Control of a flexible spacecraft using discrete IDA-PBC design ⋆

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Abstract: This paper has two objectives, first synthesize a discrete-time IDA-PBC for an underactuated port-Hamiltonian system, and second stabilize the angular position of an experimental testbed used in aerospace engineering. Based on the energetic integrator, the discrete-time methodology that exactly preserves the passivity property is presented for a linear Hamiltonian system with physical damping. A stability condition is given when taking the desired Hamiltonian as Lyapunov candidate function. The model of the spacecraft is composed of a rigid central body actuated by a torque motor around the vertical axis with two flexible appendages and a local mass at the tip of each appendage. Experiments are carried out to assess the validity of the more theoretical design methodology. The results show that the performances of our design results are better compared to an emulation controller obtained by sample and hold or Tustin transformation of the continuous-time controller.

1. INTRODUCTION

In recent years, the control of flexible systems has been given widespread attention in the literature due to practical applications in several domains such as flexible robot arms, helicopter rotors, satellite solar array and like systems. Optimal controllers for linear and nonlinear models have been presented in [Ben-Ascher et al., 1992, Karray et al., 1997], control laws based on linearization and nonlinear inversion have been designed in [Singh, 1988]. Lyapunov stability and dissipativity theory have been used to design controllers for the maneuver of flexible spacecraft [Junkins and Bang, 1993]. Recently, the design of a composite adaptive control system using the backstepping technique has also been considered [Ki-Seok and Youdan, 2003, Liu et al., 2012]. All controllers are mostly developed in continuous-time. However, they are implemented digitally and the effects of sampling may have a negative impact on the performance of the closed-loop system. This paper contributes to the synthesis of a new discrete-time controller based on the passivity theory called IDA-PBC (Interconnection Damping Assignment-Passivity Based Control) to control the orientation of the main body of the spacecraft.

In continuous time, this controller was first published in [Ortega et al., 2002a]. This approach is decomposed in two steps: first the energy shaping control u_{es} is designed to assign the desired energy function as the total energy of the system; second, the damping injection control u_{di} is designed to achieve asymptotic stability at a desired equilibrium point, which corresponds to an isolated and strict minimum of the desired energy function. In the case of underactuated mechanical systems [Ortega et al., 2002a], this approach leads to a partial differential equation (PDE) called *matching conditions* to be solved in order to establish the controller. Many useful works treat these problems assuming that the open loop is conservative i.e. no physical damping (e.g., friction) is taken into account. In [Gomez-Estern and van der Schaft, 2004], a necessary and sufficient stability condition is proposed when the physical dissipation in not neglected.

In discrete time, there are, to the authors' best knowledge, fewer works on the synthesis of the IDA-PBC controllers. In [Laila and Astolfi, 2005], the authors have been treating the case of separable Hamiltonian systems through forward Euler scheme. Then, they generalized it to nonseparable and under-actuated Hamiltonian systems, see [Laila and Astolfi, 2006]. It is mentioned that "the energy is not conserved by the forward Euler scheme, but the Hamiltonian structure of the plant can be better preserved". In [Gören-Sümer and Yalçin, 2011], a discrete gradient is used to construct discrete-time control for under-actuated Hamiltonian systems. These results contribute to the discrete-time IDA-PBC design and improve the performances of strictly emulated strategies but are strongly limited by the fact that the passivity property is preserved for small integration timestep only. Recently, we have proposed in [Aoues et al., 2013b] a discrete-time controller for fully-actuated Hamiltonian systems that exactly preserves passivity, using a Greenspan gradient proposed in [Greenspan, 1974]. All these references deal with conservative mechanical systems (no physical damping in the open loop).

The purpose of the paper is twofold. Firstly, to propose a new discrete-time IDA-PBC for damped linear Hamiltonian systems while exactly preserving the passivity property. Then, we provide a stability condition for closed-loop system using the desired energy as Lyapunov function. Secondly, we synthesize a discrete control law of a flexible spacecraft to reach the desired angular position of the hub body (see Figure 1). The performances of the proposed controller are compared with those of the emulation controller.

The remaining part of the paper is organized as follows. After some background on the linear PHS and the methodology of the

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continuous-time IDA-PBC in Section 2, Section 3 describes the discrete-time IDA-PBC design. The modeling and control of flexible spacecraft system is detailed in Section 4. Experimental tests and some comparisons for different integrations timesteps are given in Section 5. Finally, a conclusion and some perspectives are presented in Section 6.

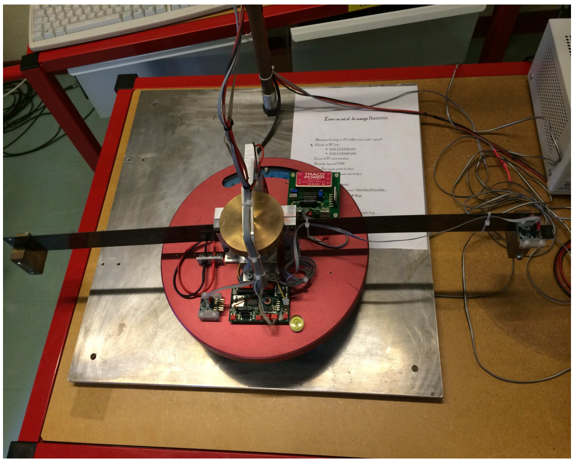

Fig. 1. Picture of a one degree of freedom flexible spacecraft

2. HAMILTONIAN FRAMEWORK & IDA-PBC DESIGN

In this section, we briefly recall some essential concepts studied in this paper. Firstly, we recall some basic properties of the linear port-Hamiltonian framework [Maschke and van der Schaft, 1992, van der Schaft, 1999]. In particular, we are interested into the dissipative equality, which we aim at preserving at the discrete level. Secondly, we introduce the methodology of the control law synthesis called IDA-PBC in continuous time.

2.1 Port-Hamiltonian Systems

We are interested in linear systems with specific structure called *port-Hamiltonian systems* (denoted PHS throughout the paper). Here, we are restricted to the canonical PHS with quadratic energy given by

$$H(q,p) = T(p) + V(q) = \frac{1}{2}p^T M^{-1} p + \frac{1}{2} q^T K q \qquad (1)$$

with $T(p)$ and $V(q)$ are the kinetic and the potential energy, respectively. $M = M^T > 0$ is the inertia matrix and $K = K^T > 0$ is the stiffness matrix.

The dynamics of the continuous-time PHS with physical damping can be written as [van der Schaft, 1999]

$$\Sigma : \begin{cases} \begin{bmatrix} \dot q \\ \dot p \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -R \end{bmatrix} \begin{bmatrix} Kq \\ M^{-1}p \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u(t) \\ y(t) = G^T M^{-1} p \end{cases} \qquad (2)$$

where $q \in \mathbb{R}^n$ is the generalized displacement, $p = M\dot q \in \mathbb{R}^n$ the generalized momentum, $R \in \mathbb{R}^{n\times n}$ is the dissipation matrix, $G \in \mathbb{R}^{n\times m}$ is the input force matrix, with Gu denoting the generalized forces resulting from the control inputs $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^m$ is the conjugate port-output of the system.

The time derivative of H along the trajectories of (2) is

$$\frac{\mathrm{d}}{\mathrm{d}t} H(q,p) = -p^T M^{-1} R M^{-1} p + y^T u \leq y^T u \qquad (3)$$

The first term of (3) design the dissipated energy of the system (2) and the product $y^T u$ design the power exchange going through the port variables u and y. From (3), the system is said to be passive. If $R = 0$, the system is conservative (*i.e.* lossless and passive).

Furthermore, for control issues, two essentials cases have to be distinguished with respect to the length m of the input vector field. The PHS (2) is said to be fully actuated when $m = n$, and underactuated when $m < n$ which is known to be the more difficult case. The details of this approaches are explained in the following section.

2.2 Continuous-time IDA-PBC design

We are concerned with control law synthesis following the IDA-PBC design introduced in [Ortega et al., 2002b]. In the context of linear systems given by (2), IDA-PBC approach aims at designing the desired closed-loop energy as

$$H_d(q,p) = T_d(p) + V_d(q) = \frac{1}{2}p^T M_d^{-1} p + \frac{1}{2} q^T K_d q \qquad (4)$$

together with the closed-loop dynamics

$$\begin{bmatrix} \dot q \\ \dot p \end{bmatrix} = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & -G K_{di} G^T - R M^{-1} M_d \end{bmatrix} \begin{bmatrix} K_d q \\ M_d^{-1} p \end{bmatrix} \qquad (5)$$

with $M_d, K_d \in \mathbb{R}^{n\times n}$ are the new mass and stiffness matrices and $K_{di} \in \mathbb{R}^{m\times m}$ is the damping gain matrix.

The control input $u = u_{es} + u_{di}$ is obtained by solving the model matching (2) = (5)

$$\begin{aligned} &\begin{bmatrix} 0 & I_n \\ -I_n & -R \end{bmatrix} \begin{bmatrix} Kq \\ M^{-1}p \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u \\ &\quad = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & -G K_{di} G^T - R M^{-1} M_d \end{bmatrix} \begin{bmatrix} K_d q \\ M_d^{-1} p \end{bmatrix} \end{aligned} \qquad (6)$$

The first row of (6) is directly satisfied, and the second row can be written as

$$G\, u = \left\{ Kq - M_d M^{-1} K_d q - G K_{di} G^T M_d^{-1} p \right\} \qquad (7)$$

If G is invertible (fully actuated case) the control law is easily computed by left multiplying both sides by G^{-1} and, if G is not invertible (underactuated case $m < n$), the following set of constraint equations must be satisfied

$$G^{\perp} \left\{ Kq - M_d M^{-1} K_d q - G K_{di} G^T M_d^{-1} p \right\} = 0 \qquad (8)$$

with $G^{\perp}$ is a full rank left annihilator of G, i.e $G^{\perp} G = 0$. In this paper, G and $G^{\perp}$ have the following form

$$G = \begin{bmatrix} I_m \\ 0_{(n-m)\times m} \end{bmatrix} \quad \text{and} \quad G^{\perp} = \begin{bmatrix} 0_{(n-m)\times m} & I_{n-m} \end{bmatrix}$$

The equation (8) must be solvable with respect to M_d and K_d in order to derive the controller.

Back to (7), the first step, called *energy shaping*, is defined by

$$u_{es} = G^{\dagger} \left\{ Kq - M_d M^{-1} K_d q \right\} \qquad (9)$$

$G^{\dagger}$ is the Moore-Penrose pseudo-inverse of the matrix G, that is, $G^{\dagger} = \left(G^T G\right)^{-1} G^T$. The second step, called *damping injection*, consists of adding friction to the system in order to achieve asymptotic stabilization of the desired equilibrium. This controller is given by a negative output feedback,

$$u_{di} = -G^{\dagger} \left\{ G K_{di} G^T M_d^{-1} p \right\} = -K_{di} G^T M_d^{-1} p, \quad K_{di} > 0. \qquad (10)$$

The complete control law writes $u = u_{es} + u_{di}$.

In the presence of the natural damping ($R \neq 0$), it is important to note that the asymptotic stability is ensured if the following condition is satisfied

$$C + D > 0 \qquad (11)$$

with

$$C := \frac{1}{2}\left(RM^{-1}M_d + M_dM^{-1}R\right) \quad \text{and} \quad D := GK_{di}G^T$$

Finally, taking H_d as Lyapunov function, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}H_d(q,p) = -p^T M_d^{-1}(C+D)M_d^{-1}p \qquad (12)$$

Then, $\dot{H}_d < 0$ if the condition (11) is guaranteed.

3. DISCRETE-TIME IDA-PBC DESIGN

In this section, we present our main contribution : discrete-time IDA-PBC design for linear Hamiltonian systems. Firstly, we present the discrete-time PHS based on the energetic scheme (midpoint). Secondly, we use the concept of passivity to design the discrete-time IDA-PBC method for linear Hamiltonian systems with physical damping. Finally, we validate the controller on the flexible spacecraft.

3.1 Discrete-time linear PHS

Using the midpoint scheme, the discrete gradient of the kinetic and potential energy is given by

$$\overline{\nabla}_q V = K\frac{q_{n+1}+q_n}{2}; \quad \overline{\nabla}_p T = M^{-1}\frac{p_{n+1}+p_n}{2} \qquad (13)$$

where $x_n = n\Delta t$ with Δt present the integration timestep. The derivatives of the state variables in (2) are approximated by a forward Euler scheme as follows,

$$\dot{q} = \frac{q_{n+1}-q_n}{\Delta t}; \quad \dot{p} = \frac{p_{n+1}-p_n}{\Delta t} \qquad (14)$$

Based on (13) and (14), the discrete-time description of the continuous system (2) is given by the following definition.

Definition 3.1. A (canonical) discrete-time linear PHS with physical damping is defined by

$$\begin{cases} \begin{bmatrix} \dfrac{q_{n+1}-q_n}{\Delta t} \\ \dfrac{p_{n+1}-p_n}{\Delta t} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -R \end{bmatrix} \begin{bmatrix} K\dfrac{q_{n+1}+q_n}{2} \\ M^{-1}\dfrac{p_{n+1}+p_n}{2} \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u_n \\ \qquad y_n = G^T M^{-1}\dfrac{p_{n+1}+p_n}{2} \left(\equiv G^T \dfrac{q_{n+1}-q_n}{\Delta t}\right) \end{cases} \qquad (15)$$

The explicit form of (15) can be directly derived. But in this study, the choice fell on an implicit form which preserves the Hamiltonian structure, to allow the usage of all advantages of the Hamiltonian formalism. For simulation purposes, the explicit form of (15) is computed off line (i.e. q_{n+1} and p_{n+1} are given as linear functions of q_n, p_n and u_n). It enables fast on-line computations.

Remark 3.1. Note that (13) together with (14) amounts Crank-Nicholson numerical scheme, which is known to be unconditionally stable and second order accurate.

Proposition 1. The discrete linear PHS (15) exactly preserves (independently of Δt) the passivity property w.r.t the same storage function H and the same dissipation R.

Proof. The discrete energy balance equation for the system (15) yields

$$\Delta H_n = [T(p_{n+1}) + V(q_{n+1})] - [T(p_n) + V(q_n)] \qquad (16)$$

Rearranging the terms, it remains to see that

$$\begin{aligned} T(p_{n+1}) - T(p_n) &= \frac{1}{2}(p_{n+1}-p_n)^T M^{-1}(p_{n+1}+p_n) \\ &\overset{(15)}{=} (p_{n+1}-p_n)^T \frac{q_{n+1}-q_n}{\Delta t} \end{aligned}$$

and

$$\begin{aligned} V(q_{n+1}) - V(q_n) &= \frac{1}{2}(q_{n+1}-q_n)^T K (q_{n+1}+q_n) \\ &\overset{(15)}{=} -(q_{n+1}-q_n)^T \frac{p_{n+1}-p_n}{\Delta t} \\ &\quad + (q_{n+1}-q_n)^T \left\{-RM^{-1}\frac{p_{n+1}+p_n}{2} + Gu_n\right\} \end{aligned}$$

Then,

$$\Delta H_n = -\Delta t\left(\frac{p_{n+1}+p_n}{2}\right)^T M^{-1}RM^{-1}\left(\frac{p_{n+1}+p_n}{2}\right) + \Delta t\, y_n^T u_n \qquad (17)$$

The previous relation (17) translates the fact that the stored energy on the interval $[n, n+1]$ equals the energy supplied through the port variables (positively or negatively) decreased by the dissipated energy during the integration timestep Δt. The conservative case ($R = 0$) is detailed in [Aoues et al., 2013a].

3.2 Discrete-time IDA-PBC

A discrete-time IDA-PBC plays the same role as the controller designed in continuous-time. The first step, called *energy shaping*, fixes the desired energy H_d which has a strict local minimum at the desired equilibrium. The second step, called *damping injection* designed to achieve asymptotic stability at the desired equilibrium point q^*, which is an isolated and strict minimum of the desired energy function.

According to the open-loop dynamics given by (15), the desired closed loop is considered as

$$\begin{bmatrix} \dfrac{q_{n+1}-q_n}{\Delta t} \\ \dfrac{p_{n+1}-p_n}{\Delta t} \end{bmatrix} = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_dM^{-1} & -(C+D) \end{bmatrix} \begin{bmatrix} K_d\dfrac{q_{n+1}+q_n}{2} \\ M_d^{-1}\dfrac{p_{n+1}+p_n}{2} \end{bmatrix} \qquad (18)$$

The discrete-time controller can be easily computed by a similar line as that in the continuous time, by solving the discrete model matching (15)=(18), i.e.

$$\begin{aligned} &\begin{bmatrix} 0 & I_n \\ -I_n & -R \end{bmatrix} \begin{bmatrix} K\dfrac{q_{n+1}+q_n}{2} \\ M^{-1}\dfrac{p_{n+1}+p_n}{2} \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u_n \\ &\quad = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_dM^{-1} & -GK_{di}G^T - RM^{-1}M_d \end{bmatrix} \begin{bmatrix} K_d\dfrac{q_{n+1}+q_n}{2} \\ M_d^{-1}\dfrac{p_{n+1}+p_n}{2} \end{bmatrix} \end{aligned}$$

We assume $G^\perp$ a full rank left annihilator of G and satisfying $G^\perp G = 0$. From the second row of the previous relation,

$$G^\perp \left\{ K\frac{q_{n+1}+q_n}{2} - M_dM^{-1}K_d\frac{q_{n+1}+q_n}{2} - GK_{di}G^TM_d^{-1}\frac{p_{n+1}+p_n}{2} \right\} = 0 \qquad (19)$$

If (19) is solvable, the energy shaping controller in discrete time is derived as,

$$(u_{es})_n = G^\dagger \left\{ K\frac{q_{n+1}+q_n}{2} - M_dM^{-1}K_d\frac{q_{n+1}+q_n}{2} \right\} \qquad (20)$$

It is important to note that the closed-loop dynamics when applying $(u_{es})_n$ is also a Hamiltonian system, and hence the discrete passivity of the new dynamics is also preserved.

The discrete damping injection controller $(u_{di})_n$ which guarantees the asymptotic stability of the system is obtained as

$$\begin{aligned}(u_{di})_n &= -G^{\dagger}\left\{GK_{di}G^T M_d^{-1}\frac{p_{n+1}+p_n}{2}\right\} \\ &= -K_{di}G^T M_d^{-1}\frac{p_{n+1}+p_n}{2}\end{aligned} \quad (21)$$

with damping gain $K_{di} = K_{di}^T > 0$. The control law u_n must be implemented using the explicit form derived from (15).

However, the approach proposed here exactly preserves all properties of the continuous-time model. Thus, the desired Hamiltonian H_d is a Lyapunov function that allows to prove the stability of the closed-loop system (18).

Proposition 2. Consider the discrete dynamics (15) and the desired closed-loop dynamics (18), where V_d has an isolated minimum at q^*. Then $(q^*, 0)$ is an asymptotically stable equilibrium of the closed-loop system with $u_n = (u_{es})_n + (u_{di})_n$ given by (20) and (21) if and only if this condition is satisfied,

$$(C+D) := \frac{1}{2}\left(RM^{-1}M_d + M_d M^{-1}R\right) + GK_{di}G^T > 0 \quad (22)$$

Proof. We use Lyapunov's second theorem to prove the statement of the proposition 2. Let $L(x) = H_d(x) - H_d(x^*)$ be the Lyapunov candidate, where $x^* = (q^*, 0)$. Then L is positive definite in a neighborhood of x^* and $\Delta L_n = (\Delta H_d)_n < 0$ applaying the discrete-time controller $u_n = (u_{es})_n + (u_{di})_n$. Since a straightforward calculation leads to

$$\begin{aligned}\Delta L_n &\triangleq L(q_{n+1}, p_{n+1}) - L(q_n, p_n) \\ &= -\Delta t\left(\frac{p_{n+1}+p_n}{2}\right)^T M_d^{-1}(C+D)M_d^{-1}\left(\frac{p_{n+1}+p_n}{2}\right)\end{aligned} \quad (23)$$

Hence x^* is asymptotically stable.

Remark 3.2. When the controller is applied to open-loop conservative models, i.e. with no physical damping, the asymptotic stability of the closed-loop system is obtained naturally by adding the desired damping matrix $D = GK_{di}G^T > 0$.

Remark 3.3. The procedure proposed in this paper is restricted to the linear case because the system to be controlled (a flexible spacecraft) is linear. Otherwise, we replace the midpoint scheme (13) by the average discrete gradient [Harten et al., 1983].

4. CONTROLLER DESIGN OF FLEXIBLE SPACECRAFT

This section presents a dynamical model and controller design for a flexible spacecraft system depicted in Figure 2. This testbed works in the horizontal plane $P_i = (0, x_i, y_i)$ where P_i is an inertial frame, and it is composed of:

- a rigid hub articulated w.r.t the inertial frame by a pivot-joint around a vertical axis $(0, z_i)$. $P = (0, x, y)$ is the hub body frame. The half-side and the inertia (around $(0, z_i)$) of this hub are denoted r and J_m, respectively,
- a torque motor, driving the hub in rotation around the vertical axis $(0, z_i)$,
- 2 identical flexible beams (in the horizontal plane) cantilevered on the hub. The sizes of each beam are l (length), b (width) and h (thickness in the horizontal plane) and the Young modulus of the beam material is denoted E,
- 2 local masses m fitted at the tip of each beam,

Let us denote:

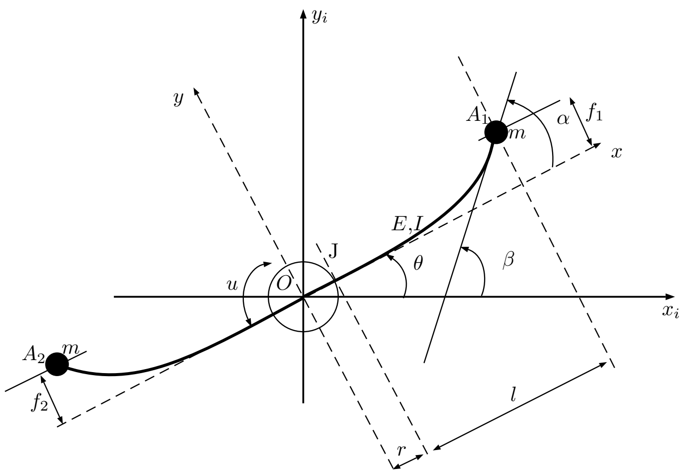

Fig. 2. Bamoss simplified sketch

- $u(Nm)$: the driving torque,
- $\theta(rd)$: angular position of the hub,
- f_1 and $f_2(m)$: lateral deflections at the free end of each beam,
- $\alpha(rd)$: the angular deviation (slope, w.r.t. to equilibrium position) at the free-end of the beam,
- $\beta = \theta + \alpha$.

All results are derived under the following assumptions :

- only motions in the horizontal plane are considered,
- masses and inertia of beams are neglected (inertia of local masses at each beam tip are also neglected),
- only small motions are considered.

Using Lagrange equations, the simplified model is given by the second order equations as follows [Alazard and Bouttes, 2009]

$$M\ddot{q} + R\dot{q} + Kq = Fu \quad (24)$$

with $q = [\theta \quad f_1 \quad f_2]^T$ is generalized coordinate, M represents the mass matrix, K represents the stiffness matrix and R is the dissipation matrix. Fu is the generalized forces vector. $F = [1\ 0\ 0]^T$ since u works only on θ. The total kinetic energy due to the rotation of the hub and the translation of two masses is given by $T(\dot{q}) = \frac{1}{2}\dot{q}^T M \dot{q}$ and the potential energy is $V(q) = \frac{1}{2}q^T K q$.

4.1 Hamiltonian model of flexible spacecraft

To obtain the Hamiltonian model of this system, we introduce the momentum $p = M\dot{q}$ and the total Hamiltonian is defined by the sum of the kinetic energy $T(p)$ and the potential energy $V(q)$

$$H(q,p) = \frac{1}{2}p^T M^{-1} p + \frac{1}{2}q^T K q \quad (25)$$

The port-Hamiltonian model is described by

$$\begin{cases}\dot{x} = \begin{bmatrix}0 & I_3 \\ -I_3 & -R\end{bmatrix}\begin{bmatrix}Kq \\ M^{-1}p\end{bmatrix} + \begin{bmatrix}0_{3\times 1} \\ F\end{bmatrix}u; \quad x = [q^T \quad p^T]^T \\ y = F^T M_d^{-1} p\end{cases}$$

where $F = [1, 0, 0]^T$ is the input vector and the matrices

$$M = \begin{bmatrix}J_t & mL & -mL \\ mL & m & 0 \\ -mL & 0 & m\end{bmatrix}; \ K = \begin{bmatrix}0 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k\end{bmatrix}; \ R = \begin{bmatrix}0.007 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}$$

where $J_t = J_m + 2mL$ present the total inertia seen from the pivot joint axis $(0, z)$ and $k = 3Ebh^3/12l^3$ is the radial stiffness of one beam.

4.2 Controller deign

Now, we apply the procedure given in Section 3 to design a discrete-time controller for the system. The main objective is to asymptotically stabilize the angular position at its desired equilibrium point θ^*.
Since the desired equilibrium angular position is already stable, we propose to shape potential energy only leaving kinetic energy unchanged

$$M_d = M$$

The potential equation (8) to be solved is given by

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \begin{bmatrix} \theta \\ f_1 \\ f_2 \end{bmatrix} - \underbrace{M_d M^{-1}}_{I} \begin{bmatrix} \nabla_\theta V_d \\ \nabla_{f_1} V_d \\ \nabla_{f_2} V_d \end{bmatrix} \right\} = 0 \qquad (26)$$

where the newly shaped potential energy has the quadratic form given by

$$V_d(q) = \frac{\alpha}{2}(\theta - \theta^*)^2 + \frac{k}{2} f_1^2 + \frac{k}{2} f_2^2 , \qquad (27)$$

where the parameter $\alpha = 0.35 > 0$ and the reference input θ^*.
Remark 4.1. The choice of this energy function is done in a way to avoid appearance of variables that cannot be measured in practice. The controller can therefore be implemented directly without designing any observer to estimate the unmeasured variables.

From this solution, the energy shaping controller u_{es} and damping controller u_{di} are directly derived as

$$u(t) = -\alpha(\theta - \theta^*) - K_{di}\dot{\theta}, \quad K_{di} = 0.25 . \qquad (28)$$

From (15), (20) and (21), the discrete controller is given by

$$u_n = -\alpha \left(\frac{\theta_{n+1} + \theta_n}{2} - \theta^* \right) - K_{di} \frac{\theta_{n+1} - \theta_n}{\Delta t} . \qquad (29)$$

The emulation approach is obtained from the continuous-time controller (28) sampled at time n.

5. EXPERIMENTAL RESULTS

In this section we give some experimental results to support the theory of proposed control method in this paper. The parameters of the system are given by the following Table 1.

E	J_m	l	m	r	h	b
$200.1 N/m^2$	$0.015 Kgm^2$	$0.286 m$	$0.3 Kg$	$5 cm$	$0.64 mm$	$4 cm$

Table 1. Numerical values of the parameters of a flexible spacecraft

These results compare between two controllers: emulation and energetic controller. The main subject here is to test the performances of each controller for a higher integration timestep Δt. In Figure 3, the experimental responses are presented under the following conditions:

- integration timestep is $\Delta t = 0.01s$,
- at time $t < 5s$, the system is at rest,
- at time $t = 5s$, the feedback loop is closed in order to reach the desired angular position $\theta^* = 10$ degrees.

We note that the emulation and energetic controller have the same behavior, all responses converges with the same rate at the equilibrium point θ^*.

The Figure 4 (obtained in the same conditions as Figure 3 with $\Delta t = 0.004s$), shows clearly that the emulation controller is sensitive to the large integration timestep. This controller immediately destroys the closed-loop stability in discrete time due to the discretization error. However, the responses obtained by the energetic controller are still stable.

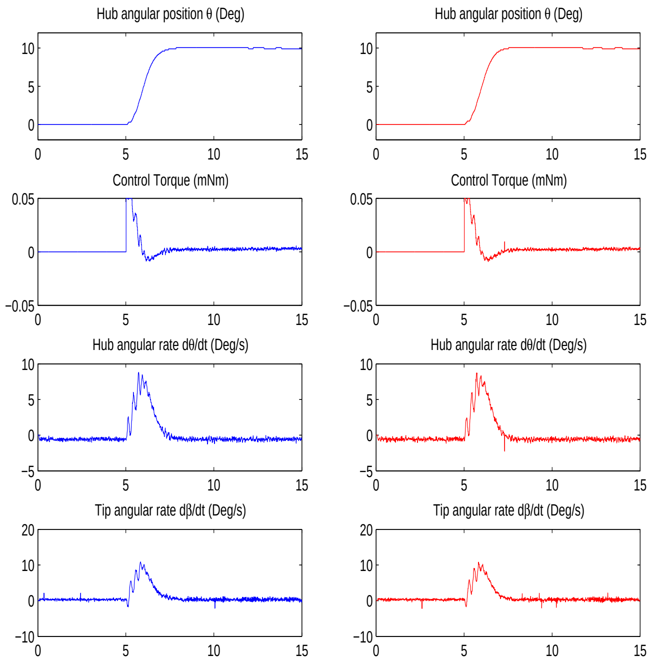

Fig. 3. Emulation controller (left) and energetic controller (right) for $\Delta t = 0.01s$

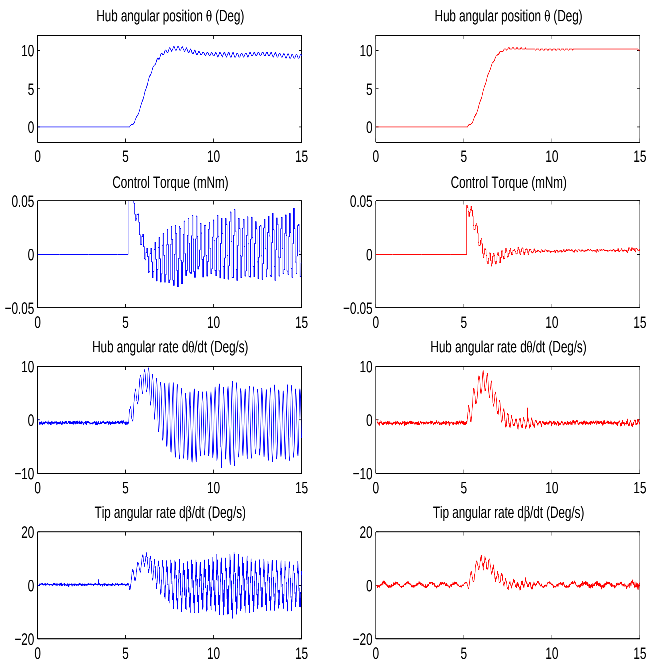

Fig. 4. Emulation controller (left) and energetic controller (right) for $\Delta t = 0.04s$

Finally, we make another test under the following conditions (see (Figure 5):

- integration timestep $\Delta t = 0.035$,

- at time $t < 5s$, symmetric mode (excited by the initial conditions),
- at time $t = 5s$, the feedback loop is closed to reach the desired angular position $\theta^* = 15$ degrees.

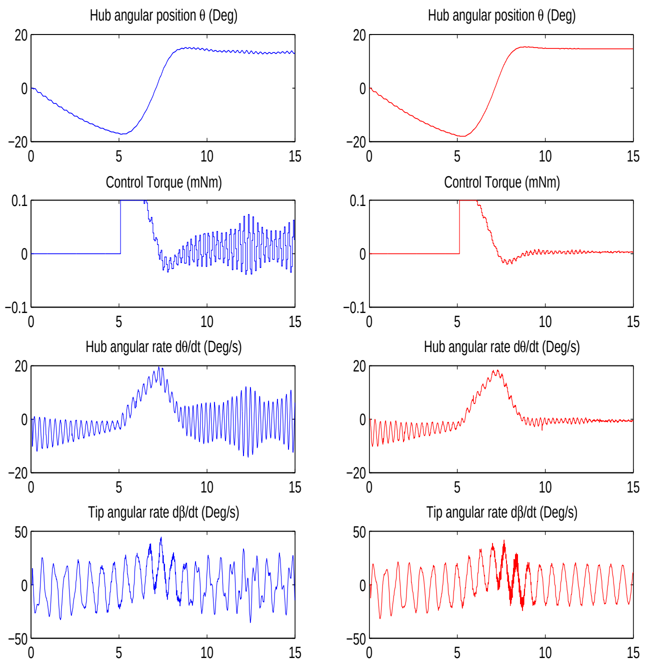

Fig. 5. Emulation controller (left) and energetic controller (right) for $\Delta t = 0.035s$

At time $t < 5s$ (open loop), we observe the oscillations phenomena due to the symmetrical flexible mode. At time $t \geq 5s$ (closed loop), the response obtained by the energetic controller converges to the desired equilibrium point $\theta^* = 15$ in approximately 3 seconds, while the symmetric mode (excited by the initial conditions) continues to vibrate as in open loop since this mode is uncontrollable (see the response of $\mathrm{d}\beta/\mathrm{d}t$). However, the emulation controller became unstable. We can conclude that the energetic controller proposed in this study yields good performances compared with the emulation controller considered as regular implementation in the literature.

6. CONCLUSION AND FUTURE WORKS

In this paper, the discrete-time IDA-PBC control technique for underactuated mechanical systems has been extended to incorporate open-loop damping. Based on the energetic scheme, it is shown that the passivity property is exactly preserved at the discrete level. The stability condition reported in this paper is the first result on the discrete-time IDA-PBC design according to the best of the authors' knowledge. Based on the PHS of a flexible spacecraft, the discrete controller has been designed to reach the desired angular position of the hub body. Experimental results show that the performances of the emulated control decrease while the integration timestep increases. On the contrary, the energetic controller proposed in this paper ensures the stability of the closed loop for the large integration timesteps.

As future work, we aim at taking into account the nonlinearity of a flexible spacecraft and design the discrete-time IDA-PBC control based on the nonlinear discrete gradient. The observer can be explored to estimate the remaining variables.

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