Enthalpy and total energy *vs.* elevation in the Italian Alps (*)

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Summary. — In this work quarterly and yearly averages of enthalpy and total energy (moist static energy) of air were calculated from literature climatological data of 21 weather stations in the Italian Alps, at altitudes ranging from 200 to 3500 m a.s.l., and it was verified that the mean total energy is almost invariant with elevation, as previously found in North America, and there is no obvious difference between easterly or westerly sections of the Alps. The invariance with the elevation of the total energy for unit mass of air can be implied from the hydrostatic equilibrium in the adiabatic case. When averaged over three months or a year, positive and negative energy inputs level out, and mean total energy is almost altitudinally and zonally invariant.

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1. – Enthalpy and total energy *vs.* elevation

It is well known that, owing to the thermal gradient of the atmosphere, the enthalpy is anticorrelated with elevation. Recently, it was shown that the total, or moist static energy averaged over a long time (for instance, one year) is almost invariant zonally and with the elevation, depending mainly on the latitude [1].

Neglecting the small contribution of the kinetic energy, the above statement can be written as

$$c_p T + L_v q + gZ = H + gZ = \text{const}\,, \tag{1}$$

where L_v is the evaporation latent heat and q the specific humidity (kg kg^{-1}).

On this basis and estimating the value of enthalpy H of moist air from leaf physiognomy of fossil plants, Forrest *et al.* [1] obtained the potential energy gZ and then the palaeoelevation of sites considered, and Wolfe *et al.* [2] found evidence of high altitudes in Nevada during the Miocene.

(*) The author of this paper has agreed to not receive the proofs for correction.

The mountain environment hosts an almost endless variety of different micro-climates, depending not on the elevation alone, but also on exposure, slope, shadows, vegetation. By consequence, the crude statement of the invariance of total energy with elevation can hold only if large averages in space and time are considered.

2. – Enthalpy and total energy in the Italian Alps

In order to ascertain if the hypotesis of the invariance of total energy with elevation can hold in the Italian Alps, we considered 21 weather stations ranging in elevation from 205 (Rovereto) to 3488 m a.s.l. (Pian Rosà) and with a span in longitude from 7° E (Susa) to 12° 20′ E (Dobbiaco), stations whose climatological records are available in the literature [3].

The 2 stations in the central Alps were considered together with the eastern Alps in order to have in this section one station at high altitude. Starting from monthly data for temperature and humidity, the mean values of the enthalpy were calculated for winter (D J F), summer (J J A) and the whole year.

We have at first to observe that such calculations are conceptually incorrect, because our data for temperature and humidity are averaged, and not contemporary as they should be in order to allow to calculate correctly the water vapor pressure and then the specific humidity [4]. The consequent error can nevertheless be accepted, because: i) in any month the range of specific humidity is small, ii) the maximum (minimum) of relative humidity occurs mainly at minimum (maximum) of temperature, and iii) the contribution to the enthalpy of $L_v q$ is small compared with $c_p T$.

Anyway, in order to estimate the importance of such error, the values of water vapor pressure calculated from averaged temperature and relative humidity were compared with monthly mean values directly available in (3) for 10 stations, and an acceptable agreement was found for most of them.

The values of atmospheric pressure with elevation, $p(Z)$, and saturation water vapor pressure with temperature $e_s(t)$ were calculated with approximate formulae common in routine meteorological work [5]:

$$p = p_0 / \exp[Z/7.387],$$

TABLE I. – *Western Alps.*

Station	Elevation (m a.s.l.)	Enthalpy (kJ/kg)			Total energy (kJ/kg)
		Winter	Summer	Year	Year
Biella	412	296.3	333.6	312.3	316.4
Susa	513	295.6	329.3	311.1	316.2
Aosta	582	294.2	329.5	310.1	315.8
Oropa	1163	293.2	324.4	306.2	317.6
Piccolo San Bernardo	2160	280.1	308.0	292.4	313.5
Corno Valdobbia	2548	284.2	308.6	294.7	319.7
Pian Rosà	3488	276.9	294.7	285.2	319.4

TABLE II. – *Central and eastern Alps.*

Station	Elevation (m a.s.l.)	Enthalpy (kJ/kg)			Total energy (kJ/kg)
		Winter	Summer	Year	Year
Rovereto	205	296.6	337.0	314.1	316.1
Bolzano	236	296.8	337.1	314.4	316.7
Trento	307	295.9	334.2	313.2	316.2
Sondrio	363	294.6	337.6	313.5	317.1
Belluno	404	294.1	330.9	310.7	314.6
Monte Venda	575	295.9	330.7	311.3	317.0
Tarvisio	758	290.0	326.1	306.2	313.6
Auronzo	871	289.4	324.4	305.7	314.3
Colle Isarco	1098	290.4	322.7	304.7	315.5
Cortina d'Ampezzo	1210	289.9	320.6	303.1	314.9
Dobbiaco	1222	291.1	324.5	305.7	317.6
Monte Grappa	1716	290.7	318.9	303.2	320.0
Monte Paganella	2108	287.2	312.6	298.1	318.8
Stelvio	2543	280.4	308.9	292.8	317.7

TABLE III. – *Alps* (*all*).

Station	Elevation (m a.s.l.)	Enthalpy (kJ/kg)			Total energy (kJ/kg)
		Winter	Summer	Year	Year
Rovereto	205	296.6	337.0	314.1	316.1
Bolzano	236	296.8	337.1	314.4	316.7
Trento	307	295.9	334.2	313.2	316.2
Sondrio	363	294.6	337.6	313.5	317.1
Belluno	404	294.1	330.9	310.7	314.6
Biella	412	296.3	333.6	312.3	316.4
Susa	513	295.6	329.3	311.1	316.2
Monte Venda	575	295.9	330.7	311.3	317.0
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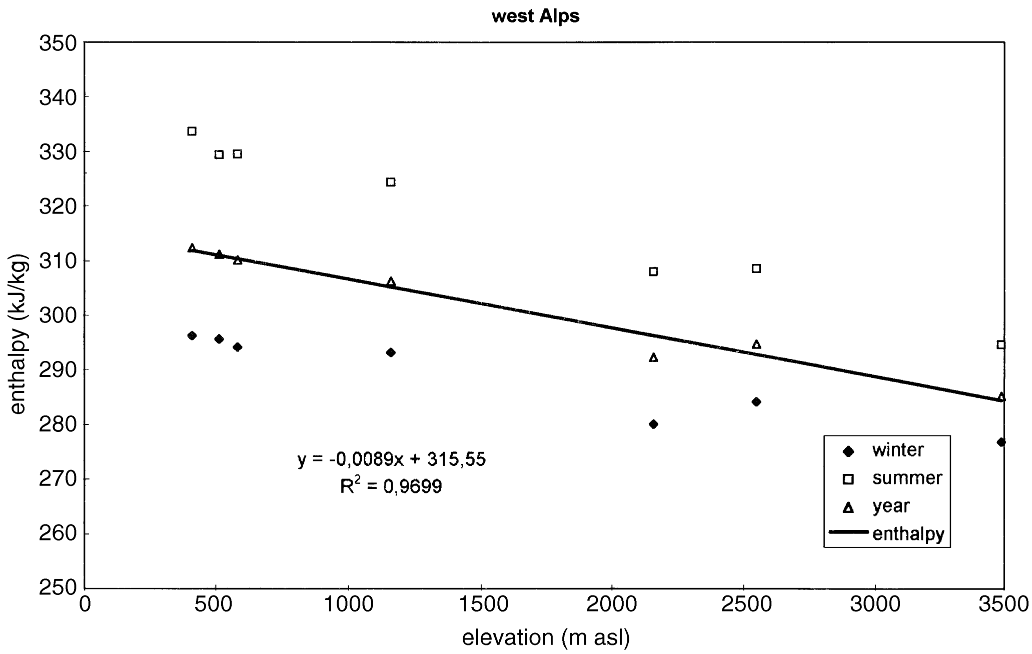


Fig. 1.

where Z is in km, and

$$e_s = 6\cdot 10 \exp[0.03t].$$

The results are shown in tables I to III and in fig. 1 to 4.

Table I refers to 7 stations in the western Alps, and contains the values of enthalpy for summer, winter and the whole year, and the yearly values of total energy.

In fig. 1 the plots of quarterly and yearly values of enthalpy are almost parallel, and the regression line for the yearly values *vs.* elevation have the following equation:

$$y = -0.0089x + 315.55\,, \qquad R^2 = 0.97\,,$$

where x is the elevation in m a.s.l. and y the enthalpy in kJ kg^{-1}.

Table II and fig. 2 show the same items for the central and easterly section of the Alps. The same considerations hold, and the regression line has the equation

$$y = -0.0086x + 315.23\,, \qquad R^2 = 0.94\,.$$

The values of the total energy from tables I and II are very similar, and, actually, a “Student t-test” shows that the two series are not statistically different at both confidence levels of 95% and 99%. By consequence, we can accept that the total energy is zonally invariant, and the values for all stations are merged in table III and fig. 3. The new regression equation is

$$y = -0.0088x + 315.4\,, \qquad R^2 = 0.96\,.$$

As we can see in the last column of table III and in fig. 4, the total energy, again

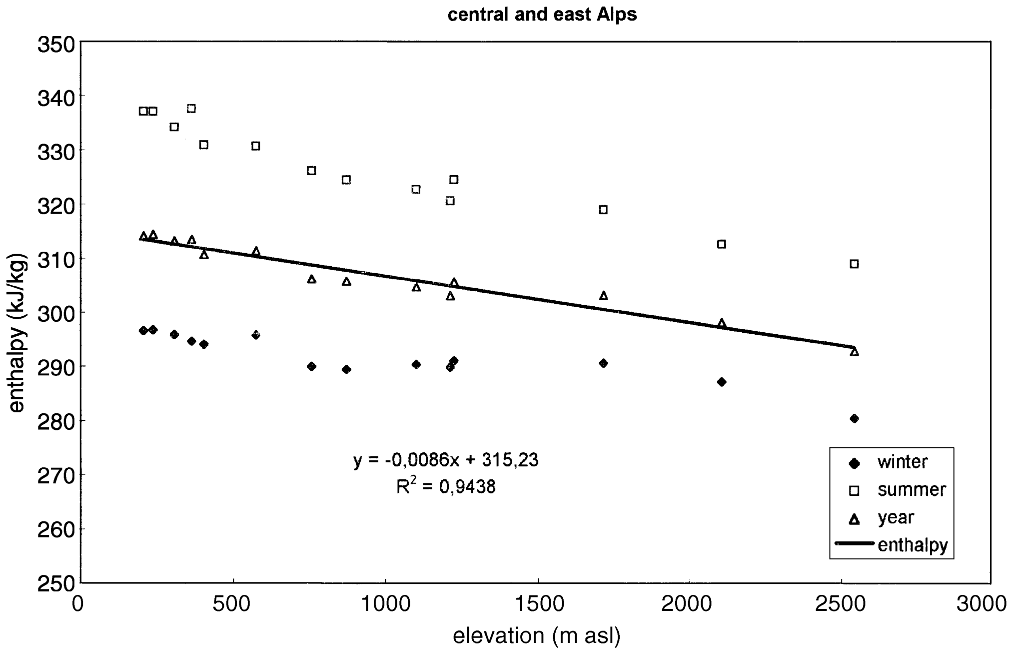


Fig. 2.

represented by y, shows a very small trend to increase with the elevation:

$$y = 0.001x + 315.4\ , \qquad R^2 = 0.26\ .$$

Because of flaws inherent in our calculations and the small value of the determination coefficient R^2, perhaps we could forget this trend and assume the total energy to be constant with elevation.

3. – Total energy and hydrostatic equilibrium

The invariance of the moist static energy with the elevation can be reduced to the expression of hydrostatic equilibrium in the adiabatic case.

Differentiating eq. (1) with respect to z, we obtain

$$c_{\mathrm{v}}\,\mathrm{d}T/\mathrm{d}z + v\,\mathrm{d}p/\mathrm{d}z + p\,\mathrm{d}v/\mathrm{d}z + g = 0\ ,$$

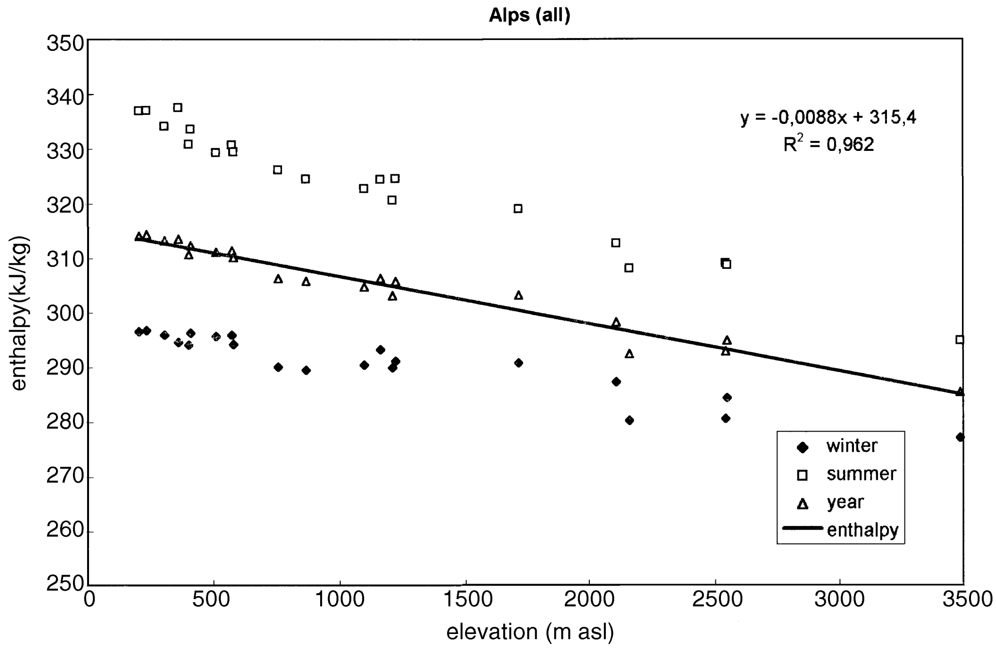
$$-v\,\mathrm{d}p/\mathrm{d}z = c_{\mathrm{v}}\,\mathrm{d}T/\mathrm{d}z + p\,\mathrm{d}v/\mathrm{d}z + g\ ,$$

where v is the specific volume ($\mathrm{m}^3\ \mathrm{kg}^{-1}$), but $\mathrm{d}q = \mathrm{d}e + p\,\mathrm{d}v = c_{\mathrm{v}}\,\mathrm{d}T + p\,\mathrm{d}v$, so

$$-v\,\mathrm{d}p/\mathrm{d}z = \mathrm{d}e/\mathrm{d}z + p\,\mathrm{d}v/\mathrm{d}z + g = \mathrm{d}q/\mathrm{d}z + g\ .$$

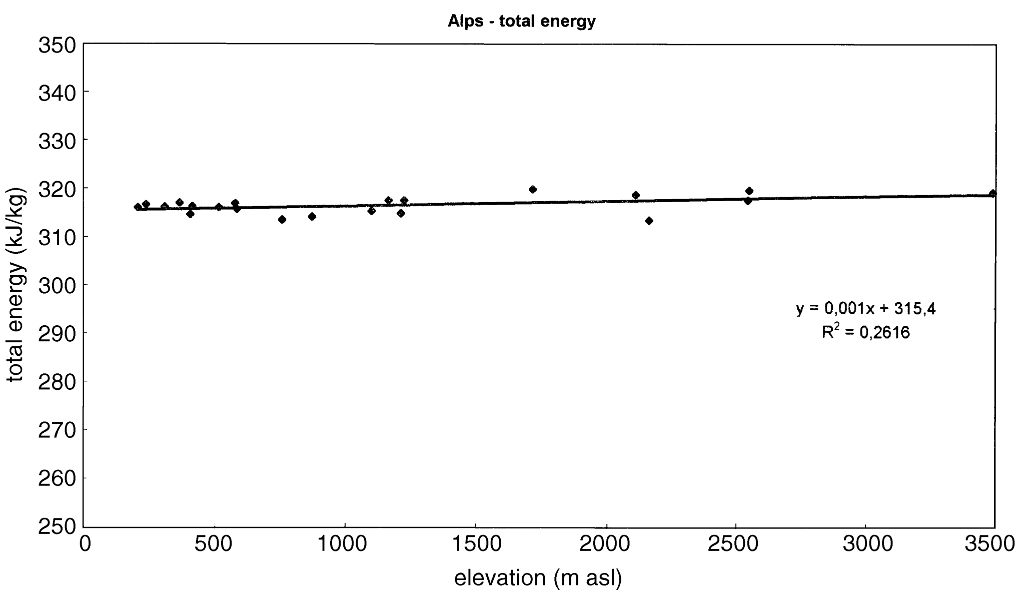
If $\mathrm{d}q = 0$ (adiabatic case) or $q(z) = \mathrm{const}$, $\mathrm{d}q/\mathrm{d}z = 0$ and being $v = 1/\varrho$, we obtain the hydrostatic equation

$$\mathrm{d}p/\mathrm{d}z = -g\varrho\ .$$



Alps (all)
y = -0,0088x + 315,4
R2 = 0,962
enthalpy(kJ/kg)
elevation (m asl)
winter
summer
year
enthalpy

Fig. 3.



Alps - total energy
y = 0,001x + 315,4
R2 = 0,2616
total energy (kJ/kg)
elevation (m asl)

Fig. 4.

4. – Conclusions

The hypothesis that the total (moist static) energy is almost invariant with the elevation is theoretically justified if data are averaged over a time long enough to approximate the adiabatic case.

From the last column of table III we can assume that in the south side of the Alps it is

$$H + gZ = 316.6 \pm 1.88 \ \text{kJ kg}^{-1} .$$

With this standard deviation, at the confidence level of 95% the incertitude for the elevation is ± 376 m or ± 3.5 °C for the mean yearly temperature. This incertitude is a serious limitation for the usefulness of the invariance hypothesis.

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