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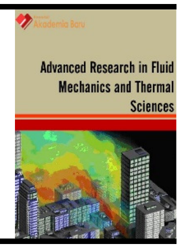

Experimental Evaluation of Kolmogorov's -5/3 and 2/3 Power Laws in the Developing Turbulent Round Jet

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ABSTRACT

The current work investigates the validity of two cornerstone results of the Kolmogorov K41 theory of turbulence in terms of the typical power law representations viz. the -5/3 law for turbulence spectra and the 2/3 law for second order structure functions. The developing region of the jet has been chosen since it is an equilibrium flow once fully developed (but not necessarily in the development phase), it becomes fully developed over lengths that are practical on a laboratory scale and it is a high-intensity flow with accessibly resolvable scales in time and space. The developing region of the jet is thus the perfect testbed for these investigations, which can herein be accurately mapped using our in-house laser Doppler anemometry (LDA) system. The high turbulence intensity and high shear flow is challenging from a measurement technical perspective, which is perhaps why this flow is so underexplored. This software-driven LDA system was developed specifically to optimize measurement of high shear and high turbulence intensities accurately in challenging flows such as the turbulent round jet in air. The jet was investigated experimentally both in the developing (non-equilibrium) and in the developed regions (equilibrium). Velocity static moments at each point are first presented to show the time averaged flow behavior while the spatial energy spectra and second order structure functions are computed to evaluate the power laws postulated by Kolmogorov. Measurements from both the developed and from the developing parts of the jet are presented to show validity of the measurement technique and unveil the actual spectral shapes in the developing non-equilibrium region, respectively.

1. Introduction

Turbulence has for a long time been regarded as the last remaining unsolved problem of classical physics according to Richard Feynman. The postulation of Kolmogorov's universality assumptions which are also known as the K41 theory, resulted in two principal results viz. the 2/3 law for second

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order structure functions and the -5/3 law for turbulence spectra [1]. Kolmogorov's 2/3 law states that "in a turbulent flow at very high Reynolds number, the mean-square velocity increments $<(\delta u(l))^2>$ between two points separated by a distance ℓ behave approximately as the 2/3 power of the distance" [2]. It results from the derivation of the second-order structure function, based on Kolmogorov's classical assumption of turbulence dynamics [3-6]

$$S_2(l) = C\varepsilon^{2/3} l^{2/3} \quad (1)$$

where C is a universal constant (to be determined from experiments), ε is dissipation rate and ℓ is the length scale. Obukhov [2] later derived the turbulent kinetic energy spectrum from (1)

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3} \quad (2)$$

where C_k is the Kolmogorov constant (to be determined from experiments) and k= $1/\ell$ is the wave number.

The theory stated by Kolmogorov [3-6] is formally based on three main hypotheses valid for sufficiently high Reynolds number flows; (1) local homogeneity/isotropy for $r<<L$, (2) where the scales for which $r<<L$ are uniquely determined by viscosity, ν, and dissipation, ε, (3) in addition to a range, $L>>r>>\eta$, that is uniquely characterized by ε, but not ν. L is the length scale characterizing the large scale turbulent structures. *Implicit* to all of these hypotheses is the "local equilibrium" hypothesis [7], which is based on the analogy of small scale turbulence to molecules in a thermodynamic system, wherein the molecules are decoupled from the macroscopic state and at a state of equilibrium. This is also by Kolmogorov assumed to be valid for the (significantly larger) small scales of turbulence.

The stationary round jet has been chosen for our investigations since it is known to be nearly at equilibrium, at least in the fully developed region, where it has also been proven to have good agreement with Kolmogorov's theory as the very first flow displaying the -5/3 slope of the energy spectrum [8-10]. However, we have specifically chosen to conduct our investigations in the developing region of the round turbulent jet, where the flow transitions from non-equilibrium (developing region) to an equilibrium state (developed region) in view of the great theoretical and practical interest in this flow regime.

This flow is challenging from a measurement technical perspective, due to high turbulence intensities and high shear, while at the same time developing (from non-equilibrium to equilibrium) over a relatively short distance that makes it practical and manageable to work with on a typical lab scale. This has typically not been the case with the classical decaying grid turbulence experiments, with the exception of the fractal grid wake experiments of Vassilicos *et al.* (see [11] for a comprehensive summary of their work).

The Laser Doppler Anemometer (LDA) has been specifically chosen for our investigation due to its capability to resolve velocity measurement without disturbing the flow [12] and for its ability to accurately distinguish the spatial velocity components from each other, even at high turbulence intensities [13]. On top of that, our system is also driven by an in-house developed software, which has been validated against corresponding measurements using a commercial LDA system [14] and spatial structure functions based on data measured using a side scattering LDA configuration [15].

Having our own fully functional state-of-the-art LDA system in-house, we can accurately test these laws in challenging high shear and high intensity turbulent flows. This may provide extended information of the range of validity of the underlying assumptions that Kolmogorov stated (listed above) and specifically of the local equilibrium hypothesis.

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2. Methodology

The jet, which is a replica of the one used by [16], consisted of a settling chamber with an inner and an outer nozzle designed to condition the flow to follow as closely as possible a laminar top-hat profile at the jet exit with diameter D=10 mm. A laser with a wavelength of 532 nm, was split into a pair of separate beams and directed through Bragg cells. The experimental setup, depicted in Fig. 1, was enclosed in a large tent of dimensions 2.5 x 3 x 10 m, with the jet positioned at the back of the enclosure to ensure a sufficient distance downstream for the measurement. The laser power was set to its maximum intensity, 1.29 W, with an amplifying current setting of 60 μA. The jet input pressure was set to 1 bar, corresponding to a jet exit velocity ≈ 43 m/s and Re ≈ 29000. The seeding pressure of 1.4 bar was chosen to obtain the optimum number of bursts in terms of maximized data rate. A frequency shift of 3 MHz was set on the Bragg cell and a total number of 400 records was taken at each measurement point. All the above-mentioned parameters were used throughout the whole measurement sequence, but with varying sampling rate (MHz) and record length (s) according to the varying conditions across the jet flow. The total number of samples of the burst signal were kept the same though i.e. 25 MS. Measurements of the axial component of velocities were acquired at several radial points along three downstream positions, x/D=10, x/D=15 and x/D=30 as illustrated in Fig. 2.

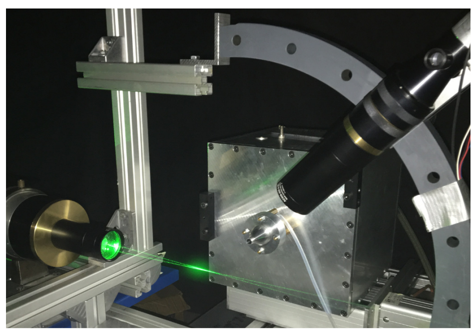

Fig. 1. Experiment setup showing jet exit, with the detector (lens focal length, f=200mm) positioned in 45° forward scattering

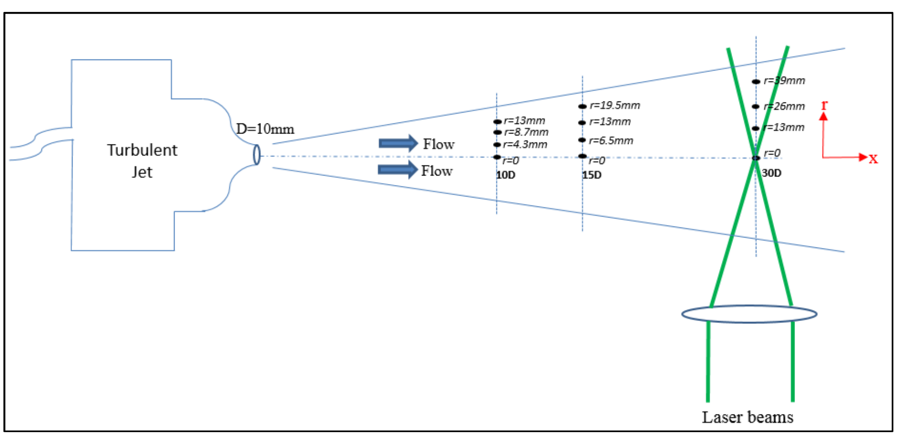

Fig. 2. Top view of the setup showing the measurement point distribution in the downstream x-direction and in the radial, r-direction

3. Results and Discussions

3.1 Velocity static moments

The raw signals obtained from the measurements were processed using our in-house software which provides the arrival time, residence time and velocities of each particle. The mean and variance of the velocity were calculated using residence time-weighting [16] to provide non-biased statistics of the LDA burst signal. Figures 3 and 4 show the radial profiles of mean velocity and variance at different downstream positions. As expected from theory and also some experimental work, e.g. [13,15], the highest velocity is spotted at the centreline for each mean velocity profile and the profiles seem to follow the expected nearly Gaussian shape, spread out and tapered in the downstream direction.

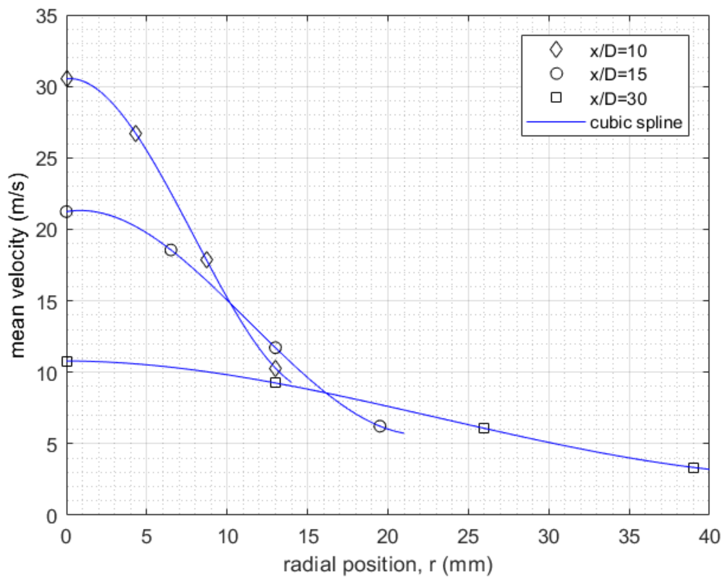

Fig. 3. Radial profiles of mean velocity at *x/D* = 10, 15 and 30 with cubic spline data interpolation

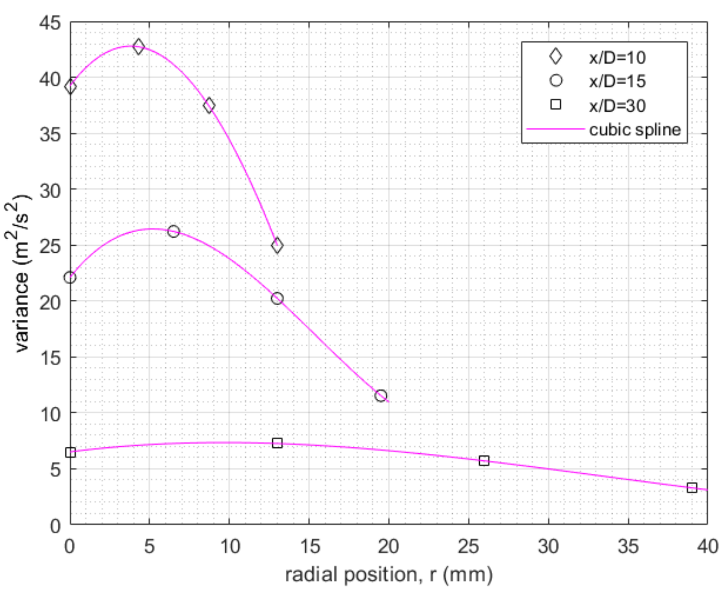

Fig. 4. Radial profiles of variance at *x/D* = 10, 15 and 30 with cubic spline data interpolation

3.2 Spatial turbulence kinetic energy spectra

Figures 5 to 7 show turbulent kinetic energy spectra as a function of wavenumber for different off-axis positions in the wave number (spatial) domain. The mapping from the temporal to the spatial domain was done using the instantaneous velocity magnitude (instead of the mean streamwise velocity as proposed by Taylor), which has been shown to yield a correct mapping between space and time using the so-called convection record [15]. Note that each spectrum was deliberately normalized to 1 in the low frequency asymptote for a clearer comparison in terms of its shape and slope with respect to Kolmogorov's -5/3 law. In the fully developed region at 30D, all spectra show a convincing collapse indicating that the turbulent kinetic energy is distributed nearly equally across the scales, independently of radial position from the jet centreline. It shows that the second order moments of turbulence in this fully developed region have finally reached a state of self-similarity. The clear -5/3 slope across a significant range, also observed in [14, 17-19], indicates the possibility of local turbulence equilibrium in the inertial subrange at this position in the flow, in accordance with the central, yet implicit assumption of Kolmogorov's local equilibrium of the small scales [7].

Meanwhile, the shape of the spectra varies with radial position in the developing region viz. at 10D and 15D, showing that the spatial structure of the velocity varies with different radial positions in the developing region of the jet. Higher energy is observed at lower wave numbers and spectra are shifted to lower wavenumbers, which explains that the smallest scales are growing away from the jet exit. The tendency to follow a -5/3 slope at higher wave number is the lowest at 10D, indicating that the local equilibrium assumption is not valid in this developing region of the jet.

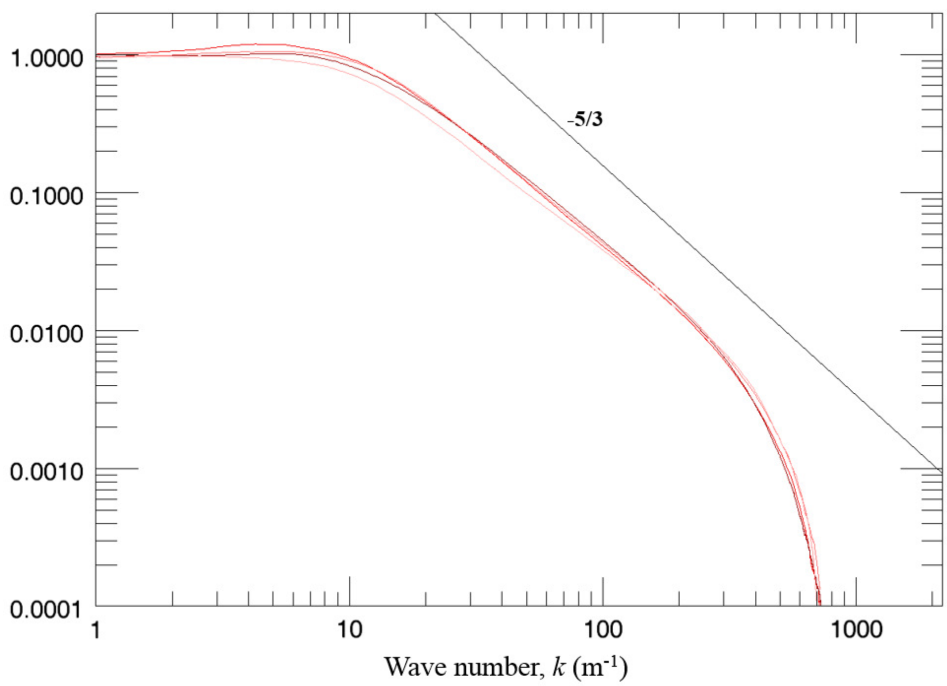

Fig. 5. Spatial turbulence kinetic energy spectra at 30D downstream. From *heavy red to light red*: off-axis position 0, 13, 26, 39 mm

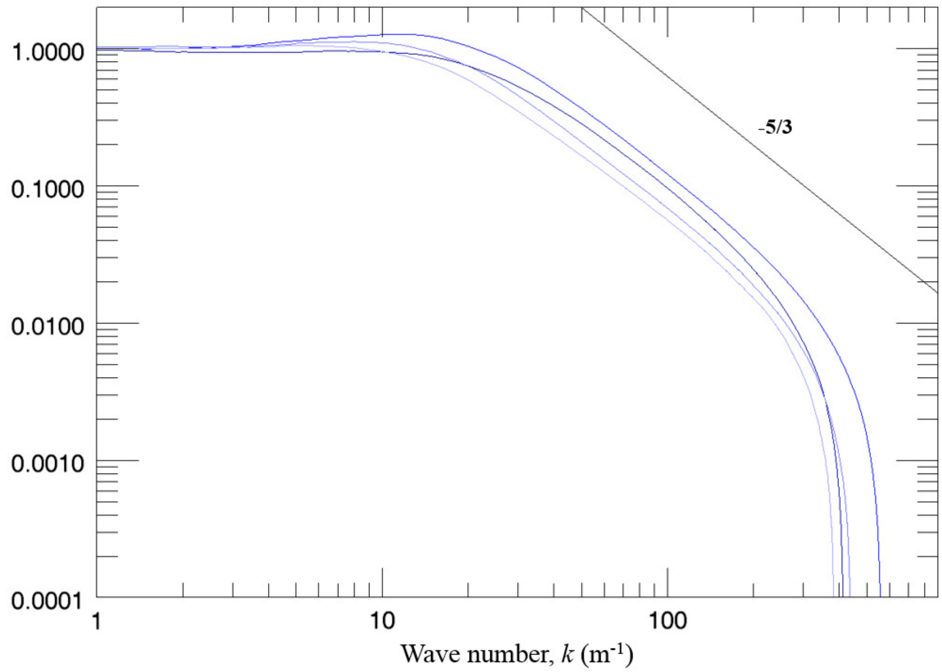

Fig. 6. Spatial turbulence kinetic energy spectra at 15D downstream. From *heavy blue to light blue*: off-axis position 0, 6.5, 13, 19.5 mm

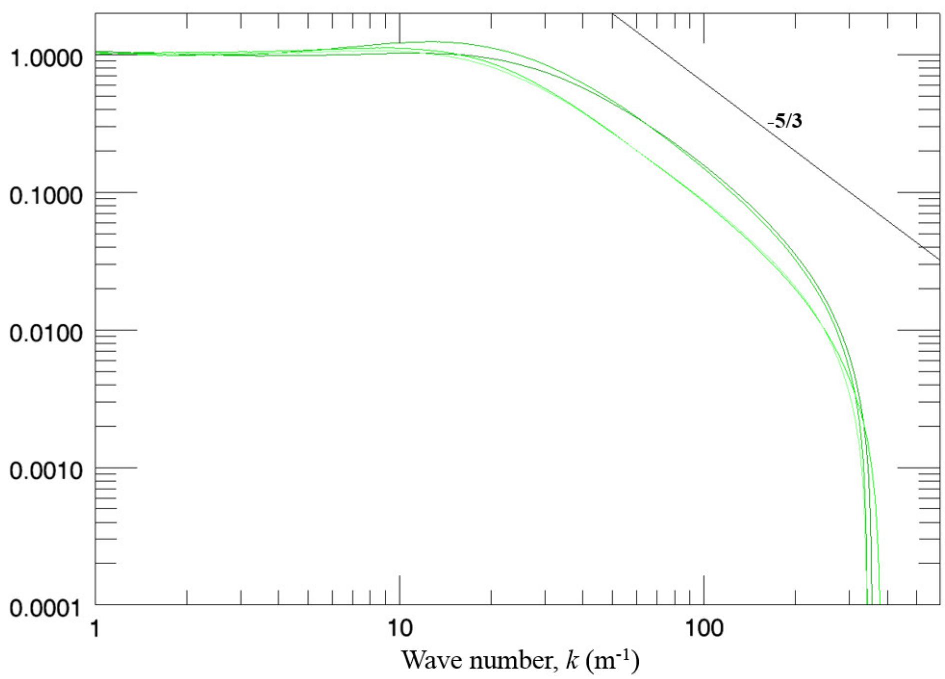

Fig. 7. Spatial turbulence kinetic energy spectra at 10D downstream. From *heavy green to light green*: off-axis position 0, 4.3, 8.7, 13 mm

3.3 Spatial second order structure function

Figure 8 shows the spatial second order structure functions across the range of measuring points. Also here, the temporal-to-spatial mapping has been done based on the convection record principle where the instantaneous velocity magnitude has been employed [15], rather than the average streamwise velocity as proposed by Taylor. Each curve demonstrates that the large scales contain the most energy. The large scales are seen to grow as they move downstream. More large-scale activity is also relatively noticed in the outer part of the jet compared to in the centreline. The curves in the equilibrium region (30D) have a greater tendency to follow the expected 2/3 slope. Meanwhile, in the non-equilibrium region, the curves show slopes deviating from the 2/3 expected from the theory of Kolmogorov. This is another indication of the development of the turbulent scales in this region. The lower slope at 10D and 15D show that the large velocity increments have not yet been produced by the cascade process.

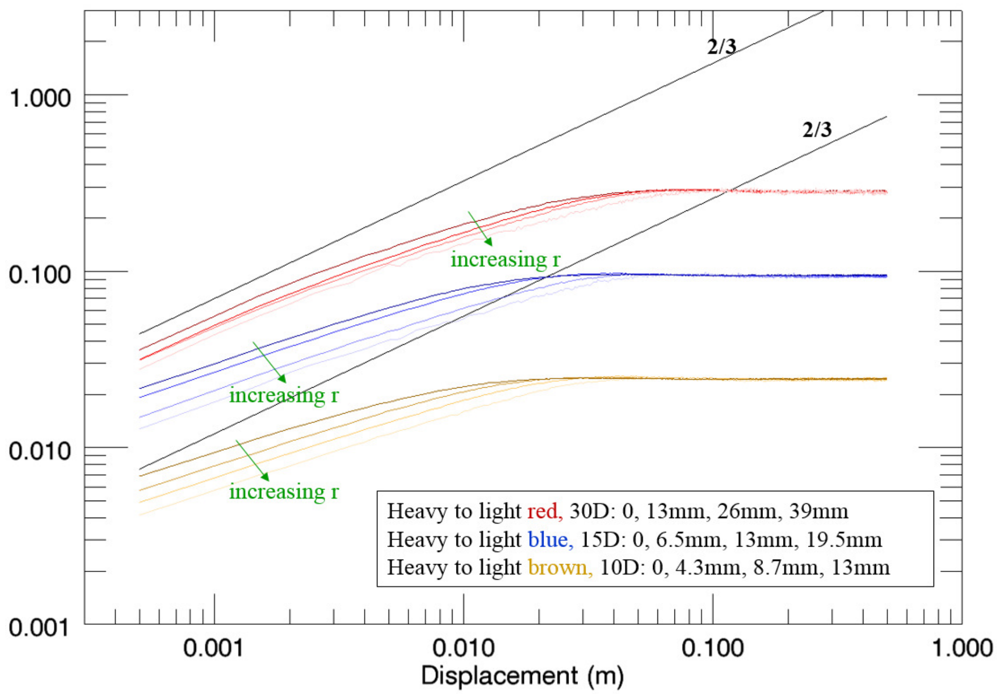

Fig. 8. Second order spatial structure functions. Curves of the same downstream position were shifted vertically for a clear comparison

4. Conclusions

The in-house LDA system was proven to successfully produce experimental results validating the Kolmogorov -5/3 and 2/3 laws in the fully developed region, while revealing interesting non-equilibrium features in the developing part of the jet flow. A more detailed mapping of the developing (non-equilibrium) region could provide further valuable insight into the limitations of the Kolmogorov -5/3 and 2/3 power laws and the underlying theory. This may not only help evaluate in which situations the restrictive assumptions suggested by Kolmogorov need to be relaxed but could also provide direct knowledge of the time scales required for the cascade process to develop into an equilibrium state.

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