CORRECTION TO "ON MINIMUM SUM REPRESENTATIONS FOR WEIGHTED VOTING GAMES"

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ABSTRACT. A proposal in a weighted voting game is accepted if the sum of the (non-negative) weights of the "yea" voters is at least as large as a given quota. Several authors have considered representations of weighted voting games with minimum sum, where the weights and the quota are restricted to be integers. Here we correct the classification of all weighted voting games consisting of 9 voters which do not admit a unique minimum sum integer weight representation.

1. INTRODUCTION

A weighted voting game is a yes-no voting system specified by non-negative voting weights $w_i \in \mathbb{R}_{\geq 0}$ for the voters and a quota $q \in \mathbb{R}_{>0}$. A proposal is accepted iff $w(Y) := \sum_{i \in Y} w_i \geq q$, where Y is the set of voters which are in favor of the proposal. Restricting weights and quota to integers poses the question for minimum sum representations, where the sum of weights of all voters is minimized. For at most 7 voters these representations are unique. For 8 voters there are exactly 154 weighted voting games with two minimum sum integer weight representations. Two voters i and j are called equivalent if for all subsets $S \subseteq \{1, \ldots, n\} \backslash \{i, j\}$ we have that $w(S \cup \{i\})$ and $w(S \cup \{j\})$ either both are strictly less than q or both values are at least q. Adding the extra condition that equivalent voters should get the same weight we speak of minimum sum representations preserving types. For $n = 8$ voters those representations are always unique, which is different for $n \geq 9$ voters. In [Kurz(2012)] we have determined the number of different weighted voting games with $n = 9$ voters and then extracted those that do not admit a unique minimum sum representation. In our implementation of the proposed enumeration algorithm we unfortunately relied on read and write accesses to a hard disk without further checking the validity of these operations. From the independent determination of the number of weighted games with $n = 9$ voters in [Kartak et al.(2015)] we have learned that some weighted games were missed during the enumeration process. We have carefully traced back the differences and pinned down the technical reason for the erroneous. The algorithmic approach is not affected. Rerunning the algorithm, checking hard disk operations, we are able to verify the number of 993 061 482 weighted voting games. In the next section we report the corrected statistics with respect to minimum sum representations.

2. CORRECTED RESULTS

Instead of 989 913 344 there are 993 061 482 weighted voting games for $n = 9$ voters so that tables 3, 4 and 5 from [Kurz(2012)] need to be corrected, see tables 1-3. The data of Table 6 remains valid.

In some cases the power distribution of a weighted game can be completed to a representation by choosing a suitable quota. As additional information, we have enumerated the number of those cases for the Shapley-Shubik SSI, Penrose-Banzhaf BZI, Johnston Jo, Public Good PGI, and Deegan-Packel index DP in Table 4. In

type	2	3	$\sum$
1	62432	624	63056
2	0	492	492
3	12838	0	12838
4	0	200	200
$\sum$	75270	1316	76586

Table 1. Number of weighted voting games for 9 voters without a unique minimum sum representation by type and number of representations.

equivalence classes	9	8	7	6	5	4
2 representations	5718	35864	24715	7659	1234	80
3 representations	0	402	500	330	76	8

Table 2. Number of weighted voting games for 9 voters without a unique minimum sum representation by the number of representations and equivalence classes of voters.

equivalence classes	9	8	7	6	5	$\sum$
2 representations	5718	4992	2134	392	14	13250

Table 3. Number of weighted voting games for 9 voters without a unique minimum sum representation preserving types by the number of equivalence classes of voters.

the line named "all" we have stated to total number of weighted games, i.e., the fraction of games permitting such a representation drastically decreases.

n	1	2	3	4	5	6	7	8	9
SSI	1	3	8	25	104	733	7780	113804	2445026
BZI	1	3	8	25	108	841	13570	404047	20625696
Jo	1	3	8	22	74	289	1328	6927	42187
PGI	1	3	8	22	71	213	788	4413	49437
DP	1	3	8	25	87	278	1019	5695	64002
all	1	3	8	25	117	1111	29373	2730164	993061482

Table 4. Number of different power vectors of weighted voting games.

References

[Kartak et al.(2015)] Kartak, V. M., Ripatti, A. V., Scheithauer, G., Kurz, S., 2015. Minimal proper non-IRUP instances of the one-dimensional cutting stock problem. Discrete Applied Mathematics 187, 120–129.

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