A(4,3)=mufe*alpha(k)*exp(alpha(k)*y2);
    A(4,4)=k*ww*exp(-k*ww*y2);
    A(4,5)=-k*ww*exp(k*ww*y2);
    A(5,4)=exp(-k*y3*ww);
    A(5,5)=exp(k*y3*ww);
    A(5,6)=-exp(-k*y3*ww);
    A(6,4)=exp(-k*y3*ww)*k*ww;
    A(6,5)=-exp(k*ww*y3)*k*ww;
    A(6,6)=-exp(-k*y3*ww)*mufe*k*ww;

    b=A\c;

    c12(k)=b(1);
```

```
    c21(k)=b(2);
    c22(k)=b(3);
    c31(k)=b(4);
    c32(k)=b(5);
    c41(k)=b(6);

    Aair1(k)=b(1)*exp(k*y*ww);
    Aal(k)=b(2)*exp(-alpha(k)*y)+b(3)*exp(alpha(k)*y);
    Afe(k)=b(4)*exp(-k*ww*y)+b(5)*exp(k*ww*y);
    Aair2(k)=b(6)*exp(-k*ww*y);
end

%getting the value of Az,k(x) in terms of his position
%depending on the region where x is

if y<0
    Azf=Aair1;
    Az0=bb(1);
end

if y*(y-y2)<=0
    Azf=Aal;
    Az0=b(2)*exp(-aalpha*y)+b(3)*exp(aalpha*y);
end

if (y-y2)*(y-y3)<0
    Azf=Afe;
    Az0=bb(4)*y+bb(5);
end

if y>=y3
    Azf=Aair2;
    Az0=bb(6);
end

%obtaining the fft as mentioned in chapter 5.2
AAx=zeros(1,N);

for k=2:N
    AAx(k)=Azf(k-1)+Azf(N+1-k);
end

AAx(1)=Az0;
AAx=fft(AAx);

x=-l/2:l/N:l/2;

Ax=zeros(1,N+1);

for i=1:N/2
    Ax(i)=AAx(N/2+i);
    Ax(N/2+i)=AAx(i);
```

end

Ax(N+1)=AAx(N/2+1);

Ax=Ax-Ax(1);

plot(x,abs(Ax)),xlabel('width(m)'),ylabel('magnetic vector potential (T/m)'), title('y=4 mm');

ANNEX III. EFFICIENT ALGORITHM TO OBTAIN B_Y

```
clc;
close all;

I=10;
l=4; %length
Jm = sqrt(2)*I/l; %current density
f = 85000; %frequency
w=2*pi*f;
mu0=4*pi*1e-7;
ww=2*pi/l;

%air properties

sigmaair=0;
muair=mu0;

N=2^11;

%ferrite properties
mufe = 2400;

%aluminium properties
mual=mu0;
sigmaal=3.77*1e7;
y1=0;
y2=0.0035;
y3=0.0075;

%as mentioned, here is not necesary obtaining the case k=0

%solving when k is not 0
alpha=zeros(1,N-1);
c12=zeros(1,N-1);
c21=zeros(1,N-1);
c22=zeros(1,N-1);
c31=zeros(1,N-1);
c32=zeros(1,N-1);
c41=zeros(1,N-1);
Aair1=zeros(1,N-1);
Aal=zeros(1,N-1);
Afe=zeros(1,N-1);
Aair2=zeros(1,N-1);
Azf=zeros(1,N-1);

A=zeros(6,6);
c=zeros(6,1);
c(6)=-mufe*mu0*Jm;

y=0.004;
```

```
for k=1:N-1
    alpha(k)=sqrt(k*k*ww*ww+1j*w*mual*sigmaal);
    A(1,1)=1;
    A(1,2)=-1;
    A(1,3)=-1;
    A(2,1)=k*ww;
    A(2,2)=alpha(k);
    A(2,3)=-alpha(k);
    A(3,2)=exp(-alpha(k)*y2);
    A(3,3)=exp(alpha(k)*y2);
    A(3,4)=-exp(-k*ww*y2);
    A(3,5)=-exp(k*y2*ww);
    A(4,2)=-mufe*alpha(k)*exp(-alpha(k)*y2);
    A(4,3)=mufe*alpha(k)*exp(alpha(k)*y2);
    A(4,4)=k*ww*exp(-k*ww*y2);
    A(4,5)=-k*ww*exp(k*ww*y2);
    A(5,4)=exp(-k*y3*ww);
    A(5,5)=exp(k*y3*ww);
    A(5,6)=-exp(-k*y3*ww);
    A(6,4)=exp(-k*y3*ww)*k*ww;
    A(6,5)=-exp(k*ww*y3)*k*ww;
    A(6,6)=-exp(-k*y3*ww)*mufe*k*ww;

    b=A\c;

    c12(k)=b(1);
    c21(k)=b(2);
    c22(k)=b(3);
    c31(k)=b(4);
    c32(k)=b(5);
    c41(k)=b(6);

    Aair1(k)=b(1)*exp(k*y*ww);
    Aal(k)=b(2)*exp(-alpha(k)*y)+b(3)*exp(alpha(k)*y);
    Afe(k)=b(4)*exp(-k*ww*y)+b(5)*exp(k*ww*y);
    Aair2(k)=b(6)*exp(-k*ww*y);
end

%getting the value of Az,k(x) in terms of his position
%depending on the region where x is
if y<0
    Azf=Aair1;
    Az0=bb(1);
end

if y*(y-y2)<=0
    Azf=Aal;
end

if (y-y2)*(y-y3)<0
    Azf=Afe;
end
```

```
if y>=y3
    Azf=Aair2;
end

%obtaining the fft as mentioned in chapter 5.3

x=-l/2:l/N:l/2;
BBy=zeros(1,N);
By=zeros(1,N+1);

for k=2:N
    BBy(k)=1j*ww*(((k-1)*Azf(k-1))-((N+1-k)*Azf(N+1-k)));
end

BBy=fft(BBy);

for i=1:N/2
    By(i)=BBy(N/2+i);
    By(N/2+i)=BBy(i);
end

plot(x, abs(By)),xlabel('width(m)'),ylabel('magnetic flux density on y-axis(T)'), title('y=4 mm');
```

ANNEX IV. EFFICIENT ALGORITHM TO OBTAIN B_X

```
clc;
close all;

clc;
close all;

I=10;
l=4; %length
Jm = sqrt(2)*I/l; %current density
f = 85000; %frequency
w=2*pi*f;
mu0=4*pi*1e-7;
ww=2*pi/l;

%air properties

sigmaair=0;
muair=mu0;

N=2^11;

%ferrite properties
mufe = 2400;

%aluminium properties
mual=mu0;
sigmaal=3.77*1e7;
y1=0;
y2=0.0035;
y3=0.0075;

%solving when k=0

aalpha=sqrt(1j*w*mual*sigmaal);

AA=zeros(6,6);
AA(1,1)=1;
AA(1,2)=-1;
AA(1,3)=-1;
AA(2,2)=1;
AA(2,3)=-1;
AA(3,2)=exp(-aalpha*y2);
AA(3,3)=exp(aalpha*y2);
AA(3,4)=-y2;
AA(3,5)=-1;
AA(4,2)=-mufe*aalpha*exp(-aalpha*y2);
AA(4,3)=mufe*aalpha*exp(aalpha*y2);
AA(4,4)=-1;
AA(5,4)=y3;
AA(5,5)=1;
```

```matlab
AA(5,6)=-1;
AA(6,4)=1;

cc=zeros(6,1);

cc(6)=mu0*Jm*mufe;

bb=AA\cc;

alpha=zeros(1,N-1);
c12=zeros(1,N-1);
c21=zeros(1,N-1);
c22=zeros(1,N-1);
c31=zeros(1,N-1);
c32=zeros(1,N-1);
c41=zeros(1,N-1);

Bxair1=zeros(1,N-1);
Bxal=zeros(1,N-1);
Bxfe=zeros(1,N-1);
Bxair2=zeros(1,N-1);
Bxf=zeros(1,N-1);

A=zeros(6,6);
c=zeros(6,1);
c(6)=-mufe*mu0*Jm;

y=0.004;

for k=1:N-1
    alpha(k)=sqrt(k*k*ww*ww+1j*w*mual*sigmaal);

    A(1,1)=1;
    A(1,2)=-1;
    A(1,3)=-1;
    A(2,1)=k*ww;
    A(2,2)=alpha(k);
    A(2,3)=-alpha(k);
    A(3,2)=exp(-alpha(k)*y2);
    A(3,3)=exp(alpha(k)*y2);
    A(3,4)=-exp(-k*ww*y2);
    A(3,5)=-exp(k*y2*ww);
    A(4,2)=-mufe*alpha(k)*exp(-alpha(k)*y2);
    A(4,3)=mufe*alpha(k)*exp(alpha(k)*y2);
    A(4,4)=k*ww*exp(-k*ww*y2);
    A(4,5)=-k*ww*exp(k*ww*y2);
    A(5,4)=exp(-k*y3*ww);
    A(5,5)=exp(k*y3*ww);
    A(5,6)=-exp(-k*y3*ww);
    A(6,4)=exp(-k*y3*ww)*k*ww;
    A(6,5)=-exp(k*ww*y3)*k*ww;
    A(6,6)=-exp(-k*y3*ww)*mufe*k*ww;
```

```
    b=A\c;

    c12(k)=b(1);
    c21(k)=b(2);
    c22(k)=b(3);
    c31(k)=b(4);
    c32(k)=b(5);
    c41(k)=b(6);

    Bxair1(k)=k*ww*b(1)*exp(k*y*ww);
    Bxal(k)=-alpha(k)*b(2)*exp(-alpha(k)*y)+alpha(k)*b(3)*exp(alpha(k)*y);
    Bxfe(k)=-k*ww*b(4)*exp(-k*ww*y)+k*ww*b(5)*exp(k*ww*y);
    Bxair2(k)=-k*ww*b(6)*exp(-k*ww*y);
end

%getting the value of Az,k(x) in terms of his position
%depending on the region where x is
if y<0
    Bxf=Bxair1;
    Bx0=0;
end

if y*(y-y2)<=0
    Bxf=Bxal;
    Bx0=-aalpha*b(2)*exp(-aalpha*y)+aalpha*b(3)*exp(aalpha*y);
end

if (y-y2)*(y-y3)<0
    Bxf=Bxfe;
    Bx0=bb(4);
end

if y>=y3
    Bxf=Bxair2;
    Bx0=0;
end

%obtaining the fft as mentioned in chapter 5.2
BBx=zeros(1,N);

for k=2:N
    BBx(k)=Bxf(k-1)+Bxf(N+1-k);
end

BBx(1)=Bx0;

BBx=fft(BBx);


x=-l/2:l/N:l/2;
```

```
Bx=zeros(1,N+1);

for i=1:N/2
    Bx(i)=BBx(N/2+i);
    Bx(N/2+i)=BBx(i);
end

Bx(N+1)=BBx(N/2+1);

Bx=Bx-Bx(1);

plot(x,abs(Bx)),xlabel('width(m)'),ylabel('magnetic flux density on x-axis(T)'), title('y=4 mm')
```