Observer-based Robust Nonlinear Control Design

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Kurzfassung

Die Regelung und Überwachung dynamischer Systeme kann voraussetzen, dass Informationen über interne Systemzustände bekannt sind. Die Verwendung von Sensoren zur Erfassung aller Systemzustände kann erhöhte Kosten zur Folge haben und die Systemzuverlässigkeit negativ beeinflussen. Weitere Probleme ergeben sich dadurch, dass ggf. nicht jeder Systemzustand sensorisch erfasst werden kann. Der Beobachter erlaubt die Rekonstruktion aller Systemzustände auf Grundlage weniger Messungen. Neben Systemzuständen können externe Eingangsgrößen wie Reibmomente und Störungen geschätzt werden. Als Konsequenz ermöglicht der Beobachter eine gegenüber Störungen robuste Regelung und Fehlerdiagnose technischer Systeme.

Der Proportional-Integral-Observer (PIO) kann mittels bestehender Entwurfsverfahren einfach implementiert werden. Durch Anpassen der Rückkopplungsmatrix eignet sich der PIO zur kombinierten Schätzung von Zuständen und unbekannten Eingangsgrößen. In diesem Zusammenhang spielt die Wahl einer betragsmäßig großen Rückkopplungsverstärkungsmatrix, als sogenannter High Gain Ansatz, eine entscheidende Rolle. Weiterhin hängt die Performance des PIO von der unbekannten Charakteristik der zu schätzenden Eingangsgröße ab. Diese Arbeit befasst sich mit der Entwicklung optimierter Entwurfsverfahren für den Proportional-Integral-Observer und der Entwicklung und Anwendung beobachterbasierter Konzepte zur robusten Regelung nichtlinearer Systeme.

In dieser Arbeit wird der modifizierte Advanced PIO (MAPIO) als erweiterte Version des Advanced PIO (APIO) eingeführt. Der Schätzfehler von MAPIO wird über ein Gütefunktional abgebildet. Das Gütefunktional wird durch Anpassung der Rückkopplungsverstärkungsmatrix an die Charakteristik der unbekannten Eingangsgröße minimiert. Die Performance der modifizierten Beobachterentwurfsansätze wird anhand eines praktischen Beispiels bewertet. Geschätzt wird eine unbekannte Kontaktkraft mit nichtlinearer Charakteristik, die auf ein mechanisches System wirkt. Anhand eines Simulationsbeispiels im offenen und geschlossenen Regelkreis wird die Performance von MAPIO gegenüber vorherigen Verfahren APIO und PIO verifiziert.

Basierend auf der Idee des Funnel Reglers wird ein neuartiges Entwurfskonzept für den Proportional-Integral-Observer vorgestellt. Die Nachteile des PIO-Konzeptes mit hohem Verstärkungsfaktor können überwunden werden und Schätzungen schneller dynamischer Verhaltensweisen lassen sich realisieren. Der Vorteil der neuartigen Funnel PIO Methode ist, dass der Schätzfehler in einem definierten Bereich, der sogenannten Funnel-Area, verbleibt. In dieser Arbeit wird gezeigt, dass der vorgeschlagene Funnel PIO Algorithmus eine adaptive PIO Verstärkungsberechnung ermöglicht, die auch in Gegenwart von Messrauschen situativ eingestellt werden kann.
Der Stabilitätsnachweis von Funnel PIO wird mittels der Lyapunov Theorie untersucht. Die Wirksamkeit der vorgeschlagenen Methode wird durch Simulation und

experimentelle Ergebnisse validiert. Eine auf einen elastischen Balken wirkende äußere Kraft mit nichtlinearer Charakteristik wird geschätzt. Ein nichtlineares MIMO System wird verwendet, um die Wirksamkeit der vorgeschlagenen Methode im geschlossenen Regelkreis zu verifizieren.

In dieser Arbeit werden zwei neue PI-Observer basierte robuste Regelungen (PIO-basierte Sliding Mode und PIO-basierte Backstepping Regelung) vorgestellt. Die Positionsregelung eines hydraulischen Differentialzylinders in Gegenwart von Modellunsicherheiten, Störungen und Messrauschen wird untersucht. Zur Anwendung der PIO-basierten Störgrößenschätzung wird eine Ein-/Ausgangs-Linearisierung des nichtlinearen Modells vorgenommen. Die Stabilität des geschlossenen Regelkreises wird in beiden Fällen mit der Lyapunov Theorie bewiesen. Die vorgeschlagenen Methoden werden experimentell validiert und die Ergebnisse werden mit dem Standard Sliding Mode Regler und einem P-Regler in Gegenwart von Messrauschen, Modellunsicherheiten und externen Störungen verglichen.

Abstract

Due to observers ability in the estimation of internal system states, observers play an important role in the field of control and monitoring of dynamical systems. In reality, using sensors to measure the desired system states may be costly and/or affects the reliability of technical systems. Besides, some signals are impractical or inaccessible to be measured and using of sensors leads to significant errors such as stochastic noise. The solution of using observers is well-known since 1964. Besides the estimation of system states, some observers are able to estimate unknown inputs affecting the system dynamics such as disturbance forces or torques. These features are helpful for supervision and fault diagnosis tasks by monitoring the sensors and system components or for advanced control purposes by realizing observer-based control for practical systems.

Among the state and disturbance observers, Proportional-Integral-Observer (PIO) is highly appreciated because of its simple structure and design procedure. Furthermore, using sufficiently high gain PIO, a robust estimation of system states and unknown inputs can be achieved. Besides taking the advantages of high gain design, the disadvantages of large overshoot and strong influence from measurement noise (as typical drawbacks of high gain utilization) in the control and estimation performance can not be neglected. Recently, some researches have been done to overcome the disadvantages of high gain observers and to adaptively adjust the gain of observer based on the resulting actual performance. Considering the advantages and disadvantages of high gain PIO besides the recent developments, it is evident that there are still open problems and questions to be solved in the area of optimal design of PIO and robust nonlinear control approaches based on PIO. On the other hand, the PI-Observer can be used in combination with linear/nonlinear control approaches (due to its simple structure and capability to estimate the system states and disturbances) to improve the performance and robustness of the closed-loop control results. Therefore, this thesis focuses on development and improvement of high gain Proportional-Integral-Observer as well as utilization of this observer in combination with well-known robust control approaches for possible general application in nonlinear systems.

The Modified Advanced PIO (MAPIO) is introduced in this work as the extended version of Advanced PIO (APIO) to tune the gain of PIO according to the current situation. A cost function is defined so that the estimation performance and the related energy can be evaluated. Comparison between advanced observer design approaches has been done in the task of reconstructing the nonlinear characteristics and estimating the external inputs (contact forces) acting to elastic mechanical structures. Simulation results in open-loop and closed-loop cases verified that the performance of MAPIO in the task of unknown input estimation is more robust to different levels of measurement noise in comparison to previous methods e.g. APIO and standard high/low gain PIO.

Furthermore, a new gain design approach of Proportional-Integral-Observer is proposed to overcome the disadvantages of high gain PIO and to realize the estimation of fast dynamical behaviors like unknown impact force. The dynamics of this force input is assumed as unknown. The idea of funnel control is taking into consideration to design the PIO gain. The important advantage of the proposed approach compared to previously published PIO gain design is the self-adjustment of observer gains according to the actual estimation situation inside the predefined funnel area. In this thesis it is shown that the proposed funnel PI-Observer algorithm allows adaptive PIO gain calculation, being able to be situatively adjusted even in the presence of measurement noise. Stability proof of funnel PI-Observer is investigated according to the switching observer condition and Lyapunov theory. The effectiveness of the proposed method is evaluated by simulation and experimental results using an elastic beam test rig. Furthermore, a nonlinear MIMO mechanical system is used to verify the effectiveness of the proposed method in the closed-loop context.

Additionally, this thesis provides two new PI-Observer-based robust controllers as PIO-based sliding mode control and PIO-based backstepping control to improve the position tracking performance of a hydraulic differential cylinder system in the presence of uncertainties e.g. modeling errors, disturbances, and measurement noise. To use the linear PIO for estimation of system states and unknown inputs, the input-output feedback linearization approach is used to linearize the nonlinear model of hydraulic differential cylinder system. Thereupon the result of state and unknown input estimation is integrated into the structure of robust control design (here SMC and backstepping control) to eliminate the effects of uncertainties and disturbances. The introduced PIO-based robust controllers guarantee the ultimate boundness of the tracking error in the presence of uncertainties. The closed-loop stability is proved using Lyapunov theory in both cases. The proposed methods are experimentally validated and the results are compared with the standard SMC and industrial standard approach P-Controller in the presence of measurement noise, model uncertainties, and external disturbances. A general comparison of SMC and backstepping control approaches is provided in the last part of this work.

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Abbreviations

PIO	Proportional-Integral-Observer
FPI	Funnel Proportional-Integral-Observer
APIO	Advanced Proportional-Integral-Observer
MAPIO	Modified Advanced Proportional-Integral-Observer
KF	Kalman Filter
EKF	Extended Kalman Filter
UKF	Unscented Kalman Filter
DO	Disturbance Observer
UIO	Unknown input Observer
PO	Perturbation Observer
LQE	Linear Quadratic Estimator
LQR	Linear Quadratic Regulator
GRV	Gaussian Random Variable
TDC	Time-Delayed Controller
DOBC	Disturbance Observer-based Control
LTR	Loop Transfer Recovery
FDI	Fault Detection and Isolation
LMI	Linear Matrix Inequality
LPV	Linear Parameter Varying
GPI	Generalized Proportional Integral
SMC	Sliding Mode Control
BC	Backstepping Controller
MPC	Model Predictive Control
SISO	Single-Input Single-Output
MIMO	Multi-Input Multi-Output
FC	Funnel Control
FEM	Finite Element Method
MSE	Mean Squared Error
UIEE	Unknown Input Estimation Error
SSE	State Estimation Error
VSCS	Variable Structure Control Systems
CLF	Control Lyapunov Function
EFL	Exact Feedback Linearization
PIO-EFL	Combination of Exact Feedback Linearization and PIO
FPIO-EFL	Combination of Exact Feedback Linearization and funnel PIO
PIO-SMC	Combination of Sliding Mode Control approach and PIO
PIO-BC	Combination of backstepping control approach and PIO

1 Introduction

1.1 Motivation and problem statement

Effects of friction, backlash, or impacts often occur in dynamical systems. Also due to these effects, the system behavior in operation becomes nonlinear. These effects can not be precisely modeled (by structure and/or parameters) which affects the design of control approaches. Several nonlinear control methods are introduced and developed during the past decades to improve the robustness of closed-loop system (feedback linearization, sliding mode control, backstepping approach, etc. [SL91, Isi95, Kha02]) with the general purposes of stability, small tracking error, disturbance attenuation, noise rejection, and insensitivity to the plant modeling errors [RT06]. The well-known restriction of mentioned classical methods is requirement of system states as measurements which is from a practical point of view costly. Furthermore, most of the nonlinear control approaches are based on the assumption, that the system model is known and related to the actual control situation (load, pressure, oil temperature, etc.). Often the existing disturbance and the related dynamical behavior, i.e. the friction and/or the load torque are not considered. This results from the fact that they are unknown, not measured, or are not measurable. For example, the well-known feedback linearization method [MVKB12] assumes an accurate nonlinear model to design a nonlinear control law to eliminate the coupled disturbance. It should be mentioned that approaches assuming exact models provide nonoptimal behavior in practical realizations (due to imperfect modeling). As a consequence, effective and safe nonlinear control approaches, which are robust to external disturbances and model uncertainties, are required.

In [LS14] limitations of nonlinear approaches applied to nonlinear systems are summarized. Nonlinear control methods are developed to control the nonlinear systems usually based on exact nonlinear system descriptions [Kha96]. Goal of the related research works is to realize robust and practical solutions for nonlinear systems. To achieve the availability of all modeled (also internal) states and assuming full state feedback, observer-based robust nonlinear control approaches have been discussed. In [HD03a] an exact feedforward linearization approach based on differential flatness for nonlinear system control is proposed, showing robustness problems as the classical feedback linearization method. In [HD03b] the robustness with respect to uncertainties and disturbances is detailed. In [HD08], a robust nonlinear predictive control based on exact feedforward linearization is introduced. It was demonstrated that the nonlinear flatness-based control methods are applicable and rational. No assumptions with respect to disturbance characteristics and availability of all states are discussed. Comparing different methods applied to solve the robustness problem of control methods, modeling errors or disturbances acting to the system are typically considered. Therefore, known bounds and/or dynamical properties have to be assumed [MT93]. Besides the robustness, the availability of (internal) modeled states has been considered and assumed in nonlinear control design.

Summarizing open problems of nonlinear control it has to be stated that besides the requirement of known models, assumptions have to be made according to the availability of internal (not measured) states as well as bounds with respect to modeling errors and external effects like disturbances. Some control approaches to improve the robustness against uncertainties and to access the system states are based on observers. As example the proposed control approach in [EF09] is based on an adaptive observer design partially linearizing the considered systems. Considering no disturbances, the adaptive control approach is suitable for a general class of nonlinear systems. In [CK06] the authors proposed a control approach with a full-order nonlinear observer to control the motion of an engine exhaust valve actuator.

According to [FBF11] the desired control performance in the presence of uncertainties can be achieved by (a) applying a robust control approach such as sliding mode control, backstepping control, H_∞ control, etc. able to compensate the uncertainties or (b) estimation of uncertainties using observers to be compensated by a classical designed control law stabilizing the nominal system e.g. in [LS14]. The performance of mentioned approaches depends on the system structure. In [KR16] a higher order sliding mode observer is used for a hydraulic actuated piston to estimate the system states and unknown load force used in a classical cascaded control structure. The effects of friction force are compensated using a static friction model.

Due to observers ability to estimate internal states of the system, observers are widely used in different aspects of control field. Observers can be augmented or replaced with sensors to increase the accuracy/reliability and to decrease the cost simultaneously. Based on the structure, order, gain, and class of system considered for observers design, observers can be categorized as the following:

- Linear and nonlinear observers
- Full-order, reduced-order, and minimal-order observers
- Observers for linear system, nonlinear system, linear system with delay, nonlinear system modeled by a multiple model approach, bilinear system, etc.
- continuous and discrete observers
- State and unknown input observers
- Stochastic and deterministic observers
- High gain and standard observers

Besides the estimation of system states, some observers are able to estimate unknown inputs affecting the system dynamics such as disturbance forces or torques.

This features are helpful for (1) supervision and fault diagnosis tasks by monitoring the sensors and system components [PC97, YDL15, CDSM02] or for (2) advanced control purposes by realizing observer-based control for practical systems [TLLL11,LYCC14]. Among the state and disturbance observers, Proportional-Integral-Observer (PIO) is highly appreciated because of its simple structure and design procedure. Furthermore, using sufficiently high gain PIO a robust estimation of system states and unknown inputs can be achieved. Besides taking the advantages of high gain design, the disadvantages of large overshoot and strong influence from measurement noise (as typical drawbacks of high gain utilization) in the control and estimation performance can not be neglected. Recently, some researches have been done to overcome the disadvantages of high gain observers. For example a recently published extension introduced as Advanced PI-Observer (APIO) in [LS12], adapts the gain design based on the resulting actual performance. Despite the strong researches and works in this area, there are still aspects to be improved. Therefore, the main part and objective of this work is dedicated to the optimal design of high gain PIO and development of robust nonlinear control approaches combined with the estimation results of PIO for possible general application in nonlinear systems. To achieve the principle objective the following tasks are arranged:

1. Investigation and analysis of the well-known Proportional-Integral-Observer
2. Investigation and improvement of existing adaptive APIO algorithm
3. Development of a new adaptive algorithm for design of high gain PIO
4. Enhancement of disturbance attenuation and system performance robustness by proposing a combination of PIO and nonlinear robust control approaches with stability analysis

1.2 Thesis organization

In this thesis, development and improvement of Proportional-Integral-Observer as well as utilization of this observer in combination with well-known robust control approaches are discussed. The thesis consist of six chapters according to the published/submitted journal papers ([BS17a], [BS17b], [BS17e]), conference papers ([BS14], [BS15a], [BS15b], [BS17d], [BS17c]), and workshop presentations ([BSW14], [BSW15a], [BSW15b], [BSW16], [BSW17]).

Accordingly, in the current chapter the overall overview and scope of this thesis including the main challenges, problems, and motivation points are discussed. The second chapter provides a review of the principal unknown input observation with elaboration of Proportional-Integral-Observer. In this chapter the structure, design goals and methods, and integration of PI-Observer in different system types are focused. Although using of terms seems to be very flexible in the discussed area, this chapter outlines a new perspective and a precise distinction between fundamental concepts of observer, filter, and estimator from a scientific and pragmatic point of view. In addition, predictor-corrector scheme including a predictor step (an explicit method to obtain a rough approximation as a first step) and a corrector step (an implicit method to refine the predicted value as a second step), is briefly reviewed by considering a new perspective of observers/filters structure. The chapter is continued with a description of some actual advanced applications of PI-Observer and high-gain design in an abstract level.

The third chapter of thesis provides the detailed insight into Proportional-Integral-Observer with its general structure and convergence conditions. Besides, a new development of high gain PIO design is proposed as Modified Advanced Proportional-Integral-Observer (MAPIO). The new approach is an improved version of previous introduced APIO [LS12] with adaptive gain scheduling approach. Moreover, comparison of the proposed MAPIO with the well-known PIO and APIO is provided by open-loop and closed-loop simulation examples. Furthermore, the advantages of using extended PIO (MAPIO) in comparison to the well-known PIO are investigated with respect to reconstruction of a complex nonlinear unknown input. Therefore, a clamped beam example is considered and reconstruction of complex nonlinear spring behavior is performed to prove the performance and advantages of MAPIO. In both open-loop and closed-loop simulation examples, the effect of additional measurement noise is investigated to illustrate the benefits of using MAPIO with adaptive high gain scheduling design.

The forth chapter focuses on a novel gain design approach of Proportional-Integral-Observer for contact force estimation with fast and unknown dynamics. The estimation of fast dynamical behavior requires high observer gains. The novel funnel PIO approach takes advantage of the funnel idea to adjust the PIO gains according to the actual situation and to maintain the estimation error in a prescribed funnel area. The introduced approach shows significant advantages with respect to estimation of system states and unknown inputs in the presence of measurement noise which is a considerable property for practical application of high gain observers. Both simulation and experimental results verify and validate the advantages of the introduced funnel PIO compared to known high gain PIO. Stability of the proposed adaptive algorithm with respect to switching PI-Observers is discussed based on Lyapunov theory. Furthermore, the advantages of using proposed funnel PIO is illustrated in closed-loop simulation for a MIMO system example.

The fifth chapter focuses on the design of observer-based robust nonlinear control approaches for nonlinear systems affected by uncertainties including modeling error, external disturbances, and measurement noise to assure suitable tracking performance as well as robustness against unknown inputs. The task of system state and unknown input estimation is performed by a high gain linear Proportional-Integral-Observer. Therefore, input-output feedback linearization is used to linearize the nonlinear system model to be used for linear PI-Observer structure. Two robust control approaches are considered in this chapter: (1) sliding mode control and (2) backstepping control. Estimation of system states and unknown inputs are integrated into the structure of robust nonlinear control approaches to achieve a control law that provides desired performance for closed-loop system in the presence of uncertainties, and compensates the effects of external disturbances, plant parameter changes, unmodeled dynamics, measurement noise, etc. Additionally, parameter selection of the proposed PIO-based controllers is elaborately considered by defining a performance/energy criterion. Stability of the closed-loop system is established using Lyapunov method in each cases. Furthermore, a complete robustness evaluation considering different level of measurement noise, modeling errors, and external disturbances is performed in this chapter. Experimental results using a hydraulic differential cylinder test rig validate the advantages of introduced combined approach compare to the standard sliding mode controller (as a robust controller for nonlinear processes subject to external disturbances and heavy model uncertainties) and P-Controller (as a standard classical industrial approach for hydraulic systems). Consequently, integration of unknown input observer estimation results into the structure of robust control approaches leads to enhance the disturbance attenuation and system performance robustness.

In the sixth chapter the summary and conclusion of the whole thesis are discussed. Furthermore, final remarks and future works are outlined for the next steps.

2 Unknown input observations: state-of-the-art with special attention to the Proportional-Integral-Observer

The contents, figures, and tables presented in this chapter are prepared for publication as a journal paper "Unknown input observations: state-of-the-art with special attention to the Proportional-Integral-Observer" and published in the conference paper "Proportional-Integral-Observer: A brief survey with special attention to the actual methods using ACC benchmark" [BS15a].

2.1 Introduction and problem classification

Observers are mathematical or algorithmic expressions with dynamical behavior, designed to combine system knowledge and measurements for reconstruction of additional information related to assumed underlying dynamical models. Observers constitute an important component module in the control and maintenance field because of their ability to estimate non-measured states. Using sensors to measure internal non-measurable modeled system states may be costly or unreliable, so alternative solutions using observers are advantages. Besides the estimation of system states, some observers are able to estimate unknown inputs affecting the system dynamics such as disturbances. The new information obtained can be used for monitoring and fault diagnosis tasks as well as advanced control purposes.
A general model for a linear time invariant system with additive unknown inputs (disturbances, unmodeled dynamics, or other nonlinearites) can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Nd(x,t) + Eg(x,t), \\ y(t) &= Cx(t) + Du(t) + h(t), \end{aligned} \tag{2.1}$$

with the state vector $x(t) \in R^n$, input vector $u(t) \in R^m$, measurement vector $y(t) \in R^r$, unknown input $d(x,t) \in R^l$, measurement noise $h(t) \in R^r$, and unmodeled dynamics $g(x,t) \in R^p$. Here the unknown input $d(x,t)$ and the unknown input matrix N are used to model unknown inputs assumed as additive inputs. Matrices A, B, C, and D are assumed as known and of appropriate dimensions $n \times n$, $n \times m$, $r \times n$, and $r \times m$, respectively.
Observers are widely used to estimate system states $x(t)$ as $\hat{x}(t)$. The common primary observers used to estimate system states are Kalman Filter [Kal60] and Luenberger Observer [Lue64] in discrete and continuous time, respectively. Luenberger observer is intensively used in the classical control field because of its capability to estimate system states. It has a feedback loop to reconstruct system states based on measured outputs.
On the other hand, in reality even if a system model is available, the system is

affected by unknown inputs such as disturbances and noises, as well as model uncertainties interpreted as additive unknown inputs. Later to improve the non-efficient proportional observers to deal with unknown effects, new observers have been proposed. The goal of designing such an observer is to observe unknown inputs, disturbances, noise, uncertainties, nonlinearities, or modeling errors. Proportional-Integral-Observer, known as PI-Observer, has been firstly introduced in [Woj78] for SISO-linear time invariant systems by augmenting an integral term into the structure of Luenberger Observer. Unlike proportional observer which uses only the current information of the estimation error ($e(t) = x(t) - \hat{x}(t)$), PI-Observer also uses the past information of the estimation error by using the additional integral term. Later PI-Observer was improved in [Kac79] and [SC85] for MIMO-linear systems to improve the estimation robustness against step disturbance and variations in the system parameters. The idea of using a linear model, described the disturbances acting upon linear systems, is proposed in [Joh76]. The authors in [ML77] came up with the conditions and proofs for modeling disturbances as a linear model with respect to linear systems. Constructing a disturbance model for more general use is proposed in [SYM95] to improve the observer performance. Based on the work in [Mül88] the authors proposed a general linear model for disturbance while no information about the unknown input is neither used for design nor for estimation. The proposed extended observer scheme can estimate the additive unknown inputs (as additive nonlinearities resulting from unmodelled dynamics) using certain assumptions like observability of the nonlinear inputs from the output.
This chapter provides a comprehensive survey of state and unknown input observation. Up to now several surveys have been published with the contents of linear and nonlinear observers. In [VS00] the author presents a view leading to a generalization of state, disturbance, nonlinearity, coupling and fault observers with internal proportional and integral feedback. Special attention is given to the essential features of PI-Observer for reconstruction of control system and from the author so-called effect system variables. The state observer and its order reduction in connection to state feedback control design are briefly presented in [SS15]. The subsequent development of new observer structures for disturbance estimation and fault detection is pointed out by the authors. Furthermore, the problem of designing a decentralized PI observer with prescribed degree of convergence for a set of interconnected systems is reviewed by the authors. A survey of observers for nonlinear dynamical systems is given in [KKXX13] including fundamental concepts of system observability defined by numerical model/differential equations with special attention to optimal filters e.g. Kalman Filter and H_∞ filter. In [CYGL16] the authors provide a general overview of disturbance observer-based control (DOBC). This survey gives a systematic summary on existing disturbance attenuation approaches e.g. active disturbance rejection control (ADRC), disturbance accommodation control (DAC), and composite hierarchical anti-disturbance control (CHADC).
The novelty of this section compared with previous publications and reviews is its comprehensiveness and completeness. Unlike other surveys in the related field, in

this contribution unknown input observation approaches (e.g. Proportional-Integral-Observer, disturbance observer, unknown input observer, and perturbation observer) with more elaboration on Proportional-Integral-Observer is taking into consideration. Accordingly the focus of this section is on PI-Observer approach with respect to structure, design goals and methods, and integration to different system structures. High-gain approaches and actual advanced applications of PI-Observer are taking into consideration to summarize different aspects in this field. A new perspective of filters and observers in the form of predictor-corrector algorithm is pointed out to inspire the mindset about observer/filter structure, so new ideas can be presented based on the introduced structure. This section starts with some general and basic observer/filter structures e.g. Luenberger observer and Kalman Filter to illustrate the predictor-corrector scheme in both deterministic and stochastic cases. Furthermore, a precise definition of basic concepts is given to avoid the misuse of terms. Finally a clear distinction between estimator, observer, and filter definitions is given. It is worth mentioning that the main part of this survey is dedicated to PI-Observer and its formulation for different system types as well as recent applications of this observer.

2.2 Fundamental and definition of basic concepts

In different articles and books (e.g. [VS00, SS15, KKXX13]) various titles have been used for defining the observation task. Different wording has been used over decades, in different communities, and with different purposes. In literature the observation task is differently denoted. For example "estimation" is used to define different tasks such as variable/parameter estimation, system state estimation, disturbance estimation, fault estimation, etc. This denotation maybe present the concept correctly but from a scientific and also from a pragmatic point of view is confusing and not generalizable. However, using of terms seems to be very flexible in the discussed area. In the following part a definition of three basic concepts is precisely brought forward to avoid misunderstanding of contents. Furthermore, it allows redefinition of the wording.

2.2.1 Estimator

According to [BSLK04]: *"Estimation is the process of inferring the values of a quantity of interest from indirect, inaccurate, and uncertain measurement. Estimation can be viewed as a scheme for information extraction and enhancement: Based on measurements, to maximize knowledge about a parameter, a state, a signal, an image, and so on."*.
An estimator realizes the estimation of parameters using measured or empirical data

which contains a random component. The parameters are related to the underlying component of a system which indirectly affects the measurements. Consequently the purpose of an estimator is to estimate the underlying parameters using measurements as accurate as possible [Sen95,KSH00]. The estimation theory can be divided into two categories [Sen95]: Parametric estimators and non-parametric estimators. In the first category estimation is achieved based on knowledge or assumptions of data and desired parameter such as probability density function. The second category deals with data without additional assumptions and therefore is more significant related to the robustness compared to the first category.
Estimators are widely used in the field of control theory particularly in the field of adaptive and optimal control [Sim06]. Estimation of some or all of the state components can be also achieved using some extended estimator schemes which are commonly known as Kalman Filter or Kalman Estimator. The extended estimator simultaneously estimates the state and underlying system parameters.

2.2.2 Observer

According to [Oga01]: *"Estimation of unmeasurable state variables is commonly called observation. A device (or a computer program) that estimates or observes the state variables is called a state observer, or simply an observer."*
The purpose of an observer is to observe the state of system for monitoring and fault detection task or other intentions. State observer is a system provides the internal state observation by using the input and output of a real system. As example, considering a general model for a linear time invariant system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{2.2}$$

with the state vector $x(t) \in R^n$, input vector $u(t) \in R^m$, measurement vector $y(t) \in R^r$, and matrices A, B, C, and D as known and of appropriate dimensions $n \times n$, $n \times m$, $r \times n$, and $r \times m$, respectively. The task of state observer is to reconstruct the system state $x(t)$ as $\hat{x}(t)$ by using the input $u(t)$, the output $y(t)$, and the system model (A, B, C, D matrices).
The state observer is computer-implemented in most cases and has many applications in control approaches especially in the field of supervision and control. Observers provide valuable information about the physical process in the supervision task as well as improve the control performance by delivering required information or increasing the quality of existing information for control.

2.2.3 Filter

According to [Jaz07]: *"The problem of determining the state of a system from noisy measurements is called filtering. It is of central importance in engineering, since it*

is required for the control of systems. Furthermore, a large class of (system) identification problems can be regarded as problems of filtering."
A filter can be assumed as a process, device, or module which can remove or attenuate undesired components and features from a signal to achieve desired spectral characteristics of the signal. Filtering process can be categorized in the signal processing field [RGY78]. It can be considered as a process to eliminate some aspect of a signal regarding the desired features. These aspects in most cases are considered as interfering signals or noise. Filters can be classified in different ways such as linear/nonlinear, analog/digital, discrete/continuous-time, passive/active, etc. One of the most important filter structure in the field of control is Kalman Filter which is detailed in the following section.

2.2.4 Kalman Filter and its extensions as an exception

According to [BW01]: *"The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error."*
Kalman filter is the optimal linear filter in sense of minimizing the variance estimate of states with the assumptions that the system model perfectly matches the real system, the entering noise is white (uncorrelated), and the covariances of the noise are exactly known [AM12]. In other words, for systems fulfilling the requirements the KF is the best choice. If the assumptions are not met, the Kalman filter is again the optimal linear estimator in the sense that no other linear filter gives a smaller variance on the estimation error. Kalman Filter is firstly introduced in [Kal60] to formulate and solve the Wiener Filter problem which is proper for filtering, smoothing, and prediction of wide-sense stationary signals. It is worth noting that Kalman Filter in discrete or continuous form [BJ87] and its extensions [JU97] are introduced as a fundamental algorithm to solve a broad range of estimation problems. Indeed in [Wie49] the authors improved a research on the extrapolation, interpolation and smoothing of stationary time series. In [Str59] the authors investigated the theory of optimal nonlinear filtering of random functions by selecting the useful signals from noise in nonlinear systems. Furthermore, in [Str60] the Markov process theory in the theory of optimal nonlinear filtering is used. The authors in [Kal60] perused the linear filtering and prediction problems and focused on solving the Wiener problem from another point of view as mentioned before.
For linear well-defined system influenced by Gaussian noise (system/measurement noise) a statistically optimal solution can be reached using Kalman Filter. The algorithm is able to remove or attenuate the effects of system/measurement noise (w_k and v_k) to reconstruct the state value x_k from the real measurement z_k. The Kalman Filter assumes that the state at time step k is calculated from the previous

state step x_{k-1} according to

$$\begin{aligned} x_k &= Ax_{k-1} + Bu_k + w_{k-1}, \\ z_k &= Hx_k + v_k, \end{aligned} \tag{2.3}$$

with A, B, and H matrices which denote the available model and normally considered as constant matrices. It is a recursive algorithm which has the ability to infer the desired parameters from indirect, inaccurate, and uncertain data affected by noise or uncertainties fulfilling given statistical conditions. If system/measurement noise (w_k and v_k) are not Gaussian, Kalman Filter is however the best linear estimator to reconstruct the true state value x_k with a random mean and standard deviation of noise which minimizes the mean square error of the estimated states [Gre11, AM12]. It is named as “filter” due to the design realizing the “best” estimation of states $\hat{x}_k$ when the measured signals are noisy. In this sense the task of Kalman Filter is “filtering out” the noise.
Consequently Kalman Filter, known also as linear quadratic estimator (LQE), is generally used to estimate the unknown state variables. One of the most important applications of Kalman Filter is utilization of noisy input/output data of a system to optimal estimation of underlying system states. Because of this application for estimation of system states, Kalman Filter can be considered also as an discrete/continuous linear observer. Therefore Kalman Filter can be categorized in the filter, observer, and estimator area simultaneously and for different purposes such as state estimation, fault detection/diagnosis, etc. In [FS99] the authors proposed a multihypothesis bank of Kalman filters to detect the size and location of damages after identification using Hilbert transform.
Besides the estimation of system states new approaches based on Kalman Filter are also able to estimate the unknown input in discrete time. In [GDM07, SLC15b] the authors propose the unknown input filter design based on Kalman Filter when no information on the unknown input is available and when partial information on the inputs exists, respectively. In [SLC15a] the authors discuss the existence condition and relationship of the proposed approaches in [GDM07, SLC15b] with the classical Kalman filter. The domains and interferences of estimators, observers, and filters are illustrated in Figure 2.1.

2.3 Predictor-corrector algorithm

The main idea of using predictor-corrector methods is combining explicit and implicit techniques to achieve a suitable convergence performance [PFTV92]. It involves a predictor step with an explicit method to obtain a rough approximation as a first step. Afterward in the corrector step, an implicit method is performed to refine the predicted value from the prediction step (often denoted as innovation, which is based on measurements). The procedure of predictor-corrector is repeated

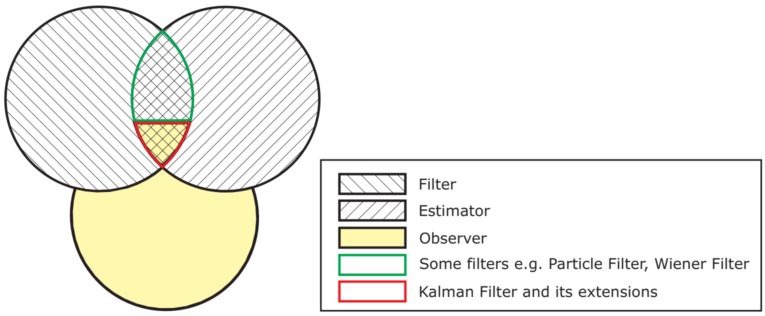


Figure 2.1: Illustration of basic concepts domain and interference

within a time-discrete simulation framework. By suitable design of feedback gain matrices, convergence of the error variance to a minimum can be ensured. Several methods can be used in each prediction and correction steps but the sufficient and straight-forward solution uses the simplest possible pair of methods e.g. the known Heun's method which contains of Euler method (explicit method) and the trapezoidal rule (implicit method) [SM03].
The strategy of predictor-corrector scheme is widely used in mathematics and particularly numerical analysis [Pre07]. Practically, the structure of observers can also be considered as innovation of the predictor-corrector mechanism uses the discrepancy between the system output as measurements and the observer output to refine the estimation. Also here the feedback gain is designed so that the error dynamics between the estimated $\hat{x}(k)$ and real $x(k)$ value vanishes and the system describing the error dynamics is asymptotic stable.
Indeed the strategy of predictor-corrector means that the observers (especially Kalman Filter) incorporate feedback into the equations describing the time behavior of the estimation. Therefore, predictor-corrector algorithm is capable to reconstruct the system state and refine the convergence of solution. In the following part the basic predictor-corrector observation algorithms with the special attention to Kalman Filter are investigated.

2.3.1 Deterministic/Luenburger Observer

Some control methods assume that the intended system states are available. However, in practical applications often only a few sensors could be used. Therefore, implementation of some control methods is significantly limited by the number of available measurements/sensors. To overcome this limitation Luenberger observer is introduced in [Lue64] for the first time. Luenberger observer is a linear observer used for reconstructing the system states by using measured inputs and outputs.

Based on (2.2) the structure of Luenberger observer can be written as

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + L[y(t) - \hat{y}(t)] + Bu(t), \\ \hat{y}(t) &= C\hat{x}(t) + Du(t), \end{aligned} \tag{2.4}$$

with observer gain L, observer state $\hat{x}(t)$, and observer output $\hat{y}(t)$. If the estimation error known as observer error ($e(t) = \hat{x}(t) - x(t)$) converges to zero when time goes to infinity, the observer is called asymptotically stable. The error dynamics of the Luenberger observer is described by $\dot{e}(t) = (A - LC)e(t)$ with the initial value $e(0) = \hat{x}(0) - x(0)$. The error dynamics is asymptotically stable if and only if $Re\{\lambda_i\}$ for all eigenvalues λ_i of matrix $(A - LC)$ is less than zero. Consequently, the dynamical behavior of observer can be regulated by adjusting the observer gain L, if the system is observable.

An extension of Luenberger observer for nonlinear single-input single-output systems is introduced in [Zei87] with reference to the well-known extended Kalman filter algorithm and based upon a local linearization around the reconstructed state. This idea is based on the assignment of nonlinear eigenvalue problems without solving the nonlinear partial differential equations. Later extended Luenberger observer for non-linear multivariable systems is introduced in [BZ88] using a transformation into the nonlinear observer canonical form and an extended linearization approach. This method also requires analytic calculations of derivatives and matrix inversions. Further improvement is proposed in [CDMG93] as Luenberger-like observer for nonlinear systems to improve the structure and performance of observation. The authors illustrate that by using an appropriate Lyapunov-like equation to design the observer gain, the state of nonlinear system can be asymptotically observed.

2.3.2 Stochastic/Kalman Filter

If the measurements contain statistical noise and other inaccuracies, a stochastic filter/Kalman Filter can be applied to estimate the system states. As mentioned in section 2.2.4, Kalman Filter is actually a set of mathematical equations which is optimal in the sense of minimizing the estimation error covariance when some presumed conditions are met. This filter/estimator firstly proposed in [Kal60] works in a two-step process: prediction of actual state and error covariance, correction of estimated state and error covariance.

Based on system model a prediction is firstly performed, then the correction part is executed using a suitable designed gain known as Kalman gain. Based on the formulation (2.3) first of all determination of required parameters (system noise covariance Q and measurement noise covariance R) and initial values must be done. The random variables w_k and v_k are assumed to be independent, white and with normal probability distributions

$$\begin{aligned} p(w) &\sim N(0, Q), \\ p(v) &\sim N(0, R). \end{aligned} \tag{2.5}$$

In practice, the process and measurement noise covariance matrices might change during the time, however in some issues they are assumed to be constant. There are two sets of equations for prediction and correction process (Table. 2.1). The time update projects the current state estimation ahead in time while the measurement update adjusts the projected estimation by an actual measurement at that time. Kalman filter solution is optimal (minimizes errors in some respect) if the following

Table 2.1: Kalman Filter prediction/correction procedure

Prediction (time update)	Correction (measurement update)
(1) *Project the state ahead* $\hat{x}_k^- = A\hat{x}_{k-1} + Bu_k$ (2) *Project the error covariance ahead* $P_k^- = AP_{k-1}A^T + Q$	(1) *Compute the Kalman gain* $K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$ (2) *Update estimate with measurement* $\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$ (3) *Update the error covariance* $P_k = (I - K_k H)P_k^-$

conditions are satisfied: (1) The system model is precise, (2) system/measurement noises are white noise, and (3) the covariance of noise are precisely known [MH12]. Under these conditions, there is a unique "best" estimation $\hat{x}_k$. Some conditions can be relaxed e.g. if the Gaussian assumption is removed, the Kalman Filter is however the best linear estimator minimizing the mean square error of the estimated states. Extended Kalman Filter (EKF) has been introduced for nonlinear systems by linearizing the estimation around the current estimation and using partial derivatives of the system/measurement equation [Sor60, Cos94]. In the presence of strong nonlinearities, extended Kalman filter leads to poor estimation results because of propagation of covariance from one measurement sample time to the next through the linearization [JU97]. In other words using of Jacobians, representing all partial derivatives of the nonlinear system (model), may lead to sub-optimal performance and sometimes divergence of the extended Kalman Filter. Therefore Unscented Kalman Filter (UKF) is introduced in [Cos94] to improve the estimation performance as well as to remove the requirement of Jacobians calculation. The UKF exactly addresses the problem of EKF by using a deterministic sampling approach. The UKF approximates the state distribution by a GRV (Gaussian Random Variable) as well as EKF, but it is represented by using a minimal set of carefully chosen sample points. It is worth noting that the computational complexity of UKF and EKF are of the same order.

The time-discrete formulation in combination with the predictor-corrector scheme allows uncomplicated practical realization due to the recursive structure of implementation as a finite difference scheme. With respect to the goal of minimizing the error covariance $P_k = E[e_k e_k^T]$, calculation of the Kalman gain K_k (also known as blending factor) can be recursively realized using the finite difference descrip-

tion [Bro83]. The equations of Kalman Filter (Table. 2.1) can be algebraically manipulated to several forms and consequently a new equation calculating the blending factor can be achieved. The final goal is to minimize the error covariance P_k. Therefore Kalman gain is one of the particular and popular form of the predictor-corrector strategy [BW01]. The process and measurement noise are assumed to be independent (of each other), white, and normal distributed. In practice, the process noise covariance and measurement noise covariance matrices can change at each time step or measurement update. Nevertheless, according to [BW01] they are assumed as constant values. The system matrix A and measurement matrix H in (2.3) could/should be updated by the analysis of the measurements, so appears as a function of time.
As an example of other design methods for blending factor the well-known H_∞ filter can be considered [LJ10]. Unlike the Kalman Filter which tries to gain the minimum mean-square estimation error, H_∞ filter tries to minimize the consequence of most nonlinearities to increase the robustness. On the other side, H_∞ filter can work with deterministic systems and it is robust against variation of the system model. It provides a trade-off between mean-square-error and peak error performance criterion to improve the system robustness. Therefore, it is also known as robust filter. Furthermore some other methods by manipulating the algebraic equations of Kalman Filter can be find in [UJDW00] and [Hab07]. In Table 2.2 a brief survey of world wide principal observers/filters, their properties, and developers are presented.

2.4 Unknown input observation

In this section disturbance observers (DOB), unknown input observers (UIO), and perturbation observers (PO) are analyzed due to their advantages in disturbance observation and related use for fault detection and isolation.

2.4.1 Disturbance observer (DOB)

In some references disturbance observer is introduced as extended observer scheme extending state observer by adding a disturbance model and describing the additional disturbing input [AM07, Dav72, Joh76]. In this case the extension is used to model constant [AM07], sinusoidal, or polynomial effects [Dav72, Joh76] assumed as additive known (modeled) inputs. Using of linear models for disturbances acting upon linear systems is introduced in [Dav72] and [Joh76] by modeling the disturbance using a suitable signal process (i.e. the disturbance can be modeled by a linear differential equations with constant coefficients driven by the difference between the tracking signal and the corresponding output of the controlled system) with the purpose of disturbance accommodation. Subsequently, [ML77] improved the proposed method by giving the conditions and proofs for modeling disturbances

Table 2.2: Survey of different observers/filters and their properties

	Approach-Property	**Developer (first publication//name)**	**State estimation**	**Disturbance estimation**	**Parameter estimation**	**Robustness performance and stability improvement**	**Linear observer**	**Nonlinear observer**	**High gain design**	**Adaptive structure**	**Requires noise information**	**Requires accurate system model**	**Based on disturbance model**	**Complexity**	**Design in time domain**	**Design in frequency domain**	**Design with standard methods**
Kalman Filter	State estimation of a system with white and Gaussian noise	1960 Kalman	x		x	x	x			x	x	x		x	x		
Luenberger Observer	State estimation of a linear system	1964 Luenberger	x				x					x			x		x
Disturbance Observer	External disturbance estimation	1972, Davison 1976, Johnson		x		x	x		x			x	x	x	x	x	
PI Observer	State and unknown input estimation	SISO: 1978 Wojciechowski MIMO: 1979,Kaczorek 1985, Shafai and Carroll	x	x		x	x								x		x
High-gain observer	States and unknown input estimation with high gain	Since 1993 Söffker,Müller. Since 1995 as PI-Observer, Müller, Lückel and Söffker.	x	x		x	x								x		x
		2010 Liu and Söffker	x	x		x	x		x	x				x	x		x
Extended KF	State and parameters estimation for a noisy dynamic system	1964 H. Cox	x		x	x		x	x	x	x			x	x		
Unscented KF		1995 S.J. Julier	x		x	x		x	x	x	x			x	x		

as linear models for more general use that no information about the disturbance is available. Most of the recently published articles define the disturbance observer in frequency domain [LP00, KMO07], therefore, this section elaborates on frequency domain.
The structure of DOB is illustrated in Figure 2.2 with the real control input and output ref and c, respectively following the work in [SVD02]. The external disturbance input is denoted by d and its estimated $\hat{d}$, and the measurement noise is denoted by w. In addition the real plant P and the inverse of nominal plant transformation P_n^{-1} are considered. The low pass filter Q is the key component of DOB added to compensate the causality of the inverse model P_n^{-1} with respect to relative degree of the nominal plant. This issue should be considered in the utilization of DOB for non-minimum-phase systems because the inverse of non-minimum-phase systems leads to a non-causal system. According to [SVD02] the performance of disturbance attenuation for systems described by a linear model relies on the numerator order of Q filter as well as the robustness depends on the relative order and the denominator order of Q filter (further details are given in [SVD02]) which can be designed as

$$Q(s) = \frac{c_{k-1}(\tau s)^{k-1}+...+c_0}{(\tau s)^l+a_{l-1}(\tau s)^{l-1}+...+a_1(\tau s)+a_0}, \tag{2.6}$$

where $c_0 = a_0$ and $l - k + 1 \geq r$ (relative degree of nominal plant r) so that the transfer function $Q(s)P_n(s)^{-1}$ becomes proper. The constant design parameters a_i should be chosen such that the polynomial $s^l + a_{l-1}s^{l-1} + ... + a_0$ is Hurwitz. The positive filter constant τ determines the bandwidth of the Q filter. An actual design of Q filter for DOB is performed in [PLY+11] for moving target tracking of an unmanned firearm robot.
Disturbance observer (DOB) operates as a filter in frequency domain to reduce the effects of disturbances. According to [PLY+11] implementation of DOB in real-time systems seems to be difficult because the plant model sometimes is not accurate enough and the inverse of the plant model can not be achieved in some cases due to a causality problem. However, by using a low pass filter Q and by considering the difference between the real plant and the system model as unknown disturbance, implementation of DOB could be successful. For example in [SVD02] a DOB-based robot tracking controller with a new structure for designing of low pass filter is proposed, which enables the utilization of DOB for non-minimum-phase systems. Actually DOB endeavors to compensate the difference between the nominal model output and the plant output affected by disturbances.
Estimation of unknown disturbances in addition to the system states is the key idea of using disturbance observers. An actual utilization of DOB is given in [PLY+11] to minimize the vibration of a moving object. A valuable work on relationship between time domain (based on [Joh71]) and frequency domain (based on [SVD02]) disturbance observer is provided in [SCY16]. The authors concluded that the traditional DOB in frequency domain [SVD02], using the low pass filter with unity gain, is able to estimate the special kind of disturbances satisfying the matching condition (the

disturbance affects the system in the same channel as the known input of the system). From the other side the classical time domain DOB [Joh71] generates higher order observer to deal with disturbances and to estimate the system states.

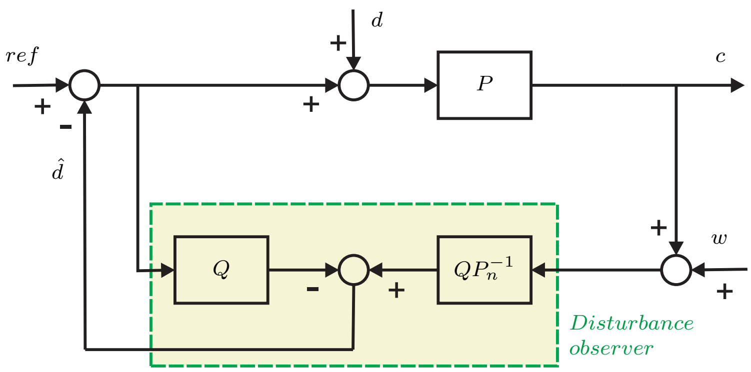


Figure 2.2: Block diagram of disturbance observer (redrawn from [PLY+11,SJK12])

2.4.2 Unknown input observer (UIO)

The term unknown input observer (UIO) is widely used in literature, especially in the Fault Detection and Isolation field (FDI) [Ger98, PC97, FD97]. Concluding the literature review and according to the assumptions to be made for the unmeasurable (unknown) inputs, a distinction between two classes of UIO can be considered. In the first category a-priori assumptions are made concerning the dynamical behavior [Joh75, MH74], in the second class no assumptions about the unknown terms are made [YW88, WWD75].
The system with additive unknown input can be described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t), \\ y(t) &= Cx(t), \end{aligned} \tag{2.7}$$

with state vector $x(t) \in R^n$, output vector $y(t) \in R^m$, known input vector $u(t) \in R^r$, and unknown input vector $d(t) \in R^q$. Matrices A, B, and C are known and of appropriate dimensions. The following definition is used as a starting point of the discussion [Che95].
Definition: “An observer is defined as an unknown input observer for the system described by (2.7), if its state estimation error vector $e(t)$ approaches zero asymptotically, regardless of the presence of the unknown input (disturbance) in the system.”
Without loss of generality the unknown input distribution matrix E is assumed as a full column rank matrix. Otherwise a rank decomposition procedure can be considered as

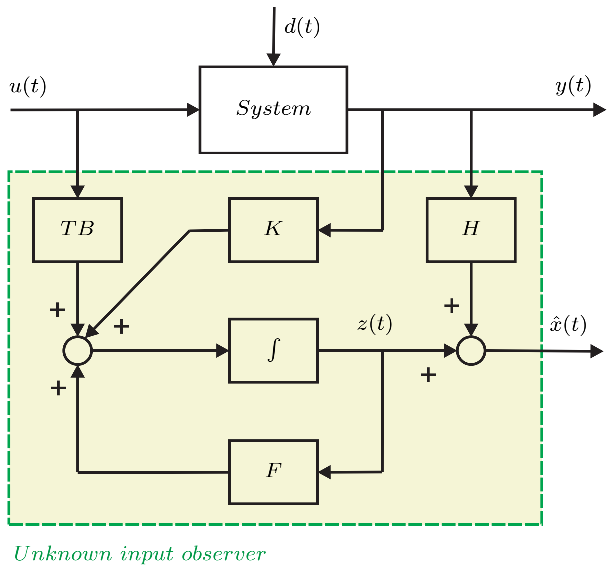


Figure 2.3: Block diagram of unknown input observer, redrawn from [Che95]

$$Ed(t) = E_1 E_2 d(t), \tag{2.8}$$

with full column rank matrix E_1 and unknown input $E_2 d(t)$ [Che95] illustrated in Figure 2.3. Following [Che95] the structure of a full-order UIO can be described as

$$\begin{aligned} \dot{z}(t) &= Fz(t) + TBu(t) + Ky(t), \\ \hat{x}(t) &= z(t) + Hy(t), \end{aligned} \tag{2.9}$$

with estimated vector $\hat{x}(t)$ and full-order observer state $z(t)$. Matrices F, T, K, and H are design matrices to achieve unknown input decoupling. By applying (2.9) to system (2.7) the estimation error

$$\begin{aligned} \dot{e}(t) = \; & (A - HCA - K_1C)e(t) + \underbrace{[F - (A - HCA - K_1C)]}_{D_1} z(t) \\ & + \underbrace{[K_2 - (A - HCA - K_1C)H]}_{D_2} y(t) + \underbrace{[T - (I - HC)]B}_{D_3} u(t) + \underbrace{(HC - I)E}_{D_4} d(t), \end{aligned} \tag{2.10}$$

with

$$K = K_1 + K_2, \tag{2.11}$$

as the gain of observer can be obtained. When the observer is designed in such a way that the following conditions are established

$$\begin{aligned} &D_1: && F = A - HCA - K_1C, \\ &D_1, D_2: && K_2 = FH, \\ &D_3: && T = I - HC, \\ &D_4: && (HC - I)E = 0, \end{aligned} \tag{2.12}$$

then the state estimation error is achieved as

$$\dot{e}(t) = Fe(t). \tag{2.13}$$

By choosing the appropriate stable eigenvalue for matrix F the estimation error $e(t)$ will be asymptotically stable, i.e. $\hat{x} \to x$. Designing of UIO means to solve (2.11) and (2.12) while matrix F has to be appropriately chosen including the stability of the error dynamics (2.13) [Che95].
According to [Che95] the necessary and sufficient conditions for existence of the UIO (2.9) are considered as the following.

- $rank(CE) = rank(E)$
- Pair (C, A_1) is a detectable pair, where $A_1 = A - E[(CE)^T CE]^{-1}(CE)^T CA$

The proof is given in [Che95] using a special solution for H as H^*

$$H^* = E[(CE)^T CE]^{-1}(CE)^T. \tag{2.14}$$

According to the first condition, the maximum number of disturbances (number of independent columns of matrix E) should be less than the number of independent measurements (number of independent rows of matrix C). Similarly, according to the second condition the transmission zeros from the unknown inputs to the measurements should be stable. Accordingly, the matrix K_1 has to be designed to stabilize the matrix F and to design the whole structure of UIO which is also perused in [Che95].

2.4.3 Perturbation observer (PO)

According to [KC04] perturbation denotes lumped uncertainty, defined as one or several sources of parametric and/or unmodelled dynamics uncertainty and affects the nominal model of a system used for designing a controller. Perturbation observer (PO) is adjusted with the purpose of estimating the perturbation adaptively. Disturbance observer introduced in section 2.4.1 is a special kind of PO in frequency domain. The disturbance observer (DOB) and Time-Delayed Controller (TDC) are the representative formulation of PO in frequency domain and time-domain, respectively. By considering a plant model as

$$\dot{x}(t) = a(x,t) + b(x,t)(u + \omega), \tag{2.15}$$

when $x, a, b \in R^{n\times 1}$, perturbation ω denotes the lumped matched uncertainty. Therefore in the case of single-input system, the perturbation can be calculated as

$$\omega(t) = b^+(x,t)(\dot{x}(t) - a(x,t)) - u(t), \tag{2.16}$$

Table 2.3: Survey of unknown input observation techniques and their properties

		System/disturbance description and estimated signal	Input/Output behavior of observer	Properties	Some recent applications
Disturbance observer (DOB)		$y(s) = P_n(s)(u(s) + d(s))$ $\hat{d}(s) = [P_n^{-1}(s)\hat{y}(s) - u(s)]\, Q(s)$	$\{u(s), y(s), P_n(s)\} \rightarrow \left\{\hat{d}(s)\right\}$	- Able to estimate disturbances, no state estimation - Formulation in frequency domain - Able to estimate external disturbances	[LHCT17, CLTT17, NSL+17, LCTT17]
Unknown input observer (UIO)		$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t)$ $y(t) = Cx(t)$ $\dot{z}(t) = Fz(t) + TBu(t) + Ky(t)$ $\hat{x}(t) = z(t) + Hy(t)$	$\{u(t), y(t), A, B, C, E\} \rightarrow \left\{\hat{d}(t), \hat{x}(t)\right\}$	- Able to estimate disturbances and system states - Formulation in time domain - Able to estimate external disturbances	[LWA+17, GLC16, ZJC16, ZLJ17]
Perturbation observer (PO)	Time domain (TDC)	$\dot{x}(t) = a(x,t) + b(x,t)(u + \omega)$ $\omega(t) = b^{+}(x,t)(\dot{x}(t) - a(x,t)) - u(t)$ $\hat{\omega}(t) = b^{+}(x, t-h)(\dot{x}(t-h) - a(x, t-h)) - u(t-h)$	$\{x, \dot{x}, u, a, b\} \rightarrow \{\hat{\omega}(t)\}$	- General form of DOB - Formulation in frequency/time domain - Able to estimate external and internal (unmodeled plant variations) disturbances	[AGZM17, YJZ+16, YHH+17, YSS+16]
	Frequency domain (DOB)	$y(s) = P_n(s)(u(s) + \omega(s))$ $\omega(s) = P_n^{-1}(s)y(s) - u(s)$ $\hat{\omega}(s) = Q(s)(P_n^{-1}(s)y(s) - u(s))$	$\{u, y, P_n\} \rightarrow \{\hat{\omega}(s)\}$		
	Discrete time (including TDC & DOB)	$x(k+1) = Ax(k) + B(u(k) + \omega(k))$ $\omega_{eq}(x, u, k) = B^{+}(x(k+1) - Ax(k)) - u(k)$ $\hat{\omega}(k) = Q\omega_{eq}(x, u, k-1)$	$\{x(k), x(k-1), u(k), A, B\} \rightarrow \{\hat{\omega}(k)\}$		
PI-Observer (PIO)		$\dot{x}(t) = Ax(t) + Bu(t) + Nd(x,t) + Eg(x,t)$ $y(t) = Cx(t) + h(t)$ $\dot{\hat{x}}(t) = A\hat{x}(t) + NH\hat{v}(t) + Bu(t) + L_1(y(t) - \hat{y}(t))$ $\dot{\hat{v}}(t) = V\hat{v}(t) + L_2(y(t) - \hat{y}(t))$ $\hat{d}(t) = H\hat{v}(t)$ $\hat{y}(t) = C\hat{x}(t)$	$\{u(t), y(t), A, B, C\} \rightarrow \left\{\hat{d}(t), \hat{x}(t)\right\}$	- Able to estimate disturbances and system states - Formulation in time domain - Able to estimate external and internal (unmodeled plant variations) disturbances	[KS17, YCKW17, RHRB17, NNGR17]

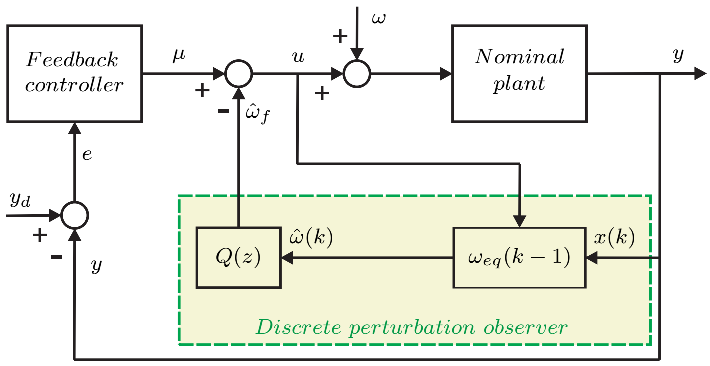


Figure 2.4: Block diagram of Discrete perturbation observer (redrawn from [KC04])

with pseudo-inverse $b^+(x,t)$. By assuming smooth change in the perturbation variation, TDC perturbation estimation can be calculated as

$$\hat{\omega}(t) = b^+(x, t-h)(\dot{x}(t-h) - a(x, t-h)) - u(t-h), \tag{2.17}$$

with sampling time h. Similarly, if the relation between system input and output is available as

$$y(s) = P_n(s)(u(s) + \omega(s)), \tag{2.18}$$

the perturbation estimation can be achieved in the frequency domain as

$$\omega(s) = P_n^{-1}(s)y(s) - u(s). \tag{2.19}$$

By using a low pass filter $Q(s)$ for decreasing the high frequency noise the typical form of DOB can be achieved similar to section 2.4.1

$$\hat{\omega}(s) = Q(s)(P_n^{-1}(s)y(s) - u(s)). \tag{2.20}$$

As illustrated in (2.17) TDC is a mapping of $(x, \dot{x}, u) \rightarrow \hat{\omega}$ when the full state is available while DOB is a mapping of $(u, y) \rightarrow \hat{\omega}$. In this section the perturbation observer is introduced as a discrete-time version [KC04] which includes TDC and DOB simultaneously. The discrete state space model of a system can be considered as

$$x(k+1) = A_r x(k) + B_r u(k) + B_\omega(f(k) + d(k)), \tag{2.21}$$

where $f(k)$ and $d(k)$ denote the unmodeled dynamics and external disturbance, respectively. By considering the additive parameter perturbation $A_r = A + \Delta A$ and $B_r = B + \Delta B$ the following plant model

$$x(k+1) = Ax(k) + Bu(k) + \underbrace{\Delta A x(k) + \Delta B u(k) + B_\omega(f(k) + d(k))}_{unknown\ vectors}, \tag{2.22}$$

with $x \in R^{n\times 1}$, $\{A, \Delta A\} \in R^{n\times n}$, $\{B, \Delta B, B_\omega\} \in R^{n\times r}$, and $\{u, f, d\} \in R^{r\times 1}$ can be concluded. By considering the following assumptions (referring to [KC04])
- The pairs (A_r, B_r) and (A, B) are controllable.
- The matching condition is satisfied for the unknown vector in (2.22), so the perturbation enters the plant with the same input distribution matrix $b(x, t)$ as the control input $u(t)$, so $\Delta Ax(k) + \Delta Bu(k) + B_\omega(f(k) + d(k)) = B\omega(k)$.

The plant model can be rewritten as

$$x(k + 1) = Ax(k) + B(u(k) + \omega(k)), \tag{2.23}$$

with the perturbation vector $\omega \in R^{r\times 1}$

$$\omega(k) = B^+(\Delta Ax(k) + \Delta Bu(k)) + B^+B_\omega(f(k) + d(k)). \tag{2.24}$$

By taking the pseudo-inverse of B matrix an equivalent quantity to the perturbation can be calculated as

$$\omega_{eq}(x, u, k) = B^+(x(k + 1) - Ax(k)) - u(k). \tag{2.25}$$

Finally a causal discrete perturbation observer can be considered as

$$\hat{\omega}(k) = \omega_{eq}(x, u, k - 1). \tag{2.26}$$

In reality implementation of (2.25) is difficult due to pseudo-inverse B^+. To avoid high frequency noise a low-pass filter Q is added to (2.26) and is modified as

$$\hat{\omega}(k) = Q\omega_{eq}(x, u, k - 1), \tag{2.27}$$

which has to be properly designed. The block diagram of the perturbation observer is illustrated in Figure 2.4. More information about the analysis of robustness and sensitivity is provided in [KC04]. The low-pass filter Q clarifies the stability, robustness, and sensitivity of the perturbation observer combined with a discrete controller regarding the change of filter parameters e.g. numerator order, denominator order, and cut-off frequency.

Comparison of unknown input observation techniques are detailed in Table 2.3 by considering the system description, input/output signals and matrices, general properties, and the recent main applications of each approach.

2.5 Proportional-Integral-Observer (PI-Observer)

2.6 PI-Observer formulation for different type of systems

According to section 2.3.1 the Luenberger observer is a linear observer used for reconstruction of system states by using the measured inputs and outputs. This kind of observer has no ability to estimate the unknown inputs affecting the system. In the presence of unknown input the Proportional-Integral-Observer (PI-Observer) can be used to estimate uncertainties, disturbances, and modeling errors (considered as an unknown input package affects the system). The PI-Observer can be used in combination with linear and nonlinear control methods to improve the robustness against unknown inputs affecting the system. In this section PI-Observer design for different type of systems is investigated to realize the formulation of PI-Observer for different system structures.

2.6.1 Linear system

A general model for a linear time invariant system with unknown inputs (disturbances, unmodeled dynamics, or other nonlinearites) is given as

$$\begin{aligned} \dot{x}(t) = &\quad Ax(t) + Bu(t) + Nd(x,t) + Eg(x,t), \\ y(t) = &\quad Cx(t) + h(t), \end{aligned} \tag{2.28}$$

with the state vector $x(t) \in R^n$, input vector $u(t) \in R^m$, measurement vector $y(t) \in R^r$, unknown input $d(x,t) \in R^l$, measurement noise $h(t) \in R^r$, and unmodeled dynamics $g(x,t) \in R^p$. Here the unknown input $d(x,t)$ and the input matrix N are used to model the additive unknown inputs. Matrices A, B, and C are assumed as known and of appropriate dimensions. The following assumptions are considered for the unknown input $d(x,t)$.

- $d(x,t)$ is as a bounded signal $|d(x,t)| \leq \alpha$.
- $d(x,t)$ has small variation rate $|d(x,t_2) - d(x,t_1)| \leq \beta$ for $\Delta t = t_2 - t_1 \to 0$.

The unknown input $d(x,t)$ can be assumed as a function of time $d(t)$[1]. One approach proposed in [SYM95] to approximate the additive acting dynamics $d(t)$ uses the approximation

$$\begin{aligned} &\dot{v} = Vv(t), \\ &d(t) \approx Hv(t), \end{aligned} \tag{2.29}$$

in which V acts as a linear model describing the disturbance behavior. By considering the system states and unknown inputs as the states of an extended system, the

[1]Note that the unknown input d is allowed to be a function of known inputs $u(t)$ and states $x(t)$ as $d(x,u,t)$. Due to the fact that no information of the dynamical behavior is available, it is written as $d(t)$ without loss of generality.

extended state space representation can be written as

$$\begin{aligned}
\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} A & NH \\ 0 & V \end{bmatrix}}_{A_e} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_e} u(t) + \begin{bmatrix} Eg(t) \\ 0 \end{bmatrix}, \\
y(t) &= \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}.
\end{aligned} \tag{2.30}$$

Matrices H and V are used to calibrate the input's relation between the real system and the model H as well as the disturbance dynamic model (2.29). If the dynamic of the uncertainty is known, an appropriate model, respectively appropriate matrices V and H can be built up [Joh76, HW85]. For example in [Joh68] constant disturbance is considered. In [Joh71] determination of V and H is examined for a scalar uncertainty $d(t)$ assumed to be composed of known Laplace-transformable functions with unknown coefficients. In [Dav72] a disturbance with the states described by a differential equation is taking into consideration. According to [Joh71], sufficiently slowly varying disturbances can be assumed as piecewise constant. As introduced in [SYM95], an adequate choice is $V = 0$ or $V \Rightarrow 0$ and $H = I$. The task of a high-gain observer-based scheme is to approximate step-wise the unknown input as constant (it is clear that in the reality the disturbance is not always a constant value and this assumption is considered to simplify the observer design procedure). The system states $\hat{x}$ and unknown inputs $\hat{v}$ using PI-Observer are estimated as

$$\begin{aligned}
\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{v}}(t) \end{bmatrix} &= \begin{bmatrix} A & N \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \underbrace{\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}}_{L} (y(t) - \hat{y}(t)), \\
\hat{y}(t) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix}.
\end{aligned} \tag{2.31}$$

The matrix L in (2.31) has to be designed as the observer gain (containing the proportional gain and the integral gain). The principal convergence behavior of PI-observer scheme is discussed in [SYM95] and the combination of PI-Observers as a suitably guided observer bank in [LS12].
A necessary requirement for PI-Observer design is the full observability of the extended system (2.30). The condition

$$rank \begin{bmatrix} \lambda_i I_n - A & -N \\ 0 & \lambda_i I_r \\ C & 0 \end{bmatrix} = n + r, \tag{2.32}$$

has to be fulfilled for all λ_i of A_e. Here n and r denote the number of states and number of unknown inputs, respectively. This condition leads to the fact that the dimension of unknown input vector $n(t)$ should be equal or less than the number of

independent measurements (proof is provided in [ML77]).
The error dynamics of the extended system (2.30) and (2.31) can be written as

$$\begin{bmatrix} \dot{e}_{est}(t) \\ \dot{f}_e(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A - L_1C & N \\ -L_2C & 0 \end{bmatrix}}_{A_{e,obs}} \begin{bmatrix} e_{est}(t) \\ f_e(t) \end{bmatrix} - \begin{bmatrix} Eg(x,t) \\ \dot{d}(x,t) \end{bmatrix} + \underbrace{\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}}_{L} h(t), \tag{2.33}$$

with $e_{est}(t) = \hat{x}(t) - x(t)$ and $f_e(t) = \hat{d}(t) - d(x,t)$. To converge the estimation error the condition

$$CA^{k-1}N = 0, \ with \ k = 2, \ ..., \ n, \tag{2.34}$$

has to be fulfilled [MB00].
For investigation of PI-Observer stability in time domain the estimation error of state and unknown input are considered according to (2.33) and the observer gain L has to be designed to achieve $e_{est}(t) \to 0$ and $f_e(t) \to 0$. In [MB00] sufficient conditions for asymptotic stability of PI-Observer in time domain are given as $\left\|\dot{d}\right\| \leq g$ and related to the high gains $L_1 \to aL_{10}$, $L_2 \to aL_{20}$, $a \to \infty$. Furthermore, for a given bound of

$$\left\|e^{A_e - LC_e}\right\| \leq ce^{-bt}, \quad \text{with } c, \ b > 0, \tag{2.35}$$

the errors $e_{est}(t)$ and $f_e(t)$ are bounded by

$$\left\| \begin{bmatrix} e_{est}(t) \\ f_e(t) \end{bmatrix} \right\| \leq ce^{-bt} \left\| \begin{bmatrix} e_{est0} \\ f_{e0} \end{bmatrix} \right\| + \frac{c}{b}(1 - e^{-bt}) \left\|\dot{d}\right\| \quad \text{and} \tag{2.36}$$

$$\|f_e\| \leq \frac{c}{b} g, \quad \text{for} \quad t \to \infty. \tag{2.37}$$

2.6.2 Linear system with delay

Time-delay systems are more complex in stability analysis and observer design than regular systems [Nic01]. With this interpretation, designing of PI-Observer for time-delay systems is a challenging point compared to the normal linear/nonlinear systems. Designing an asymptotic observer for this type of systems is elaborated by some authors such as [CS06] and later in [SSS08]. In this section a formulation of PI-Observer integrated with a time-delay system is perused based on the work in [SSS09]. A general time-delay system can be described as

$$\begin{aligned} \dot{x}(t) = & \ A_0x(t) + A_1x(t-\tau) + B_0u(t) + B_1u(t-\tau) + Ed(t), \\ y(t) = & \ C_0x(t) + C_1x(t-\tau) + Fd(t), \\ x(t) = & \ \phi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{2.38}$$

with the state variable $x(t) \in R^n$, input variable $u(t) \in R^m$, output variable $y(t) \in R^p$, disturbance vector $d(t) \in R^q$, and continuous initial value function vector $\phi(t) \in C[-\tau; 0]$. A constant known time-delay duration can be assumed as $0 \leq \tau < \infty$. In (2.38) matrices E and F contain some uncertainties or modeling errors. The extended system, comprises the linear time-delay system and a PI-Observer, can be written as

$$\begin{aligned} \dot{\hat{x}}(t) = \;& A_0\hat{x}(t) + A_1\hat{x}(t-\tau) + B_0u(t) + B_1u(t-\tau) \\ & -B_0w(t) - B1w(t-\tau) + L_p[y(t) - C_0\hat{x}(t) - C_1\hat{x}(t-\tau)], \\ \dot{w}(t) = \;& L_i[y(t) - C_0\dot{x}(t) - C_1\dot{x}(t-\tau)] + w(t) + w(t-\tau), \end{aligned} \tag{2.39}$$

with state estimation $\hat{x}(t) \in R^n$, proportional and integral gain L_p and L_i respectively, which can be designed based on several designing methods. By considering the estimation error $e(t) = x(t) - \hat{x}(t)$ the error dynamic is calculated as

$$\begin{aligned} \dot{e}(t) = \;& (A_0 - L_pC_0)e(t) + (A_1 - L_pC_1)e(t-\tau) \\ & +B_0w(t) + B_1w(t-\tau) + (E - L_pF)d(t) \\ \dot{w}(t) = \;& L_iC_0e(t) + L_iC_1e(t-\tau) + w(t) + w(t-\tau) - L_iFd(t). \end{aligned} \tag{2.40}$$

The proof of asymptotically stability as well as generalized algorithm for multi-delay systems is provided also in [SSS09].

2.6.3 Nonlinear system

Estimation of unknown inputs with the purpose of control is known as disturbance observer-based control (DOBC) method which has been developed and implemented for linear system in two past decays [NOM87]. When a nonlinear system with unknown inputs should be controlled, the challenging points are analysis and design of a complex controller containing a nonlinear control and a nonlinear disturbance observer. Sliding model-based nonlinear disturbance observer or Lyapunov-based disturbance observer [CBGO00] have been introduced as alternative solutions. On the other hand well-known linear PI-Observer can be extended to nonlinear structures to be used in combination with nonlinear systems. Based on the proposed structure in [Che04], the class of nonlinear system to be mentioned is defined as

$$\begin{aligned} \dot{x}(t) = \;& f(x(t)) + g_1(x(t))u + g_2(x(t))d(t), \\ y(t) = \;& h(x(t)), \end{aligned} \tag{2.41}$$

with $x(t) \in R^n$ as state, $u(t) \in R$ as input, and $d \in R$ as unknown input. Nonlinear functions $f(x)$, $g_1(x)$, $g_2(x)$ are assumed as smooth functions in terms of x. Furthermore it is assumed that the dynamics of the unknown input or disturbance can be approximated as linear system, described by

$$\begin{aligned} \dot{\xi} = \;& A\xi, \\ d = \;& C\xi, \end{aligned} \tag{2.42}$$

with A and C matrices which represent the disturbance model and are defined based on the disturbance information. The observer structure to estimate unknown inputs acting to the system can be assumed as

$$\begin{aligned} \dot{z}(t) &= (A - l(x)g_2(x)C)z + Ap(x) - l(x)(g_2(x)Cp(x) + f(x) + g_1(x)u), \\ \hat{\xi} &= z + p(x), \\ \hat{d} &= C\hat{\xi}. \end{aligned} \tag{2.43}$$

Here $l(x)$ is a nonlinear gain function and $z \in R^m$ is considered as the internal state variable. Nonlinear function $p(x) \in R^m$ should be designed (design method provided in [Che04]). The nonlinear observer gain $l(x)$ can be calculated as

$$l(x) = \frac{\partial p(x)}{\partial x}. \tag{2.44}$$

The disturbance observer (2.43) for considered system (2.41) affected by disturbance (2.42) can exponentially track the disturbance by choosing a suitable nonlinear gain function $l(x)$ such that

$$\dot{e}(t) = (A - l(x)g_2(x)C)e(t), \tag{2.45}$$

with estimation error

$$e(t) = \xi - \hat{\xi}, \tag{2.46}$$

is exponentially stable. The introduced method is proposed in [Che04] with the proof of global exponential stability of the proposed nonlinear observer scheme.

2.6.4 Nonlinear system modeled by a multiple model approach

A multiple model approach can be used to model the complex nonlinear systems for analysis, control, and observer design purposes. Decomposition of the operating space into finite operating zones using multiple model approach provides the advantage of defining finite simple/linear submodels. It is worth noting that using of multiple model approach is appealing because of its intrinsic simplicity to model the complex nonlinear systems [NBC95]. On the other hand there are parameter uncertainties can be assumed to be norm bounded but with unknown time-varying behavior. Therefore, integration of PI-Observer into the structure of multiple model system can be realized. For the class of uncertain and nonlinear system to be described using a multiple model approach it is assumed that the i-th submodel can be written as [OMRM08]

$$\begin{aligned} \dot{x}_i(t) &= (A_i + \Delta A_i)x_i(t) + (B_i + \Delta B_i)u(t) + D_i w(t), \\ y_i(t) &= C_i x_i(t), \\ y(t) &= \textstyle\sum_{i=1}^{L} \mu_i(\xi(t))y_i(t) + Ww(t), \end{aligned} \tag{2.47}$$

with the state vector $x_i \in R^{n_i}$ and output $y_i \in R^p$ for i^{th} submodel, input $u \in R^m$, output $y \in R^p$, and unknown input/disturbance $w \in R^r$. The matrices A_i, B_i, C_i, D_i, and W are known and of appropriate dimensions. The parametric uncertainties are denoted by ΔA_i and ΔB_i. The decision variable $\xi(t)$ is assumed to be known. The contribution of submodels are quantified using the weighting function $\mu_i(\xi(t))$ which satisfies the condition

$$\begin{array}{l} \sum_{i=1}^{L} \mu_i(\xi(t)) = 1, \\ 0 \leq \mu_i(\xi(t)) \leq 1. \end{array} \tag{2.48}$$

The decoupled multiple model (3.12) can be rewritten in the following form proposed in [OMRM08].

$$\begin{array}{ll} \dot{x}(t) & = (\tilde{A} + \Delta\tilde{A})x(t) + (\tilde{B} + \Delta\tilde{B})u(t) + \tilde{D}w, \\ \dot{z}(t) & = \tilde{C}(t)x(t) + Ww(t), \\ y(t) & = \tilde{C}(t)x(t) + Ww(t), \end{array} \tag{2.49}$$

with

$$\begin{array}{l} \tilde{A} = diag\left\{A_1 \ ... \ A_i \ ...A_L\right\}, \ \tilde{B} = \left[B_1^T \ ... \ B_i^T \ ...B_L^T\right]^T, \\ \tilde{D} = \left[D_1^T \ ... \ D_i^T \ ...D_L^T\right]^T, \ \tilde{C}(t) = \sum_{i=1}^{L} \mu(t)\tilde{C}_i, \ \tilde{C}_i = \left[0 \ ... \ C_i \ ...0\right], \end{array} \tag{2.50}$$

and with the supplementary variable

$$z(t) = \int_0^t y(\xi)d\xi, \tag{2.51}$$

which is needed for designing of PI-Observer. The multiple model (2.49) can be changed to the following format [OMRM08]

$$\begin{array}{ll} \dot{x}_a(t) & = (\tilde{A}_a(t) + \bar{C}_1\Delta\tilde{A}\bar{C}_1^T)x_a(t) + \bar{C}_1(\tilde{B} + \Delta\tilde{B})u(t) + \tilde{D}_a w(t), \\ y(t) & = \tilde{C}(t)\bar{C}_1^T x_a(t) + Ww(t), \\ z(t) & = \bar{C}_2^T x_a(t), \end{array} \tag{2.52}$$

with

$$\begin{array}{ll} x_a(t) = & \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}, \ \tilde{A}_a(t) = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}(t) & 0 \end{bmatrix}, \\ \tilde{D}_a = & \begin{bmatrix} \tilde{D} \\ W \end{bmatrix}, \ \bar{C}_1 = \begin{bmatrix} I & 0 \end{bmatrix}^T, \ \bar{C}_2 = \begin{bmatrix} I & 0 \end{bmatrix}^T. \end{array} \tag{2.53}$$

Based on the augmented decoupled multiple model defined in (2.52) the structure of PI-Observer can be defined as [HG05]

$$\begin{array}{ll} \dot{\hat{x}}(t) = & \tilde{A}_a(t)\hat{x}_a(t) + \bar{C}_1\tilde{B}u(t) + K_P(y(t) - \hat{y}(t)) + K_I(z(t) - \hat{z}(t)), \\ \hat{y}(t) = & \tilde{C}(t)\bar{C}_1^T\hat{x}_a(t), \\ \hat{z}(t) = & \bar{C}_2^T\hat{x}_a(t), \end{array} \tag{2.54}$$

with gain matrices K_p and K_I to be designed. The error dynamics can be calculated as

$$\begin{aligned} \dot{e}_a(t) = & \ (\tilde{A}_a(t) - K_P C(t)\bar{C}_1^T - K_I \bar{C}_2^T) e_a(t) \\ & + \bar{C}_1 \Delta \tilde{A} x(t) + \bar{C}_1 \Delta \tilde{B} u(t) + (\tilde{D}_a - K_P W) w(t), \end{aligned} \tag{2.55}$$

with $e_a(t) = x_a(t) - \hat{x}_a(t)$ as the state estimation error. The stability of PI-Observer estimation can be investigated by considering two assumptions [OMRM08]. At first the considered multiple model (2.49) with uncertain parts is assumed to be stable. Secondly, the input energy and the uncertainties are considered with bounded energy signals (i.e. $\|u(t)\|_2^2 < \infty$ and $\|w(t)\|_2^2 < \infty$). Based on these assumptions the error dynamics is stable if K_P and K_I are suitably chosen so that the term $(\tilde{A}_a(t) - K_P C(t)\bar{C}_1^T - K_I \bar{C}_2^T)$ is stable. Therefore by designing suitable gain matrices K_I and K_P the influence of $[w^T(t) \ \ u^T(T)]$ on the estimation error is eliminated and the estimation error $e_a(t)$ remains in a bounded area for any bounded uncertainties $(\|w(t)\|_2^2 < \infty)$.

2.6.5 Bilinear system (as a specific class of nonlinear system)

Bilinear systems are highly regarded because of their special importance in the nuclear reactor, ecological, and biological systems, as well as in heat exchangers. Bilinear systems can be classified between linear and nonlinear systems [Kha96]. The observability of bilinear systems can be affected by system inputs. Therefore designing an observer for bilinear systems is more difficult than for linear systems [Moh91]. The authors in [YS97] proposed a disturbance decoupled observer for bilinear systems based on the concept of decoupling estimation error from the input. The authors in [SC12] have proposed a method to construct the PI-Observer for bilinear systems based on Lyapunov stability. A general model of bilinear systems can be assumed as

$$\begin{aligned} \dot{x} = & \ Ax + Bu + \textstyle\sum_{i=1}^{m} D^i u_i x + Ew, \\ y = & \ Cx, \end{aligned} \tag{2.56}$$

with the states $x \in R^n$, the inputs $u \in R^m$, the unknown constant disturbance vector $w \in R^q$, the outputs $y \in R^p$, and D^i as a constant matrix of appropriate dimension. The known and constant matrices A, B, C, and E are considered with the appropriate dimensions. The structure of PI-Observer to reconstruct the system states and unknown input is defined as

$$\begin{aligned} \dot{\hat{x}} = & \ A\hat{x} + Bu + \textstyle\sum_{i=1}^{m} D^i u_i \hat{x} + E\hat{w} + K_p(y - C\hat{x}), \\ \dot{\hat{w}} = & \ K_I(y - C\hat{x}). \end{aligned} \tag{2.57}$$

Comparison of (2.57) with the PI-Observer for linear system illustrates that PI-Observer for bilinear systems affected by the term $\sum_{i=1}^{m} D^i u_i \hat{x}$ which makes the design of PI-Observer more complex. Based on (2.57) the error dynamics of estimated states and disturbance are obtained as

$$\begin{aligned} \dot{e} = & \ (A - K_pC + \textstyle\sum_{i=1}^{m} D^i u_i)e + E_d, \\ \dot{d} = & \ -K_I C_e, \end{aligned} \tag{2.58}$$

where e and d denote the state estimation error and disturbance estimation error, respectively. The additional term $\sum_{i=1}^{m} D^i u_i$ (3.13) can be interpreted as dependency of the state estimation errors to the system inputs. In [SC12] an additional constraint on the input vector u is proposed which augments a condition for existence of PI-Observer. For an observable bilinear system (3.9) with initial condition $x_0(t)$, according to [SC12] PI-Observer can be used to estimate the system state $x(t)$ and unknown input w if the condition

$$\textstyle\sum_{i=1}^{m} D^i u_i < \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}, \tag{2.59}$$

is satisfied when for any $Q > 0$ the positive definite solution P is achieved from

$$\begin{aligned} & A_v^T P + PA_v = -Q, \\ & A_v = A_x - K_x C_x,\ A_x = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix},\ K_x = \begin{bmatrix} K_p \\ K_I \end{bmatrix},\ C_x = \begin{bmatrix} C & 0 \end{bmatrix}. \end{aligned} \tag{2.60}$$

2.7 High gain scheduled PI-Observer

Proportional-Integral-Observer can be used either to improve the accuracy of steady-state estimation or to enhance the estimation robustness in the presence of unknown inputs. Based on literature, PI-Observer can be used for various purposes such as

- Improving the stability margin in Loop Transfer Recovery (LTR) design with the especial attention to time recovery effect of PI-Observer [BS90]
- Identifying and estimating of system nonlinearities and model uncertainties assumed as additive nonlinear input [SYM95]
- Fault Detection and Isolation (FDI) for both sensors and/or actuators [NSS97]
- Removing or attenuation of disturbance in control loops [GS11]
- State estimation with the purpose of robust control of nonlinear I/O-linearizable system [LS14]

On the other hand, in the past half century several design methods for different types of system (linear, nonlinear, bilinear, etc.) and with different purposes have been proposed. For instance Linear Quadratic Regulator (LQR) method [SYM95] and

eigenstructure assignment design [DLT01] are proposed to estimate the states and system nonlinearities using the weighting matrices for suitable scaling of high gains. H-infinity norm minimization method has been proposed in [SPN02] to design the PI-Observer gain with the purpose of disturbance attenuation and fault detection. Linear Matrix Inequality-based (LMI) robust design assuming convex bounded uncertainties, known as convex optimization method, has been proposed to improve the robustness against system nonlinearities and model uncertainties [JHH08].
Increasing the gain of PI-Observer will improve the estimation performance [SYM95]. In spite of improvement, to mitigate large overshooting behavior and to reduce influence from measurement noise and unmodeled dynamics, high-gain observers are usually prevented. Therefore, in recent works providing an adaptive algorithm to define the suitable gains regarding the current influence from different aspects is considered. An extension in [LS12] introduced as Advanced PI-Observer (API-Observer), adapts the gain design based on the actual performance. Online adaption of observer gain is embedded in the numerical integration procedure. It contains three parallel PI-Observers with different gains to schedule the 'relative optimal gain'. A cost function is defined so that the estimation performance and the related energy can be evaluated. Therefore the introduced algorithm searches between limited number of gains to find the relative optimal one. In consequence the performance is adequately improved compare to PI-Observer and especially in the presence of noise.
To achieve an absolute optimal gain which is the best possible gain with respect to criterion evaluated at each step of numerical integration procedure, Modified API-Observer (MAPI-Observer) has been proposed in [BS15a]. Unlike API-Observer, MAPI-Observer attempts to find the 'absolute optimal gain' within a suitable defined continuous interval of gain options and not only between a few limited ones. Besides, improvement of observer efficiency by performing a suitable gain at the current integration step can be achieved using MAPI-Observer approach. Therefore MAPI-Observer can be considered as a modified version of PI-Observer by applying adaptive gains in combination with an observer bank (firstly introduced in [LS12]). In addition to the approach introduced in [LS12] the MAPI-Observer approach combines the adaption of gains with the integration of the observer scheme (more detail is given in chapter 3).

2.8 Actual advanced applications using PI-Observer

2.8.1 Advantages and disadvantages of PI-Observer

Along the importance of control techniques, observers are of particular interest in recent researches due to their role in realizing control. Based on the evidence in literature, PI-Observer is regarded as one of the most admissible approaches in the field of control and unknown input estimation. It is worth noting that PI-Observer-based control and PI-Observer-based fault diagnosis are still current key research areas. The superiority of using PI-Observer approach can be summarized as follows.

- Full order easy to design linear observer scheme
- Ability to estimate the system states and unknown inputs simultaneously unlike disturbance observers or state observers
- Robust approach in the presence of unknown inputs compared with some other methods such as Luenberger observer approach
- Simple linear structure compared with nonlinear methods with the same functionality
- No assumptions with respect to dynamics of the disturbances are required.
- No information about the system/measurement noise are assumed unlike Kalman Filter and its extensions.

On the other hand, PI-Observer approach also has some disadvantages such as

- The number of independent disturbances to be considered must be smaller than the number of independent outputs/measurements available.
- The location that the unknown inputs affect the system is assumed as known.
- A nominal linear state space model of the system should be available (for the most of observers is usually required).
- Generally it can cope with slowly-varying/piecewise-constant disturbances, high-gain design is required in the case of wide-varying disturbances.

Recent main applications of PI-Observer (since 2008) approach are briefly summarized in Figure 2.5. As illustrated the main applications can be divided into four main categories: (1) fault detection, (2) estimation, (3) synchronization, and (4) control.

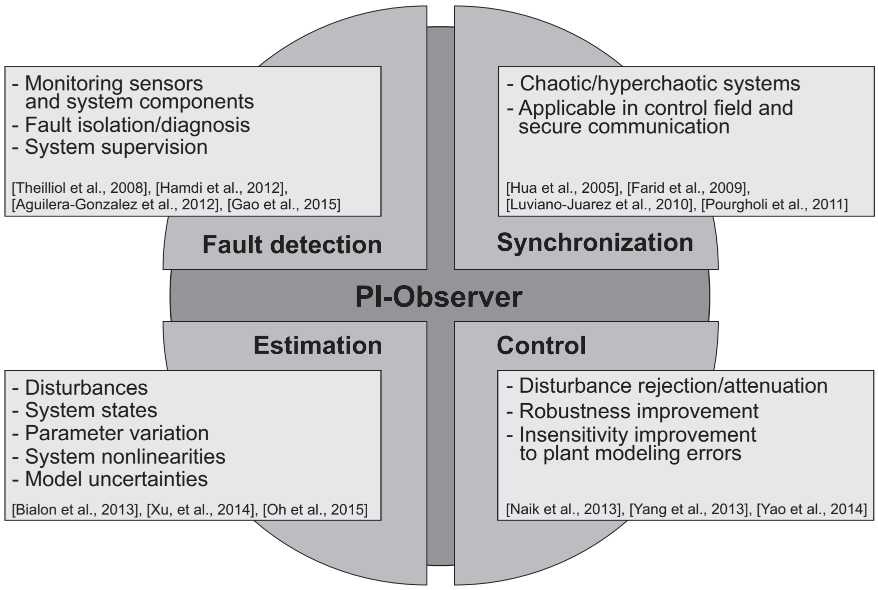


Figure 2.5: Recent main applications of PI-Observer approach (since 2008)

2.8.2 Fault detection

Proportional-Integral-Observer is one of the useful observer-based approaches used for model-based fault diagnosis when a deterministic system model is available. In the case of stochastic system model, Kalman Filter can be used [GCD15]. The scheme of observer-based fault diagnosis is illustrated in Figure 2.6 and includes fault detection, isolation, and identification, where f_a, f_c, f_s, d, and w denote the actuator fault, component fault, sensor fault, process disturbance, and measurement noise, respectively. This structure has been investigated and summarized as a recent survey of Fault Detection and Isolation (FDI) in [GCD15]. The authors in [HRM+12] investigated the problem of state estimation and fault detection for dynamic systems that can be modeled by a Linear Parameter Varying (LPV) descriptor structure. The PI-Observer is used to estimate the system states and unknown inputs or disturbances. The estimation results are used for detection, isolation, and estimation of actuator faults that affects the system inputs. The actuator fault may occur in the system because of material aging or abnormal operation. They can be described as additive and/or multiplicative faults which can affect the system performance or even lead to instability of the whole system [TJZ08]. Furthermore, in [AGTAM+12] a PI-observer-based approach with a bank of observers for fault isolation purpose is proposed as illustrated in Figure 2.6. The idea behind is to individualize a single residual which is sensitive to the fault considered and robust against other faults and uncertainties called as ‘structure residual fault isolation’ [Ger88]. The proposed method reconstruct the sensor fault with the assumption of auxiliary state and represents the dynamic behavior of fault.

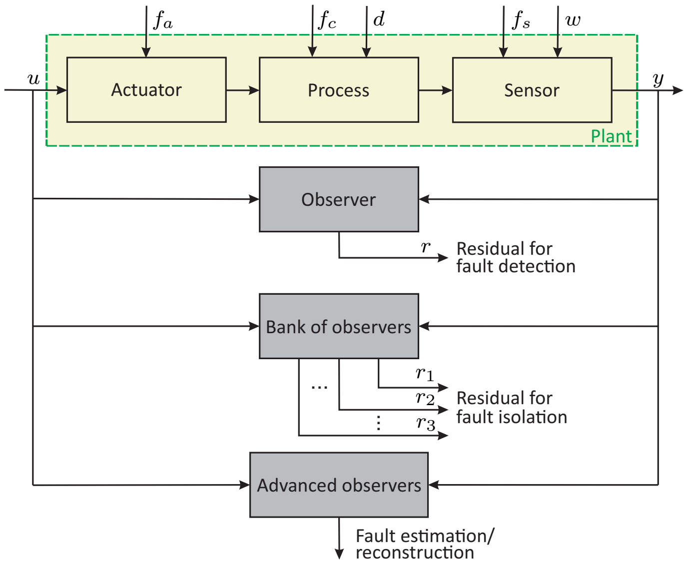


Figure 2.6: Scheme of model-based fault diagnosis (redrawn from [GCD15])

2.8.3 Estimation

PI-Observer has the ability to estimate state variables and disturbances in linear control systems. State variables reconstruction of a linear, time-invariant dynamical system (induction motor) can be considered as a recent application of PI-Observer performed in [BLPN13b]. The authors have also introduced an adaptive mechanism for parameter selection of PI-Observer [BLPN13a]. PI-Observer is also used for the state of charge estimation of Lithium-Ion batteries in [XMC$^+$14] as a recent work. Robust separate estimation of transmitted torque on each clutch of the dual clutch transmission is performed in [OC15] for ground vehicles. The estimations are obtained through three subcomponents: shaft model-based observer, unknown input observer, and a model reference PI-observer. The results of subcomponents are processed and fused together to calculate the final estimation of the torque transmitted through the first and second clutches with high accuracy.

2.8.4 Synchronization

Synchronization of chaos means adjusting of minimum two chaotic systems (either equivalent or nonequivalent) to a common behavior due to a coupling or other purposes. Synchronization problem of chaotic systems via observer method is taking into consideration in recent researches. According to [HG05] synchronization is adjusting of a second system which has the synchronized state with the first system in limited time. To solve the synchronization problem it is normally considered that the states of the first system are completely known. On the other hand, due to existence of noise and disturbances in the output channel considered for chaos synchronization, PI-Observer is proposed to solve the synchronization problem [HG05]. A robust PI adaptive observer is designed in [PM11] for synchronization of chaotic systems and the stability conditions based on Lyapunov technique are derived. Since PI-Observer gain is usually obtained from off-line calculations, the idea behind is to calculate the observer gain adaptively to make the estimation results robust against small perturbations in the control performance and to avoid the estimation error divergence. In [LJCRSR10] the authors have proposed a Generalized Proportional Integral (GPI) observer for the accurate estimation of phase variables and perturbation input of the nonlinear output dynamics. Synchronization problem of chaotic systems is solved in the presence of external disturbances using designing PI-fuzzy observer in [FIAZ09]. The authors use a general Takagi-Sugeno fuzzy model to describe the chaotic systems by considering one premise variable in fuzzy rules. Based on the proposed fuzzy model, PI-fuzzy observer is designed using Linear Matrix Inequality (LMI) approach. Stability analysis of the proposed method is also investigated by the authors to show the convergence of error system in the presence of external disturbances.

2.8.5 Control

Observer-based control design is investigated by many researchers to improve the robustness of classical linear/nonlinear controllers especially in the presence of unknown inputs and model uncertainties. All mentioned applications of PI-Observer (i.e. fault detection, estimation, and synchronization) are more or less intended for control purposes. Utilization of PI-Observer in linear/nonlinear control structures is of considerable importance because of its capability to estimate system nonlinearities and uncertainties. For example in [NSI$^+$13] a PI-Observer-based model predictive controller (MPC) is proposed to integrate the estimation of system states and uncertainties into the structure of controller. The proposed approach is practically evaluated by applying MPC to DC servomotor for position control purpose. Furthermore, PI-Observer approach for nonlinear systems has been used in [YLY13] to overcome the effect of mismatched uncertainties by designing a novel sliding surface. The proposed method behaves the same as the baseline sliding-mode controller (SMC) in the absence of disturbances and uncertainties. The chattering problem is substantially alleviated using observer-based SMC compared with traditional SMC and integral SMC. In [YJM14] a nonlinear robust controller is proposed for a hydraulic system using backstepping technique combined with uncertainties observation. The prescribed tracking transient performance and the final tracking accuracy of the proposed approach are evaluated by experimental results of an electrohydraulic system. Combination of a robust control method with uncertainties observation is performed in [LS14]. The authors introduced a robust control approach for a class of I/O-linearizable nonlinear systems affected by nonlinearities. A high-gain PI-observer is used to estimate the system states and unknown inputs to provide the requirements of input-output linearization approach. Feasibility of the proposed approach is perused using a nonlinear multi-input multi-output mechanical system as a simulation example.

2.9 Summary and conclusion

In this chapter a review of the principal unknown input observation is provided with special attention to the Proportional-Integral-Observer. Integration of PI-Observer in different system types is surveyed, including: linear system, linear system with delay, nonlinear system, nonlinear system modeled by a multiple model approach, and bilinear system (as a specific class of nonlinear systems). Furthermore, this survey incorporates some recent advanced applications and high-gain approaches of PI-Observer to summarize different aspects in this field. The significant advantages of this chapter compared with previous publications is its comprehensive and completeness in representing different observers/filters properties. Although using of terms seems to be very flexible in the discussed area, a precise distinction between fundamental concepts of observer, filter, and estimator is given from a scientific and pragmatic point of view. In addition, predictor-corrector scheme including a predictor step (an explicit method to obtain a rough approximation as a first step) and a corrector step (an implicit method to refine the predicted value as a second step), is briefly reviewed by considering a new perspective of observers/filters structure.

3 New design of high-gain Proportional-Integral-Observer

Because of non-efficiency of proportional observers in the presence of unknown input acting to the system, Proportional-Integral-Observer (PIO) has been proposed with the purpose of unknown input estimation. One advantage of this method is its ability to produce an acceptable estimation of system states in the presence of unknown inputs. Accordingly, PIO can be used for monitoring and fault diagnosis tasks as well as advanced control purposes. The name 'PIO' was firstly proposed in [Woj78] for SISO-linear time-invariant systems by augmenting an integral term into the structure of Luenberger Observer. Later it was improved in [Kac79] and [SC85] for MIMO-linear systems to improve the estimation robustness. Some extensions in the structure of PIO has been introduced in [LM79]. The authors proposed a general linear model for disturbance while no information about the unknown input is neither used for design nor for estimation. Based on this strategy by increasing the gain of PIO, the estimation performance will be improved. Due to high gains, the performance is influenced by measurement noise. This result was firstly proposed in [Mül88] as a first observation and later including proofs in [LS12].

The contents, figures, and tables presented in this chapter published in the conference papers "Proportional-Integral-Observer: A brief survey with special attention to the actual methods using ACC benchmark" [BS15a], "Reconstruction of nonlinear characteristics by means of advanced observer design approaches" [BS15b], and "High-gain scheduling of the Proportional-Integral-Observer" [BS14].

3.1 Structure of high-gain PI-Observer

Proportional-Integral-Observer can be used to estimate uncertainties, disturbances, and modeling errors considered as unknown input to the system. This observer contains two feedback loops, proportional and integral, to reconstruct system states as well as to estimate unknown input, respectively. The integral loop, as an additional degree of freedom in comparison to Luenberger observer, enables PIO to improve the steady-state estimation accuracy besides the estimation of unknown inputs.
The structure of PI-Observer is illustrated in Figure 3.1. As it is shown in this figure, the position of unknown input affecting the system (matrix N) is assumed as known. As explained, PI-Observer can be used either to improve the accuracy of steady-state estimation or to enhance the estimation robustness in the presence of unknown inputs. The equations of PI-Observer for the linear system and related observer gains can be considered according to section 2.6.1.

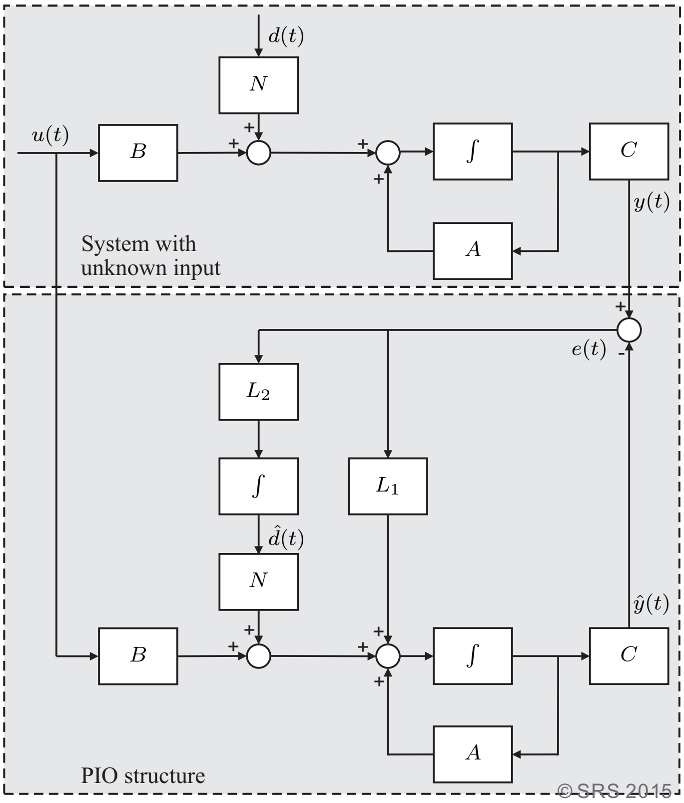


Figure 3.1: Structure of PI-Observer integrated to the real system

3.1.1 Convergence of estimation errors

In the following convergence of estimation error in the frequency domain is considered to investigate the consequence of using high gain PI-Observer in estimation performance and noise influence. According to [LS12] and based on (2.31), the error dynamics is calculated as

$$\begin{bmatrix} \dot{e}(t) \\ \dot{f}_e(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A - L_1 C & N \\ -L_2 C & 0 \end{bmatrix}}_{A_{e,obs}} \begin{bmatrix} e(t) \\ f_e(t) \end{bmatrix} - \begin{bmatrix} Eg(t) \\ \dot{d}(t) \end{bmatrix} + \underbrace{\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}}_{L} h(t), \qquad (3.1)$$

when $e(t) = \hat{x}(t) - x(t)$ and $f_e(t) = \hat{d}(t) - d(t)$ are estimation errors of state and unknown input, respectively. The objective is to design a desired observer gain L leading the estimation error to zero ($e \to 0$, $f_e \to 0$). The error dynamics in (3.1) is affected by the term $\dot{d}(t)$. In [Kra06], the authors have proposed an approximative decoupling $\dot{d}(t)$ to $e(t)$ and $f_e(t)$ by applying high gain matrix L. The error dynamics in (3.1) (stationary behavior) is described in frequency domain as

$$\begin{aligned} e(s) &= G^{-1}Nf_e(s) - G^{-1}Eg(s) + G^{-1}L_1h(s)\ , \\ f_e(s) &= -[sI + L_2CG^{-1}N]^{-1}s\,d(s) \\ &\quad + [sI + L_2CG^{-1}N]^{-1}L_2CG^{-1}Eg(s) \\ &\quad + [sI + L_2CG^{-1}N]^{-1}L_2(I - CG^{-1}L_1)h(s), \end{aligned} \tag{3.2}$$

with $G = [sI - (A - L_1C)]$, state estimation error $e(s)$, and unknown input estimation error $f_e(s)$ in the frequency domain. To minimize the influence from the disturbance to the state estimation error in (3.2), the transfer function from $sd(s)$ to $f_e(s)$ should satisfy $\|[sI + L_2CG^{-1}N]^{-1}\|_\infty \leq \gamma,\ \ \gamma \longrightarrow Minimum.$

Without loss of generality a full rank of matrix G can be assumed. By considering a high gain design for L_2 and less value of gain L_1 in G regarding high gain L_2 ($\|L_2\|_F \gg \|L_1\|_F$[1]), the considered parameter γ is achieved very small. Assuming that the unknown input $\|s\,d(s)\|_F$ is bounded, the estimation error $\|f_e(s)\|_F$ can be reduced to an arbitrary small value (but not to zero), if the measurement noise and the unmodeled dynamics are not taken into account.
From the remain parts in (3.2), it is obvious that high gain design of $\|L_2\|_F$ increases the influence from measurement noise $h(s)$ and unmodeled dynamics $g(s)$ to the estimation error $\|f_e(s)\|_F$ simultaneously. Consequently, a compromise is required to achieve the best estimation performance.

To design the high gain PIO feedback matrix L in Eqn. (2.31), Linear Quadratic Regulator (LQR) method is performed by solving the algebraic matrix Riccati equation. For a stable observer, suitable observer gains can be calculated, if for given positive definite matrices Q and R the Riccati equation

$$A_eP + PA_e^T + Q - PC_e^TR^{-1}C_eP \ = \ 0, \tag{3.3}$$

has a unique positive definite solution matrix P. The observer feedback matrix is then calculated with $L = PC_e^TR^{-1}$.

As discussed, high observer gains, which are evaluated by $\|L_2\|_F$, will possibly lead to non-negligible influence from measurement noise and unmodeled dynamics. On the other hand, the ratio between $\|L_2\|_F$ and $\|L_1\|_F$, $\delta = \|L_2\|_F/\|L_1\|_F$, should be large to compensate the effect from unknown input dynamics. These principle

[1] The norm $\|\cdot\|_F$ denotes here the Frobenius norm, $\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2} = \sqrt{trace(A^*A)}$ for A in $R^{m\times n}$.

aspects can be seen from the discussion in Eqn. (3.2). Without loss of generality the weighting matrices in Eqn. (3.3) are chosen as

$$Q = \begin{bmatrix} I_n & 0_{n\times r} \\ 0_{r\times n} & qI_r \end{bmatrix}, \quad R = I_m, \tag{3.4}$$

with only one scalar design parameter $q > 0$. It is proven in [LS12] that the parameter q can reflect almost all relevant aspects to be considered for suitable estimation of unknown inputs. According to [Liu11]:

"***Theorem 1***: *For increasing design parameter q, the ratio δ, and the norm $\|\boldsymbol{L_2}\|_F$ will increase correspondingly.*

Mathematical description:

For two general design parameters q_a and q_b, the corresponding solution matrices $\boldsymbol{P^a}$ and $\boldsymbol{P^b}$ are denoted by

$$\boldsymbol{P^a} = \begin{bmatrix} \boldsymbol{P}^a_{11} & \boldsymbol{P}^a_{12} \\ \boldsymbol{P}^{a\,T}_{12} & \boldsymbol{P}^a_{22} \end{bmatrix} \text{ and } \boldsymbol{P^b} = \begin{bmatrix} \boldsymbol{P}^b_{11} & \boldsymbol{P}^b_{12} \\ \boldsymbol{P}^{b\,T}_{12} & \boldsymbol{P}^b_{22} \end{bmatrix}. \tag{3.5}$$

Similarly,
$\boldsymbol{L}^a = \begin{bmatrix} \boldsymbol{L}^a_1 \\ \boldsymbol{L}^a_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}^a_{11}\boldsymbol{C}^T \\ \boldsymbol{P}^{a\,T}_{12}\boldsymbol{C}^T \end{bmatrix}$, and $\boldsymbol{L}^b = \begin{bmatrix} \boldsymbol{L}^b_1 \\ \boldsymbol{L}^b_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}^b_{11}\boldsymbol{C}^T \\ \boldsymbol{P}^{b\,T}_{12}\boldsymbol{C}^T \end{bmatrix}$ *are defined.*

With assumed parameters $q_a > q_b > 0$, it follows that

i) $\|\boldsymbol{L}^a_2\|_F > \|\boldsymbol{L}^b_2\|_F$ *and correspondingly*

ii) $\delta^a = \|\boldsymbol{L}^a_2\|_F / \|\boldsymbol{L}^a_1\|_F > \delta^b = \|\boldsymbol{L}^b_2\|_F / \|\boldsymbol{L}^b_1\|_F$."

Detailed proofs and further applications are introduced in [Liu11]. Therefore, to reach suitable estimation error for both states and unknown inputs, the design parameter q has to be adaptively chosen at each step of the integration procedure. The relation between estimation error of unknown input and design parameter q is illustrated in Figure 3.2. The influences from unmodel dynamics, measurement noise, and unknown input on the estimation error are clearly shown in this figure.

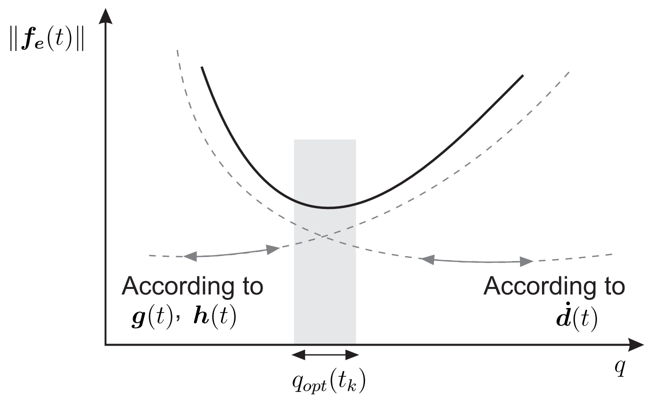


Figure 3.2: Relations between estimation error $\|\boldsymbol{f_e}(t)\|$ and design parameter q

3.2 Modified advanced PI-Observer design

Increasing the gain of PI-Observer will improve the estimation performance. However in the presence of measurement noise, the performance would be influenced by the measurement noise [SYM95]. A recently published extension introduced as Advanced PI-Observer (API-Observer) in [LS12], adapts the PIO gain based on the resulting actual performance. The online adaption of observer gain is embedded in the numerical integration. It contains three parallel PI-Observer with different gains to schedule the 'relative optimal gain'. To evaluate the estimation performance and to choose the appropriate parameter q at each step of integration procedure, a cost function is defined as

$$J(q), \quad J = \mu \frac{1}{h} \int_{t-h}^{t} e_y(\tau)^T e_y(\tau) d\tau + q, \tag{3.6}$$

with μ parameter to normalize the calculation at each step, variable h as the current step size of the numerical integration time for the observer calculation, and estimation error $e_y(t) = y(t) - \hat{y}(t)$ which implicitly contains the design parameter q. Here the design parameter q is considered to reduce the effects from unmodeled dynamics and measurement noise on the estimation performance. The goal is achieving an acceptable relative minimum level of estimation error at each step of integration procedure. Therefore the introduced algorithm searches between the limited number of gains to find the relative optimal one. In consequence of changing and scheduling the gain of PI-Observer based on the cost function, the performance is adequately improved in comparison to PI-Observer and especially in the presence of noise.
To achieve an absolute optimal gain which is the best possible gain with respect to the criterion evaluated at each step of numerical integration procedure, Modified API-Observer (MAPI-Observer) was proposed [BS15a]. Unlike API-Observer,

MAPI-Observer attempts to find the 'absolute minimal level of estimation error' within a suitable defined continuous interval of gain options and not only between a few limited ones. Therefore, MAPI-Observer is a modified version of PI-Observer by applying adaptive gains in combination with an observer bank (firstly introduced in [LS12]).

3.2.1 Structure of modified advanced PI-Observer

To achieve an absolute optimal gain, defined as the best possible gain with respect to criterion evaluated at each step of numerical integration procedure, Modified API-Observer (MAPI-Observer) is proposed in this work [BS14]. In Figure 3.3 MAPI-Observer algorithm with q and J parameters (observer design parameter and cost function, respectively) for three different observers is illustrated. Furthermore two other parameters α and β are considered ($0 < \alpha < \beta < 1$) as additional design parameters which can improve the performance of output estimation. As mentioned in section 2.7, API-Observer is based on a bank of PI-Observers with limited numbers of gains to find the relative optimal one. Unlike API-Observer, MAPI-Observer attempts to find the absolute optimal design parameter q within a suitable defined continuous interval of options and not only between a few limited options. The proposed method receives the first relative optimal design parameter q and the desired step size of integration as inputs from API-Observer algorithm. Thereafter it finds the second relative optimal design parameter q by shrinking the interval between three assumed parameters. Consequently the absolute optimal parameter q can be calculated by searching in the interval between mentioned two relative optimal parameters. Optimization methods can be used to achieve the absolute one in this interval. The MAPI-Observer has also the capability to implement the designed parameter q at the present integration time and not in the next step of integration procedure.

3.2.2 Workflow and convergence of modified advanced PI-Observer

The algorithm starts by three parallel PI-Observer with predefined gains q_m, $q_r = \beta q_m$, and $q_l = \alpha q_m$ ($0 < \alpha < 1$ and $\beta > 1$ are design parameters). The estimated values of system output using three parallel PI-Observer go through the structure of APIO to calculate the cost functions $J(q_m)$, $J(q_l)$, and $J(q_r)$. The cost functions are compared and the minimum cost function is selected. If the minimum cost function is equal to the cost function achieved by design parameter q_m the APIO algorithm goes to the next step of integration procedure (it means that the selected q_m is in the gray area illustrated in Figure 3.2). If this condition is not fulfilled the same procedure is repeated by substituting $q_m = q_{opt}$, $q_r = \beta q_{opt}$, and $q_l = \alpha q_{opt}$.
By considering the modification algorithm introduced as MAPIO in this work, in the

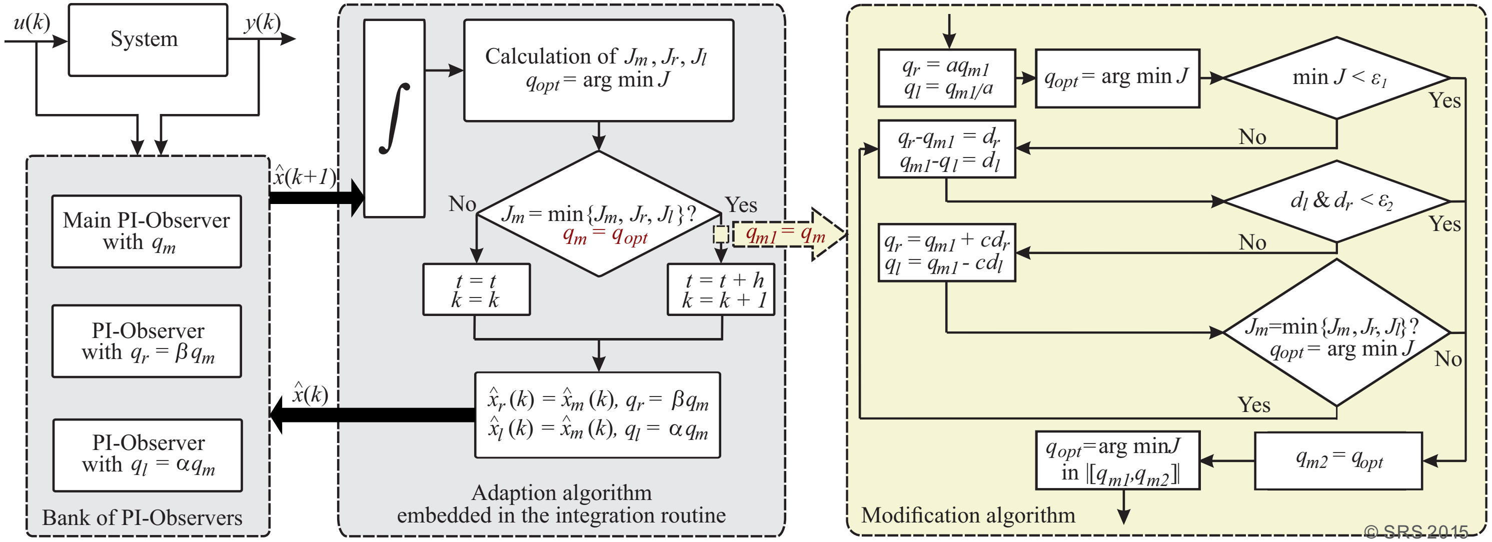


Figure 3.3: High gain scheduling of PI-Observer [LS12, BS15b]

case that the minimum cost function is equal to the cost function achieved by design parameter q_m, the procedure of MAPIO starts by substituting $q_{m1} = q_m$, $q_r = aq_{m1}$, and $q_l = q_{m1}/a$ (in this work a is considered equal to 10). The cost functions are calculated for the new design parameters and new estimated values for the system output. If the minimum cost function is smaller than the predefined small value ϵ_1 the procedure can be stopped and go to the end of MAPIO algorithm otherwise the left and right interval are calculated as $d_r = q_r - q_{m1}$ and $d_l = q_{m1} - q_l$. If d_r and d_l are smaller than predefined design parameter ϵ_2 the procedure can be terminated and continued with q_{m1} because in this case the gray area in Figure 3.2 is very small and the values of q_{m1}, q_r, and q_l are very close to each other. If the mentioned condition is not fulfilled new design parameters q_l and q_r are defined closer to q_{m1} ($q_r = q_{m1} + cd_r$ and $q_l = q_{m1} - cd_l$, in this work c is considered equal to 3/4) and the cost functions are checked again. In the case that the minimum cost function is not related to the design parameter q_{m1} the value of achieved parameter is assigned to q_{m2} as $q_{m2} = q_{opt}$ which leads to better estimation error. By considering the behavior of changing q parameter according to Figure 3.2, it can be concluded that the best design parameter is in the interval of $|[q_{m1}, q_{m2}]|$ and can be calculated by using a simple search in this interval or any optimization solution.
Considering the conditions in the procedure of MAPIO algorithm, it can be concluded that the observer gain (parameter q) is changed only in the case that the cost function J_{k+1} is smaller than the current cost function J_k. Therefore, the change and improvement in the structure of MAPIO is towards the direction that minimizes the cost function J compared to APIO which ensures the convergence of estimation error (further information about the APIO stability refer to [Liu11]).

3.2.3 Verification in open-loop simulation examples

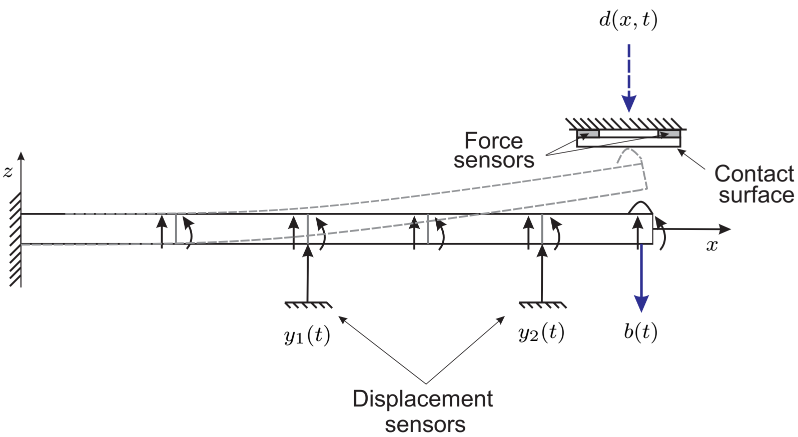


Figure 3.4: Elastic beam model

In this study verification of the introduced approaches is given using contact force estimation of an elastic beam system. The elastic beam system used here is shown in Figure 3.4. For observer design the model of the system (without unknown input) has to be obtained. Therefore the elastic beam is modeled using Finite Element Method (FEM) to achieve a linear model of the system. For the example system the length of each element is 98 mm and the cross-sectional area is 125 mm^2. The displacements z_i, the angles θ_i $(i = 1, ..., 5)$ the corresponding velocities, and angular velocities are considered as the system states to define the state space representation. Therefore the linear state space model of this elastic beam can be described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b(t) + Nd(x,t), \\ y(t) &= Cx(t), \end{aligned} \tag{3.7}$$

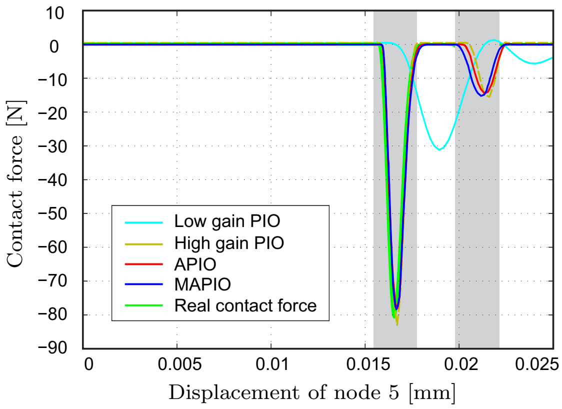
with the state vector

$$x(t) = [z_1\ \theta_1\ ...\ z_5\ \theta_5\ \dot{z}_1\ \dot{\theta}_1\ ...\ \dot{z}_5\ \dot{\theta}_5]. \tag{3.8}$$

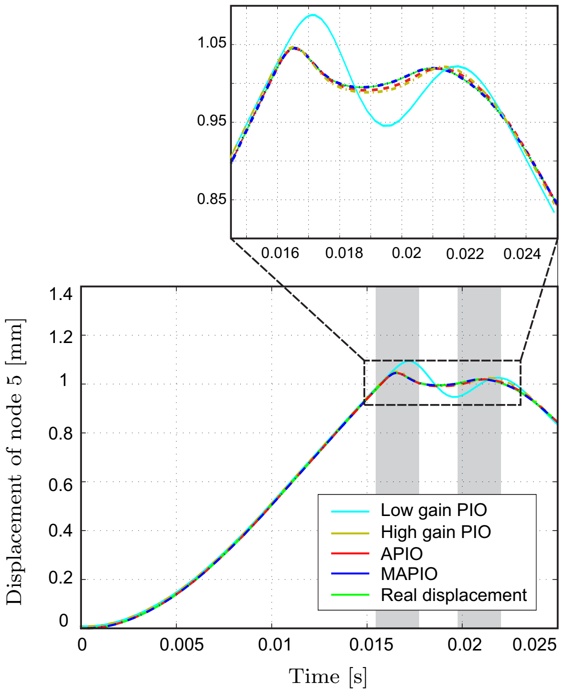
One known input $b(t)$ and one unknown input $d(x,t)$ acting at the moment of contact between vibrating beam and contact device are considered. The displacements of the second and forth nodes ($y_1(t) = x_3(t)$, $y_2(t) = x_7(t)$) are used as measurements. The goal is to estimate the unknown input $d(x,t)$ as a disturbance acting on the fifth node when the elastic beam is in contact. In this contribution the unknown input $d(x,t)$ is considered as a bounded signal with smooth variation rate. The relevant matrices of elastic beam system are the system matrix $A = \begin{bmatrix} 0_{10\times10} & I_{10\times10} \\ -M^{-1}K & -M^{-1}D \end{bmatrix}$, input matrix $N = \begin{bmatrix} 0_{18\times1} \\ 1 \\ 0 \end{bmatrix}$, and the output matrix $C = \begin{bmatrix} 0_{1\times2} & 1 & 0_{1\times17} \\ 0_{1\times8} & 1 & 0_{1\times11} \end{bmatrix}$. The stiffness matrix K and the mass matrix M are calculated using Finite Element Theory. The damping matrix is taken as $D = \xi K$, with a suitable ξ chosen by using the Raleigh damping hypothesis. The goal is to estimate the unknown contact force and system states simultaneously and by using the system measurements.

Comparison of low/high gain PIO, APIO, and MAPIO approaches

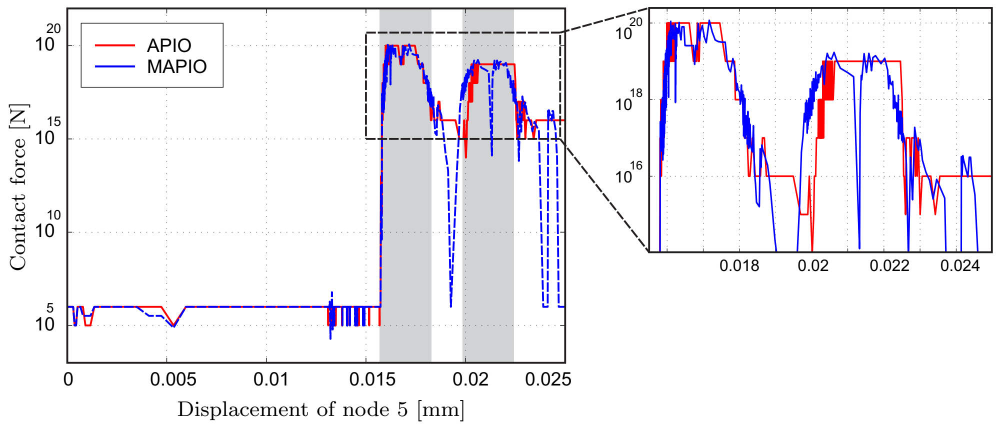
The goal of this section is to compare the introduced MAPI-Observer with previous approach as API-Observer [LS12] and low/high gain PI-Observer with respect to the task of nonlinear behavior estimation to show the advantage of MAPI-Observer. To achieve this goal some simulations are considered for the introduced simulated elastic beam. First of all estimation of contact force and system states without considering measurement noise is simulated. The results are illustrated in Figure 3.5(a) and Figure 3.5(b) for contact force estimation and displacement estimation of the elastic beam fifth node, consequently. Furthermore, gain adjustment of PI-Observer



(a) Real contact force and its estimations



(b) Real displacement of node 5 and its estimations

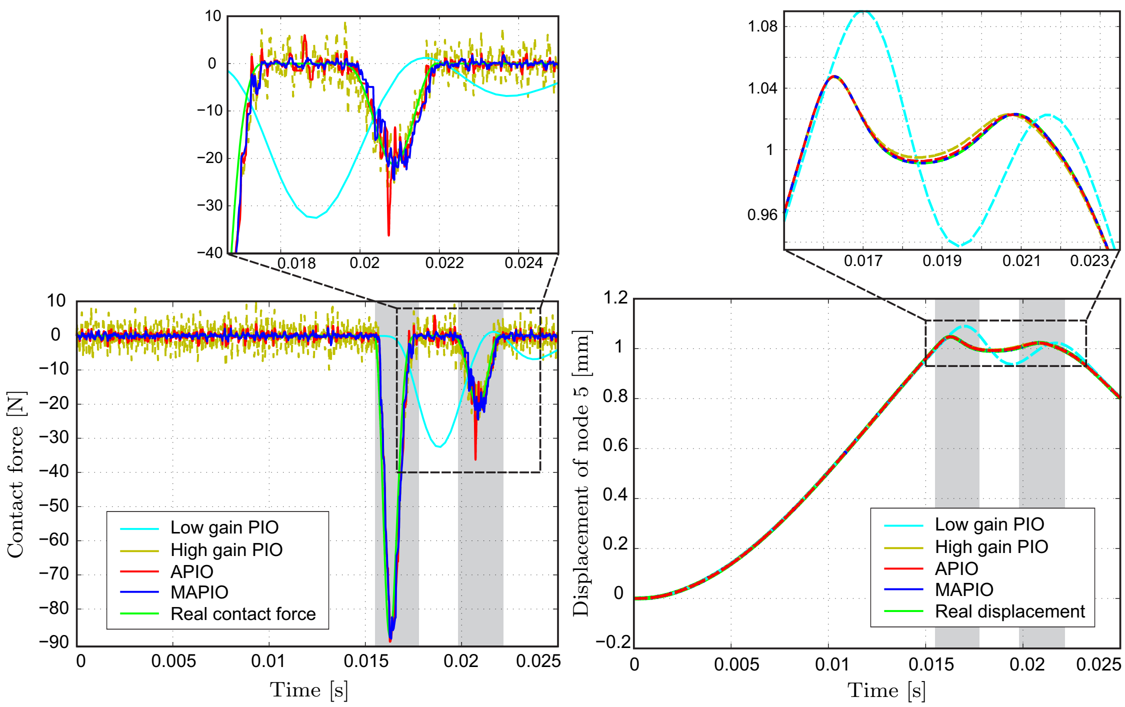


(c) Gain adjustment of APIO and MAPIO

Figure 3.5: Comparison of APIO and MAPIO without measurement noise

in this case is illustrated in Figure 3.5(c) for APIO and MAPIO approaches. From the results shown in Figure 3.5 the effect of increasing gains can be seen by comparison of low gain and high gain PI-Observer, here only the high gain approach is able to estimate the unknown input and displacement at node 5, as known from the literature [SYM95]. By increasing the observer gain, estimation result for PI-Observer with a constant high gain is comparable with MAPI-Observer. It is obvious that by using MAPIO approach, the gain of PIO can be accurately adjusted compare to the APIO approach especially in the moment that the elastic beam has contact with the obstacle or in other words when the system is affected by the unknown input $d(x, t)$ (time $\approx$ 0.016 s and time $\approx$ 0.02 s).

Additional simulations are done to analyze the sensitivity with respect to mea-

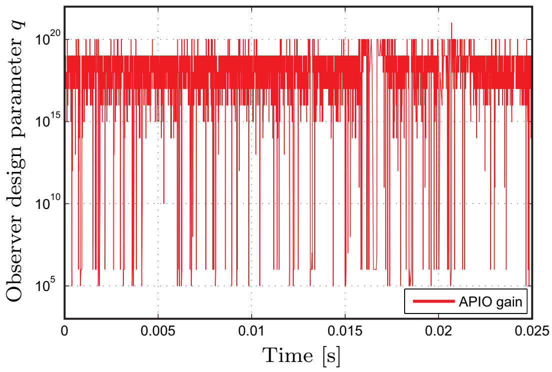


(a) Real contact force and its estimations

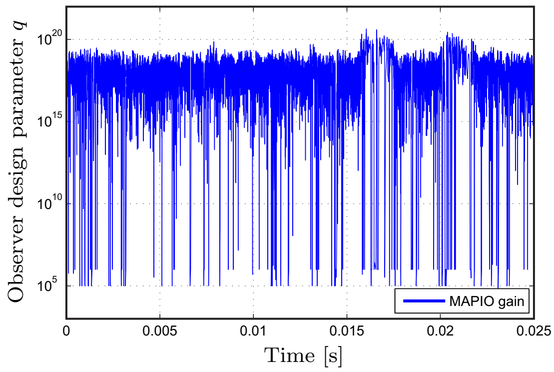
(b) Real displacement of node 5 and its estimations

Figure 3.6: Comparison of APIO and MAPIO in the presence of measurement noise

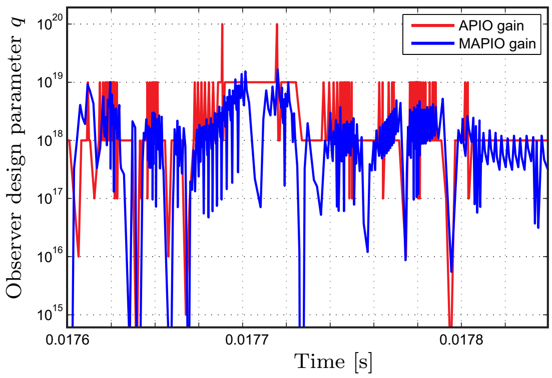
surement noise. The gain adjustment is noteworthy and required in the case that the system is affected by measurement noise to avoid the drawbacks of high gain observer design. This claim can be evaluated by the results shown in Figure 3.6. As illustrated in Figure 3.6, precise adjustment of PIO gain leads to better unknown input estimation performance and decreases the influence from measurement noise. From the results presented in Figure 3.6(a) the main effect resulting from the introduced gain adaption of MAPIO observer can be seen. The results clearly indicate that constant high gain PI-Observer estimation is strongly affected by the measurement noise. This effects strongly results from the high gain approach, which can be detected in comparison with the low gain PI-Observer results not effected by the noise (but not able to estimate the disturbances). In Figure 3.6(b) estimation of fifth node displacement is illustrated. According to the results it can be concluded that in the task of system state estimation MAPIO outperforms also the APIO. From the other side, it can be concluded that measurement noise has no significant influence on the estimation of system states because low gains are used in this situation and for estimation of system states. Gain adjustment of PI-Observer is illustrated in Figure 3.7 for APIO and MAPIO approaches.



(a) API-Observer gain adjustment



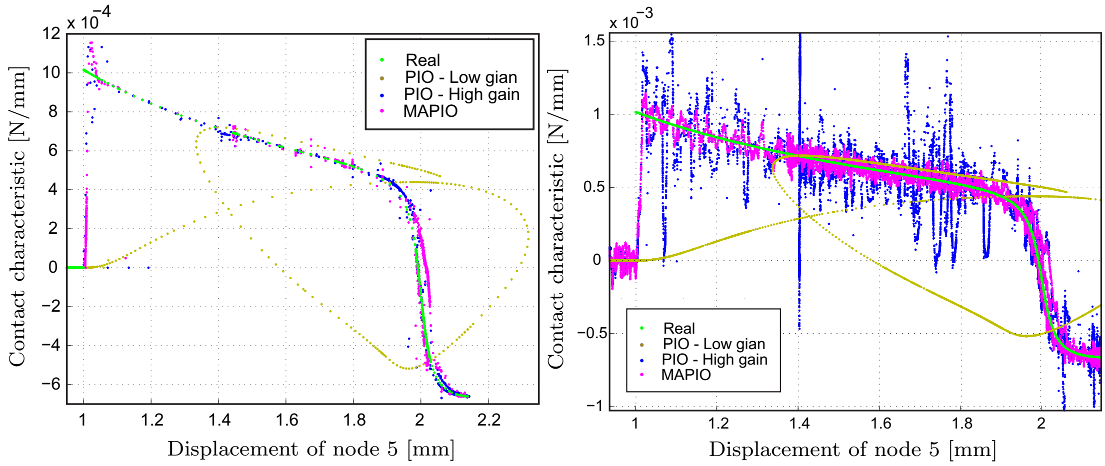
(b) MAPI-Observer gain adjustment



(c) Comparision of APIO and MAPIO gains adjustment

Figure 3.7: Comparison of APIO and MAPIO gains in the presence of measurement noise

Reconstruction of nonlinear characteristics by means of advanced observer design approaches



(a) W/o measurement noise

(b) W/ measurement noise

Figure 3.8: Reconstructed behavior of the nonlinearity (here: nonlinear spring behavior), based on both: estimation of one state (here tip displacement) and the unknown input (here: the contact force) - Complex characteristic behavior

As an example for complex contact characteristics a numerically example illustrating a complex contact behavior (as an elastic-plastic rubber contact behavior) is simulated. Assuming that the behavior is neither known nor can be measured, only the implicit measurements and the model-based observer approaches are used for reconstruction. Here estimations of the tip force in combination with the estimation of tip displacement is used to reconstruct the underlying displacement-related spring behavior.

Contact characteristics reconstruction results are illustrated in Figure 3.8(a) and Figure 3.8(b). The results show that in the task of complex contact characteristics (artificial nonlinearity) reconstruction, MAPI-Observer by scheduling the gain of PI-Observer produces precise results like high gain PI-Observer without considering the measurement noise. All approaches allow distinction between the backlash part and the contact part. The estimation of the nonlinear characteristic can be realized using high gain PI-Observer and MAPI-Observer approaches. Additionally it can be stated that also in the presence of noise the estimated contact characteristics produced by MAPI-Observer is not affected by measurement noise as much as high gain PI-Observer. A numerical comparison between advanced observer design approaches has been done in the task of estimating the external inputs (contact forces) acting to elastic mechanical structures. The numerical comparison is illustrated in

Table 3.1: Comparison of different approaches in unknown input (contact force) estimation task

Method	W/O noise		W/ noise	
	MSE	Max. |error|	MSE	Max. |error|
Low gain PIO	5.799e1	8.009e1	4.368e2	8.668e1
High gain PIO	1.606e1	1.729e1	9.274	2.143e1
MAPIO	1.047e1	1.963e1	4.782	2.320e1

Table. 3.1 and is graphically shown in Figure 3.9.

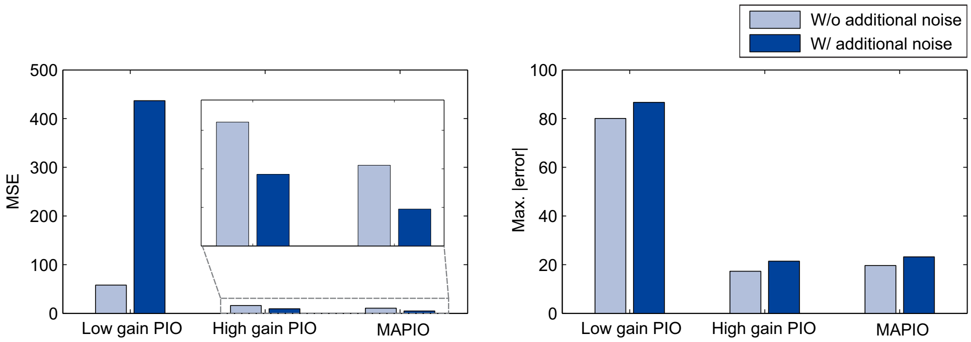


Figure 3.9: Comparison of different approaches in unknown input (contact force) estimation task

3.2.4 Verification in closed-loop simulation examples

In this section ACC Benchmark introduced by [WB92] is considered to evaluate the proposed observer. The two-mass-spring system illustrated in Figure 3.10 is an uncertain dynamical system comprises two bodies with the masses m_1 and m_2 which are connected by a spring with the stiffness k. The introduced model is defined as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -k/m_1 & k/m_1 & 0 & 0\\ k/m_2 & -k/m_2 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1/m_1\\ 0\end{bmatrix}(u+w_1) + \begin{bmatrix}0\\ 0\\ 0\\ 1/m_2\end{bmatrix}w_2, \quad (3.9)$$
$$y = x_2 + n,$$

with x_1 and x_2 denoting the positions of the masses 1 and 2, x_3 and x_4 the velocities of the masses 1 and 2, u the control input acting on the masses 1, w_1 and w_2 the unknown input forces acting on the masses 1 and 2 (as friction forces resp.), and n the measurement noise.

This benchmark problem is mostly used to evaluate the robustness of control design. Therefore, a close loop system consisting of an observer to estimate the

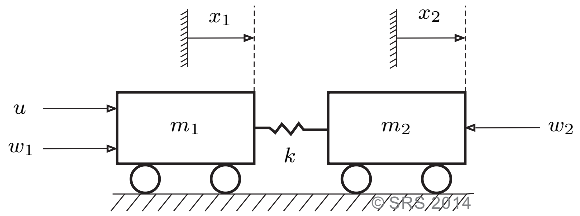


Figure 3.10: Two-mass-spring system, ACC Benchmark [WB92]

system states as well as the unknown inputs, and a full-state feedback control have been considered. Based on the extended state space representation of linear system according to (2.30) and disturbance dynamics as $\dot{v} = Vv(t) + \delta x(t)$ the extended model is rewritten as

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} A & NH \\ \delta & V \end{bmatrix}}_{A_e} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_e} u(t) + \begin{bmatrix} Eg(t) \\ 0 \end{bmatrix}, \\ y(t) &= \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}, \end{aligned} \tag{3.10}$$

where δ couples system states to unknown disturbances and is usually a matrix containing elements with very small values considered to design an external feedback [Dav72]. The δ is chosen as zero in the case of observer design. Based on observer formulation for linear systems (2.31) the states $x(t)$ and the unknown input $d(t)$ can be estimated using a high-gain observer design

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{v}}(t) \end{bmatrix} &= \begin{bmatrix} A & NH \\ 0 & V \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \underbrace{\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}}_{L} (y(t) - \hat{y}(t)), \\ \hat{y}(t) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix}. \end{aligned} \tag{3.11}$$

The high-gain PI-Observer feedback matrices L can be designed using LQR method. For the time-invariant cases this task can be realized by solving Algebraic Riccati equation. For an asymptotic stable observer, positive definite matrices Q and R are considered as

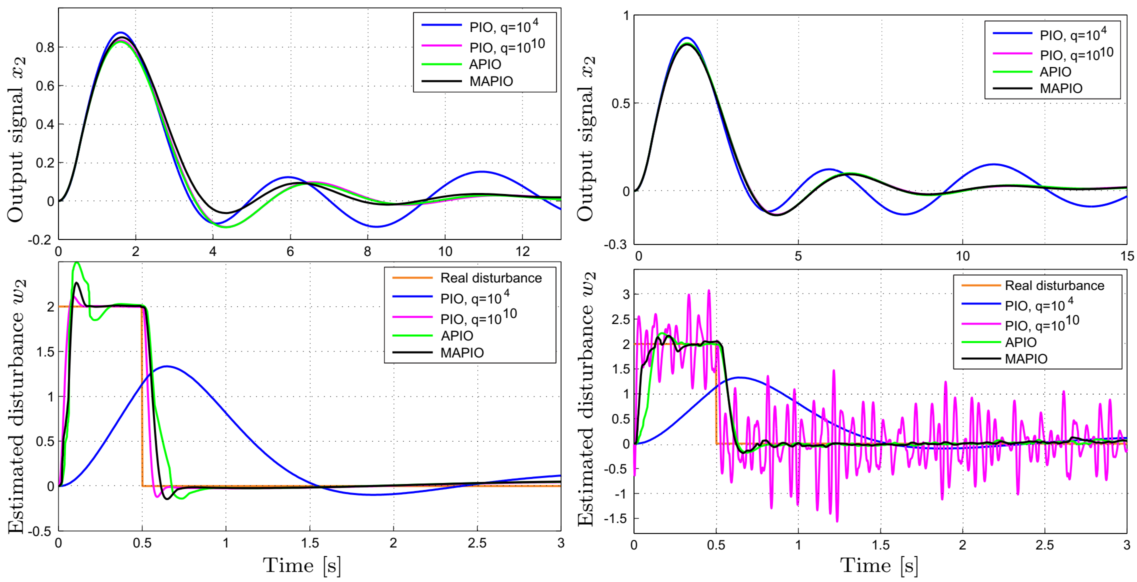
$$Q = \begin{bmatrix} I_4 & 0_{4\times1} \\ 0_{1\times4} & q \end{bmatrix}, \qquad R = I, \tag{3.12}$$

with the scalar design parameter q which can be scheduled to improve the system robustness and output performance. This task can be done using API-Observer or MAPI-Observer methods. As mentioned in (3.11) with considering δ parameter the extended system will be fully controllable and a feedback control can be proposed as

$$u = -K_x\hat{x}(t) - K_v\hat{v}(t). \tag{3.13}$$

Control gains K_x and K_v have been designed using LQR method and with positive definite matrices Q and R which have appropriate dimensions. For evaluation part two scenarios has been considered. For all scenarios, combined with different observer strategies, the performance of the observer part is of interest. To avoid effects from feedback, the same control feedback is applied for all combinations. Therefore, comparison between three introduced methods is limited only to the performance of observers.

First of all simulation results without considering measurement noise and with an



(a) W/o measurement noise

(b) W measurement noise

Figure 3.11: Estimation results and system output of the closed-loop system using ACC Benchmark example

external unit impulse disturbance w_2 applied on the mass 2 (Scenario 1) are illustrated in Figure 3.11(a). In the case without consideration of measurement noise it can be concluded that PI-Observer with high gain in combination with the proposed controller has suitable settling time in the output response. But on the other hand, in disturbance estimation results it can be seen that the design gain parameter q has to be properly chosen. With considering a low gain, PI-Observer is not able to estimate the impulse disturbance and consequently has low performance with oscillation in the output signal. However by increasing the observer gain, estimation result for PI-Observer is comparable with API-Observer and MAPI-Observer. As regards the difficulty of determining suitable gain of PI-Observer especially in practical applications, API-Observer and MAPI-Observer are more considerable. From the results it becomes obvious that disturbance estimations and output response using API-Observer and MAPI-Observer are almost as good as the results using PI-Observer with a constant high gain.

In Scenario 2 impulse disturbance w_2 applied on the mass 2 and in the presence of

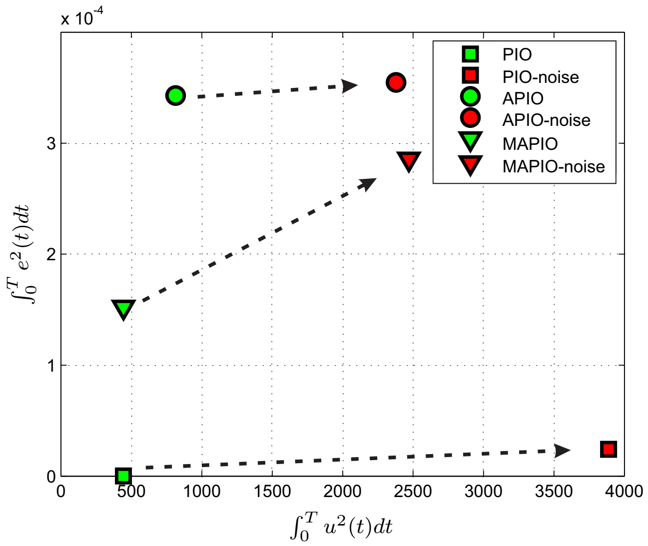
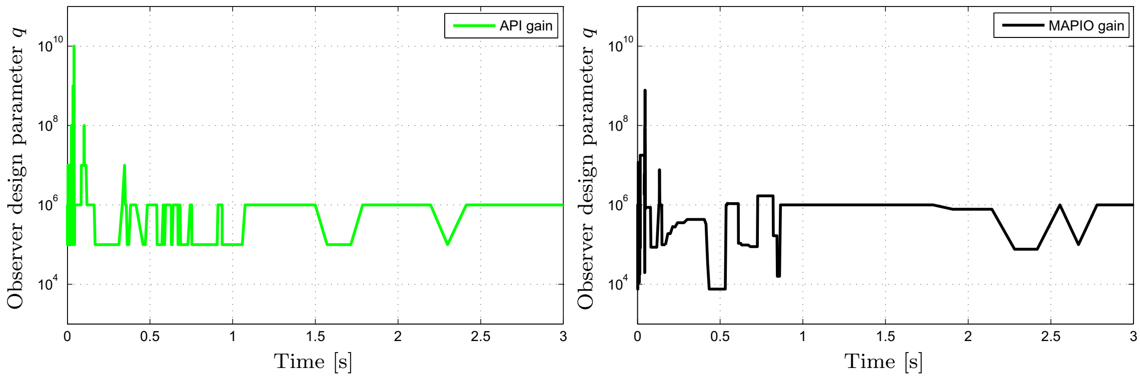


Figure 3.12: Comparison by means of criterion C

measurement noise is taking into consideration. Results presented in Figure 3.11(b) clearly indicate that with high constant gain for PI-Observer estimation, the disturbance estimation is strongly affected by the measurement noise. This effect is expected and also described in previous publications. In contrast, estimation result using low constant gain for PI-Observer illustrates no influence from the measurement noise and also no acceptable performance in the disturbance estimation. On the contrary, API-Observer and MAPI-Observer have capability to estimate the disturbance also in the presence of noise because of combining the advantages of both high and low gain based on the desired performance evaluation. From the simulation results it can be seen that MAPI-Observer reacts faster and with smaller time delay than API-Observer. Furthermore, in the disturbance estimation process MAPI-Observer acts almost equal or better than API-Observer.

From the output response shown in Figure 3.11(b) it can be concluded that in combination with the proposed controller, PI-Observer with high gain has suitable settling time and output response (the effect of noise is not visible in the output signal regarding the level of additional measurement noise). However judging only based on the output response is not completely reliable. In this work evaluation of the observer-based control results using the introduced example is done based on a suitable criterion $C = [\int_0^T e^2(t)dt, \int_0^T u^2(t)dt]$ which has to be minimized [LS12]. This criterion considers both the control error $e(t)$ and the input energy u(t). The interval length T is considered as 10 sec which denotes the time window, where the performance is considered and compared.

Results with/without noise and with the same controller for three different ob-



(a) (b)

Figure 3.13: Gain adjustment of APIO and MAPIO in the case of no measurement noise

servers are illustrated in Figure 3.12. With the same input energy the controller which has lower output error or correspondingly with the same output error which uses less input energy (closer to the origin) has better performance. From the results it is evident that without considering measurement noise, output error using PI-Observer is always less than the output error for other two observers but in the presence of noise PI-Observer requires the maximum energy and has the maximum distance from the origin in comparison to the API/MAPI-Observer. On the other hand MAPI-Observer requires always the minimum input energy. Output error using MAPI-Observer is always less than the output error using API-Observer. Results with smaller change in the trajectory under consideration of noise have more robustness. From Figure 3.12 it can be concluded that MAPI-Observer has better performance in comparison to API-Observer, however API-Observer has more robustness because of its smaller change in the trajectory under consideration of noise. Gain adjustment of APIO and MAPIO approaches is illustrated in Figure 3.13

3.3 Summary and discussion

In this chapter high gain Proportional-Integral-Observer is introduced considering its structure and conditions for the convergence of estimation error. Furthermore, MAPIO is proposed as the extended version of APIO to tune the gain of PIO according to the current situation. A cost function is defined so that the estimation performance and the related energy can be evaluated. The MAPI-Observer approach combines the adaption of observer gain with integration of the observer scheme. In this chapter a comparison between advanced observer design approaches has been done in the task of reconstructing the nonlinear characteristics and estimating the external inputs (contact forces) acting to elastic mechanical structures. Verification of introduced approaches is also given on node displacements estimation of an elastic beam system. Simulation results using PI-Observer and related extension are illustrated and advantage of using adaptive high gain PI-Observer in contact characteristics estimation is verified.
Besides comparison of different approaches in open-loop estimation task, a closed-loop simulation example known as two-mass-spring system (ACC Benchmark) is used to illustrate the advantages of the proposed approach in the closed-loop context. From the results, it can be concluded that in the task of unknown input estimation MAPIO is more successful by considering different level of measurement noise in comparison to previous methods known as APIO and standard PIO.

4 Adaptive gain scheduling of Proportional-Integral-Observer using funnel adjustment concept

Estimation of contact force is a nontrivial task because of the fast and unknown dynamics of the contact characteristic. According to the capabilities of PIO in estimation of external inputs with unknown dynamical behavior, the focus of this section is utilization of PIO for contact force estimation purpose. The contact force estimation here works as a typical task dealing with unknown effect in the control of elastic mechanical systems. From a structural point of view this task is identical with the estimation and compensation of friction forces or torques in motor driven drives, unknown effects acting to the endeffector of robots, etc. The first assumption to describe this problem class is that the number of (modeled, internal) states n is much larger than the number of independent measurements and inputs available for control. This class is also described by the fact that the system model is known, the unknown inputs are unknown, and the location of unknown and control inputs are different. To achieve an adequate estimation of states and unknown input, suitable observer gains have to be carefully designed.

As mentioned in chapter 3 estimation of contact force usually requires high observer gain to fulfill the estimation of fast dynamical behavior. On the other hand, high observer gain deals with the influence of gains measurement noise [LS12]. In contrast, low observer gain leads to unaffected results from measurement noise, whereas estimation of contact force with fast dynamics is infeasible. Therefore a compromise or an adjustment of observer gains is required. In several research contributions [AK09, EE03], switching gain approach is addressed to cope with the problem of high gain observers. The algorithm is based on two sets of observer design (one set with high observer gain and the other with low observer gain) regarding a switching condition. This idea can be applicable only if the moment of force contact is known to switch from low gain to high gain.

In chapter 3 a switching gain procedure is introduced and verified as MAPIO (the modified version of APIO) to adaptively change the gain of PIO to avoid the drawbacks of high gain observers especially in the presence of measurement noise. The difficulty of using MAPIO is its implementation with respect to real-time applications/real experiences in closed-loop system because of its execution time which can be improved by using a fast processor. However, estimation task of fast unknown dynamics in the context of closed-loop system requires a fast adaptive change in the observer gain regarding the actual state of the system. Considering the required high execution time for APIO and MAPIO leads to elaborate a more flexible and fast procedure to design and achieve an adaptive high gain PIO. Therefore, funnel adaptive PIO is proposed in this chapter. The structure of PIO is taken and the observer gains are chosen using an adaptive adjustment method. The idea is to use the funnel control method [IRS02] as a high gain adaptive (time-varying) adjustment approach. The observer gain increases (aggressive reaction) only when the estimation error increases and has less distance to prescribed boundaries (funnel function). Correspondingly, the gain decreases (more relaxed reaction) when no aggressive reaction is required for estimation error compensation (less estimation error).

The contents, figures, and tables presented in this chapter are prepared for publication in the journal paper "Contact force estimation of an elastic mechanical structure using a novel adaptive funnel PI-Observer approach" [BS17a] and published in the conference paper "Proportional-Integral-Observer with adaptive high-gain design using funnel adjustment concept" [BS17c].

4.1 Adaptive funnel adjustment of PI-Observer gain

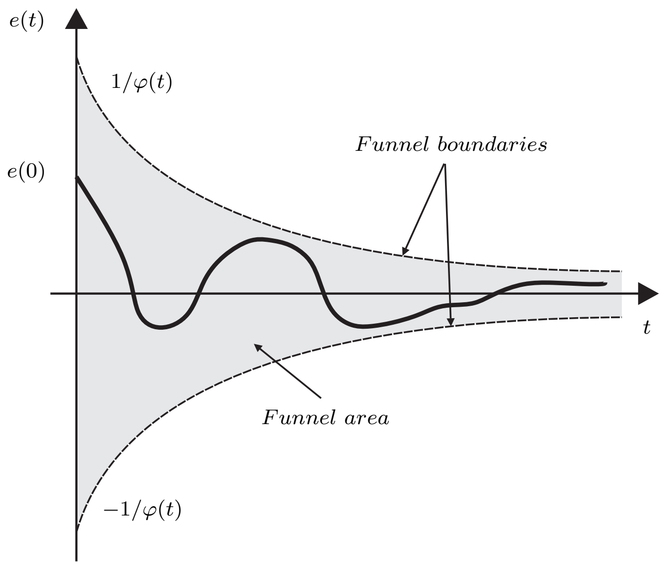


Figure 4.1: Basic idea of funnel adjustment [IRS02]

Tracking with prescribed transient accuracy was firstly proposed in [MD91]. The authors have designed a high gain-based switching discrete controller that contributes a satisfactory predefined transient behavior. On the other hand, the switching discrete behavior and non-decreasing gain are not desirable in industrial applications. In [IRS02] the authors proposed a high gain adaptive (time-varying) control concept known as Funnel Control (FC). Funnel Control is a proportional (memory-less i.e. no dynamics) approach applicable for wide range of linear/nonlinear systems under certain conditions [IRS02, IRT05]. Despite other adaptive control methods, identification or estimation of system parameters is not required. Therefore Funnel Control can be considered as an nonidentifier-based adaptive control strategy with predefined transient accuracy [ITT04].

Assuming certain conditions funnel control is applicable for the systems with relative degree one or two [IRS02]. Here without considering the conditions of funnel control approach the idea of funnel adjustment method is considered. In Figure 4.1 the main idea of funnel function is illustrated with the funnel boundary

$$\partial F_{\varphi}(t) = \frac{1}{\varphi(t)}, \tag{4.1}$$

which contains an arbitrary chosen bounded, continuous, and positive function $\varphi(t)$ for $t \geq o$ and $sup_{t\geq 0}\varphi(t) < \infty$ [IRT05]. The funnel is defined as

$$F_{\varphi} : t \rightarrow \{e(t)| \ \ \varphi(t)\|e\| < 1\}, \tag{4.2}$$

which encloses the error $e(t)$ for $t > 0$ when the initial error $e(0)$ is surrounded by the funnel boundaries. Here $e(t)$ is considered as the estimation error corresponding to the output reconstruction as

$$e(t) = \sigma(\hat{y} - y), \tag{4.3}$$

with constant σ used to make the estimation error large enough for the following steps. The funnel gain can be calculated as

$$k(t) = \frac{1}{1-\varphi(t)\|e(t)\|}, \tag{4.4}$$

to ensure that the error $e(t)$ evolves inside the funnel area. When the error $e(t)$ tends close to the upper or lower funnel boundaries, the funnel gain $k(t)$ increases. Correspondingly the funnel gain decreases when the error becomes smaller. Therefore, funnel gain $k(t)$ is only large if more aggressive reaction is required. This contribution elaborates more on the design of PIO gain based on the funnel idea. The purpose of proposed algorithm is to find suitable observer gains to achieve reasonable/acceptable estimation errors for both state and unknown input (contact force) for each estimation step and especially in the presence of measurement noise and requirement of high gain PI-Observer. To achieve this goal in the following part detailed steps of gain adjustment are explained.

4.2 Structure of funnel PI-Observer

According to section 3.1.1 the design parameter q can reflect almost all relevant aspects to be considered for suitable estimation of unknown inputs. To reach suitable estimation error for both states and unknown inputs, the design parameter q has to be adaptively chosen at each step of integration procedure [LS12]. This idea has been investigated in [LS12] and improved in [BS15b] by using a bank of PI-Observers with different design parameters q based on an optimization method. In this contri-

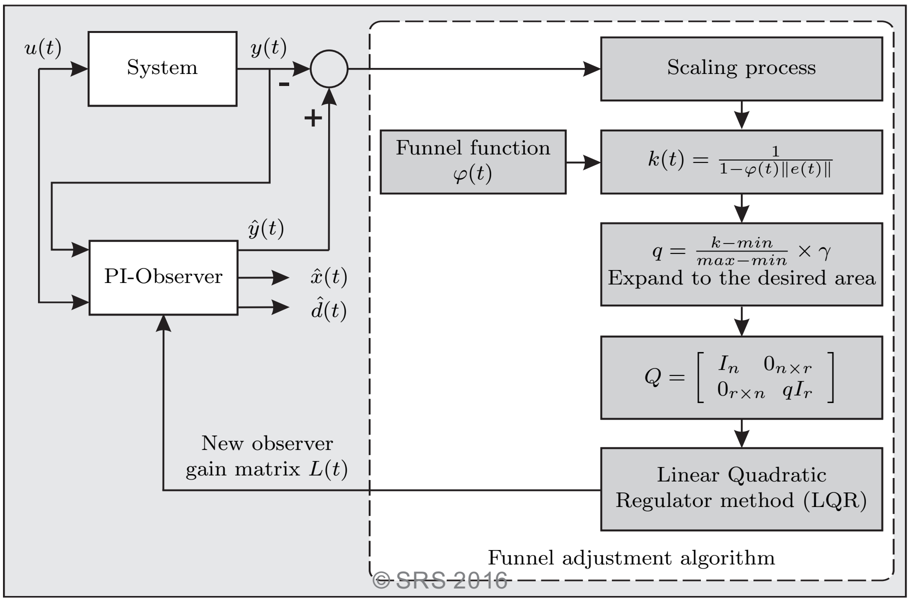
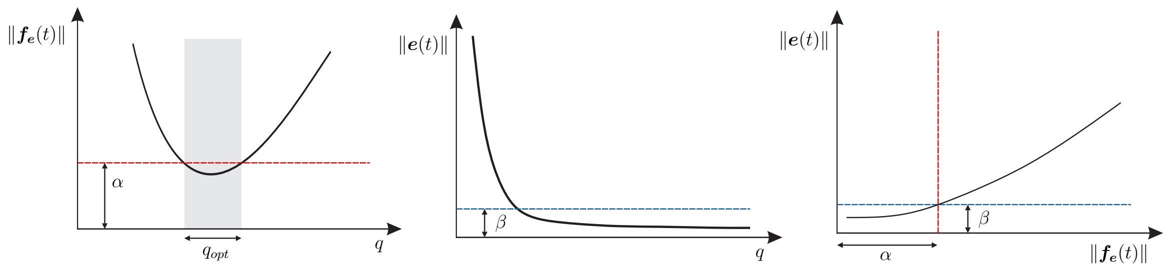


Figure 4.2: Sketch of proposed adaptive funnel PIO algorithm (Here min, max, and γ have to be defined according to the desired performance of the estimation results. They are considered as design parameters to define the suitable range of design parameter q)

bution a novel algorithm using funnel adjustment approach is proposed to achieve a simple high gain adaptive structure. It is worth noting that utilization of funnel idea leads to take the advantages of high observer gain only when high gain is required. Design of parameter q is adjusted based on the funnel gain $k(t)$. So at first $k(t)$ is calculated by considering the output estimation error and according to the funnel gain design approach in (4.4). Afterward, the calculated funnel gain $k(t)$ is additionally scaled to a reasonable interval and then replaced with design parameter q. The sketch of proposed algorithm, denoted as funnel PIO in the sequel, with online adaption of observer gains process is given in Figure 4.2. It is worth mentioning that in this contribution only the idea of funnel control is of interest, and subsequently the funnel conditions are not considered by authors (further details can be found in [IRS02]). Using this idea allows to calculate the design parameter q adaptively at each step of estimation procedure.

In Figure 4.3(b) the relation between state estimation error $\|\boldsymbol{e}(t)\|$ and design parameter q is illustrated. The state (output) estimation error is preserved under the boundary shown by β which indicates the funnel boundary introduced in Figure 4.1. According to Figure 4.3(c) when $\|\boldsymbol{e}(t)\|$ is bounded by boundary β the unknown input estimation error $\|\boldsymbol{f_e}(t)\|$ is maintained by the boundary α. Indeed, according to Figure 4.3(a) when the estimation error of the system state $\|\boldsymbol{e}(t)\|$ is preserved in a prescribed funnel area β, the estimation error of the unknown input $\|\boldsymbol{f_e}(t)\|$ is also preserved in the area of optimal q parameter (boundary α). This shows the relation between funnel PIO and the optimal design of q parameter in Figure 3.2. Therefore, using of proposed funnel PIO leads to optimal selection of q parameter as discussed in MAPIO structure. In the following sections an evaluation of the proposed funnel PIO approach is given in experimental and simulation results using an elastic beam system.



(a) Relation between unknown input estimation error $\|\boldsymbol{f_e}(t)\|$ and design parameter q

(b) Relation between state estimation error $\|\boldsymbol{e}(t)\|$ and design parameter q

(c) Relation between unknown input estimation error $\|\boldsymbol{f_e}(t)\|$ and state estimation error $\|\boldsymbol{e}(t)\|$

Figure 4.3: Illustration of estimation errors for unknown input and system states with regards to design parameter q and the boundaries

4.3 Stability of estimation error dynamics

According to the funnel PI-Observer algorithm introduced in Figure 4.2, the gain of PI-Observer is varied stepwise and adaptively. Therefore, adjustment of PI-Observer gain at each step turns the estimation problem to a switching PI-Observer issue which is illustrated in Figure 4.4. Accordingly, the PI-observer gain can be switched between different designed values $L_1, L_2, ..., L_p$. Generally speaking, changing of observer gain at each step of integration procedure can lead to an unstable total estimation error. Even if every observer gain matrix is designed so that the error dynamics converges to a small value asymptotically, the possibility of unstable estimation error of the general system still remains. Therefore, stability analysis for the whole state and unknown input estimation procedure is required. According to Eqn. (3.1) the error dynamics of the extended system for the certain step i of switching PI-Observer can be rewritten as

$$\begin{bmatrix} \dot{e}(t) \\ \dot{f}_e(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A - L_{1i}C & N \\ -L_{2i}C & 0 \end{bmatrix}}_{A_{e,obs(i)}} \begin{bmatrix} e(t) \\ f_e(t) \end{bmatrix} - \begin{bmatrix} Eg(t) \\ \dot{d}(t) \end{bmatrix} + \underbrace{\begin{bmatrix} L_{1i} \\ L_{2i} \end{bmatrix}}_{L} h(t),$$

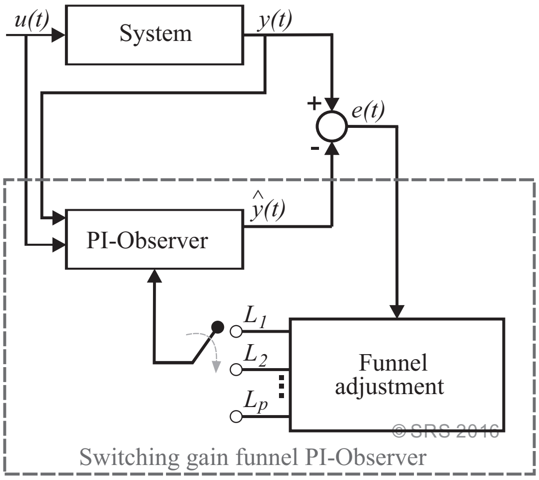


Figure 4.4: Block diagram of the switching PI-Observer

which consists of internal and external uncertainties (modeling errors, disturbances, etc.). In this chapter the goal is to estimate the unknown contact force which acts to the system as a disturbance.

Assumption: The lumped disturbance is constant i.e. $\dot{d}(t) = 0$.

Remark: If the lumped disturbance $d(t)$ is slowly time varying i.e. $\dot{d}(t) \simeq 0$ then the aforementioned assumption is no longer necessary.

Therefore, by assuming just the external uncertainties the error dynamics can be revised as

$$\dot{E}(t) = \begin{bmatrix} \dot{e}(t) \\ \dot{f}_e(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A - L_{1i}C & N \\ -L_{2i}C & 0 \end{bmatrix}}_{A_{e,obs(i)}} \begin{bmatrix} e(t) \\ f_e(t) \end{bmatrix}. \tag{4.5}$$

The Lyapunov function candidate and its derivative can be defined as

$$\begin{aligned} V(t) &= E^T(t)PE(t), \\ \dot{V}(t) &= E^T(t)A^T_{e,obs(i)}PE(t) + E^T(t)PA_{e,obs(i)}E(t) \\ &= E^T(t)(A^T_{e,obs(i)}P + PA_{e,obs(i)})E(t). \end{aligned} \tag{4.6}$$

By defining

$$P = \begin{bmatrix} P_0 & 0 \\ 0 & P_0 \end{bmatrix}, \tag{4.7}$$

the derivative of Lyapunov function can be calculated as

$$\begin{aligned}\dot{V}(t) &= E^T(t)(\underbrace{\left[\begin{array}{c|c} (A-L_{1i}C)^T & -(L_{2i}C)^T \\ \hline N^T & 0 \end{array}\right]}_{A^T_{e,obs(i)}} \begin{bmatrix} P_0 & 0 \\ 0 & P_0 \end{bmatrix} \\ &+ \begin{bmatrix} P_0 & 0 \\ 0 & P_0 \end{bmatrix} \underbrace{\left[\begin{array}{c|c} A-L_{1i}C & N \\ \hline -L_{2i}C & 0 \end{array}\right]}_{A_{e,obs(i)}})E(t) \\ &= E^T(t)\Psi E(t),\end{aligned} \tag{4.8}$$

when

$$\Psi = \left[\begin{array}{c|c} \underbrace{(A-L_{1i}C)^T P_0 + P_0(A-L_{1i}C)}_{\psi_{11}} & \underbrace{P_0 N - (L_{2i}C)^T P_0}_{\psi_2} \\ \hline \underbrace{N^T P_0 - P_0(L_{2i}C)}_{\psi_2^T} & \underbrace{0}_{\psi_{22}} \end{array}\right], \tag{4.9}$$

which has to be less than zero to make the derivative of the Lyapunov function candidate negative definite. This proves the convergence of the error dynamics toward zero. To achieve this goal the conditions

$$\begin{cases} 1. \quad (A-L_{1i}C)^T P_0 + P_0(A-L_{1i}C) \overset{!}{<} 0 \\ 2. \quad -\psi_2\psi_2^T \overset{!}{<} 0 \end{cases} \tag{4.10}$$

have to be satisfied. Considering the positive term $\psi_2\psi_2^T$ the second condition is always fulfilled. To assure the stability of the error dynamics for the whole estimation procedure the condition

$$\begin{cases} \tilde{A}^T P_0 + P_0\tilde{A} \overset{!}{<} 0 \\ \tilde{A} = A - L_{1i}C, \end{cases} \tag{4.11}$$

should be satisfied. This converts the problem of switching PI-Observer stability to the stability proof of switching Luenberger observer with observer gain L_{1i}.
The stability proof of switching observers and switching systems are taking into consideration in several contributions, e.g. in [Lun00] the authors elaborate several conditions for the design of switching observers (especially switching Luenberger observer) which guarantee the asymptotic stability of the observer error. According to [Lun00] for Luenberger observer:
"If all the observer gains are chosen/designed to have the Euclidean norm of the estimation error $\|e(t)\|$ as a Lyapunov function of the error dynamics, then the total estimation error vanishes asymptotically ($\lim_{t\to\infty}\|e(t)\| = 0$)."
To prove this assertion the authors consider that the system is always observable.

As illustrated in Figure 4.5 the gain of observer is changed at time t_k $(k = 1, 2, ...)$ between different values L_{1i} $(i = 1, 2, ..., p)$ achieved by the funnel PI-Observer algorithm. The system state does not change by switching procedure at step k. In other words

$$x(t_k - 0) = x(t_k + 0), \tag{4.12}$$

so for the ith operating condition the estimation of system states is

$$\begin{aligned} &\tfrac{d\hat{x}}{dt} = (A - L_{1i}C)\hat{x} + Bu(t) + L_{1i}y(t) \\ &\hat{x}(0) = \hat{x}_0, \end{aligned} \tag{4.13}$$

which leads to the following estimation error dynamics

$$\begin{aligned} &\dot{e}(t) = (A - L_{1i}C)e(t), \\ &e(0) = x_0 - \hat{x}_0. \end{aligned} \tag{4.14}$$

It is assumed that the system is in the time interval $t_{k-1} < t < t_k$ with observer gain L_{1i-1} and after the switching time k for the observer design the system is in the time interval $t_k < t < t_{k+1}$ with observer gain L_{1i}. As explained, in this section the observer matrix is designed using Linear Quadratic Regulator (LQR) approach for different design parameter. Therefore, according to the properties and conditions of LQR approach, there is a piecewise Lyapunov function of the estimation error dynamics at each step of switching gain design. Considering the switching moment t_k as shown in Figure 4.5 and according to the fact that at each step the Euclidean norm of the estimation error is a Lyapunov function of the error dynamics, it can be concluded [Lun00] that

$$\frac{d\left\|e(t)\right\|}{dt} < 0 \quad for \begin{cases} t_{k-1} \leq t < t_k \\ t_k \leq t < t_{k+1} \end{cases}, \tag{4.15}$$

holds. By considering the principle that the state of the system does not change during the switching procedure [Lun00] it can be concluded that

$$e(t_k - 0) = e(t_k + 0), \tag{4.16}$$

as a result

$$\left\|e(t_k - 0)\right\| = \left\|e(t_k + 0)\right\|. \tag{4.17}$$

Therefore, $\|e(t)\|$ decreases monotonous in the whole time interval $t_{k-1} < t < t_{k+1}$ as shown in Figure 4.5. This result can be generalized to every switching time t_k $(k = 1, 2, ...)$.

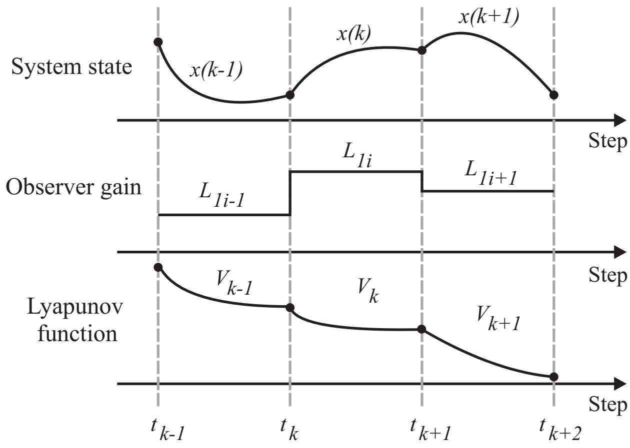


Figure 4.5: Switching procedure (system state, observer gain, and Lyapunov function

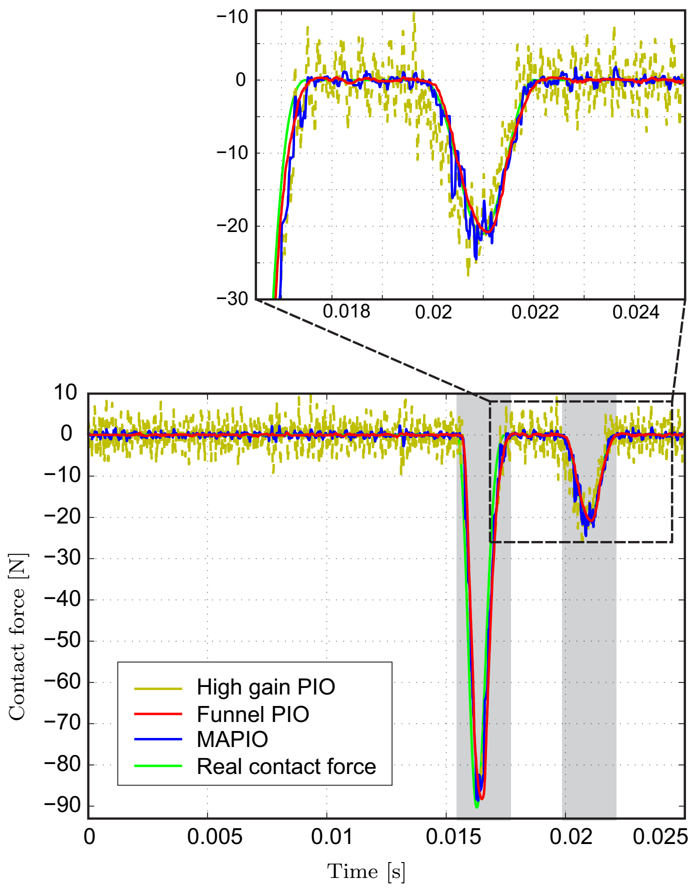


Figure 4.6: Real contact force and its estimations, with measurement noise (simulation result)

4.4 Evaluation using simulation and experimental results of an elastic beam test rig

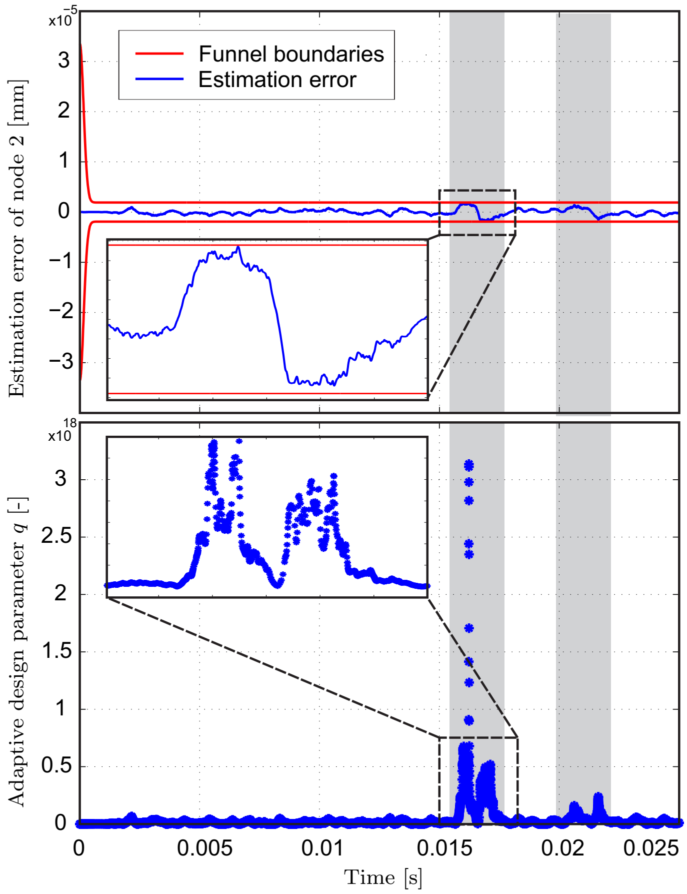


Figure 4.7: (a) A time-varying error bound with estimation error, (b) Funnel adaptive gain adjustment of PI-Observer (simulation result)

4.4.1 Simulation results

In this study verification of the introduced approaches is given using contact force estimation of an elastic beam system. The elastic beam system used here is introduced in section 3.2.3. The goal is to estimate the unknown input $d(x, t)$ as a disturbance acting on the fifth node when the elastic beam is in contact.

As clarified, in this contribution the funnel idea is utilized to adaptively adjust the gain of PIO. To achieve this goal the funnel function is considered according to Table 4.1. In Figure 4.6 comparison of estimated contact force using high gain PIO and funnel PIO is given to show at a glance the difference of approaches: adaption of the high gain PIO introduced as funnel PIO leads to better contact force estimation

Table 4.1: Parameters of adaptive funnel PIO algorithm (open-loop evaluation)

	Funel function	min	max	γ	a_1	a_2	a_3
Simulation	$\varphi(t) = \dfrac{1}{(a_1 e^{-a_2 t^2} + a_3)}$	1	1.5	e10	1	2e7	0.06
Experiment	$\varphi(t) = \dfrac{1}{(a_1 e^{-a_2 t^2} + a_3)^2}$	0.5	2	e5.5	50	1	0.04

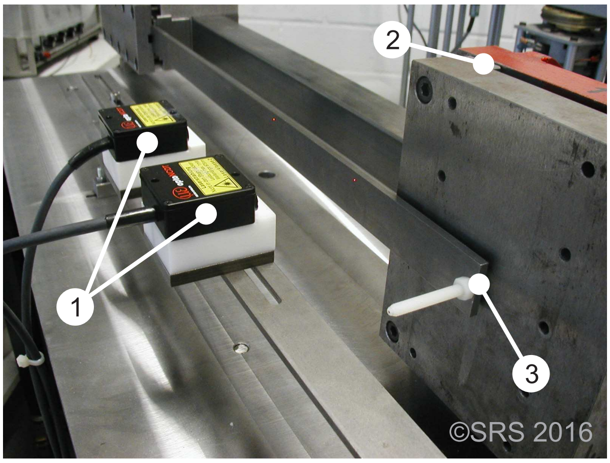


Figure 4.8: Test rig of elastic beam system at the Chair of Dynamics and Control (UDuE), 1. laser displacement sensors, 2. piezo force sensors, and 3. contact tip.

than high gain PIO and MAPIO (less influence from measurement noise).
Estimation error and related funnel function are illustrated in Figure 4.7 which clearly shows that the estimation error of the available output remains in the prescribed funnel function. The error band is about 2×10^{-4} which illustrates a satisfactory estimation error for system output. The adaptation observer gain (design parameter q) is also displayed in Figure 4.7. It is obvious that the design gain parameter increases in the presence of contact force (t≃0.017s and t≃0.022s). Correspondingly, it is adaptively adjusted more relaxed when the estimation error has large distance to the funnel boundaries.

4.4.2 Experimental validation

An elastic beam test rig [Kra06] is shown in Figure 4.8. The elastic beam system is modeled using Finite Element Method as mentioned in section 3.2.3. Two measurements, the displacements at the second and the fifth nodes ($y_1(t) = x_3(t)$, $y_2(t) = x_9(t)$), are measured using laser sensors. The task is to estimate the not measured contact force $d(t)$ acting on the last node of the beam which is unknown with respect to the time behavior.

Here the adaptive funnel PIO is performed due to its ability to adjust the ob-

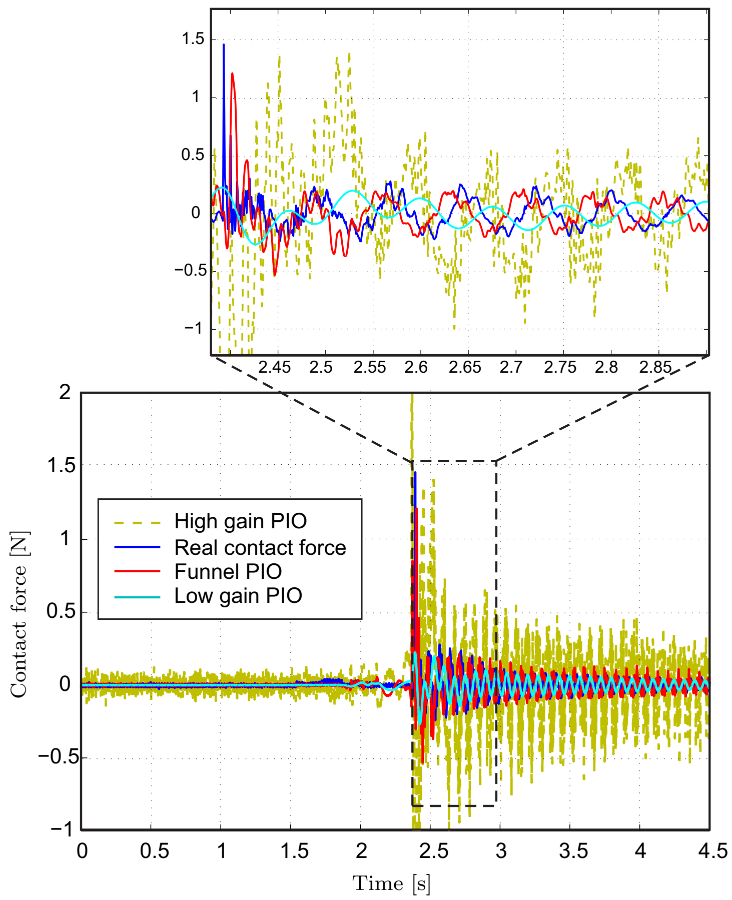


Figure 4.9: Measured real contact force and its estimations (experimental result)

server gain. Experimental results are achieved for different observer approaches, high/low gain PIO and funnel PIO. Especially the results at t≃2.4s are of interest, in the moment the contact occurs. The experimental results shown in Figure 4.9 illustrate inefficient estimation quality regarding low gain PIO due to the fast dynamical behavior of the contact force. On the other hand utilization of high gain PIO represents strong influence from measurement noise in the unknown input estimation task. The novel proposed approach, introduced as funnel PIO, adjusts the PIO gain adaptively so that the estimation follows rapidly the contact force whit designed adaptive gain. Besides, when no aggressive high gain is required (absence of contact force), the proposed method treats as relaxed low gain PIO and is not impacted by measurement noise (Figure 4.9).

Estimation error and related funnel function illustrated in Figure 4.10 clearly show that the output estimation error of the second node remains in the prescribed funnel area. Accordingly, the adaption behavior of observer gain (design parameter q) in Figure 4.10 demonstrates the advantages of proposed funnel PIO method. It is worth mentioning that during the time interval when the unexpected contact exists

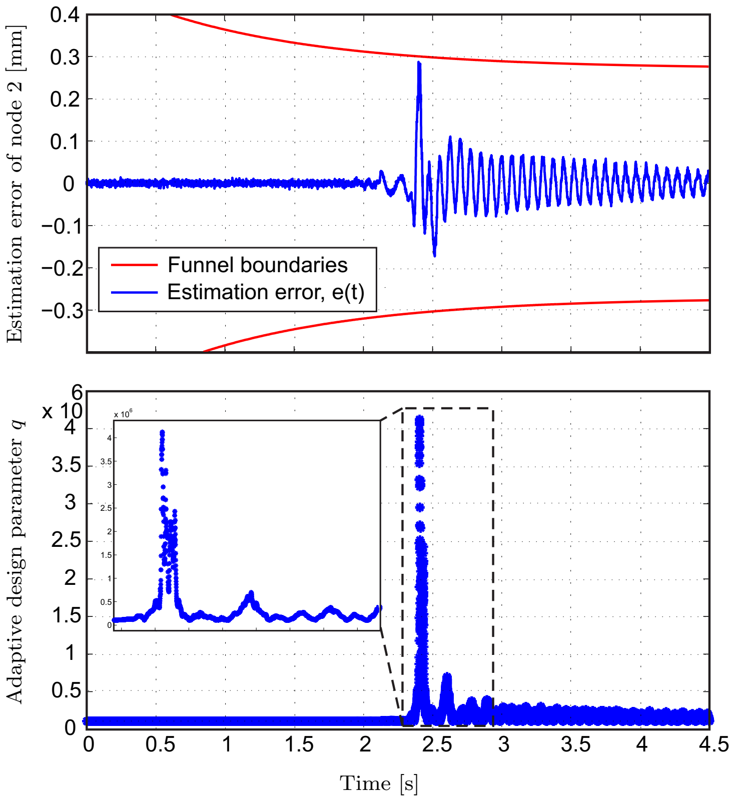


Figure 4.10: (a) A time-varying error bound with estimation error, (b) Funnel adaptive gain adjustment of PI-Observer (experimental result)

(begin at t≃2.4s) the gain is adjusted to be larger, otherwise it is adjusted more relaxed.
Single-sided amplitude spectrum of contact force estimation is illustrated in Figure 4.11 to detail further performance differences of the approaches. It can be concluded that low gain PIO is not suitable to estimate the fast dynamical behavior of the real contact force, as known. Furthermore, it can be stated that other approaches (high gain PIO and funnel PIO) show a roughly similar behavior with respect to the dynamics of the real contact force. On the other hand funnel PIO behavior shows less higher dynamical parts of the contact force (less influence of measurement noise). It should be mentioned that no further filtering is applied.
In Table 4.2 a comparison of different observer approaches in state and unknown input estimations for both simulation and experimental results are numerically represented. Besides, the numerical results are normalized to the interval [0,1] and graphically illustrated in a spy diagram (Figure 4.12). Accordingly, it can be concluded that funnel PIO has the best performance in state and unknown input estimation. On the other hand high gain PIO is more or less successful in state estimation while it has the worse performance in the unknown input estimation task.

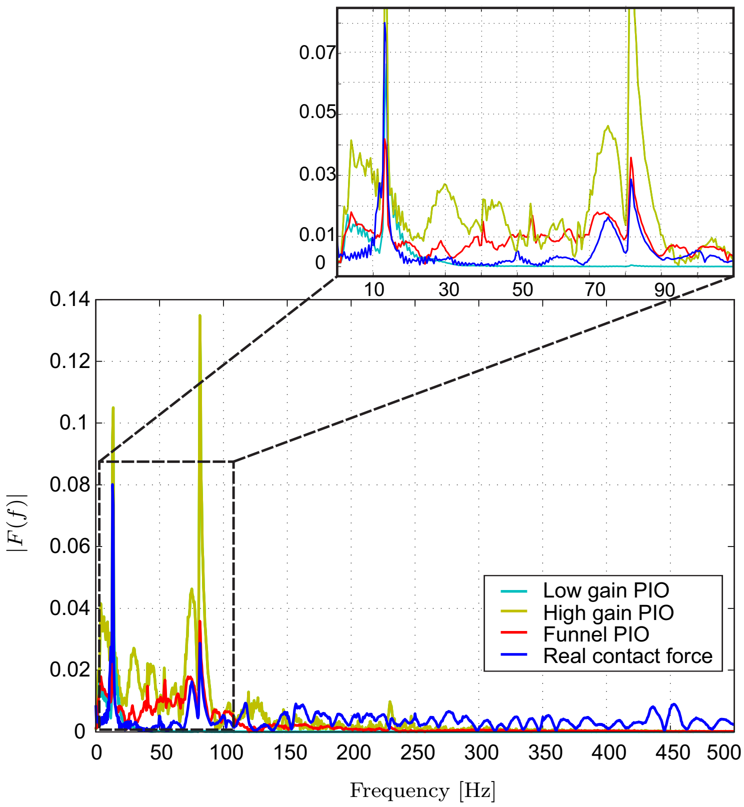


Figure 4.11: Single-sided amplitude spectrum of contact force estimation (experimental result)

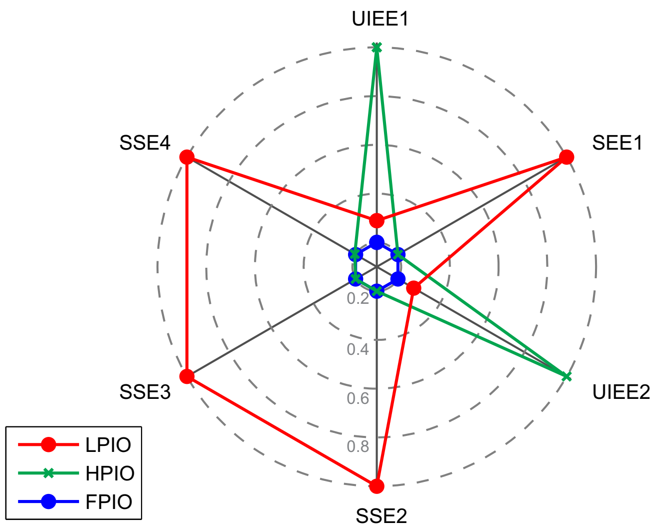


Figure 4.12: Performance of different observers (experimental results)

Table 4.2: Comparison of different observer approaches

	Simulation			Experiment		
Approach/$\int_0^T e^2 dt$	UIEE1[1]	SEE1[2]	SEE2[3]	UIEE2[1]	SEE3[2]	SEE4[3]
Funnel PIO	1.51e5	3.64e-7	2.94e-9	1.27e1	1.36e1	3.71e1
Low gain PIO	3.34e5	7.04e-3	1.44e-3	4.49e1	2.57e4	1.48e3
High gain PIO	1.80e6	3.79e-7	4.80e-9	3.55e2	1.46e1	4.56e1

[1] Unknown input estimation error (estimation of unknown contact force)
[2] State estimation error of the second measurement
[3] State estimation error of the first measurement

4.5 Funnel PI-Observer-based robust control approach of a MIMO mass-spring system

The high-gain PI-Observer as a state and disturbance observer can be applied to design an observer-based nonlinear robust control [BS89]. Due to the simple linear structure of PI-observer, the exact feedback linearization method is the best complementary nonlinear method to be combined with this observer. Therefore, the key point of this section is to combine the advantages of exact feedback linearization control method with those of high-gain PI-Observer. Accordingly, the structure of PIO is taken and the observer gains are chosen using an adaptive adjustment method introduced in section 4.2. The idea is to use the funnel control method [IRS02] as a high-gain adaptive (time-varying) adjustment approach. The advantage of the proposed approach compared to previously published PIO gain design is the self adjustment of the observer gains according to the actual estimation situation. The results of state and disturbance estimation are incorporated into the structure of exact feedback linearization method to improve the closed-loop robustness. The effectiveness of the proposed approach is verified by simulation results of a MIMO mass-spring system. Briefly speaking, the goals and objectives of this section can be summarized as:

- Utilization of funnel control concept as a high-gain adaptive (time-varying) adjustment approach to adaptively design the gain of PI-Observer at each step of integration time and to decrease the influence from measurement noise in combination with high-gain approaches
- Application of input-output feedback linearization approach to linearize the nonlinear MIMO system to be used in combination with the proposed funnel PI-Observer and state feedback control as an observer-based control approach

- Investigation and comparison of the closed-loop robustness in the presence of external disturbances and measurement noise for the introduced funnel PI-Observer-based control approach using a performance/energy criterion

4.5.1 Funnel PI-Observer-based robust control design

An example of a nonlinear MIMO mechanical system, based on the benchmark system [AM99] shown in Figure 4.13, is given to verify the proposed method. According to [LS14] the simulation example is composed of three equal bodies with mass m that slide along the horizontal axis x. The friction force is neglected in the following system model. The first mass is connected to a fixed point at $x = 0$ using a nonlinear elastic spring. The second/third mass is connected to the first/second one with a similar nonlinear elastic spring as the previous one. The control inputs are the external forces $u_1(t)$ and $u_2(t)$ affecting the first and second mass, respectively. The

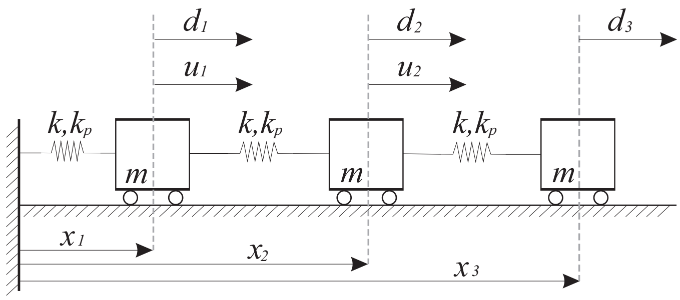


Figure 4.13: Nonlinear MIMO mechanical system example

system equations are

$$
\begin{aligned}
m\ddot{x}_1 &= k(-2x_1 + x_2) + k_p[-x_1^3 + (x_2 - x_1)^3] + u_1 + d_1, \\
m\ddot{x}_2 &= k(x_1 - 2x_2 + x_3) + k_p[(x_3 - x_2)^3 - (x_2 - x_1)^3] + u_2 + d_2, \\
m\ddot{x}_3 &= k(x_2 - x_3) + k_p(x_2 - x_3)^3 + d_3,
\end{aligned}
$$

$$
\begin{aligned}
y_{meas} &= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T, \text{ and} \\
y_{contr} &= \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T = \begin{bmatrix} x_1 & x_3 \end{bmatrix}^T.
\end{aligned} \tag{4.18}
$$

The parameters used in simulation are $m = 0.5\ kg$, $k = 217.0\ N/m$, and $kp = 63.5\ N/m^3$. The dynamics of the disturbances in the inputs d_1, d_2, d_3 are assumed as unknown to the control design but present in the simulation as $d_1 = 5$,

$d_2 = 10\sin(5t)$, and $d_3 = 20\sin(10t)$. The system can be linearized by input-output linearization approach as

$$\begin{aligned} \ddot{y}_1 &= v_1 + \frac{d_1}{m} = v_1 + \bar{\eta}_1, \\ {y_2}^{(4)} &= v_2 + [\frac{k}{m} + \frac{3k_p}{m}(x_2 - x_3)^2](\frac{d_2}{m} - \frac{d_3}{m}) = v_2 + \bar{\eta}_2, \end{aligned} \tag{4.19}$$

if the inputs are chosen as

$$\begin{aligned} &u_1 = m\left[v_1 - \tfrac{k}{m}(-2x_1 + x_2) - \tfrac{k_p}{m}\left[-x_1^3 + (x_2 - x_1)^3\right]\right], \\ &u_2 = \tfrac{m}{\frac{k}{m} + \frac{3k_p}{m}(x_2 - x_3)^2} \times \{v_2 - \left[\tfrac{k}{m} + \tfrac{3k_p}{m}(x_2 - x_3)^2\right] \times \\ &\left[\tfrac{k}{m}(x_1 - 3x_2 + 2x_3) + \tfrac{k_p}{m}\left[2(x_3 - x_2)^3 - (x_2 - x_1)^3\right]\right] - 6\tfrac{k_p}{m}(x_2 - x_3)(\dot{x}_2 - \dot{x}_3)^2\}. \end{aligned} \tag{4.20}$$

The remaining zero/internal dynamic

$$\ddot{x}_2 = \tfrac{1}{m}[k(x_1 - 2x_2 + x_3) + k_p((x_3 - x_2)^3 - (x_2 - x_1)^3) + u_2 + d_2],$$

is stable, if the disturbance d_2 is bounded. Two funnel PI-Observers are designed for the transformed decoupled dynamics (4.19) (FPIO1 and FPIO2 respectively)

$$\begin{aligned} \dot{z}_a &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_a + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{\bar{\eta}}_1 + L_{1a}(y_1 - \hat{y}_1), \\ \dot{\hat{\bar{\eta}}}_1 &= L_{2a}(y_1 - \hat{y}_1), \\ \hat{y}_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} z_a, \end{aligned} \tag{4.21}$$

and

$$\begin{aligned} \dot{z}_b &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} z_b + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \hat{\bar{\eta}}_2 + L_{1b}(y_2 - \hat{y}_2), \\ \dot{\hat{\bar{\eta}}}_2 &= L_{2b}(y_2 - \hat{y}_2), \\ \hat{y}_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} z_b, \end{aligned} \tag{4.22}$$

with the state vectors $z_a = \begin{bmatrix} \hat{y}_1 \\ \dot{\hat{y}}_1 \end{bmatrix}$ and $z_b = \begin{bmatrix} \hat{y}_2 \\ \dot{\hat{y}}_2 \\ \ddot{\hat{y}}_2 \\ \hat{y}_2^{(3)} \end{bmatrix}$, to estimate the transformed states and disturbances, namely $\hat{x}_1$, $\dot{\hat{x}}_1$, $\hat{\bar{\eta}}_1$, $\hat{x}_3$, $\dot{\hat{x}}_3$, $\ddot{\hat{x}}_3$, $\hat{x}_3^{(3)}$, and $\hat{\bar{\eta}}_2$. To construct the inputs in (4.20), besides the displacements x_1, x_2, and x_3 the velocities $\dot{x}_2$ and $\dot{x}_3$ are also required. As a transformed coordinate, the velocity $\dot{x}_3$ can be estimated using the funnel PI-Observer (4.22). To estimate the velocity $\dot{x}_2$, an additional funnel PI-Observer is designed (FPIO3) by

$$\begin{aligned}\dot{z}_c =& \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_c + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_3 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{\bar{\eta}}_3 + L_{1c}(x_2 - \hat{x}_2), \\ \dot{\hat{\bar{\eta}}}_3 =& L_{2c}(x_2 - \hat{x}_2),\end{aligned} \tag{4.23}$$

with the state vector $z_c = \begin{bmatrix} \hat{x}_2 \\ \hat{\dot{x}}_2 \end{bmatrix}$, input
$v_3 = \frac{k}{m}\left[(x_1 - 2x_2 + x_3) + k_p[(x_3 - x_2)^3 - (x_2 - x_1)^3] + u_2\right]$,
and $\bar{\eta}_3 = \frac{d_2}{m}$. Using the estimations of the three funnel PI-Observers mentioned above, the system (4.18) can be transformed into an input-output linearized form with nonlinear feedback (4.20). To realize the robust control, linear control methods can be applied to the linearized model (4.19), for example as linear state feedback control (achieved by pole placement control method)

$$\begin{aligned} v_1 =& \quad -20\hat{\dot{x}}_1 - 100(x_1 - x_{1ref}) - \hat{\bar{\eta}}_1, \\ v_2 =& \quad -200\hat{x}_3^{(3)} - 15000\hat{\ddot{x}}_3 - 500000\hat{\dot{x}}_3 - 6250000(x_3 - x_{3ref}) - \hat{\bar{\eta}}_2. \end{aligned} \tag{4.24}$$

The desired values taken in the simulation are $x_{1ref} = 0.25$ and $x_{3ref} = 0.3$. The additional measurement noise with different levels is added to the measurements x_1, x_2, and x_3. Estimations of the unknown disturbances d_1, d_2, and d_3 are calculated from the estimations $\hat{\bar{\eta}}_1$, $\hat{\bar{\eta}}_2$, and $\hat{\bar{\eta}}_3$. The block diagram of the proposed method is illustrated in Figure 4.14 (Stability proof of the closed-loop system refer to [LS14]).

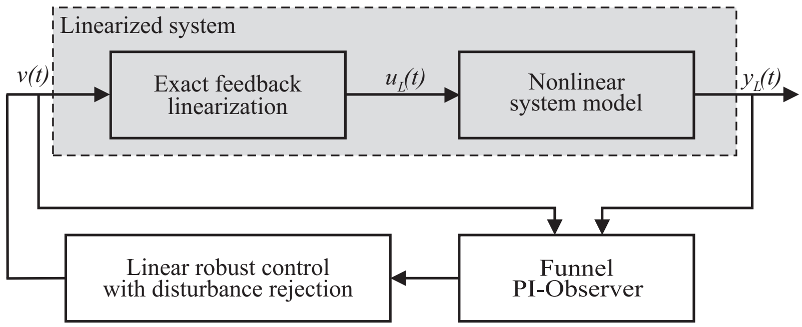


Figure 4.14: Sketch of the proposed closed-loop control approach

Table 4.3: Parameters of adaptive funnel PIO algorithm (closed-loop evaluation)

	min	max	γ	a_1	a_2	a_3
FPIO1	1	10	e4	10	180	0.18
FPIO2	1	2.6	e13	50	180	0.05
FPIO3	1	10	e7	10	180	0.01

4.5.2 Simulation results and discussion

As clarified, in this contribution the funnel idea is utilized to adaptively adjust the PIO gain. It means that the structure of funnel PI-Observer is the same as PI-Observer and the gain designing procedure is different as explained in the previous sections. The funnel function considered for designing of three mentioned PI-Observer structures is supposed as

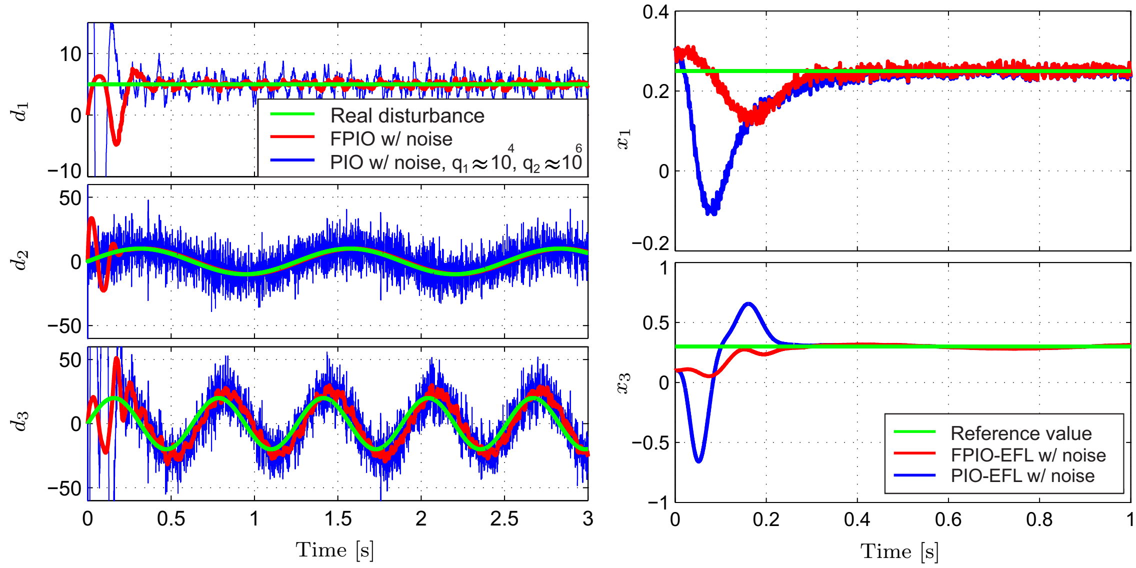
$$\varphi(t) = \frac{1}{(a_1 e^{-a_2 t^2} + a_3)^2}, \tag{4.25}$$

with constant design parameters a_1, a_2, and a_3 according to Table 4.3 for the three funnel PIO. In Figure 4.15 comparison of the estimated disturbances using high gain PIO and funnel PIO is given to show at a glance the difference of approaches: adaption of the high gain PIO introduced as funnel PIO leads to better disturbance estimation than high gain PIO (less influence from measurement noise).

To examine the performance with respect to sensitivity to noise or model uncertainties, two representative control results (system outputs) are illustrated in Figure 4.15 for different control approaches (PIO-EFL and FPIO-EFL). From the results it can be concluded that FPIO-based exact feedback linearization approach shows better performance and less tracking error compared to the other illustrated approach. However judging only based on the output performance (position of the first and third masses) is not comprehensive. Therefore, to perform the comparison of different controllers comprehensively the criterion

$$C_{criteria} = [\textstyle\int_0^T e^2(t)dt, \int_0^T u^2(t)dt], \tag{4.26}$$

with the relation between input energy $\int_0^T u^2(t)dt$ and control error $\int_0^T e^2(t)dt$ is used. The principle relation is illustrated in Figure 4.16. The interval length T denotes the time window where the performance is compared. With the same input energy, the result which has lower output error or correspondingly with the same output error which uses less input energy (closer to the origin) has better performance. In Figure 4.16 a comparison of different control approaches in the presence of additional measurement noise and by considering two state feedback controllers (achieved by pole placement control method) is represented. As illustrated, FPIO-EFL has always the best tracking performance compared to PIO-EFL. It is worth



(a) Estimation of the disturbances

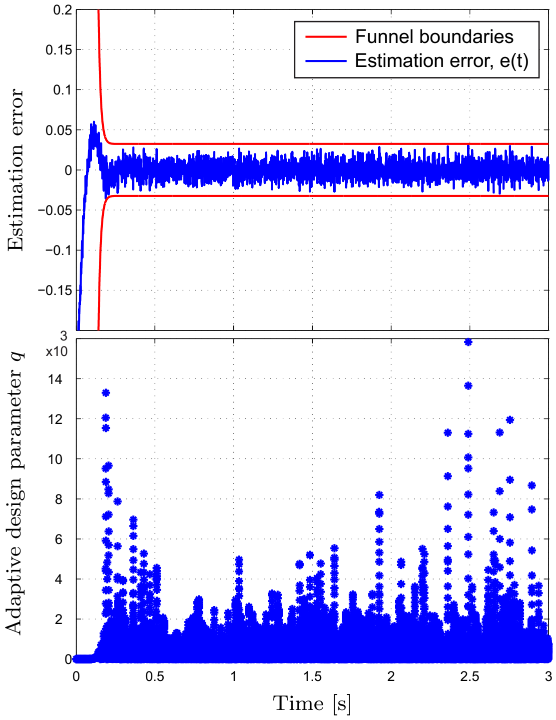
(b) Comparison of control results, system outputs

Figure 4.15: Illustration of effects resulting from added noise (to measurements)

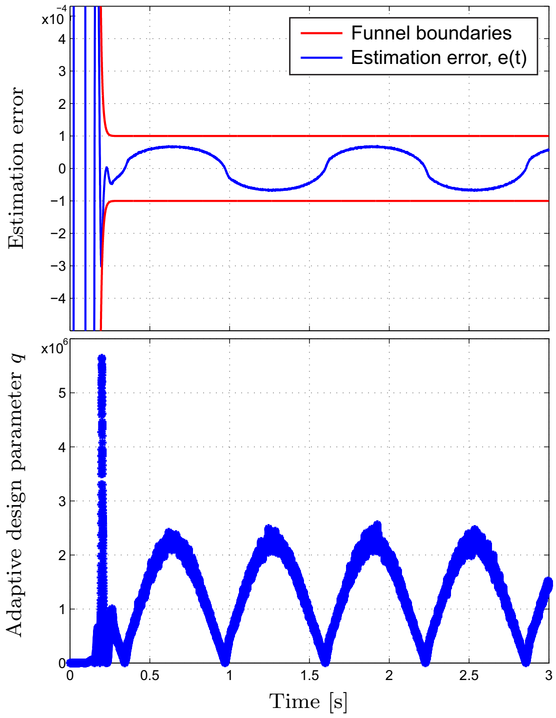
mentioning that when robustness is taking into consideration, the proposed FPIO-EFL has convenient and better tracking performance compared to the PIO-EFL regarding to the performance variation in the presence of additional measurement noise.

4.6 Summary and discussion

In this chapter, an adaptive estimation approach is proposed and applied for the task of contact force estimation (with fast dynamics) of an elastic beam. The proposed funnel PIO approach takes advantage of the funnel idea to adjust the PIO gains according to the actual situation and to maintain the estimation error in a prescribed funnel area. The introduced approach shows significant advantages with respect to estimation of system states and unknown inputs in the presence of measurement noise which is a considerable property for practical application of high gain observers. Both simulation and experimental results of an elastic beam verify and validate the advantages of the introduced funnel PIO compared to the known high gain PIO. The stability of the proposed adaptive algorithm with respect to switching funnel PI-Observers is discussed based on Lyapunov theory.
Furthermore, in this chapter, an observer-based robust control approach is proposed



(a) Funnel adaptive gain adjustment of PI-Observer for output y_1



(b) Funnel adaptive gain adjustment of PI-Observer for output y_2

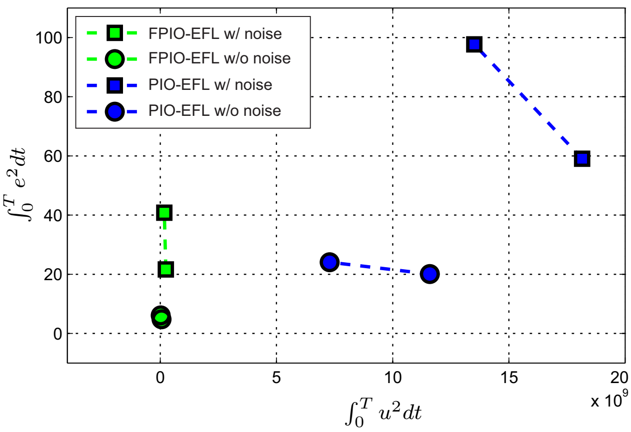


Figure 4.16: Comparison of different control methods by means of performance criterion, with consideration of measurement noise

for an input-output linearizable nonlinear system with unknown inputs (e.g., disturbances and model uncertainties). The system and unknown input estimations achieved by funnel PI-Observer is used together with the system measurements to realize the exact feedback linearization (EFL) approach. A robust disturbance rejection control is realized by using state feedback of the linearized model and estimations of the unknown inputs. The new introduced approach FPIO-EFL shows significant advantages compared to PIO-EFL with respect to estimation of system states and unknown inputs in the presence of measurement noise and integration of the estimation results in the structure of EFL approach. Simulation results of a mechanical MIMO system verify the advantages and effects of the introduced funnel PIO-based robust control method.

5 PI-Observer-based robust nonlinear control design: establishing new approaches

Motion control of power trains, drive trains, or even actuators has been in the focus of several scientific and industrial efforts of the last decade. Advanced control approaches are based on knowledge (models resp.) about the system to be controlled, realized by mathematical models (e.g. sets of differential equations).

In this chapter the introduced high-gain PI-Observer (chapter 3.1) is applied in the experimental context. The experimental application of PI-Observer approach is performed to estimate the system nonlinearities and uncertainties and to integrate the estimation results into the structure of Sliding Mode Control (SMC) and backstepping control (BC) approaches. These novel approaches are experimentally evaluated using a hydraulic differential cylinder test rig. Briefly speaking, the goals and objectives of this chapter to control a hydraulic differential cylinder test rig can be summarized as:

- Investigation and implementation of novel PI-Observer-based Sliding mode control and PI-Observer-based backsteppin control approaches
- Stability proof considering the convergence of controller position tracking error and unknown input observation error simultaneously
- Design and selection of SMC and BC parameters by defining and elaborating a performance/energy criterion
- Investigation of closed-loop robustness against modeling errors and external disturbances by using an improved implementation environment
- Comparison and discussion of experimental results of hydraulic differential cylinder test rig by considering the proposed approaches (in both approaches PI-Observer is used to estimate the uncertainties)
- Enhancement of disturbance attenuation and system performance robustness using the proposed combination of linear observer and nonlinear robust controllers

The contents, figures, and tables presented in this chapter are prepared for publication in the journal papers "Robust control of a hydraulic cylinder using an observer-based sliding mode control: theoretical development and experimental validation" [BS17e] and "Proportional-Integral-Observer-based backstepping approach for position control of a hydraulic differential cylinder system with model uncertainties and disturbances" [BS17b], and is published in the conference paper "Robust control approach for a hydraulic differential cylinder system using a Proportional-Integral-Observer-based backstepping controller [BS17d]."

5.1 Structure of considered class of nonlinear systems

The general class of system with nonlinear relation between the input variables (initial conditions or external inputs) and output or state variables, is called nonlinear system. Common description of nonlinear systems are by (nonlinear) state equations, differential equations, or difference equations. Throughout this chapter, the discussion is restricted to the class of nonlinear systems which are linear with respect to the manipulated input (control-affine system). The considered class of nonlinear systems with unknown inputs, called also continuous time smooth nonlinear system, is described by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \underbrace{\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}(t)}_{\text{Nominal model}} + \boldsymbol{E}\boldsymbol{d}(\boldsymbol{x}, t), \\ \boldsymbol{y}(t) &= \boldsymbol{h}(\boldsymbol{x}),\end{aligned} \tag{5.1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ denotes the state vector, $\boldsymbol{u}(t) \in \mathbb{R}^l$ the input vector, $\boldsymbol{y}(t) \in \mathbb{R}^m$ the output to be controlled. The vector $\boldsymbol{d}(\boldsymbol{x}, t) \in \mathbb{R}^s$ with $s \leq n$ together with the constant matrix $\boldsymbol{E} \in \mathbb{R}^{n \times s}$ represents the unknown inputs. Disturbances, modeling errors, parameter uncertainties, or other uncertainties to the nominal model can be summarized under $\boldsymbol{E}\boldsymbol{d}(\boldsymbol{x}, t)$, possibly in all the dynamical equations (when the rank of $\boldsymbol{E}$ equals n).

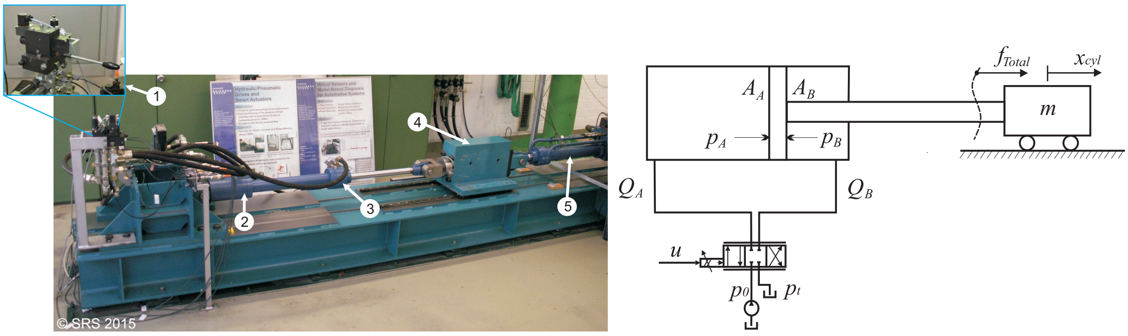
The objective is to design a controller that stabilizes the system behavior (5.1) or in other words, to realize stable tracking and regulation control in the presence of disturbances $\boldsymbol{d}(\boldsymbol{x}, t)$. In the following several assumptions are considered regarding the system structure (5.1).

- The vector fields $\boldsymbol{f}(\cdot) \in \mathbb{R}^n$, $\boldsymbol{d}(\cdot) \in \mathbb{R}^s$, $\boldsymbol{g}(\cdot) \in \mathbb{R}^{n \times l}$, and $\boldsymbol{h}(\cdot) \in \mathbb{R}^m$ are smooth[1].
- The system has an equilibrium at point $\boldsymbol{x} = 0$.
- The unknown inputs $\boldsymbol{d}(\boldsymbol{x}, t)$ and the corresponding derivatives are bounded, but the bounds and the related dynamical behavior are unknown.
- The nominal model of the system is available and is input-output linearizable. Besides, the internal dynamics are stable.
- In MIMO cases, the number of inputs is equal to the number of outputs, namely $l = m$ (in this chapter just a SISO system is considered).

[1] A smooth function is a function that has continuous derivatives up to desired order over some domain.

5.2 Model of a hydraulic differential cylinder system

Hydraulic cylinders are actuators with strongly nonlinear behavior. They are widely used in several industrial areas, such as heavy machines, cranes, robots, etc. The dynamical behavior of hydraulic differential cylinders can be described by a coupled set of nonlinear differential equations [JK12]. A model of a hydraulic differential cylinder with a proportional control valve as shown in Figure 5.1 [JK12] is given by



(a) Test rig of hydraulic differential cylinder system at the Chair of Dynamics and Control (UDuE), 1. directional control valve, 2. oil supply in chamber A, 3. oil supply in chamber B, 4. moving mass, and 5. load cylinder

(b) Sketch of the hydraulic differential cylinder system

Figure 5.1: Hydraulic differential cylinder system

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} x_2 \\ \dfrac{1}{m(x_1)}\left[(x_3 - \dfrac{x_4}{\varphi})A_A\right] \\ \dfrac{E_{oil}(x_3)}{V_A(x_1)}(-A_A x_2) \\ \dfrac{E_{oil}(x_4)}{V_B(x_1)}\left(\dfrac{A_A}{\varphi}x_2\right) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{E_{oil}(x_3)}{V_A(x_1)}Q_A(x_3) \\ \dfrac{E_{oil}(x_4)}{V_B(x_1)}Q_B(x_4) \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ d(t) \\ 0 \\ 0 \end{bmatrix}, \tag{5.2}$$

$$= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u(t) + \boldsymbol{d}(t),$$

$$y(t) = h(\boldsymbol{x}) = x_1(t),$$

with the variant mass

$$m(x_1) = m_{basic} + \rho_{fl}(V_A(x_1) + V_B(x_1)), \tag{5.3}$$

Table 5.1: Definition of parameters and variables

Variable	Physical meaning	Value (Unit)
$x_1(t) = x_{cyl}(t)$	Displacement of the mass cart	- (m)
$x_2(t) = \dot{x}_{cyl}(t)$	Velocity of the mass cart	- (m/s)
$x_3(t) = p_A(t)$	Pressure in chamber A	- (pa)
$x_4(t) = p_B(t)$	Pressure in chamber B	- (pa)
$f_d(t)$	External force acting on the piston	- (N)
m_{basic}	Basic mass of the cart	279.6 (kg)
ρ_{fl}	Density of the hydraulic oil	870 (kg/mm^3)
p_0	Supply pressure	8×10^6 (pa)
p_t	Tank pressure	5×10^5 (pa)
A_A	Cylinder piston area	3117.2 (mm^2)
A_B	Cylinder ring area	1526.8 (mm^2)
$\varphi = \dfrac{A_A}{A_B}$	Area ratio	2.042 (-)
$E_{oil,max}$	Max. bulk modulus of elasticity	1.8×10^9 (pa)
p_{max}	Max. supply pressure	2.8 $\times10^7$ (pa)
V_{cA}	Pipeline and dead volume (A)	198.6 (cm^3)
V_{cB}	Pipeline and dead volume (B)	297.8 (cm^3)
H	Stroke of the cylinder	0.5 (m)
I_{max}	Max. input current	0.63 (A)
Q_N	Nominal valve flow	85 (L/min)
Δp_N	Pressure drop of valve	9×10^5 (pa)

the volumes V_A, V_B in chambers A and B as

$$V_A(x_1(t)) = V_{cA} + x_1(t)A_A,$$
$$V_B(x_1(t)) = V_{cB} + (H - x_1(t))A_B,\ 0 \le x_1(t) \le H,$$

the disturbance $d(t)$ as

$$d(t) = \frac{f_{Total}(t)}{m(x_1)}, \tag{5.4}$$

the hydraulic flows

$$Q_A(x_3(t)) = \begin{cases} B_\nu sgn(p_0 - x_3(t))\sqrt{\mid p_0 - x_3(t) \mid}, & u \ge 0 \\ B_\nu sgn(x_3(t) - p_t)\sqrt{\mid x_3(t) - p_t \mid}, & u < 0 \end{cases},$$
$$Q_B(x_4(t)) = \begin{cases} -B_\nu sgn(x_4(t) - p_t)\sqrt{\mid x_4(t) - p_t \mid}, & u \ge 0 \\ -B_\nu sgn(p_0 - x_4(t))\sqrt{\mid p_0 - x_4(t) \mid}, & u < 0 \end{cases},$$

with

$$B_\nu = \frac{Q_N}{\sqrt{0.5\Delta p_N}}, \tag{5.5}$$

and the bulk modulus of elasticity

$$E_{oil}(p) = \frac{1}{2}E_{oil,max}\log_{10}(90\frac{p}{p_{max}} + 3).$$

The input $u(t)$ is the electrical current which is limited as $-I_{max} \leq u(t) \leq I_{max}$. The flow characteristic of the valve is assumed to be proportional and no internal and external leakage effects are considered. The friction of spool, piston, and cart are neglected in the modeling of cylinder valve. The variables and constants are defined in Table 5.1.

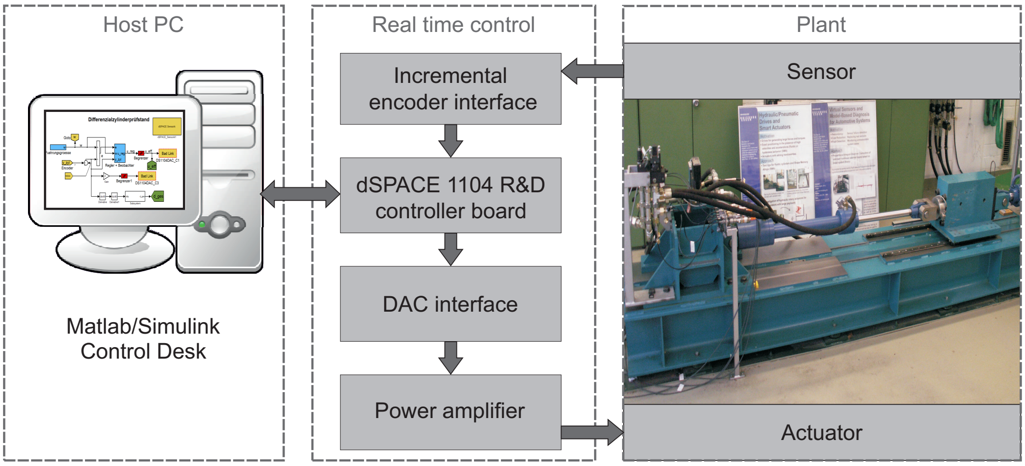
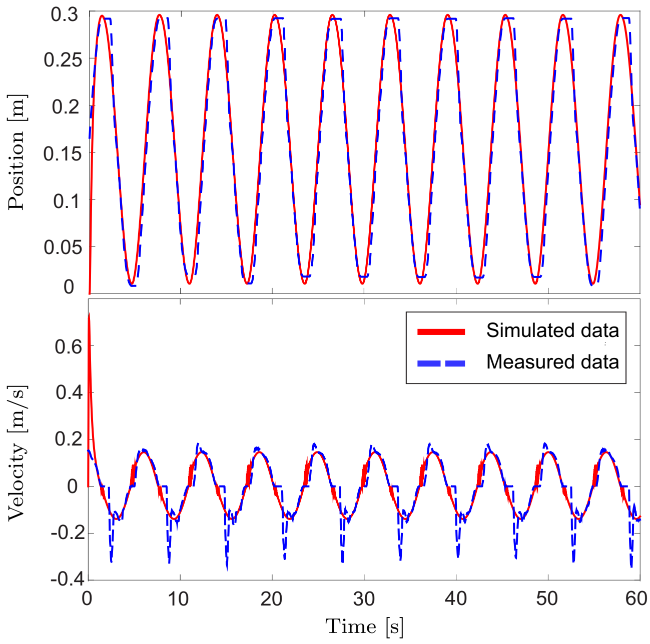


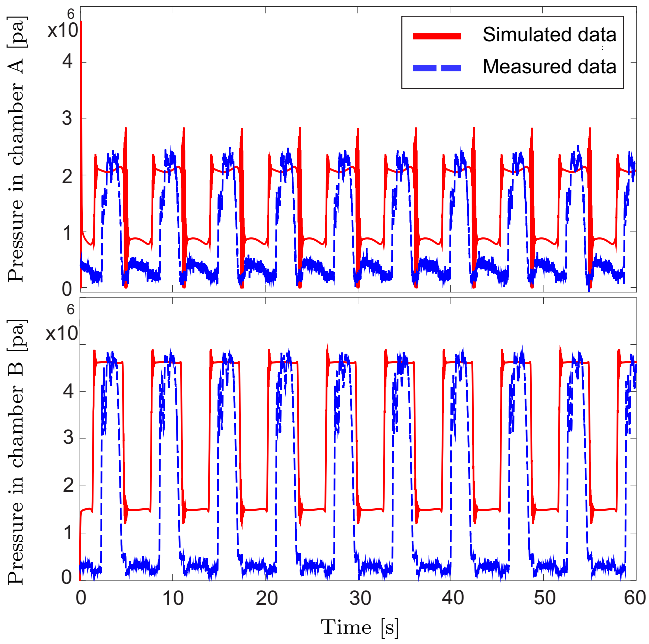
Figure 5.2: Block diagram of experimental setup

5.2.1 Model Verification

Before using the introduced model in the procedure of observer/controller design for the hydraulic differential test rig, the introduced nonlinear system model has to be validated to verify the reliability of the model. Afterward, the simulated model can be used to verify the control and observer designs before implementation in real time on the test rig. As a key component of model-based approaches, simulation of system model enables the designer to find the design errors, validate requirements, and verify the control system performance before implementation. This procedure is practical in the development proceeding through the simulation. Therefore, in



(a) Position and velocity of the cylinder



(b) Pressure in chambers A and B

Figure 5.3: Comparison of simulated and measured data for 60 seconds

this section the results of simulated model are compared with the results of real experimental implementation from the hydraulic differential cylinder test rig. Afterward, the considered model is used to verify the control/observer design before implementation on the real system. To implement the different control methods in real time, a compact system is designed as shown in Figure 5.2.
To validate the system model, a simple P-Controller is considered in the simulation and experimental cases. The experimental and simulation results carried out using regular working conditions. The reference signal is considered as a sinusoidal signal with the same frequency, amplitude, and bias for the both simulation and experiment. The comparison between simulation and experimental results is illustrated in Figure 5.3. In this figure it is obvious that in the moment of changing direction of the cylinder, there is an unexpected peak in the velocity results of experimental test. This unexpected behavior can be considered as effect of friction force as an unknown input or an external disturbance.

5.3 PIO-based sliding mode controller

Due to the nonlinear valve geometry as well as the nonlinear temperature and pressure depending viscosity of the fluid, hydraulic drives are categorized as strong nonlinear systems [JK12]. Typical control approaches in this field are based on Proportional-Integral SISO control. Reviews e.g. [JK12] state the dominance of these kind of approaches. On the other hand, robust nonlinear controllers for hydraulic systems are widely discussed because of the strong nonlinear electromechanical behavior with a large extent of model uncertainties. These uncertainties can be categorized as parametric uncertainties (e.g. large variations in load and hydraulic parameters such as bulk modulus due to component wear or temperature change) or uncertain nonlinearities (e.g. external disturbance, leakage, and friction). Due to the above mentioned facts, in [SLF13] SMC is recommended for controlling the hydraulic systems because once the state of system reach the predefined sliding surface, thereafter the system performance is not dependent on the system parameters, disturbances, or other nonlinearities.

Sliding Mode Control (SMC) approach is a particular approach in the field of robust control design which has been widely employed because of its ability to eliminate disturbances and to handle effects of uncertainties. The SMC approach has been studied for over 40 years [Itk76, Utk78, ES98] because of its applicability to complex high-order nonlinear dynamic plants under uncertain conditions. The SMC is well-known because of its low sensitivity to unknown disturbances and variations of plant parameters. It is worth mentioning that the exogenous perturbations that can be compensated by the traditional SMC have to satisfy the so-called matching condition [Dra69] means acting in the same channel as the system input. The SMC approach is a specific type of Variable Structure Control Systems (VSCS) combining feedback control laws and decision rules. It contains two main parts: (a) design of the switching function also using measurements of the system to be controlled and (b) selection of the control law which is not necessarily discontinuous.

Using unknown input observers like disturbance observers to estimate the effects in real-time and integration of estimation results into the SMC structure is highly regarded [YLY13, GSP13] because (a) the SMC approach frequently relies on the availability of the state measurements [IS97] and (b) even though the traditional SMC approach is invariant to the class of bounded matched uncertainties, in the presence of large extent of uncertainties in practical applications, implementation of traditional SMC is challenging due to the limited bandwidth actuators [Wel02]. In [ZKZL13] a sliding mode controller is proposed for time-delay systems affected by system nonlinearities and stochastic perturbation. Asymptotic stability of the overall closed-loop system besides the disturbance attenuation strategy is proposed using LMI approach. In [BGK09] a high gain sliding mode observer is designed to reconstruct the states of the system. The estimation results are considered for designing of a sliding mode controller for a class of mismatched uncertain systems. In [WYL15] the disturbance, considered as the effects of parameter uncertainties and external interferences, is modeled as a kind of unknown derivative-bounded disturbance. A nonlinear disturbance observer is used to estimate the modeled disturbances integrated to the SMC structure.

It is worth noting that most of the studies on SMC concentrate on the matched uncertainties condition. So the disturbances or uncertainties occurs in the same channel as the system input. Since the conventional SMC approach is inefficient in the presence of mismatched uncertainties, newer studies focus on the mismatched uncertainties in SMC approach [YLY13, WC08, ZSX10, Cho07]. In [WC08] an adaptive SMC is proposed for stabilizing a class of dynamic systems with matched/mismatched uncertainties using an adaptive mechanism embedded in the sliding surface function design. In [ZSX10] an adaptive SMC is proposed via a convex optimization technique for a fuzzy system affected by mismatched disturbance. In [Cho07] Integral SMC is combined with linear matrix inequality method to dominate the mismatched norm bounded uncertainties in the state matrix as well as the input matrix. Integration of uncertainties estimation into the structure of SMC has been proposed in [YLY13]. In [GSP13] the approach has been extended by combining the SMC with Disturbance Observer (DO). This combination provides the possibility to reduce the magnitude of discontinuous component in the control law and thereby the chattering problem as detailed in [GSP13].

In the following part a combination of the well-known SMC method with a linear high-gain Proportional-Integral-Observer (PIO), able to estimate both system states and unknown effects, is discussed to improve the tracking performance of the closed-loop system. The key property of the new approach is that neither direct measurements of the disturbances are needed, nor measurements of the system states are used. Modeling errors and disturbances are estimated as unknown inputs together with the system states using a high-gain PIO [LS12], which solves the problems mentioned above. Thereupon the results are integrated into the structure of sliding surface design to eliminate the effect of uncertainties. The block diagram of the proposed PIO-SMC is illustrated in Figure 5.4. In the following the robust nonlinear control based on combination of PIO and SMC is performed and the combination is explained for the considered motion/position control task. The system model is linearized using input-output linearization method. Furthermore, PIO is used to estimate disturbances and model uncertainties such as friction force, mass acceleration forces, and modeling errors for the motion control task. The estimation is integrated into the SMC structure to realize the robust motion control of hydraulic differential cylinder system.

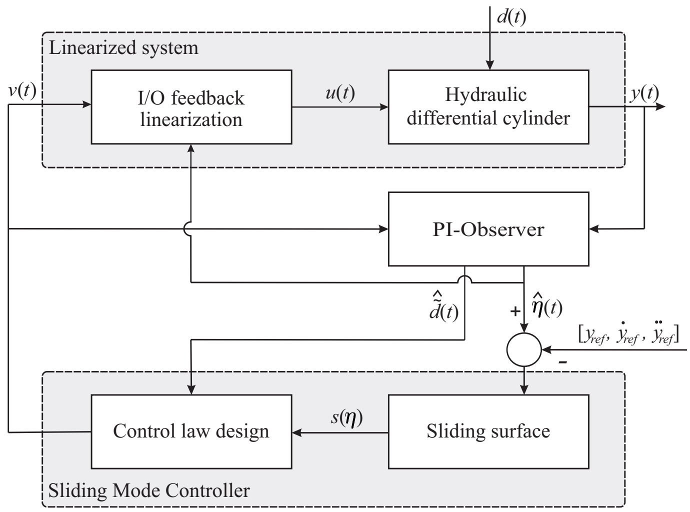


Figure 5.4: Block diagram of the proposed PIO-SMC method

5.3.1 Linearization of hydraulic cylinder model

The main idea of input-output linearization approach is to transform the nonlinear system dynamics to a fully or partly linear one. Afterward, linear control or observer techniques can be applied to the linear dynamical model of the system. Therefore, a nonlinear transformation has to be performed to express the system model in a new coordinates as a linear model. To find the mentioned nonlinear transformation first of all the derivative of output signal is calculated so many times until an explicit relation between the input signal u and the output signal y is achieved. The number of differentiations to achieve this explicit relation is called system relative degree. If the relative degree of the system is equal to the system order it is called exact input-output linearization approach. The nonlinear model of the hydraulic differential

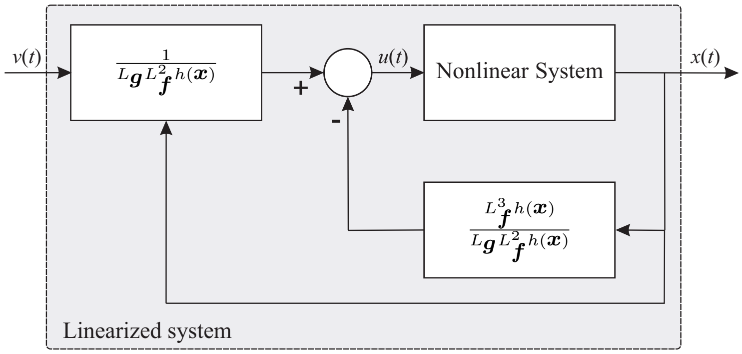


Figure 5.5: Input-output linearization of the nonlinear system model

cylinder with proportional valve is linearized by input-output linearization (Figure 5.5). The system (5.2) has one input $u(t) = x_{v,effect}(t)$ and one output to be controlled $y(t) = x_1(t)$. For the nonlinear model (5.2), the input-output linearized model is described by

$$\dddot{y}(t) = v(t) + \tilde{d}(t), \tag{5.6}$$

with

$$\begin{aligned} v(t) &= L^3_{\boldsymbol{f}} h(\boldsymbol{x}) + L_{\boldsymbol{g}} L^2_{\boldsymbol{f}} h(\boldsymbol{x}) u(t), \\ L^3_{\boldsymbol{f}} h(\boldsymbol{x}) &= \frac{A_A x_2}{\varphi^2 m^2(x_1) V_A(x_1) V_B(x_1)} \\ &\quad [\rho_{fl}(A_B - A_A)(x_3 \varphi^2 V_A(x_1) V_B(x_1) - x_4 \varphi V_A(x_1) V_B(x_1)) \\ &\quad - A_A \varphi^2 m(x_1) V_B(x_1) E_{oil}(x_3) - A_A m(x_1) V_A(x_1) E_{oil}(x_4)], \\ L_{\boldsymbol{g}} L^2_{\boldsymbol{f}} h(\boldsymbol{x}) &= \frac{A_A}{\varphi m(x_1) V_A(x_1) V_B(x_1)} \\ &\quad [\varphi V_B(x_1) E_{oil}(x_3) Q_B(x_3) - V_A(x_1) E_{oil}(x_4) Q_A(x_4)], \end{aligned} \tag{5.7}$$

and

$$\tilde{d}(t) = L_{\boldsymbol{d}} L^2_{\boldsymbol{f}} h(\boldsymbol{x}) + \frac{d^2}{dt} L_{\boldsymbol{d}} h(\boldsymbol{x}) + \frac{d}{dt} L_{\boldsymbol{d}} L_{\boldsymbol{f}} h(\boldsymbol{x}) = \dot{d}(t), \tag{5.8}$$

where $L_{\boldsymbol{f}}(\cdot)$, $L_{\boldsymbol{g}}(\cdot)$, and $L_{\boldsymbol{d}}(\cdot)$ denote the Lie derivatives[2] and $\boldsymbol{f}$, $\boldsymbol{g}$, $\boldsymbol{d}$, and h are defined according to (5.2). The nonlinear system model also has to be analyzed with respect to the system dynamics. The zero dynamics of the system (5.2) can be written by

$$\dot{z}(t) = -\frac{m(x_1 = 0)}{A_A} \tilde{d}(t), \tag{5.9}$$

[2]Here $L^i_{\boldsymbol{\mathcal{G}}} \boldsymbol{\mathcal{H}}$ denotes the $i-$th Lie derivative of $\boldsymbol{\mathcal{H}}$ with respect to $\boldsymbol{\mathcal{G}}$, where the Lie derivatives are defined by $L^0_{\boldsymbol{\mathcal{G}}} \boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}$, $L^i_{\boldsymbol{\mathcal{G}}} \boldsymbol{\mathcal{H}} = L_{\boldsymbol{\mathcal{G}}} L^{i-1}_{\boldsymbol{\mathcal{G}}} \boldsymbol{\mathcal{H}} = \nabla (L^{i-1}_{\boldsymbol{\mathcal{G}}} \boldsymbol{\mathcal{H}}) \boldsymbol{\mathcal{G}} (i = 1, 2, \ldots)$ with $\nabla \boldsymbol{\mathcal{G}} \triangleq \frac{\partial \boldsymbol{\mathcal{G}}}{\partial \boldsymbol{x}}$ and $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}(\boldsymbol{x})$ and $\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{G}}(\boldsymbol{x})$ are functions of $\boldsymbol{x}$.

with $z(t) = x_3(t) - \dfrac{x_4(t)}{\varphi}$. To guarantee the zero output the input $u^*(t)$ can be considered as

$$u^*(t) = \frac{-L^3_{\boldsymbol{f}}h(\boldsymbol{x}) - \tilde{d}(t)}{L_{\boldsymbol{g}}L^2_{\boldsymbol{f}}h(\boldsymbol{x})}. \tag{5.10}$$

Assuming $\tilde{d}(t)$ and error $||\tilde{d}(t) - \hat{\tilde{d}}(t)||$ as bounded (the first one is one of the usual assumptions in this field and the second one can be achieved according to the stability proof of PI-Observer) it can be concluded that the zero dynamics stay bounded (the disturbance in the original coordinates $d(t)$ is assumed with smooth variation rate to be differentiable). The linearized dynamics (5.6) is described as

$$\begin{aligned}\dot{\boldsymbol{\eta}}(t) &= \begin{bmatrix}\dot{y}(t)\\ \ddot{y}(t)\\ \dddot{y}(t)\end{bmatrix} = \begin{bmatrix}\eta_2(t)\\ \eta_3(t)\\ L^3_{\boldsymbol{f}}h(\boldsymbol{x})\end{bmatrix} + \begin{bmatrix}0\\ 0\\ L_{\boldsymbol{g}}L^2_{\boldsymbol{f}}h(\boldsymbol{x})\end{bmatrix}u(t) + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\tilde{d}(t)\\ &= \tilde{\boldsymbol{\alpha}}(\boldsymbol{\eta}) + \tilde{\boldsymbol{\beta}}(\boldsymbol{\eta})u(t) + \tilde{\boldsymbol{N}}\tilde{d}(t).\end{aligned} \tag{5.11}$$

5.3.2 PI-Observer design for the linearized model of hydraulic differential cylinder system

The linear PI-Observer can be used for the state and unknown input estimation of linearized system (5.11). The linear system model (5.11) can be rewrite as

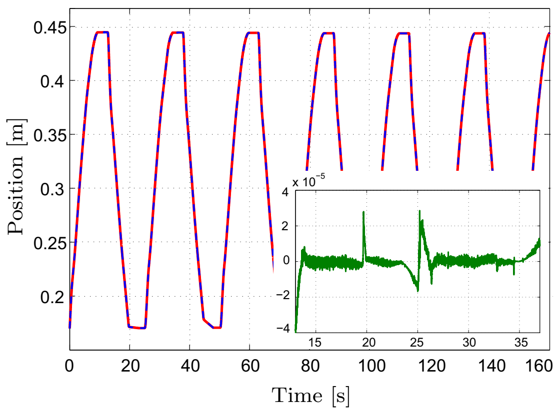
$$\dot{\boldsymbol{\eta}}(t) = \begin{bmatrix}\dot{y}(t)\\ \ddot{y}(t)\\ \dddot{y}(t)\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}}_{\mathbf{A}}\boldsymbol{\eta}(t) + \underbrace{\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}}_{\mathbf{B}}v(t) + \underbrace{\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}}_{\mathbf{N}}\tilde{d}(t), \tag{5.12}$$

with

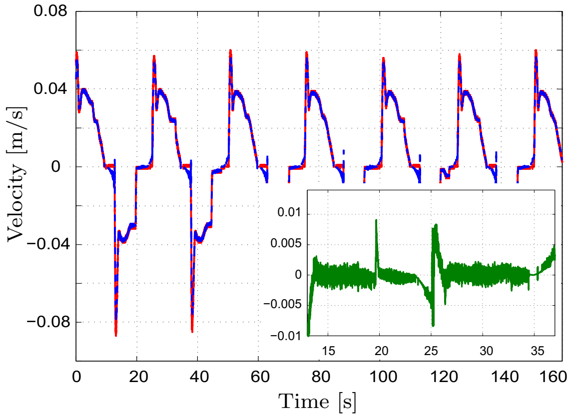
$$v(t) = L^3_{\boldsymbol{f}}h(\boldsymbol{x}) + L_{\boldsymbol{g}}L^2_{\boldsymbol{f}}h(\boldsymbol{x})u(t). \tag{5.13}$$

A linear PI-Observer can be designed for linearized system model (5.12) as

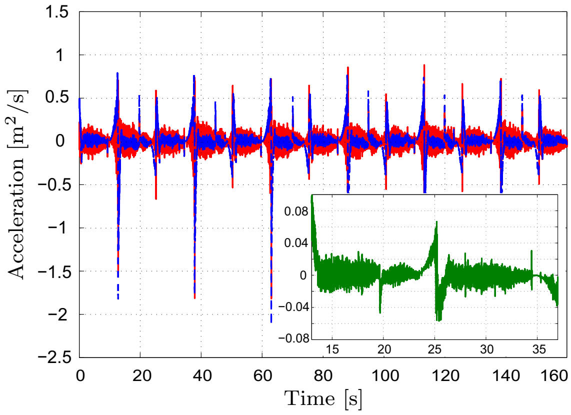
$$\begin{aligned}\dot{\hat{\boldsymbol{\eta}}}(t) &= \boldsymbol{A}\hat{\boldsymbol{\eta}}(t) + \boldsymbol{B}v(t) + \boldsymbol{N}\hat{\tilde{d}}(t) + \boldsymbol{L}_1(y(t) - \hat{y}(t)),\\ \dot{\hat{\tilde{d}}}(t) &= \boldsymbol{L}_2(y(t) - \hat{y}(t)),\\ \hat{y}(t) &= \begin{bmatrix}1 & 0 & 0\end{bmatrix}\hat{\boldsymbol{\eta}}(t),\end{aligned} \tag{5.14}$$



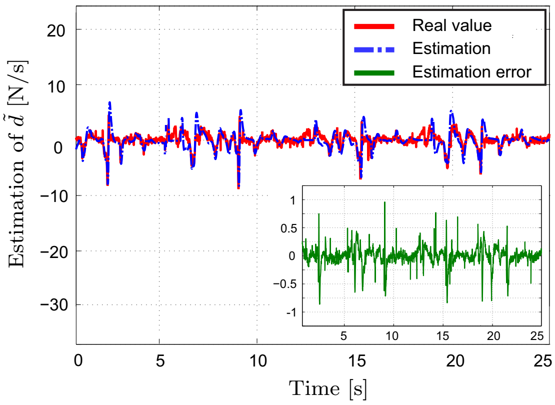
(a) Estimation of cylinder position η_1



(b) Estimation of cylinder velocity η_2



(c) Estimation of cylinder acceleration η_3



(d) Estimation of transformed unknown input $\tilde{d}$

Figure 5.6: Estimation results using linear high gain PI-Observer

with suitable observer gain matrices $\boldsymbol{L}_1$ (proportional observer gain) and $\boldsymbol{L}_2$ (integral observer gain). The transformed states $\boldsymbol{\eta}$ and the disturbance $\tilde{d}(t)$ can be estimated as $\hat{\boldsymbol{\eta}}(t)$ and $\hat{\tilde{d}}(t)$. The states in the original coordinates are available from measurement and estimation of $\hat{\eta}_2(t) = \hat{\dot{x}}_{cyl}(t)$. The derivative of the disturbance in the original coordinate $\dot{d}(t) = \tilde{d}(t)$ is also estimated by (5.14). According to chapter 3, high-gain PI-Observer can be utilized for appropriate estimation of unknown inputs together with the linearized system states. The estimation results using the introduced PI-Observer in (5.14) is illustrated in Figure 5.6. In the following section combination of PI-Observer estimation results with SMC structure is proposed and validated with experimental results.

5.3.3 Sliding mode control design

Sliding mode control is a specific type of variable structure control systems (VSCS) with combination of feedback control law and decision rule. Decision rule part contains of a switching function with some measures of the current system as input and a particular value as output which should be used at that moment for feedback controller. Variable structure control system is used to constrain the system state to the situate in the defined neighborhood of the switching function. Accordingly, sliding mode design approach contains two main part: (a) designing of the switching function and (b) selection of the control law which is not necessarily discontinuous. Finally, the feedback control can switch from one continuous structure to another based on the current position in the state space and it can be considered as a hybrid dynamical system. Implementation of SMC requires more precision in comparison to the other nonlinear control methods because of chattering problem, energy loss, plant damage, and excitation of unmodeled dynamics caused by hard sliding mode action.
As explained, the main idea of SMC is design of a switching control. The main purpose of using switching control is to drive the nonlinear system states on a predefined trajectory and to maintain the states on this trajectory for the subsequent time. This trajectory (called sliding surface or sliding manifold) defines the rule for proper switching conditions. Design of the switching control to drive the system state to the sliding surface can be done using the so-called Lyapunov approach.
A general system state space model can be considered as

$$\dot{x}(t) = f(x, t, u), \tag{5.15}$$

with a sliding surface $s(x) = 0$ and switching gain

$$u = \begin{cases} u^+(x,t), & s(x) > 0 \\ u^-(x,t), & s(x) < 0, \end{cases} \tag{5.16}$$

where $u^+(x,t)$ and $u^-(x,t)$ are continuous functions, with $u^+(x,t) \neq u^-(x,t)$. The switch controller is designed as a discontinuous function of the system state which drives the system state to the sliding surface (see Figure 5.7). It is worth mentioning that SMC is one of the efficient and useful robust controller approaches for complex high-order nonlinear dynamic plant affected by uncertainties and unknown factors. The advantages of using this control approach is its low sensitivity to parameter variations and disturbances that leads to elimination of exact model requirement.
According to [YLY13, CC10], to track the desired trajectory (to force the tracking error $e_t = y - y_{ref}$ to approach the sliding surface) as well as to attenuate the disturbance effect asymptotically, the sliding surface in this work is designed as

$$\begin{aligned} s(\boldsymbol{\eta}) &= c^T(\boldsymbol{\eta} - \boldsymbol{\eta}_{ref}) \\ &= c_1(\eta_1 - y_{ref}) + c_2(\eta_2 - \dot{y}_{ref}) + c_3(\eta_3 - \ddot{y}_{ref}) \\ &= c_1 e_t + c_2 \dot{e}_t + c_3 \ddot{e}_t, \end{aligned} \tag{5.17}$$

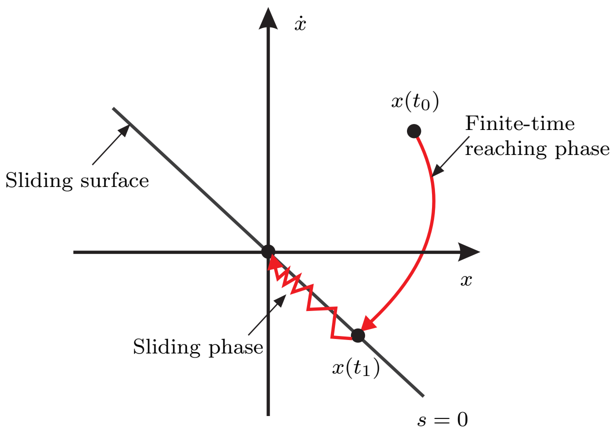


Figure 5.7: Phase trajectory in sliding mode

with sliding surface design parameters c_i $(i = 1, 2, 3)$ to be adjusted. As stated in [Utk78] the design parameters c_i are chosen as positive constant such that the polynomial $p_o(s) = c_3 s^2 + c_2 s + c_1$ is Hurwitz so that asymptotic stability of the sliding motion can be guaranteed. In this contribution according to [Utk78], parameter c_3 is considered as $c_3 = 1$. Here y_{ref} denotes a reference or desired signal has to be tracked by the closed control loop. Based on the sliding surface design introduced in [Utk78] the derivative of the sliding surface $\dot{s}(\boldsymbol{\eta})$ can be calculated as

$$\begin{aligned} \dot{s}(\boldsymbol{\eta}) &= -ksgn(s(\boldsymbol{\eta})) = grad^T s(\boldsymbol{\eta}).\dot{\boldsymbol{\eta}} \\ &= \quad grad^T s(\boldsymbol{\eta}).(\tilde{\boldsymbol{\alpha}}(\boldsymbol{\eta}) + \tilde{\boldsymbol{\beta}}(\boldsymbol{\eta})u(t) + \tilde{\boldsymbol{N}}\hat{\tilde{d}}(t)). \end{aligned} \tag{5.18}$$

The control input $u(t)$ is achieved directly from (5.18) as

$$u(t) = -\frac{c_1\eta_2 + c_2\eta_3 + \hat{\tilde{d}} + k\ sgn(s(\boldsymbol{\eta})) + L^3_{\boldsymbol{f}} h(\boldsymbol{x})}{L_{\boldsymbol{g}} L^2_{\boldsymbol{f}} h(\boldsymbol{x})}, \tag{5.19}$$

with constant design parameters c_1, c_2, and k. The introduced approach requires the estimation of $\tilde{d}$. In this contribution the transformed disturbance $\tilde{d}$ is estimated as $\hat{\tilde{d}}$ using the introduced PI-Observer in section 5.3.2.
To make the control function (5.19) continuous/smooth and to avoid the high-frequency oscillation of the input the discontinuous function $sgn(s(\boldsymbol{\eta}))$ in (5.18) is replaced by

$$sat(\frac{s(\boldsymbol{\eta})}{\sigma}) = \begin{cases} \frac{s(\boldsymbol{\eta})}{\sigma}, & \left|\frac{s(\boldsymbol{\eta})}{\sigma}\right| \leq 1 \\ sign(\frac{s(\boldsymbol{\eta})}{\sigma}), & \left|\frac{s(\boldsymbol{\eta})}{\sigma}\right| > 1, \end{cases} \tag{5.20}$$

in which σ denotes the boundary layer which can be chosen to achieve the smooth control action and the desired control performance simultaneously. In this work σ is considered equal to 0.05.

5.3.4 Stability analysis

In the following part the stability proof of the proposed method is given.

Lemma 1 [SYM95]: Assume that the unknown input $\boldsymbol{d}(\boldsymbol{x}, \boldsymbol{u}, t)$ is bounded. Then there exists a high-gain PI-Observer for system (5.12) such that $\boldsymbol{e}_{est}(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t) \rightarrow 0$ and $f_e(t) = \hat{d} - d \rightarrow 0$ for $t > 0$ for any initial states $\boldsymbol{x}(0)$, $\hat{\boldsymbol{x}}(0)$, and $\hat{\boldsymbol{d}}(0)$ if

- pair $(\boldsymbol{A}, \boldsymbol{C})$ is observable,
- $\text{rank}(\begin{bmatrix} \boldsymbol{A} & \boldsymbol{N} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix}) = n + r$, and
- $\boldsymbol{C}\boldsymbol{A}^i\boldsymbol{N} = 0$ for $i = 0, 1, ..., k-2$, where k is the observability index of pair $(\boldsymbol{A}, \boldsymbol{C})$.

Lemma 2 [LS12]: The feedback matrices $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$ are required to stabilize the extended system described by the matrix $\boldsymbol{A}_e$ and are also required to minimize the influence from the unknown inputs to the estimation errors $\boldsymbol{e}_{est}(t)$ and $f_e(t)$. Therefore the two requirements

- $\text{Re}\{\lambda_i\} < 0$, for all the eigenvalues of matrix $\begin{bmatrix} \boldsymbol{A} - \boldsymbol{L}_1\boldsymbol{C} & \boldsymbol{N} \\ -\boldsymbol{L}_2\boldsymbol{C} & \boldsymbol{0} \end{bmatrix}$ which illustrates the dynamics of the estimation error for the extended system, and
- $\|\boldsymbol{L}_2\|_F \gg \|\boldsymbol{L}_1\|_F$[3]

for the PI-Observer gain matrices design have to be fulfilled.

Lemma 3 [Kha96]: Considering a nonlinear system $\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{w})$ which is input-to-state stable (ISS), if $\lim_{t\rightarrow\infty} \boldsymbol{w}(t) = 0$, then $\lim_{t\rightarrow\infty} \boldsymbol{x}(t) = 0$.

Assumption 1: By using PIO the state and disturbance estimation errors are bounded defined as $\boldsymbol{e}_{est}^* = sup_{t>0}\,|\boldsymbol{e}_{est}(t)|$ with $\boldsymbol{e}_{est}(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$ and ${f_e}^* = sup_{t>0}\,|f_e(t)|$ with $f_e(t) = \hat{\tilde{d}} - \tilde{d}$.

Theorem 1: Suppose that *Assumption 1* is satisfied for the considered linearized system (5.11). If the switching gain k in control law (5.19) is designed as a positive value therefore, the control-loop system with SMC is asymptotically stable.

[3]The norm $\|\cdot\|_F$ denotes here the Frobenius norm, $\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2} = \sqrt{trace(A^*A)}$ for A in $R^{m\times n}$.

Proof 1: Considering a candidate Lyapunov function as

$$V(s) = \tfrac{1}{2}s^2, \tag{5.21}$$

by replacing $\dot{s}$ from (5.18) and by substituting the control law (5.19) the derivative of $V(s)$ is achieved as

$$\begin{aligned} \dot{V}(s) &= s\dot{s} \\ &= s\ (grad^T s(\boldsymbol{\eta}).(\tilde{\boldsymbol{\alpha}}(\boldsymbol{\eta}) + \tilde{\boldsymbol{\beta}}(\boldsymbol{\eta})u(t) + \tilde{\boldsymbol{N}}\tilde{d}(t))) \\ &\leq -|s|\,(k + f_e^*) \\ &\leq -\sqrt{2}(k + f_e^*)V^{1/2}. \end{aligned} \tag{5.22}$$

By designing an appropriate positive switching gain k for the sliding mode controller, it can be derived from (5.22) that the proposed control law can force the system state to reach the defined sliding surface $s = 0$. The condition $s = 0$ implies that

$$c_1(\eta_1 - y_{ref}) + c_2(\eta_2 - \dot{y}_{ref}) + c_3(\eta_3 - \ddot{y}_{ref}) = 0, \tag{5.23}$$

or

$$c_1(\eta_1 - y_{ref}) + c_2(\dot{\eta}_1 - \dot{y}_{ref}) + c_3(\ddot{\eta}_1 - \ddot{y}_{ref}) = 0. \tag{5.24}$$

Combining (5.24) with the observer dynamics yields (the closed-loop system with PI-Observer and SMC)

$$\begin{cases} c_1(\eta_1 - y_{ref}) + c_2(\dot{\eta}_1 - \dot{y}_{ref}) + (\ddot{\eta}_1 - \ddot{y}_{ref}) = 0 \\ \dot{\boldsymbol{e}}_{est}(t) = (\boldsymbol{A} - \boldsymbol{L}_1\boldsymbol{C})\boldsymbol{e}_{est}(t) + \boldsymbol{N} f_e \\ \dot{f}_e = -\boldsymbol{L}_2\boldsymbol{C}\boldsymbol{e}_{est}. \end{cases} \tag{5.25}$$

To proof the stability of system (5.25) containing the system states and the observer dynamics, let assume $\boldsymbol{\xi} = [\xi_1, \xi_2, \xi_3, \xi_4]^T = [\eta_1 - y_{ref}, \dot{\eta}_1 - \dot{y}_{ref}, \boldsymbol{e}_{est}, f_e]^T$ therefore

$$\begin{aligned} \dot{\boldsymbol{\xi}}(t) &= \boldsymbol{A}_{\boldsymbol{\xi}}\boldsymbol{\xi}(t), \\ \boldsymbol{A}_{\boldsymbol{\xi}} &= \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ -c_1 & -c_2 & 0 & 0 \\ \hline 0 & 0 & \boldsymbol{A} - \boldsymbol{L}_1\boldsymbol{C} & \boldsymbol{N} \\ 0 & 0 & -\boldsymbol{L}_2\boldsymbol{C} & 0 \end{array}\right]. \end{aligned} \tag{5.26}$$

Under the conditions that the polynomial $s^2 + c_2 s + c_1$ and the matrix $\begin{bmatrix} \boldsymbol{A} - \boldsymbol{L}_1\boldsymbol{C} & \boldsymbol{N} \\ -\boldsymbol{L}_2\boldsymbol{C} & \boldsymbol{0} \end{bmatrix}$ are Hurwitz (assuming the sliding surface design condition, *Lemma 1*, and *Lemma 2*), the matrix $\boldsymbol{A}_{\boldsymbol{\xi}}$ in (5.26) is also Hurwitz so the system $\dot{\boldsymbol{\xi}}(t) = \boldsymbol{A}_{\boldsymbol{\xi}}\boldsymbol{\xi}(t)$ is exponentially stable. Therefore, according to Lemma 5.5 in [Kha96] system (5.26) is ISS. Consequently, it can be derived from *Lemma 3* and design condition of sliding surface that the states of system (5.26) satisfy $\lim_{t\to\infty} \boldsymbol{\xi}(t) = 0$. Subsequently the steady state reference tracking error together with the observer estimation errors converge to zero under the proposed observer-based control law.

Table 5.2: Experimental conditions considered for the evaluation of proposed PIO-based robust control approaches

Case I	Case II	Case III
$\Delta m = 0$:	$\Delta m = 0$:	$\Delta m = 0$:
no mass uncertainty	no mass uncertainty	no mass uncertainty
$f_{cylinder} = 0$:	$f_{cylinder} = 0$:	$f_{cylinder} = 0$:
no disturbance force	no disturbance force	no disturbance force
No measurement error	Measurement error (about $\pm 2.5\%$)	Measurement error (about $\pm 5\%$)
Case IV	Case V	
$\Delta m = 100kg$	$\Delta m = 100kg$	
$f_{cylinder} \neq 0$	$f_{cylinder} \neq 0$	
No measurement error	Measurement error (about $\pm 2.5\%$)	

5.3.5 Experimental validation and discussion

The robust control design is validated using the test rig shown in Figure 5.1 and related system model (5.2). The disturbances and uncertainties are the friction force $f_{fric}(\boldsymbol{x}, t)$ between the mass and its bearing surface and the disturbance force $f_{cylinder}(\boldsymbol{x}, t)$ generated from a 2nd hydraulic cylinder with passive dynamics acting oppositely (see Figure 5.1). An uncertainty of the moved mass Δm can be considered additionally. Leakages between the cylinder chambers as well as external oil leakages are neglected in this consideration; their influence on the cylinder dynamics is negligible. The implementation of the proposed robust control design is carried out using regular working conditions. For evaluating the robustness performance additional noise of different levels is added to related measurements.

For experimental comparison three different controller types, P-control, SMC, and PIO-SMC, are applied. To illustrate the results different cases according to Table 5.2, are performed to verify the performance of SMC approach in combination with PIO. It is worth noting that an appropriate design of SMC parameters is required to achieve a good performance. Two parameter sets k and C (c_1, c_2 are linear dependent) have to be designed. To tune the control parameters the criterion

$$C_{criteria} = [\int_0^T e^2(t)dt, \int_0^T u^2(t)dt], \tag{5.27}$$

with the relation between input energy $\int_0^T u^2(t)dt$ and control error $\int_0^T e^2(t)dt$ is used like the previous chapters. The principle relation is illustrated in Figure 5.8. The interval length T denotes the time window where the performance is compared. The comparison is done using different parameters for sliding surface and for both approaches: SMC and SMC combined with PIO, so four arbitrary parameter sets for c_1, c_2, and three different k are considered according to Table 5.3 to evaluate the principal perform of this kind of control in comparison to others. Consequently,

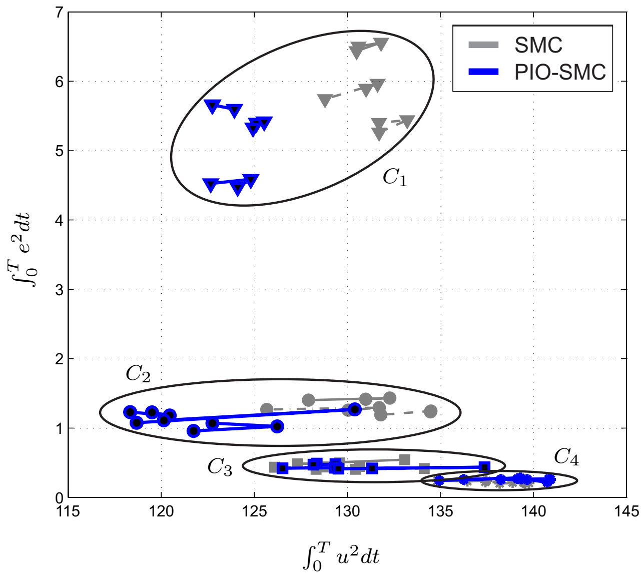


Figure 5.8: Comparison of design parameters by means of criterion $C_{criteria}$ to tune the design parameters of SMC/PIO-SMC approaches (Case I)

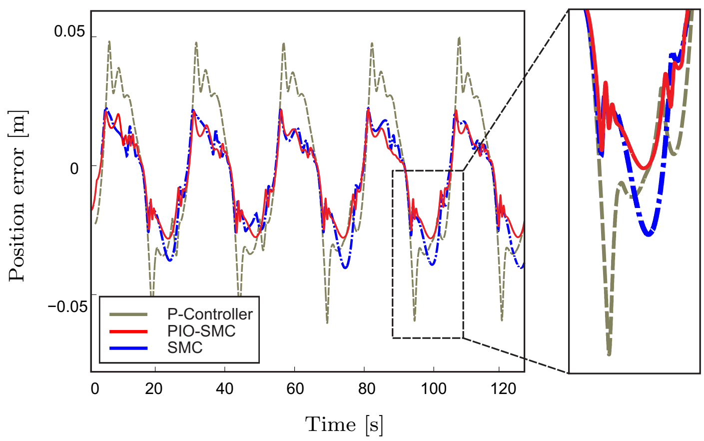


Figure 5.9: Position control error w/o additional measurement noise (Case I) for different approaches with sinusoidal signal as reference signal

Table 5.3: Parameters of sliding surface and switching gain

$c^T = [c_1, c_2, c_3]^T$	k_1	k_2	k_3	$C_{criteria}$
$[4, 4, 1]^T$	770	820	870	C_1
$[9, 6, 1]^T$	770	820	870	C_2
$[16, 8, 1]^T$	770	820	870	C_3
$[25, 10, 1]^T$	770	820	870	C_4

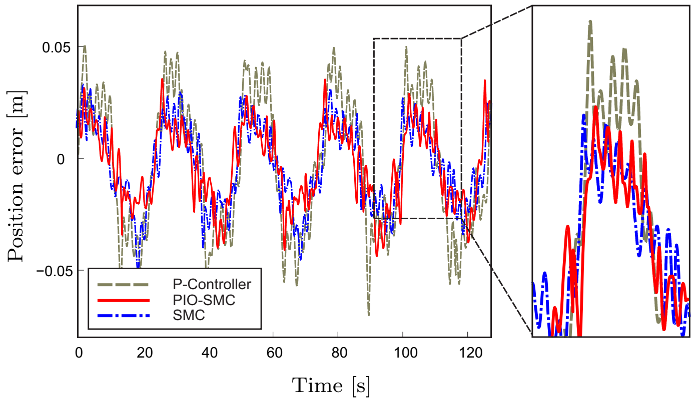


Figure 5.10: Position control error w/ additional measurement noise (Case III) for different approaches with sinusoidal signal as reference signal

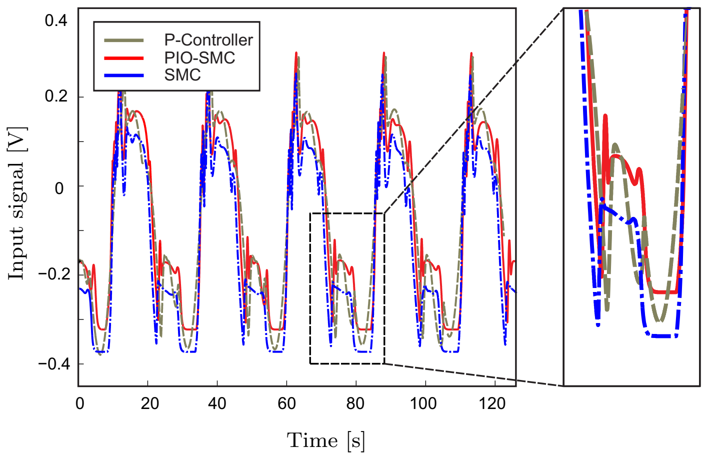
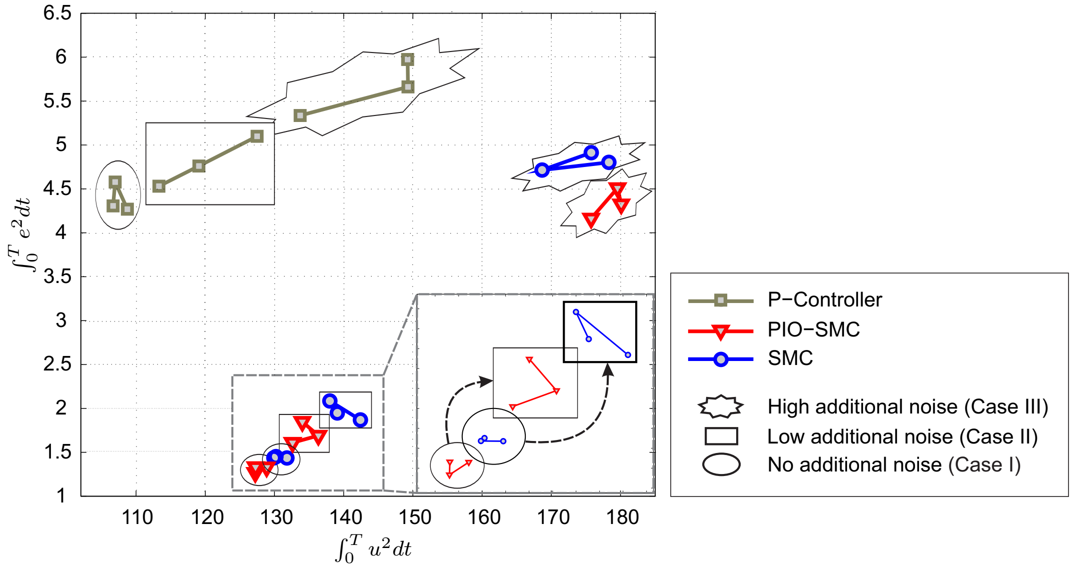


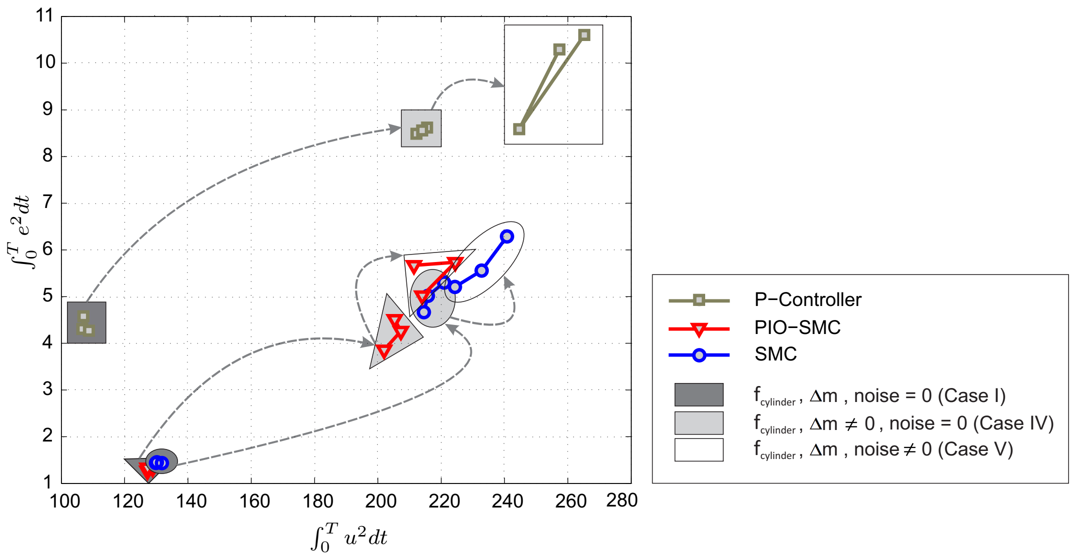
Figure 5.11: Input signal w/o additional measurement noise (Case I) for different approaches with sinusoidal signal as reference signal

the results of performance criterion (5.27) are calculated based on the experimental results for 12 different situations for both SMC and PIO-SMC. With the same input energy, the result which has lower output error or correspondingly with the same output error which uses less input energy (closer to the origin) has better performance. From the results it is evident that for the considered situations and parameters, PIO-SMC always shows better performance compare to SMC. This comparison clearly states the superiority of PIO in combination with SMC. To tune the design parameters a compromise between total input energy and control error can be considered. The results of the comparison provides that both methods (SMC and PIO-SMC) generally have better performance with C_2 by considering the mentioned compromise. Therefore, the implementation task in the following parts is performed using parameter set C_2 for design parameters.

To examine the performance with respect to sensitivity to noise or model uncer-



(a) Considering different level of additional noise



(b) Considering model uncertainties and unknown effects

Figure 5.12: Comparison of different control methods (PIO-SMC, SMC, and P-Controller) by means of criterion (5.27)

tainties, two representative control results are illustrated in Figure 5.9 and Figure 5.10 as well as the input signal in Figure 5.11 for different controller types (P-

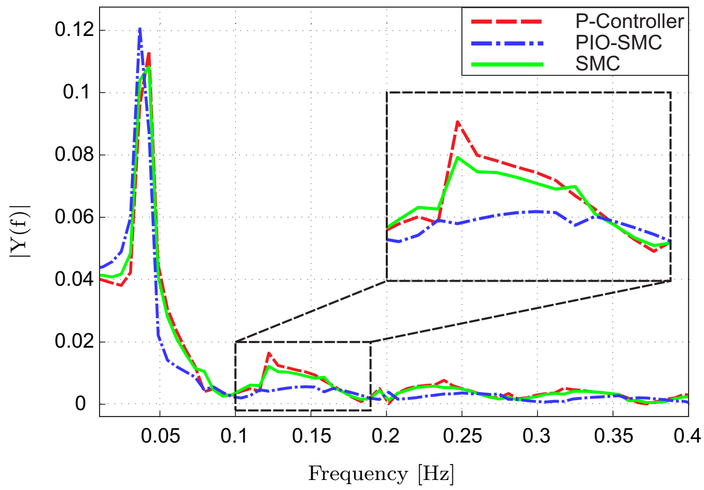


Figure 5.13: Single-sided amplitude spectrum of $y(t)$ for different approaches (Case IV)

Controller, SMC, and PIO-SMC). The reference signal is considered as a sinusoidal signal. From the results it can be concluded that for this dynamical motion control task, PIO-SMC shows better performance and less tracking error. However judging only based on the output performance (position of hydraulic cylinder) is not comprehensive. Therefore, the introduced criterion (5.27) is used for different control methods and by considering different level (without/with low/high) of measurement noise. Results are illustrated in Figure 5.12(a). It is evident that without considering additional measurement noise, output error using PIO-SMC is always less than those obtained by other methods. By increasing the additional noise PIO-SMC method provides better performance compare to normal SMC. However, the results of PIO-SMC are very sensitive to high level noise. On the other hand P-Controller requires always less input energy but shows worse performance in comparison to other ones. When the total input energy is taking into consideration, P-Controller will be a suitable choice.

In Figure 5.12(b) comparison of different control approaches in the presence of mass uncertainty Δm and passive dynamics force $f_{cylinder}(\boldsymbol{x}, t)$ is illustrated. The P-Controller has inadequate performance in the presence of uncertainties either in tracking error or in required input energy. Furthermore, the better performance of PIO-SMC is considerable compare to SMC approach.

Single-sided amplitude spectrum of $y(t)$ is illustrated in Figure 5.13 to detail further performance differences. It can be concluded that all three approaches show a roughly similar behavior with respect to the dynamics of desired reference motion (harmonic reference function with frequency$\approx 0.04Hz$). The detailed analysis of the frequency domain behavior shows differences with respect to higher order dynamics of the control error as shown in Figure 5.13. Here the PIO-SMC behavior shows less higher dynamical parts of the output signal (frequency$\approx 0.12Hz$). It should be

mentioned that no further filtering is applied to all approaches. The same result can be concluded for the control output (the systems input) signal.
In Table 5.4 a comparison of different methods is presented. It can be concluded that PIO-SMC requires less measurements in comparison to SMC approach because of using PI-Observer to estimate the system states. It is worth mentioning that the velocity of cylinder is required for defining the sliding surface which has to be measured in the SMC approach. As respects, PIO-SMC approach can use the estimated velocity provided by PIO. The requirement of using only one (easy to have) measurement is - in comparison to the two measurements required for SMC approach - with respect to practical application a strong advantage worth to mention. Accordingly, the following statements can be concluded for the experimental results of this section:

- By considering additional noise the performance is decreased. However, the best achievable results are obtained using PIO-SMC.
- In the presence of low level additional noise, PIO-SMC is sensitive as SMC (same robustness).
- In the presence of high additional noise level, PIO-SMC is more sensitive than SMC (more variations in the results).
- The improvement obtained by PIO (comparing SMC and PIO-SMC) can be clearly shown (with/without noise).
- By considering model uncertainties and unknown inputs, PIO-SMC always shows the best results either in tracking performance or required input energy.
- By considering different level of additional noise, P-Controller requires always less total input energy but on the other hand in the tracking task shows worse performance.
- The maximum input energy used and the maximum tracking error produced by PIO-SMC are less than SMC and P-Controller approaches (see Table 5.4 and Figure 5.14).
- The PIO-SMC is based on the use of only one sensor (same as P-control).

Table 5.4: Robust comparing different position control methods

		Number of sensors used	$\int e^2 dt$	$\int u^2 dt$	Max. |error|[m]	Max. |input|[V]
Without additional measurement noise	P-Controller	1	3.639	109.875	0.0611	0.3053
	SMC	2	1.6464	135.0846	0.0443	0.2999
	PIO-SMC	1	1.2692	123.1733	0.0285	0.2990
With additional measurement noise	P-Controller	1	4.452	111.269	0.0717	0.3519
	SMC	2	1.8787	138.754	0.0507	0.3076
	PIO-SMC	1	1.4459	132.8480	0.0438	0.3091

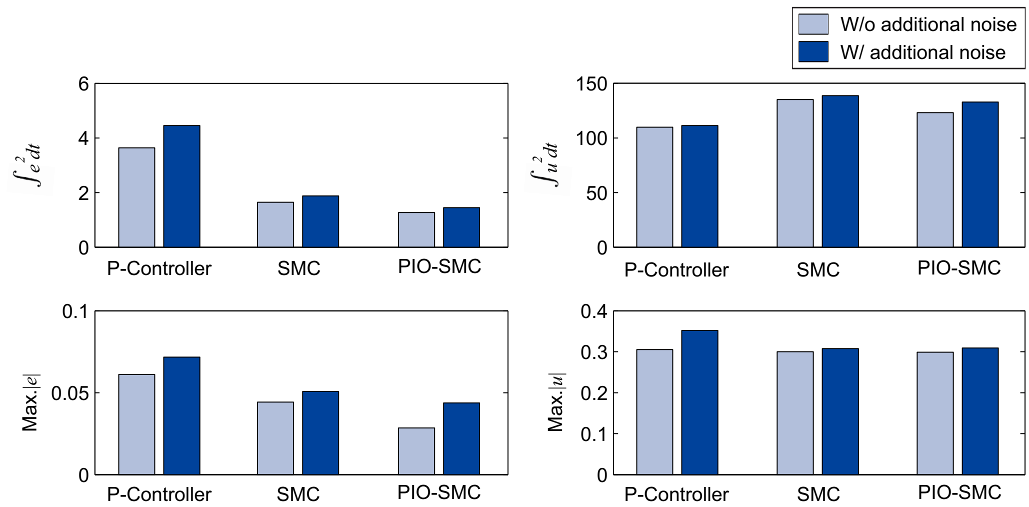


Figure 5.14: Comparison of different position control methods considering the effects of measurement noise

Summarizing the results of the new approach it has to be stated that the PIO-SMC approach shows strong advantages in comparison to existing approaches. This mainly results from the option to choose a linearized system model and therefore the nonrequirement of exact models and assumptions of bounds. In comparison with SMC this approach combines two advantages: inherent control robustness features of SMC as well as the shown robustness of the PIO. The results obtained from PIO-SMC clearly show the superiority of the introduced novel approach as stated in detail in the current section.

5.4 PIO-based backstepping controller

Backstepping method provides a designing control approach to track a reference signal by specifying an adequate Lyapunov function and by recursively designing the intermediate control laws for so-called "virtual controls" [KKK92, KKM91]. At each step the stability of the considered subsystem should be ensured using the specified Lyapunov function. The method is used to achieve the asymptotic tracking of reference signals while the global stability of the system is guaranteed.

In the following section the task of system states and unknown input estimation is performed using linear high gain Proportional-Integral-Observer. Therefore, the simplified model of the hydraulic differential cylinder is used according to [Liu11] to be used for linear PI-Observer structure. From the other hand, backstepping controller is utilized for nonlinear system model to construct the Lyapunov function and to design the control input simultaneously so that the stability or the negativeness of the derivative of every-step Lyapunov function is fulfilled. The main contribution of this section is improved structure of backstepping controller leads to the novel stability proof and new conditions for the whole control loop together with the PI-Observer estimations.

Table 5.5: Symbols used in the design of BC

$x_{1d} = w(t)$	Tracking signal
x_{2d}, x_{3d}	Virtual controller of x_2, x_3, respectively
e_1, e_2, e_3	Error of $x_1 - x_{1d}, x_2 - x_{2d}, \bar{x}_3 - \bar{x}_{3d}$, respectively
V_1, V_2, V_3	Lyapunov functions
k_1, k_2, k_3	Positive parameters for design of virtual controllers
ρ_1, ρ_2, ρ_3	Positive parameters for Lyapunov functions V_1, V_2, V_3

5.4.1 Backstepping control design

The result of PI-Observer is integrated into the structure of BC design to eliminate the effect of mismatched uncertainties. By defining the new system state $\bar{x}_3 = A_A x_3 - A_B x_4$ as the pressure difference between chambers A and B, the nonlinear model (5.2) can be rewritten as

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= \bar{x}_3(t)/m(x_1(t)) + d(t), \\ \dot{\bar{x}}_3 &= f(\boldsymbol{x})u(t) + g(\boldsymbol{x}), \\ y(t) &= x_1(t), \end{aligned} \tag{5.28}$$

with

$$\begin{aligned} f(\boldsymbol{x}) &= \frac{E_{oil}(x_3)}{V_A(x_1)} Q_A(x_3) A_A - \frac{E_{oil}(x_4)}{V_B(x_1)} Q_B(x_4) A_B, \\ g(\boldsymbol{x}) &= -\frac{E_{oil}(x_3)}{V_A(x_1)} \left(A_A^2 x_2\right) - \frac{E_{oil}(x_4)}{V_B(x_1)} \left(A_B^2 x_2\right). \end{aligned} \tag{5.29}$$

In the following the main steps of BC design are listed. The symbols used in the deduction are shown in Table 5.5. In this subsection, the backstepping controller is designed to compensate the unknown disturbance, to stabilize the close loop system (PIO and BC), and to track the given desired signal simultaneously. The block diagram of the proposed PIO-BC is shown in Figure 5.15.

Assumption 1: The disturbance d and its derivative $\dot{d}$ are bounded such that $|\dot{d}| \leq |\dot{d}|_{max}$ but the related bounds are unknown.

Theorem 1: Considering the position tracking error $e_1 = x_1 - x_{1d}$ and the system model (5.28), by considering the positive parameters $k_1, k_2, k_3, \rho_1, \rho_2, \rho_3$ defined in

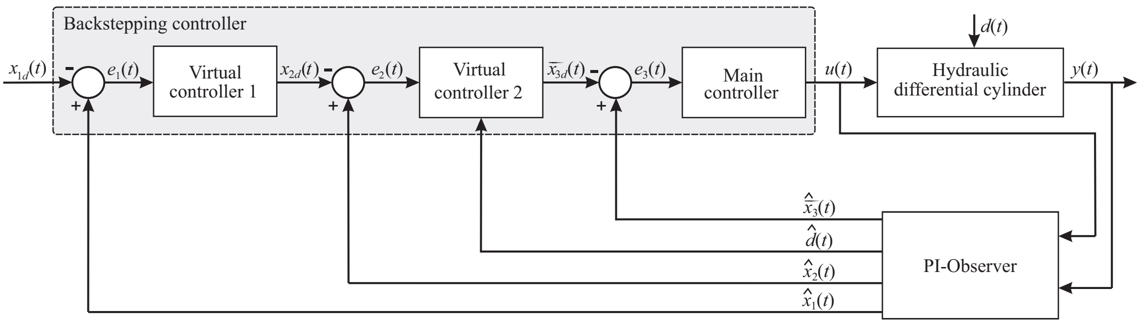


Figure 5.15: Block diagram of the proposed PIO-based backstepping control method

Table 5.5, the goals of disturbance compensation, system state stabilization, and desired signal tracking are achieved by defining the control law

$$\begin{aligned} &x_{2d} = \dot{x}_{1d} - k_1 e_1, \\ &\bar{x}_{3d} = m(x_1)(-\hat{d}(t) + \ddot{x}_{1d} - k_1 x_2 + k_1 \dot{x}_{1d} - \tfrac{\rho_1}{\rho_2} e_1 - k_2 e_2), \\ &u(t) = (-k_3 e_3 - \tfrac{\rho_2}{m\rho_3} e_2 + \dot{\bar{x}}_{3d} - g(\boldsymbol{x}))/f(\boldsymbol{x}). \end{aligned} \tag{5.30}$$

5.4.2 Lyapunov stability

Proof 1: **Step 1**: The derivative of e_1 with respect to time gives

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 - \dot{x}_{1d}. \tag{5.31}$$

The control Lyapunov function (CLF) candidate V_1 can be defined as

$$V_1 = \tfrac{1}{2}\rho_1 e_1^2. \tag{5.32}$$

The derivative of V_1 with respect to time is given by

$$\dot{V}_1 = \rho_1 e_1 \dot{e}_1 = \rho_1 e_1 (e_2 + x_{2d} - \dot{x}_{1d}) \tag{5.33}$$

Substituting x_{2d} from (5.30) into (5.33) results in

$$\dot{V}_1 = \underbrace{-\rho_1 k_1 e_1^2}_{negative\ term} + \underbrace{\rho_1 e_1 e_2}_{cross\ term}. \tag{5.34}$$

Step 2: The derivative of e_2 with respect to time is

$$\dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} = \bar{x}_3(t)/m(x_1(t)) + d(t) - \dot{x}_{2d}. \tag{5.35}$$

The second control Lyapunov function (CLF) candidate V_2 can be defined as

$$V_2 = V_1 + \tfrac{1}{2}\rho_2 e_2^2 + \tfrac{1}{2} f_e^2(t), \tag{5.36}$$

considering the estimation error of the unknown input together with the tracking error. According to (3.1) and by considering $f_e(t) = \hat{d}(t) - d(t)$ the derivative of V_2 with respect to time is given by

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \rho_2 e_2 \dot{e}_2 + f_e \dot{f}_e \\ &= -\rho_1 k_1 e_1^2 + \rho_1 e_1 e_2 + \rho_2 e_2 [\tfrac{1}{m(x_1)}(e_3 + \bar{x}_{3d}) + d(t) - \ddot{x}_{1d} + k_1(x_2 - \dot{x}_{1d})] + f_e \dot{f}_e. \end{aligned} \tag{5.37}$$

Substituting $\bar{x}_{3d}$ from (5.30) and $\hat{d}(t)$ from (2.31) into (5.37) results in

$$\dot{V}_2 = \underbrace{-\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2}_{negative\ term} + \underbrace{\frac{\rho_2}{m(x_1)} e_2 e_3}_{cross\ term} + \underbrace{f_e \dot{f}_e - \rho_2 e_2 f_e}_{robust\ term}. \tag{5.38}$$

Step 3: The derivative of e_3 with respect to time is

$$\dot{e}_3 = \dot{\bar{x}}_3 - \dot{\bar{x}}_{3d} = f(\boldsymbol{x})u(t) + g(\boldsymbol{x}) - \dot{\bar{x}}_{3d}. \tag{5.39}$$

The third control Lyapunov function (CLF) candidate V_3 which is the overall Lyapunov candidate can be defined as

$$V_3 = V_2 + \tfrac{1}{2}\rho_3 e_3^2 \tag{5.40}$$

The derivative of V_3 with respect to time is given by

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + \rho_3 e_3 \dot{e}_3 \\ &= -\rho_1 k_1 e_1^2 - \rho_2 k_2 e_2^2 + \tfrac{\rho_2}{m} e_2 e_3 + f_e \dot{f}_e - \rho_2 e_2 f_e + \rho_3 e_3 (f(\boldsymbol{x})u(t) + g(\boldsymbol{x}) - \dot{\bar{x}}_{3d}). \end{aligned} \tag{5.41}$$

Substituting $u(t)$ from (5.30) into (5.41) results in

$$\dot{V}_3 = -k_1 \rho_1 e_1^2 - k_2 \rho_2 e_2^2 - k_3 \rho_3 e_3^2 + f_e \dot{f}_e - \rho_2 e_2 f_e, \tag{5.42}$$

which can be rewritten as

$$\begin{aligned} \dot{V}_3 &= -k_1 \rho_1 e_1^2 - k_3 \rho_3 e_3^2 - k_2 \rho_2 (e_2 + \tfrac{f_e}{2k_2})^2 + \tfrac{\rho_2 f_e^2}{4k_2} + f_e \dot{f}_e \\ &= \underbrace{-k_1 \rho_1 e_1^2 - k_3 \rho_3 e_3^2 - k_2 \rho_2 (e_2 + \frac{f_e}{2k_2})^2 - \frac{k_2}{\rho_2} \dot{f}_e^2}_{negative\ term} + \underbrace{\frac{\rho_2}{4k_2} (f_e + \frac{2k_2 \dot{f}_e}{\rho_2})^2}_{robust\ term} \\ &\leq -k_1 \rho_1 e_1^2 - k_3 \rho_3 e_3^2 - k_2 \rho_2 (e_2 + \tfrac{f_e}{2k_2})^2 - \tfrac{k_2}{\rho_2} \dot{f}_e^2 + \tfrac{\rho_2}{4k_2} (f_{emax} + \tfrac{2k_2 \dot{f}_{emax}}{\rho_2})^2 \\ &\leq -(1-\theta)\dot{V}_{30} - \theta \dot{V}_{30} + \tfrac{\rho_2}{4k_2} (f_{emax} + \tfrac{2k_2 \dot{f}_{emax}}{\rho_2})^2, \end{aligned} \tag{5.43}$$

where

$$\dot{V}_{30} = k_1 \rho_1 e_1^2 + k_3 \rho_3 e_3^2 + k_2 \rho_2 (e_2 + \tfrac{f_e}{2k_2})^2 + \tfrac{k_2}{\rho_2} \dot{f}_e^2, \tag{5.44}$$

and $0 < \theta < 1$. By considering the closed-loop error as $e_{cl} = [e_1 e_2 e_3 f_e]$, if $\|e_{cl}\| \geq \psi_r$ where

$$\psi_r = \{e_{cl} \mid \theta \dot{V}_{30} = \frac{\rho_2}{4k_2}(f_{emax} + \frac{2k_2 \dot{f}_{emax}}{\rho_2})^2\}, \tag{5.45}$$

then

$$\dot{V}_3 = -(1-\theta)\dot{V}_{30} \leq 0. \tag{5.46}$$

Therefore, e_{cl} is globally uniformly bounded if $k_1, k_2, k_3, \rho_1, \rho_2, \rho_3 > 0$ holds. By assuming the unknown input as a piecewise constant signal, the size of the ball ψ_r in (5.45) mainly depends on the design parameters ρ_2 and k_2. It is worth mentioning that (5.40) is the Lyapunov function of the total system in (5.28) and the control law given in (5.30) makes the derivative of Lyapunov function in (5.40) negative semi-definite. Consequently, the closed-loop system with the proposed PIO-BC is stable.

5.4.3 Experimental validation and discussion

The proposed robust control design is experimentally validated using the test rig shown in Figure 5.1. The system model is described by (5.28). The disturbances and uncertainties are the friction force $f_{fric}(\boldsymbol{x}, t)$ between the mass and its bearing surface and the disturbance force $f_{cylinder}(\boldsymbol{x}, t)$ generated from a 2nd hydraulic cylinder with passive dynamics acting oppositely (see Figure 5.1). An uncertainty of the moved mass Δm can be considered additionally. Leakages between the cylinder chambers as well as external oil leakages are neglected in this consideration; their influence on the cylinder dynamics is negligible. The experiments of the robust control design are carried out using regular working conditions. For evaluating the robustness performance additional measurement noise is added to the related measurement. For experimental comparison three different controller types, P-Controller, PI-Observer-based sliding mode controller (PIO-SMC), and PI-Observer-based backstepping controller (PIO-BC) are considered. To illustrate the results different cases are performed according to Table 5.2, to verify the performance of backstepping approach in combination with PIO.

Parameter selection of BC approach

It is worth noting that an appropriate design of PIO-BC parameters is required to achieve a good controller performance. As explained in section 5.4.1 there are two groups of parameters for design of virtual controllers k_1, k_2, k_3 and for Lyapunov functions ρ_1, ρ_2, ρ_3. In order to tune the design parameters the criterion

$$C_{criteria} = [\int_0^T e^2(t)dt, \int_0^T u^2(t)dt], \tag{5.47}$$

Table 5.6: Parameters of BC approach

ρ_1	ρ_2	ρ_3	$[k_1, k_2, k_3]$	$C_{criteria}$
3e10	2e5	7e-5	[85, 0.001, 0.01] [40, 0.001, 0.01] [85, 0.1, 0.01] [85, 0.001, 1] [40, 0.1, 1]	Set1
25e8	e6	7e-5	[85, 0.001, 0.01] [40, 0.001, 0.01] [85, 0.1, 0.01] [85, 0.001, 1] [40, 0.1, 1]	Set2
25e8	2e5	6e-5	[85, 0.001, 0.01] [40, 0.001, 0.01] [85, 0.1, 0.01] [85, 0.001, 1] [40, 0.1, 1]	Set3
25e8	2e5	7e-5	[85, 0.001, 0.01] [40, 0.001, 0.01] [85, 0.1, 0.01] [85, 0.001, 1] [40, 0.1, 1]	Initial set

considering the input energy $\int_0^T u^2(t)dt$ (Integral Square Input (ISI)) and control error $\int_0^T e^2(t)dt$ (Integral Square Error (ISE)) is used. The interval length T denotes the time window where the performance is compared. With the same input energy, the result which has lower output error or correspondingly with the same output error which uses less input energy (closer to the origin) has better performance. Due to the high number of parameters the procedure of parameter selection is started from an appropriate initial set. The initial values are achieved according to the working point of system and considering stability conditions. As illustrated in Figure 5.16 the initial parameter set is considered for the fixed Lyapunov parameters ρ_1, ρ_2, ρ_3 and for five combinations of different virtual control parameters k_1, k_2, k_3 (see Table 5.6). Afterward, the influence of changing parameters are investigated by considering set1, set2, and set3 which means changing of ρ_1, ρ_2, and ρ_3, correspondingly. To choose the design parameters a compromise between total input energy and control error is considered. From the principle relation shown in Figure 5.16 it is evident that

- changing of parameter ρ_1 has a positive effect on the performance of the controller.
- changing of parameter ρ_2 has a negative effect in combination with some of the design parameters k_1, k_2, k_3.

- changing of parameter ρ_3 has almost no influence on the performance of the controller.

In this work Set1 is selected for the further implementations.

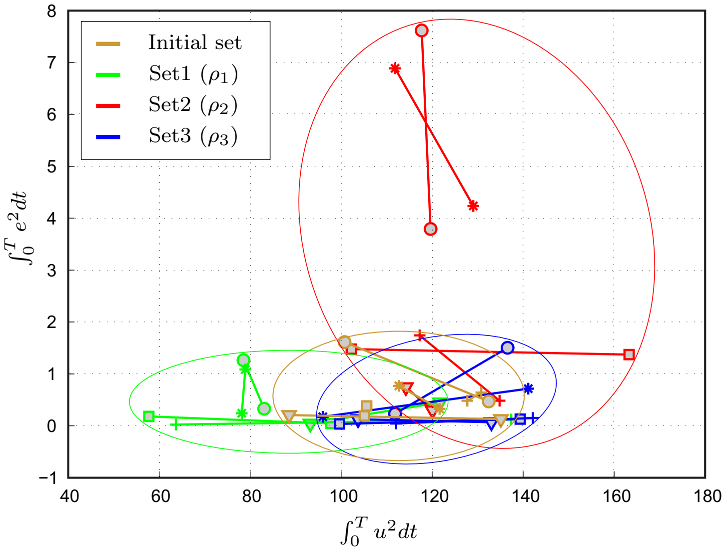


Figure 5.16: Comparison of design parameters by means of criterion $C_{criteria}$ to tune the design parameters of PIO-BC (Case I)

Results and comparison

In this subsection comparison of P-Controller, PIO-SMC, and PIO-BC in the sense of performance and robustness against measurement noise, model uncertainties, and external disturbances is considered. Correspondingly, different conditions are considered for experimental results according to Table 5.2. The PIO-SMC results are achieved using the same test rig and same conditions (Table 5.2) which deals with controlling of nonlinear hydraulic cylinder system with uncertainties using input-output feedback linearization method, sliding mode controller, and PI-Observer. In Figure 5.17 the position tracking error of the proposed method compare to the PIO-SMC and P-Controller for case I is illustrated. The reference signal is considered as a sinusoidal signal. It is evident that the proposed PIO-BC has the best position tracking performance according to the illustrated experimental result in Figure 5.17.

Furthermore, to compare the robustness of different approaches, the criterion in (5.47) is used by considering different level (without/with low/high) of measurement noise according to Table 5.2 for cases I-III. Experimental results shown in Figure 5.18(a) indicate that the proposed method has the best performance (minimum total error) compared to PIO-SMC and P-Controller while P-Controller outperforms the

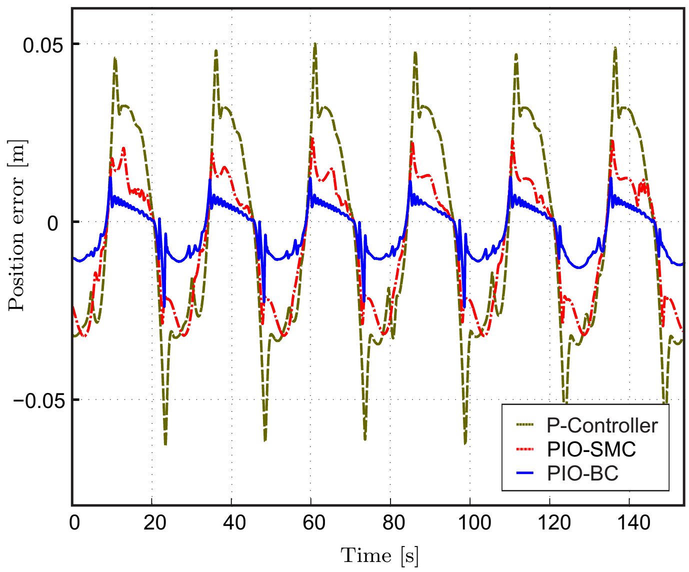
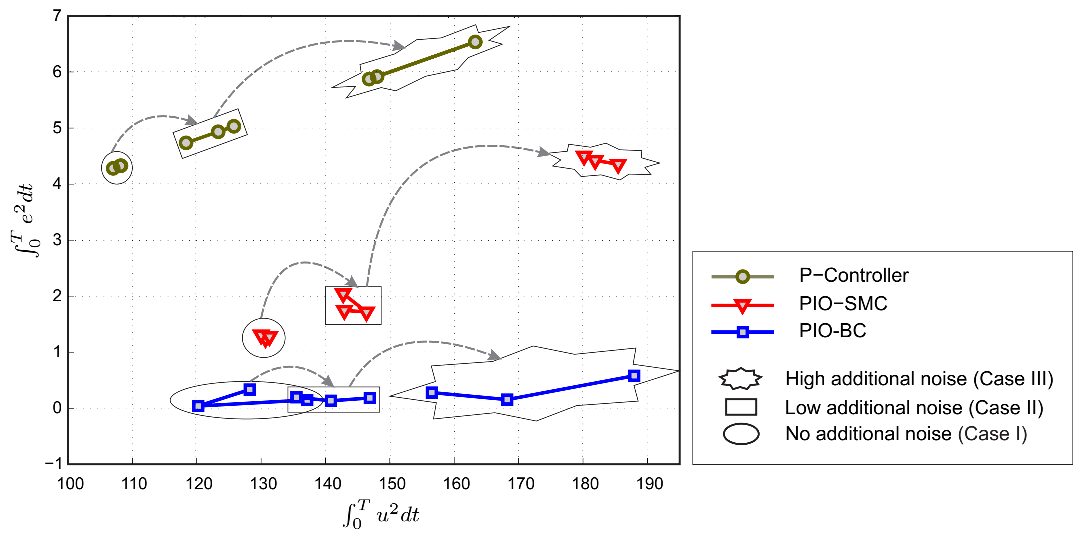


Figure 5.17: Position control error w/o additional measurement noise (Case I) for different approaches with sinusoidal signal as reference signal

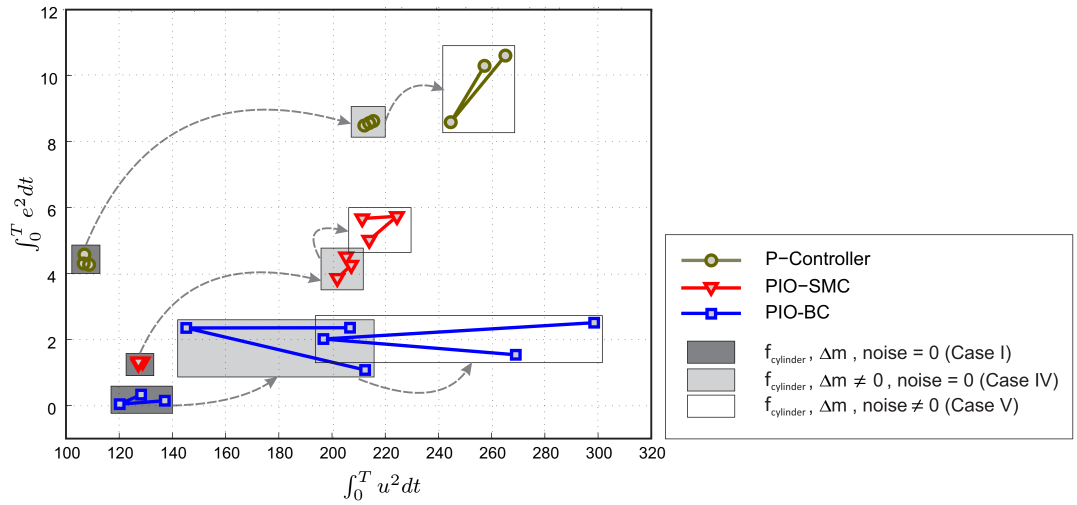
other two approaches when total input energy is taking into consideration. By increasing the level of additional measurement noise, the proposed PIO-BC preserves more or less the same performance whereas more input energy is required. In contrast, in the presence of additional measurement noise PIO-SMC and P-Controller reach unsatisfactory performance compared to the proposed method. By increasing the level of noise the performance is decreased (case III). Therefore, the proposed method is less sensitive to the different level of additional measurement noise compared to PIO-SMC and P-Controller.

For further robustness evaluation, some external disturbances (passive dynamics force $f_{cylinder}(\boldsymbol{x}, t)$) and model uncertainties (mass uncertainty Δm) are considered according to Table 5.2. As illustrated in Figure 5.18(b) PIO-BC leads to a better tracking performance compare to the considered approaches even when the external disturbance and internal model uncertainty affect the system. By considering the robustness of the controlled system in dealing with unknown internal/external effects, it is evident that the proposed method has more sensitive performance because of more variation in the behavior of tracking performance/required input energy (see Figure 5.18(b)). This fact can not be detected from the PIO-SMC behavior. P-Controller requires always less total input energy but on the other hand in the tracking task shows worse performance. However, PIO-BC always shows the best results in tracking performance either in the presence of measurement noise or external disturbance/model uncertainty.

In Table 5.7 a comparison of different methods in number of used sensors, maximum error, and maximum input energy is presented. It can be concluded that PIO-BC requires less measurements in comparison with SMC approach. Furthermore, it can



(a) Considering different level of additional noise



(b) Considering model uncertainties and unknown effects

Figure 5.18: Comparison of different control methods (PIO-SMC, PIO-BC, and P-Controller) by means of criterion (5.47)

be stated that the proposed method uses the estimation of system states provided by PI-Observer instead of measuring the whole set of system state variables. With respect to practical application this is a strong advantage worth to mention. The total input energy and tracking error obtained using the proposed method is always less than PIO-SMC and the tracking performance is always better than P-Controller.

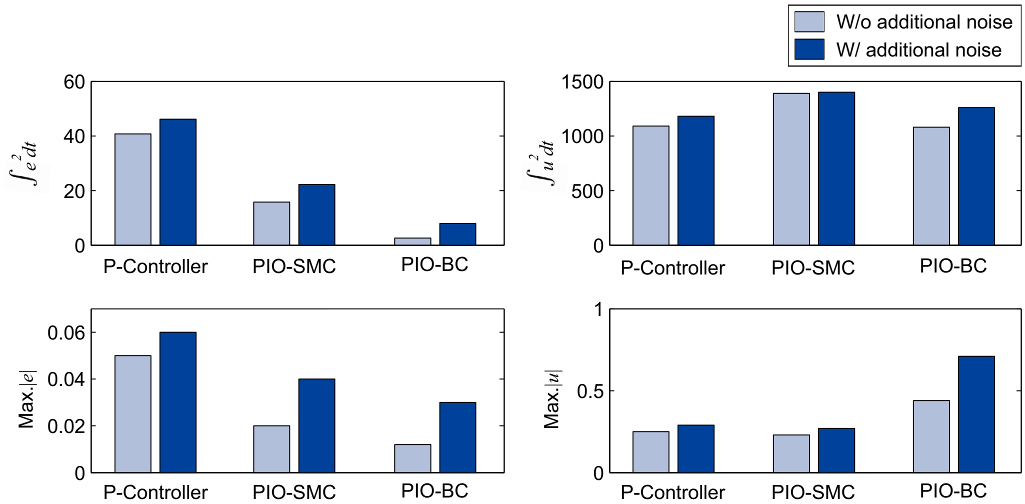


Figure 5.19: Comparison of different position control methods considering the effects of measurement noise

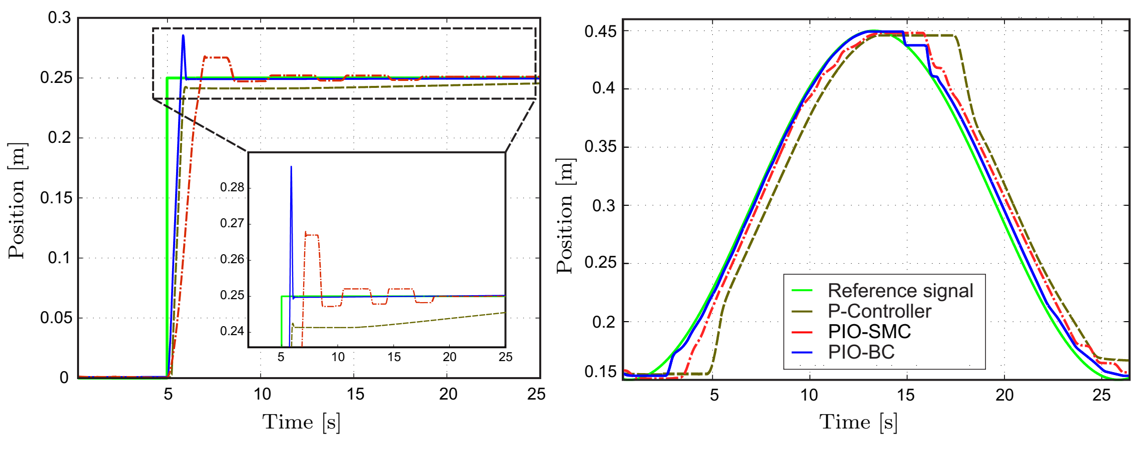
Table 5.7: Robust comparing different position control methods

		Number of sensors used	$\int e^2dt$	$\int u^2dt$	Max. |error|[m]	Max. |input|[V]
Without additional measurement noise	P-Controller	1	40.74	1.09e3	0.05	0.25
	PIO-SMC	1	15.79	1.39e3	0.02	0.23
	PIO-BC	1	2.59	1.08e3	0.012	0.44
With additional measurement noise	P-Controller	1	46.11	1.18e3	0.06	0.29
	PIO-SMC	1	22.26	1.40e3	0.04	0.27
	PIO-BC	1	7.96	1.26e3	0.03	0.71

The point which has to be mentioned is that maximum input required by PIO-BC is more than other approaches which has to be considered in terms of implementation (see Figure 5.19).

To effectively evaluate the performance of PIO-SMC and PIO-BC, the fundamental difference between SMC and BC could be identified. Sliding mode control is an useful approach for degradation of the system order to a lower one by defining a suitable sliding surface. It can facilitate the controller design because of more variety and experience in the field of control and stability analysis for low-order systems. From the scientific analysis point of view, relation between the sliding surface and the original variables of the system must be clarified to analyze the stability of controlled system. From the other side, backstepping control is a well-known approach with construction of the Lyapunov function and designing of the control input simultaneously. To achieve the stability through a suitable definition and differentiation of candidate Lyapunov functions at each step, cancellation of the indefinite cross terms (e.g. in (5.34) and (5.38)) is required (typical feature of backstepping controller). This appears as the most prominent difference between backstepping control and sliding mode control. The cross term cancellation in backstepping approach leads to

possible unsatisfactory robustness which is not expected in sliding mode approach regarding the input-output analysis. This is in accordance with the experimental results illustrated in Figure 5.18(b).
Furthermore, in the proposed sliding mode control approach the rate of convergence can be controlled by choosing suitable sliding surface parameters $c_i(i = 1, 2, 3)$ implicitly and sliding control law design parameter k explicitly. On the other side the convergence rate of backstepping approach introduced in this section depends on the Lyapunov functions parameters ρ_1, ρ_2, ρ_3 implicitly and the virtual controllers parameters k_1, k_2, k_3 explicitly. In Figure 5.20(a) and Figure 5.20(b) the response of closed-loop system using proposed approaches are illustrated for step signal and sinusoidal signal as reference signal, respectively. It it obvious that the proposed PIO-BC has better convergence rate compare to the other considered approaches and based on the selected parameters introduced in Table 5.3 and Table 5.6. This can be improved by changing the design parameters of all approaches.
In Table 5.8 a general comparison of traditional sliding mode control and backstepping control approach is surveyed to show the advantages and disadvantages of each approach. The properties are preserved in the case that PI-Observer is combined with the introduced robust control approaches for estimation of system states and unknown inputs.



(a) Step signal as reference

(b) Sinusoidal signal as reference

Figure 5.20: Comparison of convergence speed for different control methods (PIO-SMC, PIO-BC, and P-Controller)

Table 5.8: Comparison of traditional Sliding Mode and backstepping controllers

Properties	Sliding Mode Control	Backstepping Control
Matching condition	Insensitive to matched uncertainties. In the case of mismatched uncertainties the sliding motion depends on the uncertainties and the condition for the well-known robustness of SMC does not hold.	** Insensitive to matched/mismatched uncertainties
Chattering problem	High frequency oscillation around the predefined switching manifold(s) because of switching condition of SMC or parasitic dynamics (fast actuator and sensor dynamics). Solution: Nonlinear gains, dynamic extension, or higher order SMC.	** No chattering problem
Switching control strategy (discontinuous control action)	Control is shaped as a high frequency discontinuous signal.	** No switching control strategy
Design and computational complexity (so-called explosion of complexity)	** Choise of sliding surface and determination of existence condition and stability condition (switching manifold selection and discontinuous control design)	Design of control laws and stability together. The complexity increases with system order because of performing a repeated differentiations of the nonlinear functions.
Implementation difficulty	** Degradation of system order	Depends on system order
Convergence rate	Depends on the design parameters of sliding surface and sliding control law design	Depends on the design parameters of virtual controllers and Lyapunov functions
Recursive structure/procedure	No recursive structure	The design procedure can be started from a known stable system and back out new controllers. The stability of each outer system can be progressively reached till the final external control is achieved.

** Indicates the utility and efficiency of the method in the desired property

5.5 Summary and conclusion

This chapter provides two novel PI-Observer-based robust controllers as PIO-SMC and PIO-BC to improve the position tracking performance of a hydraulic differential cylinder system in the presence of uncertainties e.g. modeling errors, disturbances, and measurement noise. The proposed methods consist of a Proportional-Integral-Observer as a state and unknown input observer to be combined with SMC or BC. Since linear PI-Observer is utilized, input-output linearization method is considered for the linearization of nonlinear hydraulic cylinder system model. Thereupon the result of state and unknown input estimation is integrated into the structure

of robust control design (here SMC and BC) to eliminate the effect of related uncertainties. The introduced PIO-based robust controllers guarantee the ultimate boundness of the tracking error in the presence of uncertainties. The closed-loop stability is proven using Lyapunov theory in both cases. Evaluation of the proposed methods is experimentally shown using a hydraulic differential cylinder test rig. Experimental results validate the advantages of introduced PIO-SMC in comparison to standard SMC and industrial standard approach P-Controller in the presence of measurement noise, model uncertainties, and external disturbances.
Furthermore, PIO-BC approach is compared with PIO-SMC and standard P-Controller. The results illustrate significant improvement with respect to sensitivity to different level of measurement noise. Robustness of the closed-loop system in the presence of external disturbances (passive dynamics force $f_{cylinder}(\boldsymbol{x}, t)$) and model uncertainties (mass uncertainty Δm) is evaluated for the three considered approaches. Although the proposed PIO-BC approach has no satisfactory robustness performance in this evaluation, it has always a better tracking error performance compared to other two approaches. Finally, this chapter is concluded with comparison of PIO-BC and PIO-SMC from the design process perspective which shows the advantages and disadvantages of both considered approaches. The introduced approaches illustrate significant theoretical and practical advantages with respect to all criterion relevant for industrial applications like less sensitivity or improved robustness with respect to model uncertainties and/or noise. The illustrated approaches can be implemented on industrial DSP or PLC systems.

6 Summary, conclusion, and future work

6.1 Summary and conclusion

This thesis provides a review of the principal unknown input observation with elaboration of Proportional-Integral-Observer. The structure, design goals and methods, and integration of PI-Observer in different system types are focused. Furthermore, fundamental concepts of observer, filter, and estimator are distinguished with the main issues in each field to avoid the misuse of terms. In addition, the first chapter of this thesis outlines a new perspective of filters and observers with respect to predictor-corrector algorithm to inspire the reader about observer/filter structure, so new ideas can be presented based on the introduced structure. A description of some actual advanced applications of the PI-Observer and high-gain design in an abstract level is detailed and investigated.

The literature review presented in this work, emphasizes the role of observers particularly Proportional-Integral-Observer to be combined with linear/nonlinear control approaches to achieve a robust control method to deal with unknown effects such as internal/external disturbances, model uncertainties, measurement noise, unmodeled dynamics, etc. Based on analysis of PI-Observer design, two objectives are considered as the key parts of this thesis: (a) investigation and improvement of high gain PI-Observer by adaptively design of observer gain and (b) decrement of unknown effects and enhancement of system performance/robustness by combining the PI-Observer approach and nonlinear robust control methods. Both of the research directions are discussed and accordingly the solutions are investigated and implemented to achieve the aforementioned goals.

The structure and convergence conditions of PI-Observer are detailed and the advantages and disadvantages of using high gain is focused. This analysis leads to the point that the gain of PI-Observer has to be adaptively selected at each step of integration procedure (gain compromise is required to overcome the disadvantages of high gain utilization). Therefore, a new development of high gain PI-Observer design is proposed as Modified Advanced Proportional-Integral-Observer (MAPIO) as an improved version of previous introduced Advanced Proportional-Integral-Observer (APIO) with adaptive gain scheduling approach. Unlike API-Observer, MAPI-Observer attempts to find the 'absolute minimal level of estimation error' within a suitable defined continuous interval of gain options and not only between a few limited ones. In consequence of changing and scheduling the gain of PI-Observer based on the cost function, the performance is adequately improved in comparison to PI-Observer and Advanced PI-Observer especially in the presence of measurement noise. The adaption procedure and its evaluation are detailed by simulation results in open-loop and closed-loop contexts. An illustrative example of an elastic beam is considered where the task of PI-Observer is estimation of contact force as an unknown input to the system. Furthermore, a two-mass-spring system (ACC Benchmark) is used to illustrate the advantages of the proposed approach in the closed-loop context. Different scenarios are defined to compare the proposed approach with the previous Advanced PI-Observer and low/high gain PI-Observer. From the results, it can be concluded that the performance of Modified Advanced PI-Observer in the task of unknown input estimation is more robust to different level of measurement noise in comparison to previous methods. It can also be concluded that the gain of proposed observer approach is adaptively changed according to different situations and has to be defined online at each step of integration procedure. That leads to the difficulty of implementation with respect to real-time applications.

To overcome the disadvantages of high gain design of PI-Observer and difficulties of Modified advanced PI-Observer, an adaptive estimation approach is proposed based on funnel control idea. The proposed funnel PI-Observer approach takes the advantage of the funnel idea to adjust the PI-Observer gains according to the actual situation and to maintain the estimation error in a prescribed funnel area. The proposed approach shows significant advantages with respect to estimation of system states and unknown inputs in the presence of measurement noise. Both simulation and experimental results of an elastic beam system verify and validate the advantages of the introduced funnel PI-Observer compared to the known low/high gain PI-Observer. The stability of the proposed adaptive algorithm with respect to switching funnel PI-Observers is discussed based on Lyapunov theory.
Furthermore, application of funnel PI-Observer for nonlinear systems (robust control design) is taking into consideration for an input-output linearizable nonlinear system with unknown inputs. The system and unknown input estimations achieved by funnel PI-Observer is used together with the system measurements to realize the exact feedback linearization (EFL) approach. A robust disturbance rejection control is realized by using state feedback of the linearized model and estimations of the unknown inputs. Simulation results are carried out using a nonlinear MIMO mass-spring system for two approaches (funnel PIO-EFL and PIO-EFL). For both approaches the same control design is considered and combined with different observers. The new introduced approach FPIO-EFL shows significant advantages compared to PIO-EFL with respect to estimation of system states and unknown inputs in the presence of measurement noise and integration of the estimation results in the structure of EFL approach. To perform the comparison of proposed controller comprehensively, a criterion based on the control error and the corresponding input energy is considered. From the simulation results it can be concluded that the proposed FPIO-EFL has always better tracking performance compared to PIO-EFL. It is worth mentioning that when robustness is taking into consideration, the proposed FPIO-EFL has convenient and better tracking performance compared to the PIO-EFL regarding the performance variation in the presence of additional measurement noise.

The application of PI-Observer-based robust nonlinear control is performed using

the hydraulic differential cylinder test rig to assure suitable tracking performance as well as robustness against unknown inputs. The task of system states and unknown input estimation is performed by a high gain linear Proportional-Integral-Observer. Therefore, input-output feedback linearization is used to linearize the nonlinear system model to be used for linear PI-Observer structure. Two robust control approaches are considered as (1) sliding mode control and (2) backstepping control. The estimation of system states and unknown inputs are integrated into the structure of robust nonlinear control approaches to achieve a control law that provides the desired performance to the closed-loop system in the presence of uncertainties, and compensates the effects of external disturbances, plant parameter changes, unmodeled dynamics, measurement noise, etc. Additionally, parameter selection of the proposed PIO-based controllers is elaborately considered by defining a performance/energy criterion. The stability of the closed-loop system is established using Lyapunov method for both cases. Furthermore, a complete robustness evaluation considering different level of measurement noise, modeling errors, and external disturbances is experimentally evaluated.
Experimental results validate the advantages of using introduced combined approach compare to the standard sliding mode controller (as a robust controller for nonlinear processes subject to external disturbances and heavy model uncertainties) and P-Controller (as a standard classical industrial approach for hydraulic systems). Consequently, integration of unknown input observer estimation results into the structure of robust control approaches leads to enhance the disturbance attenuation and system performance robustness. In addition, a comprehensive comparison between the introduced PIO-based sliding mode control and PIO-based backstepping control is experimentally performed with focusing on the fundamental difference between sliding mode control and backstepping control approaches. The experimental results illustrate that in the presence of additional measurement noise PIO-based backstepping control approach acts more robust while in the presence of model uncertainties and disturbances, PIO-based sliding mode control is more robust than the other approaches. However, PIO-based backstepping approach has always the best tracking performance. This appears as the most prominent difference between backstepping control and sliding mode control. The cross term cancellation in backstepping approach leads to possible unsatisfactory robustness which is not expected in sliding mode approach regarding the input-output analysis (in the case that the zero dynamics are stable or at least bounded).

To summarizing the whole work, the advantages of gain scheduling for Proportional-Integral-Observer is shown by improving the Advanced PIO approach as Modified advanced PIO and using funnel theory to adaptively design the high gains. Furthermore, integration of PI-Observer estimation results into the structure of nonlinear control approaches (sliding mode control and backstepping control) evidence the feasibility of improving the robustness and control performance. In this work the following new methods are developed and the described new results are obtained:

- Design and development of Modified Advanced PI-Observer (MAPIO) as an improved version of Advanced PI-Observer (APIO) with adaptive gain scheduling procedure
- Comparison of the proposed MAPIO with previous advanced observers in open-loop and closed-loop simulation results
- Proposing a new gain design approach of Proportional-Integral-Observer as funnel PI-Observer algorithm able to self adjustment of observer gains according to the actual estimation situation inside the predefined funnel area
- Stability proof of the proposed funnel PI-Observer according to the switching observer condition and Lyapunov theory
- Evaluation of the proposed funnel PI-Observer by simulation and experimental results using an elastic beam test rig (open-loop estimation) and a nonlinear MIMO mechanical system (closed-loop estimation combined with input-output feedback linearization approach)
- Investigation and implementation of novel PI-Observer-based Sliding mode control and PI-Observer-based backsteppin control approaches
- Stability proof of proposed approaches combined with PI-Observer structure using Lyapunov theory
- Implementation of the proposed PIO-based SMC and PIO-based BC using the hydraulic differential cylinder test rig
- Design and selection of SMC and BC parameters by defining and elaborating a performance/energy criterion
- Enhancement of disturbance attenuation and system performance robustness using the proposed combination of linear observer and nonlinear robust controllers
- Interpretation and analysis of system performance and robustness according to the basic features of SMC and BC approaches

6.2 Future work

For future work, the following points are considered and suggested:

The efficiency of the proposed funnel PI-Observer is evaluated by simulation and experimental results for the contact force estimation of an elastic beam system (open loop situation). Furthermore, the advantages of using this observer is evaluated in the closed-loop system structure by combining the funnel PI-Observer with the input-output feedback linearization and state feedback control approach. Implementation of the proposed approach in the real time context and in combination with linear/nonlinear control approaches (hardware in the loop) can be considered as the future work.

The proposed funnel PI-Observer contains different design parameters for the funnel function. For the future work an adaptive and flexible approach can be designed for parameter selection of the funnel PI-Observer method. In addition, the funnel function can be adaptively changed during the run time and according to the desired situation of estimation procedure.

In this work the PI-Observer estimation results are integrated into the structure of sliding mode control and backstepping control. For the future work following this study, combination of PI-Observer or funnel PI-Observer with adaptive version of the mentioned controllers can be considered. For example using of high order SMC or integral SMC combined with observer estimation results can be considered to improve the control performance and robustness.

Bibliography

[AGTAM+12] Aguilera-Gonzalez, A. ; Theilliol, D. ; Adam-Medina, M. ; Astorga-Zaragoza, C. M. ; Rodrigues, M.: Sensor fault and unknown input estimation based on Proportional-Integral-Observer applied to LPV descriptor systems. In: *IFAC Proceedings Volumes* 45 (2012), No. 20, pp. 1059–1064

[AGZM17] Al-Ghanimi, A. ; Zheng, J. ; Man, Z.: A fast non-singular terminal sliding mode control based on perturbation estimation for piezoelectric actuators systems. In: *International Journal of Control* 90 (2017), No. 3, pp. 480–491

[AK09] Ahrens, J. H. ; Khalil, H. K.: High-gain observers in the presence of measurement noise: A switched-gain approach. In: *Automatica* 45 (2009), No. 4, pp. 936–943

[AM99] Astolfi, A. ; Menini, L.: Further results on decoupling with stability for hamiltonian systems. In: *Springer* (1999)

[AM07] Anderson, B. D. ; Moore, J. B.: Optimal control: Linear quadratic methods. In: *Courier Corporation* (2007)

[AM12] Anderson, B. D. ; Moore, J. B.: Optimal filtering. In: *Courier Corporation* (2012)

[BGK09] Bandyopadhyay, B. ; Gandhi, P. S. ; Kurode, S.: Sliding mode observer-based sliding mode controller for slosh-free motion through PID scheme. In: *IEEE Transactions on Industrial Electronics* 56 (2009), No. 9, pp. 3432–3442

[BJ87] Bucy, R. S. ; Joseph, P. D.: Filtering for stochastic processes with applications to guidance. In: *American Mathematical Soc.* 326 (1987)

[BLPN13a] Białoń, T. ; Lewicki, A. ; Pasko, M. ; Niestrój, R.: Parameter selection of an adaptive PI state observer for an induction motor. In: *Bulletin of the Polish Academy of Sciences: Technical Sciences* 61 (2013), No. 3, pp. 599–603

[BLPN13b] Białoń, T. ; Lewicki, A. ; Pasko, M. ; Niestrój, R.: PI-Observer stability and application in an induction motor control system. In: *Bulletin of the Polish Academy of Sciences: Technical Sciences* 61 (2013), No. 3, pp. 595–598

[Bro83] Brown, R. G.: Introduction to random signal analysis and Kalman filtering. In: *John Wiley & Sons* (1983)

[BS89] Beale, S. ; Shafai, B.: Robust control system design with a Proportional-Integral-Observer. In: *International Journal of Control* 50 (1989), No. 1, pp. 97–111

[BS90] Beale, S. ; Shafai, B.: Loop transfer recovery via a Proportional-Integral-Observer. In: *IEEE International Conference on Systems Engineering* (1990), pp. 468–473

[BS14] Bakhshande, F. ; Söffker, D.: High-gain scheduling of the Proportional-Integral-Observer. In: *PAMM, Wiley Online Library* (2014)

[BS15a] Bakhshande, F. ; Söffker, D.: Proportional-Integral-Observer: A brief survey with special attention to the actual methods using ACC Benchmark. In: *IFAC-PapersOnLine* 48 (2015), No. 1, pp. 532–537

[BS15b] Bakhshande, F. ; Söffker, D.: Reconstruction of nonlinear characteristics by means of advanced observer design approaches. In: *ASME Dynamic Systems and Control Conference (DSC)* 2 (2015), pp. V002T23A007

[BS17a] Bakhshande, F. ; Söffker, D.: Contact force estimation of an elastic mechanical structure using a novel adaptive funnel PI-Observer approach. In: *ASME Journal of Dynamic Systems, Measurement and Control* (2017), submitted

[BS17b] Bakhshande, F. ; Söffker, D.: Proportional-Integral-Observer-based backstepping approach for position control of a hydraulic differential cylinder system with model uncertainties and disturbances. In: *ASME Journal of Dynamic Systems, Measurement and Control* (2017), submitted

[BS17c] Bakhshande, F. ; Söffker, D.: Proportional-Integral-Observer with adaptive high-gain design using funnel adjustment concept. In: *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* 6 (2017), pp. V006T10A011

[BS17d] Bakhshande, F. ; Söffker, D.: Robust control approach for a hydraulic differential cylinder system using a Proportional-Integral-Observer-based backstepping control. In: *American Control Conference* (2017), pp. 3102–3107

[BS17e] Bakhshande, F. ; Söffker, D.: Robust control of a hydraulic cylinder using an observer-based sliding mode control: Theoretical development and experimental validation. In: *IEEE/ASME Transactions on Mechatronics* (2017), submitted

[BSLK04] Bar-Shalom, Y. ; Li, X. R. ; Kirubarajan, T.: Estimation with applications to tracking and navigation: Theory algorithms and software. In: *John Wiley & Sons* (2004)

[BW01] Bishop, G. ; Welch, G.: An introduction to the kalman filter. In: *Proc of SIGGRAPH, Course* 8 (2001), No. 27599-23175, pp. 41

[BZ88] Birk, J. ; Zeitz, M.: Extended Luenberger observer for nonlinear multivariable systems. In: *International Journal of Control* 47 (1988), No. 6, pp. 1823–1836

[CBGO00] Chen, W. ; Ballance, D. J. ; Gawthrop, P. J. ; O'Reilly, J.: A nonlinear disturbance observer for robotic manipulators. In: *IEEE Transactions on Industrial Electronics* 47 (2000), No. 4, pp. 932–938

[CC10] Chen, M. ; Chen, W.: Sliding mode control for a class of uncertain nonlinear system based on disturbance observer. In: *International Journal of Adaptive Control and Signal Processing* 24 (2010), No. 1, pp. 51–64

[CDMG93] Ciccarella, G. ; Dalla Mora, M. ; Germani, A.: A Luenberger-like observer for nonlinear systems. In: *International Journal of Control* 57 (1993), No. 3, pp. 537–556

[CDSM02] Commault, C. ; Dion, J. ; Sename, O. ; Motyeian, R.: Observer-based fault detection and isolation for structured systems. In: *IEEE Transactions on Automatic Control* 47 (2002), No. 12, pp. 2074–2079

[Che95] Chen, J.: Robust residual generation for model-based fault diagnosis of dynamic systems. (1995), University of York, PhD Thesis

[Che04] Chen, W.: Disturbance observer-based control for nonlinear systems. In: *IEEE/ASME Transactions on Mechatronics* 9 (2004), No. 4, pp. 706–710

[Cho07] Choi, H. H.: LMI-based sliding surface design for integral sliding mode control of mismatched uncertain systems. In: *IEEE Transactions on Automatic Control* 52 (2007), No. 4, pp. 736–742

[CK06] Chladny, R. R. ; Koch, C. R.: Flatness-based tracking of an electromechanical VVT actuator with magnetic flux sensor. In: *IEEE International Conference on Control Applications* (2006), pp. 1663–1668

[CLTT17] Chen, S. L. ; Li, X. ; Teo, C. S. ; Tan, K. K.: Composite jerk feedforward and disturbance observer for robust tracking of flexible systems. In: *Automatica* 80 (2017), pp. 253–260

[Cos94] Costa, P. J.: Adaptive model architecture and extended Kalman-Bucy filters. In: *IEEE Transactions on Aerospace and Electronic Systems* 30 (1994), No. 2, pp. 525–533

[CS06] Chen, W. ; Saif, M.: An iterative learning observer for fault detection and accommodation in nonlinear time-delay systems. In: *International Journal of Robust and Nonlinear Control* 16 (2006), No. 1, pp. 1–19

[CYGL16] Chen, W. H. ; Yang, J. ; Guo, L. ; Li, S.: Disturbance-observer-based control and related methods, an overview. In: *IEEE Transactions on Industrial Electronics* 63 (2016), No. 2, pp. 1083–1095

[Dav72] Davison, E. J.: The output control of linear time-invariant multivariable systems with unmeasurable arbitrary disturbances. In: *Automatic Control, IEEE Transactions on* 17 (1972), No. 5, pp. 621–630

[DLT01] Duan, G. R. ; Liu, G. P. ; Thompson, S.: Eigenstructure assignment design for Proportional-Integral-Observer: continuous-time case. In: *IEE Proceedings Control Theory and Applications* 148 (2001), No. 3, pp. 263–267

[Dra69] Draženović, B.: The invariance conditions in variable structure systems. In: *Automatica* 5 (1969), No. 3, pp. 287–295

[EE03] Elbeheiry, E. M. ; Elmaraghy, H. A.: Robotic manipulators state observation via one-time gain switching. In: *Journal of Intelligent and Robotic Systems* 38 (2003), No. 3-4, pp. 313–344

[EF09] Efimov, D. ; Fradkov, A.: Robust and adaptive observer-based partial stabilization for a class of nonlinear systems. In: *IEEE Transactions on Automatic Control* 54 (2009), No. 7, pp. 1591–1595

[ES98] Edwards, C. ; Spurgeon, S.: Sliding mode control: Theory and applications. In: *CRC Press* (1998)

[FBF11] Ferreira, A. ; Bejarano, F. J. ; Fridman, L. M.: Robust control with exact uncertainties compensation: With or without chattering? In: *IEEE Transactions on Control Systems Technology* 19 (2011), No. 5, pp. 969–975

[FD97] Frank, P. M. ; Ding, X.: Survey of robust residual generation and evaluation methods in observer-based fault detection systems. In: *Journal of Process Control* 7 (1997), No. 6, pp. 403–424

[FIAZ09] Farid, R ; Ibrahim, A ; Abo-Zalam, B: Chaos synchronization based on PI fuzzy observer. In: *Proceedings of the 10th WSEAS International Conference on Fuzzy systems* (2009), pp. 84–89

[FS99] Feldman, M. ; Seibold, S.: Damage diagnosis of rotors: Application of Hilbert transform and multihypothesis testing. In: *Journal of Vibration and Control* 5 (1999), No. 3, pp. 421–442

[GCD15] Gao, Z. ; Cecati, C. ; Ding, S. X.: A survey of fault diagnosis and fault-tolerant techniques-Part I: Fault diagnosis with model-based and signal-based approaches. In: *IEEE Transactions on Industrial Electronics* 62 (2015), No. 6, pp. 3757–3767

[GDM07] Gillijns, S. ; De Moor, B.: Unbiased minimum-variance input and state estimation for linear discrete-time systems. In: *Automatica* 43 (2007), No. 1, pp. 111–116

[Ger88] Gertler, J. J.: Survey of model-based failure detection and isolation in complex plants. In: *IEEE Control systems magazine* 8 (1988), No. 6, pp. 3–11

[Ger98] Gertler, J.: Fault detection and diagnosis in engineering systems. In: *CRC press* (1998)

[GLC16] Gao, Z. ; Liu, X. ; Chen, M. Z.: Unknown input observer-based robust fault estimation for systems corrupted by partially decoupled disturbances. In: *IEEE Transactions on Industrial Electronics* 63 (2016), No. 4, pp. 2537–2547

[Gre11] Grewal, M. S.: Kalman filtering. In: *Springer* (2011)

[GS11] Ghadami, R ; Shafai, B: Decentralized PI-Observer-based control of nonlinear interconnected systems with disturbance attenuation. In: *American Control Conference (ACC)* (2011), pp. 4705–4710

[GSP13] Ginoya, D. ; Shendge, P. D. ; Phadke, S. B.: Sliding mode control for mismatched uncertain systems using an extended disturbance observer. In: *IEEE Transactions on Industrial Electronics* 61 (2013), No. 4, pp. 1983–1992

[Hab07] Habibi, S.: The smooth variable structure filter. In: *Proceedings of the IEEE* 95 (2007), No. 5, pp. 1026–1059

[HD03a] HAGENMEYER, V. ; DELALEAU, E.: Exact feedforward linearization based on differential flatness. In: *International Journal of Control* 76 (2003), No. 6, pp. 537–556

[HD03b] HAGENMEYER, V. ; DELALEAU, E.: Robustness analysis of exact feedforward linearization based on differential flatness. In: *Automatica* 39 (2003), No. 11, pp. 1941–1946

[HD08] HAGENMEYER, V. ; DELALEAU, E.: Continuous-time nonlinear flatness-based predictive control: An exact feedforward linearisation setting with an induction drive example. In: *International Journal of Control* 81 (2008), No. 10, pp. 1645–1663

[HG05] HUA, C. ; GUAN, X.: Synchronization of chaotic systems based on PI-Observer design. In: *Physics letters A* 334 (2005), No. 5, pp. 382–389

[HRM+12] HAMDI, H. ; RODRIGUES, M. ; MECHMECHE, C. ; THEILLIOL, D. ; BRAIEK, N. B.: Fault detection and isolation in linear parameter-varying descriptor systems via Proportional-Integral-Observer. In: *International journal of adaptive control and signal processing* 26 (2012), No. 3, pp. 224–240

[HW85] HIPPE, P. ; WURMTHALER, C.: Zustandsregelung: Theoretische Grundlagen und anwendungsorientierte Regelungskonzepte. In: *Springer* (1985)

[IRS02] ILCHMANN, A. ; RYAN, E. P. ; SANGWIN, C. J.: Tracking with prescribed transient behaviour. In: *ESAIM: Control, Optimisation and Calculus of Variations* 7 (2002), pp. 471–493

[IRT05] ILCHMANN, A. ; RYAN, E. P. ; TRENN, S.: Tracking control: Performance funnels and prescribed transient behaviour. In: *Systems & Control Letters* 54 (2005), No. 7, pp. 655–670

[IS97] IORDANOU, H. N. ; SURGENOR, B. W.: Experimental evaluation of the robustness of discrete sliding mode control versus linear quadratic control. In: *IEEE Transactions on control systems technology* 5 (1997), No. 2, pp. 254–260

[Isi95] ISIDORI, A.: Nonlinear control systems. In: *Springer Verlag* (1995)

[Itk76] ITKIS, U.: Control systems of variable structure. In: *Halsted Press* (1976)

[ITT04] Ilchmann, A. ; Thuto, M. ; Townley, S.: Input constrained adaptive tracking with applications to exothermic chemical reaction models. In: *SIAM journal on control and optimization* 43 (2004), No. 1, pp. 154–173

[Jaz07] Jazwinski, A. H.: Stochastic processes and filtering theory. In: *Courier Corporation* (2007)

[JHH08] Jung, J. ; Han, S. ; Huh, K.: Robust Proportional-Integral Kalman filter design using a convex optimization method. In: *Journal of Mechanical Science and Technology* 22 (2008), No. 5, pp. 879–886

[JK12] Jelali, M. ; Kroll, A.: Hydraulic servo-systems: Modelling, identification and control. In: *Springer Science & Business Media* (2012)

[Joh68] Johnson, C. D.: Optimal control of the linear regulator with constant disturbances. In: *IEEE Transactions on Automatic Control* 13 (1968), No. 4, pp. 416–421

[Joh71] Johnson, C. D.: Accomodation of external disturbances in linear regulator and servomechanism problems. In: *IEEE Transactions on automatic control* 16 (1971), No. 6, pp. 635–644

[Joh75] Johnson, C. D.: On observers for systems with unknown and inaccessible inputs. In: *International Journal of Control* 21 (1975), No. 5, pp. 825–831

[Joh76] Johnson, C. D.: Theory of disturbance accommodating controllers. In: *Control and Dynamic Systems* 12 (1976), pp. 387–489

[JU97] Julier, S. J. ; Uhlmann, J. K.: A new extension of the Kalman filter to nonlinear systems. In: *Int. symp. aerospace/defense sensing, simul. and controls* 3 (1997), pp. 2–3

[Kac79] Kaczorek, T.: Proportional-Integral-Observer for linear multivariable time-varying systems. In: *at-Automatisierungstechnik* 27 (1979), No. 1-12, pp. 359–363

[Kal60] Kalman, R. E.: A new approach to linear filtering and prediction problems. In: *Journal of Fluids Engineering* 82 (1960), No. 1, pp. 35–45

[KC04] Kwon, S. ; Chung, W. K.: Perturbation compensator based robust tracking control and state estimation of mechanical systems. In: *Springer Science & Business Media* 307 (2004)

[Kha96] Khalil, H. K.: Noninear Systems. In: *Prentice-Hall, New Jersey* (1996)

[Kha02] Khalil, H. K.: Nonlinear systems (3rd Edition.). In: *Prentice-Hall* (2002)

[KKK92] Krstić, M. ; Kanellakopoulos, I. ; Kokotović, P. V.: Adaptive nonlinear control without overparametrization. In: *Systems & Control Letters* 19 (1992), No. 3, pp. 177–185

[KKM91] Kanellakopoulos, I. ; Kokotovic, P. V. ; Morse, A. S.: Systematic design of adaptive controllers for feedback linearizable systems. In: *IEEE Transactions on Automatic control* 36 (1991), No. 11, pp. 1241–1253

[KKXX13] Kang, W. ; Krener, A. J. ; Xiao, M. ; Xu, L.: A survey of observers for nonlinear dynamical systems. In: *Data Assimilation for Atmospheric, Oceanic and Hydrologic Applications (Vol. II)*. Springer, 2013, pp. 1–25

[KMO07] Katsura, S. ; Matsumoto, Y. ; Ohnishi, K.: Modeling of force sensing and validation of disturbance observer for force control. In: *IEEE Transactions on Industrial Electronics* 54 (2007), No. 1, pp. 530–538

[KR16] Koch, S. ; Reichhartinger, M.: Observer-based sliding mode control of hydraulic cylinders in the presence of unknown load forces. In: *e & i Elektrotechnik und Informationstechnik* 133 (2016), No. 6, pp. 253–260

[Kra06] Krajcin, I: Einsatz des PI-Beobachters zur modellbasierten Diagnose und Regelung elastischer mechanischer Strukturen. (2006), University of Duisburg-Essen, Ph.D. Thesis

[KS17] Kim, I. H. ; Son, Y. I.: Regulation of a DC/DC boost converter under parametric uncertainty and input voltage variation using nested reduced-order PI-Observers. In: *IEEE Transactions on Industrial Electronics* 64 (2017), No. 1, pp. 552–562

[KSH00] Kailath, T. ; Sayed, A. H. ; Hassibi, B.: Linear estimation. In: *New Jersey, Prentice Hall* (2000)

[LCTT17] Li, X. ; Chen, S. L. ; Teo, C. S. ; Tan, K.: Data-based tuning of reduced-order inverse model in both disturbance observer and feedforward with application to tray indexing. In: *IEEE Transactions on Industrial Electronics* 64 (2017), pp. 5492–5501

[LHCT17] Liang, W. ; Huang, S. ; Chen, S. ; Tan, K. K.: Force estimation and failure detection based on disturbance observer for an ear surgical device. In: *ISA transactions* 66 (2017), pp. 476–484

[Liu11] Liu, Y.: Robust nonlinear control design with Proportional-Integral-Observer technique. (2011), University of Duisburg-Essen, Ph.D. Thesis

[LJ10] Li, W. ; Jia, Y.: H-infinity filtering for a class of nonlinear discrete-time systems based on unscented transform. In: *Signal Processing* 90 (2010), No. 12, pp. 3301–3307

[LJCRSR10] Luviano-Juarez, A. ; Cortes-Romero, J. ; Sira-Ramirez, H.: Synchronization of chaotic oscillators by means of generalized Proportional-Integral-Observers. In: *International Journal of Bifurcation and Chaos* 20 (2010), No. 05, pp. 1509–1517

[LM79] Lückel, J. ; Müller, P. C.: Verallgemeinerte Störgrößenaufschaltung bei unvollständiger Zustandskompensation am Beispiel einer aktiven Federung. In: *at-Automatisierungstechnik* 27 (1979), No. 1-12, pp. 281–288

[LP00] Liu, C. ; Peng, H.: Disturbance observer based tracking control. In: *Journal of Dynamic Systems, Measurement, and Control* 122 (2000), No. 2, pp. 332–335

[LS12] Liu, Y. ; Söffker, D.: Variable high-gain disturbance observer design with online adaption of observer gains embedded in numerical integration. In: *Mathematics and Computers in Simulation* 82 (2012), No. 5, pp. 847–857

[LS14] Liu, Y. ; Söffker, D.: Robust control approach for input-output linearizable nonlinear systems using high-gain disturbance observer. In: *International Journal of Robust and Nonlinear Control* 24 (2014), No. 2, pp. 326–339

[Lue64] Luenberger, D. G.: Observing the state of a linear system. In: *IEEE Transactions on Military Electronics* 8 (1964), No. 2, pp. 74–80

[Lun00] Lunze, J.: An example for the design of switching observers. In: *at-Automatisierungstechnik Methoden und Anwendungen der Steuerungs-, Regelungs-und Informationstechnik* 48 (2000), No. 11/2000, pp. 556–562

[LWA+17] Li, S. ; Wang, H. ; Aitouche, A. ; Tian, Y. ; Christov, N.: Robust unknown input observer design for state estimation and fault detection using linear parameter varying model. In: *Journal of Physics: Conference Series* 783 (2017), No. 1, pp. 012001

[LYCC14] Li, S. ; Yang, J. ; Chen, W. ; Chen, X.: Disturbance observer-based control: Methods and applications. In: *CRC press* (2014)

[MB00] Müller, P. C. ; Başpinar, C.: Convergence of nonlinearity estimations by linear estimators. In: *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik* 80 (2000), No. S2, pp. 325–326

[MD91] Miller, D. E. ; Davison, E. J.: An adaptive controller which provides an arbitrarily good transient and steady-state response. In: *IEEE Transactions on Automatic Control* 36 (1991), No. 1, pp. 68–81

[MH74] Meditch, J. S. ; Hostetter, G. H.: Observers for systems with unknown and inaccessible inputs. In: *International Journal of Control* 19 (1974), No. 3, pp. 473–480

[MH12] Matisko, P. ; Havlena, V.: Optimality tests and adaptive Kalman filter. In: *IFAC Proceedings Volumes* 45 (2012), No. 16, pp. 1523–1528

[ML77] Müller, P. C. ; Lückel, J.: Zur Theorie der Störgrößenaufschaltung in linearen Mehrgrößenregelsystemen/Disturbance rejection control in linear multivariable systems. In: *at-Automatisierungstechnik* 25 (1977), No. 1-12, pp. 54–59

[Moh91] Mohler, R. R.: Nonlinear systems: Applications to bilinear control. In: *Prentice-Hall, Inc.* 2 (1991)

[MT93] Marino, R. ; Tomei, P.: Robust stabilization of feedback linearizable time-varying uncertain nonlinear systems. In: *Automatica* 29 (1993), No. 1, pp. 181–189

[Mül88] Müller, P. C.: Control of nonlinear systems by applying disturbance rejection control techniques. In: *International Conference on Control* (1988), pp. 734–737

[MVKB12] Mintsa, H. A. ; Venugopal, R. ; Kenné, J. ; Belleau, C.: Feedback linearization-based position control of an electrohydraulic servo system with supply pressure uncertainty. In: *IEEE Transactions on Control Systems Technology* 20 (2012), No. 4, pp. 1092–1099

[NBC95] Narendra, K. S. ; Balakrishnan, J. ; Ciliz, M. K.: Adaptation and learning using multiple models, switching, and tuning. In: *IEEE control systems* 15 (1995), No. 3, pp. 37–51

[Nic01] Niculescu, S.: Delay effects on stability: A robust control approach. In: *Springer* 269 (2001)

[NNGR17] Najeh, T. ; Njima, C. B. ; Garna, T. ; Ragot, J.: Input fault detection and estimation using PI-Observer based on the ARX-Laguerre model. In: *The International Journal of Advanced Manufacturing Technology* 90 (2017), No. 5-8, pp. 1317–1336

[NOM87] Nakao, M. ; Ohnishi, K. ; Miyachi, K.: A robust decentralized joint control based on interference estimation. In: *IEEE International Conference on Robotics and Automation* 4 (1987), pp. 326–331

[NSI+13] Naik, V. V. ; Sonawane, D. N. ; Ingole, D. D. ; Ginoya, D. L. ; Patki, V. V.: Design and implementation of Proportional-Integral-Observer-based linear model predictive controller. In: *International Journal on Control System and Instrumentation* 4 (2013), No. 1, pp. 23

[NSL+17] Noshadi, A. ; Shi, J. ; Lee, W. S. ; Shi, P. ; Kalam, A.: Robust control of an active magnetic bearing system using H-inf and disturbance observer-based control. In: *Journal of Vibration and Control* 23 (2017), No. 11, pp. 1857–1870

[NSS97] Niemann, H. H. ; Stoustrup, J. ; Shafai, .B.: Fault detection using Proportional-Integral-Observer. In: *Proc. Control* (1997), pp. 604–609

[OC15] Oh, J. J. ; Choi, S. B.: Real-time estimation of transmitted torque on each clutch for ground vehicles with dual clutch transmission. In: *IEEE/ASME Transactions on Mechatronics* 20 (2015), No. 1, pp. 24–36

[Oga01] Ogata, K.: Modern control engineering. In: *Prentice Hall PTR* (2001)

[OMRM08] Orjuela, R. ; Marx, B. ; Ragot, J ; Maquin, D.: Proportional-Integral-Observer design for nonlinear uncertain systems modelled by a multiple model approach. In: *IEEE Conference on Decision and Control, CDC* (2008), pp. 3577–3582

[PC97] Patton, R. J. ; Chen, J.: Observer-based fault detection and isolation: Robustness and applications. In: *Control Engineering Practice* 5 (1997), No. 5, pp. 671–682

[PFTV92] Press, W. H. ; Flannery, B. P. ; Teukolsky, S. A. ; Vetterling, W. T.: Integration of ordinary differential equations. In: *Numerical Recipes in C, The Art of Scientific Computing* (1992), pp. 707–752

[PLY+11] Park, H. ; Lee, K. ; Yoon, K. ; Lee, W. ; Bae, J. ; Lee, M.: The design of a disturbance observer for moving target tracking of an unmanned firearm robot. In: *11th International Conference on Control, Automation and Systems (ICCAS)* (2011), pp. 1221–1225

[PM11] Pourgholi, M. ; Majd, V. J.: A novel robust Proportional-Integral (PI) adaptive observer design for chaos synchronization. In: *Chinese Physics B* 20 (2011), No. 12, pp. 120503

[Pre07] Press, W. H.: Numerical recipes 3rd edition: The art of scientific computing. In: *Cambridge university press* (2007)

[RGY78] Rabiner, L. R. ; Gold, B. ; Yuen, C. K.: Theory and Application of Digital Signal Processing. In: *IEEE Transactions on Systems, Man, and Cybernetics.* 8 (1978), No. 2, pp. 146

[RHRB17] Rabaoui, B. ; Hamdi, H. ; Rodrigues, M. ; Braiek, N. B.: A polytopic Proportional-Integral-Observer design for fault diagnosis. In: *International Conference on Advanced Systems and Electrical Technologies* (2017), pp. 14–17

[RT06] Respondek, W. ; Tall, I. A.: Feedback equivalence of nonlinear control systems: A survey on formal approach. In: *Chaos in Automatic Control* (2006), pp. 137–262

[SC85] Shafai, B. ; Carroll, R. L.: Design of Proportional-Integral-Observer for linear time-varying multivariable systems. In: *24th IEEE Conference on Decision and Control* 24 (1985), pp. 597–599

[SC12] Shafai, B. ; Cheng, Y.: Design of PI-Observer for bilinear systems with unknown input disturbance. In: *IEEE International Conference on Control Applications (CCA)* (2012), pp. 910–915

[SCY16] Su, J. ; Chen, W. H. ; Yang, J.: On relationship between time-domain and frequency-domain disturbance observers and its applications. In: *Journal of Dynamic Systems, Measurement, and Control* 138 (2016), No. 9, pp. 091013

[Sen95] Sengijpta, S. K.: Fundamentals of statistical signal processing: Estimation theory. In: *Technometrics* 37 (1995), No. 4, pp. 465–466

[Sim06] Simon, D.: Optimal state estimation: Kalman, H infinity, and nonlinear approaches. In: *John Wiley & Sons* (2006)

[SJK12] Son, Y. I. ; Jeong, G. J. ; Kim, I. H.: Disturbance observer based position control of a one-link manipulator under input time delay. In: *International Journal of Innovative Computing, Information and Control* 8 (2012), No. 2, pp. 1371–1384

[SL91] Slotine, J.J. E. ; Li, W.: Applied nonlinear control. In: *Prentice-Hall* (1991)

[SLC15a] Su, J. ; Li, B. ; Chen, W. H.: On existence, optimality and asymptotic stability of the Kalman filter with partially observed inputs. In: *Automatica* 53 (2015), pp. 149–154

[SLC15b] Su, J. ; Li, B. ; Chen, W. H.: Simultaneous state and input estimation with partial information on the inputs. In: *Systems Science & Control Engineering* 3 (2015), No. 1, pp. 445–452

[SLF13] Su, X. ; Li, W. ; Fan, Q.: Sliding mode robustness control strategy of shearer height adjusting system underground coal mines. In: *Journal of Theoretical & Applied Information Technology* 50 (2013), No. 2

[SM03] Süli, E. ; Mayers, D. F.: An introduction to numerical analysis. In: *Cambridge university press* (2003)

[Sor60] Sorenson, H. W.: Kalman filtering: theory and application. In: *IEEE* (1960)

[SPN02] Shafai, B. ; Pi, C. T. ; Nork, S.: Simultaneous disturbance attenuation and fault detection using Proportional-Integral-Observer. In: *Proceedings of American Control Conference* 2 (2002), pp. 1647–1649

[SS15] Shafai, B. ; Saif, M.: Proportional-Integral-Observer in robust control, fault detection, and decentralized control of dynamic systems. In: *Control and Systems Engineering*. Springer, 2015, pp. 13–43

[SSS08] Sadaka, H. ; Shafai, B. ; Sipahi, R.: PI-Observer design for linear time-delay systems. In: *World Automation Congress* (2008), pp. 1–6

[SSS09] Sadaka, H. ; Shafai, B. ; Sipahi, R.: Robust PI-Observer design for linear time-delay systems. In: *Control Applications,(CCA) & Intelligent Control,(ISIC)* (2009), pp. 1209–1213

[Str59] Stratonovich, R. L.: Optimum nonlinear systems which bring about a separation of a signal with constant parameters from noise. In: *Radiofizika* 2 (1959), No. 6, pp. 892–901

[Str60] STRATONOVICH, R. L.: Conditional markov processes. In: *Theory of Probability & Its Applications* 5 (1960), No. 2, pp. 156–178

[SVD02] SCHRIJVER, E. ; VAN DIJK, J.: Disturbance observers for rigid mechanical systems: Equivalence, stability, and design. In: *Journal of Dynamic Systems, Measurement, and Control* 124 (2002), No. 4, pp. 539–548

[SYM95] SÖFFKER, D. ; YU, T. J. ; MÜLLER, P. C.: State estimation of dynamical systems with nonlinearities by using Proportional-Integral-Observer. In: *International Journal of Systems Science* 26 (1995), No. 9, pp. 1571–1582

[TJZ08] THEILLIOL, D. ; JOIN, C. ; ZHANG, Y.: Actuator fault tolerant control design based on a reconfigurable reference input. In: *International Journal of Applied Mathematics and Computer Science* 18 (2008), No. 4, pp. 553–560

[TLLL11] TONG, S. ; LI, Y. ; LI, Y. ; LIU, Y.: Observer-based adaptive fuzzy backstepping control for a class of stochastic nonlinear strict-feedback systems. In: *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)* 41 (2011), No. 6, pp. 1693–1704

[UJDW00] UHLMANN, J. ; JULIER, S. ; DURRANT-WHYTE, H. F.: A new method for the non linear transformation of means and covariances in filters and estimations. In: *IEEE Transactions on automatic control* 45 (2000)

[Utk78] UTKIN, V. I.: Sliding modes and their application in variable structure systems. In: *Mir Publishers* (1978)

[VS00] VAN SCHRICK, D.: Zur Regelung und Überwachung mit PI-Beobachtern- Eine verallgemeinernde Beschreibung (Control and supervision with PI-Observers-A generalizing description). In: *at-Automatisierungstechnik* 48 (2000), No. 10/2000, pp. 496

[WB92] WIE, B. ; BERNSTEIN, D. S.: Benchmark problems for robust control design. In: *Journal of Guidance, Control, and Dynamics* 15 (1992), No. 5, pp. 1057–1059

[WC08] WEN, C. ; CHENG, C.: Design of sliding surface for mismatched uncertain systems to achieve asymptotical stability. In: *Journal of the Franklin Institute* 345 (2008), No. 8, pp. 926–941

[Wel02] WELLS, S. R.: Application of sliding mode methods to the design of reconfigurable flight control systems / DTIC Document. 2002. – Technical Report

[Wie49] WIENER, N.: Extrapolation, interpolation, and smoothing of stationary time series. In: *MIT press Cambridge* 2 (1949)

[Woj78] WOJCIECHOWSKI, B.: Analysis and synthesis of Proportional-Integral-Observer for single-input-single-output time-invariant continuous systems. (1978), Gliwice, Poland, Ph.D. Thesis

[WWD75] WANG, S. ; WANG, E. ; DORATO, P.: Observing the states of systems with unmeasurable disturbances. In: *IEEE Transactions on Automatic Control* 20 (1975), No. 5, pp. 716–717

[WYL15] WANG, N. ; YAO, X. ; LI, W.: Nonlinear disturbance-observer-based sliding mode control for flexible air-breathing hypersonic vehicles. In: *Mathematical Problems in Engineering* (2015)

[XMC⁺14] XU, J. ; MI, C. C. ; CAO, B. ; DENG, J. ; CHEN, Z. ; LI, S.: The state of charge estimation of lithium-ion batteries based on a Proportional-Integral-Observer. In: *IEEE Transactions on Vehicular Technology* 63 (2014), No. 4, pp. 1614–1621

[YCKW17] YOUSSEF, T. ; CHADLI, M. ; KARIMI, H. R. ; WANG, R.: Actuator and sensor faults estimation based on Proportional-integral-Observer for TS fuzzy model. In: *Journal of the Franklin Institute* 354 (2017), No. 6, pp. 2524–2542

[YDL15] YANG, Y ; DING, S. X. ; LI, L: On observer-based fault detection for nonlinear systems. In: *Systems & Control Letters* 82 (2015), pp. 18–25

[YHH⁺17] YANG, B. ; HU, Y. ; HUANG, H. ; SHU, H. ; YU, T. ; JIANG, L.: Perturbation estimation based robust state feedback control for grid connected DFIG wind energy conversion system. In: *International Journal of Hydrogen Energy* 42 (2017), No. 33, pp. 20994–21005

[YJM14] YAO, J. ; JIAO, Z. ; MA, D.: Extended-state-observer-based output feedback nonlinear robust control of hydraulic systems with backstepping. In: *IEEE Transactions on Industrial Electronics* 61 (2014), No. 11, pp. 6285–6293

[YJZ⁺16] YANG, B. ; JIANG, L. ; ZHANG, C. K. ; SANG, Y ; YU, T. ; WU, Q. H.: Perturbation observer-based adaptive passive control for nonlinear systems with uncertainties and disturbances. In: *Transactions of the Institute of Measurement and Control* 40 (2016), pp. 1223–1236

[YLY13] YANG, J. ; LI, S. ; YU, X.: Sliding mode control for systems with mismatched uncertainties via a disturbance observer. In: *IEEE Transactions on Industrial Electronics* 60 (2013), No. 1, pp. 160–169

[YS97] Yang, H. ; Saif, M.: State observation, failure detection and isolation (FDI) in bilinear systems. In: *International Journal of Control* 67 (1997), No. 6, pp. 901–920

[YSS+16] Yang, B. ; Sang, Y. ; Shi, K. ; Yao, W. ; Jiang, L. ; Yu, T.: Design and real-time implementation of perturbation observer based sliding-mode control for VSC-HVDC systems. In: *Control Engineering Practice* 56 (2016), pp. 13–26

[YW88] Yang, F. ; Wilde, R. W.: Observers for linear systems with unknown inputs. In: *IEEE Transactions on Automatic Control* 33 (1988), No. 7, pp. 677–681

[Zei87] Zeitz, M.: The extended Luenberger observer for nonlinear systems. In: *Systems & Control Letters* 9 (1987), No. 2, pp. 149–156

[ZJC16] Zhang, K. ; Jiang, B. ; Cocquempot, V.: Fuzzy unknown input observer-based robust fault estimation design for discrete-time fuzzy systems. In: *Signal Processing* 128 (2016), pp. 40–47

[ZKZL13] Zhang, P. ; Kao, Y. ; Zhu, J. ; Li, W.: Observer-based sliding mode control for uncertain stochastic systems with time-varying delays. In: *Mathematical Problems in Engineering* (2013)

[ZLJ17] Zhang, K. ; Liu, G. ; Jiang, B.: Robust unknown input observer-based fault estimation of leader-follower linear multi-agent systems. In: *Circuits, Systems, and Signal Processing* 36 (2017), No. 2, pp. 525–542

[ZSX10] Zhang, J. ; Shi, P. ; Xia, Y.: Robust adaptive sliding mode control for fuzzy systems with mismatched uncertainties. In: *IEEE Transactions on Fuzzy Systems* 18 (2010), No. 4, pp. 700–711

Intermediate results presented/published in the following conferences/journals or prepared for submission to following journals are stated as an integral part of this thesis:

Articles

[BS14] Bakhshande, F; Söffker, D.: High-Gain Scheduling of the Proportional-Integral-Observer. In: *PAMM, Wiley Online Library*, Vol. 14, 2014, pp. 927-928

[BS15a] Bakhshande, F; Söffker, D.: Proportional-Integral-Observer: A Brief Survey with Special Attention to the Actual Methods using ACC Benchmark. In: *IFAC-PapersOnLine*, Vol. 48(1), 2015, pp. 532-537

[BS15b] Bakhshande, F; Söffker, D.: Reconstruction of Nonlinear Characteristics by Means of Advanced Observer Design Approaches. In: *ASME 2015 Dynamic Systems and Control (DSC) Conference*, Ohio, USA, Vol. 2, 2015, pp. V002T23A007

[BS17a] Bakhshande, F; Söffker, D.: Contact Force Estimation of an Elastic Mechanical Structure using a Novel Adaptive Funnel PI-Observer Approach. In: *ASME Journal of Dynamic Systems, Measurement and Control*, submitted

[BS17b] Bakhshande, F; Söffker, D.: Proportional-Integral-Observer-based Backstepping Approach for Position Control of a Hydraulic Differential Cylinder System with Model Uncertainties and Disturbances. In: *ASME Journal of Dynamic Systems, Measurement and Control*, submitted

[BS17c] Bakhshande, F; Söffker, D.: Proportional-Integral-Observer with Adaptive High-Gain Design using Funnel Adjustment Concept. In: *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, Ohio, USA, 2017, pp. V006T10A011

[BS17d] Bakhshande, F; Söffker, D.: Robust control approach for a Hydraulic Differential Cylinder System using a Proportional-Integral-Observer-based Backstepping Control. In: *American Control Conference 2017*, IEEE, pp. 3102-3107

[BS17e] Bakhshande, F; Söffker, D.: Robust Control of a Hydraulic Cylinder using an Observer-based Sliding Mode Control: Theoretical Development and Experimental Validation. In: *IEEE/ASME Transactions on Mechatronics*, submitted

[BS17g] Spiller, M; Bakhshande, F; Söffker, D.: A comparison of augmented state and minimum variance unbiased joint input-state estimation for practitioners. In: *Journal of the Franklin Institute*, submitted

[BS17h] Spiller, M; Bakhshande, F; Söffker, D.: The uncertainty learning filter: a revised smooth variable structure filter. In: *Signal Processing*, accepted

[BS18] Bakhshande, F; Söffker, D.: Variable Step Size Kalman Filter using Event Handling Algorithm for Switching Systems. In: *ASME 2018 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, accepted

Workshop presentations

[BSW17] Bakhshande, F; Söffker, D.: Robust Control of a Hydraulic Cylinder using Observer-based Nonlinear Controllers: Theoretical Development and Experimental Validation. In: *51. Regelungstechnisches Kolloquium*, Boppard, February 15-17, 2017

[BSW16] Bakhshande, F; Söffker, D.: Adaptive Gain Scheduling of Proportional-Integral-Observer using Funnel Adjustment Concept. In: *GAMM FA Dynamik und Regelungstheorie*, Freiberg, May 12-13, 2016

[BSW15a] Bakhshande, F; Söffker, D.: Robust Estimation of Unknown Inputs by using Adaptive Observer. In: *18. Workshop GAMM FA Dynamik und Regelungstheorie*, Hamburg, March 14-15, 2015

[BSW15b] Bakhshande, F; Söffker, D.: Robust Control of a Hydraulic Cylinder using Observer-based Sliding Mode Control Method. In: *GAMM FA Dynamik und Regelungstheorie*, Duisburg, October 1-2, 2015

[BSW14] Bakhshande, F; Söffker, D.: High-Gain Scheduling of the Proportional-Integral-Observer. In: *GAMM FA Dynamik und Regelungstheorie*, Anif/Salzburg, September 24, 2014

In the context of research projects at the Chair of Dynamics and Control, the following student thesis has been supervised by Fateme Bakhshande and Univ.-Prof. Dr.-Ing. Dirk Söffker. Development steps and results of the research projects and the student thesis are integrated with each other and hence are also part of this thesis.

[Bach15] Bach, R., Conception and DSP-based implementation of an observer-based robust position control on a hydraulic cylinder, Master Thesis, October 2015

In the context of research projects at the Chair of Dynamics and Control, the following student thesis has been supervised by Fateme Bakhshande and Univ.-Prof. Dr.-Ing. Dirk Söffker.

[Spil17] Spiller, M., A Survey on Filtering and Matching Methods for Object Tracking in Image Sequences, Master Thesis, January 2017