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The Economics of the Sabbath

Paper

THE ECONOMICS OF THE SABBATH

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A U S T R I A

Abstract

Our paper is inspired by the observation that - contrary to the US-experience - in many European countries working time schedules for different groups of workers are strictly regulated for the purpose of protecting "common leisure time" (like holidays, sundays, recreation periods during the night and so on ...)[1]

As a first step towards a more general analysis, a simple model analyzing the pattern of working time and leisure for two types of workers is developed. The basic assumptions are: productivity in general differs between periods where both are at work simultaneously ("common working time") and periods where only one is at work "private working time"). Moreover, utility of "common" (= overlapping) leisure time differs from utility of "private" leisure. The findings show that from a welfare point of view people work too long and get a shortage of common leisure whenever productivity during common working time is lower than outside common working time for any single individual. If the reverse holds, workers will end up with too short a working day and a lack of private leisure. The long-run development of the productivity gap is given a tentative historical interpretation.

I. Introduction

Although not completely absent in the U.S., labor market regulations with regard to both the length of an individual labor day **and** the time during which work is allowed to be done are familiar only in Europe. Our paper focuses on the welfare impact of those regulations. In particular, we show that the outcome in a competitive framework may be inefficient, but regulations, e.g. set by unions, may not be efficient, either.

The working schedules of the two agents determine common working time and private working time in production as well as the partition of leisure in common and private free time. Dealing with the choice of the working day, two aspects have to be considered in particular. The resulting overlap of individual working sche-

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[1] We use the term "sabbath" in the general sense of "common leisure time" being protected by normative prescriptions. In fact, the historical "sabbath" can be seen not only as an important religious institution but also as one of the earliest attempts of human mankind to protect a public good (common "leisure" time) by social regulation and - for obvious reasons -as one of the earliest examples of "social policy" as well!

dules will affect labor productivity in production on the one hand, and the composition of leisure on the other. We assume that productivity in general differs between common and private working time for two reasons: First, synergy effects determine the productivity of agents working alone or working in a team. Second, there are economies of scope between simultaneous production of one agent and consumption of the other one (e.g. for personal services of for shopping facilities). Furthermore, we suggest that agents value leisure time differently depending on whether it is consumed as a private good or as a common good.

It is shown that if the economies of scope effect is dominated by the synergy effect in production so that common working time productivity exceeds private working time productivity, in a competitive framework both individuals will choose too short a working day, i.e. they will both start working too late in the morning and will stop working too soon in the evening. The economic intuition behind this result is that they neglect negative externalities on the other agent's income (leading a social planner to extend high productive common working time) as well as consumption externalities (inducing the planner to increase private leisure time at the expense of common leisure time).

If the reverse holds, so that private working time productivity exceeds common working time productivity, two possibilities have to be distinguished: First, if productivity is only slightly higher at private working time than at common working time, in a private arrangement agents will continue to choose too short a total working day. Yet as the sign of the income externality is reversed now, in a social arrangement they would have to increase private working time and reduce common working time. Second, if private working time productivity exceeds common working time productivity substantially, people will tend to work too long both in the morning and in the evening. In this case a social planner will unambiguously shorten individual working schedules and increase the supply of both common and

private leisure time. Thus, contrary to what is sometimes stated, the existence of the public good "common leisure time" is not a sufficient condition for the underprovision with common free time. Only if in addition private working time at production yields sufficiently higher earnings than common working time production, this conjecture is found to be correct.

II. The Model

The model investigates the choice of working hours for two individuals, s and t, in a simple static framework. During the working day two periods have to be distinguished: if both individuals work at the same time (common working time), productivity per man and hour worked, in general differs from productivity obtained if only one of them is at work (private working time). In principle, there are two types of effects that account for this productivity gap. First, technological reasons might influence the optimal utilization of capital during the course of the day: According to the law of diminishing returns of labor if applied to a given stock of capital, we should expect the productivity of private working time to exceed productivity of common working time. However, technological reasons might lead to the opposite productivity gap, too: if e.g. weather is a necessary input for production, high productivity will demand common input of labor if weather conditions are favorable. Second, marketing of goods might require that a product that has been made at common working time can be sold only at private working time: If buying the good or a personal service is time consuming, private leisure time of one agent is complementary to private working time of the other one. In this case the productivity effect between standard and nonstandard working hours will be determined by the ratio of common and private working time. It is reasonable to suppose that this relationship will be influenced by social variables like the labor force participation of women, too. If a substantial part of women does not supply labor at the labor market, the economics of scope effect between private working time and private leisure time will be

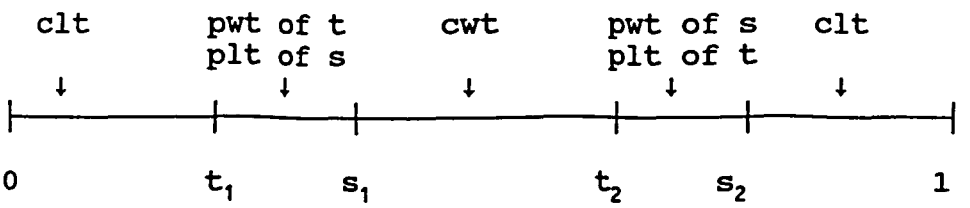
weak. If, on the other hand, many women go to work, the economics of scope effect will get stronger as there is a lack of shopping time now.

It is obvious that we do not require workers to be employed at the same firm: thus a higher wage at private working time might steem from the possibility that t sells his good while s goes shopping after her work has been done as well as t's marginal product of labor is high because he is the only one using a given stock of capital.

Finally, we assume that output markets are competitive, thus profits are zero and wages equal productivity.

Fig.1 shows the course of the day, the length of which is taken to be unity. Determining private and common leisure and working time for both individuals, Hotellings idea of a "linear city" (1929) is used to analyze decisions on working time. t_1 and t_2 indicate the start and the end of work for t, the same holds for s_1 and s_2 for s. Without loss of generality, we assume that t starts to work in the morning and stops only after s has begun her working day. In addition, we suppose that s finishes work at night.

Fig.1: The course of a day for the workers t and s



For each worker, the day can be split into 4 parts: working hours at common working time (cwt), working hour at private working time (pwt), private leisure (plt) and common leisure time (clt). Common working time is given by $t_2 - s_1$ for both t and s. (s_1-t_1) is private working time for t and private leisure for s simultaneously:

$$L_s = s_1 - t_1. \quad (1)$$

Similarily, private leisure time of t,

$$L_t = s_2 - t_2 \quad (2)$$

coincides with private working time at work for s. Common leisure K for both s and t is:

$$K = t_1 + 1 - s_2. \quad (3)$$

Nobody being indifferent between working day and night, we assume that starting to work at the very beginning of the day is extremely costly for agents. However, in the course of the morning, the disutility of going to work declines until it finally disappears at all. Similarily, agents are not indifferent with regard to the point of time ending work in the evening. Disutility of staying at work sharply increases towards the end of the day. An "asymmetry" variable A is introduced to capture this idea. We assume that when the point of starting (finishing) work is shifted towards the morning (night), finally a saturation level is reached. Above the saturation level, marginal utility from an increase in A is zero, below the saturation level utility declines at increasing rates if A is reduced. Asymmetry factors for s and t are:

$$A^s = 1 - s_2 \quad (4)$$

$$A^t = t_1. \quad (5)$$

It will become clear in the next section that the asymmetry factor is necessary in order to derive a unique solution for the working schedule in this model.

Finally, total working time is $t_2 - t_1$ for t and $s_2 - s_1$ for s. Agents are assumed to be homogeneous, so that marginal productivity neither at common working time nor at private working time differs across workers. Individual income is the sum of

common working time and private working time earnings, total income Y is $Y_t + Y_s$:

$$Y_s = f[(t_2 - s_1), (s_2 - t_2)], \tag{6}$$

$$Y_t = f[(t_2 - s_1), (s_1 - t_1)]. \tag{7}$$

Our model is concerned with the allocation of time for four different uses (private and common free time, common and private working time). The main problem arises from the existence of externalities of both consumption and production activities of one agent on production and consumption possibilities of the other one. In a market solution without a social planner, each agent individually maximizes utility by choosing points of starting and ending the working day while neglecting spillovers to the other agent. In the course of this, three kinds of externalities arise: Deciding to start his work at t_1, t has an influence on both s's private free time as well as on her common leisure time. Moreover, t's ending work at t_2 affects s's productivity at work since t_2 determines common working time and private working time in production. Similarily, s's decision will have an impact on t's income, his common and his private free time, too. Therefore, we can expect individualistic decisions to be not efficient from a social point of view in general.

To recapitulate, utility of s and t is supposed to be dependent on consumption (which is given by income in this model), common and private leisure and the asymmetry effect:

$$\begin{aligned} U^s &= U^s(Y^s, L^s, A^s, K), \\ U^t &= U^t(Y^t, L^t, A^t, K). \end{aligned} \tag{8}$$

We assume that utility of workers is concave in Y, L, K and A, so that $U_i>0$, $U_{ii}<0$ and $U_{ij}>0$ for $i,j=Y,L,A,K$ and $j\neq i$[2]. We can now

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[2]If someone doubts whether common and private leisure time are different goods, he should simply note that for many social activities simultaneous consumption of leisure is a necessary

proceed to analyze the choice of working schedules by a social planner and by individually maximizing workers.

II.I. The noncooperative market solution

In a private arrangement, agents would choose individual working schedules to maximize utility:

$$\underset{s_1,s_2}{\text{Max}}\ U^s(Y^s,\ L^s,\ A^s,\ K)$$

s.t. (1), (3) (4) and (6),

$$\underset{t_1,t_2}{\text{Max}}\ U^t(Y^t,\ L^t,\ A^t,\ K).$$

s.t. (2), (3) (5) and (7).

We assume that in the process of maximization each individual takes the working-time decision of the other one as given. For the Nash-solution the following first-order conditions with respect to optimal points of starting and ending work are derived:

$$\tilde{}_{t1}: \quad U^t_{t1} = -U^t_Y f_2 + U^t_K + U^t_A = 0 \qquad (9)$$

$$\tilde{}_{t2}: \quad U^t_{t2} = U^t_Y f_1 - U^t_L = 0 \qquad (10)$$

$$\tilde{}_{s1}: \quad U^s_{s1} = -U^s_Y f_1 + U^s_L = 0 \qquad (11)$$

$$\tilde{}_{s2}: \quad U^s_{s2} = U^s_Y f_2 - U^s_K - U^s_A = 0 \qquad (12)$$

U^t_{t1}, U^t_{t2} indicate the marginal utility of increasing t_1 or t_2, respectively, for individual t. Similar interpretations hold for U^s_{s1} and U^s_{s2}. f_1 and f_2 are marginal products of labor at common working time and at private working time, respectively. (9) states that t will extend his working day in the morning until

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condition (e.g. participating in democratic activities, common activities within the family etc.), while for other activities during leisure time it is necessary that other people are at work (e.g. shopping facilities or personal services as they are offered at restaurants etc).

his marginal utility of income at private working time equals marginal costs of going to work. In our model, opportunity costs of private working time are the sum of marginal utility of common free time and the asymmetry effect. (12) yields a similar condition for s's ending her working day in the evening. According to (10), t chooses the end of his individual working schedule in order to equate marginal utility of income at common working time and marginal utility of private leisure time. (11) shows that the same condition holds for s's choice of starting her working day.

(9) - (12) characterize the noncooperative market solution realized in a private framework. It will be stable if no agent has an incentive to deviate from his time schedule, given the working hours of the other one. In this noncooperative Nash-equilibrium, common working time and common leisure time are identical for both s and t. As agents are only concerned with their own utility, externalities in production and consumption are not taken into consideration in this type of market solution.

II.II. Working schedules determined by a social planner

A social planner who is perfectly informed about workers' utility functions will maximize social welfare by choosing optimal working schedules for s and t:

$$\underset{t_1,t_2,s_1,s_2}{\text{Max}} \; U^s(Y^s, L^s, A^s, K) + U^t(Y^t, L^t, A^t, K) \qquad (13)$$

s.t. (1)-(7).

First-order conditions yield:

$$t_1^*: \quad -U^t_Y f_2 + U^t_K + U^t_A = U^s_L - U^s_K \qquad (14)$$

$$t_2^*: \quad U^t_Y f_1 - U^t_L = U^s_Y(f_2 - f_1) \qquad (15)$$

$$s_1^*: \quad -U^s_Y f_1 + U^s_L = U^t_Y(f_1 - f_2) \qquad (16)$$

$$s_2^*: \quad U^s_Y f_2 - U^s_K - U^s_A = U^t_K - U^t_L. \qquad (17)$$

Since utility is strictly concave in all arguments, second order conditions are satisfied for both the private and the social maximazion problem.

In (14)-(17), the l.h.s. refer to variables that have already been taken into consideration by the individually maximizing agents, the r.h.s. indicate consumption and production externalities. (14) and (17) state that when selecting the start and the end of the total working schedule, the social planner takes into consideration that increasing t_1 transforms private free time into common free time for s. Similarly, reducing s_2 will change t's private leisure time into common leisure time. Therefore, in a social arrangement, private marginal valuations of the two types of free time have to be considered when choosing t_1 and s_2. Concerning the determination of t_2 and s_1, (15) and (16) show that private decisions will be corrected in a social framework if labor productivity differs between common and private working time, so that income externalities are present.

In order to analyze how the noncooperative equilibrium differs from the socially optimal one, it is useful to investigate two special cases first, one of them neglecting differences in labor productivity between common and private working time, the other one abstracting from the distinction between private and common consumption of leisure.

(i) No externalities in production, externalities in consumption

If for technological reasons the marginal product of labor is constant during the whole working day, $f_1 = f_2$ holds, so that first-order conditions for the social planner boil down to:

$$t_1^*: \quad -U^t_Y f_2 + U^t_K + U^t_A = U^s_L - U^s_K \qquad (18)$$

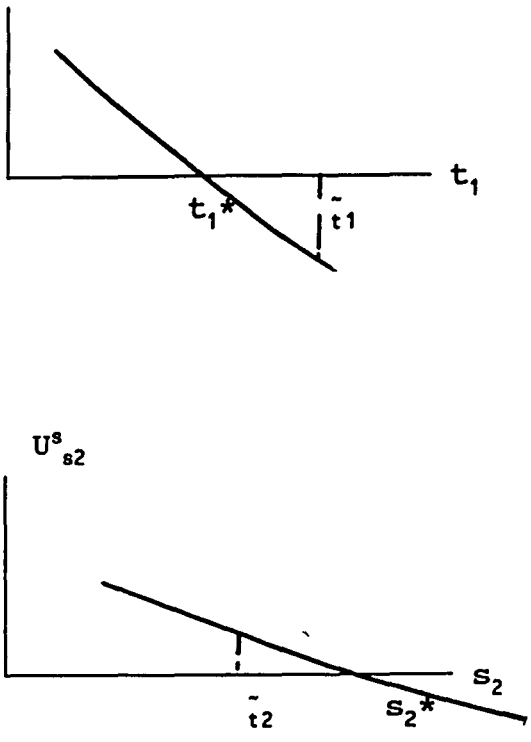
$$t_2^*: \quad U^t_Y f_1 - U^t_L = 0 \qquad (19)$$

$$s_1^*: \quad -U^s_Y f_1 + U^s_L = 0 \qquad (20)$$

$$s_2^*: \quad U^s_Y f_2 - U^s_K - U^s_A = U^t_K - U^t_L. \qquad (21)$$

According to (18) and (21), the difference of t_1 and s_2 between a social and a private choice of working schedules will depend upon the sign of U_K-U_L, i.e. whether agents have a higher valuation for common or for private leisure time at the margin. From first-order conditions for individual optimization, (9)-(12), it follows that $U_K-U_L=-U_A<0$ holds for both agents, reflecting that marginal utility of private leisure time must exceed marginal utility of common free time for both individuals since leisure in the very beginning of the day and in the evening enters utility for two independent reasons: firstly it is useful for common activities, secondly agents have a distaste for going to work at a unpleasant time of the day. Inserting this result in (18) and (21), we find: $dU^t/dt_1=U^s_A>0$, $dU^s/ds_2=-U^t_A<0$ in a socially chosen optimum. Fig.2 shows the selection of t_1 and s_2 in a private ($\tilde{t}_1$ and $\tilde{s}_2$) as well as a social (t_1^* and s_2^*) arrangement. In the noncooperative market solution, agents choose the beginning and the end of the total working day so that $dU^t/dt_1 = dU^s/ds_2 = 0$. Thus we recognize that the social planner will unambiguously prolong the working day both in the morning and in the evening, i.e. t_1 will be decreased and s_2 will be increased relatively to the values chosen by individualistic maximization: As the asymmetry effect of going to work at an unfavorable point of time is present only for one of the two agents at a given point of time, he (she) does not take account that the other one has a higher valuation of private free time than of common free time at the margin. For this reason, private decisions lead to a underprovision of private leisure time. Taking this market failure into consideration, the length of private working time for both individuals will be increased in a socially oriented organization. Thus the planner transforms common leisure time into private leisure time for one agent, and into private working time for the other one.

Fig.2: U^t_{t1}



Note that first-order conditions of neither t_2 nor s_1 are affected by the social planner because there are no income externalities due to changes of labor productivity. Nevertheless agents will react to the planners intervention: t will end working sooner and s will start working later in a socially oriented organization relatively to the private framework in order to restore optimal allocation of time: If t has to go to work sooner in the morning, his income will increase. Subsequently, he will substitute income by private leisure and shorten his working day in the afternoon. At the same time s, who is forced to stay at work longer, will decide to start working later, thus substituting income by private free time at the margin.

To sum up, if labor productivity is constant during the whole working day, workers will always end up lacking private free time in the noncooperative market solution. Thus the social planner will extend private leisure time by prolonging agents'

private working time. Subsequently, individuals will react to this regulation by increasing their own amount of private free time even more. This policy bringing forth a prolongation of private leisure, utility of both individuals can be increased.

In the absence of the asymmetry effect no externalities at all would be present in this special case, so that the social planner would not change private decisions. However, it is to be noted that in this case no unique solution for the partition of the day will exist: Since agents would be indifferent with respect to the "location" of the working day in the linear city as long as both the length and the overlap of individual working schedules are unaffected, an infinite number of solutions would yield the same level of utility for both workers. We have introduced the asymmetry effect in order to derive a unique solution for the partition of the day in our model.

(ii) No externalities in consumption, externalities in production

If agents are indifferent between common and private leisure time, utility functions are reduced to $U^i=U(Y^i,A^i,F^i)$, i=s,t, where F is total free time (i.e. $F^s=s_1+1-s_2$, $F^t=t_1+1-t_2$). First-order conditions for the social planner are derived as follows:

$$t_1^*: \quad -U^t_Y f_2 + U^t_F + U^t_A = 0 \tag{22}$$

$$t_2^*: \quad U^t_Y f_1 - U^t_F = U^s_Y(f_2 - f_1) \tag{23}$$

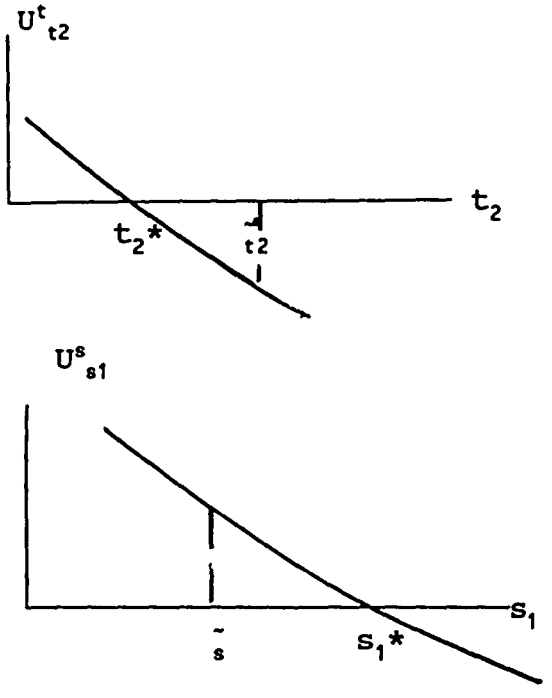
$$s_1^*: \quad -U^s_Y f_1 + U^s_F = U^t_Y(f_1 - f_2) \tag{24}$$

$$s_2^*: \quad U^s_Y f_2 - U^s_F - U^s_A = 0. \tag{25}$$

In order to determine the sign of the externality on income in (23) and (24), we investigate first-order conditions of the individual optimization problem in this case, yielding $U_Y(f_2-f_1)=U_A$ for both agents. If private working time productivity exceeds common working time productivity, underprovision of common working time will result: s chooses too late a start of her working day since she is not concerned with t's increase in

income if the start of her individual working day is moved towards the morning. Similarly, t stops to work too soon, neglecting the negative income spillover for s. Thus it becomes socially desirable to regulate individual decisions by extending high productive private working time at the expense of less productive common working time in both directions by increasing s_1 and decreasing t_2. At the same time both s and t will get an additional supply of leisure time. Fig.3. shows the selection of t_2 and s_1 in a social and in a private arrangement.

Fig.3:



If common working time productivity is increased relatively to private working time productivity, agents will shift the points of starting (ending) work towards the center of the day, thus taking advantage of the high wage during common working time. In the course of this, the asymmetry effect will decline until it finishes to operate as soon as the saturation level is reached. From this moment on individually maximizing agents will allocate time in order to set $U_Y f_1 = U_Y f_2$. Thus, if due to the law of diminishing marginal returns, productivity of labor during private and during common working time declines if private or common working time is extended, respectively, agents will select

working schedules in order to equate the marginal utility of income in both parts of the working day. Of course, this implies that they are paid identical wages at common working time and at private working time at the margin. Consequently, externalities on income will disappear so that the social planner will not affect private decisions.

Finally, if for some exogenous reason $f_1>f_2$ independent of the ratio of common and private working time, people will stop working private working time at all and will agree to share the same working schedule. Again, no income externalities can arise and the social planner will not interfere with private decisions.

In this second special case maximization of social welfare yields to the same first-order conditions for s_1 and t_2 as individual maximization does, as no difference between common and private leisure time is made. Yet again individuals will adapt to this situation: If $f_2>f_1$ (which is the only relevant case for regulations), agents will react to the planner's reduction of common working time by even more extending private working time at the expense of common leisure time.

To sum up, if people do not distinguish between common and private free time but only between working time and leisure, the social planner will only intervene if productivity of private working time exceeds common working time productivity. In this case he will reduce common working time and extend private working time beyond the level chosen by individually maximizing agents. Workers react by further prolonging private working time. Both the workers will end up with a higher level of utility.

(iii) Externalities in production and consumption

In general, all kinds of externalities will be present at the same time. Necessary conditions for the social planner are

summed up by (14)-(17). In order to determine the direction of consumption and production externalities, again first-order conditions of private maximization, (9)-(12), are used, telling us that $U_K-U_L=(f_2-f_1)U_Y-U_A$ for both s and t. There are three special cases of interest that have to be investigated in turn.

(i) $f_1>f_2$, $-U_A + U_Y(f_2-f_1) = U_K-U_L < 0$.

If common working time productivity exceeds private working time productivity, it follows that $U_K-U_L<0$, indicating a shortage of private free time in the noncooperative market solution. A social planner will unambiguously prolong agents' working days: t has to work longer in the afternoon in order to increase high productive common working time for s. At the same time he has to get to work sooner, so that s's consumption of private leisure time is increased. The same reasoning applies to s: She has to work longer in the morning as well as in the evening, too. However, it is clear that agents always prefer the social to the private solution as it yields a Pareto superior solution for the allocation of time.

(ii) $f_1<f_2$, $U_Y(f_2-f_1) < U_A$, $U_K-U_L < 0$.

If working at private working time is more productive than working at common working time, marginal utility of private leisure continues to exceed marginal utility of common leisure time if the asymmetry effect outweighs the difference of utility to be gained from the productivity pap in production at the margin. This situation is characterized by a shortage of private working time as well as scarcity of private free time. Consequently, in a social arrangement t_1 and t_2 will be decreased and s_1 and s_2 will be increased relatively to the private solution, shifting individual working schedules towards the morning and the night, respectively. The total working day will be prolonged to compensate for the higher valuation of private leisure time than of common leisure time at the margin. However, t will have to stop working sooner and s will have to start working later in order

to reduce common working time and to gain more productive private working time.

(iii) $f_1<f_2$, $U_Y(f_2-f_1) > U_A$, $U_K-U_L > 0$.

Finally, if private working time productivity exceeds common working time productivity sufficiently, a shortage of common free time will finally arise. Furthermore, private maximizing workers will extend common working time beyond a level that is desirable from a welfare point of view. Therefore agents choose too long a working schedule in the noncooperative market solution. The social planner will shorten individual working days both in the morning and in the evening: t then stops his working day sooner so that less productive common working time is reduced for s. In the morning, t gets to work later as now s has a higher marginal valuation of common free time than of private free time at the margin.

It is interesting to note that the kind of market failure arising due to externalities mainly depends on the productivity gap between private working time and common working time. If $f_2>f_1$ sufficiently, so that productivity of private working time exceeds common working time productivity substantially, individually maximizing agents would choose too long a working day by both starting to work too soon and ending to work too late, producing an excessive level of incomes and a shortfall of common leisure time. A social planner would shorten individual working days, reducing income and increasing the supply of common leisure at the same time. On the other hand, if common working time productivity is higher than productivity at private working time, working schedules chosen in a private arrangement are too short form a social point of view. In this case both t and s perform too little work, not taking into account the other individual's preferences with respect to income and leisure. In addition, agents will lack private free time. Thus it becomes socially desirable to extend the working day of both individuals, increasing income as well as private free time and re-

ducing the amount of common leisure available.

Final Remark: A Historical Interpretation

It is very plausible to suppose that the productivity gap follows a historically determined pattern: it will be negative in agriculturally dominated societies, and rising during periods of industrialization. Finally, it may even become positive in a service economy.

As long as agriculture is the dominant sector of the economy, "nature", especially weather, dictates when work has to be done. B. Moore (1987) describes how the inhabitants of the Japanese village during the rule of the Tokuwaga Gender managed the problem of cultivating rice: in order to meet the very demanding requirement for labor inputs when the harvest had to be brought in or when watering and planting paddy-fields, the whole village had to cooperate. When industry became more important, the significance of the weather for working hours quickly declined. Producing goods now is nearly equally productive during the whole day. In fact, working hours rose and common leisure was reduced. For many reasons working hours grew to an extremely high level, calling for the activities of unions. At last, with the rise of the service sector, the productivity gap probably is reversed: personal services can only be rendered if the consumer has got enough leisure to buy them. Thus the trade sector as well as the service sector have a very strong demand for private working time. As a consequence, common leisure time tends to become more scarce in the long run. From a social point of view individuals get up "too early" and stop work "too late". At the same time agents are working too long.

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