FOUR ESSAYS IN DYNAMIC MACROECONOMICS

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Four Essays in Dynamic Macroeconomics

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Abstract

The dissertation contains essays concerning the linkages between macroeconomy and financial market or the conduct of monetary policy via DSGE modelling. The dissertation contributes to the questions of fitting macroeconomic models to the data, and so contributes to our understanding of the driving forces of fluctuations in macroeconomic and financial variables.

Chapter one offers an introduction to my thesis and outlines in detail the main results and methodologies.

In **Chapter two** I introduce a statistical measure for model evaluation and selection based on the full information of sample second moments in data. A model is said to outperform its counterpart if it produces closer similarity in simulated data variance-covariance matrix when compared with the actual data. The "distance method" is generally feasible and simple to conduct. A flexible price two-sector open economy model is studied to match the observed puzzles of international finance data. The statistical distance approach favours a model with dominant role played by the expectational errors in foreign exchange market which breaks the international interest rate parity.

Chapter three applies the distance approach to a New Keynesian model augmented with habit formation and backward-looking component of pricing behaviour. A macro-finance model of yield curve is developed to showcase the dynamics of implied forward

yields. This exercise, with the distance approach, reiterate the inability of macro model in explaining yield curve dynamics. The method also reveals remarkable interconnection between real quantity and bond yield slope.

In **Chapter four** I study a general equilibrium business cycle model with sticky prices and labour market rigidities. With costly matching on labour market, output responds in a hump-shaped and persistent manner to monetary shocks and the resulting Phillips curve seems to radically change the scope for monetary policy because (i) there are speed limit effects for policy and (ii) there is a cost channel for monetary policy. Labour reforms such as in mid-1980s UK can trigger more effective monetary policy. Research on monetary policy shall pay greater attention to output when labour market adjustments are persistent.

Chapter five analyzes the link between money and financial spread, which is oft missed in specification of monetary policy making analysis. When liquidity provision by banks dominates the demand for money from the real economy, money may contain information of future output and inflation due to its impact on financial spreads. I use a sign-restriction Bayesian VAR estimation to separate the liquidity provision impact from money market equilibrium. The decomposition exercise shows supply shocks dominate the money-price nexus in the short to medium term. It also uncovers distinctive policy stance of two central banks.

Finally **Chapter six** concludes, providing a brief summary of the research work as well as a discussion of potential limitations and possible directions for future research.

JEL Classifications: C10; E24; E44; E52; F41.

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Contents

Chapter 1
Introduction

Dynamic macroeconomics has developed at vast speed in recent years. The last two decades, accompanied by theoretical developments and computing power, have witnessed a remarkable development of work in dynamic stochastic general equilibrium models (DSGE) for macroeconomic research. The basic building blocks of these models are well specified decision rules for all agents on the basis of microeconomic foundations.[1] More than just being used for theoretical explorations, as in its earlier stages,[2] DSGE modelling has become very popular as an empirical technique in recent years as well, for instance for short- to medium-term forecasting for central banks (Del Negro and Schorfheide, 2006). In this thesis, I study several aspects of modern business cycles, especially on theoretical and empirical linkages between macroeconomic and financial variables, with DSGE modelling techniques.

The introductory chapter presents a brief discussion of these important but interconnected issues, among which I highlight the main themes of my doctoral research: (1) the evaluation of the empirical fit of DSGE models; (2) the role of exogenous shocks, or forcing processes, and (3) the relative role of nominal versus real rigidities and (4) the role of policy decision rules in changing the dynamics of macro-financial linkages. From the point of view of an empirical macroeconomist, I firstly define the scope and methodology

1 See the Nobel prize lectures by Robert Lucas (1996) and by Finn Kydland and Ed Prescott (2004).

2 See Altug, Chadha and Nolan (2003) for a summary of the early stages of DSGE modelling.

of DSGE framework by summarizing its advancements up to date. Section 1.1 contains a short literature review and motivates the introduction of a new method to evaluate DSGE model and conduct model selection for Chapter two. In the subsequent section I discuss the theoretical components of the dissertation, namely, the role of internal propagation and external shocks in development of dynamic models. Finally in section 1.3, the structure of the dissertation is introduced.

1.1 Dynamic stochastic general equilibrium model with calibration

The first version of modern DSGE analysis is the Real Business Cycle (RBC) originated by Kydland and Prescott (1982) who proposed that dynamic general equilibrium models should be used to evaluate the structure of the macroeconomy. In this case the representative household solves the dynamic programming problem of balancing consumption, savings, work and leisure over time and results in decision rules for each that approximately mimic consumption, investment, work hours and real wages at the business cycle frequency.

Over time macroeconomists began to consider other uncertainties beyond technology disturbances and dynamic models were augmented with other types of exogenous disturbances, such as preference (demand) shocks, monetary policy shocks, mark-up shocks and fiscal shocks, etc. More recently, DSGE modelling become almost a new orthodoxy for an important subset of macroeconomists, monetary policy makers, after several key contribution in this area, including Rotemberg and Woodford's (1997) introduction of New-

Keynesian DSGE models by adding staggered prices, Smets and Wouters's (2003) influential work of DSGE evaluation on Euro Area economy, and Del Negro and Schorfheide's (2006), among others', incorporation of VAR analysis and Bayesian approach to estimating "deep parameters" within the context of DSGE modeling.

The upshot of this work is that modern dynamic macroeconomics has now added DSGE techniques to its toolkit and constructs and analyzes these "larger, computationally-demanding models with multiple disturbances" as standard (Karagedikli *et al*, 2009). There are many critiques of DSGE modeling, see for example, the criticisms of Marcus Miller, Paul De Grauwe and Charles Goodhart, which essentially boil down to either a criticism of the use of the representative agent or the lack of financial frictions. Addressing the fundamental concerns lies beyond the scope of the dissertation, and the focus remains on model evaluation and the taking of the calibrated model to the data.

As suggested by Canova and Ortega (2000), model evaluation and selection themselves can be seen as integrated parts of calibration process. Conventionally a dynamic stochastic model can be solved analytically given an initial calibration of parameter value and exogenous shocks. The paths of endogenous variables can thus be depicted in simulation. The fit of the model is evaluated by 'selecting a metric and comparing the outcomes of the model relative to a set of "stylized facts"', including 'sample statistics of the actual data such as means, variances, correlations' and also impulse response functions of the VAR structure.

Ortega (1996) argued that this approach is in nature *ad-hoc* and lacks rigorous statistical foundation. As opposed to this "Informal Approach", a series of formal statistically grounded procedures have been initiated. Canova and Ortega (2000) provided a comprehensive survey of this pioneering work. Some of these procedures focus on the frequency domain of models and data (Watson, 1993, Diebold, Ohanian and Berkowitz, 1995). Although most of the literature relies on econometric estimation instead of calibration (Christiano and Eichenbaum, 1992, Ireland, 2004). Is there a method in between the two so that computational burden is minimized while distributional information can be largely utilized? An attempt has been made by Canova and Sala (2009) and others by matching impulse responses. In this monograph we refer to Bhattacharjee and Thoenissen (2007) for a model selection procedure in spirit of Ortega's (1996) informal approach. By simply accounting for unconditional second moments contained in variance covariance matrix (VCM) of endogenous variables, we apply some metrics to calculate distance measures used for model evaluation and selection practice.

The method is further applied to two DSGE models, namely, one with New Keynesian Phillips Curve (NKPC) model and the other with open economy macroeconomic model. In the fairly simple and stylized NKPC model, I use inference on the VCM approach to help calibrate the exogenous shocks. For the open economy model, the newly introduced distance measures indicate the best choice of exogenous shocks and deep parameters in a complicated model setup. The method may well be a highly efficient method for empirical analysis of dynamic economy.

1.2 Theoretical issues in a dynamic world

Turning to the theoretical part of the dissertation, several questions stand out as a research agenda for my research. On the linkage of macroeconomic and financial variables, I am particularly interested in following questions: (1) Do DSGE models explain the nexus of macro and financial indicators in business cycles? And to what extent has it evolved over time and across different cycles? (2) How do nominal and real rigidities improve our understanding of these stylized facts? (3) How can we obtain meaningful conclusion on the importance of exogenous shocks and structural parameters in matching facts in actual data? (4) How has monetary policy making interacted with the macro-financial interconnection? What do we learn from the data towards a more efficient monetary policy rule? I try to help formulate an answer one or more of these questions in each of subsequent chapters, by not restricted to these open discussion.

For the theoretical building blocks, I follow the micro-founded approach to developing aggregate behavioural equations but do not confine myself to New Keynesian or real (flex-price) models, much depends on the requirements of specific economic issue. Definitionally, the DSGE modeling framework attributes fluctuations to multiple disturbances that lead to changes in relative prices and quantities, or what are termed state variables. The state variable can be forward-looking or pre-determined. So the thesis also focuses on the specification of exogenous shocks, although the identification procedure of shocks is far from being established as a scientific tool for DSGE macroeconomist.

Calvo (1983) and Yun (1996) built the foundation of New Keynesian macroeconomics by formulating staggered price and wages in an economy with monopolistically competitive firms. Nominal rigidities like these have been seen a textbook answer for the question why money is not neutral in short to medium-term, although the new generation of NK school, the New Neoclassical Synthesis does admit the neutrality of money in long-run (Dixon, 2007).

On top of nominal rigidities, I investigate two types of real rigidities, namely, habit formation in consumption preference (Fuhrer, 2000) and a labour market search and match mechanism (Mortensen and Pissarides, 1994 and Walsh, 2003). These features are appealing as theory and also proved to be relevant in our understanding of UK business cycles. While most DSGE models have been criticized for their complete market settings, I also incorporate incomplete financial market as part of my research agenda. Without digging into more theoretical solution to these questions, I instead focus my efforts on the explaining power of various shocks due to the limitation of empirical tools. Indeed, there are so many of macroeconomic puzzles, much more than what we have learned, that we cannot resolve in a few stylized models. Due to the complexity of interaction among individual agents and the ongoing problems of aggregation across many agents, DSGE models can only serve as a stepping stone for the better understanding of certain aspects of the macroeconomy. This is the case especially when monetary policy plays important role in the nexus between macro and finance linkages.

Based on the analysis of impulse responses of exogenous shocks, there are several contribution made in the dissertation. The most important one is the observed missing role of money in monetary policy analysis and its consequences for policy making, as in chapter five. The chapter shows, with US and UK data, that increase in broad money can be due to both demand and supply factors. But the money supply channel via liquidity provision is absent in a typical NKPC model therefore the consequences for monetary policy may be misleading. Other contributions on the theoretical wing include: (1) Expectational errors in foreign exchange markets are of great significance in understanding puzzles on international risk sharing; (2) A macro-finance yield curve model can explain both macro data and yield curve slope, but not yield curve curvature or level and (3) A labour search and match mechanism make monetary policy making radically different from that of pure inflation-propagation one, and the model implies a bias to unemployment due to greater welfare losses caused by the period of labour market search in a recession. These theoretical findings are suggested by numerical simulation and are also supported by empirical evidence.

Finally, the final stage of my PhD research coincided with the financial turmoil originating in the subprime debt crisis and subsequent credit crunch, which motivated the research work in chapter five. The crisis and recession, although destructive to aggregate financial wealth, by injecting considerable variance into the economy has added to economists' understanding of the macro-financial linkages amid a changing world. In this case, I highlight the role of liquidity shocks in the money market for the supply of loans and its likely impact on asset prices. This is a vivid example why a sensible way of macroeconomic research is to eye the development of exogenous shocks and structure jointly, the main building block of DSGE analysis.

1.3 Outline of the dissertation

The main body of my dissertation is structured as follows. Each of chapter two to chapter five of the dissertation is an independent essay, which tries to answer one or more questions I have raised in this introduction. They each examine issues concerned with the linkages between macroeconomy and financial market or the conduct of monetary policy from both theoretical and empirical angles. Three of the four papers (Chapter two, Chapter three and Chapter four) use dynamic stochastic general equilibrium (DSGE) models and two of the papers (Chapter two and Chapter five) develop testing methodologies for the empirical fit of these models. The dissertation contributes to the questions of fitting macroeconomic models to the data, using a weak interface with the data rather than full-scale estimation, and so contributes to our understanding of the driving forces of fluctuations in macroeconomic and financial variables.

In Chapter two a new statistical measure by Bhattacharjee and Thoenissen (2007) is applied for model evaluation and selection based on the full information of sample second moments in data. A model is said to outperform its counterpart if it produces closer similarity between the simulated data variance-covariance matrix (VCM) when compared with the actual data VCM. I adopt a number of metrics to infer on the statistical divergences in VCMs under the assumption of normally distributed innovations.

The method by Bhattacharjee and Thoenissen (2007) is designed to enhance understanding of rather complicated sophisticated DSGE models which might be intractable in estimation-based measurement. I use a flexible price two-sector open economy model to match the widely-observed puzzles of a poor degree of international risk sharing and exchange rate disconnect. The model identifies a solution to the puzzle by allowing a concurrence of shocks in productivity, preference and interest rate parity condition in two open economies featuring traded and non-traded sectors and an incomplete cross border financial market. Among a group of candidate calibration for the UK-US open economy business cycle data, the statistical distance approach favours a model with a dominant role played by the expectational errors in foreign exchange market which breaks the international interest rate parity condition and hence allows relative consumption to diverge widely from the real exchange rate, which is pinned down by the interest rate parity condition.

Chapter three applies the distance approach to a New Keynesian model augmented with habit formation and backward-looking component of pricing behaviour. A macro-finance model of yield curve is developed to showcase the dynamics of implied forward yields, under pure expectation hypothesis. This exercise, with the distance approach, echoes some well-established empirical findings of NK Phillips curve (see Chadha and Holly, 2006), namely, inability of macro model in explaining yield curve dynamics without careful specification of financial factors. I assess the role played by exogenous shocks, including their changing magnitude through decades. In particular I find the method implies remarkable interconnection between output and the bond yield slope, which may serve as the reason why it can be used to predict business cycle turning point.

In Chapter four I study the implications, for monetary policy and output dynamics, of a general equilibrium business cycle model with sticky prices and labour market rigidities. With costly matching from vacancies to employment, output responds in a hump-shaped and persistent manner to monetary shocks and the resulting Phillips curve seems to change the scope for monetary policy. There are important consequences of output deviations as compared with a model admitting inflation effects alone. This is because (i) there are speed limit effects for policy and (ii) there is a cost channel for monetary policy. Based on simulations I find that labour reforms, like in the UK in the mid-1980s, trigger more effective monetary policy but still cannot explain the very slow response of unemployment to monetary shocks. This work motivates the need for monetary policy to pay greater attention to output when labour market adjustments are persistent, than in the case when inflation alone is thought to be important.

Chapter five explores the missing role of money in monetary policy-making by considering the link between money and the it's cost, the financial spread. When liquidity provision by banks dominates the demand for money from the real economy, money is likely to contain information about future output and inflation because of its impact on financial spreads. I illustrate such linkage graphically and in a small DSGE model but use a sign-restriction Bayesian VAR estimation to unwind the liquidity provision impact from money market equilibrium. The decomposition analysis shows supply shocks dominate the money-price nexus in UK, US and Euro area in the short to medium term. It also provides clues about the distinctive policy stance of three central banks. I conclude that financial spread may be an important policy criterion when the role of liquidity provision dominates as a supply shock to broad money.

Finally Chapter six concludes, providing a brief summary of the research work contained in the thesis as well as a discussion of potential limitations and possible directions for future research.

Chapter 2
Productivity, Preferences and UIP deviations in an Open Economy Business Cycle Model

2.1 Introduction

[3]It is well documented that international risk sharing and the real exchange rate seem to divert far from the levels that would be associated with their complete market allocations. Many authors, originating with Backus and Smith (1993) and Backus, Kehoe and Kydland (1995),[4] have pointed to a lack of aggregate risk sharing across open economies and as an analogue many have also commented on the disconnect between the relative price of goods and their relative consumption, see, for example, Obstfeld and Rogoff (2000) for a summary. We concentrate on a flexible price solution to the problem in the vein on Baxter and Crucini (1995) and Stockman and Tesar (1995) but also allow for financial market imperfections, following Devereux and Engel (2002). We find, within the context of a new methodology for model evaluation of calibrated models, that a two-sector open economy replete with financial market imperfections and driven by productivity, preference and exchange rates that are allowed to deviate stochastically from UIP may provide a reasonably satisfactory contribution to the solution of these puzzles.

3 A paper co-authored with Jagjit S. Chadha and Arnab Bhattacharjee based on this chapter has been accepted by *Open Economies Review* for publication in 2010.

4 Simply put the Backus-Kehoe-Kydland puzzle is that it is income rather than consumption that is more closely correlated across open economies, which suggests that payoffs from idiosyncratic foreign (domestic) income shocks are not being used to smooth domestic (foreign) consumption. The Backus-Smith puzzle is the analogous puzzle that relative consumption across open economies does not arbitrage relative price (real exchange rate) differences.

To understand the puzzles, a few benchmark models have been established. Chari, Kehoe and McGrattan (2002) use price stickiness to achieve volatile and persistent real exchange rate, but fails in resolving Backus-Kehoe-Kydland puzzle. Benigno and Thoenissen (2008) and Stockman and Tesar (1995) both incorporate a non-traded sector and find productivity shocks alone could explain some of irregularities in the data. The later paper even tried a preference shock and justified its role in explaining relative price and relative consumption puzzle.

Motivated by above-mentioned projects, we use a two-sector version of Chari et al (2002), developed by Benigno and Thoenissen (2008), in which there are infinitely-lived representative optimizing households, a two-sector production sector for traded and non-traded goods, where the law of one price holds but where there are also incomplete financial markets. As is well known, under a complete markets environment, cross-country holdings of assets should be sufficient to ensure that consumption rather than income is highly correlated in open economies and that relative consumption responds to changes in relative prices.[5] Because considerable evidence has suggested that international portfolios are home-biased (Tesar and Werner, 1995) and imply that an important channel for risk sharing may be impeded, to some extent, a popular treatment is to introduce incomplete markets by assuming that portfolio diversification relies only on non-state contingent bonds, as in Kehoe and Perri (2002), and accordingly we adopt this feature.[6]

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5 Baxter and Jermann (1997) conclude, under a wealth holding model with a production sector, that domestic individuals should hold only foreign shares against loss caused for labour income by a domestic negative shock.

6 Recently authors such as Sorensen *et al.* (2007) have documented a reduction in home bias but continue to draw a clear link between home bias and

Full price flexibility is maintained in the model but real rigidities are present in the form of a home bias in both consumption and the use of both traded and non-traded goods in output. The model we adopt also allows for costly capital accumulation, an interest rate spread and the possibility of a country being a net creditor (or debtor).[7] The model is driven by three types of shocks: to both traded and non-traded sector productivity; to preferences in the allocation of time between work and leisure of the representative household, and by deviations of the exchange rate from the path expected by relative interest rates (see, Frankel, 1996, and Sarno and Taylor, 2002).

A further contribution of this chapter is the implementation of summary statistics on the distance of each model simulation to the data in the sense of Geweke's (1999) 'weak' interface with the data, provided by Bhattacharjee and Thoenissen (2007). We define a model as a structural set of equations, which are parameterised, and simulated with forcing variables defined over a given variance-covariance matrix (VCM) of shocks. The model then produces an artificial economy which can be thought of as lying some distance from our systematic observations on real-world economies (Watson, 1993). In this sense, the open-economy puzzles drive a large wedge between theory and observation and so we construct a number of empirical measures of this wedge across models and choice of forcing variables to understand which models provide a more satisfactory resolution of the puzzles.

risk sharing. Our set-up is sufficiently flexible to allow us to alter the cost of borrowing from abroad.

7 The importance of these creditor or debtor positions have been explored comprehensively by Lane and Milesi-Ferretti (2002).

Our results suggest that some form of financial market incompleteness will probably be required to solve the open-economy puzzles (as suggested by Engel, 2000). A key result is that price stickiness may not necessarily be required to resolve the puzzles. It turns out that reasonable answers can be found with reference to traded and non-traded forcing processes and by allowing the exchange rate to deviate from the UIP condition. In the former case, with a dominant role for traded over non-traded productivity shocks, in an incomplete financial market, domestic households raise consumption for traded and non-traded goods compared to overseas but the real exchange rate depreciates if the terms of trade effect outweighs the Harrod-Balassa-Samuelson effect (Corsetti *et al.*, 2004). In the case of preference (for work over leisure) shocks, the labor supply curve shifts out and hence demand for goods increases (Hall, 1997) but with an elastic investment supply schedule, and hence output, there is little response in the real exchange rate. And deviations from the uncovered interest rate parity equation for the exchange rate can operate to drive the exchange rate to appreciate even if domestic interest rates fall. Consumption increases in response to the fall in real rates and investment also increases, with wage growth attenuated by the exchange rate appreciation and this results in a reduction in net foreign assets (a current account deficit). Finally, it can also be shown that a combination of these shocks seems to explain the puzzles best.

2.1.1 Some simple observations

We examine open economy data from 24 OECD and emerging country economies. Figure

2.1 gives the descriptive statistics of HP filtered cyclical data and illustrates some clues that the behavior of the current account over the cycle is likely to help explain the puzzles. We note that (i) the real exchange rate is considerably more volatile than relative consumption; (ii) that relative output still seems more correlated than relative consumption; (iii) that current and trade account dynamics follow each other closely and (iv) that the current account is (mostly) countercyclical.

Figure 2.1 is set over four panels. The top left hand panel of Figure 2.1 shows the extent to which the real exchange rate seems noisy and significantly more volatile than its fundamentals would imply. The range for observed volatility of the real exchange rate is between 1-9, with an average, over this dataset of nearly 4. Researchers have explained this high volatility from many dimensions in the literature.[8] And certainly, we find that compared to relative consumption, which ranges from 0.5 to just under 3, the real exchange rate does look 'disconnected'. The top right hand side panel of Figure 2.1 scatters the correlation of national consumption of the economies with US consumption against the correlation of output with US output and suggests in general that output is more closely related across countries than consumption, which implies somewhat less than perfect risk sharing.

8 These explanations include price stickiness and the famous case of exchange rate overshooting (Dornbusch, 1976).

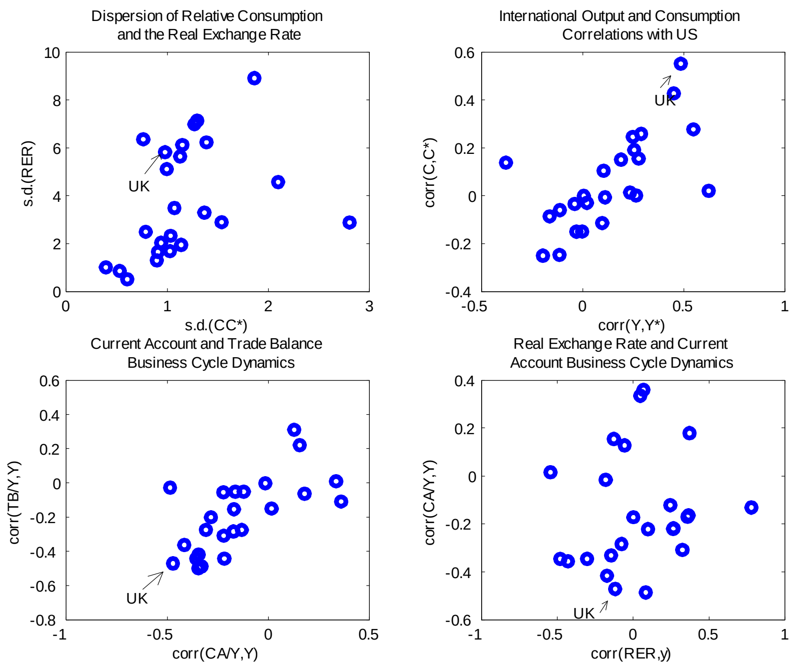


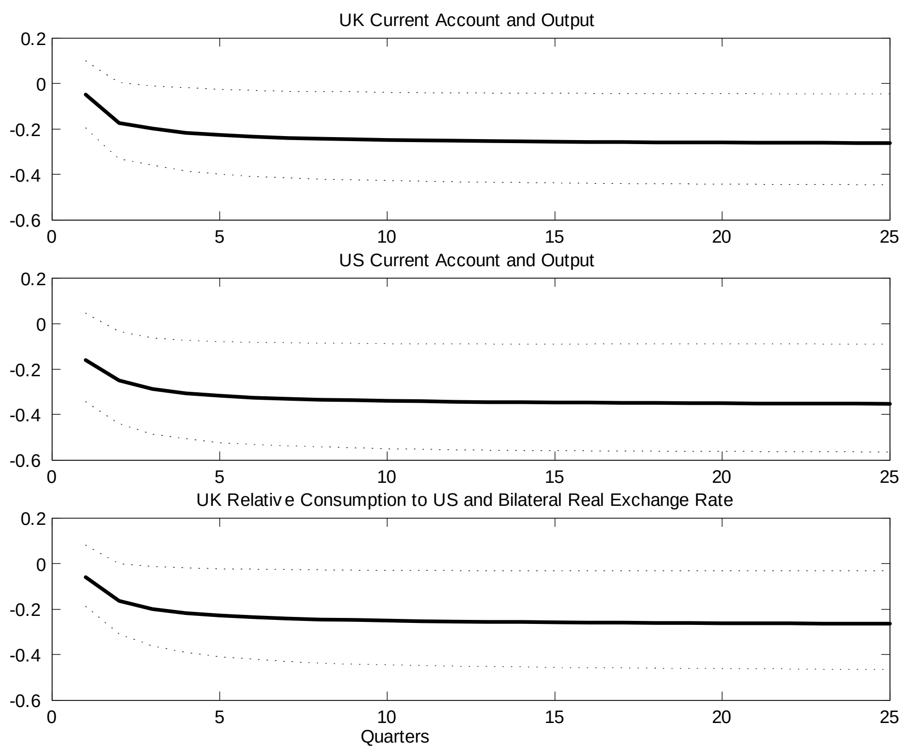
Figure 2.1 - International Economy Stylised Facts

Note: Quarterly data from 1980 to 1998 for 24 OECD and emerging market economies is obtained from the IMF IFS database. s.d. denotes standard deviation of HP-filtered series of the variables. corr denotes the correlation coefficient between two HP-filtered series. RER denotes bilateral real exchange rate. C, C*, Y, Y* are household consumption and real GDP of small open economy and US respectively. CC* is the relative consumption to US. TB/Y is the ratio of trade balance to output and CA/Y the ratio of current account to output.

The left hand lower panel of Figure 2.1 shows the close correspondence between the business cycle dynamics of the current account and the trade balance over the business cycle across these economies - suggesting a strong role for intertemporal trade over the business cycle with some deviation from complete markets as the balance on the trade account is not offset by returns from assets held overseas.[9]

[9] The finding that the current account is likely to play an important role in the resolution of puzzles has two implications for our work, we will want to adopt a model where current account dynamics play an important role and assess the fit of any models we develop with, inter alia, their match to current account data.

Finally, the lower right hand side panel of Figure 2.1 suggests that the current account tends to be countercyclical (with a deficit under an economic expansion). But that the real exchange rate looks as likely to appreciate or depreciate over the same economic cycle. Put alternatively, there is a higher demand for foreign assets during an expansion (with current account output correlations negative) but that the real exchange rate plays a limited role in choking off that higher demand.



Note: Forecast error correlation calculated with Den Haan (1996) code. Solid lines denote correlations and the dotted lines represent a 90% conf idence interv al of point estimate.

Figure 2.2 - Price Stickiness

A second modelling question concerns whether price stickiness is required for the resolution of the puzzles. Figure 2.2 shows the forecast error correlation of up to 25 quarters of US and UK current account and real exchange rate and relative consumption and the real exchange rate (den Haan, 2000). The panels show that over the long run, these quantities are countercyclical but over the short term, all three measures somewhat less so. As price stickiness can be expected to play a less important role in long run dynamics, than in short run, there is some initial motivation for excluding this feature from our model.

The rest of the chapter is organized as follows. Section 2.2 describes the model, section 2.3 outlines the solution technique and model calibration, section 2.4 offers the model results, section 2.5 compares the model to the data VCM and section 2.6 concludes. Appendices A and B offer more detail on model, shock selection and the evaluation methodology.

2.2 The Model

This section describes the baseline model. Essentially, we take the flexible price two-country, two sector model derived by Benigno and Thoenissen (2008) and emphasize the specification of driving forces as in Chadha, Janssen and Nolan (2001). The model is driven variously by forcing variables in domestic and overseas traded and non-traded productivity shocks, domestic and overseas preference shocks and by deviations from the UIP condition for the exchange rate.

2.2.1 Consumer behavior

We adopt a two-country model. Consumers are infinitely lived. The world economy is populated by a continuum of agents on the interval $[0, 1]$, with the segment $[0, n)$ belonging to the country H (Home) and the population on segment $[n, 1]$ belonging to the F (Foreign) country. Preferences for the Home consumer (with an identical set-up for the foreign consumer) are described by the utility function:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j, \xi_{C,s}) V(l_s^j) \right], \tag{2.1}$$

where E_t denotes the expectation conditional on the information set at date t, and β is the intertemporal discount factor, with $0 < \beta < 1$. The Home consumer obtains utility from consumption, C^j, and receives disutility from supplying labor, l^j. $\xi_{C,s}$ is a stochastic disturbance affecting the utility the agent receives from a unit of consumption.

The asset market structure in the model is standard and is described in detail in Benigno (2001) and Benigno and Thoenissen (2008). Home individuals are able to trade two nominal bonds denominated in the domestic and foreign currency. The bonds are issued by residents in both countries in order to finance their consumption expenditure. Foreign residents, on the other hand, can allocate their wealth only in bonds denominated in the foreign currency. Home households face a cost when they take a position in the foreign bond market. As in Benigno (2001), this transaction cost depends on the net foreign asset position of the home economy.[10]

The Home consumer maximizes utility subject to the following budget constraint:

$$P_t C_t^j + \frac{B_{H,t}^j}{(1+i_t)} + \frac{S_t B_{F,t}^j}{(1+i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)} = B_{H,t-1}^j + S_t B_{F,t-1}^j + P_t w_t l_t^j + \Pi_t^j \tag{2.2}$$

where P_t is the price index corresponding to the basket of final goods C, w is the real

10 Alternative ways of closing open economy models are discussed in Schmitt-Grohe and Uribe (2003).

wage earned by agent in return for supplying labor and Π are dividends received by the agent from holding an equal share of the economy's intermediate goods producing firms.

Home agents can hold two types of nominal, non-state contingent bonds. B_H^j denotes agent j's holdings of Home-currency denominated bonds. The one-period return from these bonds is denoted by $(1 + i_t)$. S denotes the nominal exchange rate, defined as Home currency price of a unit of foreign currency. B_F^j denotes agent j's holdings of Foreign-currency denominated bonds. The one-period return from foreign-currency denominated bonds is $(1 + i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)$, where $(1 + i_t^*)$ is the gross rate of return and $\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)$ is a proportional cost associated with foreign currency-denominated bond holding that depends on the economy-wide holdings of foreign-currency denominated bonds.[11]

The first order condition of the representative consumer can be summarized as follows:

$$U_{c,t} = (1 + i_t)\beta E_t \left[U_{c,t+1} \frac{P_t}{P_{t+1}}\right] \tag{2.3}$$

$$U_{c,t+1} = (1 + i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)\beta E_t \left[U_{c,t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}}\right]. \tag{2.4}$$

$$U_{c,s} w_t = V_l(l_s) \tag{2.5}$$

where $U_{c,t} \equiv U_c(C_t, \xi_{C,t}, 1 - l_t)$ and where there is an analogous intertemporal condition to (2.3) for the Foreign consumer. As in Benigno (2001), we assume that all individuals

[11] The factor of proportionality $\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)$ is equal to unity only when economy-wide bond holdings are at their initial steady state level, thus ensuring that in the long-run the economy returns to its initial steady state level of bond holdings.

belonging to the same country have the same level of initial wealth. This assumption, along with the fact that all individuals face the same labor demand and own an equal share of all firms, implies that within the same country all individuals face the same budget constraint and so they will choose identical paths for consumption. As a result, we are able to drop the j superscript and focus on a representative individual for each country.

2.2.2 The supply side

There are three layers of production in this economy. Final goods are produced by a competitive final goods producing sector using Home traded and non-traded intermediate goods as well as foreign-produced traded intermediate-goods. Final goods are non-traded and are either consumed or used as investment goods to augment the domestic capital stock. Intermediate goods producers combine labor and capital according to a constant returns to scale production technology. Each country produces two types of intermediate goods, a differentiated traded good and a non-traded good.

2.2.2.1 Final good producers Let Y be the output of final goods produced in the home country. Final goods producers combine domestic and foreign-produced intermediate goods to produce Y in a two-step process. The final good Y is made up of traded, y_T, and non-traded inputs, y_{NT}, combined in the following manner:

$$Y = \left[\omega^{\frac{1}{\kappa}} y_T^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} y_N^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}, \tag{2.6}$$

where ω is the share of traded goods in the final good, and κ is the intratemporal elasticity of substitution between traded and non-traded intermediate goods. The traded component, $y_{T,}$ is, in turn, produced using home and foreign-produced traded goods (y_H and y_F respectively) in the following manner:

$$y_T = \left[v^{\frac{1}{\theta}} y_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} y_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{2.7}$$

where v is the domestic share of home produced traded intermediate goods in total traded intermediate goods and θ is the elasticity of substitution between home and foreign-produced traded goods. Final goods producers are competitive and maximize profits, where P is the aggregate or sectoral price index and Y the aggregate output; therefore, they maximise the profits

$$\max_{y_N, y_H, y_F} PY - P_T y_T - P_N y_N, \tag{2.8}$$

subject to (2.7), where traded goods' output is maximized subject to the value of home and foreign traded goods.

This maximization yields the following input demand functions for the home and foreign (not shown but identical) firm:

$$\begin{aligned} y_N &= (1-\omega) \left(\frac{P_N}{P} \right)^{-\kappa} Y \qquad (2.9) \\ y_H &= \omega v \left(\frac{P_H}{P_T} \right)^{-\theta} \left(\frac{P_T}{P} \right)^{-\kappa} Y \\ y_F &= \omega (1-v) \left(\frac{P_F}{P_T} \right)^{-\theta} \left(\frac{P_T}{P} \right)^{-\kappa} Y. \end{aligned}$$

The price index that corresponds to the above maximization problem is:

$$
\begin{aligned}
P_T^{1-\theta} &= [vP_H^{1-\theta} + (1-v)P_F^{1-\theta}] \qquad (2.10)\\
P^{1-\kappa} &= [\omega P_T^{1-\kappa} + (1-\omega)\,P_N^{1-\kappa}],
\end{aligned}
$$

And the goods produced in the final goods sector are only used domestically, either for consumption or investment, x_t, for home and overseas:

$$
Y_t = C_t + x_t. \qquad (2.11)
$$

2.2.2.2 Traded-intermediate goods sector Firms in the traded intermediate goods sector produce goods using capital and labor services. The typical firm maximizes the following profit function:

$$
\max P_{H_t} y_{H_t} + S_t P_{H_t}^{*} y_H^{*} - P_t w_t l_{H,t} - P_t x_{H,t}, \qquad (2.12)
$$

or because the law of one price holds at the wholesale level,

$$
\max_{H_t} P_{H_t}\left(y_{H_t} + y_H^{*}\right) - P_t w_t l_{H,t} - P_t x_{H,t}.
$$

This maximization is subject to:

$$
\begin{aligned}
y_{H_t} + y_{H_t}^{*} &= F(k_{H,t-1}, l_{H,t}) = \left(A_t l_{H,t}\right)^{\alpha} k_{H,t-1}^{1-\alpha} \qquad (2.13)\\
k_{H,t} &= (1-\delta)k_{H,t-1} + x_{H,t} - \phi\left(\frac{x_{Ht}}{k_{Ht-1}}\right) k_{Ht-1},
\end{aligned}
$$

where $\phi(.)$ denotes the cost for installing investment goods.[12]

Then, the stochastic maximization problem of the domestic intermediate goods firm is given by:

$$L = \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \frac{U_{c,t}}{P_t} \left\{ \begin{array}{c} \left[P_{H,t} \left(A_t l_t \right)^{\alpha} (k_{H,t-1})^{1-\alpha} - P_t w_t l_{H,t} - P_t x_{H,t} \right] \\ +\lambda_t \left[\begin{array}{c} (1-\delta) k_{H,t-1} + x_{H,t} \\ -\phi \left(\frac{x_{H,t}}{k_{Ht-1}} \right) k_{H,t-1} - k_{H,t} \end{array} \right] \end{array} \right\}. \tag{2.14}$$

The first order conditions with respect to the labor input, investment and capital are given by:

$$P_t w_t = \alpha P_{H,t} (A_t)^{\alpha} (\frac{k_{H,t-1}}{l_{H,t}})^{1-\alpha}, \tag{2.15}$$

$$P_t = \lambda_t - \phi' \left(\frac{x_{H,t}}{k_{H,t-1}} \right) \lambda_t, \tag{2.16}$$

$$\lambda_t = \mathrm{E}_t \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \left\{ \begin{array}{c} P_{H_{t+1}} (1-\alpha) \left(\frac{A_{t+1} l_{H,t+1}}{k_{Ht}} \right)^{\alpha} + \\ \lambda_{t+1} \left[(1-\delta) - \phi \left(\frac{x_{Ht+1}}{k_{H,t}} \right) + \phi' \left(\frac{x_{H,t+1}}{k_{H,t}} \right) \frac{x_{H,t+1}}{k_{H,t}} \right] \end{array} \right\}. \tag{2.17}$$

And using the expression for $P_{H,t}$ from the wage equation yields:

$$\lambda_t = \mathrm{E}_t \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \left\{ \begin{array}{c} \frac{(1-\alpha)}{\alpha} \left(\frac{l_{t+1}}{k_t} \right) P_{t+1} w_{t+1} + \\ \lambda_{t+1} \left[(1-\delta) - \phi \left(\frac{x_{H,t+1}}{k_{H,t}} \right) + \phi' \left(\frac{x_{H,t+1}}{k_{H,t}} \right) \frac{x_{H,t+1}}{k_{H,t}} \right] \end{array} \right\}.$$

Next, we substitute in the expression for λ to obtain:

---

12 Following Benigno and Thoenissen (2008), in steady state: $\phi(.) = x/k, \phi'(.) = 1, \phi''(.) = b < 0$.

$$U_{c,t} = \left[1-\phi'\left(\frac{x_t}{k_{t-1}}\right)\right]\mathrm{E}_t\beta U_{c,t+1}w_{t+1}\frac{f_{k_{t+1}}}{f_{l_{t+1}}} + \tag{2.18}$$
$$\mathrm{E}_t\beta\frac{1-\phi'\left(\frac{x_t}{k_{t-1}}\right)}{1-\phi'\left(\frac{x_{t+1}}{k_t}\right)}U_{c,t+1}\left[(1-\delta)-\phi\left(\frac{x_{H,t+1}}{k_{H,t}}\right)+\phi'\left(\frac{x_{H,t+1}}{k_{H,t}}\right)\frac{x_{H,t+1}}{k_{H,t}}\right],$$

where f_{k_t} is the marginal product of capital and $f_{l_{t+1}}$ the marginal product of labor and w_{t+1} is the real wage, $U_{c,t} \equiv U_c(C_t, \xi_{C,t}, 1-l_t)$.

2.2.2.3 Non-traded-intermediate goods sector The non-traded intermediate goods producer has the similar maximization problem:

$$\max P_{N_t}y_{N_t} - P_tw_tl_{N,t} - P_tx_{N,t}, \tag{2.19}$$

which is subject to

$$y_{N_t} = F(k_{t-1,}l_{N,t}) \tag{2.20}$$
$$k_{N,t} = (1-\delta)k_{N,t-1} + x_t - \phi\left(\frac{x_{N,t}}{k_{N,t-1}}\right)k_{N,t-1},$$

If we now set up the stochastic maximization problem of the domestic intermediate goods firm:

$$L = \mathrm{E}_t\sum_{t=0}^{\infty}\beta^t\frac{U_{c,t}}{P_t}\left\{\begin{array}{c}\left[\begin{array}{c}P_{N,t}\left(A_{N,t}l_{N,t}\right)^{\alpha}\left(k_{N,t-1}\right)^{1-\alpha}\\ -P_tw_tl_{Nt} - P_tx_{N,t}\end{array}\right]\\ +\lambda_t\left[\begin{array}{c}(1-\delta)k_{N,t-1}+x_{N,t}\\ -\phi\left(\frac{x_{N,t}}{k_{N,t-1}}\right)k_{N,t-1}-k_{N,t}\end{array}\right]\end{array}\right\}. \tag{2.21}$$

The first order condition with respect to labor input is then given by:

$$P_tw_t = \alpha P_{N,t}(A_{N,t})^{\alpha}(\frac{k_{N,t-1}}{l_{N,t}})^{1-\alpha}.$$

The first order condition with respect to investment is:

$$P_t = \lambda_t - \phi'\left(\frac{x_{N,t}}{k_{N,t-1}}\right)\lambda_t.$$

The first order condition with respect to capital is:

$$\lambda_t = \mathrm{E}_t\beta\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\left\{\begin{array}{c} P_{N_{t+1}}(1-\alpha)\left(\frac{A_{t+1}l_{N,t+1}}{k_{N,t}}\right)^{\alpha}+ \\ \lambda_{t+1}\left[(1-\delta)-\phi\left(\frac{x_{N,t+1}}{k_{N,t}}\right)+\phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)\frac{x_{N,t+1}}{k_{N,t}}\right]\end{array}\right\}, \quad (2.22)$$

and using the expression for P_N from the wage equation yields:

$$\lambda_t = \mathrm{E}_t\beta\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\left\{\begin{array}{c} \frac{(1-\alpha)}{\alpha}\left(\frac{l_{N_{t+1}}}{k_{N_t}}\right)P_{t+1}w_{t+1}+ \\ \lambda_{t+1}\left[(1-\delta)-\phi\left(\frac{x_{N,t+1}}{k_{N,t}}\right)+\phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)\frac{x_{N,t+1}}{k_{N,t}}\right]\end{array}\right\}.$$

As before, substituting in the expression for λ, we have

$$\begin{aligned} U_{c,t} \quad = \quad & \left[1-\phi'\left(\frac{x_{N,t}}{k_{Nt-1}}\right)\right]\mathrm{E}\beta U_{c,t+1}w_{t+1}\frac{f_{k_{t+1}}}{f_{l_{t+1}}}+ \qquad (2.23) \\ & \mathrm{E}_t\beta\frac{1-\phi'\left(\frac{x_{N,t}}{k_{N,t-1}}\right)}{1-\phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)}U_{c,t+1}\left[(1-\delta)-\phi\left(\frac{x_{N.t+1}}{k_{N,t}}\right)+\phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)\frac{x_{N,t+1}}{k_{N,t}}\right]. \end{aligned}$$

2.2.3 The real exchange rate

In this model, the real exchange rate is defined as:

$$RS_t = \frac{S_t P_t^*}{P_t} \qquad (2.24)$$

and can deviate from purchasing power parity (PPP) as a result of three channels. As in Benigno and Thoenissen (2008), allowing for the possibility of home bias in consumption ($v > v^*$), via the terms of trade channel (because of home bias) and via the internal real exchange rate channel (because of non-traded goods), (2.24) can be expanded to give:

$$\frac{S_t P_t^*}{P_t} = \frac{S_t P_{H,t}^*}{P_{H,t}}\frac{P_{H,t}}{P_{T,t}}\frac{P_{T,t}^*}{P_{H,t}^*}\frac{P_{T,t}}{P_t}\frac{P_t^*}{P_{T,t}^*},$$

which when linearized around the steady state, where $\frac{SP^*}{P}$ equals unity, can be shown to be equal to:

$$\widehat{RS}_t = (v - v^*)\hat{T}_t + (\omega - 1)\,\hat{R}_t + (1 - \omega^*)\,\hat{R}_t^*. \tag{2.25}$$

The deviation of the real exchange rate around its steady state depends on deviations of the home and foreign retail to wholesale price ratios, the terms of trade, T, defined as $\frac{P_F}{P_H}$, and the relative price of non-traded to traded goods, R.

2.2.4 The current account

The current account is defined as changes in foreign asset holding, within the incomplete financial market. Home and foreign agents trade intermediate goods and the trade balance is used to buy foreign bonds and so the flow budget constraint shows the current account dynamics below. The left hand side is the changes in foreign asset holding. The right hand side shows the total production (first two terms) minus consumption and investment, yielding adjustment of bond wealth:

$$\frac{S_t B_t^F}{P_t\,(1+i_t^*)}\frac{1}{\Theta\left(\frac{S_t B_t^F}{P_t}\right)} - \frac{S_t B_{t-1}^F}{P_t} = \frac{P_{H_t}}{P_t}\,(y_{H_t} + y_H^*) + \frac{P_{N_t}}{P_t} y_{N_t} - C_t - x_t. \tag{2.26}$$

2.2.5 Forcing variables

We adopt the specification of Stockman and Tesar (1995) and Chadha, Janssen and Nolan (2001) by investigating the role of both productivity and preference shocks for an open economy. We use both traded sector and non-traded sector productivity, which drive the input and hence product price, shocks to the allocation of time spent in work over leisure, which affects labor supply, and to stochastic deviations in the UIP condition, which directly affects the terms of trade. Each shock originates from a different sector but allows us to attribute exchange rate volatility to more than one exogenous factor. In total, we enable seven shocks (two sectoral and a preference shock in each of two countries, plus UIP deviations) and try to locate the importance in explaining open economy business cycles. The construction of each shock process is explained in Appendix A.

2.3 Solution and Model Calibration

2.3.1 Solution method

Before solving the model, it is log-linearized around the steady state to obtain a set of equations describing the equilibrium fluctuations of the model. The log-linearization yields a system of linear difference equations which we list in an appendix and can be expressed as a singular dynamic system of the following form:

$$\mathbf{A}\mathrm{E}_t\mathbf{y}(t+1 \mid t) = \mathbf{B}\mathbf{y}(t) + \mathbf{C}\mathbf{x}(t)$$

where $\mathbf{y}(t)$ is ordered so that the non-predetermined variables appear first and the predetermined variables appear last, and $\mathbf{x}(t)$ is a martingale difference sequence. There are up to seven shocks in $\mathbf{C}$. The variance-covariance as well as the autocorrelation matrices associated with these shocks are described in Table 2.1. Given an initial parametrization of the model, which we describe in the next section, we solve this system using the King and Watson (1998) solution algorithm.

2.3.2 Data and calibration

Table 2.1 summarizes the calibration parameters for the baseline simulation of the model. We collect both quarterly and annual data and calibrate the model for the pair of countries – the UK and the US. Values of parameters are either estimated from US or UK data or taken from extant literature. An annual risk free rate of 4% and depreciation at 10% is assumed. Labor share is calibrated at 0.67 for the UK and the US. We take the consumption and leisure curvature of 2 (Corsetti *et al.*, 2004) and 4 (Chadha *et al.*, 2001). The elasticity of substitution between home and foreign goods in UK is 1.5 as in Chari *et al.* (2002). For the trade-off between traded and non-traded goods we adopt the elasticity suggested by Corsetti *et al.* (2004) of 0.74. UK and US trade data reveals the shares of UK produced goods in UK and US production to be 0.73 and 0.0157. Traded goods weights in all household consumption are estimated to be 0.3 and 0.24, smaller than that of Corsetti *et al.* (2004), 0.45 to 0.5. Cost of financial intermediation is 70bp as in Selaive and Tuesta (2003). The cost of investment, $b = 2$, is chosen to match the relative volatility of investment.[13] Steady state of net foreign asset is set to be 0 or 0.5 which means, respectively, that the UK has a balanced current account or is a creditor.

We have at most seven exogenous shocks in our experiments. The vector of shocks Π_t are assumed to follow a VAR(1) process:

[13] However we also run experiments with $b = 5$, which are available on request and covered in the robustness exercise of Figure 2.7.

$$\Pi_{t+1} = A\Pi_t + U_{t+1}$$
$$U_{t+1} \sim N(0, \Sigma)$$

Table 2.1 - Quarterly Calibration for Small Open Economy Model

Parameter	Values	Description
β	0.99	Discount factor
δ	0.025	Depreciation factor
α	0.67	Labor share
ρ	2	CRRA
η	-4	Elasticity of marginal value of time
θ	1.5	Elasticity: Home/Foreign traded goods
κ	0.74	Elasticity: Traded/Non-traded goods
(υ, υ^*)	$(0.73, 0.02)$	Home prod. share of tradeables (home, overseas)
(ω, ω^*)	$(0.45, 0.45)$	Share of tradeables in output (home, overseas)
ε	70 basis points	Interest spread (quarterly)
$\overline{a}$	0	Steady state Net Foreign Asset
b	10	Cost of capital adjustment
(ρ_A, ρ_{A^*})	0.918	Persistence of traded productivity shocks
(σ_A, σ_{A^*})	$(1.17\%, 1.41\%)$	Volatility of traded productivity shocks
(ρ_{AN}, ρ_{AN^*})	0.945	Persistence of non-traded productivity shocks
$(\sigma_{AN}, \sigma_{AN^*})$	$(0.51\%, 0.56\%)$	Volatility of non-traded productivity shocks
(ρ_ξ, ρ_{ξ^*})	0.937	Persistence of preference shocks
$(\sigma_\xi, \sigma_{\xi^*})$	$(0.82\%, 0.82\%)$	Volatility of preference shocks
$(\rho_{UIP^H}, \rho_{UIP^L})$	0.88 or 0.38	Persistence of UIP deviations (high or low)

Note: We have an utility function similar to Chadha et al. (2001). The elasticity of intertemporal substitution in leisure $\frac{1}{\eta-1}$ is -0.2; the elasticity of labor supply in this model is around 4; the discount factor β, CRRA ρ, depreciation coefficient δ and labor share α are taken from standard open economy and real business cycle literature such as Corsetti et al. (2005), Chari et al. (2002); we take elasticity of substitution among consumables θ, κ from Corsetti et al.; the share of traded goods ω, ω^* are taken as 0.45 in accordance with the literature; for home bias feature in traded goods, we take average value share of UK produced goods in UK and US GDP, υ and υ^*, respectively; interest spread ε is a yield discount when holding foreign bond and is calibrated as 280 base points annually by Selaive and Tuesta (2003); the cost of capital adjustment b is calibrated to match UK output volatility; we set Net Foreign Asset position $\overline{a}$ as zero in benchmark case; the persistence and volatility of shocks are estimated on UK data.

2.4 Model Results

We now turn to the evaluation of the structural linear model by its simulation and comparison to our observations on the economy. To put it in a simple way, the section tries to evaluate the open economy model not only on some random dimensions, but on all those covariances among variables. A typical DSGE model builder runs simulations with a calibrated model, but conduct the model-data comparison only on limited dimensions. Although it is acceptable due to model complexity, it remains as an weakness of DSGE analysis. In this section and Appendix C, a new method by Bhattacharjee and Thoenissen (2007) is shown to overcome the weakness.

2.4.1 Methodology

Conventional tools such as the impulse response function and variance decomposition help us understand the dynamics of an artificial economy. The standard practice is also to assess models against some selected second moments of the data. But in this chapter we utilize criteria that takes into account all the second moments and evaluate model performance based on formal statistical measures. We indicate a better model, as one that can render a better match between VCM of the data and the VCM simulated by the model. In order to pin down some parameter value or decide on certain features of a model, we work on a class of candidate models (or calibrations). By examining the corresponding match for candidate models, we call any improvement towards the criterion a gain in marginal information. We also evaluate the gain on a particular parameter, by which we can signal the importance of any one feature of the model. Strictly speaking, we cannot guarantee the marginal information gain is reliable, or nearer to the 'true' model, unless we are quite certain about the rest of the model. The proposition of a marginal information gain we make is therefore a 'weak-form' of model selection (see Geweke, 1999).

The criteria we use involve the statistical divergence of the two VCMs. We develop formal and also intuitive distance measures elsewhere but some details are available in Appendix C. A higher value of distance denotes a model that is further from our measure on 'true' data process.[14] The data required to evaluate the open economy model is of high dimension and a relatively short sample, which tends to make model evaluation and selection very challenging problems. We calculate for each candidate model a distance and compare across each measure. We are cautious in making a proposition of model selection, especially for a particular parameter constellation, but feel able to make some statements on the validity of the joint choices on model and shock processes.

As argued by Bhattacharjee and Thoenissen (2007), this class of proposed approach have several appealing features. First of all, the method is extremely easy to perform. Secondly, it is less vulnerable to Lucas Critique as it does not augment the model with any stochastic error other than those the theory suggests, such as Watson (1993) and Ireland's (2004) estimation method. Some methods are based on VAR analysis, including

[14] In developing this approach, we use Monte Carlo simulations on some artificial models. We find: (1) this approach works very well , particularly if the multivariate normality is approximately tenable; (2) our approach helps overcome small-sample bias, and (3) the model selection outcome depends quite strongly on the sub-block of the full VCM chosen for comparison. In other words, the choice of state variables is very crucial, and has to be made carefully based on the specifics of the application considered.

Rotemburg and Woodford (2007), which receives no support from micro-foundation of macroeconomics.

However, the method is also constrained by some shortcomings. Particularly, the method ignores conditional moments of the data, therefore is less convincing than estimation-based approached if conducted for simpler models. Del Negro and Schorfheide (2006), among others that rely on Bayesian method, is superior in this sense that Bayesian estimation uses the full information of time series data. Secondly, even some candidate models are ordered by distance metrics, it is a weak proposition to say how much model 'A' is better than model 'B'. Future study need to quantify the improvement via marginal information gain.

2.4.2 Impulse responses

The impulse response functions are based on the seven-shock model.[15] In this calibration, the foreign country has the same properties as home, such as shares of traded and home goods on market. As an alternate specification, we consider $v = 0.85$ and $v^* = 0.15$, the home produced share of tradeables in intermediate goods production at home and overseas respectively, in order to highlight the effect of foreign sector.

2.4.2.1 Traded productivity shocks Figure 2.3 plots the response of quantities and relative prices to a traded productivity shock in the home country. The response of real exchange rate depends on two effects: the terms of trade and the Harrod-Balassa-Samuelson

15 The construction and estimation of shocks is outlined in appendix A: traded and non-traded productivity shocks in A.1, preference shocks in A.2 and stochastic deviations from UIP in A.3.

(HBS) effect. The former requires an adjustment in relative traded prices, which requires a depreciation in the real exchange rate in the long run. However, the latter effect drives up wages in both the traded and non-traded sector but with no productivity improvement in the non-traded sector, non-traded prices will rise and hence so will the real exchange rate. This effect is especially strong, see section 2.2.3, when there is a home bias in consumption, which acts to accentuate the real exchange rate change. Finally, the lack of complete risk sharing means that consumption is more elastic to a productivity shock than under a complete markets allocation. The combination of forward-looking domestic consumption responding to higher productivity (income) but an attenuated overall investment response - where traded sector investment rises but non-traded sector investment falls - leads to the accumulation of foreign debt to finance current demand.

2.4.2.2 Non-traded productivity shocks Following a non-traded productivity shock (Figure 2.4), investment and labor increase. Home households enjoy somewhat higher consumption in this case, more so than in the case of traded sector productivity shock. In this case, the terms of trade effect and HBS effect are the same, causing the real exchange rate to depreciate. Although the response of relative consumption is positive, it is not large enough to bring about a current account deficit, because there is a larger response from the labor input, and hence there is net lending overseas. In general the impulse responses suggest that strong traded-sector productivity shocks can lead to the matching of some elements of the open economy. A lack of complete risk sharing raises consumption at home compared to abroad and a strong preference for home goods consumption also amplifies the extent to which output increases.

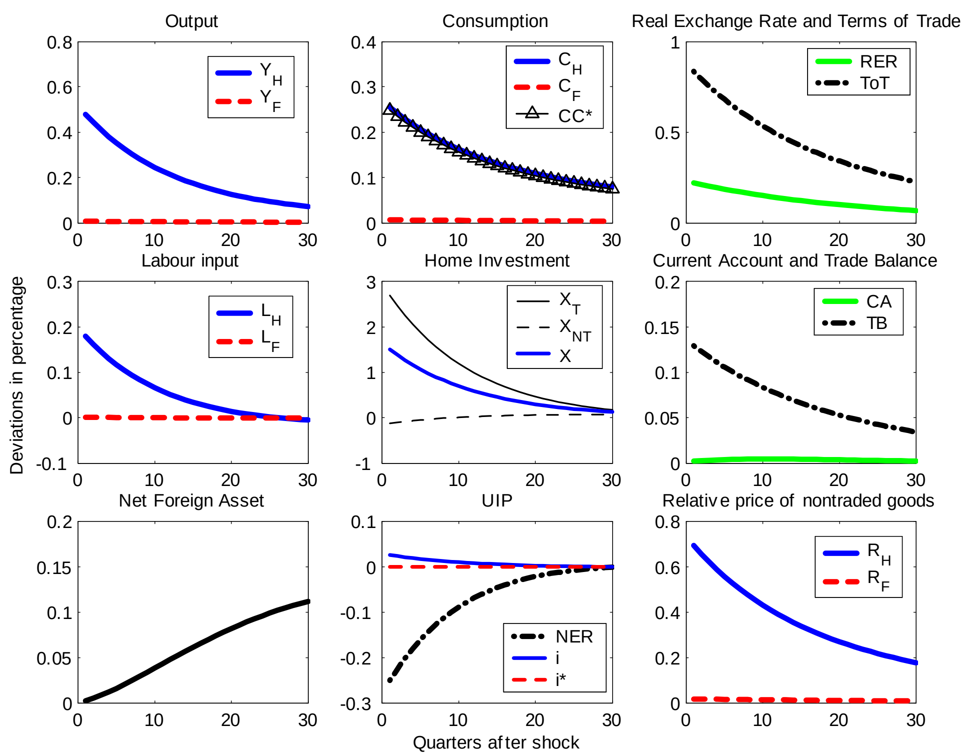


Figure 2.3 - Response to Traded Productivity Shock

Notes for Figures 2.3 to 2.6: The impulse responses show percentage deviation from steady state from period 1 when there is a 1% shock to traded productivity: RER - real exchange rate; CA, TB - current account and trade balance measured as percentage of output; NER - nominal exchange rate; i, i* - interest rate of small open economy and US; CC* - relative consumption to US; subscript H denotes home country whereas F denotes foreign country; subscript T denotes traded sector and NT denotes non-traded sector.

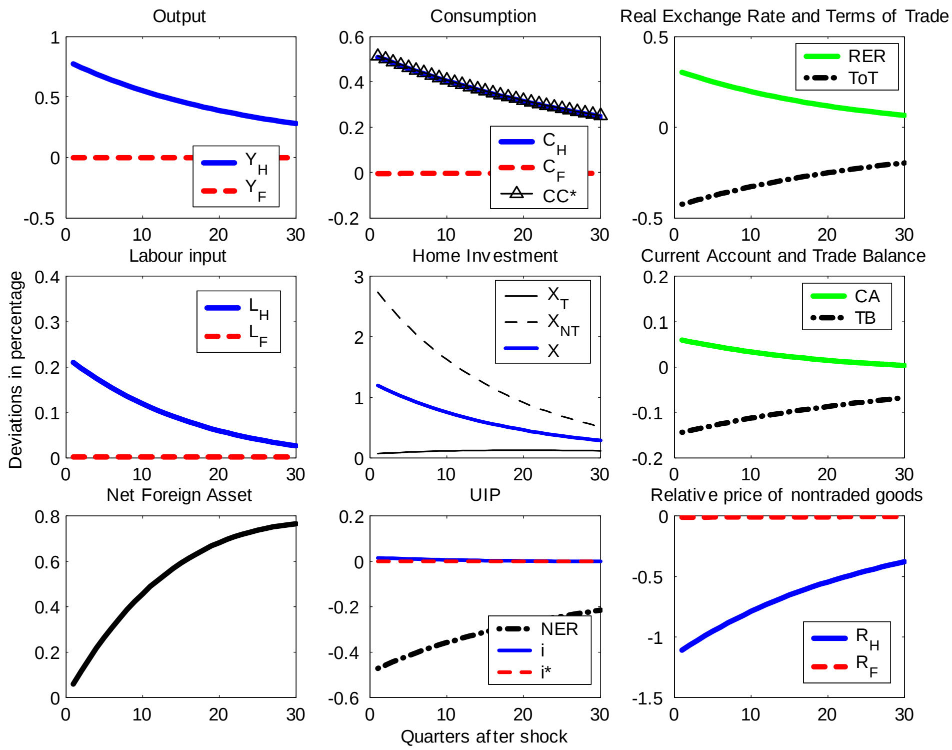


Figure 2.4 - Response to Non-Traded Productivity Shock

2.4.2.3 Preference shocks In principle, preference shocks might be thought to contribute a solution to the Backus-Smith puzzle simply as marginal utility is now, *inter alia*, a function of the preference shocks rather than just consumption growth: $RS = \frac{U_C^*}{U_C}$,where we note that the real exchange rate can be thought of as related to the ratio of marginal utilities in consumption (in a complete markets set-up). But these preference shocks by themselves may not provide a resolution as they seems to imply relatively acyclical current account dynamics and a reduction of real exchange rates along with higher domestic supply (see Chadha *et al.*, 2001). This is because they alter the equilibrium point in the household trade-off between leisure and consumption. Following Hall (1997) such shocks simply suggest that the household decides to allocate more (or less) time to work, which finances consumption, rather than leisure. As one would expect preference shocks help increase the volatility of the labor input by introducing exogenous shifts in work and may act to solve the puzzle of the Backus-Smith correlation (Figure 2.5). A home preference shock drives up labor input and consumption and reduces relative prices, if the supply response is elastic. So unless home agents become elastic in the substitution of leisure across periods, increased consumption is also met by an increase in investment and the current account remains acyclical.

2.4.2.4 Stochastic deviations from UIP Following the suggestion of Devereux and Engel (2002), we explore the implication of stochastic deviations from the uncovered interest rate parity (UIP) condition for the determination of exchange rate changes. These shocks, motivated by the poor empirical performance of UIP equations, (see Sarno and Taylor, 2002 for an indicative survey) imply that the exchange rate does not move equiproportionately to interest rate differentials and in fact it often moves in the opposite direction. These stochastic deviations, which can be thought of as excess returns in a particularly currency mean that the exchange rate can disconnect from the relative interest rates. The impulse responses show that a shock that brings about an initial exchange rate appreciation is similar to a demand shock in that it depresses traded and non-traded wages via competition with overseas traded-sector wages. To deal with the temporary fall in wages, consumption - which is tilted up by the fall in domestic interest rates - is maintained by overseas borrowing and investment is stimulated by the fall in wages.

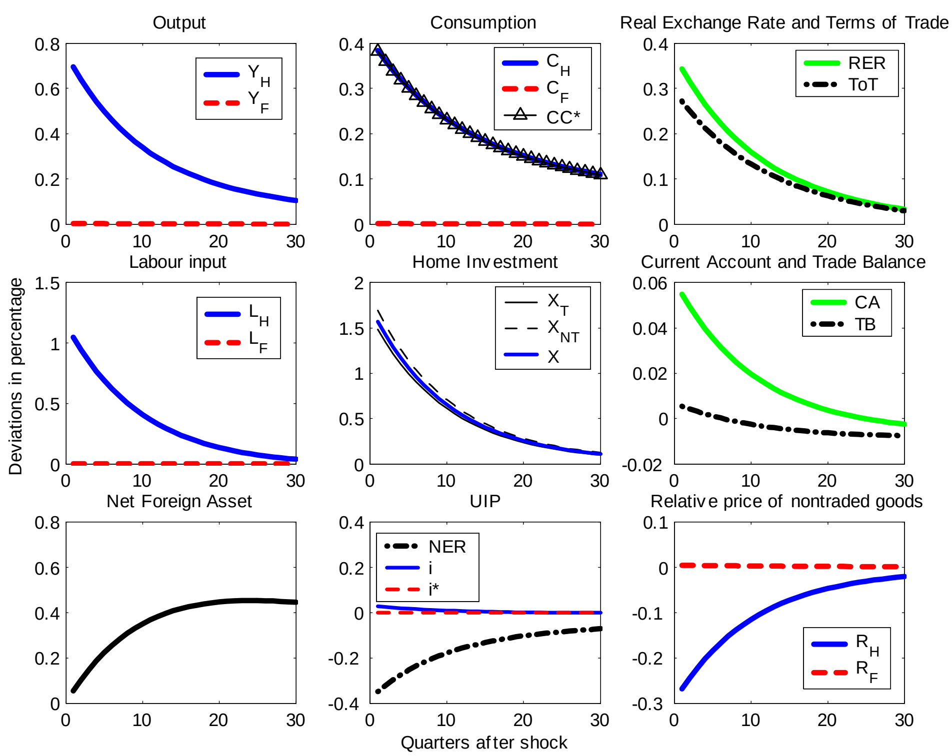


Figure 2.5 - Response to Preference Shock

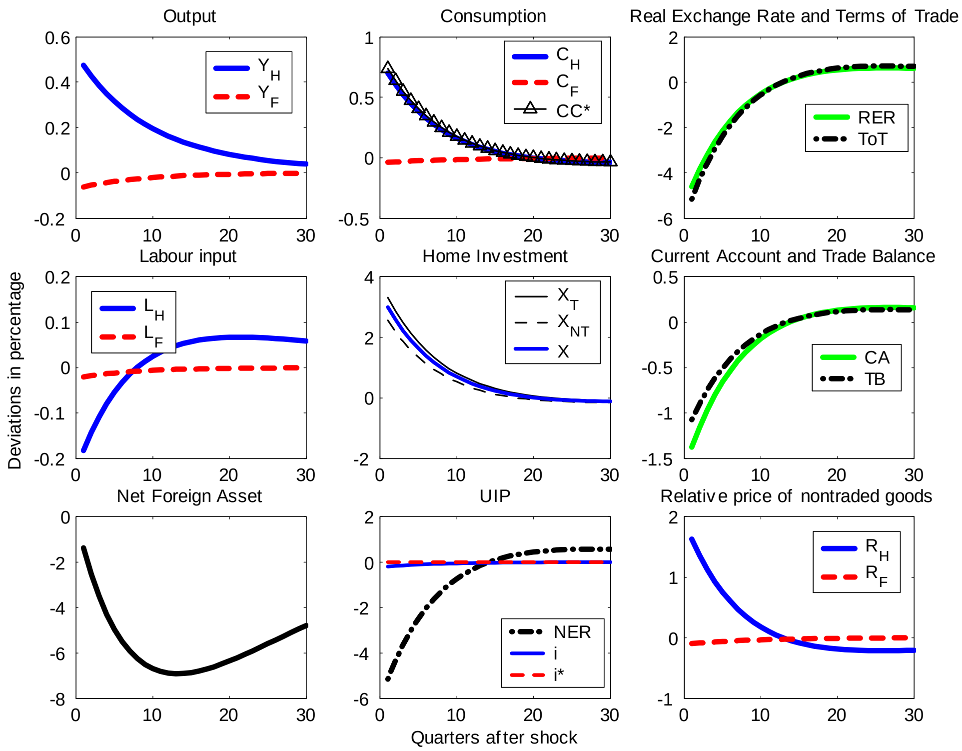


Figure 2.6 - Response to UIP Deviation

2.4.3 Variance decomposition

Table 2.2 shows the decomposition of unconditional variances for relative consumption, the real exchange rate and the current account from the model simulation. The first four columns show the contribution from each of the seven shocks in explaining the variance of these three key variables in the case of persistent, temporary UIP deviations and when the home economy is a creditor or debtor. The final three columns then exclude one type of shock in turn and shows the resulting contribution by the remaining shocks. Table 2.2 illustrates that both sets of productivity shocks and UIP deviations are likely to play a dominant role in explaining the variance of the key open economy variables, the former for relative consumption and the latter for the real exchange rate and the current account.

The Table shows the dominant role that UIP deviations play under the baseline calibration in explaining the variance of the current account and real exchange rate over the business cycle. It also suggests that productivity shocks, particularly in the non-traded sector, might play an important role in explaining fluctuations in relative consumption and also for the real exchange rate and the current account when UIP deviations are excluded. Preference shocks play a negligible role in explaining the variances of these key variables unless we exclude productivity shocks altogether in which case they can explain over 20% of the variance in relative consumption. The finding that productivity shocks are important for quantities and relative prices even in the presence of exchange rate volatility is similar to other studies, such as Straub and Tchakarov (2004).

2.4.4 Simulated moments

In Table 2.3, we present second moments of the artificial simulated model for the benchmark calibration. The first column gives the moments from the UK data over the period 1980-2006. The next four columns correspond to the cases of persistent UIP deviations, temporary UIP and for the persistent UIP case also when the economy is a steady-state creditor or debtor - with assets or debts at 50% of GDP in each case, respectively. In the final three columns, we remove one set of shocks from the baseline calibration in order to understand how the artificial model data changes.[16]

The baseline calibration captures well the main moments of the data: consumption, labor inputs and wages are smooth relative to output and investment, the real exchange rate and the terms of trade are markedly volatile. The correlations of the main quantities and relative prices with output are all correctly signed (apart from interest rates). The model produces the positive relationship between the terms of trade and the real exchange rate found in the data, as well as the exchange rate disconnect, with relative consumption negatively correlated. Finally, although higher than the correlation observed in data (0.16), the model does not predict that relative consumption will be perfectly correlated (with estimates in the range 0.5 to 0.7) and thus goes some distance towards understanding the lack of complete risk sharing.

This is because the non-state contingent bond is used to smooth investment and consumption following a shock.[17] In the event of a temporary productivity shock, which has little impact on permanent income, the home country consumer borrows from abroad, which raises overseas interest rates and lowers overseas consumption as well, which leads to a correlation in relative consumption. But when there are persistent productivity shocks, permanent income falls somewhat and so there is not as strong a need to borrow from abroad to smooth consumption or investment, which then means that overseas interest rates do not rise and lower overseas consumption. Hence there is something of a fall in

16 In earlier versions of this paper we also presented results for the estimated spill-over of productivity and preference shocks but as we found that these do not change the moments qualitatively we have removed them from this version.

17 Unlike an asset that can be bought to insure prior to shocks.

the consumption correlation when there are non-state contingent bonds and persistent productivity shocks.

The persistence of the UIP shocks plays an important role in explaining both the relative variance of the real exchange rate and to a lessor extent that of relative consumption, which falls from 5.2% to 2.5% and from 1.1 to 0.9, respectively when we reduce the AR(1) persistence of UIP deviations from 0.88 to 0.38. Note also that the relative consumption becomes nearly acyclical (-0.02) when the UIP shocks fall in persistence. Moving towards a model where the steady-state level of net foreign assets is not zero does not alter the basic picture but when the home country is treated as a debtor investment, the real exchange rate and the terms of trade become more volatile and the current account becomes considerably less volatile.

If we examine the model with or without UIP deviations (compare column 2 to the final column), it appears that UIP deviations play a clear role in helping to explain the exchange rate disconnect. This is because the exchange rate can be driven whether there are movements in relative interest rates or not, which in turn depend mostly on planned relative consumption levels. An absence of UIP deviations from the model thus drives the correlation of relative consumption with real exchange rate to 0.76 rather than the data estimate of -0.61 or the benchmark model estimate of -0.65. Note also that in the model without UIP deviations, consumption, investment, labor inputs, real exchange rates and the terms of trade are somewhat too volatile. The main role of preference shocks it to raise the volatility of the labor input and lower that of the wage rate.

The overall performance of baseline calibrated model in terms of explaining the puzzles is reasonable. Specifically, we find that: (1) the model enables different shocks to interact and seems to solve the Backus-Smith puzzle in that it does not forecast perfect consumption correlation across the two economies with the help of a non-traded sector and incomplete financial markets; (2) this model stresses the HBS effect and therefore generates volatile real exchange rates; and (3) countercyclical current account is a robust result, as the current account moves together with real exchange rate. In other words it seems to match the OECD and emerging economy experience suggested in Figure 2.1.

Table 2.2 - Variance Decomposition of Current Account, Real Exchange Rate and Relative Consumption

	(a) All shocks				(b) Excluding shocks:		
	Persistent UIP Dev.	Temporary UIP Dev.	Net Creditor[1]	Net Debtor[1]	Productivity	Preference	UIP
Current Account (CA/Y)							
Traded productivity, Home	0.2%	1.6%	0.1%	0.4%	–	0.2%	28.6%
Non-Traded productivity, Home	0.4%	3.1%	1.0%	3.1%	–	0.4%	55.6%
Traded productivity, Foreign	0.0%	0.1%	0.0%	0.0%	–	0.0%	1.5%
Non-Traded productivity, Foreign	0.0%	0.0%	0.0%	0.0%	–	0.0%	0.1%
Preference, Home	0.1%	0.5%	0.1%	0.1%	0.1%	–	9.0%
Preference, Foreign	0.0%	0.3%	0.0%	0.1%	0.0%	–	5.2%
UIP	99.3%	94.4%	98.8%	96.3%	99.9%	99.4%	–
Real Exchange Rate (RER)							
Traded productivity, Home	0.8%	5.6%	0.9%	0.8%	–	0.8%	35.9%
Non-Traded productivity, Home	0.0%	0.1%	0.0%	0.0%	–	0.0%	0.7%
Traded productivity, Foreign	0.1%	0.6%	0.1%	0.1%	–	0.1%	3.8%
Non-Traded productivity, Foreign	0.5%	3.2%	0.5%	0.4%	–	0.5%	20.1%
Preference, Home	0.4%	2.7%	0.4%	0.4%	0.4%	–	17.4%
Preference, Foreign	0.5%	3.5%	0.5%	0.5%	0.5%	–	5.2%
UIP	97.7%	84.3%	97.6%	97.8%	99.1%	98.6%	–
Relative Consumption (CC^*)							
Traded productivity, Home	14.4%	16.0%	14.5%	14.3%	–	15.8%	16.2%
Non-Traded productivity, Home	62.8%	70.0%	62.6%	63.0%	–	69.2%	71.0%
Traded productivity, Foreign	0.8%	0.9%	0.8%	0.8%	–	0.9%	0.9%
Non-Traded productivity, Foreign	1.3%	1.4%	1.3%	1.3%	–	1.4%	1.4%
Preference, Home	4.7%	5.3%	4.8%	4.7%	22.8%	–	5.3%
Preference, Foreign	4.5%	5.0%	4.5%	4.5%	21.9%	–	5.1%
UIP	11.5%	1.3%	11.6%	11.4%	55.4%	12.6%	–

Table 2.3 - Results of benchmark UK/US calibration

	UK Data	(a) All shocks				(b) Excluding shocks:		
		Persistent UIP Dev.	Temporary UIP Dev.	Net Creditor[1]	Net Debtor[1]	Productivity	Preference	UIP
Relative volatility to output (interest rate and CA/Y take raw value)								
Consumption	*0.78*	0.92	0.64	0.91	0.93	1.09	1.04	0.59
Investment	*2.30*	3.99	2.84	3.91	4.08	4.62	4.58	2.60
Interest rate	*1.01*	0.42	0.37	0.40	0.43	0.42	0.42	0.05
Labour	*0.95*	0.90	1.00	0.89	0.90	1.13	0.36	1.02
Wage	*0.86*	0.89	0.65	0.88	0.90	1.01	1.02	0.61
RER	*4.89*	5.16	1.91	5.13	5.20	6.71	6.21	0.67
ToT	*1.66*	5.88	2.47	5.85	5.91	7.50	7.08	1.46
CA/Y	*1.06*	1.99	0.61	4.12	2.80	1.99	1.99	0.12
CC*	*1.27*	1.13	0.94	1.12	1.14	1.22	1.24	0.92
Correlation with output								
Consumption	*0.79*	0.90	0.97	0.90	0.89	0.91	0.90	1.00
Investment	*0.79*	0.88	0.93	0.89	0.88	0.89	0.90	0.96
Interest rate	*0.19*	−0.33	−0.08	−0.33	−0.33	−0.51	−0.43	0.90
Labour	*0.78*	0.56	0.79	0.57	0.56	0.55	0.14	0.82
Wage	*0.13*	0.56	0.32	0.56	0.55	0.37	0.94	0.27
RER	*-0.10*	−0.46	0.00	−0.46	−0.45	−0.63	−0.58	0.57
ToT	*-0.13*	−0.47	−0.05	−0.47	−0.46	−0.64	−0.59	0.19
CA/Y	*-0.30*	−0.52	−0.22	−0.36	−0.22	−0.65	−0.64	−0.13
CC*	*0.19*	0.64	0.48	0.65	0.64	0.83	0.65	0.46
Correlation with RER								
ToT	*0.10*	0.98	0.81	0.98	0.98	1.00	0.98	0.21
CC*	*-0.61*	−0.65	−0.02	−0.65	−0.66	−0.83	−0.75	0.76

2.5 Model-data comparison

A typical business cycle exercise examines the volatility of key economic variables and their correlation with output - as a measure of their business cycle behavior. At the very least such an examination neglects the cross-correlations in other elements in the VCM that may matter to us, which in this case is the relationship between exchange rates, relative consumption and the current account. Our model selection is thus based on the comparison of the VCM of seven key endogenous variables simulated by our model to the actual data, see Appendix B for some further details. To illustrate our point, we consider the open economy sub-set of the variables for this exercise. In this section we obtain six statistical measures of distance of the model-generated data from the sample observations and the results are given in Table 2.4. The smaller statistics indicate a better fit of data to model and we find for the main model selection criterion the models with persistent UIP deviations with debtor status are closest to the observed data.

2.5.1 Model selection based on VCM

If we choose to define a preferred model as that with the least deviation from the data, there may be a number of possible metrics we can employ. Our model selection from a class of candidate models is based on the comparison of the VCM of endogenous variables simulated by our model to that of the actual data, see Appendix B for further details on the distance measures used.[18] We consider a sub-set of the model variables that are closely

[18] A copy of the procedures written in MATLAB will be made available on request.

related to the open economy puzzles highlighted in Figure 2.1: relative consumption, real exchange rate, relative output, home current account and home trade balance.

Specifically, as well as basic criteria such as root mean squared error (RMSE) and mean absolute error (MAE), we use two likelihood based methods to determine how different the two matrices are: (1) the Box-Bartlett test (Bartlett, 1937; Box, 1949); (2) the distance measure flowing from the Kullback-Leibler (Kullback and Leibler, 1951) Information Criteria (KLIC) method. As shown in Bhattacharjee and Thoenissen (2007), we can also use the hypothesis testing method of Nagao (1973) and a revised test by Ledoit and Wolf (2002), which are designed to test an equality hypothesis of VCMs.[19] The key differences between these classes of approach are explained in the Appendix B but essentially the basic criteria of RMSE and MAE are akin to an approximate eyeballing of the data whereas the Box-Barlett test, KLIC methods, Nagao and Ledoit-Wolf allow for sampling variability and the KLIC also allows sampling variability in the simulated model.

For each case, we obtain six statistical measures of distance of the simulated model from the sample VCM for our 7 key variables. The results are given in Table 2.4. We assess the distance with different degrees of persistence in the UIP deviations and varying the NFA position. The smaller statistics indicate a better fit of data to the model. The best calibration according to each of the six criteria is therefore marked with an asterisk. If

[19] The original Nagao's (1973) test is also an LR type test. The Ledoit and Wolf (2002) method aims to deal with the special cases where data dimension is larger than number of observations (or relatively small sample data). Such a property makes the data VCMs rank-deficient. Although we have rank-deficient VCMs in DSGE models for a different reason, where variables are greater in number than shocks and predetermined variables taken together, we utilize this method to deal with rank-deficiency problem. Note that canonical LR methods cannot be directly applied to rank-deficient VCMs; see Bhattacharjee and Thoenissen (2007) for further discussion. We outline our distance metrics in Appendix B.

we examine the first three columns of results we will note that simple eyeballing of the data might lead us to prefer models with less persistent UIP shocks. But, when sampling and model uncertainty is accounted for, the other tests suggest we should prefer more persistence in the UIP deviations. We find models with persistent UIP deviations are closest to the observed data. Furthermore when we allow the steady-state debt position to move from creditor to debtor status we find that the best fit - smallest distance - occurs when the home economy is a debtor.

There are two main findings that stand out. Firstly, the distance measures suggest that persistent UIP deviations are helpful in generating a VCM similar to that of UK/US open economy data. We have shown in the impulse responses that deviations from UIP are the only forcing variable which helps resolve the Backus-Smith puzzle, by driving up large swings of real exchange rate and generating a volatile and countercyclical current account. More dominant UIP deviations are required to replicate the observed data. Secondly, we find that a non-zero NFA position is also helpful for improved goodness of fit, with net debtor calibration for the UK being slightly better than the net creditor case. However, negative or positive NFA position improve the model fit quite differently. A net debtor calibration mainly contributes to a better fit associated with current account dynamics. A net creditor calibration improves the goodness of fit for UK and US output and consumption data. In a two-country model, a net creditor UK means a net debtor US (as in the real world). This realistic calibration can better explain relative output and consumption but also generate a volatile current account on both sides and thus increase distance measures for the overall fit. Therefore, based on the VCM distance approach, we highlight a net creditor and persistent UIP deviation calibration for the UK/US small open economy model.

Table 2.4 - Model Selection: Distance Measures between Data and Model VCMs

VCM Distance Calculation Method	All Shocks Persistent UIP Dev.	Temporary UIP Dev.	i.i.d UIP Dev.	Net Creditor	Net Debtor
RMSE	0.0462%	0.0431%	0.0444%	0.0620%	$0.0388\%*$
MAE	0.0241%	0.0122%	$0.0121\%*$	0.0313%	0.0204%
Box-Bartlett	92053	99787	99841	44379	$43947*$
Kullback-Leibler	432	468	469	208	$206*$
Nagao	2.74×10^7	3.18×10^7	3.18×10^7	5.12×10^6	$4.95 \times 10^6*$
Ledoit-Wolf	2.70×10^7	3.14×10^7	3.14×10^7	5.02×10^6	$4.85 \times 10^6*$

Notes to Tables 2.2 to 2.4: The quarterly data is the HP filtered series of OECD MEI, 1980-2006: RER is real exchange rate; CA/Y is current account to GDP ratio; ToT is terms of trade and is import price over export price; CC* is relative consumption to US. The base case calibration is as Table 2.1. The UK is the home country. Net creditor calibration is set at $\overline{a} = 0.5$ while net debtor sets $\overline{a} = -0.5$ for a small open economy; the UIP shock has an AR(1) coefficient of $\rho_{UIP} = 0.88$ in the persistent case as in Kollmann (2003), whereas the temporary case and i.i.d cases correspond to $\rho_{UIP} = 0.38$ and $\rho_{UIP} = 0$ respectively; in both net creditor and net debtor case the UIP deviations are persistent; RMSE denotes root mean squared errors; MAE denotes mean squared errors; each of the distance metrics is discussed in Appendix B; the asterisk (*) denotes minimum distance measure across all the five calibrations.

2.5.2 Sensitivity analysis

Sensitivity analysis is shown in Figures 2.7-2.12 and is based on the seven-shock model with the basic calibration given in Table 2.1. We simulate the model and allow some deep parameters to change and check the sensitivity of some key moments with respect to several main statistical measures: the Backus-Smith correlation, the extent of exchange rate disconnect, the correlation between the trade and current account and the cyclicality of the current account. The vertical solid line(s) denotes the initial calibration.

First, we consider frictions in the model: costly investment and costly foreign asset holding. In Figure 2.7, although higher cost of investment alters volatility of open economy variables, it does not change the basic correlation structure. In Figure 2.8, costly foreign asset holding make the channel of risk sharing smaller, therefore the Backus-Smith correlation tends to zero. However, this will happen when the cost is extremely high. As the model has very simple assumption for financial markets, we emphasize its qualitative implication instead of its value denoted by basis points.

Secondly, we discuss the characteristics of the market and production. Steady state NFA does not alter real exchange rate dynamics significantly but it is crucial for current account dynamics. For a net debtor, a positive traded TFP shock leads to current account deficit. For example, upon a positive traded productivity shock, output increases, the real exchange rate appreciates, Home country borrows and a current account deficit results. But as a debtor there is requirement for paying interest, making the borrowing incentive lower and thus the extent to which the current account is countercyclical is mitigated, as shown in Figure 2.11.

Thirdly, we consider varying sources of dynamics - the exogenous forcing variables. The UIP shock in the baseline calibration is highly persistent and by examining different degrees of persistence in UIP deviations as in Figure 2.10, we find real effects only in the case of highly persistent shocks. Adding UIP deviations reinforces the pattern of correlation we find in the data. When we vary the relative magnitude of non-traded productivity shocks in Figure 2.11, it leads to changes in the key correlations. Relatively strong traded compared to non-traded productivity shocks contribute to negative Backus-Smith correlation and countercyclical current account. Turning to Figure 2.12, as preference shocks are strengthened, the negative correlation on both counts is weakened.

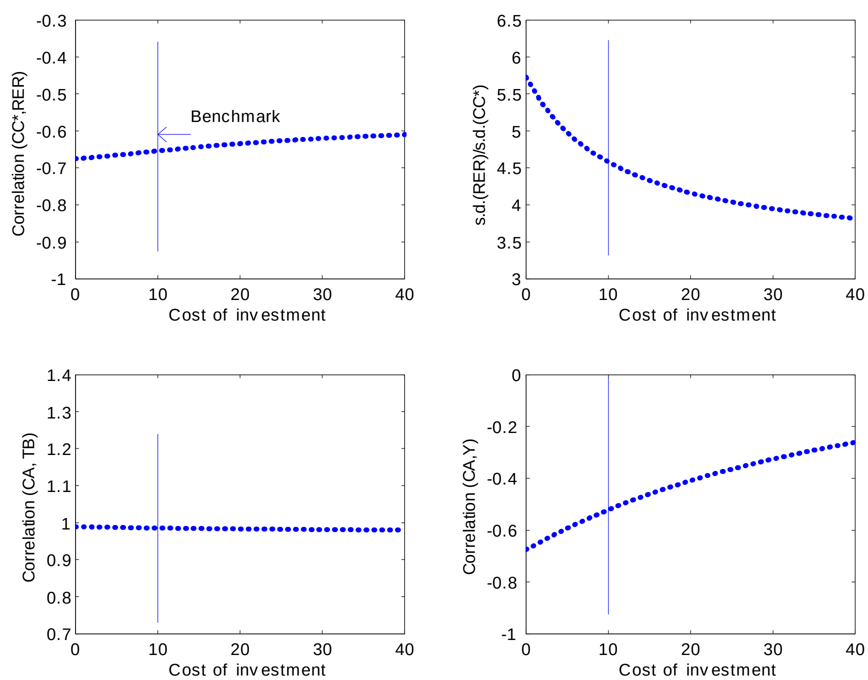


Figure 2.7: Sensitivity - Investment Cost (b)

Note: The charts 2.7 to 2.12 show the sensitivity to several coefficients. The vertical line denotes the benchmark calibration of Table 2.1. CC* denotes relative consumption to US; RER: real exchange rate; CA: current account; TB: trade balance; Y: real output. CA and TB are measured as ratio to GDP.

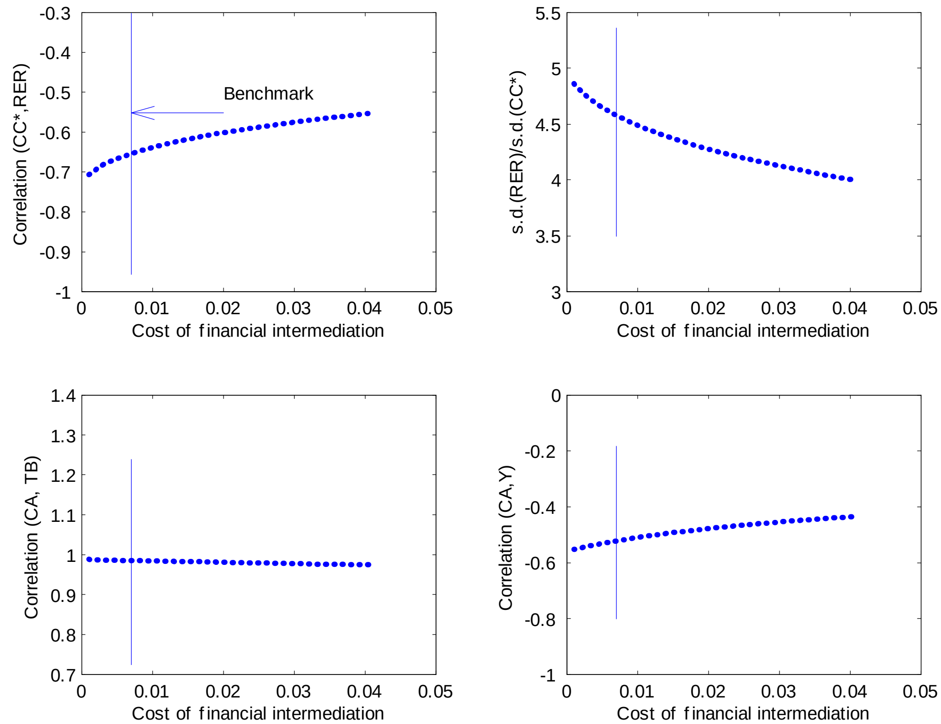


Figure 2.8: Sensitivity - Financial Intermediation Costs (ϵ)

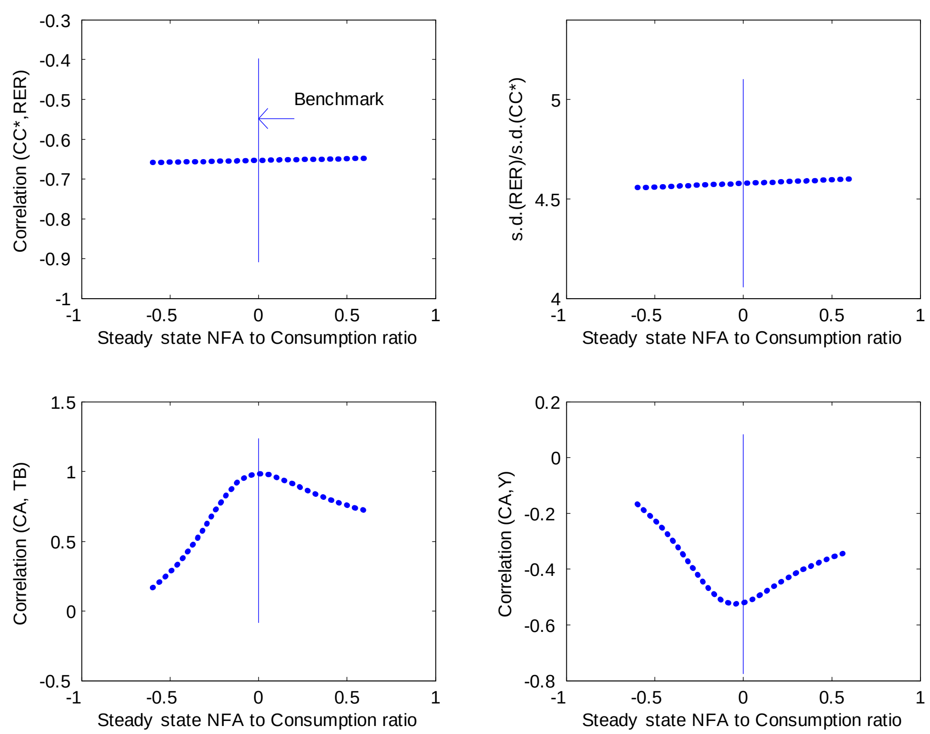


Figure 2.9: Sensitivity - NFA ratio ($\overline{a}$)

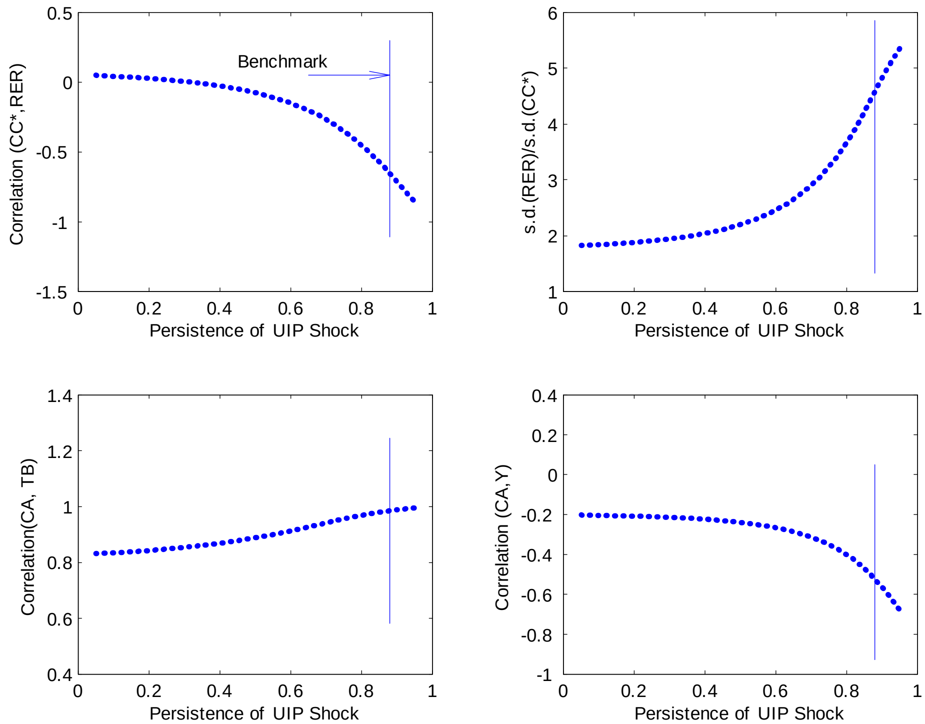


Figure 2.10: Sensitivity - UIP Deviation (ρ_{UIP})

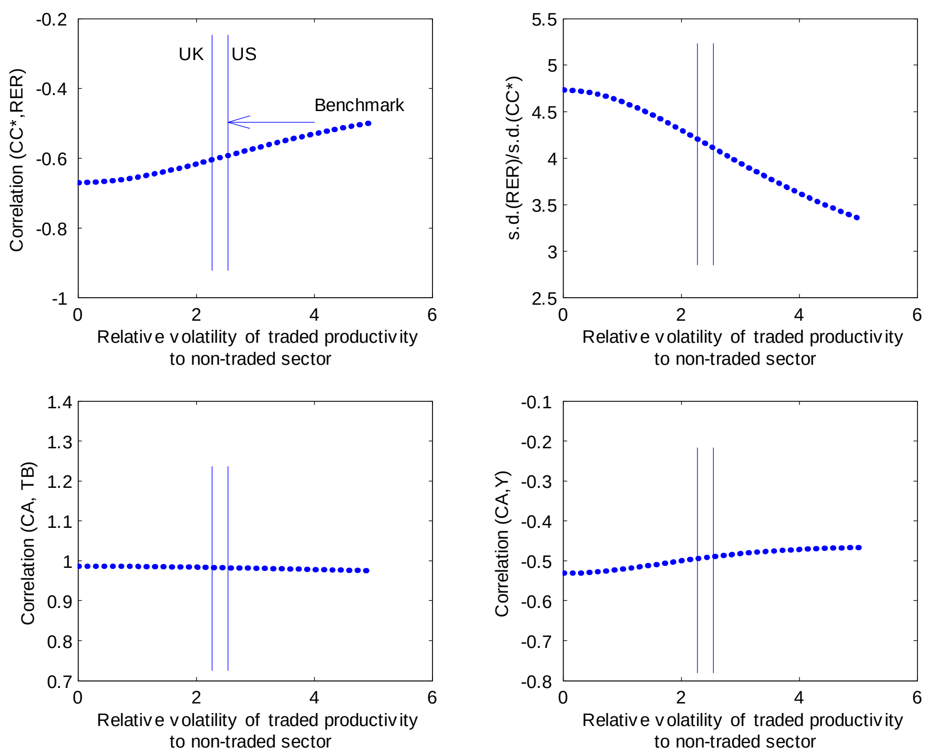


Figure 2.11: Sensitivity - Traded Sector Volatility (σ_A/σ_{AN})

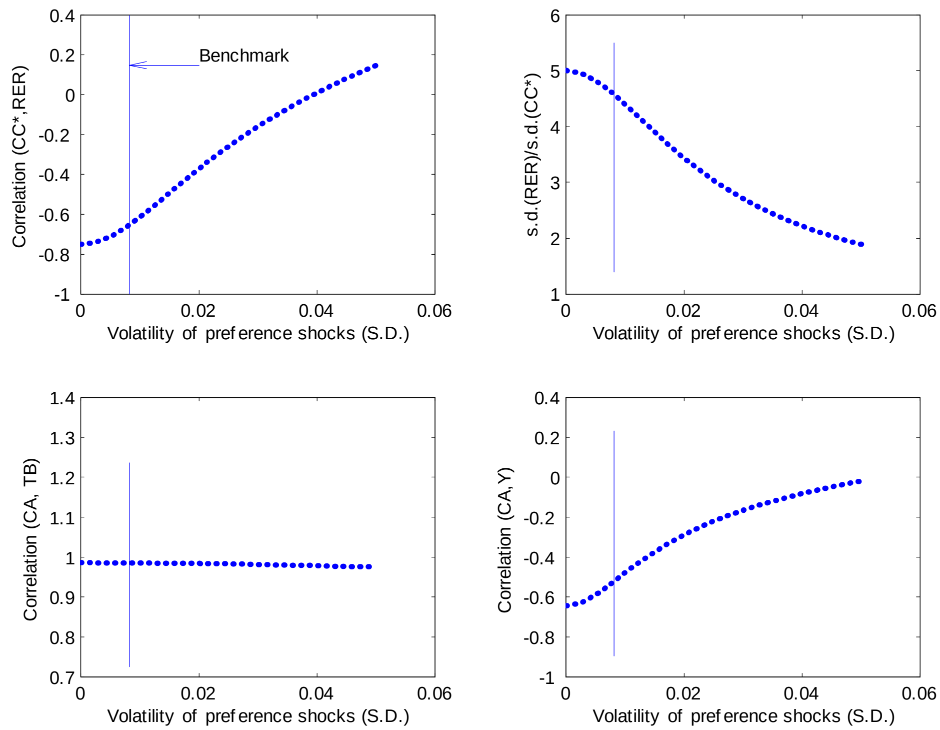


Figure 2.12: Sensitivity - Preference Shock Volatility (σ_P)

2.6 Conclusion

Open-economy general equilibrium models offer an attractive laboratory in which to examine the stylised facts, whether empirical regularities or puzzles, observed in the data. However, there is not much methods to formally evaluate the goodness of model fit. This chapter looks into the two areas by applying a new empirical method to a large open economy model.

The puzzles we focus on are indeed covering many aspects of open economies, including current account dynamics, real exchange rate disconnect and international risk sharing puzzle. Existing literature has tried to explain a certain angle of these facts. We propose a model with much richer structure and sources of perturbation. We examine the properties of a two-sector real business cycle model with incomplete financial markets. The model is driven by a number of driving forces or shocks including both domestic and overseas traded and non-traded productivity, to the work-leisure margin at home and overseas and to deviations in the exchange rate from the level suggested by the UIP equation. Given the complexity of the open economy model, it is appropriate to use some informal measurement for model selection. The method picks the covariance matrices of all (or selected group of) variables as a benchmark and calculate some metrics to quantify the goodness of fit.

We find evidence to support the proposition that when all these shocks perturb the model economy there is some move towards resolution of the Backus-Smith puzzle, and the model also stresses the Harrod-Balassa-Samuelson effect. The most important modelling choices - over and above a standard one-sector small economy RBC model - involve the adoption of a two sector model, allowing for shocks to non-traded as well as traded sector productivity, the employment of incomplete markets with the existence of a non-state contingent bond and of stochastic deviations from the UIP equation for the exchange rate. The aspects of the model induce greater real exchange rate variability and yet alongside the absence of complete risk sharing ensure that consumption need not simultaneously jump to arbitrage price differentials.

Finally we note that the modelling approach we use is flexible and simple to use. Our methodology facilitates examination of deep parameters and shock processes for small open economies, and allows the researcher to examine some simple summary statistics when assessing model fit. The distance measures might usefully be applied more generally to the question of the fit of data to DSGE models.

Chapter 3
Taking a Macro-finance Model to UK Yield Curve

3.1 Introduction

It is becoming clear in the literature that a pure macroeconomic standpoint is not sufficient for yield curve analysis, at least for DSGE model builders, because yield curve models contain increasingly more risk-adjusted terms into the longer end (Rudebusch and Wu, 2008). The established dichotomy of the term structure analysis assigns the short-end to macroeconomic domain and the long-end to the finance domain, since the policy rate governs the short-term interest rates but long-term interest rates contain risk premium originated from some of financial factors (Rudebusch and Wu, 2008). This chapter starts from such doctrine but explores to what extent the macroeconomic fundamentals can explain bond spread within a standard macro-finance framework.

Why is the yield curve important for macroeconomists and monetary policy makers? The most influential empirical observation in yield curve research is that it provides leading information for future recessions (Estrella and Mishkin, 1997).[20] In reality, the yield curve contains rich information on both interest rate and inflation expectation from the private sector and therefore represents an important vehicle and measure of the monetary policy transmission (see Chadha and Holly, 2006, Kozicki and Tinsley, 2008). For example,

20 Friedman and Kuttner (1991) found similar leading information for paper-bill spread.

Chadha and Holly (2006) find high persistence in an New Keynesian model can help explain the shape of volatility yield curve. Despite ample empirical evidence, the theoretical mechanism explaining the link is not yet fully understood. Estrella (2005) has pioneered work along this direction by incorporating the term structure into a DSGE model and investigating the predictive power of term premium for real activity. However, Gurkaynak *et al* (2003) argue that such treatment in theory can hardly explain the high sensitivity of long-term interest rate in empirical business cycles, as macroeconomic shocks die out quickly and pass on inadequate variation into the long-end of yield curve. Similar work on the separate dynamics of inflation expectation component in yield curve includes Chadha et al (2007). Tovar (2008) further points out that the missing link between long-term interest rate and short-term real activity in DSGE models prevents it from being an useful tool for monetary policy makers. This burgeoning literature all seemingly tell us a DSGE model overlooking financial factors is not preferable in general but not especially for yield curve analysis.

For these reasons, both financial economists and monetary economists use factor models in explaining empirical yield curve dynamics, while the latter proxies unobservable factors by macroeconomic conditions (see, Rudebusch and Wu, 2008), the use of structural linear combinations of macro data may not be substantially meaningful. The chapter therefore makes the contribution to existing literature by scrutinizing the advantages and disadvantages of DSGE macro models in matching the features of the yield curve, namely, the slope, curvature and level of nominal yield curve and its two building blocks, real yield curve and inflation expectation yield curve. I consider both forward-looking and backward-looking properties in Phillips curve and aggregate demand schedule. Special focus is put on the ability of various macroeconomic shocks in explaining unconditional second moments of key macro variables and yield curve features. The model is also augmented with shocks to yield curve variables themselves to represent financial factors that are excluded from the pure DSGE macro models.

Several findings stand out after a series of simulation based analysis with the DSGE model carefully calibrated. I find monetary policy shocks, as one would expect, play an important role in variation of key variables and yield curve, which is similar to results of Lemke (2008). A model dominated by monetary policy shocks can also explain the predictive power of bond yield spreads for business cycles. I also find evidence of somewhat more dominant role of supply shocks in the recent sub-sample of UK data. As for yield curve variables, the slope and level of the curve are closely linked to macroeconomic impulses but the curvature is mostly driven by other factors. This distinguishes yield curve slope as a critical information source for signaling business cycle fluctuation.

The rest of the chapter is organized as following. Section 3.2 introduces a standard New Keynesian model with sufficient flexibility in calibration, with which I can solve for expectation of forward interest rates and link canonical macroeconomic models to yield curve under pure expectation hypothesis. I also discuss yield curve dynamics of calibrated model following macroeconomic shocks. In section 3.3 I describe the source data from UK and present some stylised facts of cyclical yield curve features. Section 3.4 uses a model selection method based on variance-covariance matrices of key variables to explore the accountability of macro shocks in explaining yield curve volatilities. The last section gives a general assessment of the framework I use and highlight several new findings for further debate.

3.2 A Hybrid NK Model

The model is a New Keynesian AS-AD model similar to Bekaert *et al* (2006) and Chadha and Holly (2006). It is consisted of three key equations: (1) An optimisation IS curve as the aggregate demand schedule; (2) A New Keynesian Phillips Curve (NKPC) as the aggregate supply schedule; and (3) A feedback nominal interest rate rule as the monetary policy. In this model, we use log-deviation, $\widehat{X}_t$, for real variables such as output. The inflation and interest rate are assumed to fluctuate around a linear trend or a constant.

The Aggregate Demand (**IS equation)** is the Euler equation that is derived from a representative agents' utility maximisation problem. The Euler equation gives the pricing kernel (where nominal interest rate in gross term and cyclical form are: $R_{t+1} = 1+\bar{i}+i_t$):

$$E_t \left[M_{t+1} R_{t+1} \right] = 1$$

$$M_{t+1} = \beta \frac{U'(C_{t+1}) P_t}{U'(C_t) P_{t+1}}$$

For a logarithm utility $U(C_t) \equiv \ln C_t$, imposing the resource constraint $C_t = Y_t$ and log-linearising this equation takes us to the interest rate measured as deviation from a constant level:

$$\begin{aligned} -i_t &= \widehat{Y}_t - E_t \widehat{Y}_{t+1} - E_t \widehat{\pi}_{t+1} \\ \widehat{Y}_t &= E_t \widehat{Y}_{t+1} - \left(\widehat{i}_t - E_t \widehat{\pi}_{t+1} \right) \end{aligned} \quad (3.1)$$

This is the IS equation in the simplest case. Recently some empirical research achieves plausible results with habit formation specification in preference, including for the monetary policy and bond pricing, see Fuhrer (2000) and Wachter (2006). In this forward-looking IS equation, we introduce habit formation to bring in endogenous persistence in demand channel. If the current period utility takes the form $U(C_t) \equiv \ln \frac{C_t}{C_{t-1}}$[21], the IS equation becomes:

$$\widehat{Y}_t = \frac{1}{2} E_t \widehat{Y}_{t+1} + \frac{1}{2} \widehat{Y}_{t-1} - \frac{1}{2} \left(\widehat{i}_t - E_t \widehat{\pi}_{t+1} \right) \quad (3.2)$$

We would rather to use a general form for the IS equation for allowing a flexible calibration in the yield curve model:

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21 The general form of utility function is $U(C_t) \equiv C_{t-1}^{\eta} \frac{C_t^{1-\sigma}}{1-\sigma}$.

$$\widehat{Y}_t = \mu E_t \widehat{Y}_{t+1} + (1-\mu)\,\widehat{Y}_{t-1} - \delta\left(\widehat{i}_t - E_t \widehat{\pi}_{t+1}\right) + \epsilon_{IS,t} \tag{3.3}$$

where $\epsilon_{IS,t}$ denotes a demand shock such as preference shocks or nominal income shocks. The coefficient μ and δ is determined by the curvature and habit formation parameters of consumption in the utility function.

The Aggregate Supply (**AS equation**) is formulated by a Calvo-type price stickiness model showing the relation between marginal cost, inflation and its lag/lead. In a standard Calvo (1983) model, some of the firms are assumed to be unable to change price facing new market conditions. They fix the price or charge an inflation-led mark-up based on historical information:

$$P_t(i) = P_{t-1}(i)\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\tau} \tag{3.4}$$

The coefficient τ ranges from 0 to 1, showing degree of indexation to previous inflation. A higher value of τ makes the aggregate pricing more backward-looking and explicitly introduces inflation inertia. While a zero τ is assumed in Calvo's model. It generates a $\widehat{\pi}_{t-1}$ term in the NKPC:

$$\widehat{\pi}_t = \beta\phi E_t \widehat{\pi}_{t+1} + (1-\phi)\,\widehat{\pi}_{t-1} + \varpi \widehat{S}_t + \epsilon_{AS,t} \tag{3.5}$$

where $\epsilon_{AS,t}$ is the markup shock, or cost-push shock, to represent unexpected changes in price level, such as food and energy price inflation in most recent economic cycles.

Gali and Gertler (1999) argue that replacing marginal cost by output gap is acceptable only in a frictionless labour market. They also suggest that output gap shall not be approximated by detrending the output. In our model, we take a rather simple treatment by using an output gap, but taking a AR(1)-type, cyclical potential output $\widehat{Y}_t^N$ (or flexible price output, supply capacity) implied by the habit formation (see Bekeart *et al*, 2006). The persistence of potential output ρ_S is determined by the habit formation coefficient and we further introduce an i.i.d shock to potential output to reflect supply side deviations.

$$\begin{aligned} \widehat{\pi}_t &= \beta\phi E_t\widehat{\pi}_{t+1} + (1-\phi)\,\widehat{\pi}_{t-1} + \kappa\left(\widehat{Y}_t - \widehat{Y}_t^N\right) + \epsilon_{AS,t} \\ \widehat{Y}_t^N &= \rho_S\,\widehat{Y}_t^N + \epsilon_{YN,t} \end{aligned} \tag{3.6}$$

The **monetary policy** is in line with the feedback Taylor rule proposed by Clarida, Gali and Gertler (1999):

$$\widehat{i}_t = \gamma\widehat{i}_{t-1} + (1-\gamma)\,\phi_\pi E_t\widehat{\pi}_{t+1} + (1-\gamma)\,\phi_y\left(\widehat{Y}_t - \widehat{Y}_t^N\right) + \epsilon_{MP,t} \tag{3.7}$$

These three equation (3.3), (3.6), (3.7) and four exogenous shocks $\{\epsilon_{IS,t}, \epsilon_{AS,t}, \epsilon_{MP,t}, \epsilon_{YN,t}\}$ construct a model that can be solved with King and Watson (1998) algorithm. We further calculate theoretical second moments of endogenous variables with the solution to the following dynamic system:

$$\mathbf{A}E_t\mathbf{y}(t+1 \mid t) = \mathbf{B}\mathbf{y}(t) + \mathbf{C}\mathbf{x}(t)$$

The yield curve across maturity is calculated under a pure expectation hypothesis by summarizing future spot rates (forward rates). With no time-varying term premium and inflation risk premium specified in the model, our empirical analysis may deliver somewhat weak results, especially regarding the slope and curvature of yield curve.

$$i_{K,t} = \frac{1}{K}\sum_{n=1}^{K} E_t \widehat{i}_{t+n}, K = 1, 2, ...\text{(quarters)} \tag{3.8}$$

$$r_{K,t} = \frac{1}{K}\sum_{n=1}^{K} E_t \widehat{r}_{t+n}, K = 1, 2, ...\text{(quarters)} \tag{3.9}$$

$$\pi^e_{K,t} = \frac{1}{K}\sum_{n=1}^{K} E_t \widehat{r}_{t+n}, K = 1, 2, ...\text{(quarters)} \tag{3.10}$$

We define the nominal yield curve level, curvature and slope as following.

$$\begin{aligned} yc_{L,t} &= \frac{1}{3}\left(i_{10,t} + i_{20,t} + i_{40,t}\right) \\ yc_{C,t} &= 2 \cdot i_{20,t} - i_{40,t} - i_{10,t} \\ yc_{S,t} &= i_{40,t} - i_{10,t} \end{aligned}$$

3.2.1 Calibration

The analysis is conducted on carefully calibrated DSGE model abovementioned. We start from King's (2002) and Chadha and Holly's (2006) calibrations on similar models. In the rest of the chapter, I investigate the possibility of improving the initial calibration by fitting second moments of model variables.

Not like King's calibration, Bakeart *et al* allow backward-looking behavior in firm pricing and aggregate demand. Also following Fuhrer (2000) and Chadha and Holly (2006) I focus on a model with more forward-looking property in Phillips curve but some habit formation (backward-looking property) in aggregate demand. For a constant rate of risk aversion (CRRA) coefficient of 5 and a moderate level of habit formation in the consumption utility, I set the real rate elasticity of output, $\delta = 0.1$ in equation (3.3). For the same reason the forward-looking coefficient $\mu = 0.75$. The NKPC is rather forward-looking in my specification, with $\phi = 0.95$ in equation (3.6). The slope of NKPC is set to $\kappa = 0.1$ in the base case but is allowed to vary across samples, as suggested by Iakova (2007). The monetary policy takes the typical Taylor rule with persistence in nominal rate: $\gamma = 0.6$, $\phi_\pi = 1.5$, $\phi_y = 0.5$ (Equation 3.7). The discount factor for quarterly model is set as $\beta = 0.99$ to match a 4% annual risk-free rate. Table 3.1 shows the initial calibration for exogenous shocks from demand, mark-up, monetary policy and supply capacity.

Table 3.1: Initial calibration of exogenous shocks

Parameter	Description	Calibration value
ρ_{IS}	Persistence: Demand shock	0.33
ρ_{AS}	Persistence: Markup shock	0.74
ρ_{MP}	Persistence: Policy shock	0.3
ρ_{YN}	Persistence: Supply shock	0.95
σ_{IS}	S.D.: Demand shock	1.00%
σ_{AS}	S.D.: Markup shock	0.11%
σ_{MP}	S.D.: Policy shock	0.82%
σ_{YN}	S.D.: Supply shock	0.72%

3.2.2 Impulse Responses

We plot impulse responses of key macro variables and yield curve components for each of exogenous shocks.

3.2.2.1 Demand Shocks Figure 3.1 shows that the policy rate rises in response to a demand shock and the subsequent higher price level as well as positive output gap. However, it is the low persistence of demand shock instead of the feedback rule that stabilizes inflation and its expectation quickly. In our benchmark calibration, we allow a low consumption elasticity of real interest rate, therefore we see much higher jump in real output.

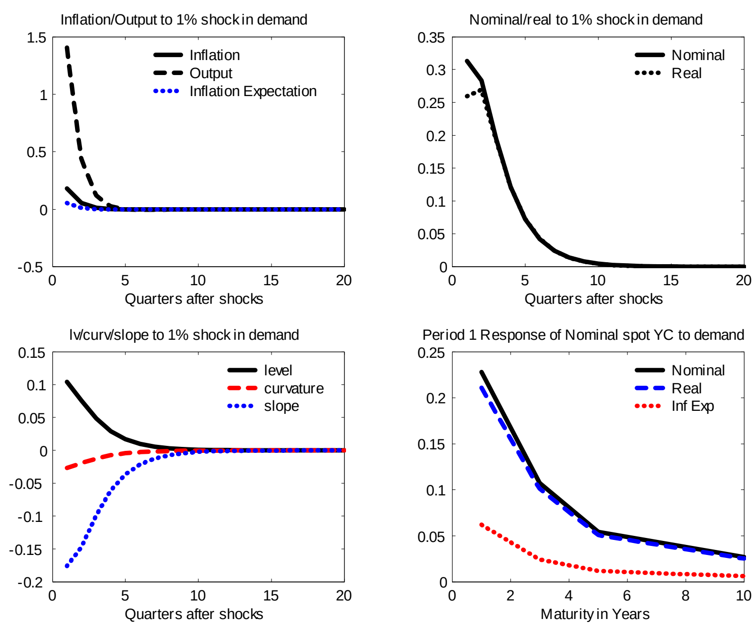

Figure 3.1 - Impulse response to demand shocks

As the demand shock is transitory, it does not affect much on the long-term interest rate, which brings about a bear-flattening to the yield curve. Along with this pattern of yield curve shift we also see a lower curvature. Overall, the yield curve level rises by about 20 bps immediately after the shock and attenuate to half-way in 3-4 quarters.

3.2.2.2 Mark-up Shocks Figure 3.2 shows that the interest rate responds strongly to a mark-up shock as it drives up the price significantly and in a persistent manner. Nominal rate hikes following the Taylor rule are lower than the inflation rate for two reasons. Firstly, inflation is expected to fall and well below the actual inflation in expectation. Secondly, the cost-push leads to a hump-shaped negative response in output. For these reasons the real interest rate is negative in the short run.

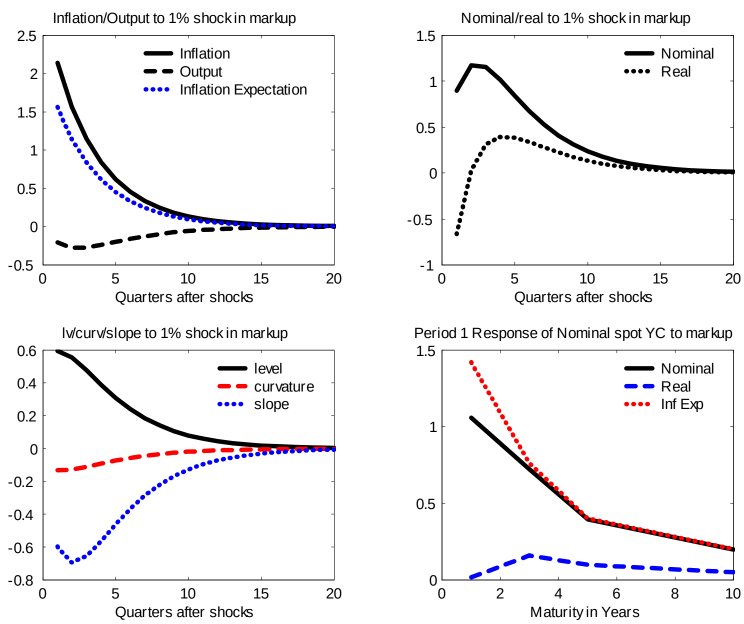

Figure 3.2 - Impulse response to mark-up shocks

Responses of the yield curve is similar to that of demand shocks but different in magnitude and inertia. The 1% mark-up shock is persistent therefore drives up the yield curve across all the maturities, with a 50-bp impact on 5-year spot rate. Yield curve level is up by 50 bps and reaches its half-way after about 5 quarters.

3.2.2.3 Monetary Shocks In Figure 3.3 we simulate the responses of an unexpected 100-bp rate cut as the monetary policy shock. In our calibration of less habit formation in the utility function, output jumps after the shock as a result of booming demand. So does it to inflation and inflation expectation, therefore real rate is falling even more than nominal interest rate. Under a calibration of higher degree of habit formation, or in other words more backward-looking property in IS curve, we find a hump-shaped response of output as suggested by Fuhrer (2000)[22]. However, such parameterisation leads to overshooting in the policy rate.

Following the rate cut it is a typical bull-steepening of the yield curve with the level falling and curvature rising as well as slope.

3.2.2.4 Supply Shocks Figure 3.4 shows the responses to a positive supply shock. Expansion in production capacity means a negative output gap, so that central bank cuts the rate to boom the demand to match the potential output level. High supply depresses the price level, which reinforces a persistent loosening in monetary policy by setting lower policy rate over a long time period. Real interest rate is simply pushed up in the deflation scenario.

The nominal yield curve shifts down quite uniformly across all the maturities led by the persistence of supply shock, leaving curvature nearly unchanged. For the same reason all the forward nominal rates are affected. However, the impact on the future rate and the

22 Inflation is also hump-shaped in some VAR evidence, for example with Euro area data, although it is not the case in our model.

long end of the yield curve disappear as we decrease the AR(1) coefficient of supply shock from 0.95 to 0.7.

3.2.2.5 Summary The simulation results are quite standard compared to existing literature. We find the main difficulty is lack of explanatory power of macro shocks in the long-end of yield curve. Under expectation hypothesis, impact of these shocks on medium to long term forward rates quickly die out. Secondly, nominal curvature hardly moves in response to shocks because the impact on future rate is passed on from one period to the next very smoothly, which is in clear contradiction to the actual data.

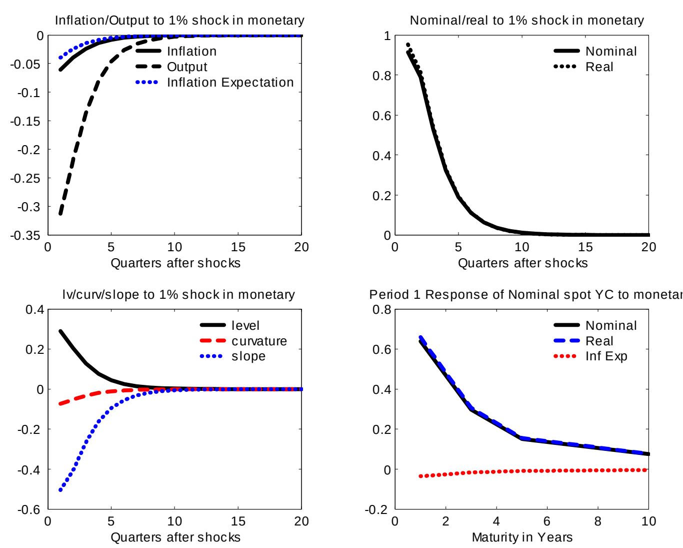

Figure 3.3 - Impulse response to policy shocks

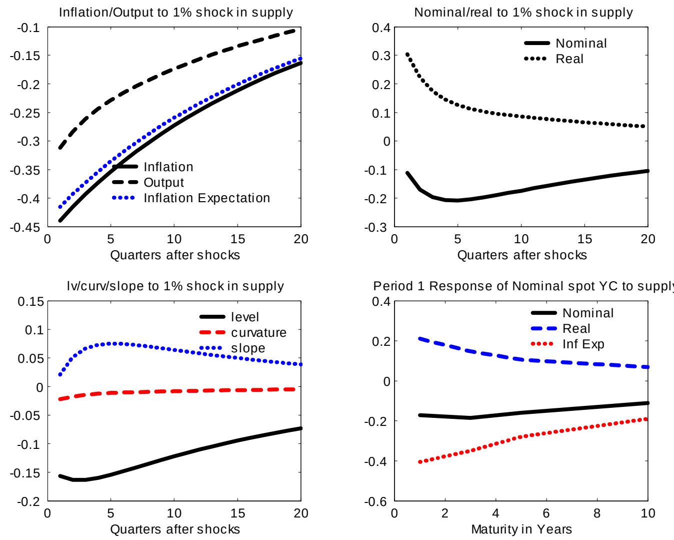

Figure 3.4 - Impulse response to supply shocks

3.3 Data and Stylized Facts

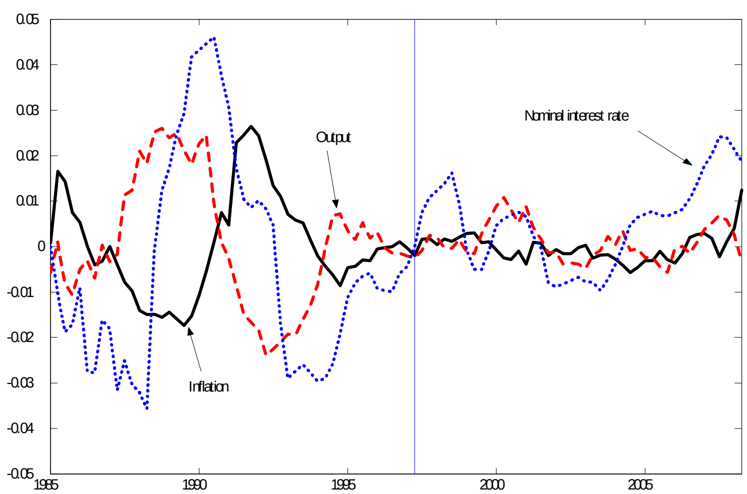

Figure 3.5 - Time series of key macro variables

I obtain UK macroeconomic and yield curve data to show some stylised facts and fit the

DSGE model. Real output is the Hodrick-Prescott filtered series of logged real GDP from OECD Main Economic Indicators. Similarly I take HP filtered CPI index as the cyclical inflation measures. They are both quarterly data for 1985Q1 to 2008Q2. Policy rate is the bank base rate after taking detrending for two separate samples: pre-1997 and post-1997. The sample partition represents a regime change of Bank of England's independence in June 1997. Yield curve data is from the Bank of England website. The spot yields are calculated from normal and inflation indexed government liabilities of the maturity 2.5 years, 5 years and 10 years.[23] The cyclical time series are plotted in Figure 3.5 and 3.6.

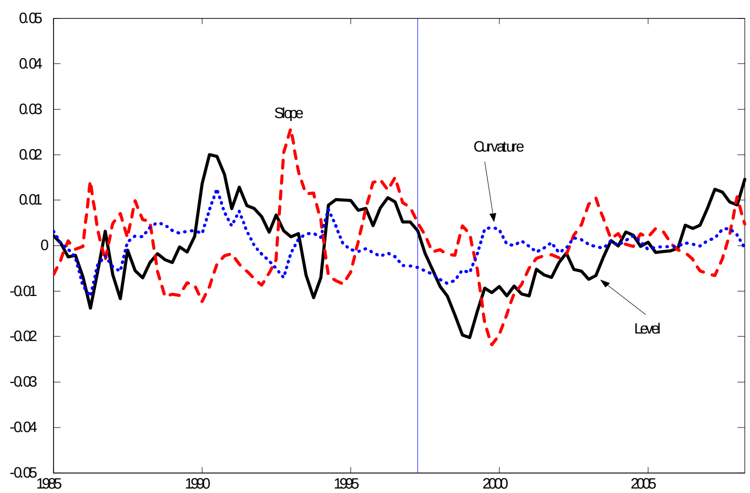

Figure 3.6 - Time series of yield curve variables

3.3.1 Stylised facts

It is very clear that variable volatilities show a distinction between the two sample periods. I investigate the composition of volatility of UK government liability yield. Figure 3.7

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23 The inflation expectation is calculated from the nominal and real yield by Bank of England, but it contains two distortions: liquidity premium and inflation risk premium, which are excluded from the analysis for this paper.

plots the three components of nominal yield variances for two data samples that are of our interests: 1997 onwards when Bank of England is granted operational independence; pre-1997 covers more than one monetary policy regime but can be seen as a period lack of credible stabilising monetary policy.[24] The volatility of nominal yields falls significantly in the late sample largely attributed to a sharp smoothing in inflation expectation, denoting a major change in policy parameter. As for an DSGE model, the volatility of endogenous variables is also dependent on the magnitude of exogenous shocks. In our case, I suppose the two sample periods represent more discrepancies in calibration for shocks rather than that for deep parameters.

[24] In 1980s UK monetary policy represent a nominal income targeting principle; during 1990 - 1992 UK became a European ERM member; for 1992 - 1997 Bank of England committed to an inflation target.

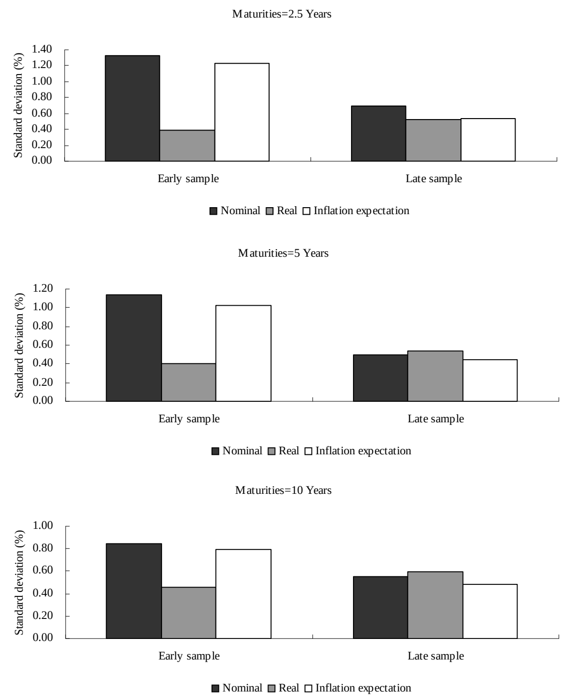

Figure 3.7 - Volatility of nominal yields

By eyeballing the graphical pattern of time series data in Figure 3.5 and 3.6, it is noticeable that yield curve variables volatility remains quite stable throughout two periods, which is in contrast to volatilities of key macro variables. Table 3.2 shows the comparison for nominal yield curve and its real part and inflation expectation part, respectively. Clearly, relative volatilities of all yield curve variables increase in late sample, but only due to the falling volatility of the key macro variables, inflation, output and policy rate. Secondly, the inflation expectation part of yield curve slope and curvature, as calculated from nominal and inflation-protected government bond yields, shows larger volatility than that of real yield curve counterparts, however the disparity is narrowing. The fact requires us to look into the determinants of yield curve slope and curvature in an DSGE model. Finally, yield curve slope and curvature show larger discrepancy among two samples than yield curve level.

Table 3.2: Relative volatilities of yield curve variables

	Sample	Nominal	Real	Inflation Expectation
Yield curve level				
$\sigma_{yc,L}/\sigma_{\pi}$	Early	0.926	0.323	0.812
	Late	1.637	1.602	1.046
$\sigma_{yc,L}/\sigma_{y}$	Early	0.744	0.260	0.653
	Late	1.256	1.229	0.803
$\sigma_{yc,L}/\sigma_{i}$	Early	0.328	0.115	0.288
	Late	0.475	0.465	0.304
Yield curve curvature				
$\sigma_{yc,C}/\sigma_{\pi}$	Early	0.424	0.251	0.527
	Late	0.855	1.571	2.066
$\sigma_{yc,C}/\sigma_{y}$	Early	0.340	0.202	0.424
	Late	0.656	1.206	1.585
$\sigma_{yc,C}/\sigma_{i}$	Early	0.150	0.089	0.187
	Late	0.248	0.456	0.600
Yield curve slope				
$\sigma_{yc,S}/\sigma_{\pi}$	Early	0.838	0.414	0.986
	Late	2.313	1.774	2.674
$\sigma_{yc,S}/\sigma_{y}$	Early	0.674	0.332	0.792
	Late	1.775	1.361	2.052
$\sigma_{yc,S}/\sigma_{i}$	Early	0.297	0.147	0.349
	Late	0.671	0.515	0.776

These observations themselves raise open questions for the DSGE model analysis framework. Refinement of calibration and interpretation of model dynamics are towards a sound understanding of the sources of these stylised facts.

3.4 Detection of Shocks in Macro- and Yield Curve Variables

Following the assumption that two types of shocks, macroeconomic shocks and financial shocks, may jointly drive yield curve variables, it is rather painful to introduce a shock identification framework. However, it is sensible to firstly calibrate the model using macro shocks only for best fit of the key macroeconomic variables, then investigate how much of other shocks, including financial shocks, are required to capture observed data of yield curve dynamics.

While focusing on calibration of exogenous shock process, deep parameters (μ,ϕ,κ,δ) and policy parameters (γ,ϕ_π,ϕ_y) are made fixed temporally for an obvious reason: the choice of these parameters in existing literature does not show large discrepancy. Although specification of shock process is highly relevant to the sample data, it is often chosen randomly in "informal approach" (Ortega, 1996) of model evaluation such as DSGE literature. It is preferable to allow limited cross-sample variation in deep and policy parameters but more flexibility of shock specification.

3.4.1 Calibration with model selection

For computational simplicity I discard full-scale estimation but adopt the calibration approach via a 'weak form' model selection procedure of Chapter 2, which was inspired by Watson (1993). Under a frequentist paradigm, sample data Variance Covariance Matrix (VCM, with a complete set of unconditional second moments) can be used to infer a 'distance' measure for any calibrated model. According to Bhattacharjee and Thoenissen (2007), I use the Kullback-Leibler information criterion (K-L) to measure the 'information loss' by interpreting the unconditional, true data second moments with the calibrated dynamic models. The best model would cause the least loss of information, represented by the lowest K-L distance, d_{KL}, therefore calibration with K-L distance can be seen as a minimum-distance procedure. For a given set of data and a part of parameters fixed, I use a Matlab optimisation routine to pin down the best-fit parameters of our interests. A detailed description of the procedure can be found in Appendix C of the dissertation.

It should be noted that the chosen parameter-set for refined calibration should not include too many members, as the procedure may lead to over-identification. Consequently the marginal improvement of model fit might be problematic. By fixing the base case calibration except for shock volatilities (σ_{IS},σ_{AS},σ_{MP},σ_{YN}), I search for the best calibration for four chosen volatility parameters in each of sample. Table 3.3 shows the search results from the minimum distance measure of covariance matrices for a trivariate system of ($\widehat{\pi}_t$,$\widehat{Y}_t$,$\widehat{i}_t$).

Table 3.3: Calibration of macro shock volatilities

Fixed parameters		Newly calibrated parameters	
Structural	Shocks	Early sample	Late sample
$\mu = 0.75$	$\rho_{IS} = 0.33$	$\sigma_{IS} = 0.923\%$	$\sigma_{IS} = 0.265\%$
$\phi = 0.95$	$\rho_{AS} = 0.74$	$\sigma_{AS} = 0.130\%$	$\sigma_{AS} = 0.109\%$
$\delta = 0.1$	$\rho_{MP} = 0.30$	$\sigma_{MP} = 2.013\%$	$\sigma_{MP} = 0.705\%$
$\kappa = 0.1$	$\rho_{YN} = 0.95$	$\sigma_{YN} = 0.000\%$	$\sigma_{YN} = 0.305\%$
$\gamma = 0.6$			
$\phi_\pi = 1.5$			
$\phi_y = 0.5$			
K-L distance (base case)		5.802	1.251
K-L distance (best-fit)		1.896	0.865

By choosing specific combinations of shock volatilities, the model could generate a simulation with covariance matrix closer to actual data. The model selection favors a dominant role of policy shock in explaining unconditional second moments, especially in early sample. It is also believed that the early sample is mostly a demand driven economy, with monetary shocks coming into demand channel indirectly. Supply side shocks in terms of price mark-up are stable over two samples. But supply shocks via capacity are more important in second sample period. The results are in general reasonable given the development of UK and global business cycles in last two decades: multiple monetary policy regimes took place in early sample including a nominal income targeting period; the most recent policy rate are mostly in line with the expectation under inflation target and independent central bank; supply shocks are believed to contain much more uncertainties in recent decade due to the changing environment of macroeconomic management, including higher level of globalization (Garganas, 2006). In the late sample, for example, the increasing role of supply shocks can be considered as evidence of capacity expansion in a globalised product market.

Macroeconomic shocks are assumed to explain all the variation in the key variables. However, best-fit parameters do not repeat success when matching the yield curve variables. In the simulated series for early sample level, yield curve curvature and slope indicators have standard deviations of 0.37%, 0.15% and 0.47%, respectively. Only slope variable achieves good fit of nearly one to one, in contrast to that of level (0.78%) and curvature (0.94%) in actual early sample data. Yield curve level and curvature are not always dampened in model simulation. For late sample, the yield curve variables become less volatile than model prediction, which may suggest some extra shocks that affect yield curve are likely to offset impact from macro shocks.

Can we calibrate the model recursively to find the best-fit over a broader parameter space? I suppose the answer would be NO. It is unwise to make inference on full parameter set as (1) the model might be misspecified model in some dimensions; (2) the VCM distance approach ignores conditional second moments and (3) the property of the minimum-covariance matrix distance approach is not fully understood.

3.4.2 More shocks in yield curve dynamics

For the purpose of second moments matching for yield curve features, it is necessary to add more shocks into the system. The shock shall only affect yield curve dynamics so that the easiest way to do is to insert a serially uncorrelated noise $u_{i,t}$, $i = L, C, S$:

$$\begin{aligned} yc_{L,t} &= \frac{1}{3}(i_{10,t} + i_{20,t} + i_{40,t}) + u_{L,t} \\ u_{L,t} &\sim N(0, \sigma_{u1}^2) \end{aligned}$$

These shocks can be roughly considered as financial shocks, such as liquidity premium deviations, noise trading and changing risk premium etc. The ultimate purpose of adding new shocks is to quantify the role of macroeconomic shocks in yield curve dynamics. Again I run minimum distance procedure to pin down the best-fit value of shock volatilities σ_{u1},σ_{u2},σ_{u3}, subject to the covariance matrice of nominal yield curve triplet. The best calibration and decomposition of volatility accountability are reported in Table 3.4.

Table 3.4: Variance decomposition of shock-augment model

Early sample	Shocks & Calibration		Decomposition of unconditional variances π	Y	i	yc_L	yc_C	yc_S
Macro-shocks	σ_{IS}	0.923%	16.6%	33.8%	2.0%	2.7%	0.6%	3.0%
	σ_{AS}	0.130%	0.9%	0.0%	0.0%	0.0%	0.0%	0.0%
	σ_{MP}	2.013%	**82.5%**	**66.1%**	**98.0%**	**89.3%**	19.0%	**93.3%**
	σ_{YN}	0.000%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Other-shocks	σ_{u1}	0.586%	0.0%	0.0%	0.0%	8.0%	0.0%	0.0%
	σ_{u2}	0.766%	0.0%	0.0%	0.0%	0.0%	**80.4%**	0.0%
	σ_{u3}	0.415%	0.0%	0.0%	0.0%	0.0%	0.0%	3.7%
Late sample	Shocks & Calibration		π	Y	i	yc_L	yc_C	yc_S
Macro-shocks	σ_{IS}	0.265%	0.7%	18.7%	0.8%	0.1%	0.0%	0.8%
	σ_{AS}	0.109%	3.0%	0.2%	0.3%	0.1%	0.0%	0.7%
	σ_{MP}	0.705%	8.0%	**80.8%**	**86.1%**	5.8%	0.7%	**53.4%**
	σ_{YN}	0.305%	**88.3%**	0.3%	12.8%	6.9%	0.1%	**30.7%**
Other-shocks	σ_{u1}	0.741%	0.0%	0.0%	0.0%	**87.2%**	0.0%	0.0%
	σ_{u2}	0.640%	0.0%	0.0%	0.0%	0.0%	**99.2%**	0.0%
	σ_{u3}	0.234%	0.0%	0.0%	0.0%	0.0%	0.0%	14.5%

This revised model clearly show a sample disparity in the accountability of macro shocks in both macroeconomic and yield curve dynamics. Firstly, monetary policy shocks seem very effective in directing macroeconomic fluctuations. Demand shocks are less influencial in late sample, while supply shock via potential output takes the turn, but not on the real activity. Regarding the yield curve, the curvature is mainly driven by unobserved factors $u_{C,t}$. But the level and slope are mostly macro shock driven variables. The disconnection in late sample for yield level might be another expression of the conundrum of the long term rate by Alan Greenspan. Monetary shocks are dominant in both samples, but less so in late sample. Particularly, yield curve slope is more relevant to macro shocks, which explains why it is useful for predicting business cycle turning points. These findings are in line with most literature of macro-finance interpretation of yield curve movements. Inflation dynamics are largely explained by supply shocks in the late sample, motivating further research of a possible break in traditional price-output correlation in previous demand-dominant period.

To summarize the overall performance of newly calibrated model, Table 3.5 gives the relative volatility of yield curve variables in calibrated model. By fitting the model with macroeconomic variables, all of three key facts (section 3.3) about relative volatilities of yield curve variables are poorly matched. In Figure 3.1 through 3.4 it is clear that markup-shock and policy shock drive stronger response than other two shocks. As smoother macro series require muted policy shocks, yield curve variables share a decrease in absolute volatilities, which is not in line with the data. Specifically, the transmission of stabilized monetary policy (rate policy) is well foreseen so the actual inflation volatility is only dependent on mark-up shocks, which are similar in the calibration of the two samples. Consequently, the inflation volatility remains stable and the overall match of relative volatility is poor. Other irregularity includes: the nominal and real yield curves are very close to each other in terms of dominant foreseeable monetary policy; the curvature is mostly determined by rate dynamics capturing the belief of a pure expectation hypothesis. In order to fill in these gaps, the shocks only relevant to yield curves are added, contributing to an joint explanation of second moments of these variables (column 4).

In short, the optimal calibration for shocks based on macroeconomic data does not guarantee an even reasonable match for yield curve variables. No doubt the existence of more factors, including financial factors, gives a straightforward answer to such an inquiry, the above exercise still serves as a vivid illustration of the incapability that macroeconomic model cannot explain yield curve dynamics alone.

Table 3.5: Relative volatilities of yield curve in simulated models

	Sample	Only macro shocks $yc^i_{L/C/S}$	$yc^r_{L/C/S}$	$yc^{\pi e}_{L/C/S}$	Plus new shocks $yc^i_{L/C/S}$
		Yield curve level			
σ_{yc}/σ_π	Early	1.778	1.810	0.293	2.375
	Late	0.798	0.762	0.459	2.473
σ_{yc}/σ_y	Early	0.468	0.477	0.077	0.625
	Late	0.566	0.541	0.326	1.755
σ_{yc}/σ_i	Early	0.291	0.297	0.048	0.389
	Late	0.311	0.297	0.179	0.963
		Yield curve curvature			
σ_{yc}/σ_π	Early	0.447	0.455	0.069	2.091
	Late	0.193	0.188	0.068	2.027
σ_{yc}/σ_y	Early	0.118	0.120	0.018	0.550
	Late	0.137	0.133	0.049	1.439
σ_{yc}/σ_i	Early	0.073	0.075	0.011	0.343
	Late	0.075	0.073	0.027	0.790
		Yield curve slope			
σ_{yc}/σ_π	Early	3.252	1.384	0.235	3.435
	Late	1.371	0.579	0.293	1.557
σ_{yc}/σ_y	Early	0.856	0.364	0.062	0.904
	Late	0.973	0.411	0.208	1.105
σ_{yc}/σ_i	Early	0.533	0.227	0.038	0.563
	Late	0.534	0.226	0.114	0.607

3.5 Conclusion

In this chapter, I manage to incorporate a DSGE modeling technique to macro-finance field, establishing links between the yield curve and simple macroeconomic state variable.

In this sense, I build a model related to the conceptual approach of the single factor model of Vasicek (1977) but allow the single factor of the policy rate to be related to a specified macroeconomic model of the type used for monetary policy analysis. This exploration is thus carried out in clean framework with only a canonical New Keynesian model. As well as a calibration and simulation study based on the connection between macro shocks and yield curve features, I also apply a minimum distance approach based on unconditional co-variance matrix to match macro and yield curve data to this highly stylized DSGE model.

Using UK data as the benchmark, the findings are relatively standard within the context of existing empirical research (see Chadha and Holly, 2006). I find a significant mismatch between the observed yield curve and that which is conditioned on a calibrated NK macroeconomic model, but mainly for yield curve curvature. I start from a benchmark calibration and use the VCM matching technique to detect a sensible set of macroeconomic shocks, or forcing processes, behind UK business cycles. The typical macro shocks, including demand, supply and policy shocks, can each explain dynamics of some yield curve features, except for very weak impact on yield curve curvature. Consequent to this identification, I am then able to gauge the magnitude of unobserved shocks that are required to fit the yield curve alone. What the results seem to suggest is that there is a limit beyond which the yield curve seems to be disconnected from the first generation of DSGE models. We thus need to add exogenous shocks to the yield curve in order to better fit its dynamics but these shocks do not seem especially helpful to understand macroeconomic dynamics.

That said there seems to be significant information from the macroeconomy for the slope term, which again fits with the basic insight of a considerable body of empirical evidence: that the slope component of the yield curve is closest to the dynamics of a DSGE model is perhaps an important observation.

The next steps for this research agenda is to understand the structural causes of level and curvature shocks to the yield curve, which may in turn be related to term premia and the interaction of the quantity and price of risk, which is not modelled in this type of DSGE framework directly but as the financial crisis suggests, perhaps should be so modelled.

Chapter 4
Labour Market Search and Monetary Policy - A Theoretical Consideration

4.1 Introduction

[25]The real business cycle revolution (see Finn Kydland and Ed Prescott's respective Nobel Prize Lectures, 2004) seems to have been most tangibly captured by monetary theoreticians and practitioners. It might reasonably be argued that this a remarkable, almost perverse, outcome because a real business cycle economy is one in which agents follow optimal decision rules at all times and in all states of nature and where output lies at the flex-price equilibrium through time. That is one in which there is typically no unemployment. In other words it is also one in which there is no obvious role for a monetary policy-maker, whose role might be defined as that of ensuring an optimal rate of convergence to the flex-price outcome following shocks, and where the costs of business cycle fluctuations are limited.[26] How such models became the workhorse for studying monetary policy problems, where sequences of short term interest rates are chosen to offset the impact of expected divergences of output from its flex-price optimum, is a question that will no doubt intrigue future historians of economic thought.

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[25] This chapter is based on a joint research project completed with Jagjit S. Chadha. An earlier version of the paper has been published as a chapter "Labour Market Search and Monetary Shocks: A Theoretical Consideration", in Unemployment: Past and Present, Philip Arestis and John McCombie (Eds), Palgrave Macmillan, 2008.

[26] In fact in a famous thought experiment, Lucas (1987) showed that under a representative agent assumption the actual costs from expected flutuations were a small fraction of average consumption.

The bolting of price-setting rigidities onto a real business cycle model by Yun (1996) using the Calvo (1983) price setting mechanism – a form of price rigidity that means only an exogenously set fraction of firms can reset prices in every period – allowed the introduction of meaningful monetary policy. In the Calvo-Yun set-up, productivity (or marginal cost) shocks impact on the optimal, or the desired, price level in each period, but as only some firms are given the signal to reset, the overall price level is suboptimal. So following a positive productivity shock, overall prices are too high, which means that output is too low (relative to the flex-price outcome) and that some firms suffer a lower than optimal profit level as their prices do not have an appropriate mark-up. The policy problem is thus to set a rule such that, contingent on such shocks, the expected sequence of interest rates ensures that the deviation of output from its flex-price optimum is minimized subject to informational constraints (see Woodford, 2003, for an elaboration). This paradigm has been hugely influential.

And yet the popular New Keynesian Phillips curve (NKPC) framework has not been especially successful in matching several aspects of the business cycle.[27] The current generation of monetary research, which has concentrated on the implications of nominal rigidities in prices or wages, has perhaps not sufficiently emphasized the possibility of real rigidities, such as habit formation in preference or labour market matching. In fact, the labour market has been a particularly sore point of contention. King and Rebelo (2000) in their overview of the real business cycle developments outline a number of prob-

[27] We shall use the terms New Keynesian and Calvo-Yun interchangeably in this paper.

lems with the standard business cycle model in terms of the labour market: it implies too high an elasticity of labour supply to wages; business cycle hours variation arises from changes in the hours-per-worker in the model (the so-called 'intensive margin'), whereas it is movements in numbers of people employed (the so-called 'extensive margin') that actually seem to determine fluctuations in total labour input; and so the model suggests a counterfactual degree of correspondence between labour inputs and its average product.

In this chapter we will highlight two aspects that the standard business cycle model need to address: (i) the observed persistence in the response of employment patterns and wage rates and monetary variables; and (ii) the hump-backed shape of the impulse response of output following a monetary policy shock. Following Walsh (2003), in which a dynamic stochastic general equilibrium (DSGE) model with labour market matching is introduced and calibrated for the US economy, we will examine the issue of labour market search in a DSGE model calibrated on UK data.[28] The main focus will be on labour market rigidities that prevent unemployed workers from finding new jobs and firms with job vacancies from filling them immediately. This feature generates both a persistence of response in the labour market to monetary shocks and adjusts the nature of the policy problem faced by the authorities. Dynamic simulations are used to investigate the role the job matching process plays in affecting the economy's dynamic adjustment to shocks. Employing a degree of nominal rigidity as well, we will investigate the implications for monetary policy. We point to ways in which this work can be used to understand the role of the labour market

[28] Using ONS sources and other estimates we are able to assess the implied second moments of UK business cycles pre- and post-reform in mid 1980s.

in explaining the UK business cycle over the past 20 years.

As mentioned, there are two reasons why we pay attention to the labour market specification in a DSGE model. Firstly, most of the existing RBC and DSGE literature use total hours as the labour input and consider the intensive margin of labour supply. In such a set-up unemployment is ignored in the analysis. Consequently, the model-generated dynamics imply highly procyclical real wages and smooth employment, as opposed to nearly acyclical real wages and volatile employment numbers in reality.[29] Therefore a more realistic model for labour supply, such as indivisible labour (Hansen, 1985), may be required. Secondly, nominal rigidities in price and wage setting are not sufficient in accounting for inertia of macroeconomic variables. In respect of the wage bargain, it might be argued that plans for long-term service may outweigh the incentive to renegotiate wages, and to some extent this undermines the microfoundations of Walrasian wage setting. Given these problems, the investigation of the implications of some sort of real rigidities, such as job-matching frictions, seems highly appropriate and may reconcile the debate of a frictionless labour market and match the persistence in the data.

Walsh (2003), Walsh (2005) and Ravenna and Walsh (2007) use a labour market search and matching model to incorporate this type of real rigidity. In Walsh (2005), this model predicts a hump-shaped response of output to a policy rate shock. The key to this result is the delay in production caused by the time required to fill job vacancies, which is a costly

[29] According to King and Rebelo (2000) and Millard *et al.*'s (1997) calibration, US and UK data of output-real-wage correlation is 0.12 or 0.01 versus 0.98 in the RBC model; US and UK relative volatility of employment to output is 0.99 or 1.11 versus 0.48 in the RBC model.

process for firms and workers. The job-matching feature also permits a considerably richer dynamic in inflation than the NKPC model.[30] This motivates us to repeat the empirical investigation on UK data, especially in relation to the past two decades, when the UK labour market became more flexible. For example, during this period, days lost in strikes have fallen markedly as a fraction of UK total hours, as a result, in part, to more rigorous legislation on union actions.[31]

This chapter contribute to the existing literature in two aspects. Firstly, it formally construct the theoretical link between labour market rigidity and preferrable emphasis of monetary policy. Without sufficient consideration on labour market specification, policy makers may prefer an inflation target instead of output and employment objectives. Secondly, the constructed model replicate, with reasonably convincing calibration for UK data, that the monetary policy transmission is more effective after UK undertook some reform in labour markets.

In the next section we will outline the response of key UK variables to a monetary shock. We will use these estimates to understand the extent to which output and unemployment respond in a persistent manner to monetary policy shocks. We then outline the key implications of the Walsh matching model for the Phillips curve, before going on to illustrate the resulting dynamics from a baseline simulation of the model and indicating

30 In Generalized Method of Moments (GMM) estimation, the Ravenna-Walsh model is not rejected for US post-war data, as opposed to its NKPC counterpart.

31 Financial Times, Work days lost to strikes soar, 12 June 2007. Based on ONS data on total hours and average weekly hours worked and FT data on days lost to strike, this ratio for 1970s, 1980s and post-1980s period are roughly 10%, 6% and 1%.

the implications for monetary policy, in terms of the weight given to inflation rather than output in the policy rule. In the Appendix we give the full set of linear equations for the Walsh (2003) model and in Tables 3.1, 3.2 and 3.3 the set of calibration parameters we adopt.

4.2 UK data responses

To explore some of the ideas about persistence and speed of adjustment to shocks, we will estimate a simple model of the UK macroeconomy. We use quarterly data from 1978 to 2006 for UK bank rate (i), the real effective exchange rate based on relative CPI (er), the constant price GDP by the expenditure approach (y), the all-items RPI index (π) and the unemployment rate (u) measured by the claimant count. We estimate a VAR of order 2 to understand the dynamics. Base rate and unemployment enter the VAR in levels, but the real exchange rate, real output and the price level are differenced. To identify the system, we use a Cholesky ordering restriction and examine the response of these variables to a monetary policy shock, where: $i \rightarrow er \rightarrow y \rightarrow \pi \rightarrow u$.

Figure 3.1 shows the responses of the key macroeconomic variables to a 100 base-point shock to the base rate. The abscissa shows the length of response in quarters and the y-axis the amplitude of the response in percentage point deviations from the mean of the series. Note first that the half-life of a 100Bp shock to interest rates is in the region of 12 quarters and represents the typical magnitude of a policy shock. The exchange rate responds earliest, and by the second quarter of the shock has undergone an appreciation in the region of 0.75 per cent, which decays back to nearly baseline after a year or so. For the UK, which is a relatively open economy with imports and exports each equal to nearly 30 per cent of GDP, inflation mirrors the jump in the exchange rate and we observe a hump-backed response from inflation which falls most around a year after the initial policy shock, by around 1.25 per cent, and it is after about four years that it returns to base.

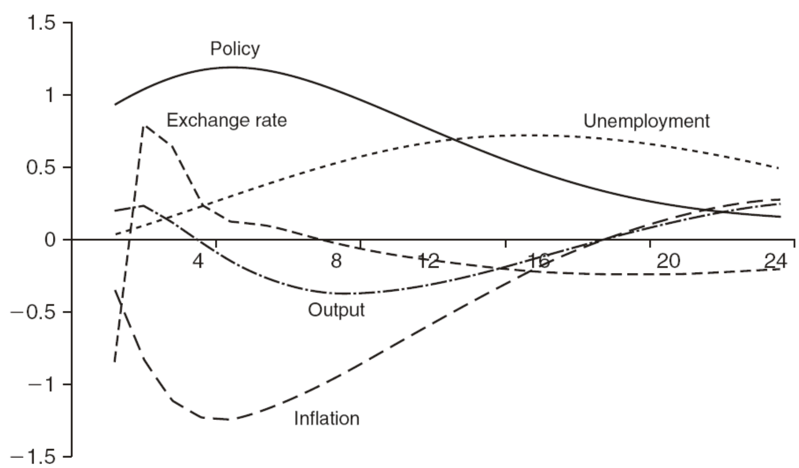

Figure 4.1 Responses to policy shock: UK

Note: We plot impulse response in a VAR model for UK business cycle variables and bank rate quarterly data 1978 to 2006. The Cholesky ordering restriction is (bank rate) ⟶(real exchange rate) ⟶(real output) ⟶(inflation) ⟶(unemployment rate) It is noticeable unemployment responds to policy rate with 4-year lag.

But the key message is that output adjusts slowly, with its initial response somewhat less than its largest response, with a 0.5 per cent fall in output around two years after the initial monetary policy shock. The impact of the shock decays to baseline in around year four. We note also that labour unemployment – that is, movements along the extensive margin

– are perturbed for quite some time following a typical monetary policy shock, with the peak response in unemployment coming at about three to four years. And in fact on these estimates unemployment still seems to be adjusting some six years after the initial interest rate shock. The Bank of England's (2005) quarterly model predicts a similar response in real variables following a contractionary monetary shock (100bp rate hike), however the peak response appears at 1 to 2 years horizon. In the later section we compare the impulse response of Walsh model and BoE model.

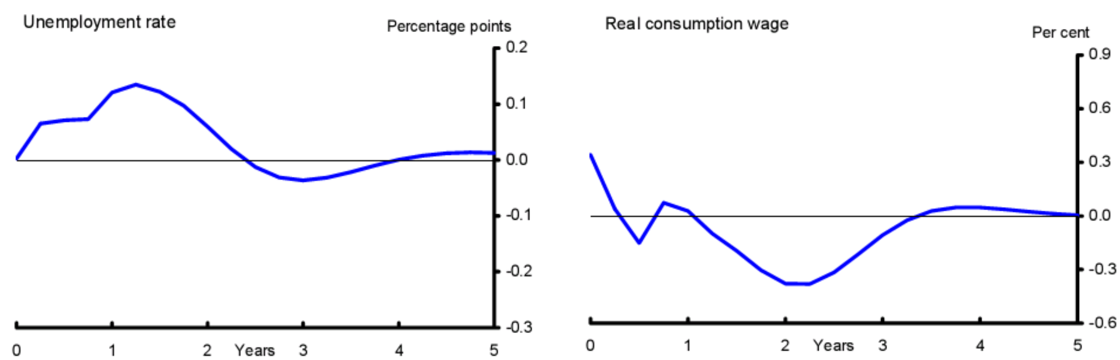

Figure 4.2 Impulse responses to a shock of 100bp rate hike in Bank of England's quarterly model

Note: The Impulse Response Functions are taken from Bank of England Quarterly Model book, page 130. Unemployment rate rises in response to a rate hike while real wages falls after first year. The model brings also sticky wages in addition to real rigidities such as union bargaining in the model. Compared to Figure 4.1, the model is successful in generating certain lag of unemployment response, but not sufficient. In fact, the norminal rigidities contribute merely to a sluggish adjustment in labour market. Relative volatility of business cycle variables remain a puzzle. This is one of the motivation to highlight a search and matching framework.

The gradual build-up of momentum on the real-side of the economy from a monetary shock, with output and unemployment responding persistently, has a number of implications. First, that even though inflation, which is likely to have a strong forward-looking element, may be stabilized from a given monetary policy shock, the real economy may still be some way from equilibrium and still imposing some losses on the representative household, which may be better off from stable inflation but still likely to suffer from losses from some degree of unemployment. Secondly, this implies that monetary policy, which places any weight on output fluctuations, will be somewhat more cautious when stabilizing inflationary shocks. We shall explore below the implications of building more labour market persistence into the basic New Keynesian (NK) framework, in particular for the design of monetary policy.

4.3 Labour search and matching model

In a standard NK model, the labour market is modelled as follows. Positive productivity shocks lower marginal costs and the Walrasian labour market continues to ensure that labour is paid at its higher rental rate. Higher real wages bid for the marginal leisure hours of workers, and output, which is Cobb-Douglas, increases. The main additional feature of this labour search and match model is the endogenous and exogenous job destruction in the labour market. Endogenous job destruction is a result of individual productivity, i.e., there is a threshold in productivity which determines a individual worker either maintain a job contract or lose it. Such a margin requirement may generate acyclical real wages as it is argued that low productivity (paid) workers are fired in downturns. There is also a possibility of exogenous job destruction. Shocks may impact on those who left positions to join the job seeker pool and go through a matching process via job applications to posted vacancies. The difficulty in the matching of job seekers and vacancies decreases the instantaneous response of output to aggregate shocks. This structure creates inertia in the dynamics of key variables.

Table 4.1 - Key variables

Variable	**Definition**
R_t :	Nominal rate
y_t :	Real output
π_t :	Inflation
n_t :	Employment
φ_t :	Survival rate
V_t :	Vacancies
k_t^f :	Probability of filling a vacancy (firms)
q_t :	Expected excess value of match
a_t :	Destruction margin (threshold productivity for continuing job contract)
u_t :	Unemployed workers (searchers)
k_t^w :	Probability of finding a job (workers)
μ_t :	Mark-up
ρ_t:	Endogenous job destruction
χ_t :	Matched jobs (job creation)
w_t :	Real wages
$\widetilde{u}_t$:	Unemployment rate (deviation from steay state, we assume 5.5% for UK)

Figure 4.3 shows a very simple case of matching duration and its implication on persistence in macroeconomic dynamics. In a world with only labour as production input and productivity is fixed, at time 0 there is a 1 per cent permanent shock to labour input. With instantaneous matching (upper panel), output and labour adjust immediately – this is analogous to the NK model case. However, with a matching probability at, say, 50 per cent per period, the matching in the labour market will continue gradually until the new steady

state is reached, thus generating significant persistence in the dynamics – with the overall time period for adjustment in the region of three to four periods.

Let us start from a typical Calvo-Yun economy where the production does not include capital accumulation $Y_t = Z_t N_t$, where Z_t denotes the exogenous technology variable, N_t is total labour hours and Y_t is output. With monopolistic competition in the final goods sector and sticky prices, we have:

$$w_t = \frac{Z_t}{\mu_t}, \tag{4.1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t, \tag{4.2}$$

$$mc_t = w_t - z_t, \tag{4.3}$$

where μ_t denotes mark-up of market prices to wholesale prices, $\kappa \equiv \frac{(1-\omega)(1-\omega\beta)}{\omega}$, which is the slope of the Phillips curve, ω is the probability that a firm receives a signal to reprice, β is the subjective discount factor, mc_t is the marginal cost and w_t is wages.

The Walsh model considers a form of job separation and a prolonged matching process. The probability of job matching $q(\theta_t)$ is a function of labour market tightness, θ_t, which is the vacancy-search ratio. The adoption of a matching technology complicates a firm's decisions on whether to post a vacancy and makes both labour market tightness and the cost of posting a vacancy determinants of the equilibrium wage. Real wages now take this form:

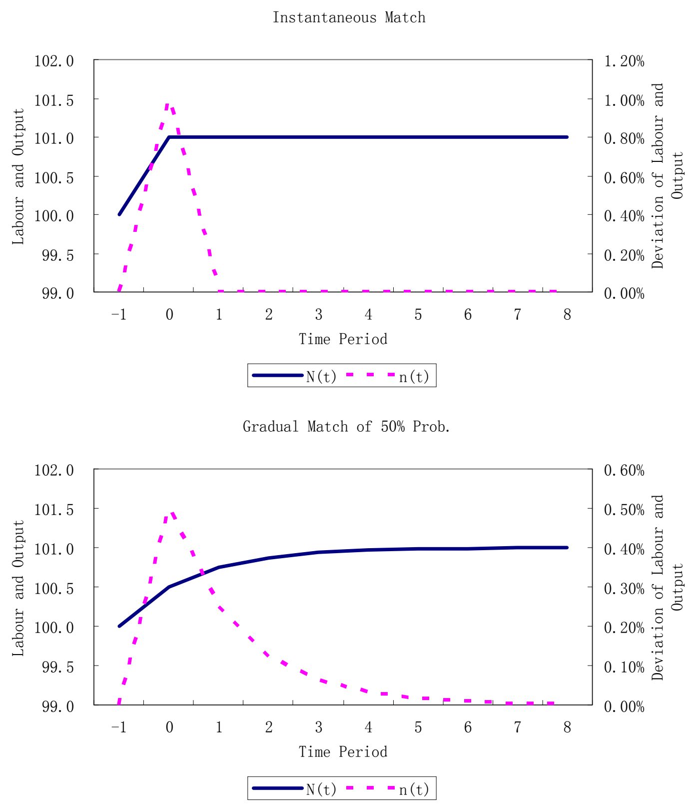

Figure 4.3 - Job matching and persistence

Note: In this simple one-factor economy, productivity is normalized to 1 therefore $Y(t) = N(t)$, lower-case variables denote deviation from steady states. For instantaneous match, labour input and output jump to the new steady state but in gradual match case it takes more than 4 periods to reach the new level.

$$w_t = \frac{Z_t}{\mu_t} - \frac{\gamma}{q(\theta_t)} + \beta(1-\rho) E_t(\frac{\lambda_{t+1}}{\lambda_t}) \frac{\gamma}{q(\theta_{t+1})}, \tag{4.4}$$

where γ is the cost of posting a vacancy, $q(\theta)$ is the steady-state probability of job matching, ρ is the probability of job separation, λ_t is the shadow value of consumption and μ_t is the mark-up. In Equation (4.4), the real wage is determined by both the marginal product and the two terms from job matching. The real wage adjusts downwards when vacancies are posted (placed) in the current period and adjusts upwards for any delay in posting a vacancy until the next period. If we substitute this key equation into the Phillips curve, augmented by matching in the labour market and by a term for impacting on the cost of search, i.e. in terms of the interest rate, we arrive at a considerably more complicated dynamic:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t, \tag{4.5}$$

$$\begin{aligned} -mc_t &= \mu_t = z_t - A\left(1-\xi\right)\widehat{\theta}_t \\ &\quad -A\beta\left(1-\rho\right)\left[1-\eta\theta q\left(\theta\right)\right]\left(i_t - E_t\pi_{t+1}\right) \\ &\quad +A\beta\left(1-\rho\right)\left[1-\xi-\eta\theta q\left(\theta\right)\right]E_t\widehat{\theta}_{t+1}, \end{aligned} \tag{4.6}$$

where (for both equations) A is given by $\mu(\frac{1}{1-\eta})\frac{\gamma}{q(\theta)}$, $mc_t(=\frac{1}{\mu_t})$ is marginal cost and the steady-state mark-up for the retail sector is μ, η is the worker's share in the wage bargain, ξ is the elasticity of vacancies with respect to matches, and $(i_t - E_t\pi_{t+1})$ is the real interest rate. The expression of the mark-up, μ_t, implies that a tighter labour market today increases marginal costs and creates an incentive for inflation, though the likelihood of a tighter labour market tomorrow reduces current marginal costs and tends to lower inflation, and higher interest rates impact directly on marginal costs and so on inflationary pressure. Inflation can also escalate on the real interest rate, which is a cost to the firms. Evidence for such a cost-channel of inflation can be found in Ravenna and Walsh (2006).

They shows a cost-channel enters the monetary policy making endogenously and under the optimal policy does not have to imply an output level close to the flexible-price output. In a search model, Equation (4.6) shows immediately that an increase in steady state matching probability $q(\theta)$ is favorable in mitigating the cost-push effect of inflation. Moreover, it also modify the determinacy problem in a typical three-variable NKPC framework.

So the mark-up, as the inverse of marginal cost, is affected by the unit labour cost (z_t), adjusted by a cost-push channel (real interest) and labour market tightness terms (second and fourth term). A tighter labour market (higher $\widehat{\theta}_t$) increases, to some extent, inflation instantaneously as the marginal cost rises in the cost of posting vacancies. Expectation of an increase in the level of future labour market tightness is also priced in current inflation but with a reverse effect. The reason is that an expectation of future labour market tightness motivates firms to post vacancies *ex ante*, pushing up current labour market tightness even further. But precautionary job matching and an increase in labour supplied ensures some pressure for wages is cut in the next period, inducing a fall in marginal costs. These two channels of job matching, as inflation drivers, are parallel to those of real wage determination in equation (4.4). These two terms decrease in vacancy posting cost and in matching efficiency.

In a comparison with Calvo-Yun, we see that the job matching terms simply disappear from the Calvo-Yun framework. With a frictionless labour market, the production adjusts simply to an exogenous shock (for instance a cost-push shock to the Phillips curve) and labour is bid into place with no further adjustment and thus the key variables do not show sufficient persistence. In contrast, a matching model implies an important role for sluggish adjustment in the labour market. The production is effected by labour market tightness across periods so that higher inertia is built into this model.

In the study of a calibrated model, we are particularly interested in impulse response to nominal rate shock and aggregate productivity shock and sensitivity analysis on deep parameters in the labour market, that is: (i) the probability of job separation via exogenous and endogenous job destruction; (ii) the probability of matching between firms and job seekers; and (iii) the cost of posting a vacancy. We will relate these parameters to UK labour market policy during the last two decades in the following section. And in the subsequent section we will explore the implications for stability from this type of Phillips curve.

4.4 UK calibration

We now turn to calibration for Walsh model and illustration of the relationship between UK business cycles, policy and the labour market within the context of a search and matching mechanism. We are mostly interested in the model behaviour over the period from the late 1980s to 2007. The stylised facts of the UK labour market (see, Millard, 2000, for example) are a good starting point. Within the context of a more flexible labour market and weakening union power (or membership) over the past two decades, several aspects of labour market performance over the UK business cycle are noticeable: (i) employment and unemployment are more volatile after the 1980s reform; but (ii) real wages and hours (average and total) became less volatile; (iii) average hours worked rose in the early years after flexible labour market reform; and (iv) unemployment duration has remained quite stable since the early 1980s.[32] Our calibration allows for these changes in labour market behaviour and so allows us to examine whether important degrees of persistence still remain in the real-side of the economy.

We rely on a calibration against current UK data to evaluate this model. Tables 4.2 give the basic calibration parameters.[33] The model is solved for impulse response analysis. We wish to match the calibrated UK model to replicate major UK business cycle quantities, including output, inflation, unemployment, total hours, wages, vacancies and job separation.

4.4.1 Labour market issues

We try to understand the consequences of several major moves in labour market policy-making, including union legislation, immigration laws, unemployment and minimum wages, and other reform, such as the 'New Deal'. See Millard *et al.* (1997) and Nickell and Quintini (2001) for comprehensive review on the reform in UK labour market.

[32] Millard (2000) shows that the relative volatility of employment and unemployment to output during 1979-1988 and 1988-1996 are (0.87, 7.66) and (0.92, 8.42), respectively, while the relative volatility of total hours are 1.19 and 1.15. The relative volatility of real wages are 0.66 and 0.40, respectively. The average unemployment duration is roughly stable within 6-9 months.

[33] Further explanation of variables and parameters are given in Tables 4.1 and 4.2. The Appendix lays out the key equations of the model, which boil down to the Phillips curve.

Table 4.2 - Key calibration parameters

Coefficient	Walsh	Description	Source of calibration
ρ	0.10	Total prob. of job separation	0.08 (ONS)
ρ^x	0.068	Steady state: exogenous unemployment	0.05
$F(\widetilde{a})$	0.0343	Steady state: endogenous job destruction	0.03
$\widetilde{a}$		Steady state: job destruction margin	0.75 (see note to the table)
$H(\widetilde{a})$	1	Expected mean productivity	
e_a^F	15	Elasticity of $F(a)$ to a	
e_a^H	1	Elasticity of $H(a)$ to a	
ξ	0.6	Vacancy share in generating a match	0.6 (Millard (2000))
k^f	0.7	Prob. of filling a vacancy	
k^w	0.6	Prob. of finding a job	0.45 (ONS)
N	0.94	Steady state labour force	
γ	0.6	Recruiting cost	0.33 (Millard (2000))
η	0.5	Worker's share in wage bargaining	0.3 (Millard (2000))
κ	0.05	Coefficient of marginal cost in NKPC	
θ	11	Demand elasticity of differentiated goods	
h	0	Utility of home production if unemployed	
δ	2	CRRA	
Θ	1.01	S.S.: money growth	
β	0.989	Discount factor	

Unemployment rate and job separation rate is not stable throughout the sample in UK so that we take values close to all-time average 8% for total separation. Walsh's (2003) calibration is based on a comprehensive literature review but for US data, we carefully choose some of his calibration in our experiments due to lack of data[34]. Calibration on

[34] For example, according to den Haan et al. (2000) calibration, we need to know how many of 'jobs counted as destroyed in a quarter fail to reappear in the following quarter' (72.3 per cent in the US) and what is the 'ratio of creation to employment' (i.e., the job-creation rate, 5.2 per cent in the US). These two variables jointly determin how likely a firm fill a vacancy.

$F(\widetilde{a})$ and $\widetilde{a}$ follow Walsh's (2003) assumption that individual productivity $a_{i,t}$ follow a log-normal distribution with mean 1 and standard deviation of 0.15. $\widetilde{a}$=0.75 corresponds to 0.03 in cumulative density function, meaning 3% of the labour force lose jobs due to an endogenous job separation. This leaves an exogenous job separation of 5%. Probability of job matching is approximated from unemployment duration and survey data on firms. Here we assume an US firm is similar to an UK firm in the probability of filling an vacancy. For an average longer duration of unemployment in UK (more than 7 months versus less than 5 months in US), the probability of finding a job is $k^w = 0.45$ (Labour Market Review 2006 by ONS Labour Market Statistics). The exogenous shocks of producitivity and money supply is taken from typical UK business cycle calibration. Millard (2000) calibrate a search model to select the workers' share in wage bargaining conditional on the best fit for unemployment data. His calibration shows $\eta = 0.3$ for full sample.

Power of Trade Unions

In the 1980s there were considerable changes in the UK labour market, in particular: (i) the power of trade unions in calling and sustaining a strike became more restricted; (2) collective wave bargaining became more decentralized. We wish to evaluate the matching model under a certain degree of union power in wage bargaining, and we find the dynamics are sensitive to this parameter. In the wage determination equations (4.4) and (4.7), the higher the power of the union, the higher the wage. The model shows explicitly how union legislation affects the macroeconomy via the wage bargaining channel.

Unemployment Benefits

In the last two decades or so, the UK has seen some decreases in unemployment benefit. In this model we specify a household production channel for those who do not find a match in the matching process and leave the labour force. We can evaluate the role of unemployment benefit as one type of income source other than a paid job. In the search model this 'outsider' productivity has an impact on real wages. The vacancies posted by firms are also affected. We could answer the following questions: Does an increase in unemployment benefits lead to a slower or faster adjustment in a labour market hit by an exogenous shock? How does a decrease in UK unemployment benefit contribute to observed macroeconomic data quantitatively?

In this model, wages are determined both from wholesale producers' profit maximization (Equation 4.4) and workers' trade-off in entering or exiting the job market. The level of unemployment income (w^u, either home production or benefits) enters workers' wage bids and thus will tend to increase the reservation wage:

$$\begin{aligned} w_t &= w^u + \frac{\eta}{1-\eta}\frac{\gamma}{q(\theta_t)} \\ &\quad - \frac{\eta}{1-\eta}\beta(1-\rho)E_t(\frac{\lambda_{t+1}}{\lambda_t})\left[1-\theta_{t+1}q(\theta_{t+1})\right]\frac{\gamma}{q(\theta_{t+1})}. \end{aligned} \tag{4.7}$$

Adjusting w^u does not affect the dynamic response directly but lower unemployment benefits become an incentive to searchers to find a match quickly. This indirect effect on the model may shift the labour supply curve downwards (with a lower intercept, i.e., the reservation wage) and turn it flatter (more elastic to real wage).

Minimum Wages

The National Minimum Wage Act of 1998 and the establishment of the Low Pay Commission have led to several increases in minimum wages (NMW) since 1999. Although only a small fraction of around 6 to 8 percent of the labour force is affected by changes in the NMW, and more than one half of them are in part-time jobs, the estimated impact on total household nominal income is notable (a 0.2 to 0.5 per cent increase approximately with a 10 per cent increase in NMW). Although the real wage is endogenous in most DSGE literature, we can specify a minimum nominal wage and explore in this matching model the extent to which the NMW increase impacts on inflationary pressure. Does flexible labour market adjustment account for a muted response in aggregate economy to an NMW adjustment?

We may look into the possibility of designing a labour-specific wage determination mechanism which is compatible with firm aggregate wage equation. Wage determination for individuals is required to surpass a threshold and is related to idiosyncratic productivity $Z_{i,t}$. It will take the following form:[35]

$$w_{i,t} = \underline{w} + \int_{\overline{Z}}^{Z_{i,t}} \left[\frac{Z_{i,t}}{\mu_t} - \frac{\gamma}{q(\theta_t)} + \beta(1-\rho) E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{\gamma}{q(\theta_{t+1})} \right] f(Z)\, dZ. \tag{4.8}$$

Immigration Policy

35 This functional form aims to illustrate the intuition of modelling labourspecific wage. In a model where the NWM is enforced, the competitive wholesale sector requires that the retail market installs a lower mark-up, as the marginal cost is inflated by NMW. So the NMW might be thought of as a change in the steady-state mark-up.

Economists and policy-makers are interested in the consequences of increased immigrants on the output-inflation trade-off. Job market slack caused by loosening immigration policy will ease inflation pressure but will increase unemployment. We are interested in understanding the consequences of new steady state labour supply and higher possibility of filling vacancies to monetary policy under a matching model.

Other Issues

There are more policy issues that might be incorporated in the discussion, such as the 'New Deal' – introduced by New Labour in 1998, which can be seen as an effort to increase the steady-state probability of finding a job k^w.[36]

4.4.2 Pre- and post-1980s calibration

In Table 4.3 we list the key parameters that are affected in labour reform scenarios. In addition to the parameter of union power, which is represented by worker's share in wage negotiation, the post-reform period is also characterized by (1) lower cost in posting a vacancy, as a result of modern technology and employment service; (2) higher possibility of a firm filling a vacancy, thanks to immigration; (3) higher possibility of a worker to achieve a match, partly as a fact of higher motivation to find a job (lower unemployment benefits) and the training to build up skills.

[36] McVicar and Podiminsky (2003) shows some evidence in UK umployment duration data for younger workers.

Table 4.3 - Baseline versus labour market reform calibration

Coefficient	**Description**	**UK pre-1980s**	**Baseline (post reform)**
k^f	Prob. of filling a vacancy (i.e., $q(\theta)$)	0.7	0.8
k^w	Prob. of finding a job	0.3	0.45
γ	Recruiting cost	0.5	0.33
η	Worker's share in wage bargaining	0.5	0.3

We make the post-reform calibration as the base case and highlight a case for pre-1980s period for UK labour market and business cycles. We calibrate the model roughly by taking existing ones and reflecting theoretical implications. Due to lack of data, for instance, we assume 70% pre-reform UK firms' vacancies can be filled in a quarter, the same as in US, while recruitment is more successful in post-reform period (up to 80% thanks to flexible labour market). The probability of finding a job in a quarter is simly the inverse of unemployment duration in quarters. After excluding recession years, the average duration of unemployment is nearly 3 quarters for early sample and 2.33 quarters for the late sample, which give similar numbers to the table. Recruiting cost and workers' bargaining power are from Walsh (2003) and Millard (2000). γ is set to 0.33 to represent the hiring cost of 1 month salary typical to the head hunt market. With no further data evidence, we roughly assume a 50% premium in this cost to illustrate a higher firing cost in early sample. On top of η =0.3 in late sample, I assume a dominant role of union in wage bargaining, η =0.5, in 1980s. Please note that we are arbitarily choosing some unavailable coefficients for the purpose of generating two distinct scenarios: an union featured labour market versus a flexible labour market, respectively for early and late sample. So, we admit these coefficients are not prcisely calibrated but argue that the impulse response of calibrated models have their theoretical implications.

4.4.3 Impulse responses

Figures 4.4 and 4.5 outline the output, employment (top panel), nominal interest rate and inflation responses to each of productivity and then of a monetary shock in a canonical NK model. In Figure 4.4, in response to a persistent productivity shock, output jumps immediately and employment jumps in a similar manner but with considerably less amplitude as hours take more of the strain. Interest rates and inflation jump in the same direction and inflation returns to base in around eight quarters. In the longer run the increase in productivity means that there is some overshoot, and inflation will tend to fall unless offset by cuts in interest rates. A key observation is that output follows closely the expected path of productivity and that interest rates work hard to stabilize inflation but that employment just mimics output.

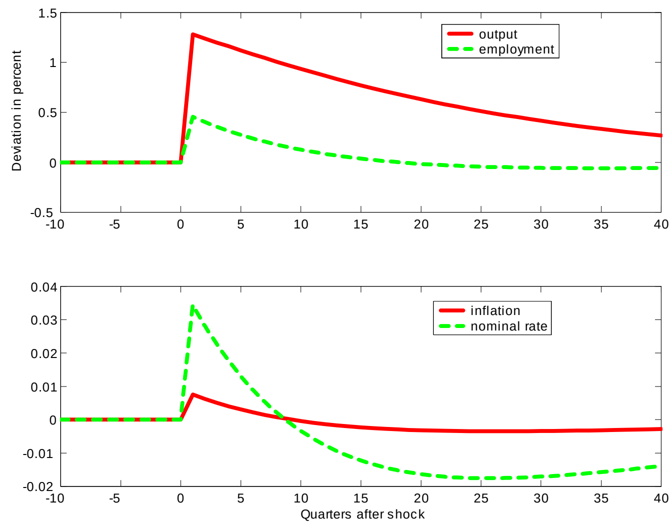

Figure 4.4 - Response of NK model to productivity shocks

Note: Impulse responses to a 1% shock in productivity.

Figure 4.5 shows the response of the economy to an unanticipated monetary shock. That is, an increase in the money supply over and above that suggested by the expected increase in nominal demand. The upper panel shows that output and employment have a temporary boom that lasts around four quarters. In the lower panel we note that there is a parallel temporary increase in inflation that drags up interest rates temporarily. But as this is a one-off money shock the economy stabilizes quickly. Again, the inflation is temporary and so is the problem for the economy.

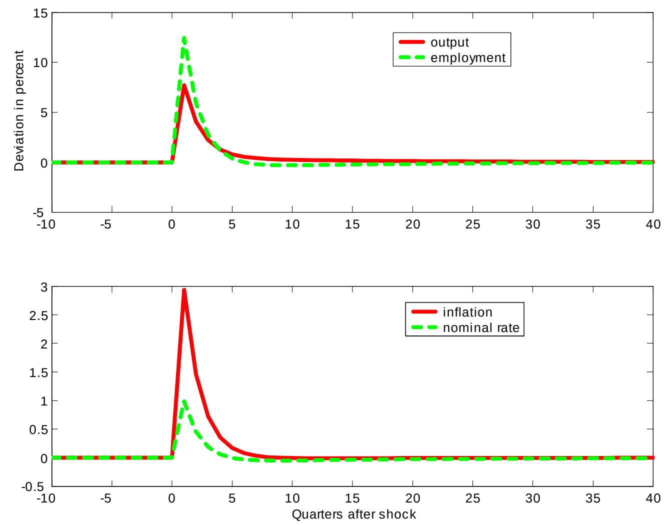

Figure 4.5 - Response of NK model to monetary shocks

Note: Impulse responses to a 100bps cut in base rate.

In the DSGE model with labour market search the key is that the employment match does not happen in a Walrasian manner. In such a set-up, for example in Calvo-Yun, workers, operating on the full employment frontier, do not simply vary their hours in response to changes in the real wage, which is interpreted as the relative price of work to leisure. But now the margin of job destruction becomes revealed, which impacts on the search and vacancy dynamics of the labour market and inversely on the rate of job destruction. And the value of finding a match is found to be closely related to the probability of finding a match. A number of calibrations are possible. But what we illustrate here is with a baseline from our estimates on ONS data and based on the parameterisations in the literature and the issues discussed in the previous section.

Figure 4.6 shows how in the model with labour market search we can end up with greater amplitude in employment following a productivity shock. Overall there is twice the amplitude of employment to a productivity shock (see top left-hand panel). In this model productivity reduces the marginal incentive for job destruction (bottom right-hand panel) and accordingly this reduces job destruction alongside a persistent reduction in vacancies. The need for search falls as the probability of worker matching is persistently raised and firms find it more difficult to locate appropriate staff. Note that the initial response to employment is a decrease, mainly due to the sticky price set-up therefore a temporary delay of increase in nominal demand. This also cause the initial response to other variables at odd to longer term dynamics: more job destruction and at the same time more hiring efforts in preparation for future's production. After the inflation peaks and aggregate demand booms, employment rises gradually following the match and real wages rise as labour market gets tighter. As a result output and employment take sufficient time to peak in their hump-backed responses.

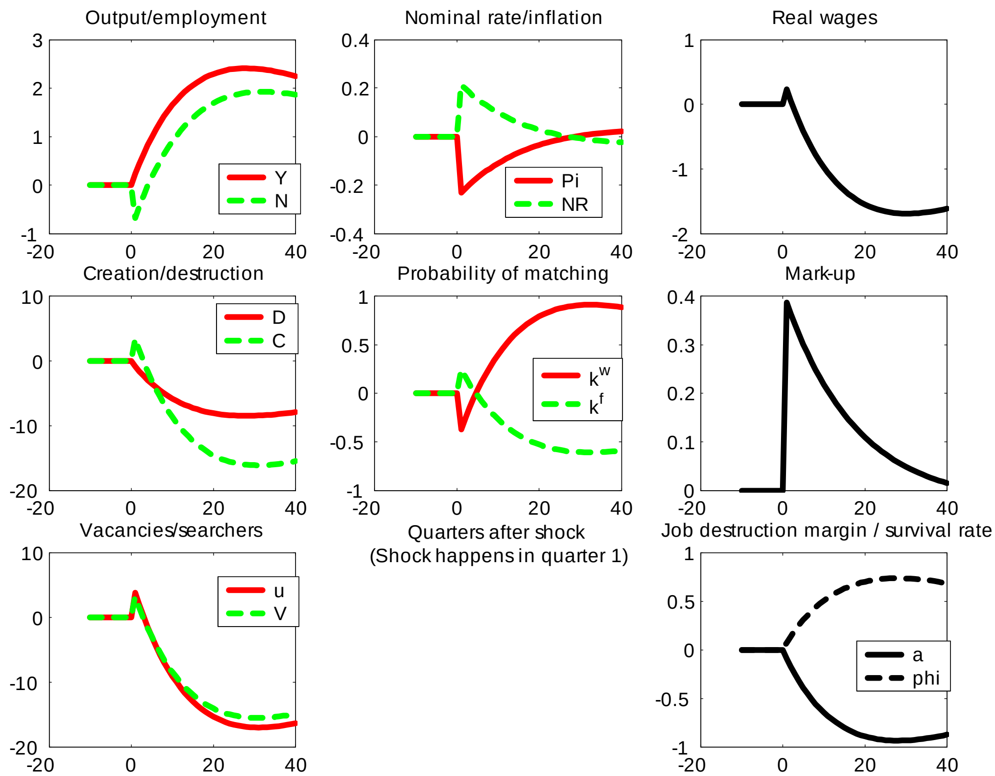

Figure 4.6 - Responses of labour market models to productivity shocks

Figure 4.7 shows the considerably less amplitude in output and employment from a monetary shock. But clearly that employment drives output fluctuations. With a expansionary monetary shock, the nominal demand rises so that the survival rate increases and the destruction rate falls temporarily, which acts to reduce vacancies and the need to search. The probability of workers finding a match increases and so employment rises. As there is no increase in productivity, output is simply bid up in the extent of additional employment. This is the reason why employment response more strongly than output.

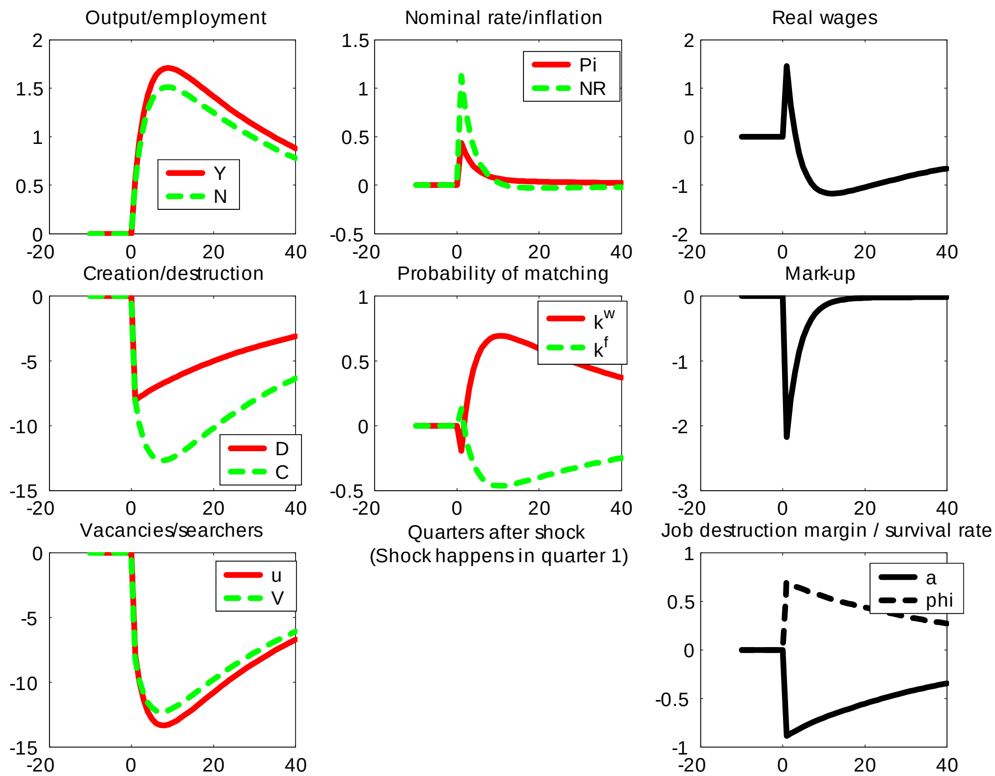

Figure 4.7 - Responses of labour market models to monetary shocks

Compared with the Calvo-Yun model, inflation behaves in a relatively similar manner when we analyse the responses in a search model. The forward-looking behaviour of inflation setting seems to take care of that. But we find that the underlying output dynamics – even when we allow for considerably more labour market flexibility over time – appear considerably more persistent, where the endogenous probability of finding a match creates an ongoing perturbation to the labour market. We also notice the inability of this model in generating the negative correlation between vacancies and unemployment, i.e., the Beveridge curve. Krause and Lubik (2003) offers a solution by introducing nominal rigidities in wage bargaining therefore making a good example of building both nominal and real rigidities in a DSGE models for labour market. In addition to the base case discussion, we further highlight a case for labour reform in the next sub-section.

4.4.4 Labour reform in a search and matching model

Going back the discussion on UK labour market, we address a further implication of the search model on monetary policy making. Under calibration for pre- and post-1980s period in UK, we find the impulse responses of labour market variables are quite distinctive. For post-1980s calibration, when labour market was more flexible as a result of a series of reform, monetary policy seems more effective in stabilizing economy and employment market.

Figure 4.8 shows the impulse responses of real wages and unemployment rate to a expansionary monetary shock. The initial jump to real quantities are similar in both cases, representing equally effective monetary policy *ex ante*. However, the post-reform calibration predicts a faster convergence in employment, the average time required for unemployment to return half-life is 27-28 quarters for pre-reform calibration but 12-13 quarters for post-reform calibration. In fact, removal of real rigidities in labour market tends to be favorable to monetary policy-making, as a shorter time period is required to stabilize the aggregate economy. Responses of real wages behave similarly. In post-reform calibration, real wages jump with a larger magnitude and quickly converge. We notice in a flexible labour market, real wages do not follow a overshooting path as pre-reform calibration implies. This may explain why real wages became less volatile in the post-1980s period.[37]

Figure 4.9 repeats the VAR analysis in Figure 4.1 but partition the sample period to early sample (1975-1990) and late sample (1991-2008). Following a contractionary monetary shock, unemployment rate increases and peaks in a 3-4 years horizon for late samples but in two years in early samples. The unrestricted VAR impulse responses are in line with the theoretical prediction in Figure 4.8, where a post-reform calibration triggers more effective monetary policy.

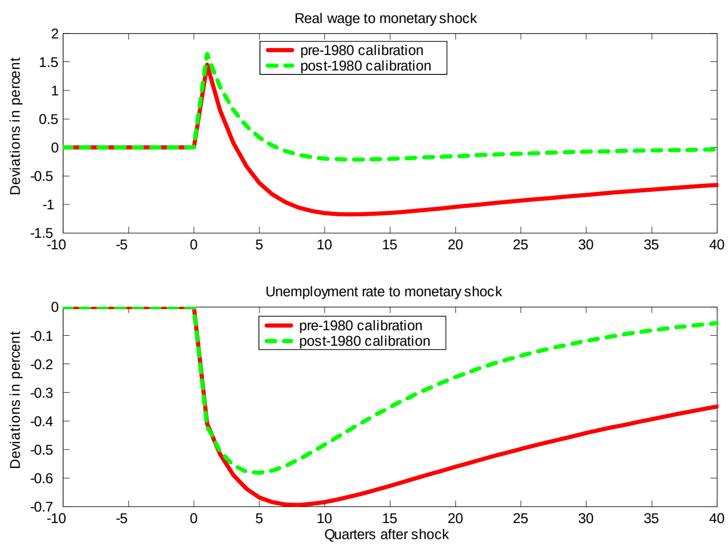

Figure 4.8 - Impulse responses to monetary shocks under pre- and post-reform calibration

We also find the coefficient for vacancy-posting cost is crucial for the sign of real wages to a productivity shock. A larger value of γ erodes the firm profitability and therefore on

37 Post-reform calibration imply a relative volatility of real wages to output as 1.02 compared to 1.14 in pre-reform calibration. While the relative volatility of unemployment rate is not sensitive to such calibration.

real wages. Although we doubt this mechanism turns the response of real wages to productivity improvement negative, the model suggests hiring cost shall be kept at a low level to enhance welfare of the working population while at the same time provide employment security (represented by high hiring and firing costs).

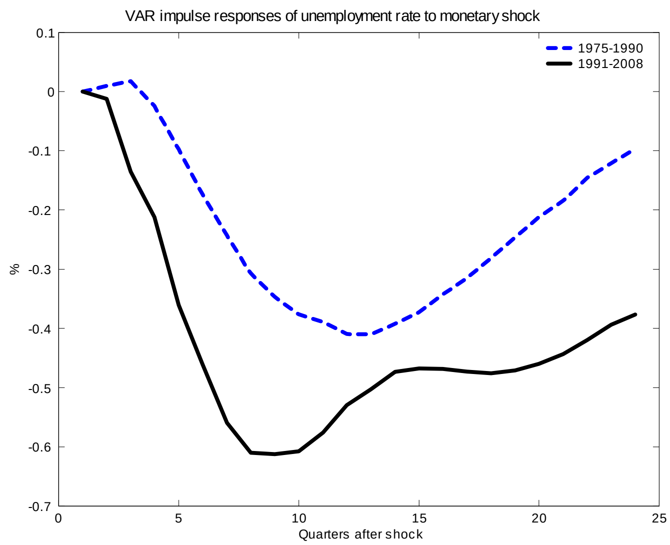

Figure 4.9 VAR impulse responses to monetary shocks under pre- and post-reform data

Note: The impulse responses to a monetary shock (100 Bps rate cut) are plotted for the same unrestricted VAR as in Section 4.2. The order of variables is [unemployment rate, inflation, real output, real exchange rate, nominal interest rate].

To sum up the findings, Walsh model clearly introduces persistence in real quantities and therefore hump-backed responses by enforcing real rigidities. However, the initial responses to monetary shocks seems too big in contrast to VAR results. Nevertheless, the model outperforms Calvo-Yun set-up in terms of more flexible assumption for labour market and richer structure of Phillips curve. In next section we reconsider the determinacy problem for a typical NKPC paradigm and see what's the implication for monetary policy.

4.5 The implications for monetary policy

The Taylor principle results most simply from an analysis of the stability conditions for the simple form of a forward-looking spending and Phillips curve equation in deterministic form:

$$y_t = E_t y_{t+1} - \lambda \left(i_t - E\pi_{t+1} \right), \tag{4.9}$$

$$\pi_t = \beta E_t \pi_{t+1} + \varkappa y_t, \tag{4.10}$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t, \tag{4.11}$$

which comprises the forward-looking spending (IS) equation in terms of output, y_t, the New Keynesian Phillips curve setting the inter-temporal trade-off for inflation, π_t, in proportion to the current level of output and the policy rule for the interest rate in terms of output and inflation. It is well known (see Chadha, 2008, for an overview) that the stability of this simply Calvo-Yun system is given by the choice of policy parameters for the weight of inflation and output in the policy rule:

$$\frac{(1-\beta)}{\varkappa}\phi_y + \phi_\pi > 1, \tag{4.12}$$

Note that independent of any choices for the parameter $\varkappa$, the slope of the Phillips curve, or the rate of time preference, β, by choosing a sufficiently high weight on inflation, i.e. $\phi_\pi > 1$, the determinacy of this system can be assured and provides some rationale for a rather simplistic form of inflation targeting.

We can examine the Phillips curve resulting from the use of a search model to see what happens to these determinacy conditions be considering the Phillips curve from combining (4.5) and (4.6). They become somewhat more complicated but show that when there are speed limit effects and a cost channel for monetary policy, the importance of targeting inflation alone is diminished. The IS equation and policy rule remain the same but the Phillips curve takes the following form:

$$\pi_t = \beta\pi_{t+1} + \varkappa A y_t - \varkappa B y_{t+1} + \varkappa\lambda\left(i_t - \pi_{t+1}\right), \tag{4.13}$$

where current inflation, π_t, reflects future inflation, π_{t+1}, the current output gap, y_t, is reduced if slack is expected in the next period, y_{t+1}, and where current real rates, $(i_t - \pi_{t+1})$, impact on the cost base, where $\varkappa$, A and B are parameters.[38] The extra terms in future output and on the cost channel of monetary policy result from the explicit modelling of the labour market. The output gap determines the level of slack in the labour market, and as we would expect when current output is high there is little labour market slack and so wages and costs are bid up. But note that there is a forward-looking element to the wage bargain. Insofar as if there is expected to be a similarly tight market in the next period some wage bargains will be brought forward to this period and there will be more

38 Note that if $B = \lambda = 0$, we recover the NK standard Phillips curve.

vacancies in the next period. In fact what we end up with here is rather than inflation tilted by the paths of future output gaps, we see in this set-up that the change in inflation is proportional to the change in output plus an intercept term in the cost channel (assuming $A = B$ and $\beta = 1$):

$$\frac{1}{\varkappa}\left(\pi_{t+1} - \pi_t\right) \approx \left(y_{t+1} - y_t\right) - \lambda\left(i_t - \pi_{t+1}\right), \tag{4.14}$$

To solve for the equivalent determinacy conditions we place the first order system $[\pi_t, y_t]'$ in compact form and examine the roots of the Jacobian. The full derivation is available on request but for the determinacy a key condition is that:

$$\frac{\left(\phi_y + \varkappa\phi_\pi\left(A - \lambda\right) + 1\right)}{\beta + \varkappa\left(A - \lambda\right)} > 1, \tag{4.15}$$

which we can compare directly with (4.12) above. Equation (4.15) clearly implies a greater efficacy from stabilizing output than inflation as each of $\varkappa$ and $(A - \lambda)$ are less than 1. Note that the term, λ, on the cost channel detracts from the stabilizing properties of strict inflation targeting.

Let us illustrate with a simple calculation taking (4.12) as our starting point. First let us set the weight on inflation in the policy rule to zero and then solve for the required weight on output:

$$\phi_y > \frac{\varkappa}{(1 - \beta)}. \tag{4.16}$$

For $\varkappa = 0.2$ and $\beta = 0.989$, $\phi_\pi = 0$, then $\phi_y > 18.2$ is required to ensure determinacy in the Tack-Yun case. Now if we solve (4.13) in the same manner:

$$\phi_y > \beta + \varkappa (A - \lambda) - 1, \tag{4.17}$$

We find that for the Walsh set-up the threshold hardly bites if $(A - \lambda) = 0.8$ as $\phi_y > 0.15$. This result implies that the concentration on output alone can stabilize a system with real labour market rigidities. Or at the very least that the efficacy of inflation targeting alone may be questioned when one allows for meaningful delays in job matching. The intuition for the result follows from a simple manipulation of the Phillips curve in (4.13), where we note that the acceleration in inflation is closely pinned down by the growth rate of output. So ensuring an absence of large peaks or recessions in output may be the best way to prevent a deleterious inflation dynamic from developing.

Recalling our findings of more effective monetary policy in a flexible labour market, we find it supportive to our argument that monetary policy shall not skewed to inflation targeting. In a search model with flexible labour market, inflation expectation over medium and long term might be stabilized as households expect the output gap may quickly vanish in recognition of an strong and credible monetary policy favouring both fully employment and an inflation target. Ravenna and Walsh's (2006) work on cost-channel of monetary policy is also supportive to this argument, as the existence of cost channel means output level may above or below the trend level to avoid bias caused by rate adjustment. Then it is sensible to allow more flexibility in the labour market and achieve maximum effect of rate cut or rate hike in shorter time period.

4.6 Concluding remarks

Nominal rigidities have been emphasized as the breaking point of New Keynesian macroeconomics. But even these features cannot explain inflation dynamics very well. In this chapter, we continue from chapter 3 to analyze one more real rigidity, the labour market search. UK's labour market reform present itself a very good case study for sluggish adjustment in labour market.

Although the search model for labour market has been studied for many years, it is very rare to see the joint impact on monetary policy analysis. How can we use the type of models favoured by central banks and a generation of macroeconomists to learn about the nexus between the labour market and overall macroeconomic fluctuations? Monetary policy-makers are mostly interested in issues that are related to inflationary pressures and the dynamics of inflation and output gap over business cycles. A calibrated DSGE model with labour market search has illustrated the extent of an interaction between the labour market and inflation-output dynamics. It turns out that the labour market is a crucial part of a prolonged adjustment mechanism for an aggregate economy.

The extent to which the labour market is flexible or not will determine, to a considerable degree, the household level utility losses from economic shocks. But we have shown that even if the labour market has been reformed, as in the UK, issues connected with labour market search will still be likely to generate considerable persistence into the adjustment of the real economy to shocks. The consideration of optimal monetary policy cannot thus simply ignore the likely endured by the representative household. We have illustrated with empirical evidence, the impulse responses from a calibrated DSGE model and a simple analytical model how the persistence of the labour market adjustment ought perhaps to be a key consideration in the setting of monetary policy.

Chapter 5
Money, Prices and Liquidity Effects: Separating Demand from Supply

5.1 Introduction

[39]The proposition that inflation is a monetary phenomenon often sits uncomfortably with the perhaps mixed evidence that money has significant information for inflation at the policy horizon.[40] A standard response to this puzzle is that the path of real output and inflation (nominal output) over the business cycle will generate a proportional demand for money balances, which will be supplied elastically by the central bank at an interest rate appropriate for the maintenance of nominal stability and that broad money will be multiplied out by the act of financial intermediation. In the long run output will be determined by real factors leaving the supply of money to pin down the price level.[41] In this chapter we take this dichotomy between the short and long run correlation between money and prices and explore the impact of decomposing broad money innovations into those that reflect demand and supply separately. We can also consider to what extent the broad money supply is not pinned down by the policy function, which acts on policy rates alone. We consider whether financial intermediaries may separately impact on the supply of money and so

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[39] This chapter is based on a joint research project to understand money in DSGE models. My co-authors include Jagjit S. Chadha and Luisa Corrado. A different version of the paper has been submitted to Journal of Economic Dynamics and Control but was not yet given a final decision by the date of submission.

[40] The breakdown of the medium link between money and nominal expenditure has been well documented and played a key role in the move away from monetary targetry. See Goodhart (1999).

[41] See Lucas (1996) for a simple exposition of this point.

generate excesses or shortages in nominal demand which impact directly on inflation.

In this chapter, we build upon the recent work of Goodhart (1999), King (2002) and Chadha *et al.* (2008) who suggest that liquidity effects may impact on monetary conditions independently of the policy function. Specifically in a model (see, Goodfriend and McCallum, 2007) where banks supply loans as a function of the marginal costs of loans provision, the external finance premium faced by borrowers is proportional to these costs and increases in the value of collateral or monitoring. Financial spreads are thus driven down by any increases in the marginal efficiency of loans production and by the resulting liquidity in the money markets, which leads to excessive levels of output in the economy. But when banks supply deposits simply to meet productive capacity, liquidity is not exogenously reflected in excessive demand. And so we find that when financial sector productivity is a dominant source of business cycle fluctuations some attention would be paid to the nexus of financial spreads and liquidity. Specifically when spreads fall (increase) and liquidity rises (falls), the monetary policy maker might have to pay particular attention to offset these expansionary (contractionary) impulses.[42]

There is a large literature on the relationship between money, prices and output.[43] To some extent the debate has been brought back into sharp relief by the recent and ongoing disturbances in money markets, which have may have disrupted the link between mon-

42 Despite the mythology about modern macroeconomics and money, the kind of disconnect between money markets and monetary policy was considered in work by Carlstrom and Fuerst (1995) and by Ireland (1996), the latter of whom found that in the presence of significant changes in the required proportion of money balances to transactions, interest rates may not operate as a good instrument of monetary policy.

43 See Christiano *et al*. (1999) for a comprehensive overview of the literature.

etary policy and broad liquidity provision. And we are interested here in using the sign restrictions suggested above to identify separately demand and supply shocks in the broad money markets. Originating with Faust (1998), Uhilg (2001) and Canova (2002) VARs can be estimated with Bayesian priors on the sign response to demand or supply shocks in the money markets. Specifically, we run VARs in broad money and measures of the external finance premium to identify primitive demand and supply shocks to the broad money market where supply shocks (a so-called liquidity effect) cause spreads and money to move in the opposite directions and demand shocks lead to spreads and money to move together.

As earlier influential work by Bernanke and Mihov (1998), we find strong evidence for a liquidity effect that can be shown to dominate monetary behaviour in both recent UK and US data. And as Lastrapes and McMillin (2004) we find significant effects from financial prices on supply factors for broad money. More work is required to decompose further the equilibrium outcomes we observe on monetary aggregates, particularly in sectoral money aggregates, but tentatively we suggest that policy, particularly in the US, may not have acted to fully offset the exogenous compression of market interest rates by financial markets. Given recent developments in financial markets, that have started to deleverage after a long period of balance sheet expansion, these results may provide a useful diagnostic on the extent to which policy was inattentive.[44]

This chapter is structured as follows. In section 5.2, we outline a simple monetary

44 See the discussion by the IMF (2008) on the implications of leverage and deleveraging in financial markets.

model in which the exogenous supply of liquidity perturbates output and inflation. In section 5.3 we outline our methodology for identifying a series of VARs in money and interest rates. In section 5.4 we outline our basic results and provide some analysis of or findings and we finish with some concluding remarks.

5.2 A Liquidity Effects Model: Money and External Finance Premia

In this section we develop a simple endowment economy model of a representative infinitely lived household.[45] The model is used to show how policy needs to account for financial disturbances, as represented by unanticipated changes in the ability of money to finance consumption. And also how money is ultimately related to changes in the external finance premium, which reflects both the nominal interest rate and a rate reflecting this liquidity provision. We sketch a simple version of this model as a quadrant diagram and relate our estimation strategy to one of the quadrants, as a reduced form of this model.

A simple model might think of a household receiving a stochastic endowment that cannot be stored, which is exogenous and it is received at the end of the period. The household thus has to decide over two stores of wealth, real money balances, $\frac{M_t}{P_t}$, and a one-period nominal bond, B_t. The nominal bond purchased at date t pays one unit of currency at date $t+1$ and has a price of $q_t \left(= \frac{1}{1+i_t}\right)$.

The household maximizes utility over an infinite horizon as is standard. The cash-in-advance economy is structured as follows. At the end of previous period a stochastic

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[45] See Lucas (1982) and Labadie (1994).

shock to liquidity, in terms of exogenous velocity υ, alters the value of money, $\upsilon_{t-1}M_{t-1}$, which changes the required money balance to effect consumption decisions and results from financial intermediation; in addition, a real endowment shock, y_t, is realised at the start of the next period. Following the money transfer, returns from maturing bonds and receipt of endowment, the representative household decides on how to allocate its wealth between money balances and nominal discount bonds.

Once the asset market has closed, the household uses its money balances acquired at the beginning of the period M_t to finance its consumption purchases, $c_t p_t$, where p_t is the price level at date t. The household then receives its nominal endowment income $p_t y_t$, which it cannot spend until the subsequent period.

The representative household maximises the following utility problem:

$$\max U = E_t \sum_{i=t}^{\infty} \beta^{i-t} u\left(c_i\right), \tag{5.1}$$

where β is the subjective rate of time preference, E_t, are expectations formed at time and $u\left(c_i\right)$ is a mapping from consumption this period to utility in the same period. Subject to the household budget constraint:

$$\frac{p_{t-1}}{p_t}c_{t-1} + \frac{q_t}{p_t}b_t + \frac{M_t}{P_t} = \frac{p_{t-1}}{p_t}y_{t-1} + \frac{b_{t-1}}{p_t} + \upsilon_{t-1}\frac{M_{t-1}}{P_t}, \tag{5.2}$$

and the cash-in-advance constraint:

$$c_t \leq \frac{M_t}{P_t} v_t. \tag{5.3}$$

The lagrange multiplier attached to the first constraint is $\lambda_{1,t}$ and to the second is $\lambda_{2,t}$. The first order conditions of this problem with respect to c_t, b_t and $\frac{M_t}{p_t}$ are given respectively by:

$$u'(c_t) = \lambda_{2,t} + E_t \beta \lambda_{1,t+1} \frac{p_t}{p_{t+1}}, \tag{5.4}$$

$$\lambda_{1,t} \frac{q_t}{p_t} = E_t \lambda_{1,t+1} \frac{\beta}{p_{t+1}}, \tag{5.5}$$

$$\frac{\lambda_{1,t}}{v_t} = \lambda_{2,t} + E_t \beta \lambda_{1,t+1} \frac{p_t}{p_{t+1}}. \tag{5.6}$$

By equating (6) to (4) we find that:

$$\lambda_{1,t} = u'(c_t) v_t$$

And so the equilibrium condition for nominal bonds is:

$$E_t \frac{u'(c_t)}{\beta u'(c_{t+1})} = E_t \frac{v_{t+1}}{v_t} \frac{p_t}{p_{t+1}} (1 + i_t), \tag{5.7}$$

which says that the household consumption path will equate the present value of consumption in successive periods subject to deviations in the nominal interest rate, inflation and financial liquidity.[46] Following Woodford (2003)[47] the appropriate Wicksellian policy will take the following form:

$$z_t \equiv E_t \frac{u'(c_t)}{\beta u'(c_{t+1})}, \tag{5.8}$$

where z_t is the intertemporal marginal rate of substitution in consumption. And so the interest rate policy rule can be written as follows:

$$1 + i_t = \phi(p_t, v_t, z_t), \tag{5.9}$$

which means that an equilibrium condition will require:

$$E_t \frac{p_{t+1}}{v_{t+1}} z_t = \frac{p_t}{v_t} \phi(p_t, v_t, z_t), \tag{5.10}$$

which means that policy maker have to consider a stable path for financial shocks as well as the price level to ensure a stationary equilibrium. We now turn to the implications for growth, inflation and spreads in this model. Adopting log utility, $u(c_t) = \ln c_t$, we can re-write (8) as:

$$E_t \frac{c_{t+1}}{c_t} = E_t \frac{v_{t+1}}{v_t} \frac{p_t}{p_{t+1}} \beta (1 + i_t), \tag{5.11}$$

which we can log-linearise to obtain:

46 This point was made by Ireland (1996).

47 See Walsh (2003) for an exposition of this point.

$$E_t \triangle c_{t+1} = i_t - E_t \pi_{t+1} + E_t \triangle v_{t+1,} \tag{5.12}$$

which is now a familiar intertemporal spending equation and tells us that consumption growth is tilted by liquidity effects on broad money as well as the interest rate. If we think in terms of a short run inflation induced by spending, we can iterate this expression forward to obtain:

$$c_t = -E_t \sum_{j=0}^{\infty} \left(i_{t+j} - \pi_{t+j+1} + \triangle v_{t+j+1} \right), \tag{5.13}$$

which can be substituted into a New Keynesian Phillips curve to obtain:

$$\pi_t = -E_t \kappa \sum_{j=0}^{\infty} \beta^j \left(i_{t+j} - \pi_{t+j+1} + \triangle v_{t+j+1} \right), \tag{5.14}$$

where κ is the slope of the Phillips curve. And tells us that inflation and consumption will be tilted by the liquidity premium as well as the policy rate adjusted for expected inflation. As expected money growth from the cash in advance constraint is:

$$E_t \triangle c_{t+1} = E_t \triangle m_{t+1} - E_t \pi_{t+1} + E_t \triangle v_{t+1} \tag{5.15}$$

$$E_t \triangle m_{t+1} = i_t$$

which tells us that in the long run higher money growth will simply drive up the nominal rate. So in the short run the policy rate and the liquidity premium will determine the deviation of consumption from its long run level and so the rate of inflation, but in the long run we might expect, with stable real rates, inflation and liquidity shocks, money growth to feed simply into the inflation component of nominal interest rates.

We can sketch this model in a four quadrant space to illustrate our basic points more fully. The north-east quadrant of Figure 5.1 shows the equilibrium in the market for central bank money, M_0, with demand, M_0^d, negatively sloped and the supply of central bank money, M_0^s, perfectly elastic with respect to the chosen policy rate, i_t. Shocks to demand for central bank money thus neither impact on policy rate nor on the level of aggregate demand in the economy. The market clearing quantity of central bank money is multiplied by MM in the south-east quadrant to arrive at a level of broad money, M_B, where we can think of this level of broad money as the outcome of a process of financial intermediation. The steeper is the MM curve the higher is the money multiplier. The south-west quadrant clears the broad money market in supply, which increases in the spread charged over the policy rate, efp_t, and demand for broad money, which from the cash in advance constraint is a function of consumption, c_t, which is itself determined by the spread. At the steady-state level of market rate interest rates, consumption, c_t, will equal its long run level, $\bar{c}$. But if the spread is above (below) the long run level consumption will be below (above) $\bar{c}$ and inflation will be below (above) any target. In this sense, higher (lower) spreads will be associated with lower (higher) inflation and consumption as in (5.13) and (5.14).

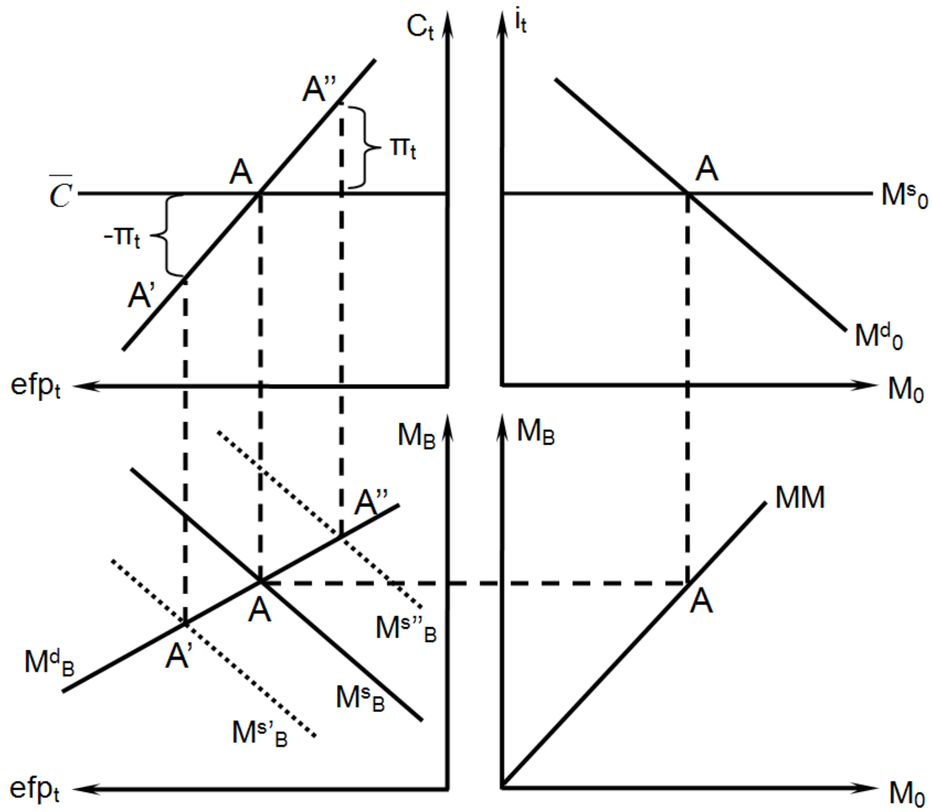

Figure 5.1 - A simple model of money and the external finance premium

Note: The model is elaborated in section 5.2. In the south-east quadrant ‘MM’ denotes the money multiplier, which can be either constant or time-varying. In the south-west quadrant the $M_B^{s'}$ and $M_B^{s''}$ denote two alternative scenarios for the supply shocks and how they affect liquidity provision. The corresponding short-term equilibria for the money market and the aggregate economy are A' or A'', away from the initial equilibrium A.

To re-iterate in the north-west quadrant inflation, π_t, results from any deviation in consumption from its long run level and we can sketch the implication from an exogenous shift in broad money supply in the south-west quadrant. A shift out (in) in the broad money supply schedule[48] will lead to a reduction (increase) in the efp and consequently to

48 We hold aside the policy response or any implied money multiplier shift to aid pictoral clarity.

an increase (decrease) in consumption and so inflation. Equally, a shock to the demand for broad money, will show up as having the same sign on the efp and the quantity of broad money or liquidity. And so we can identify shocks to the market for broad money, with the help of market interest rates to uncover demand or supply perturbations to this market and then assess the extent to which one type of shock or other is related to inflation and aggregate price level dynamics. This is the purpose of the next section.

5.3 Identifying Demand and Supply in the Money Market

In this section we describe how to identify money and supply shocks using sign restrictions with a Bayesian VAR on the model variables described in the south-west quadrant of Figure 5.1 in section 5.2. We follow Canova and De Nicoló (2002), Uhlig (2005) and Faust (1998) and adopt the standard reduced form VAR of order p:

$$Y_t = B(L)Y_{t-1} + u_t, \tag{5.16}$$

where $Y_t = (\Delta m_t, efp_t)$ is a 2×1 vector of data for the first difference of log-money, m_t, and the external finance premium, efp_t,[49] $B(L)$ is a polynomial of order p and L is the lag operator. Note in the estimation we use a stacked version of the VAR model: $Y_t = X_t B + u_t$, where X_t is a matrix of lagged model variables: Y_{t-n}, $n = 1...p$.

The main point of this exercise is to identify the structural shocks contained in the residual vector. Let $\varepsilon_{j,t}$ for $j = s, d$ denote money supply and money demand shock

[49] As stressed in Canova and de Nicoló (2002) in order to interpret the responses to shocks as short-run dynamics around a steady-state, the VAR representation must be stationary. For this reason broad money has been first-differenced.

respectively. Canonical transformations of such shocks require them to be $i.i.d.$ white noise processes having zero mean, unitary variance and to be serially uncorrelated at all leads and lags. We can therefore denote the relationship between our structural shocks $\varepsilon_{j,t}$ and the vector of VAR residuals, $u_{j,t}$, as:

$$u_{j,t} = A\varepsilon_{j,t}, \tag{5.17}$$

where A is a 2×2 matrix. The main point is that by identifying A we can automatically recover the structural shocks $\varepsilon_{j,t}$. An equivalent formulation for (5.17) is:

$$\Sigma_t = E(u_{j,t}u_{j,t}^{'}) = AE(\varepsilon_{j,t}\varepsilon_{j,t}^{'})A', \tag{5.18}$$

where Σ_t is a symmetric variance-covariance matrix and A is our vehicle to identify the structural shocks.[50] To accomplish this we focus on the a^j column of A containing the j-th identifying restriction and we consider the corresponding impulse response function. Given the structural impulse vector, a^j, the set of all structural response coefficients of the bivariate system up to horizon h, denoted as $\phi_{1,\ldots,}\phi_h$, can be computed using the estimated coefficient matrix $B(L)$ from the reduced form VAR:

[50] As stressed by Canova and De Nicoló (2002) there is a multiplicity of orthogonal decompositions. For any orthogonal matrix Q, with $QQ^{'} = I$ also $\Sigma = AQQ^{'}A^{'}$ is an admissible decomposition for Σ. One example is the Cholesky decomposition of Σ, where A is lower triangular. However alternative orderings of the variables in the system implying different representations for Σ may produce different structural systems.

$$\begin{aligned} \phi_s^j &= \sum_{n=0}^{s} B_{s-n}\phi_n^j \qquad s \geqslant 1 \;\; B_{n-s} = 0 \;\; s-n \geqslant p \\ \phi_0^j &= a^j. \end{aligned} \tag{5.19}$$

Note that the impulse vector a^j maps the innovation to the j-th structural shock into the contemporaneous impulse responses of our variables, ϕ_0.

Informal restrictions are made on the cumulative impulse response function ϕ_h, so that we define $\bar{A}_h$ as the matrix of identifying restriction for time interval h, whose elements can fulfill any of the following inequality constraints $\bar{A}_{ij,h} > 0$ or $\bar{A}_{ij,h} < 0$. Let us (safely) assume that a positive money supply shock has a positive effect on money, Δm_t, and a negative effect on the financial spread, efp. In practice such shock represents an increase in liquidity provision originated either from monetary policy or from external shocks, hence: $\bar{A}^s = \left[\begin{smallmatrix} + \\ - \end{smallmatrix}\right]$. Similarly a positive money demand shock has a positive effect on money and a positive effect on the external finance premium, hence: $\bar{A}^d = \left[\begin{smallmatrix} + \\ + \end{smallmatrix}\right]$.

Therefore the matrix $\bar{A}$ of identifying restrictions shall take the following form:

$$\bar{A} = \begin{bmatrix} + & + \\ - & + \end{bmatrix}. \tag{5.20}$$

We concentrate on the temporary impact of identified structural shocks by imposing sign restrictions for the first 6 months in the cumulative impulse response function defined through the coefficients $\phi_{h,\, h=1\ldots6}$.[51] Note that in our specification of a stationary VAR,

51 We admit that the choice of six months is arbitrary and can easily implement restrictions over different horizons, we suggest that, as 2 quarters is

the permanent impact from shocks on the growth rate of money or the external finance premium has been ruled out.

The full procedure to identify structural shocks using sign restrictions is implemented using a Bayesian VAR setting as in Uhlig (2005). We start from the MLE estimator of the reduced VAR(p) process (5.16) in stacked format: $Y_t = X_t B + u_t$, whose lag length is chosen using canonical information criteria such as AIC, Schwarz and Hannan-Quinn:

$$\widehat{B} = (X'X)^{-1} X'Y,\ \widehat{\Sigma} = \frac{1}{T}\left(Y - X\widehat{B}\right)'\left(Y - X\widehat{B}\right). \tag{5.21}$$

To fit the data with a Bayesian VAR model, we assume a standard diffuse prior on the VAR coefficients and on the covariance matrix.[52] We also assume a Gaussian process for the data, therefore the prior and posterior of (B, Σ) belong to the Normal-Wishart family. The Normal-Wishart distribution assumes that the uncertainty of (B, Σ) can be decomposed into the variation of B around a mean, $\overline{B}$, and of Σ around a positive definite mean covariance matrix, S. The mean coefficient matrix $\overline{B}$ is of size $ml \times m$ where m is the number of variables (in our model $m = 2$) and l is the optimal lag-length of the VAR while S is of size $m \times m$. The probability of the posterior distribution also depends on a positive definite matrix N of size $ml \times ml$ and a degrees of freedom real number $v \geqslant 0$ that describes the uncertainty of (B, Σ) around $\left(\overline{B}, S\right)$.

generally thought to the start of the business cycle frequency, a response of a given sign of up to six months might be thought of as comparable to the limit in the length of a money market shock.

[52] Uhlig (1994) studies the properties of different priors for estimation in non-explosive univariate AR(1) time series and each candidate prior behaves closely to a diffuse (or flat) prior in practical applications. In Uhlig (2005) this point is further explored by proving that all the decomposition of Σ plus a random orthogonal matrix Q of unitary length shall lead to the identical prior distribution of the impulse matrix (defined through the impulse vector a^j).

In the posterior Σ^{-1} follows a Normal-Wishart distribution $W\left(S^{-1}/v, v\right)$ and the column-wise vectorisation of B, $vec\left(B\right)$, follows a Normal distribution conditional on Σ: $N\left(vec\left(\overline{B}\right), \Sigma \otimes N^{-1}\right)$ where $\otimes$ is the Kronecker product. We define a weak diffuse prior for the Normal-Wishart family with $N_0 = 0, v_0 = 0$, while S_0 and $\overline{B}_0$ are arbitrary and follow Uhlig (1994) and Uhlig (2005) with the posterior: $N_T = X'X, v_0 = T, S_T = \widehat{\Sigma}$ and $\overline{B}_T = \widehat{B}$.

Given the posterior distribution of the VAR coefficient, we could simply investigate the property of an unrestricted Bayesian VAR model by running the posterior draw of (B, Σ) for K_1 times.[53] This would also allow us to calculate the cumulative impulse responses by canonical Cholesky decomposition. However, our objective is to enforce the sign restriction for the Bayesian VAR. For this purpose it is required to assign zero weight for those arbitrary parameter S_0 and $\overline{B}_0$ in the diffuse prior which do not fulfill the sign restrictions (see Dedola and Neri, 2007).

It is rather easy to achieve such a goal technically. We randomly choose an occurrence of $\left(\widetilde{B}, \widetilde{\Sigma}\right)$ from the posterior distribution, namely a random number generation from $W\left(\widehat{\Sigma}^{-1}/T, T\right)$ for $\widetilde{\Sigma}^{-1}$ and $N\left(vec(\widehat{B}), \widetilde{\Sigma} \otimes (X'X)^{-1}\right)$ for $\widetilde{B}$. For each draw k we define the set of parameters $\widetilde{B}, \widetilde{\Sigma}$ and locate the corresponding identification matrix $\widetilde{A}$. Let A_0 be any other matrix satisfying (5.17) such that $\widetilde{A} = A_0 Q$, where Q is a random orthogonal matrix obtained by QR decomposition such that $Q'Q = I$. We choose A_0 to be the Cholesky decomposition of $\widetilde{\Sigma}$ therefore $\widetilde{A}$ also fulfills (5.17) and it is the instantaneous impulse matrix we choose for the draw.

53 In this paper we set $K_1 = 500$.

For each draw k we define the set of parameters $\left(\widetilde{B}, \widetilde{\Sigma}, \widetilde{A}\right)_k$ and calculate the cumulative responses of money and external finance premium to one standard deviation of the demand and supply shocks respectively and check if they are consistent with the sign restrictions in $\bar{A}$ with impulse response coefficient, ϕ_h. We keep all the draws that pass the sign restriction, check and discard those who do not satisfy it. We repeat the procedure until we collect K_2 valid draws $\left(\widetilde{B}, \widetilde{\Sigma}, \widetilde{A}\right)_k$, $k = 1...K_2$. In this thesis we set $K_2 = 200$.

5.3.1 Constructing the Primitive Data Series with Money Supply or Money Demand Shocks

An additional exercise we are interested in undertaking is to identified money demand and supply shocks in each of the valid draws. Such shocks $\widetilde{\varepsilon}_{j,t}$ (for $j = s, d$) can be retrieved by premultiplying the residual matrix $\widetilde{u}_t$ with the inverse of the identification matrix $\widetilde{A}^{-1}$ where $\widetilde{u}_t = Y_t - X_t\widetilde{B}$ then $\widetilde{\varepsilon}_t = \widetilde{A}^{-1}\widetilde{u}_t$. Finally for each valid draw we construct the alternative data series solely dominated by either primitive supply or demand shocks in the money market:

$$\widetilde{Y}_{j,t} = Y_t - \sum_{h=0}^{t-1} \phi_h \widetilde{\varepsilon}_{i \neq j, t-h} \quad i, j = s, d$$

which filters out from the historical data Y_t the impact of the identified shocks other than shock j.[54] So $\widetilde{Y}_{d,t} = \left[\Delta\widetilde{m}_{d,t}, e\widetilde{f}p_{d,t}\right]$ denote demand shock driven series and $\widetilde{Y}_{s,t} = \left[\Delta\widetilde{m}_{s,t}, e\widetilde{f}p_{s,t}\right]$ denote supply shock driven series.

[54] As we rule out possibility of permanent impact of shocks in a stationary VAR, the shock-excluding operation turns out to be a reasonable treatment for the accounting analysis of specific shock.

The next step is to define the short-term correlation (dynamic correlation) between our decomposed data for money when the j-th shock dominates, $\Delta\widetilde{m}_{j,t}$, and actual inflation, Δp_t:

$$\widetilde{\rho}_{j,h} = \frac{cov(\Delta\widetilde{m}_{j,t}\, \Delta p_{t+h})}{\sqrt{var(\Delta\widetilde{m}_{j,t})var(\Delta p_{t+h})}} \qquad h = -24, ..., 0, ..., 24, \tag{5.22}$$

therefore we are considering the dynamic correlations up to 2-years monthly leads and lags.

The corresponding long-term counterpart can be defined as:

$$\widetilde{\rho}_{j,H} = \frac{cov(\sum_{k=1}^{H} \Delta\widetilde{m}_{j,t-k+1}\ \sum_{k=1}^{H} \Delta p_{j,t-k+1})}{\sqrt{var\left(\sum_{k=1}^{H} \Delta\widetilde{m}_{j,t-k+1}\right) var\left(\sum_{k=1}^{H} \Delta p_{j,t-k+1}\right)}} \qquad H = 0, ..., 180 \tag{5.23}$$

therefore we are considering correlations up to 15 years.

The corresponding short-term and long-term correlations based on the historical data for money, Δm_t , and inflation, Δp_t, are simply:

$$\rho_h = \frac{cov(\Delta m_t\, \Delta p_{t+h})}{\sqrt{var(\Delta m_t)var(\Delta p_{t+h})}} \qquad h = -24, ..., 0, ..., 24$$

$$\rho_H = \frac{cov(\sum_{k=1}^{H} \Delta m_{t-k+1}\ \sum_{k=1}^{H} \Delta p_{j,t-k+1})}{\sqrt{var\left(\sum_{k=1}^{H} \Delta m_{t-k+1}\right) var\left(\sum_{k=1}^{H} \Delta p_{j,t-k+1}\right)}} \qquad H = 0, ..., 180$$

In order to assess whether money is informative for inflation when either shock (supply or demand) is dominant we plot them pairwise over short and long time horizons.[55] Similarly, we draw 68% quantile error bands for inference purpose.

5.4 Empirical Results

This section describes the data used, summarises the main steps in the estimation strategy described in section 5.2 and comments the results. We particularly concentrate on the impulse responses derived from the Bayesian VAR with sign restrictions using monthly UK and US data for money and external finance premium from 1987 to 2008. We also present the analysis of the short-term and long-term correlation with respect to inflation of our primitive money data driven by either supply or demand shocks and the historical data for money.

5.4.1 Data

We run the Bayesian VAR estimation with monthly UK and US macroeconomic and money market data covering the years 1987M02-2008M07. We are interested in the full sample results and also in the two sub samples: 1987M02-1997M12 and 1998M01-2008M07. The convenient split of the data at the midpoint allows to compare the period of central bank independence under inflation targeting in the UK and the operation of Federal Reserve Policy after the Asian crisis.

Broad money for UK is the $M4$ aggregate seasonally adjusted series from the Bank

[55] In addition to short- and long-run correlation calculated from the raw data, we also convert the first-difference data back to logarithm by summing up lagged value to the beginning of observations. We therefore decompose the logarithm data using HP filter. We analyze the short-run correlation with cyclical money and long-run correlation with trend money. The advantage is to distinguish the cross-correlation over short, medium and long term. Indeed, first-difference or HP filtering for either historical data or dominant-shock alternative series are just two parallel ways of extraction of cyclical information.

of England. The US counterpart is the $M3$ aggregate seasonally adjusted series from the OECD Main Economic Indicators. The UK price level, P, is RPIX[56], seasonally adjusted series from the Office of National Statistics. The US price level is the Consumer Price Index all items, seasonally adjusted series from OECD Main Economic Indicators.

Table 5.1 - Descriptive Statistics of Model Variables

	Early Sample 1987:2-1997:12		Late Sample 1998:1-2008:7		Full Sample 1987:2-2008:7	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
			US			
$\Delta m_{3,t}$	0.29%	0.24%	0.51%	0.33%	0.40%	0.31%
Δp_t	0.28%	0.16%	0.24%	0.27%	0.26%	0.22%
efp_t	0.31%	0.26%	0.22%	0.27%	0.27%	0.27%
			UK			
$\Delta m_{4,t}$	0.77%	0.71%	0.71%	0.50%	0.74%	0.61%
Δp_t	0.32%	0.24%	0.21%	0.19%	0.27%	0.22%
efp_t	0.23%	0.23%	0.20%	0.23%	0.22%	0.23%

Note: The model variables we investigate include broad money growth (monthly), inflation (monthly) and external finance premium (level) on wholesale money market. The data sources are given in section 5.4.1. We show the mean value over the sample period and standard deviations (S.D.).

The policy rate, R_P, is simply the UK bank base rate and the US federal fund rate. The wholesale market interest rate, RIB, is the British Banker's Association (BBA) 3-month sterling London interbank offered rate (LIBOR) for UK and the 3-month dollar LIBOR, averaged of last five trading days in a month, for US.[57] The candidate short term interest rates include LIBOR, T-bill repo rate and CD rate. LIBOR is the marginal rate of refinancing among banks and also fixes the settlement price for interest rate and Eurodollar

[56] RPIX is a measure of inflation in the United Kingdom, equivalent to the all items Retail Price Index (RPI) excluding mortgage interest payments.

[57] This series is taken from Economagic.com. We also cross-check our results with other measures of the external finance premium, such as long term corporate spreads over benchmark government bond rates and find little difference in terms of the main finding.

futures contracts and so provides a reference point for a set of short term retail interest rates for the money markets. Not like T-bill rate or CD rate who have much smaller market, LIBOR is the most representative indicator of wholesale money market price. The Figure 5.2 shows the comparison of LIBOR and T-bill rate on 3-month tenor. The external finance premium, efp, is the wholesale spread $efp = RIB - R_P$, and it is defined as the difference between the interbank and the policy rate.

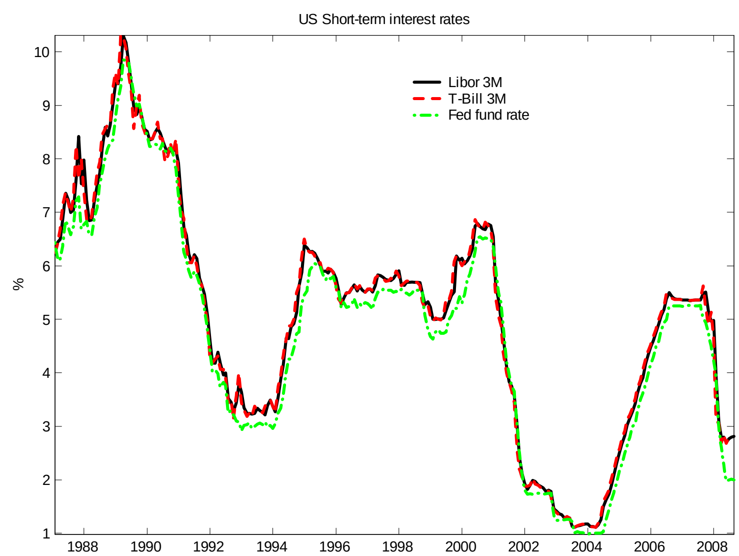

Figure 5.2 - US Short-Term Interest Rate

Parallel to our definition of the financial spread (wholesale money external finance premium[58]), TED spread and LIBOR-OIS are two most frequently used money market indicators. The LIBOR-OIS is defined similarly to ours but the cost of fund is the average of effective federal fund rates, instead of the headline rates. The TED spread is rather a

58 Usually quoted External Finance Premium is defined as the funding cost differential externally for the corporate sector. The spread between corporate bond yield and T-bill rate is a typical measure for corporate EFP, which usually fluctuate similarly to money market EFP.

credit risk premium indicator since it takes the spread between interbank loan rate and T-bill backed borrowing rate. These money market indicators are closely correlated (Figure 5.3) to show both liquidity risk and credit risk, but may sometimes be affected by counterparty risks or seasonal liquidity conditions (Taylor and Williams, 2009). These factors may potentially weaken the significance of proposed analysis on excess liquidity.

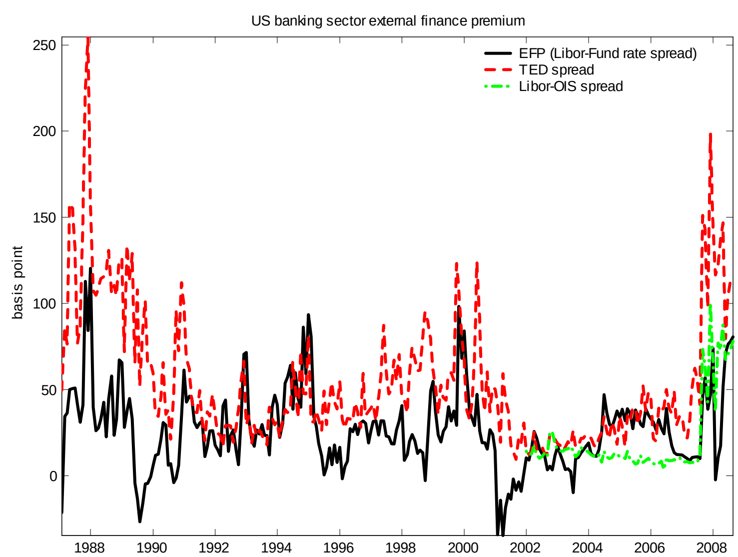

Figure 5.3 - US Money Market Indicators

Note: The yields are month end readings from Bloomberg.

5.4.2 Estimation

In this sub-section we briefly summarize the estimation strategy as a part of the overall methodology described in section 5.3. As we wish to construct a stationary VAR we consider the first difference in the logarithm of money supply and the price level. We use the level of the external finance premium (EFP) to match the theoretical model we develop in section 5.2. From an ADF unit root test, we find the p-value of the null hypothesis that the EFP is a unit root process to be between 0.16 and 0.2, suggesting weak nonstationarity of the data and so we treat the spread as stationary.

To identify the money supply and demand shocks, we follow the pure sign restriction approach suggested by Uhlig (2005). We summmarise the steps of the estimation strategy outlined in section 5.3:

(i) We assume the unrestricted VAR(p) as in (5.16) for the model variables, broad money growth and external finance premium. The sample moments are reported in Table 5.1, the money growth and inflation rates are in annual percentage terms and the EFP as a fraction of 100 basis points. It is noticeable that average of both model variables and inflation decrease from the early sample to the late sample, which denotes a structural break in the full sample model, with an exception of accelerating US broad money growth. We choose the optimal lag length for the VAR by multiple criteria and report the unrestricted VAR model information and residual diagnostic checks in Table 5.2. The optimal lags are typically within one to two quarters, similar to that of Canova and De Nicoló (2002) versus 12 months in the non-stationary VAR setting of Uhlig (2005). However, in the unrestricted VAR we obtain residuals that are hardly normally distributed according to Jarque-Bera test statistics. We also found weak serial correlation in the residuals, up to a lag of 9 and 12 months.

Table 5.2 - VAR Model Estimation

Models	Lags	Resid-ACF1	Resid-ACF12	Resid-N	Total Draws
US full sample	3	0.040	0.073	0.000	743
US late sample	2	0.000	0.090	0.000	959
UK full sample	5	0.079	0.115	0.000	1174
UK late sample	2	0.037	0.023	0.000	1318

Note: The model is $(\Delta m_t, efp_{w_t})$ for each case. The column 'Lags' shows lags in VAR selected by several information criteria. 'Resid-ACF1' shows the p-value of a Null hypothesis that there is no serial correlation in residuals at lag 1. The next column show the corresponding p-value for lag 12 months. 'Resid-N' shows the p-value for a Jarque-Bera test with the Null hypothesis of normally distributed residuals. 'Total Draws' show how many random draws are needed to get valid 200 replications. The higher the total draws, the more difficult to enforce the sign restrictions.

(*ii*) A Bayesian VAR of the same order is fitted to the data. A weak Normal-Wishart diffuse prior is assumed for the VAR parameters and the corresponding posterior distribution is formed under the sample data. The Normal-Wishart diffuse prior is particularly suitable in our case as it is a very weak prior that permits stationary, unit and explosive roots and therefore accounts for any weak nonstationarity in the data.

(*iii*) We therefore enforce the sign restrictions by taking a draw from the posterior distribution of the VAR coefficients and check whether the draw is accepted. We then compute the cumulative impulse responses and check whether the range of impulse response is compatible with the sign restrictions. By keeping valid draws and discarding invalid draws we collect 200 possible successful draws. A Bayesian VAR with sign restrictions is therefore estimated in each successful draw. We also report in Table 5.2 the total draws needed to achieve the 200 successful replications. With a larger number of total draws, it is more difficult to fit the data with the sign restriction Bayesian VAR model. In each of the models we consider, the valid draw as a percentage of the total draws is usually higher than 15%, suggesting that the sign restrictions for the VAR model are a reasonable description of the true data generating process.

(*iv*) Given the population of successful draws from the posterior distribution of the VAR coefficients it is straightforward to make inference on the coefficients, define the impulse responses and derive the related statistics, including the error bands for these statistics. We plot in the charts from Figure 5.7 to Figure 5.12 the 16th and 84th quantiles and also the median of the results from all the 200 draws. The error band is simply a ± 1 standard deviation from the median.

5.4.3 Sign restriction findings

Figures 5.4 and 5.5 show the correlation between broad money growth and inflation for US and UK data respectively. The zero mark on the abscissa represents the contemporaneous correlation and points to the right represent the lead information money growth has for inflation and to the left the lead information that inflation has for money. Figure 5.4 suggests some evidence of quite a change in the dynamic correlations in the two sub-samples in the US. In the earlier period inflation and money growth look positively related to each at leads and lags of up to one year. But in the later sample, inflation has a negative lead information for money and similarly so does money for inflation at up to one year. In the UK, Figure 5.5, the picture looks significantly more stable with inflation negatively leading money growth and money growth having positive leads for inflation. At face value this pattern of correlations suggests quite a different constellation of demand and supply shocks in the respective money markets and over time.

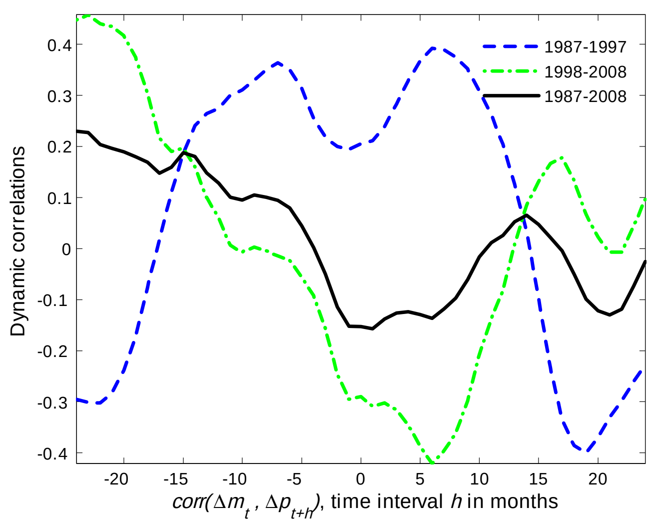

Figure 5.4 - US dynamic correlation between money and prices

Note: Dynamic correlation between US monthly money growth and inflation. We obtain HP filtered cyclical series of each variable as the link between raw monthly growth rate is noisy. For a positive correlation with

$h > 0$, money is leading inflation.

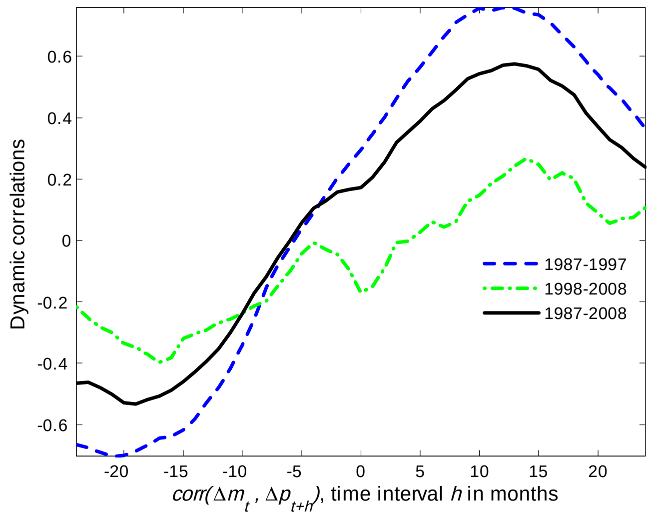

Figure 5.5 - UK dynamic correlation between money and prices

Note: Dynamic correlation between UK monthly money growth and inflation. We obtain HP filtered cyclical series of each variable as the link between raw monthly growth rate is noisy. For a positive correlation with $h > 0$, money is leading inflation.

Figures 5.6 and 5.6 show the correlation between money and prices at a successively longer horizon i.e. $corr(\frac{m_{t+h}}{m_t}, \frac{p_{t+h}}{p_t})$. In the absence of velocity or liquidity shocks, we would expect the correlation to rise with horizon (see equation 5.15). Figure 5.6 shows that in the US, we find that the correlation in the latter sample does not conform very clearly to our priors, in that at longer horizons the correlation tends to go negative, which suggests quite a large increase in velocity or liquidity in the latter period. Figure 5.7 shows that in the UK the pattern is more in line with our priors but there is some evidence of some deterioration in the positive correlation in the latter sub-period towards the end of the sample. The pattern that emerges from the US data again is one of volatility in the money-price correlation, particularly in the latter sample. Our next step is to try and uncover whether the change in the correlation can be attributed to some degree to either demand or supply shocks in the broad money market.

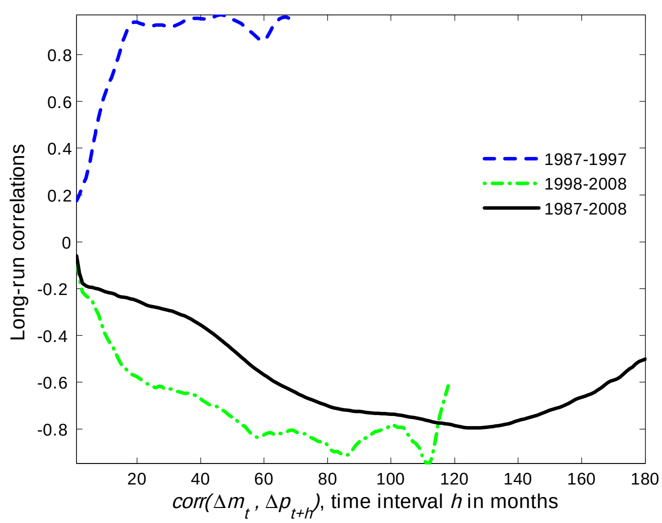

Figure 5.6 - US long run correlation between money and price

Note: Long-run correlation between the average growth for UK money growth and inflation. We obtain original logarithm series of each variable. For an increasing positive long-run correlation we find long-run neutrality for money.

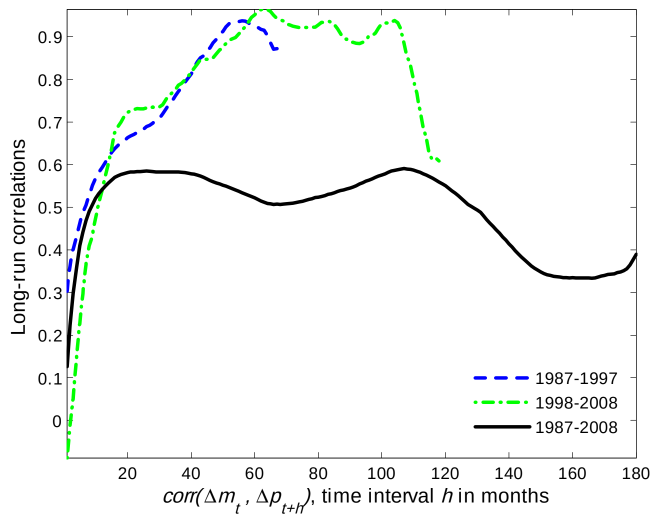

Figure 5.7 - UK long-run correlation between money and prices

Note: Long-run correlation between the average growth for UK money growth and inflation. We obtain original logarithm series of each variable. For an increasing positive long-run correlation we find long-run neutrality for money.

Figure 5.8 plots the impulse responses and the forecast error decomposition of US broad money and the EFP following the implementation of our identification scheme. A standard deviation demand shock to the broad money market is found to raise the EFP by some 8 bps and year on year growth in money by around 0.15% with the half life of the shocks estimated to be in the region of around 18 months. The lower panels suggest that demand shocks account for around 40% of fluctuations in EFP and broad money growth in this sample. A standard deviation supply shock to broad money is found to reduce the EFP by around 18 bp and increase money growth by around 0.15%. The half-life of the impact is considerably quicker with 50% of the shock dissipated in less than six months. The supply shock accounts for some 60% of the fluctuations in money growth and EFP over this sample.

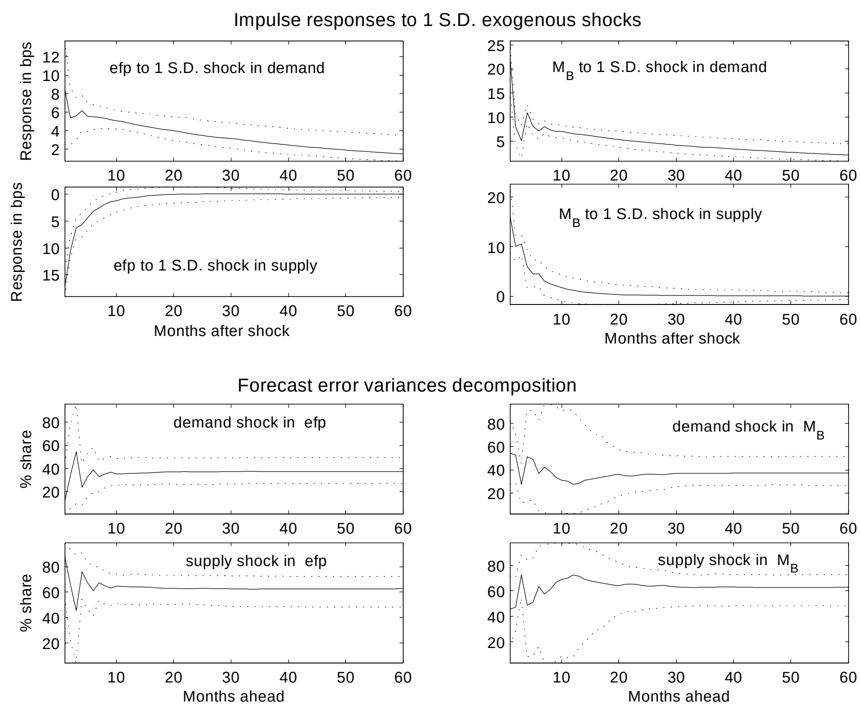

Figure 5.8 - US VAR impulse responses with sign restriction

Note: The first and second rows show the impulse responses of the model variables to a standard deviation of demand and supply shocks in money. Sign restrictions are imposed in the first 6 months. With 200 draws from a random Bayesian VAR posterior satisfying sign restrictions, the solid line is the median response and the dotted lines are ± 1 standard errors. The third and fourth row shows the h-month ahead forecast error variance decomposition. Again, solid and dotted lines denote median and ± 1 standard errors bands, respectively.

Figure 5.9 shows comparable and similar results for the UK. Two main differences stand out. There is a larger movement in the quantity of money given a movement in the EFP in the UK, suggesting flatter demand and supply curves. This is reflected in the basic moments of the data presented in Table 5.1, which show that money growth is more volatile and EFP less so in the UK compared to the US. That said more of the fluctuations in the EFP and in broad money growth can be explained by supply shocks in the UK, at nearly 80% compared to 60% in the US.

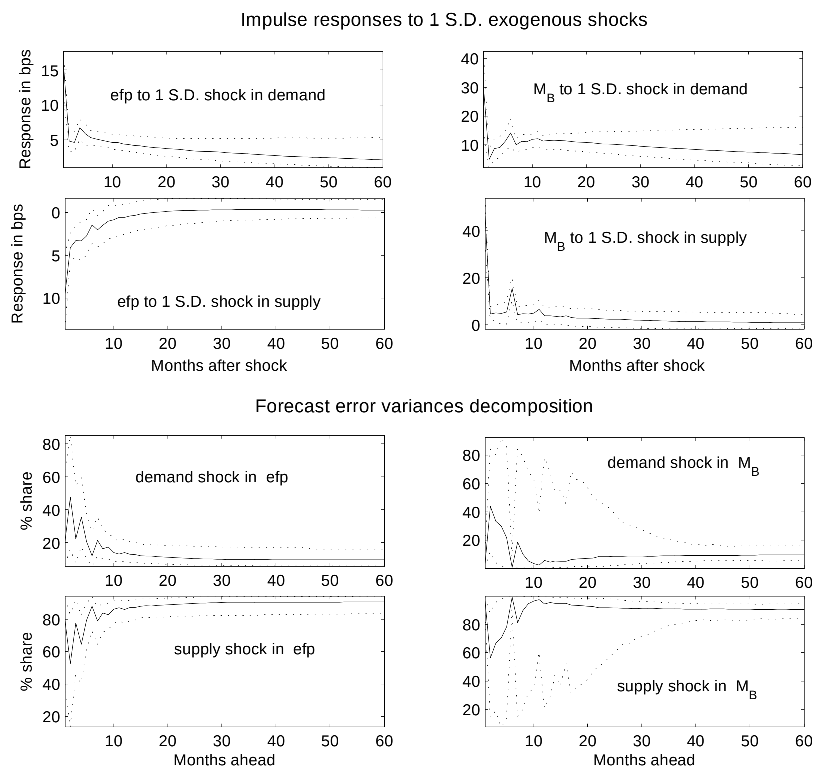

Figure 5.9 - UK VAR impulse responses with sign restriction

Note: The first and second rows show the impulse responses of the model variables to a standard deviation of demand and supply shocks in money. Sign restrictions are imposed in the first 6 months. With 200 draws from a random Bayesian VAR posterior satisfying sign restrictions, the solid line is the median response and the dotted lines are ± 1 standard errors. The third and fourth row shows the h-month ahead forecast error variance decomposition. Again, solid and dotted lines denote median and ± 1 standard errors bands, respectively.

Figures 5.10 and 5.11 replay the dynamic correlations from Figures 5.4 and 5.5 but with the correlation obtained from the data purged of demand and supply shocks, respectively. So that the contemporaneous negative correlation between money and inflation in Figures 5.4 and 5.10 for the US data seem to be something we can associate with a dominance of supply over demand shocks. Similarly for the UK data there appears to be a closer fit with the data when we consider the supply shock rather than demand shock case for the dynamic correlations.

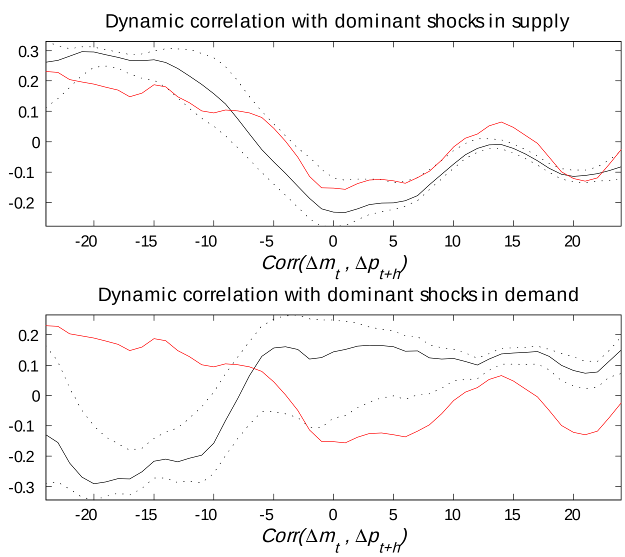

Figure 5.10 - US dynamic correlation between inflation and supply- or demand- driven money

Note: The charts plot the dynamic correlation between the original data series and the alternative series dominated by primitive shocks in money market. The red solid line represent the actual correlation while the black solid line is the median of alternative dynamic correlations. The dotted lines are ± 1 standard errors bands.

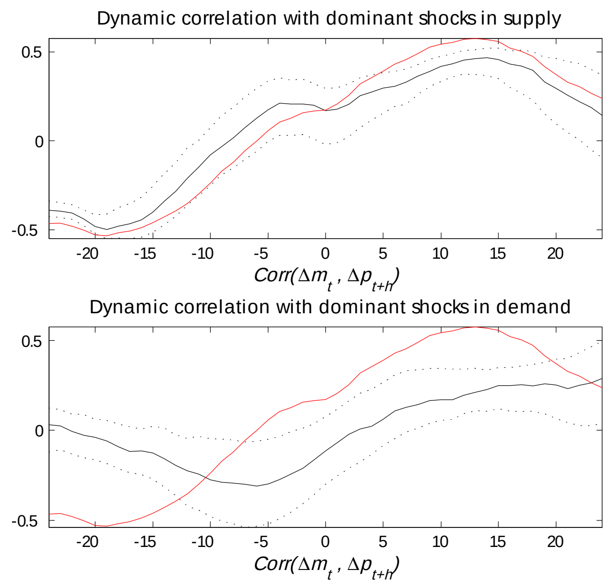

Figure 5.11 - UK dynamic correlation between inflation and supply- or demand- driven money

Note: The charts plot the dynamic correlation between the original data series and the alternative series dominated by primitive shocks in money market. The red solid line represent the actual correlation while the black solid line is the median of alternative dynamic correlations. The dotted lines are ± 1 standard errors bands.

Figures 5.12 and 5.13 replay the long run correlations from Figures 5.6 and 5.7. For the US the downturn in correlation at longer horizons and particularly in the latter sub-period seems to be well explained by demand shocks rather than supply shocks. So we have a story where supply shocks in the broad money market dominate at shorter horizons but demand shocks dominate over the longer run. For the UK the results is somewhat less clear cut with possibly both and demand and supply shocks having a role to play in the longer term correlation.

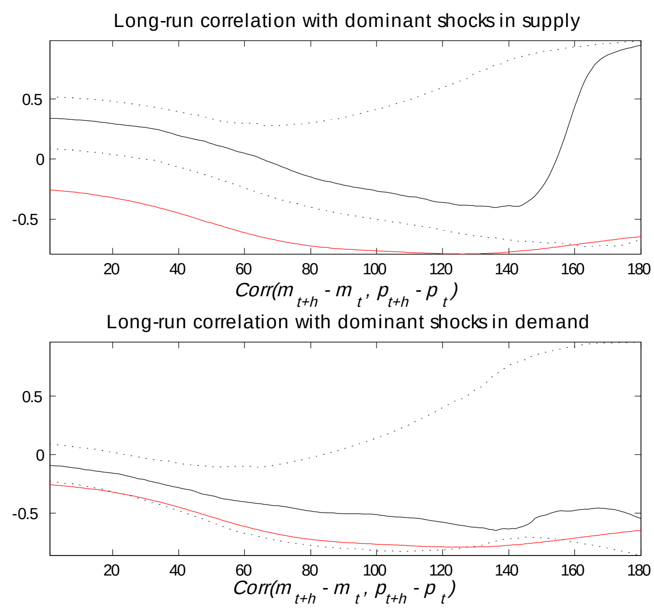

Figure 5.12 - US long-run correlation between inflation and supply- or demand- driven money

Note: The charts plot the long-run correlation of original data series and those alternative series dominated by primitive shocks in money market. The red solid line represent the actual correlation while the black solid line is the median of alternative long-run correlations. The dotted lines are ± 1 standard errors bands.

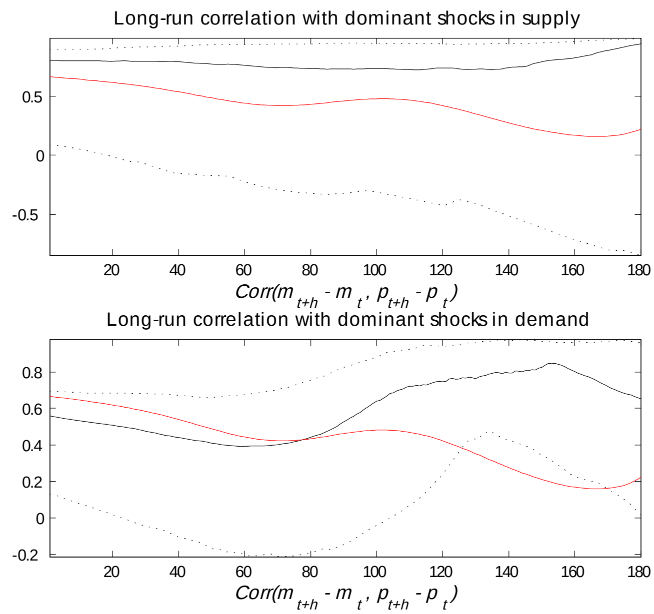

Figure 5.13 - UK long-run correlation between inflation and supply- or demand- driven money

Note: The charts plot the long-run correlation between the original data series and the alternative series dominated by primitive shocks in money market. The red solid line represent the actual correlation while the black solid line is the median of alternative long-run correlations. The dotted lines are ± 1 standard errors bands.

Concentrating on the finding that supply shocks seem the dominant explanation for fluctuations in broad money at the monthly frequency, we can use our method to uncover whether the supply shocks have been driven more by policy rate or LIBOR. Recall that the EFP equals difference between LIBOR and policy and a supply shock reduce the spread which may imply either or both of an increase in the policy rate or a reduction in LIBOR. We can interpret the former, a positive correlation between policy rates and money supply shocks, as a policy response and any negative correlation between supply shocks and LIBOR as an exogenous increase in money market supply.

In this sense Figure 5.14 is very revealing. We can estimate the correlation between our identified shocks and the LIBOR and the policy rate and plot the correlation as a kernel density. In both the full samples and the latter sample, US policy rates seem uncorrelated with the supply shocks to the money market and suggest that they emanated from the liquidity provision of the banking sector, which acted in response to a compression in financial spreads - as represented by the negative correlation in LIBOR. In the UK, Figure 5.15, the picture that emerges is somewhat different. In that over the full sample, the policy rate has been offsetting supply shocks as we locate a positive correlation but to some extent in the latter period, this attenuation has diminished to around 0.2 from 0.4.

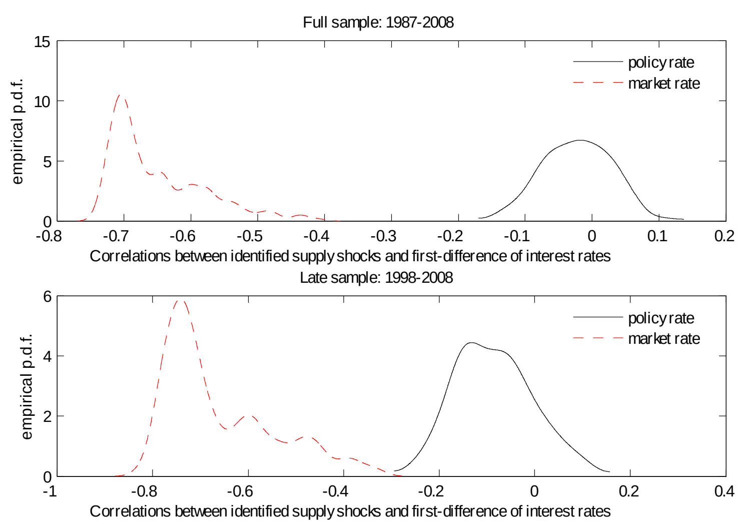

Figure 5.14 - US money supply shock accounting

Note: The chart shows whether the identified money supply shocks are associated with changes in policy rate or market rate, the two components in the financial spread, efp. The market rate is simply the interbank rate on wholesale money market. The empirical density is the kernel density estimator from the 200 valid draws.

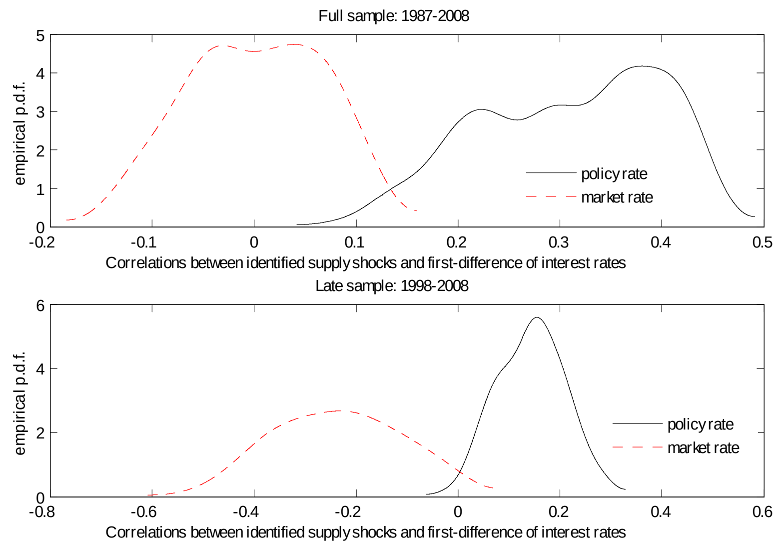

Figure 5.15 - UK money supply shock accounting

Note: The chart shows whether the identified money supply shocks are associated with changes in policy rate or market rate, the two components in the financial spread, efp. The market rate is simply the interbank rate on wholesale money market. The empirical density is the kernel density estimator from the 200 valid draws.

5.5 Conclusion

It has become a truism to state that monetary policy in the period of inflation targeting began to ignore money. This chapter as well illustrating why that might be the case - there are strong demand and supply shocks emanating in money markets which make inference on the true cause of any observed perturbation difficult - it offers a possible strategy that might be employed to uncover whether money is driven by demand or supply. The key is to view supply shocks in money market as an important indicator of both monetary policy innovations and money market liquidity provision. A demand-supply framework can well capture this big picture. By using a computationally intensive Bayesian VAR estimation with fairly pedestrian sign restrictions, which we show can fall out of a simple analysis of money markets, we can uncover primitive demand and supply shocks in the US and UK broad money market. We find that supply shocks dominate the innovations in cost of funding and the quantity of funding and particularly strong evidence in the US that these supply shocks were more closely related to financial market driven supply of funds rather than policy-induced variation. At least for 3-month tenor money market, the Fed seems reluctant to affect market liquidity by implementing proactive interest rate instruments. Considerably more work on sectoral money and individual market interest rates (of various tenors) will be required to firm up our tentative conclusions but at a moment when financial markets seem to be frozen, it is important to try and evaluate whether (a) policy (mistake) has had any role to play in the over-reach of the financial sector. Our tentative answer is yes.

Chapter 6
Conclusion

This study, developed with the considerable help of my thesis advisors, has analyzed several DSGE models to understand mainly, but not restricted to, the macroeconomic and financial linkage with traditional calibration method and a newly developed model evaluation procedure. The question was whether these models and various exogenous shocks are able to explain the observed volatilities and correlation of macro and financial variables. In addition to the empirical investigation, I develop testing methodologies for the empirical fit of these models. Finally, I use a Bayesian VAR framework to analyze the changing dynamics of macro-financial interaction observed on money market. As the short concluding chapter, I summarize the main findings and discuss for the shortcomings and possibilities of further research in these areas.

6.1 Main findings and discussion

The main results can be summarized as follows:

1. An open economy macro model with two sectors and incomplete financial market perturbed by productivity, preference and significant interest rate parity disturbances can account for observed puzzles of international finance literature, including lack of international risk sharing. Significantly perhaps there is no necessary role for various forms of price rigidities here. What is key is that the coupling of a financial friction with expec-

tational errors in the interest rate parity condition may provide a promising direction for resolutions of these puzzles (see Charles Engel, 2001, NBER Macroeconomics Annual);

2. A macro-finance model of yield curve under expectation hypothesis is in general not compatible with observed yield curve dynamics of the level of the term structure, but less so for yield curve slope, making it a good candidate for signalling business cycle fluctuations. That the DSGE models can fit the slope rather than the level may be related to the inability of these models to deal with time variation of expected steady states.

3. Labour reform such as UK in 1980s is likely to have lead to more efficient monetary policy transmission. But significant rigidities remain with employment adjusting slowly to shocks, and in such a world I suggest that policy needs to be attentive to recessions, as the search may persist for a long time period. To some extent this finding may alter the terms of trade in monetary policy for pursuing inflation targeting at the expense of output - the trade-off my not be quite so clearly in favour of price stability.

4. Orthodox monetary policy treats the evolution of the money stock as being closely tied to equilibrium outcomes on output and the price level. In this sense, observations on the money stock are unlikely to explain much about the future evolution of nominal output. But actual data suggests supply shocks, for example originating in financial sector loan production or innovation, play a significant role broad money balances over the short- to medium-term, therefore traditional inflation-targeting monetary policy might be inappropriate. In the period of global monetary easing, in the early part of this decade, the

Federal Reserve did not seem to respond so obviously to supply shocks but Bank of England and European Central Bank seem to, at least with some efforts, try offsetting such shocks.

On methodological side, there are two contribution in the thesis:

5. Unconditional second moments have been used to develop a new set of procedures for model evaluation and selection. The method features great flexibility and simplicity but works fairly well with the DSGE models explored here.

6. Finally, I repeat the shock identification procedure with Uhlig's (2005), among others, Bayesian VAR, adding to the existing literature on DSGE model identification.

6.2 Limitations and suggestion for future research

The complete research agenda is based on loosely organized topics in the chosen field, taking DSGE models to data of macro-financial variables. Without a firm key question, the whole research work suffers from inconsistency, especially the modeling technique. However this is due to the various macroeconomic puzzles I wish to resolve.

Some other shortcomings of the dissertation should be noted as well. First of all, as the main contribution, the new distance approach for model evaluation and selection is worthy of further robustness check on its properties and ability to uncover better models. In the current version, the six alternative metrics were either a rule-of-thumb type indicator or borrowed from other literature. Without an in depth understanding of it statistical properties, especially in a misspecified model, the model selection exercise may lead to incorrect inference. Some further work on the robustness of these tests is required but for reasons of space, such analysis is beyond the scope of my thesis. Separate work should be conducted on this promising empirical tool, perhaps with only the Kullback-Leibler information criterion (KLIC), as suggested by Watson (1993). The distributional assumption is in line with comparable empirical approaches and the concept of “information loss” is straightforward. An advantage of KLIC is the possibility to develop a “stronger interface”, such as a distance band, by utilizing data variability in the model simulation.

Although the distance approach has successfully indicated the best case for the open economy model and NKPC model, there are other concerns on the validity of such comparison between candidate models. In the open economy case, we compare only a sub-block of the variance covariance matrices. Candidate models have different numbers of exogenous shocks. These issues need to be justified in a structured research of the methodology. In the NKPC chapter, the method is used to fine-tune the magnitude of exogenous shocks. These attempts to improve model’s empirical fit may raise criticism due to lack of thorough justification of the distance measure approach. For instance, the sub-block problem might be resolved by incorporating a weighting matrix; simulation analysis can be conducted for misspecified models.

By construction, DSGE is a parsimonious framework so that a full-scale model fit can be very difficult. However, based on the proposed unconditional second moments method, model complexity is no longer the main issue. In chapter three through to five, more model features can be incorporated. For chapter three, a macro-finance yield curve can be constructed by revising the pure expectation hypothesis and considering investment habitat of bond as a financial shock. By doing so the yield curve curvature and level might be better explained.

In chapter four, the very slow response of unemployment is still a puzzle. Apparently further rigidity in labour market is required. Also specification of monetary policy could be modified to capture both inflation targeting and nominal income targeting, as opposed to the simple quantity rule in current version. It could very well be that some interaction of shocks may further help understand these dynamics: consider a reduction in credit availability combining with higher levels of unemployment to help explain large falls in output during recessions.

Chapter five has shown the distinctive policy stand confronting supply shock of money markets. This would be an important findings if it can also be observed in an DSGE model. Liquidity provision and various interest rate could be incorporated in a New Keynesian framework, such as Goodfriend and McCallum (2007) or that of Chadha and Corrado (2009), to address empirical distinction among US and EU data, as European Central Bank's two pillar principle provides a benchmark for such analysis.

Simulation based DSGE research is a wonderful world when computation burdens have been relaxed. And, rather like theoretical physics, the worlds created are magical, tractable and consistent from micro-foundations to superstructure. For a reasonably long time, DSGE macroeconomists have limited their analysis to the laboratory environment. However, recent years have seen more and more DSGE research work that contribute to a better understanding of aggregate macroeconomic and financial behaviour and thus may shed light on monetary policy making. And my PhD research is dedicated to this direction. Given the events of the past two years since the onset of the financial crisis, I can think no more laudable aim.

Appendix A
An Open Economy Macroeconomic Model: Specification, Solution and Approximation

Consumer optimization

In this model, the home and overseas economy are composed of infinitely lived identical representative agents. The utility function takes a non-separable form derived from consumption goods and leisure, which is similar to Stockman and Tesar (1995) and Holland and Scott (1998).

$$U_t = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\rho}}{1-\rho} \frac{(1-l_s)^{\xi_s}}{\eta \bar{\xi}} \right] \tag{A.1}$$

where ξ_t denotes a shift in preference, which generates fluctuations on demand side.

The incomplete financial market is introduced by home bonds or foreign bonds available to all agents. Home agents have income from employment and equal share of profit in final goods market. The budget constraint is:

$$P_t C_t + \frac{B_{H_t}}{(1+i_t)} + \frac{S_t B_{F_t}}{(1+i_t^*)\Theta\left(\frac{S_t B_{F_t}}{P_t}\right)} = B_{H_{t-1}} + S_t B_{F_{t-1}} + P_t w_t l_t + \Pi_t \tag{A.2}$$

where $\Theta(\cdot)$ denotes a cost for foreign bond holding. The first order conditions for this maximization problem with respect to B_{H_t}, B_{F_t} and l_t are:

$$U'_{C_t} = (1 + i_t)\beta \mathrm{E}_t \left[U'_{C_{t+1}} \frac{P_t}{P_{t+1}} \right] \tag{A.3}$$

$$U'_{C_t} = (1 + i_t^*)\Theta \left(\frac{S_t B_{F_t}}{P_t} \right) \beta \mathrm{E}_t \left[U'_{C_{t+1}} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right]. \tag{A.4}$$

$$w_t = \frac{\eta \bar{\xi}}{1 - \rho} \frac{C_t \xi_t}{1 - l_t} \tag{A.5}$$

From (A.3) and (A.4) we obtain the Uncovered Interest Parity (UIP) condition:

$$(1 + i_t) = (1 + i_t^*)\Theta \left(\frac{S_t B_{F_t}}{P_t} \right) \mathrm{E}_t \frac{S_{t+1}}{S_t} \tag{A.6}$$

Therefore the expected nominal exchange rate depend on foreign bond holding as well as the interest rate differential. We modify this result by further assuming a UIP shock ς_t originated from mis-perception in foreign exchange market, as in Kollmann (2003).

$$\mathrm{E}_t \frac{S_{t+1}}{S_t} = \varsigma_t \frac{1 + i_t}{1 + i_t^*} \Theta^{-1} \left(\frac{S_t B_{F_t}}{P_t} \right) \tag{A.7}$$

For the overseas economy there exists:

$$U'_{C_t^*} = (1 + i_t^*)\beta \mathrm{E}_t \left[U'_{C_{t+1}^*} \frac{P_t^*}{P_{t+1}^*} \right] \tag{A.8}$$

$$w_t^* = \frac{\eta \bar{\xi}}{1 - \rho} \frac{C_t^* \xi_t^*}{1 - l_t^*} \tag{A.9}$$

In the section below we disable the home bond for foreign agents. The purpose is to simplify the solution and highlight the incompleteness of financial market. For home investors (consumers), they could either invest in foreign bonds or adjust physical capital investment in response to shocks, as the net demand of bond is zero when all individuals are identical. The budget constraint becomes:

$$P_t C_t + \frac{S_t B_{F_t}}{(1+i_t^*)\Theta\left(\frac{S_t B_{F_t}}{P_t}\right)} = S_t B_{F_{t-1}} + P_t w_t l_t + \Pi_t \tag{A.10}$$

Final goods producer

On final consumption level, wholesaler combines the traded and non-traded intermediate goods in a CES fashion:

$$Y_t \equiv C_t = \left[\omega^{\frac{1}{\kappa}} c_{T_t}^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} c_{N_t}^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \tag{A.11}$$

where ω is the weight of traded goods in final consumption basket, and κ is the elasticity of substitution between traded and non-traded goods. The wholesaler seeks maximal profit by:

$$\max_{c_T, c_N} P_t C_t - P_{T_t} c_{T_t} - P_{N_t} c_{N_t} \tag{A.12}$$

The traded intermediate goods are produced by combining home produced and overseas produced components:

$$c_{T_t} = \left[\upsilon^{\frac{1}{\theta}} c_{H_t}^{\frac{\theta-1}{\theta}} + (1-\upsilon)^{\frac{1}{\theta}} c_{F_t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{A.13}$$

where υ denotes the share of traded goods produced domestically and θ is the elasticity of substitution between home produced and imported traded goods. The composite traded goods provider tries to maximize the profit:

$$\max_{c_H, c_F} P_{T_t} c_{T_t} - P_{H_t} c_{H_t} - P_{F_t} c_{F_t} \tag{A.14}$$

We assume symmetric structure for overseas economy. The corresponding weights are ω^* and υ^*. In particular υ^* is the share of home produced traded intermediate goods in overseas economy. For a two-country symmetric model we shall have $\upsilon^* = 1 - \upsilon$. The corresponding production function of foreign traded intermediate goods is:

$$c_{T_t}^* = \left[\upsilon^{*\frac{1}{\theta^*}} c_{H_t}^{*\frac{\theta^*-1}{\theta^*}} + (1-\upsilon^*)^{\frac{1}{\theta^*}} c_{F_t}^{*\frac{\theta^*-1}{\theta^*}} \right]^{\frac{\theta^*}{\theta^*-1}} \tag{A.15}$$

where c_H^* and c_F^* denotes home-produced and foreign-produced traded intermediate goods consumed in overseas.

Profit maximization problem of composite goods producers yields the demand functions of each goods.

$$c_{N_t} = (1-\omega)\left(\frac{P_{N_t}}{P_t}\right)^{-\kappa} C_t \tag{A.16}$$

$$c_{H_t} = \omega\upsilon\left(\frac{P_{H_t}}{P_{T_t}}\right)^{-\theta}\left(\frac{P_{T_t}}{P_t}\right)^{-\kappa} C_t \tag{A.17}$$

$$c_{F_t} = \omega(1-\upsilon)\left(\frac{P_{F_t}}{P_{T_t}}\right)^{-\theta}\left(\frac{P_{T_t}}{P_t}\right)^{-\kappa} C_t \tag{A.18}$$

The associated Dixit-Stiglitz price indices are:

$$P_t = \left[\omega P_{T_t}^{1-\kappa} + (1-\omega) P_{N_t}^{1-\kappa}\right]^{\frac{1}{1-\kappa}} \tag{A.19}$$

$$P_{T_t} = \left[\upsilon P_{H_t}^{1-\theta} + (1-\upsilon) P_{F_t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{A.20}$$

Correspondingly, in overseas the intermediate goods demand functions are:

$$c^*_{N_t} = (1-\omega^*)\left(\frac{P^*_{N_t}}{P^*_t}\right)^{-\kappa^*} C^*_t \tag{A.21}$$

$$c^*_{F_t} = \omega^*(1-\upsilon^*)\left(\frac{P^*_{F_t}}{P^*_{T_t}}\right)^{-\theta^*}\left(\frac{P^*_{T_t}}{P^*_t}\right)^{-\kappa^*} C^*_t \tag{A.22}$$

$$c^*_{H_t} = \omega^*\upsilon^*\left(\frac{P^*_{H_t}}{P^*_{T_t}}\right)^{-\theta^*}\left(\frac{P^*_{T_t}}{P^*_t}\right)^{-\kappa^*} C^*_t \tag{A.23}$$

And the aggregate price indices:

$$P^*_t = \left[\omega^* P_{T_t}^{*1-\kappa^*} + (1-\omega^*) P_{N_t}^{*1-\kappa^*}\right]^{\frac{1}{1-\kappa^*}} \tag{A.24}$$

$$P^*_{T_t} = \left[\upsilon^* P_{H_t}^{*1-\theta^*} + (1-\upsilon^*) P_{F_t}^{*1-\theta^*}\right]^{\frac{1}{1-\theta^*}} \tag{A.25}$$

On final goods market, the budget constraints exclude government spending or further investment goods.

$$
\begin{aligned}
Y_t^* &= C_t^* && \text{(A.26)} \\
Y_t &= C_t
\end{aligned}
$$

We sum up the intermediate level production and define it the aggregate output (GDP, or gross value added):

$$
\begin{aligned}
V_t &= y_{H_t} + y_{N_t} && \text{(A.27)} \\
V_t^* &= y_{F_t}^* + y_{N_t}^*
\end{aligned}
$$

Intermediate goods producer

In each sector, the producer of intermediate goods hires capital and labour to maximise the expected discounted value of profit.

$$
\max \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \frac{U'_{C_t} P_0}{U'_{C_0} P_t} \left[P_{H_t} y_{H_t} - P_t w_t l_{H_t} - P_{H_t} x_{H_t} \right] \tag{A.28}
$$

with respect to:

$$
\begin{aligned}
y_{H_t} &= F\left(l_{H_t}, k_{H_{t-1}}\right) = A_t l_{H_t}^{\alpha} k_{H_{t-1}}^{1-\alpha} && \text{(A.29)} \\
k_{H_t} &= (1-\delta) k_{H_{t-1}} + x_{H_t} - \phi \left(\frac{x_{H_t}}{k_{H_{t-1}}} \right) k_{H_{t-1}} && \text{(A.30)}
\end{aligned}
$$

First order conditions to labor, investment, Lagrange multiplier and capital stock provide the equilibrium solution:

$$\frac{P_{H_t}}{P_t} = \frac{w_t}{\alpha A_t \left(\frac{k_{H_{t-1}}}{l_{H_t}}\right)^{1-\alpha}}$$

$$U'_{C_t} = \beta \mathrm{E}_t \frac{P_{H_{t+1}}}{P_{t+1}} \frac{P_t}{P_{H_t}} U'_{C_{t+1}} \left\{ \begin{array}{c} (1-\alpha)\left(1-\phi'\left(\frac{x_{H_t}}{k_{H_{t-1}}}\right)\right) A_{t+1} \left(\frac{l_{H_{t+1}}}{k_{H_t}}\right)^{\alpha} + \\ \frac{1-\phi'\left(\frac{x_{H_t}}{k_{H_{t-1}}}\right)}{1-\phi'\left(\frac{x_{H_{t+1}}}{k_{H_t}}\right)} \left[(1-\delta) - \phi\left(\frac{x_{H_{t+1}}}{k_{H_t}}\right) + \phi'\left(\frac{x_{H_{t+1}}}{k_{H_t}}\right) \frac{x_{H_{t+1}}}{k_{H_t}}\right] \end{array} \right\} \tag{A.31}$$

Similarly, the producer of nontraded intermediate goods hires capital and labor to maximize the expected discounted value of profit.

$$\max \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \frac{U'_{C_t} P_0}{U'_{C_0} P_t} \left[P_{N_t} y_{N_t} - P_t w_t l_{N_t} - P_{N_t} x_{N_t}\right] \tag{A.32}$$

with respect to:

$$y_{N_t} = F\left(l_{N_t}, k_{N_{t-1}}\right) = A_{N_t} l_{N_t}^{\alpha} k_{N_{t-1}}^{1-\alpha} \tag{A.33}$$

$$k_{N_t} = (1-\delta) k_{N_{t-1}} + x_{N_t} - \phi\left(\frac{x_{N_t}}{k_{N_{t-1}}}\right) k_{N_{t-1}} \tag{A.34}$$

First order conditions to labour, investment, Lagrange multiplier and capital stock provide the equilibrium solution:

$$\frac{P_{N_t}}{P_t} = \frac{w_t}{\alpha A_{N_t}\left(\frac{k_{N_{t-1}}}{l_{N_t}}\right)^{1-\alpha}}$$

$$U'_{C_t} = \beta \mathrm{E}_t \frac{P_{N_{t+1}}}{P_{t+1}} \frac{P_t}{P_{N_t}} U'_{C_{t+1}} \left\{ \begin{array}{c} (1-\alpha)\left(1-\phi'\left(\frac{x_{N_t}}{k_{N_{t-1}}}\right)\right) A_{N_{t+1}} \left(\frac{l_{N_{t+1}}}{k_{N_t}}\right)^{\alpha} + \\ \frac{1-\phi'\left(\frac{x_{N_t}}{k_{N_{t-1}}}\right)}{1-\phi'\left(\frac{x_{N_{t+1}}}{k_{N_t}}\right)} \left[(1-\delta) - \phi\left(\frac{x_{N_{t+1}}}{k_{N_t}}\right) + \phi'\left(\frac{x_{N_{t+1}}}{k_{N_t}}\right)\frac{x_{N_{t+1}}}{k_{N_t}}\right] \end{array} \right\}$$

Relative Prices

It is useful to define two relative price measures: terms of trade (TOT, import to export price; an increase means deterioration of TOT) and relative price of non-traded goods.

$$T_t = \frac{P_{F_t}}{P_{H_t}} \qquad \text{(A.35)}$$

$$R_t = \frac{P_{N_t}}{P_{T_t}} \qquad \text{(A.36)}$$

For simplicity, we replace log-linearized price variables by expressions of these relative prices (T_t and R_t) in the remaining parts of the paper. This can be done by log-linearising the Dixit-Stiglitz price indices:

$$\begin{aligned} \widehat{P}_{T_t} - \widehat{P}_{H_t} &= (1-\upsilon)\,\widehat{T}_t \\ \widehat{P}_{T_t} - \widehat{P}_{F_t} &= -\upsilon \widehat{T}_t \\ \widehat{P}_t - \widehat{P}_{T_t} &= (1-\omega)\,\widehat{R}_t \\ \widehat{P}_t - \widehat{P}_{N_t} &= -\omega \widehat{R}_t \end{aligned} \qquad \text{(A.37)}$$

The overseas counterpart of relative prices can be shown as:

$$
\begin{aligned}
\widehat{T}_t^* &= \widehat{P^*}_{H_t} - \widehat{P^*}_{F_t} \\
\widehat{R}_t^* &= \widehat{P}_{N_t}^* - \widehat{P}_{T_t}^* \\
\widehat{P^*}_{T_t} - \widehat{P^*}_{F_t} &= \upsilon^* \widehat{T^*}_t \\
\widehat{P^*}_{T_t} - \widehat{P^*}_{H_t} &= -(1-\upsilon^*)\,\widehat{T^*}_t \\
\widehat{P^*}_t - \widehat{P^*}_{T_t} &= (1-\omega^*)\,\widehat{R^*}_t \\
\widehat{P^*}_t - \widehat{P^*}_{N_t} &= -\omega^* \widehat{R^*}_t
\end{aligned}
$$

where $S_t P_{H_t}^* = P_{H_t}$, $S_t P_{F_t}^* = P_{F_t}$, (local currency pricing) as required by the non-discriminative prices. Thus $T_t T_t^* = 1$.

Current account

In equation (A.10) we plug in the wholesale profit of home economy:

$$
\begin{aligned}
\Pi_t &= P_{H_t} y_{H_t} - P_t w_t l_{H_t} - P_{H_t} x_{H_t} + \\
&\quad P_{N_t} y_{N_t} - P_t w_t l_{N_t} - P_{N_t} x_{N_t}
\end{aligned} \tag{A.38}
$$

and the economy-wide resource constraint: $l_t = l_{H_t} + l_{N_t}$; $x_t = x_{H_t} + x_{N_t}$. We obtain:

$$
C_t + \frac{S_t B_{F_t}}{P_t(1+i_t^*)\Theta\left(\frac{S_t B_{F_t}}{P_t}\right)} = \frac{S_t B_{F_{t-1}}}{P_t} + \frac{P_{H_t}}{P_t} y_{H_t} + \frac{P_{N_t}}{P_t} y_{N_t} - \frac{P_{H_t}}{P_t} x_t \tag{A.39}
$$

Market equilibrium in the intermediate goods level requires: $y_{N_t} = c_{N_t}$; $y_{H_t} = c_{H_t} + c^*_{H_t} + x_{H_t} + x_{N_t}$. Finally, the current account equation has the following form:

$$\frac{S_t B_{F_t}}{P_t(1+i^*_t)\Theta\left(\frac{S_t B_{F_t}}{P_t}\right)} = \frac{S_t B_{F_{t-1}}}{P_t} + \frac{P_{H_t}}{P_t}c^*_{H_t} + \frac{P_{H_t}}{P_t}c_{H_t} + \frac{P_{N_t}}{P_t}c_{N_t} - C_t \tag{A.40}$$

Real exchange rate

In our model, the real exchange rate is defined as:

$$RS_t = \frac{S_t P^*_t}{P_t}$$

$$\frac{S_t P^*_t}{P_t} = \frac{S_t P^*_{H,t}}{P_{H,t}} \frac{P_{H,t}}{P_{T,t}} \frac{P^*_{T,t}}{P^*_{H,t}} \frac{P_{T,t}}{P_t} \frac{P^*_t}{P^*_{T,t}}$$

Monetary policy

Since we are characterising a nominal model we need to specify a monetary policy rule. In what follows we assume that the monetary authorities in both countries follow a strategy of setting consumer price inflation equal to zero (this would be equivalent of saying that our model is a model in which prices are perfectly flexible).

Log-linearisation

This section approximates the system of equations around a steady state. The equations appearing in the code are labeled with a prefix L.

Current account

We do not provide a functional form for the cost of financial intermediation. However, we specify following properties for this cost function: $\Theta(0) = 1$; $\Theta(b_t)$ is a differenciable decreasing function around the neighbourhood of 0, where $b_t = \frac{S_t B_{F_t}}{P_t}$ is the real foreign asset holding in home currency. We assume a constant elasticity $(-\varepsilon)$ of this cost parameter in response to changes in foreign asset position.

$$-\varepsilon = \frac{\partial\Theta\left(b_t\right)}{\Theta\left(b_t\right)} / \frac{\partial b_t}{b_t}$$

$$\varepsilon = -\frac{\Theta'\left(b_t\right) b_t}{\Theta\left(b_t\right)} \tag{A.41}$$

The steady state of current account equation is:

$$\frac{\beta\bar{b}}{\Theta\left(\bar{b}\right)} = \bar{b} + \omega^* \upsilon^* \overline{Y}^* + \omega\upsilon\overline{Y} + (1-\omega)\,\overline{Y} - \overline{Y} \tag{A.42}$$

$$\frac{\overline{Y}^*}{\overline{Y}} = \frac{\omega\left(1-\upsilon\right) + \left(\beta - 1\right)\overline{a}}{\omega^* \upsilon^*} \tag{A.43}$$

where $\overline{a} \equiv \frac{\bar{b}}{\overline{Y}} \simeq \frac{\bar{b}}{\Theta(\bar{b})\overline{Y}}$ is the steady state of net foreign asset position. Home country is either a creditor or debtor for positive or negative $\overline{a}$. Equation (A.42) shows a steady state trade and asset holding: the left hand side is the present value of increased foreign asset position to home output. The right hand side is the net export as a percentage of home output. As the steady state of economy aggregate is determined by the factor endowment, $\frac{\overline{Y}^*}{\overline{Y}}$ is known. Specifically, in symmetric case, $\overline{a} = 0$, $\overline{Y}^* = \overline{Y}$, $\omega = \omega^*$, $1 - \upsilon = \upsilon^*$.

We multiply both side of current account equation by $(1 + i_t^*)$ and approximate it using first-order Taylor series expansion:

$$\begin{aligned} lhs &= \frac{\bar{b}}{\Theta\left(\bar{b}\right)} + \frac{\Theta\left(\bar{b}\right) - \bar{b}\Theta'\left(\bar{b}\right)}{\Theta^2\left(\bar{b}\right)}\left(b_t - \bar{b}\right) \\ &= \frac{\bar{b}}{\Theta\left(\bar{b}\right)} + \frac{1+\varepsilon}{\Theta\left(\bar{b}\right)}\left(b_t - \bar{b}\right) \end{aligned} \tag{A.44}$$

$$\begin{aligned} rhs &= \frac{S_t P_{t-1} S_{t-1} B_{F_{t-1}}}{S_{t-1} P_t P_{t-1}}(1+i^*_t) + \\ &\quad \frac{P_{H_t}}{P_t}\omega^*\upsilon^*\left(\frac{P^*_{H_t}}{P^*_{T_t}}\right)^{-\theta^*}\left(\frac{P^*_{T_t}}{P^*_t}\right)^{-\kappa^*} Y^*_t(1+i^*_t) + \\ &\quad \frac{P_{H_t}}{P_t}\omega\upsilon\left(\frac{P_{H_t}}{P_{T_t}}\right)^{-\theta}\left(\frac{P_{T_t}}{P_t}\right)^{-\kappa} Y_t(1+i^*_t) + \\ &\quad \frac{P_{N_t}}{P_t}\left(1-\omega\right)\left(\frac{P_{N_t}}{P_t}\right)^{-\kappa} Y_t(1+i^*_t) - Y_t(1+i^*_t) \\ &= \frac{1}{\beta}\exp\left(\widehat{S}_t - \widehat{S}_{t-1} + \widehat{P}_{t-1} - \widehat{P}_t + \widehat{i^*}_t\right) b_{t-1} + \\ &\quad \frac{\omega^*\upsilon^*}{\beta}\frac{\overline{Y}^*}{\overline{Y}}\left[-\left(1-\upsilon\right)\widehat{T}_t - \left(1-\omega\right)\widehat{R}_t + \theta^*\left(1-\upsilon^*\right)\widehat{T}_t + \kappa\left(1-\omega^*\right)\widehat{R^*}_t + \widehat{Y^*}_t + \widehat{i^*}_t\right] + \\ &\quad \frac{\omega\upsilon}{\beta}\left[-\left(1-\theta\right)\left(1-\upsilon\right)\widehat{T}_t - \left(1-\kappa\right)\left(1-\omega\right)\widehat{R}_t + \widehat{Y}_t + \widehat{i^*}_t\right] + \\ &\quad \frac{1-\omega}{\beta}\left[\omega\widehat{R}_t - \kappa\omega\widehat{R}_t + \widehat{Y}_t + \widehat{i^*}_t\right] - \\ &\quad \frac{1}{\beta}\left[\widehat{Y}_t + \widehat{i^*}_t\right] \end{aligned} \tag{A.45}$$

The first term in right hand side can be reduced:

$$
\begin{aligned}
\frac{1}{\beta}\exp\left(\widehat{S}_t-\widehat{S}_{t-1}+\widehat{P}_{t-1}-\widehat{P}_t+\widehat{i^*}_t\right)b_{t-1} &= \frac{1}{\beta}\exp\left(\widehat{S}_t-\widehat{S}_{t-1}+\widehat{P}_{t-1}-\widehat{P}_t+\widehat{i^*}_t\right)\left(b_{t-1}-\overline{b}\right)+\\
&\quad \frac{1}{\beta}\overline{b}\exp\left(\widehat{S}_t-\widehat{S}_{t-1}+\widehat{P}_{t-1}-\widehat{P}_t+\widehat{i^*}_t\right)\\
&\simeq \left[\frac{1}{\beta}\widehat{b}_{t-1}+\frac{1}{\beta}\overline{a}+\frac{1}{\beta}\overline{a}\left(\triangle\widehat{S}_t-\pi_t+\widehat{i^*}_t\right)\right]\overline{Y}
\end{aligned}
$$

where $\widehat{b}_t = \frac{b_t-\bar{b}}{\Theta(\bar{b})\overline{Y}} \simeq \frac{b_t-\bar{b}}{\overline{Y}}$. Multiply both side by β and subtract steady state value of each side using (A.42).

$$
\begin{aligned}
\beta\left(1+\varepsilon\right)\widehat{b}_t &= \widehat{b}_{t-1}+\overline{a}\left(\triangle\widehat{S}_t-\pi_t\right)+\beta\overline{a}\left(1+\varepsilon\right)\widehat{i^*}_t+\\
&\quad \left[\theta^*\left(1-\upsilon^*\right)\left(\overline{\gamma}-\omega\upsilon\right)+\left(1-\upsilon\right)\left(\theta\omega\upsilon-\overline{\gamma}\right)\right]\widehat{T}_t+\\
&\quad \left[\omega\left(1-\kappa\right)-\overline{\gamma}\right]\left(1-\omega\right)\widehat{R}_t+\kappa^*\left(1-\omega^*\right)\left(\overline{\gamma}-\omega\upsilon\right)\widehat{R^*}_t+\\
&\quad \left(\overline{\gamma}-\omega\upsilon\right)\widehat{Y^*}_t-\omega\left(1-\upsilon\right)\widehat{Y}_t \qquad \text{(L6)}
\end{aligned}
$$

where $\overline{\gamma}=\omega+\left(\beta-1\right)\overline{a}$.

Consumption Euler and other aggregate equations

We approximate the Euler equations, UIP condition and consumption-leisure trade-off around the steady state:

$$\widehat{U'_{C_t}} = \widehat{i}_t + \mathrm{E}_t\left(\widehat{U'_{C_{t+1}}} - \pi_{t+1}\right) \tag{L1}$$

$$\widehat{U'_{C^*_t}} = \widehat{i^*}_t + \mathrm{E}_t\left(\widehat{U'_{C^*_{t+1}}} - \pi^*_{t+1}\right) \tag{L2}$$

$$\mathrm{E}_t\Delta S_{t+1} = \widehat{i}_t - \widehat{i^*}_t + \varepsilon\widehat{b}_t + \widehat{\varsigma}_t \tag{L3}$$

$$\widehat{w}_t = \widehat{C}_t + \widehat{\xi}_t + \frac{\bar{l}}{1-\bar{l}}\widehat{l}_t \tag{L4}$$

$$\widehat{w^*}_t = \widehat{C^*}_t + \widehat{\xi^*}_t + \frac{\bar{l}}{1-\bar{l}}\widehat{l^*}_t \tag{L5}$$

Log-linearisation of equilibrium inputs in production

We now log-linearise the equilibrium conditions for competitive marekets of intermediate goods that we derived in previous section.

Firstly we log-linearise equation (A.31). We specify the cost of capital adjustment as: $\phi\left(\frac{x_t}{k_{t-1}}\right) = \frac{b\left(\frac{x_t}{k_{t-1}}-\delta\right)^2}{2}$. The log-linearisation can be done by introducing some intermediate variables. The equation can become the following system of equation after introduction of several instrumental quantities $Z_{1,t}$, $Z_{2,t}$, $Z_{3,t}$.

$$\begin{aligned}
Z_{1,t} &= 1 - \phi'\left(\frac{x_{H_t}}{k_{H_{t-1}}}\right) \\
Z_{2,t} &= (1-\delta) - \phi\left(\frac{x_{H_{t+1}}}{k_{H_t}}\right) + \phi'\left(\frac{x_{H_{t+1}}}{k_{H_t}}\right)\frac{x_{H_{t+1}}}{k_{H_t}} \\
Z_{3,t} &= f_{k,t+1}Z_{1,t} + \frac{Z_{1,t}}{Z_{1,t+1}}Z_{2,t} \\
U'_{C_t} &= \beta\mathrm{E}_t\frac{P_{H_{t+1}}}{P_{t+1}}\frac{P_t}{P_{H_t}}U'_{C_{t+1}}Z_{3,t}
\end{aligned}$$

Each of these equations can be log-linearised easily by straightforward algebra. We obtain the following equations by tidying up log-linearized counterparts of all the equations.

$$\begin{aligned}
\widehat{Z}_{1,t} &= -b\delta\left(\widehat{x}_{H_t}-\widehat{k}_{H_{t-1}}\right)\\
\widehat{Z}_{2,t} &= \frac{b\delta^2\left(\widehat{x}_{H_{t+1}}-\widehat{k}_{H_t}\right)}{1-\delta}\\
\widehat{Z}_{3,t} &= \beta f_k\left(\widehat{f}_{k,t+1}+\widehat{Z}_{1,t}\right)+\beta\left(1-\delta\right)\left(\widehat{Z}_{1,t}-\widehat{Z}_{1,t+1}+\widehat{Z}_{2,t}\right)\\
\widehat{f}_{k,t} &= \widehat{A}_t+\alpha\widehat{l}_{H_t}-\alpha\widehat{k}_{H_{t-1}}
\end{aligned}$$

$$\begin{aligned}
\rho\widehat{C}_{t+1}+\psi_\xi\widehat{\xi}_{t+1}+\psi_l\widehat{l}_{t+1} &= \rho\widehat{C}_t+\psi_\xi\widehat{\xi}_t+\psi_l\widehat{l}_t+\\
&\quad\left(1-\omega\right)\left(\widehat{R}_t-\widehat{R}_{t+1}\right)+\left(1-\upsilon\right)\left(\widehat{T}_t-\widehat{T}_{t+1}\right)-\\
&\quad b\delta\left(\widehat{x}_{H_t}-\widehat{k}_{H_{t-1}}\right)-\left(\alpha\lambda+b\beta\delta\right)\widehat{k}_{H_t}+\\
&\quad b\beta\delta\widehat{x}_{H_{t+1}}+\lambda\widehat{A}_{t+1}+\alpha\lambda\widehat{l}_{H_{t+1}} \qquad \text{(L7)}
\end{aligned}$$

where $\lambda=1-\beta\left(1-\delta\right)$, $\psi_\xi=-\eta\overline{\xi}\ln\left(1-\overline{l}\right)$, $\psi_l=\eta\overline{\xi}\frac{\overline{l}}{1-\overline{l}}$.

Capital accumulation equation takes the form:

$$\widehat{k}_{H_t}=\left(1-\delta\right)\widehat{k}_{H_{t-1}}+\delta\widehat{x}_{H_t} \qquad \text{(L9)}$$

The labour input equation:

$$\widehat{w}_t=\left(\upsilon-1\right)\widehat{T}_t+\left(\omega-1\right)\widehat{R}_t+\widehat{A}_t+\left(\alpha-1\right)\widehat{l}_{H_t}+\left(1-\alpha\right)\widehat{k}_{H_{t-1}} \qquad \text{(L11)}$$

For non-traded sector, we have:

$$\begin{aligned}\rho\widehat{C}_{t+1}+\psi_{\xi}\widehat{\xi}_{t+1}+\psi_{l}\widehat{l}_{t+1} &= \rho\widehat{C}_{t}+\psi_{\xi}\widehat{\xi}_{t}+\psi_{l}\widehat{l}_{t}+\omega\left(\widehat{R}_{t+1}-\widehat{R}_{t}\right)- \\ &\quad b\delta\left(\widehat{x}_{N_t}-\widehat{k}_{N_{t-1}}\right)-\left(\alpha\lambda+b\beta\delta\right)\widehat{k}_{N_t}+ \\ &\quad b\beta\delta\widehat{x}_{N_{t+1}}+\lambda\widehat{A}_{N_{t+1}}+\alpha\lambda\widehat{l}_{N_{t+1}} \qquad \text{(L8)}\end{aligned}$$

$$\begin{aligned}\widehat{k}_{N_t} &= (1-\delta)\widehat{k}_{N_{t-1}}+\delta\widehat{x}_{N_t} \qquad \text{(L10)} \\ \widehat{w}_{t} &= \omega\widehat{R}_{t}+\widehat{A}_{N_t}+(\alpha-1)\widehat{l}_{N_t}+(1-\alpha)\widehat{k}_{N_{t-1}} \qquad \text{(L12)}\end{aligned}$$

The overseas counterparts of equation $L7$ through $L12$ are:

$$\begin{aligned}\rho\widehat{C^{*}}_{t+1}+\psi_{\xi}\widehat{\xi^{*}}_{t+1}+\psi_{l}\widehat{l^{*}}_{t+1} &= \rho\widehat{C^{*}}_{t}+\psi_{\xi}\widehat{\xi^{*}}_{t}+\psi_{l}\widehat{l^{*}}_{t}+ \\ &\quad (1-\omega^{*})\left(\widehat{R^{*}}_{t}-\widehat{R^{*}}_{t+1}\right)+\upsilon^{*}\left(\widehat{T}_{t}-\widehat{T}_{t+1}\right)- \\ &\quad b\delta\left(\widehat{x^{*}}_{F_t}-\widehat{k^{*}}_{F_{t-1}}\right)-\left(\alpha^{*}\lambda+b\beta\delta\right)\widehat{k^{*}}_{F_t}+ \\ &\quad b\beta\delta\widehat{x^{*}}_{F_{t+1}}+\lambda\widehat{A^{*}}_{F_{t+1}}+\alpha^{*}\lambda\widehat{l^{*}}_{F_{t+1}} \qquad \text{(L13)}\end{aligned}$$

$$\begin{aligned}\rho\widehat{C^{*}}_{t+1}+\psi_{\xi}\widehat{\xi^{*}}_{t+1}+\psi_{l}\widehat{l^{*}}_{t+1} &= \rho\widehat{C^{*}}_{t}+\psi_{\xi}\widehat{\xi^{*}}_{t}+\psi_{l}\widehat{l^{*}}_{t}+\omega^{*}\left(\widehat{R^{*}}_{t+1}-\widehat{R^{*}}_{t}\right)- \\ &\quad b\delta\left(\widehat{x^{*}}_{N_t}-\widehat{k^{*}}_{N_{t-1}}\right)-\left(\alpha^{*}\lambda+b\beta\delta\right)\widehat{k^{*}}_{N_t}+ \\ &\quad b\beta\delta\widehat{x^{*}}_{N_{t+1}}+\lambda\widehat{A^{*}}_{N_{t+1}}+\alpha^{*}\lambda\widehat{l^{*}}_{N_{t+1}} \qquad \text{(L14)}\end{aligned}$$

$$\widehat{k^*}_{F_t} = (1-\delta)\,\widehat{k^*}_{F_{t-1}} + \delta\widehat{x^*}_{F_t} \tag{L15}$$

$$\widehat{k^*}_{N_t} = (1-\delta)\,\widehat{k^*}_{N_{t-1}} + \delta\widehat{x^*}_{N_t} \tag{L16}$$

$$\widehat{w^*}_t = \upsilon^*\widehat{T}_t + (\omega^* - 1)\,\widehat{R^*}_t + \widehat{A^*}_{F_t} + (\alpha^* - 1)\,\widehat{l^*}_{F_t} + (1-\alpha^*)\,\widehat{k^*}_{F_{t-1}} \tag{L17}$$

$$\widehat{w^*}_t = \omega^*\widehat{R^*}_t + \widehat{A^*}_{N_t} + (\alpha^* - 1)\,\widehat{l^*}_{N_t} + (1-\alpha^*)\,\widehat{k^*}_{N_{t-1}} \tag{L18}$$

Other steady state

In the steady state, all prices are equalized and normalized to unity. We obtain following steady state:

$$\begin{aligned} 1+\bar{i} &= \frac{1}{\beta} \\ 1 &= \beta\left(\overline{f}_{H_k} + 1 - \delta\right) \\ 1 &= \beta\left(\overline{f}_{N_k} + 1 - \delta\right) \end{aligned}$$

In the steady state, capital adjustment cost is zero, and $\frac{\overline{x}_H}{\overline{k}_H} = \frac{\overline{x}_N}{\overline{k}_N} = \delta$.

$$\begin{aligned} \frac{\overline{x}_H}{\overline{y}_H} &= \frac{\overline{x}_H}{\overline{k}_H}\left(\frac{\overline{k}_H}{\overline{y}_H}\right) = \delta\left(\frac{\overline{f}_{H_k}}{1-\alpha}\right)^{-1} = \delta\left(\frac{1-\alpha}{1/\beta - 1 + \delta}\right) \\ \frac{\overline{x}_N}{\overline{y}_N} &= \delta\left(\frac{1-\alpha}{1/\beta - 1 + \delta}\right) \end{aligned}$$

$$
\begin{aligned}
\overline{y}_H &= \overline{c}_H + \overline{c}_H^* + \overline{x}_H + \overline{x}_N \\
\overline{y}_H &= \omega\upsilon\overline{Y} + \omega^*\upsilon^*\overline{Y}^* + \overline{y}_H\left(\frac{\overline{x}_H}{\overline{y}_H}\right) + (1-\omega)\,\overline{Y}\frac{\overline{x}_N}{\overline{y}_N} \\
\frac{\overline{y}_H}{\overline{Y}} &= \frac{\omega + [\beta(1+\varepsilon)-1]\,\overline{a} + \frac{\delta(1-\omega)(1-\alpha)}{1/\beta-1+\delta}}{1-\frac{\delta(1-\alpha)}{1/\beta-1+\delta}} \\
\frac{\overline{c}_H}{\overline{y}_H} &= \omega\upsilon\left(\frac{\overline{y}_H}{\overline{Y}}\right)^{-1} \\
\frac{\overline{c}_H^*}{\overline{y}_H} &= \omega^*\upsilon^*\left(\frac{\overline{y}_H}{\overline{Y}}\right)^{-1}\left(\frac{\overline{Y}^*}{\overline{Y}}\right) \\
\frac{\overline{x}_N}{\overline{y}_H} &= 1-\frac{\overline{c}_H}{\overline{y}_H}-\frac{\overline{c}_H^*}{\overline{y}_H}-\frac{\overline{x}_H}{\overline{y}_H}
\end{aligned}
$$

The labor inputs:

$$
\begin{aligned}
\overline{w} &= \alpha\frac{\overline{y}_H}{\overline{l}_H} = \alpha\frac{\overline{y}_N}{\overline{l}_N} \\
\overline{l} &= \overline{l}_H + \overline{l}_N \\
\frac{\overline{l}_N}{\overline{l}} &= \frac{\frac{\overline{y}_N}{\overline{Y}}}{\frac{\overline{y}_H}{\overline{Y}}+\frac{\overline{y}_N}{\overline{Y}}} \\
\frac{\overline{l}_H}{\overline{l}} &= 1-\frac{\overline{l}_N}{\overline{l}}
\end{aligned}
$$

Log-linearization of constraints

Production function of traded goods equals final consumption plus investment requirement: $y_{H_t} = c_{H_t} + c_{H_t}^* + x_{H_t} + x_{N_t}$.

$$\begin{aligned}\widehat{A}_t + \alpha \widehat{l}_{H_t} + (1-\alpha)\,\widehat{k}_{H_{t-1}} &= \frac{\overline{c}_H}{\overline{y}_H}\left[\theta\,(1-\upsilon)\,\widehat{T}_t + \kappa\,(1-\omega)\,\widehat{R}_t + \widehat{Y}_t\right] + \\ &\quad \frac{\overline{c}_H^*}{\overline{y}_H}\left[\theta^*\,(1-\upsilon^*)\,\widehat{T}_t + \kappa^*\,(1-\omega^*)\,\widehat{R}_t^* + \widehat{Y}_t^*\right] + \\ &\quad \frac{\overline{x}_H}{\overline{y}_H}\widehat{x}_{H_t} + \frac{\overline{x}_N}{\overline{y}_H}\widehat{x}_{N_t} \qquad \text{(L19)}\end{aligned}$$

And $y_{N_t} = c_{N_t}$

$$\widehat{A}_{N_t} + \alpha \widehat{l}_{N_t} + (1-\alpha)\,\widehat{k}_{N_{t-1}} = -\kappa\omega\widehat{R}_t + \widehat{Y}_t \qquad \text{(L20)}$$

The overseas counterparts:

$$\frac{\overline{x}_F^*}{\overline{y}_F^*} = \frac{\overline{x}_N^*}{\overline{y}_N^*} = \delta\frac{1-\alpha^*}{1/\beta - 1 + \delta}$$

$$\begin{aligned}\overline{y}_F^* &= \overline{c}_F + \overline{c}_F^* + \overline{x}_F^* + \overline{x}_N^* \\ \overline{y}_F^* &= \omega\,(1-\upsilon)\,\overline{Y} + \omega^*\,(1-\upsilon^*)\,\overline{Y}^* + \overline{y}_F\left(\frac{\overline{x}_F^*}{\overline{y}_F^*}\right) + (1-\omega^*)\,\overline{Y}^*\frac{\overline{x}_N^*}{\overline{y}_N^*} \\ \frac{\overline{y}_F^*}{\overline{Y}^*} &= \frac{\omega\,(1-\upsilon)\,/\left(\frac{\overline{Y}^*}{\overline{Y}}\right) + \omega^*\,(1-\upsilon^*) + \frac{\delta(1-\omega^*)(1-\alpha^*)}{1/\beta-1+\delta}}{1 - \frac{\delta(1-\alpha^*)}{1/\beta-1+\delta}} \\ \frac{\overline{c}_F}{\overline{y}_F^*} &= \omega\,(1-\upsilon)\left(\frac{\overline{y}_F^*}{\overline{Y}^*}\frac{\overline{Y}^*}{\overline{Y}}\right)^{-1} \\ \frac{\overline{c}_F^*}{\overline{y}_F^*} &= \omega^*\,(1-\upsilon^*)\left(\frac{\overline{y}_F^*}{\overline{Y}^*}\right)^{-1} \\ \frac{\overline{x}_N^*}{\overline{y}_F^*} &= 1 - \frac{\overline{c}_F}{\overline{y}_F^*} - \frac{\overline{c}_F^*}{\overline{y}_F^*} - \frac{\overline{x}_F^*}{\overline{y}_F^*}\end{aligned}$$

$$
\begin{aligned}
\widehat{A}^{*}_{F_t} + \alpha^{*}\widehat{l}^{*}_{F_t} + (1-\alpha^{*})\,\widehat{k}^{*}_{F_{t-1}} &= \frac{\overline{c}_F}{\overline{y}^{*}_F}\left[-\theta\upsilon\widehat{T}_t + \kappa\,(1-\omega)\,\widehat{R}_t + \widehat{Y}_t\right] + \\
&\quad \frac{\overline{c}^{*}_F}{\overline{y}^{*}_F}\left[-\theta^{*}\upsilon^{*}\widehat{T}_t + \kappa^{*}\,(1-\omega^{*})\,\widehat{R}^{*}_t + \widehat{Y}^{*}_t\right] + \\
&\quad \frac{\overline{x}^{*}_F}{\overline{y}^{*}_F}\widehat{x}^{*}_{F_t} + \frac{\overline{x}^{*}_N}{\overline{y}^{*}_F}\widehat{x}^{*}_{N_t} \qquad \text{(L21)}
\end{aligned}
$$

$$
\widehat{A}^{*}_{N_t} + \alpha^{*}\widehat{l}^{*}_{N_t} + (1-\alpha^{*})\,\widehat{k}^{*}_{N_{t-1}} = -\kappa^{*}\omega^{*}\widehat{R}^{*}_t + \widehat{Y}^{*}_t \qquad \text{(L22)}
$$

Labor inputs:

$$
\begin{aligned}
\overline{w}^{*} &= \alpha^{*}\frac{\overline{y}^{*}_F}{\overline{l}^{*}_F} = \alpha^{*}\frac{\overline{y}^{*}_N}{\overline{l}^{*}_N} \\
\overline{l}^{*} &= \overline{l}^{*}_F + \overline{l}^{*}_N \\
\frac{\overline{l}^{*}_N}{\overline{l}^{*}} &= \frac{\frac{\overline{y}^{*}_N}{\overline{Y}^{*}}}{\frac{\overline{y}^{*}_F}{\overline{Y}^{*}} + \frac{\overline{y}^{*}_N}{\overline{Y}^{*}}} \\
\frac{\overline{l}^{*}_F}{\overline{l}^{*}} &= 1 - \frac{\overline{l}^{*}_N}{\overline{l}^{*}}
\end{aligned}
$$

The constraints for final consumption:

$$
\widehat{Y}_t = \widehat{C}_t \qquad \text{(L23)}
$$

$$
\widehat{Y}^{*}_t = \widehat{C}^{*}_t \qquad \text{(L24)}
$$

The constraints for labor input:

$$\widehat{l}_t = \frac{\bar{l}_H}{\bar{l}}\widehat{l}_{H_t} + \frac{\bar{l}_N}{\bar{l}}\widehat{l}_{N_t} \tag{L25}$$

$$\widehat{l}^*_t = \frac{\bar{l}^*_F}{\bar{l}^*}\widehat{l}^*_{F_t} + \frac{\bar{l}^*_N}{\bar{l}^*}\widehat{l}^*_{N_t} \tag{L26}$$

Other equations

The real exchange rate:

$$\widehat{RS}_t = (\upsilon - \upsilon^*)\hat{T}_t + (\omega - 1)\,\hat{R}_t + (1 - \omega^*)\,\hat{R}^*_t \tag{L27}$$

Evolvement of terms of trade:

$$T_t = \frac{P_{F_t}}{P_{H_t}} = \frac{P_{F_{t-1}}}{P_{H_{t-1}}}\frac{S_t P^*_{F_t}}{S_{t-1}P^*_{F_{t-1}}}\frac{P_{H_{t-1}}}{P_{H_t}}$$

$$\widehat{T}_t = \widehat{T}_{t-1} + \Delta\widehat{S}_t + \pi^{F^*}_t - \pi^H_t \tag{L28}$$

Define relative consumption:

$$\widehat{CC^*}_t = \widehat{C}_t - \widehat{C^*}_t \tag{L29}$$

Define current account as the capital account adjustment, as home agents buy foreign bond with payment in trade abroad.

$$\widehat{CA}_t = \widehat{b}_t - \widehat{b}_{t-1} \tag{L30}$$

CPI indices:

$$\pi_t = \omega\upsilon\pi_t^H + \omega\left(1-\upsilon\right)\pi_t^{F^*} + \omega\left(1-\upsilon\right)\Delta\widehat{S}_t + \left(1-\omega\right)\pi_t^N \tag{L31}$$

$$\pi_t^* = \omega^*\upsilon^*\pi_t^H - \omega^*\upsilon^*\Delta\widehat{S}_t + \omega^*\left(1-\upsilon^*\right)\pi_t^{F^*} + \left(1-\omega^*\right)\pi_t^{N^*} \tag{L32}$$

Central banks set CPI to zeros (flexible-price monetary policy):

$$\pi_t = 0 \tag{L33}$$

$$\pi_t^* = 0 \tag{L34}$$

PPI Inflation of non-traded sector can be expressed:

$$\begin{aligned}
\pi_t^N &= \left(\widehat{P}_{N_t} - \widehat{P}_t\right) - \left(\widehat{P}_{N_{t-1}} - \widehat{P}_{t-1}\right) + \widehat{P}_t - \widehat{P}_{t-1} \\
&= \omega\left(\hat{R}_t - \hat{R}_{t-1}\right) + \pi_t
\end{aligned} \tag{L35}$$

$$\pi_t^{N^*} = \omega^*\left(\hat{R}_t^* - \hat{R}_{t-1}^*\right) + \pi_t^* \tag{L36}$$

The trade balance is defined as real net export deflated by home price:

$$\begin{aligned}
TB_t &= \frac{P_{H_t}}{P_t}c_{H_t}^* - \frac{P_{F_t}}{P_t}c_{F_t} \\
&= \frac{P_{H_t}}{P_t}\omega^*\upsilon^*\left(\frac{P_{H_t}^*}{P_{T_t}^*}\right)^{-\theta^*}\left(\frac{P_{T_t}^*}{P_t^*}\right)^{-\kappa^*}Y_t^* - \frac{P_{F_t}}{P_t}\omega\left(1-\upsilon\right)\left(\frac{P_{F_t}}{P_{T_t}}\right)^{-\theta}\left(\frac{P_{T_t}}{P_t}\right)^{-\kappa}Y_t
\end{aligned} \tag{A.46}$$

The cyclical trade balance is defined as

$$\begin{aligned}\frac{dTB_t}{\overline{Y}} &= \left[\theta^*\left(1-\upsilon^*\right)\left(\overline{\gamma}-\omega\upsilon\right)+\left(1-\upsilon\right)\left(\omega\upsilon\theta-\overline{\gamma}\right)\right]\widehat{T}_t + \\ &\left(1-\omega\right)\left(\omega-\overline{\gamma}\right)\widehat{R}_t + \\ &\left(\overline{\gamma}-\omega\upsilon\right)\left(1-\omega^*\right)\kappa\widehat{R}^*_t + \\ &\left(\overline{\gamma}-\omega\upsilon\right)\widehat{Y}^*_t-\omega\left(1-\upsilon\right)\widehat{Y}_t \qquad \text{(L37)}\end{aligned}$$

where $\overline{TB}=\left(\beta-1\right)\overline{a}\overline{Y}$.

Finally, we calculate the aggregate investment level for both countries:

$$\widehat{x}_t = \frac{\overline{x}_H}{\overline{x}}\widehat{x}_{H_t}+\frac{\overline{x}_N}{\overline{x}}\widehat{x}_{N_t} \qquad \text{(L38)}$$

$$\widehat{x}^*_t = \frac{\overline{x}^*_F}{\overline{x}^*}\widehat{x}^*_{F_t}+\frac{\overline{x}^*_N}{\overline{x}^*}\widehat{x}^*_{N_t} \qquad \text{(L39)}$$

We define the GDP as the gross value added at intermediate goods production stage deflated by consumer price index: $V_t=\frac{P_{N_t}}{P_t}y_{N_t}+\frac{P_{H_t}}{P_t}y_{H_t}=\frac{P_{N_t}}{P_t}c_{N_t}+\frac{P_{H_t}}{P_t}\left(c_{H_t}+c^*_{H_t}+x_{H_t}+x_{N_t}\right)$; $V^*_t=\frac{P^*_{N_t}}{P^*_t}y^*_{N_t}+\frac{P^*_{F_t}}{P^*_t}y^*_{F_t}=\frac{P^*_{N_t}}{P^*_t}c^*_{N_t}+\frac{P^*_{F_t}}{P^*_t}\left(c^*_{F_t}+c_{F_t}+x^*_{F_t}+x^*_{N_t}\right)$. The linearized equations are:

$$\begin{aligned}\frac{\overline{V}}{\overline{Y}}\widehat{V}_t &= \left(1-\omega+\omega\upsilon\right)\widehat{Y}_t+\left(\overline{\gamma}-\omega\upsilon\right)\widehat{Y}^*_t+\frac{\overline{x}_H}{\overline{y}_H}\frac{\overline{y}_H}{\overline{Y}}\widehat{x}_{H_t}+\left(1-\omega\right)\frac{\overline{x}_N}{\overline{y}_N}\widehat{x}_{N_t}+ \\ &\left(1-\omega\right)\left[\omega\kappa\left(\upsilon-1\right)-\frac{\overline{V}}{\overline{Y}}+1\right]\widehat{R}_t+\kappa^*\left(\overline{\gamma}-\omega\upsilon\right)\left(1-\omega^*\right)\widehat{R}^*_t+ \\ &\left[\left(1-\upsilon\right)\left(\omega\upsilon\theta-\frac{\overline{V}}{\overline{Y}}+1-\omega\right)+\theta^*\left(\overline{\gamma}-\omega\upsilon\right)\left(1-\upsilon^*\right)\right]\widehat{T}_t \qquad \text{(L40)}\end{aligned}$$

where $\frac{\overline{V}}{\overline{Y}} = (1-\omega) + \overline{\gamma} + \frac{\overline{x}_H}{\overline{y}_H}\frac{\overline{y}_H}{\overline{Y}} + (1-\omega)\frac{\overline{x}_N}{\overline{y}_N}$.

$$\begin{aligned}
\frac{\overline{V}^*}{\overline{Y}^*}\widehat{V}_t^* &= (1-\omega^*\upsilon^*)\widehat{Y}_t^* + \frac{\omega\omega^*\upsilon^*(1-\upsilon)}{\overline{\gamma}-\omega\upsilon}\widehat{Y}_t + \frac{\overline{x}_F^*}{\overline{y}_F^*}\frac{\overline{y}_F^*}{\overline{Y}^*}\widehat{x}_{F_t}^* + (1-\omega^*)\frac{\overline{x}_N^*}{\overline{y}_N^*}\widehat{x}_{N_t}^* + \\
& (1-\omega^*)\left[1-\omega^*\kappa^*\upsilon^* - \frac{\overline{V}^*}{\overline{Y}^*}\right]\widehat{R}_t^* + \kappa(1-\omega)\frac{\omega\omega^*\upsilon^*(1-\upsilon)}{\overline{\gamma}-\omega\upsilon}\widehat{R}_t + \\
& \upsilon^*\left[\frac{\overline{V}^*}{\overline{Y}^*} - 1 + \omega^* - \omega^*\theta^*(1-\upsilon^*) - \frac{\omega\omega^*\theta\upsilon(1-\upsilon)}{\overline{\gamma}-\omega\upsilon}\right]\widehat{T}_t \qquad \text{(L41)}
\end{aligned}$$

where $\frac{\overline{V}^*}{\overline{Y}^*} = 1 - \omega^*\upsilon^* + \frac{\omega\omega^*\upsilon^*(1-\upsilon)}{\overline{\gamma}-\omega\upsilon} + \frac{\overline{x}_F^*}{\overline{y}_F^*}\frac{\overline{y}_F^*}{\overline{Y}^*} + (1-\omega^*)\frac{\overline{x}_N^*}{\overline{y}_N^*}$.

Summary of the system

We have eight lagged variables therefore another eight equations enter the system. Now we have 41 endonegous variables, 8 predetermined variables and 7 exogenous shocks.

Appendix B
Measurement of Exogenous Shocks in Open Economy Macroeconomic Model

B.1 Productivity Measurement

Sectoral productivity is calculated as total factor productivity (TFP) in traded (manufacturing) or non-traded (services) sector. We use OECD STAN database 2005 Release (OECD, 2004) to construct sectoral TFP series for the UK and the US. Incomplete data on total hours and gross capital stock are complemented by total employment and capital formation data. Based on these data, TFP is measured as:

$$TFP_t^A = \log\left(\frac{Y_t^A}{\left(K_t^A\right)^{1-\alpha}\left(N_t^A\right)^{\alpha}}\right), \tag{B.1}$$

where $A = \{T$ (traded), NT (non-traded)$\}$, and α denotes the labor share in production calibrated at 0.67.

B.2 Measuring the preference shock

We follow Holland and Scott (1998) for measuring preference deviations, $\widehat{\xi}_t$. Specifically, we use the Euler equation describing the leisure-consumption trade-off to find an expression for the preference variable ξ_t.[59] In the above equilibrium the real wage must equals $\frac{U_L}{U_C}$, the marginal rate of substitution between leisure and consumption, to clear labor mar-

59 A positive preference deviation, such as $\xi_t = 1\%$, is said to be biased to leisure time.

ket. The time endowment and the utility non-separable to leisure are:

$$L_t = 1 - l_t \tag{B.2}$$

$$U = \frac{1}{1-\rho} C_t^{1-\rho} \frac{L_t^{\eta \xi_t}}{\eta} \tag{B.3}$$

Then, the leisure-consumption trade-off yields:

$$w_t = \frac{U_L}{U_C} = \frac{\eta C_t \xi_t}{(1-\rho)(1-l_t)} \tag{B.4}$$

Calibration for the parameters are taken from Table 2.1. Then, the stochastic preference shocks can be measured by using US and UK aggregate data on w_t, C_t and l_t.

Note we could also enforce the equilibrium condition for flexible wage setting:

$$w_t = F_l\left(K_{t-1}, l_t\right). \tag{B.5}$$

In this case, we can replace w_t by $\frac{\alpha Y_t}{l_t}$ to avoid the sticky wage setting which may undermine the basic assumption of the flexible price model. This measure also captures very well the idea of preference shocks in Hall (1997), since data shows an association of recession years with increase in leisure-biased preference ξ_t.

Finally, our preference shocks are the detrended series of ξ_t in logarithm:

$$\ln \xi_t = \ln \frac{\alpha Y_t (1-\rho)(1-l_t)}{\eta l_t C_t} \tag{B.6}$$

B.3 Expectational Errors in Exchange Rates

Furthermore, we allow a random shock in the UIP condition, making exchange rate volatility attributable to more factors. This treatment aims to reconcile the contrafactual interest rate parity predicted by consumption Euler equation. The uncovered interest rate parity calls for an even urgent need in recent years when carry trade increase sharply both in volume and its market impact. We have a simple model for the UIP shocks with participants in foreign exchange market allowed to let exchange rate deviate from theoretical value in the short run. The nominal exchange rate adjustment follows the covered interest rate parity condition of foreign bond holding but is also subject to a shock $x_{u,t}$:

$$E_t \Delta s_{t+1} = i_t - i_t^* + \varepsilon \widehat{B}_t + x_{u,t} \tag{B.7}$$

This representation is an extremely simple way to model UIP shock, not like other research who assume some theoretical background behind the disparity (for instance Dornbusch's overreaction model). Attemps to estimate the above relationship can be problemetic due to potential omitted variables. This partly contributes to the dominant role of expectational errors in foreign exchange market, such as the UIP deviations in our set-up. Instead of estimation, we refer to Selaive and Tuesta's (2003) finding on foreign bond holding costs, and adopt their calibration of $\varepsilon = 0.007$ (based on quarterly data). The UIP deviations are therefore computed by substituting calibrated parameters and historical data into equation (B.7), where we measure net foreign asset adjustment $\widehat{B}_t$ (bond holding) by the detrended series of Net Foreign Asset to GDP ratio: $\frac{S_t B_t}{Y_t}$.

We find the resulting UIP deviation is highly volatile but not highly persistent. However, a persistent UIP shock is required for a foreign exchange market characterised by carry trade and uncovered interest rate parity. By contrast to above estimation, Kollmann (2003) uses a two-part UIP shock $x_{u,t} = a_t + \omega_t$ and finds UIP shocks to be quite persistent. We conduct sensitivity check by allowing persistent UIP deviations versus the base case scenario.

Appendix C
Evaluating Model Fit

Canova and Ortega (2000) discuss four possible approaches in evaluating DSGE model fit. The variety of approaches arise from the different treatments of model uncertainties and data sampling uncertainties: (a) an informal approach, which ignores both sampling variability in the data and uncertainty regarding model parameters, (b) methods that consider model uncertainty but not sampling variability in the data, and (c) methods that consider sampling variability in the data but not uncertainty in model; and (d) approaches that account for both sampling variability in the data and model uncertainty.

Bhattacharjee and Thoenissen (2007) propose the modified Nagao test which belongs to the class of methods (c):

> "... we consider an approach that uses sampling variability of actual data to provide a measure of the distance between model and the data, holding the model VCM fixed. This approach is explicitly based on the context of dynamic general equilibrium macroeconomic models, where given specific calibrated or estimated values for the parameters, the model can be simulated for as many periods of time as desired. Thus, for given parameter values, the asymptotic VCM of the state variables obtained from such simulation has no sampling variability. On the other hand, the data VCM is based on a data for a finite sample period. In most applications, this period would be from 1960 or later to the most recent period for which data are available. Thus, there is substantial sampling variation in the data VCM, while the model VCM can be considered fixed for a given combination of parameter values. By computing distances for distinct combinations of possible parameter values across all the competing models, we can ignore the uncertainty regarding calibration or estimation of parameters, while taking account of sampling variability in the actual data."

Using approach (c), we propose similar methods derived from the Box-Bartlett test

(Bartlett, 1937; Box, 1949) and its variant based on the Ledoit-Wolf test (Ledoit and Wolf, 2002). In addition we explore the possibility of using parallel approaches following Canova and Ortega's (2000) guideline: eyeballing approach such as RMSE and MAE are implementations of approach (a), while Kullback-Leibler is an implementation of approach (d).

Since most DSGE models are driven by only a limited number of shocks and predetermined state variables, the model VCM is usually rank-deficient. Except for RMSE and MAE, we use a projection of both data and model VCM to lower dimensional subspace introduced by Bhattacharjee and Thoenissen (2007) to deal with the rank-deficient problem.

The methods developed here will also take into account two other common features of model selection in the stated context. First, as emphasized earlier, DSGE models are intended to be abstractions of reality and are often driven by a lesser number of shocks than the number of state variables. In other words, while actual data VCMs would be full-rank, simulated data VCMs may often have a lower rank. Our methods will explicitly take into account this possibility. Second, the metrics will be developed in such a way that enables model selection when the candidate DSGE models may be non-nested. This feature of our methodology is obviously important and enhances the applicability of the methods.

Distance metrics

We denote by $[\Sigma_0]_{m\times m}$ the full-rank data VCM estimated using n_0 data points ($\rho(\Sigma_0) = m$), where ρ is the rank of VCM. $[\Sigma_{M_1}]_{m\times m}$, $[\Sigma_{M_2}]_{m\times m}$, $[\Sigma_{M_3}]_{m\times m}$, $\ldots$ denote estimated VCMs using simulated data from a countable collection of competing models $M_1, M_2, M_3, \ldots$ and based on $n_1, n_2, n_3, \ldots$ simulated observations respectively. Some of these matrices may be rank deficient; that is, $\rho(\Sigma_{M_j}) \leq \rho(\Sigma_0) = m$.

We shall propose several alternate metrics, denoted $d(\Sigma_0, \Sigma_{M_j})$, that give scalar measures of how different any of the simulated VCMs are from Σ_0, where d is a metric measuring the distance between Σ_0 and Σ_{M_j}. These measures can then be used to select an appropriate model from all the competing ones. In the following, we focus on one competing model VCM, say Σ_M and elaborate on different possible approaches and corresponding metrics.

Naive, or Eyeballing, approach

This is not based on any distributional assumption. Root Mean Squared Errors (RMSE) and Mean Absolute Errors (MAE) are defined as:

$$RMSE = \sqrt{MSE}; MSE = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}\widetilde{\sigma}_{i,j}^2 \tag{C.1}$$

$$MAE = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m}|\widetilde{\sigma}_{i,j}| \tag{C.2}$$

where $\widetilde{\Sigma} = ((\widetilde{\sigma}_{ij}))_{m\times m} = \Sigma_M - \Sigma_0$. In terms of the typology developed in Canova and Ortega (2000), the above two metrics ignore sampling variability in both data and model VCM.

Testing approach

This approach is based on a multivariate normality assumption underlying both the estimated VCMs, Σ_0 and Σ_M. However, we consider the possibility that the model VCM may not be full rank. The idea here is to pretend that we are conducting a test of the hypothesis $H_0 : \Sigma_0 = \Sigma_M$ against the omnibus alternative $H_1 : \Sigma_0 \neq \Sigma_M$. We are not as such interested in the outcome of the test, since we do not strongly believe that any of the models will generate simulated VCMs that are statistically indistinguishable from the data VCM. However, we can still use the p-values of the tests (or the values of the test statistic itself, adjusted for degrees of freedom) to give us a metric to compare between competing models. Note that the testing approach considers sampling variation in the data VCM, but the comparison is made with a simulated VCM based on large data where sampling variability may be negligible. We consider the following cases:

Σ_M is full-rank

Here we can use a whole battery of tests developed in the multivariate statistics literature. The most popular of these tests are the Box (1949) modification to the test proposed by Bartlett (1937), and the test proposed by Nagao (1973).

Bartlett (1937) proposed the test statistic:

$$M = \sum (n_0 + n_M) \ln |\Sigma| - n_0 \ln |\Sigma_0| - n_M \ln |\Sigma_M| \tag{C.3}$$

where the pooled estimate of the common covariance matrix under the null hypothesis is

$$\Sigma = \frac{1}{n_0 + n_M} \left[n_0 \Sigma_0 + n_M \Sigma_M \right].$$

When multiplied by a scaler C^{-1} (Box, 1949):

$$C^{-1} = 1 - \frac{2m^2 + 3m - 1}{6\left(m+1\right)}\left(\frac{1}{n_0} + \frac{1}{n_M} - \frac{1}{n_0 + n_M}\right),$$

the Box's M test statistic MC^{-1} has a Chi-square distribution (df = $m\left(m+1\right)/2$) under the null hypothesis and multivariate normality assumption.

Nagao (1973) proposed a test for the null hypothesis $H_0 : \Sigma_M^* = I$ against the omnibus alternative (where I is the identity matrix) given by the test statistic:

$$N = \frac{n_M}{2} tr\left(\Sigma_M^* - I\right)^2,$$

where $tr(.)$ denotes trace of a square matrix. The test statistic has a Chi-square distribution (df = $m\left(m+1\right)/2$) under the null hypothesis and multivariate normality assumption. This test can be adopted to our situation by using the Cholesky decomposition of Σ_0, as follows:

$$\begin{aligned} \Sigma_0 &= P'P \\ \Sigma_M^* &= P'^{-1}\Sigma_M P^{-1} \\ I &= P'^{-1}\Sigma_0 P^{-1} \end{aligned}$$

so that testing $H_0 : \Sigma_0 = \Sigma_M$ is now equivalent to testing $H_0 : \Sigma_M^* = I$ against the omnibus alternative. This is equivalent to premultiplying the actual and simulated data vectors by P'^{-1}. Both the Box's M-test and Nagao's test are known to be very conservative even in small samples (seldom accept the null hypothesis); this is, however, not of any major consequence for our work since we are not interested in the exact results of the test.

Σ_M is rank deficient ($\rho(\Sigma_M) < \rho(\Sigma_0) = m$)

This is the usual case. The model here is clearly an abstraction driven by only a limited number of shocks and predetermined variables. In fact, this abstraction can also represent reality to a high degree, in the sense that often only a small number of shocks can explain a substantial part of the variation in actual data on a larger number of state variables. In most applications, only a limited number of leading eigenvalues (and their corresponding eigenvectors) account for most of the variation in the data VCM, the remaining eigenvalues are small in comparison.

While the Box-Bartlett and Nagao tests do not directly apply to this situation, we propose two simple modifications. First, we adapt an extension of Nagao's test to the rank deficient case proposed by Ledoit and Wolf (2002). Ledoit and Wolf (2002) have recently considered a situation where the number of variables is large and higher than the sample size. They modify the Nagao (1973) test to this situation and derive asymptotic theory when both the dimension of the VCM and sample size increase to ∞ at the same asymptotic rate. In particular, their test statistic is given by:

$$W = \frac{1}{m} tr\left(\Sigma_M^* - I\right)^2 - \frac{m}{\rho(\Sigma_M^*)}\left[\frac{1}{m} tr\left(\Sigma_M^*\right)\right]^2 + \frac{m}{\rho(\Sigma_M^*)}.$$

Under the null hypothesis and multivariate normality, $\frac{1}{2}\rho(\Sigma_M^*).m.W$ has a Chi-squared distribution with $m(m+1)/2$ degrees of freedom. This extension is based on an asymptotic setup where, as sample size (time periods under study) increases, the set of state variables under comparison is also augmented; this assumption is reasonable in many practical situations.

Second, following Bhattacharjee and Thoenissen (2007), we project the data VCM onto a lower dimensional subspace spanned by the shocks and free predetermined variables driving the model. The usual Box-Bartlett and Nagao tests are then employed for VCM comparisons over this lower dimensional subspace; see Bhattacharjee and Thoenissen (2007) for further details.

Measures based on distance between distributions

One possible limitation of the above testing based approach is that it ignores sampling variation in the model VCM, and therefore its applicability for moment comparison specific to known time periods may be tenuous. An alternative is the approach, indicated in Watson (1993), based on computing the Kullback-Leibler Information Criteria (KLIC) between the distributions given by the data (mean zero, VCM Σ_0) and the model (mean zero, VCM Σ_M) and choosing the best model based on this measure. The KLIC is given by:

$$I\left(\Sigma_0, \Sigma_M\right) = E_{f(.;0,\Sigma_0)} \ln \frac{f(Y;0,\Sigma_M)}{f(Y;0,\Sigma_0)} = \int_{-\infty}^{\infty} \ln \frac{f(y;0,\Sigma_M)}{f(y;0,\Sigma_0)} f(y;0,\Sigma_0)dy, \qquad \text{(C.4)}$$

where $f(.;0;\Sigma)$ denotes the density of the multivariate Gaussian distribution with mean vector zero and VCM Σ, and the expectation is taken with respect to the distribution of the data (mean zero and VCM Σ_0).

While Watson (1993) suggests use of the KLIC in full-rank situations, we extend the method to models with lower number of shocks by using density functions for singular normal distributions. Specifically, we consider the singular value decomposition (SVD) of the simulated model VCM : $\Sigma_M = \lambda_1 e_1 e_1' + \lambda_2 e_2 e_2' + ... + \lambda_p e_p e_p' + 0 e_{p+1} e_{p+1}' + ... +$

$\lambda_m e_m e_m'$, where $p = \rho(\Sigma_M) < m$ is the rank of the model VCM. The density function of this rank-deficient model (mean zero, VCM Σ_M, $\rho(\Sigma_M) = p < \rho(\Sigma_0) = m$) on the subspace spanned by only the p leading eigenvectors is:

$$f\left(\underline{y}_{m\times 1}; 0, \Sigma_M\right) = \frac{1}{(2\pi.\lambda_1.\lambda_2....\lambda_p)^{m/2}}.\exp\left(-\frac{1}{2}\underline{y}'\Sigma_M^-\underline{y}\right),$$

where a generalized inverse (g-inverse) of Σ_M is given by $\Sigma_M^- = 1/\lambda_1.\underline{e_1}.\underline{e_1'}+1/\lambda_2.\underline{e_2}.\underline{e_2'}+\ldots+1/\lambda_p.\underline{e_p}.\underline{e_p'}$. The density of the data VCM (full-rank) is computed in the usual way.

The KLIC approach, however, has a few features that are of importance. First and most importantly, KLIC does not give a strict distance metric, since it is not symmetric in its arguments. One can use symmetric versions of KLIC reported in the literature and besides this may not be a major issue in our case, since we are interested only in finding distances of different models from the data VCM, which is held constant throughout the exercise, and to this extent our approach is consistent. Second, KLIC is of course based on an assumed parametric distribution. We may assume multivariate normality or if appropriate, some other parametric distribution. Third, the KLIC is often difficult to compute particularly in a multi-dimensional case because this involves numerical integration in high dimensions. We address this issue by taking a Monte Carlo or bootstrap approach as follows.

We note that the KLIC is the expected value of difference of log-likelihoods under the two alternative distributions (given by Σ_0 and Σ_M) for samples from the distribution given by the data VCM. Empirically we can either generate a Monte Carlo sample (sample size

N_0^{MC}) with data VCM, or take bootstrap resamples (bootstrap sample size N_0^{BS}) from the actual data, and then calculate the sample mean of log likelihood ratios. By the weak law of large numbers, both these approaches will give consistent estimates of the KLIC. However, the Monte Carlo method will depend more specifically on the validity of the multivariate normality assumption, hence the bootstrap approach may be preferable in practice:

$$\widehat{I}_{Monte\,Carlo}\left(\Sigma_0, \Sigma_M\right) = \frac{1}{N_0^{MC}} \sum_{i=1}^{N_0^{MC}} \ln \frac{f(y_i; 0, \Sigma_M)}{f(y_i; 0, \Sigma_0)},$$

$$\widehat{I}_{Bootstrap}\left(\Sigma_0, \Sigma_M\right) = \frac{1}{N_0^{BS}} \sum_{i=1}^{N_0^{BS}} \ln \frac{f(y_i; 0, \Sigma_M)}{f(y_i; 0, \Sigma_0)}.$$

MATLAB codes for the implementation of the metrics used in this thesis are available from the authors on request.

Appendix D
Walsh (2003) labour search DSGE model

We take a labour search and matching model of Walsh (2003) and calibrate for UK context. A full list of variables and parameters can be found in Table 4.1 and 4.2. Following key equations construct a dynamic stochastic model that can be solved using standard MatLab code.

- The cash-in-advance constraint requires the money growth equal change of nominal demand, which combines real GDP growth and inflation:

$$\hat{\Theta}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{\pi}_t; \tag{D.1}$$

- The money growth in nominal term is taken exogenously as a policy rule:

$$\hat{\Theta}_t = \rho_m \hat{\Theta}_{t-1} + \hat{\phi}_t; \tag{D.2}$$

- The consumption Euler fo the households simply describes the aggregate demand schedule:

$$0 = \mathrm{E}_t \hat{y}_{t+1} - \hat{y}_t - \left(\frac{1}{\sigma}\right) \hat{r}_t + \left(\frac{1}{\sigma}\right) \mathrm{E}_t \hat{\pi}_{t+1}; \tag{D.3}$$

- Retail firms' pricing decisions give the typical new Keynesian Phillips curve. Note the marginal cost is different from Calvo-Yun's paradigm.

$$0 = \beta \mathrm{E}_t \hat{\pi}_{t+1} - \hat{\pi}_t - \kappa \hat{\mu}_t \tag{D.4}$$

- In equilibrium, the endogenous job destruction margin rises to an increase in either real rate or real marginal cost, but falls to an increase in productivity and firm's marginal surplus, therefore links marginal cost to labour market dynamics:

$$\hat{a}_t = \hat{r}_t + \hat{\mu}_t - \left(\frac{\mu R q}{\tilde{a}}\right) \hat{q}_t - \hat{z}_t; \tag{D.5}$$

- Solving an intertemporal optimisation of the firm, there is a present value condition (in terms of firm's surplus) for equilibrium matches:

$$\hat{q}_t = AB\left(e_{H,a}\mathrm{E}_t\hat{a}_{t+1} - \mathrm{E}_t\hat{\mu}_{t+1} - \mathrm{E}_t\hat{r}_{t+1} + \mathrm{E}_t z_{t+1}\right) + \left[\frac{(1-\eta k^w)\beta\varphi(q-A)}{q}\right]\mathrm{E}_t\hat{\varphi}_{t+1}$$
$$- \left(\frac{q+h}{q}\right)(\hat{r}_t - \mathrm{E}_t\hat{\pi}_{t+1}) - \left(\frac{\eta k^w}{1-\eta k^w}\right)\left(\frac{q+h}{q}\right)\hat{k}^w_t + (1-\eta k^w)\beta\varphi\mathrm{E}_t\hat{q}_{t+1} \quad \text{(D.6)}$$

- The firm decides the optimal vacancies to post subject to a cost:

$$\hat{k}^f_t = -\left(\frac{\eta k^w}{1-\eta k^w}\right)\hat{k}^w_t - \left(\frac{q}{q+h}\right)\hat{q}_t; \quad \text{(D.7)}$$

- The probability a vacancy is filled *per se* is the ratio of matches to vacancies. Job matches is a homogenous Cobb-Douglas-type function of searchers and vacancies.

$$\hat{k}^f_t = (\xi - 1)\,\hat{u}_t - (1-\xi)\hat{v}_t; \quad \text{(D.8)}$$

- Successful matches for both searchers and vacancies construct an identity:

$$\hat{v}_t + \hat{k}^f_t = \hat{u}_t + \hat{k}^w_t; \quad \text{(D.9)}$$

- Labour supply is a predetermined variable summing up the surviving employees and new matches:

$$\hat{n}_{t+1} = \varphi\hat{\varphi}_t + \varphi\hat{n}_t + \left(\frac{vk^f}{N}\right)\hat{v}_t + \left(\frac{vk^f}{N}\right)\hat{k}^f_t; \quad \text{(D.10)}$$

- The aggregate supply scheme is simply the total production minus the resources consummed in posting the vacancies.

$$\hat{y}_t = \left(\frac{Q}{Y}\right)(e_{H,a}\hat{a}_t + \hat{n}_t + z_t) - \left(\frac{\gamma V}{Y}\right)\hat{v}_t; \quad \text{(D.11)}$$

- The survival rate is the proportion of employees survive from both exogenous and endogenous job detruction, which is purely pin down by destruction margin:

$$\hat{\varphi}_t = -\left(\frac{\rho^n}{1-\rho^n}\right)e_{F,a}\hat{a}_t; \quad \text{(D.12)}$$

- The identity of total labour supply gives a representation of unemployed workers, i.e., the searchers:

$$\hat{u}_t = -\left(\frac{\varphi N}{u}\right)\hat{n}_t - \left(\frac{\varphi N}{u}\right)\hat{\varphi}_t; \quad \text{(D.13)}$$

Bibliography

[1] Altug, S., Chadha, J. S., Nolan, C., 2003. *Dynamic macroeconomic analysis: Theory and policy in general equilibrium*, Cambridge University Press.

[2] Backus, D. K., Kehoe, P. J., Kydland, F. E., 1995. International business cycles: theory and evidence. In T.F. Cooley (ed.) *Frontiers of Business Cycle Research*, Princeton University Press, Princeton: 331-56.

[3] Backus, D. K., Smith, G. W., 1993. Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, vol. 35: 297-316.

[4] Bartlett, M., 1937. Properties of sufficiency and statistical tests. *Proceedings of the Royal Society of London Series A* 160: 268-82.

[5] Baxter, M., Crucini, M. J., 1995. Business cycles and the asset structure of foreign trade. *International Economic Review*, vol. 36: 821-54.

[6] Baxter, M., Jermann, U. J., 1997. The international diversification puzzle is worse than you think. *American Economic Review*, vol. 87: 170-80.

[7] Bekaert, G., Cho, S., Moreno, A., 2006. New-Keynesian Macroeconomics and the term structure," *CEPR Discussion Papers* 5956.

[8] Benigno, G., Thoenissen, C., 2008. Consumption and real exchange rates with incomplete financial markets and non-traded goods. *Journal of International Money and Finance*, vol. 27: 926-48.

[9] Benigno, P., 2001. Price stability with imperfect financial integration. New York University, *Mimeo*.

[10] Bernanke, B. S., 2007. The financial accelerator and the credit channel. Speech at *The Credit Channel of Monetary Policy in the Twenty-first Century Conference*, Federal Reserve Bank of Atlanta, Atlanta, Georgia, June 15, 2007.

[11] Bernanke, B. S., Mihov, I., 1998. The liquidity effect and long-run neutrality. *Carnegie-Rochester Conference Series on Public Policy* 49, 149-194.

[12] Bhattacharjee, A., Thoenissen, C., 2007. Money and monetary policy in dynamic stochastic general equilibrium models. *Manchester School*, vol. 75(S1): 88-122.

[13] Box, G. E. P., 1949. A general distribution theory for a class of likelihood criteria. *Biometrika,* vol. 36: 317-46.

[14] Calvo, G. A., 1983. Staggered prices in a utility-maximising framework, *Journal of Monetary Economics*, 12(3), pp.983-998.

[15] Canova, F., 2002. Validating monetary DSGE models though VARs. *CEPR Working Papers* No. 3442.

[16] Canova, F., 2005. *Methods for applied macroeconomic research,* Princeton University Press.

[17] Canova, F., De Nicoló G., 2002. Monetary disturbances matter for business fluctuations in the G-7. *Journal of Monetary Economics*, 49, 1131-1159.

[18] Canova, F., Ortega, E., 2000. Testing calibrated general equilibrium models, in R. Mariano, M. Weeks and T. Schuermann (eds.), *Simulation-based inference in econometrics: Methods and applications*, Cambridge University Press.

[19] Canova, F., Ortega, E., 2000. Testing calibrated general equilibrium models. In R. Mariano, T. Shuermann and M. Weeks (eds.) *Inference using Simulation Techniques*, Cambridge University Press.

[20] Canova, F., Sala, Luca., 2009. Back to square one: identification issues in DSGE models, *CEPR Discussion Papers* 7234.

[21] Carlstrom, C. T., Fuerst, T. S., 1995. Interest rate rules vs. money growth rules: A welfare comparison in a cash-in-advance economy. *Journal of Monetary Economics* 36, 247-267.

[22] Chadha, J. S., 2008, Policy rules under monetary and the fiscal theories of the price level', forthcoming in *Cambridge Handbooks in Applied Economics*.

[23] Chadha, J. S., Corrado, L., Holly, S., 2008. Reconnecting money to inflation: the role of the external finance premium. *University of Cambridge Working Paper in Economics* CWPE/0852.

[24] Chadha, J. S., Holly, S., 2006. Macroeconomic models and the yield curve: An assessment of the fit, *Cambridge Working Papers in Economics* 0640, University of Cambridge.

[25] Chadha, J. S., Janssen, N., Nolan, C., 2001. Productivity and preferences in a small open economy. *Manchester School*, vol. 69: 57-80.

[26] Chadha, J. S., Macmillan, P., Nolan, C., 2007. Independence day for the 'Old Lady': A natural experiment on the implications of central bank independence, *Manchester School*, 75(3), 311-327.

[27] Chadha, J. S., Nolan, C., 2002. Supply shocks and the natural rate of interest: an exploration, in L. Mahadeva and P. Sinclair (eds) *Monetary Transmission in Diverse Economies*, Cambridge University Press.

[28] Chari, V. V., Kehoe, P. J., McGrattan, E. R., 2002. Can sticky price models generates volatile and persistent real exchange rate?. *Review of Economic Studies*, vol. 69: 633-63.

[29] Christiano, L. J., Eichenbaum, M., Evans, C. L., 1999. Monetary policy shocks: What have we learned and to what end? In: Taylor, J. B., Woodford, M. (eds.), *Handbook of Macroeconomics*, vol. 1, chapter 2, 65-148.

[30] Christiano, L.J., Eichenbaum, M., 1992. Current Business Cycle Theories and Aggregate Labor Market Fluctuations, *American Economic Review*, 82, 430-450.

[31] Clarida, R., Gali, J., Gertler, M., 1999. The science of monetary policy: A New Keynesian perspective, *Journal of Economic Literature*, 37(4), 1661-1707.

[32] Corsetti, G., Dedola, L., Leduc, S., 2004. International risk-sharing and the transmission of productivity shocks. ECB Working Paper Series No.308. Forthcoming in the *Review of Economic Studies*.

[33] Dedola, L., Neri, S., 2007. What does a technology shock do? A VAR analysis with model-based sign restrictions. *Journal of Monetary Economics* 54, 512-549.

[34] Del Negro, M., Schorfheide, F., 2006. How good is what you've got? DSGE-VAR as a toolkit for evaluating DSGE models, *Economic Review*, Federal Reserve Bank of Atlanta, Q2, 21-37.

[35] den Haan, W. J., 2000. The comovement between real activities and price at different business cycle frequencies. *Journal of Monetary Economics*, vol. 46: 3-30.

[36] den Haan, W. J., Ramey, G., Watson, J., 2000. Job destruction and propagation of shocks, *American Economic Review*, 90(3), pp.482-498.

[37] Devereux, M. B., Engel, C., 2002. Exchange rate pass-through, exchange rate volatility, and exchange rate disconnect. *Journal of Monetary Economics*, vol. 49: 913-40.

[38] Diebold, F., Ohanian, L. and Berkowitz, J., 1998. Dynamics general equilibrium economies: A framework for comparing models and data, *Review of Economic Studies*, 68, 433-451.

[39] Dixon, H., 2007. New Keynesian macroeconomics: Entry for New Palgrave dictionary of economics, 2nd Edition, *Cardiff Economics Working Papers* E2007/3, Cardiff Business School.

[40] Dornbusch, R., 1976. Expectations and exchange rate dynamics. *Journal of Political Economy*, vol. 84: 1161-76.

[41] Engel, C., 2000. Comment on Obstfeld and Rogoff. *NBER Macroeconomics Annual*. National Bureau of Economic Research.

[42] Estrella, A., 2005. Why does the yield curve predict output and inflation?, *Economic Journal*, 115(505), 722-744.

[43] Estrella, A., Mishkin, F. S., 1997. Is there a role for monetary aggregates in the conduct of monetary policy?, *Journal of Monetary Economics*, 40(2), 279-304.

[44] Faust, J., 1998. The robustness of identified VAR conclusions about money. *Carnegie-Rochester Series on Public Policy* 49, 207-244.

[45] Fitzgerald, D., 2003. Terms-of-trade effects, interdependence and cross-country differences in price levels. UC-Santa Cruz, *Mimeo*.

[46] Frankel, J., 1996. How well do foreign exchange markets work: might a Tobin tax help?. In M. ul Haq, I. Kaul and I. Grunberg (eds.) *The Tobin Tax: Coping with Financial Volatility*, Oxford University Press.

[47] Fuhrer, J. C., 2000. Habit formation in consumption and its implications for monetary-policy models, *American Economic Review*, 90(3), 367-390.

[48] Gali, J., Gertler, M., 1999. Inflation dynamics: A structural econometric analysis, *Journal of Monetary Economics*, 44(2), 195-222.

[49] Garganas, N. C., 2006. Macroeconomic management in an environment of aggregate supply shocks – lessons from recent experience, speech on 25 November 2006, *BIS Review* 119/2006.

[50] Geweke, J., 1999. Computational experiments and reality. *Mimeo*.

[51] Goodfriend, M., McCallum, B. T., 2007. Banking and interest rates in monetary policy analysis: A quantitative exploration. *Journal of Monetary Economics* 54, 1480-1507.

[52] Goodhart, C. A. E., 1999. Central bankers and uncertainty. *Proceedings of the British Academy 101: 1998 Lectures and Memoirs*, pp. 229-271, Oxford University Press.

[53] Guerkaynak, R. S., Sack, B., Swanson, E., 2003. The excess sensitivity of long-term interest rates: evidence and implications for macroeconomic models, *Proceedings*, Federal Reserve Bank of San Francisco, March issue.

[54] Hall, R. E., 1997. Macroeconomic fluctuations and the allocation of time. *Journal of Labor Economics*, vol. 15(S1): 223-8.

[55] Hansen, G., 1985. Indivisible labour and the business cycle, *Journal of Monetary Economics*, 16, pp.309-27.

[56] Holland, A., Scott, A., 1998. The determinants of UK business cycles. *Economic Journal,* vol. 108: 1067-92.

[57] Iakova, D., 2007. Flattening of the Phillips curve: Implications for monetary policy, *IMF Working Papers*, WP/07/76.

[58] IMF, 2008. Global Financial Stability Report, October.

[59] Ireland, P. N., 2004. A method for taking models to the data, *Journal of Economic Dynamic Dynamics and Control*, 28, 1205-1226.

[60] Ireland, P., 1996. The role of countercyclical monetary policy. *Journal of Political Economy* 104, 704-723.

[61] Kakkar, V., 2003. The relative price of non-traded goods and sectoral total factor productivity: an empirical investigation. *Review of Economics and Statistics*, vol. 85: 444-52.

[62] Karagedikli, O., Matheson, T., Smith, C., Vahey, S. P., 2009. RBCs and DSGEs: The computational approach to business cycle theory and evidence, forthcoming, *Journal of Economic Surveys*.

[63] Kehoe, P. J., Perri, F., 2002. International business cycles with endogenous incomplete markets. *Econometrica*, vol. 70: 907-28.

[64] King, M. A., 2002. No money, no inflation - the role of money in the economy. *Bank of England Quarterly Bulletin*, Summer, 162-177.

[65] King, R. G., Rebelo, S. T., 2000. Resuscitating real business cycles, in M. Woodford and J. Taylor (eds) *Handbook of Macroeconomics*, North Holland.

[66] King, R. G., Watson, M. W., 1998. The solution of singular linear difference systems under rational expectations. *International Economic Review*, vol. 39: 1015-26.

[67] King, R.G., Watson, M. W., 1998. The Solution of Singular Linear Difference Systems Under Rational Expectations, *International Economic Review*, 39, 1015-1026.

[68] Kollmann, R., 2003. Monetary policy rules in an interdependent world. *CEPR Discussion Paper* No. 4012.

[69] Kozicki, S., Tinsley, P.A., 2008. Term structure transmission of monetary policy, *The North American Journal of Economics and Finance*, 19(1), 71-92.

[70] Krause, M. U., Lubik, T. A., 2003. Real wage dynamics in a monetary business cycle model with search frictions, *Computing in Economics and Finance* 2003-230, Society for Computational Economics.

[71] Kullback, S., Leibler, R. A., 1951. On information and sufficiency. *Annals of Mathematical Statistics*, vol. 22. pp.79-86.

[72] Kydland, F., 2004. Quantitative aggregate theory, Nobel Prize Lecture.

[73] Kydland, F., Prescott, E., 1982. Time to Build and Aggregate Fluctuations, *Econometrica*, 50, 1345-1370.

[74] Labadie, P., 1994. The term structure of interest rates over the business cycle. *Journal of Economic Dynamics and Control* 18, 671-698.

[75] Lane, P. R., Milesi-Ferretti, G. M., 2002. External wealth, the trade balance, and the real exchange rate. *European Economic Review*, vol. 46: 1049-71.

[76] Lastrapes, W. D., McMillin, W. D., 2004. Cross-country variation in the liquidity effect: The role of financial markets. *Economic Journal* 114, 890-915.

[77] Ledoit, O., Wolf, M., 2002. Some hypothesis tests for the covariance matrix when the dimension is large compared to the sample size. *Annals of Statistics*, vol. 30: 1081-102.

[78] Lemke, W., 2008. An affine macro-finance term structure model for the euro area, *The North American Journal of Economics and Finance*, 19(1), 41-69.

[79] Lucas, R. E., 1982. Interest rates and currency prices in a two-country world. *Journal of Monetary Economics* 10, 335-359.

[80] Lucas, R. E., 1987. *Models of business cycles*, John Wiley.

[81] Lucas, R. E., 1996. Nobel lecture: Monetary neutrality. *Journal of Political Economy* 104, 661-682.

[82] McVicar, D., Podivinsky, J., 2003. Unemployment duration before and after new deal, In *Royal Economic Society / Royal Economic Society Annual Conference* 2003.

[83] Millard S., Scott, A., Sensier M., 1997. Labour market over the business cycle: Can theory fit the facts?, *Oxford Review of Economic Policy* 13: 70-92.

[84] Millard, S., 2000. The effects of increased labour market flexibility in the United Kingdom: Theory and Practice', In: Bank of England / *Bank of England working papers*.

[85] Mortensen, D. T., Pissarides, C. A., 1994. Job Creation and Job Destruction in the Theory of Unemployment, *Review of Economic Studies*, 61(3), 397-415.

[86] Nagao, H., 1973. On some test criteria for covariance matrix. *Annals of Statistics*, vol. 1: 700-9.

[87] Nickell, S., Quintini, G., 2002. The recent performance of the UK labour market, *Oxford Review of Economic Policy*, 18: 202-220.

[88] Obstfeld, M., Rogoff, K., 2000. The six major puzzles in international macroeconomics: is there a common cause?. *NBER Macroeconomics Annual*. National Bureau of Economic Research.

[89] OECD, 2004. STAN database.

[90] Ortega, E., 1996. Comparing Evaluation Methodologies for Stochastic Dynamic General Equilibrium Models, Working Paper No. 9820 of the Bank of Spain.

[91] Poole, W., 1970. Optimal choice of monetary policy instruments in a simple stochastic macro model. *Quarterly Journal of Economics* 84, 197-216.

[92] Prescott, E. C., 2004. The transformation of macroeconomic policy and research, Nobel Prize Lecture.

[93] Ravenna, F., Walsh, C. E., 2007. Vacancies, unemployment, and the Phillips curve, Kiel Working Papers 1362, Kiel Institute for the World Economy.

[94] Rotemberg, J. J., Woodford, M., 1997. An optimization-based econometric framework for the evaluation of monetary policy, *NBER Macroeconomics Annual*, 12(1997), 297-346.

[95] Rudebusch, G. D., Wu, T., 2008. A macro-finance model of the term structure, monetary policy and the economy, *Economic Journal,* 118(530), 906-926.

[96] Sarno, L., Taylor, M. P., 2002. *The Economics of Exchange Rates*, Cambridge University Press.

[97] Schmitt-Grohe, S., Uribe, M., 2003. Closing small open economy models. *Journal of International Economics*, vol. 61: 163-85.

[98] Selaive, J. D., Tuesta, V., 2003. Net foreign assets and imperfect pass-through: the consumption-real exchange rate anomaly. Board of Governors of the Federal Reserve System, *International Finance Discussion Papers*, No. 764.

[99] Smets, F., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the Euro area, *Journal of the European Economic Association*, 1(5), 1123-1175.

[100] Sorensen, B. E., Wu, Y-T., Yosha, O., Zhu, Y., 2007. Home bias and international risk sharing: twin puzzles separated at birth. *Journal of International Money and Finance*, vol. 26: 587-605.

[101] Stockman, A. C., Tesar, L. L., 1995. Tastes and technology in a two-country model of the business cycle: explaining international comovements. *American Economic Review*, vol. 85: 168-85.

[102] Straub, R., Tchakarov, I., 2004. Non-fundamental exchange rate volatility and welfare. *ECB Working Paper Series* No. 328.

[103] Taylor, J. B., Williams, J. C., 2009. A black swan in the money market. *American Economic Journal: Macroeconomics*, vol. 1(1), 58-83.

[104] Tesar, L. L., 1993. International risk sharing and non-traded goods. *Journal of International Economics*, vol. 35 (1-2): 69-89.

[105] Tesar, L. L., Werner, I. M., 1995. Home bias and high turnover. *Journal of International Money and Finance*, vol. 14: 467-92.

[106] Tovar, C. E., 2008. DSGE models and central banks, *BIS Working Papers* 258.

[107] Uhlig, H., 1994. What macroeconomists should know about unit roots: a Bayesian perspective. *Econometric Theory* 10, 645–671.

[108] Uhlig, H., 2001. Did the Fed surprise the markets in 2001? A case study for VARs with sign restrictions. *CESifo Working Paper Series*, No. 629.

[109] Uhlig, H., 2005. What are the effects of monetary policy on output? Results from an agnostic identification procedure. *Journal of Monetary Economics* 52, 381-419.

[110] Vasicek, O., 1977. An equilibrium characterisation of the term structure, *Journal of Financial Economics* 5, 177–188.

[111] Wachter, J. A., 2006. A consumption-based model of the term structure of interest rates, *Journal of Financial Economics*, 79(2), 365-399.

[112] Walsh, C. E., 2003. Labor market search and monetary shocks, in S. Altug, J. S. Chadha and C. Nolan (eds) *Dynamic macroeconomic analysis: Theory and policy in general equilibrium*, Cambridge University Press.

[113] Walsh, C. E., 2003. *Monetary Theory and Policy*, MIT Press.

[114] Walsh, C. E., 2005. Labor market search, sticky prices, and interest rate policies, *Review of Economic Dynamics*, 8(4), pp.829-849.

[115] Walsh, C., Ravenna, F., 2006. Optimal monetary policy with the cost channel, *Journal of Monetary Economics*, Vol. 53(2), pp.199-216.

[116] Watson, M. W., 1993. Measures of fit for calibrated models. *Journal of Political Economy*, vol. 101: 1011-41.

[117] Woodford, M., 2003. *Interest and Prices*, Princeton.

[118] Yun, T., 1996. Nominal price rigidity, money supply endogeneity and the business cycle, *Journal of Monetary Economics*, 37(2), pp.345-370.