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Paper

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*AND RATIONAL ADDICTION**

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Much of empirically observable consumption behavior seems to be in contradiction with rational choice theory. Here we refer to heavy eating followed by strict dieting, smoking, quitting and starting again, taking drugs, quitting, and starting all over again, etc. In a recent paper, Gary S. Becker and Kevin M. Murphy (1988) demonstrate that a wide variety of consumer behavior – addiction included – is consistent with utility maximization. As a consequence, addictive behavior need not be excluded from the rational choice framework. As Becker and Murphy (1988) point out, an even stronger claim holds true: Analyzing addictive behavior within the rational choice framework provides us with new insight and a better economic understanding of addiction.

Explaining addictive behavior in terms of utility maximizing consumers does not imply, however, that addicts can be considered as *happy* persons. What this framework does is to demonstrate that for a variety of reasons certain consumers can become addicted; once they are addicted, to stop consuming the addictive good would make them worse-off.

A consumer is said to be addicted to a consumption good c if an increase in past consumption of c causes present consumption to rise. In the theory of rational addiction (see George J. Stigler and Becker, 1977, Becker and Murphy, 1988, and Laurence R. Iannaccone, 1986) past consumption is summarized by a stock of *consumption capital* (habits) that together with current consumption affects current utility. Hence, a consumer can be called addicted to c if consumption of c increases with an increase in the stock of its corresponding consumption capital. This definition implicitly assumes that c accumulates a single stock (consumption capital). We call this stock *commodity-specific consumption capital.* It is this case that is extensively discussed in the literature on rational addiction. If on the contrary, a consumption good accumulates several stocks of consumption capital past consumption affects current utility through several channels. As one expects this requires a separate analysis of addiction. It is at this point where the present note steps in. In particular, we study addiction in a model where a single consumption good accumulates two stocks. This is the simplest case without commodity-specific consumption capital but it provides all the economic intuition for the more general case.

One important consequence of the relaxation of commodity-specific consumption capital is that the variety of possible consumption profiles over time is enlarged. This motivates us to explore the relationship between various degrees of addiction and the consumption profile over a person's lifetime. In the case of commodity-specific consumption capital Becker and Murphy (1988) discuss several possible outcomes: They show, for example, that in the case of very strong addiction rational consumers have to go *cold turkey*, i.e., to stop consuming the good abruptly. In other cases with lower degrees of addiction consumption follows a monotonic path. This holds true even for the case of an unstable steady state. But does this imply that cyclical consumption policies cannot be explained by the theory of rational addiction? Certainly not, as has already been demonstrated by Becker and Murphy (1988). Our note elaborates on their analysis and we fully explain the causes for the occurrence of cyclical consumption trajectories. We show that for the case of commodity-specific consumption capital consumption trajectories will always be monotonic. Hence, cyclical consumption paths expressed as stable limit cycles or damped (explosive) oscillations require a consumption good that accumulates at least two stocks. It is the interaction of these two stocks that causes *irregular* behavior. In particular, we show that only if present consumption is positively correlated with past consumption as expressed in one of the two stocks (let's call this stock *eating capital*) but is negatively correlated with the other stock (let's call this one *weight*) cyclical consumption patterns are possible. This implies that consumption cycles require two counterbalancing effects: an addictive one and a satiating one. The addictive force causes current consumption to increase as past consumption accumulates, hence the ascending part of the cycle, the satiating force causes current consumption to decline as habits accumulate, hence the descending part of the cycle.

As mentioned above cyclical consumption profiles can be expressed both in terms of damped or explosive oscillations as well as in terms of stable limit cycles. In the first case the trajectory either converges to or diverges from the steady state consumption capitals, i.e., the long-run solutions. In the latter case the long-run equilibrium is not a single point but rather an invariant manifold, i.e., the limit cycle. Economically, the existence of a stable limit cycle implies that *binges* can continue to cycle throughout much of a person's lifetime. This brings us back to the examples discussed at the very beginning of this section and explains why certain consumer might follow *binges* throughout much of their lifetime.

In Section I we introduce a model of rational addiction that is a variation of the two state variable example of Becker and Murphy (1988). Here we concentrate only on what might be called the *semi-reduced form* model that involves the allocation of consumption goods over time and the corresponding accumulation of two stocks of consumption capital. In Dockner and Feichtinger (1989) we demonstrate that this model can be derived from a more general one that includes the intertemporal choice of leisure time and goods allocation subject to a wealth constraint. In Section II we discuss the relationships between complementarity over time, addiction and satiation and use these results to give precise information about monotonic or cyclical consumption profiles. In Section III we present a numerical example with strictly concave preferences that leads to a stable limit cycle as optimal policy and Section IV concludes this note.

I. The Model

We consider a representative consumer who derives at each instant of time t utility from consumption of goods $c_1(t)$ and $c_2(t)$ and stocks of consumption capital, $S_1(t)$ and $S_2(t)$, as in

$$U(t) = U(c_1(t), c_2(t), S_1(t), S_2(t)). \tag{1}$$

We assume that the utility function $U(c_1, c_2, S_1, S_2)$ is strictly concave jointly in (c_1, c_2, S_1, S_2). Moreover, the cross partial derivative $U_{S_1S_2}$ is set equal to zero, i.e., marginal utility with respect to consumption capital S_i is independent of the level of S_j, $i \neq j$.

The stocks $(S_1(t), S_2(t))$ are measures of past consumption of $(c_1(t), c_2(t))$ that affect current utility through an accumulation process, i.e.,

$$\dot{S}_1(t) = f_1(c_1(t), c_2(t)) - \delta_1 S_1(t), \tag{2}$$

$$\dot{S}_2(t) = f_2(c_1(t), c_2(t)) - \delta_2 S_2(t). \tag{3}$$

Relations (2) and (3) are neoclassical accumulation equations with δ_i as constant depreciation rates and the functions f_i are specified depending on whether the consumption capitals are commodity-specific or not. Throughout the paper we assume that $\delta_1 > \delta_2$.

Assuming an infinitely lived consumer with a constant rate of time preference, r, who maximizes the discounted stream of utility subject to the accumulation equations (2) and (3), the dynamic household problem becomes

$$\max_{c_1(t), c_2(t)} \int_0^\infty e^{-rt} U(c_1(t), c_2(t), S_1(t), S_2(t)) dt \tag{4}$$

subject to (2), (3) and given initial conditions $S_i(0) = S_{i0} \geq 0$.

Model (2) - (4) describes the optimal allocation of the consumption good(s) over time when utility depends on both current and past levels of consumption. It can be derived from a more general model that includes the intertemporal choice of leisure as well as a wealth constraint (see Dockner and Feichtinger, 1989, for details).

The general specification (2) - (4) allows for two important examples as special cases: One with commodity-specific consumption capital and one without it. Adopting the definition of Iannaccone (1986) we call the consumption capitals (habits) commodity-specific if for each S_i there exists a unique commodity c_i, such that f_i and U_{c_i}/U_{S_i} depend only on (c_i, S_i). In our case with two consumption capitals this implies that each consumption good accumulates a single stock, $f_i(c_i(t), c_j(t)) = c_i(t)$, and the marginal rate of substitution between current and past consumption is a function of (c_i, S_i). Thus, the utility function is of the multiplicative separable form $U(c_1, c_2, S_1, S_2) = \tilde{U}_1(c_1, S_1)\tilde{U}_2(c_2, S_2)$. Applying a logarithmic transformation and keeping in mind that this does not change the basic ordinal properties of consumer preferences we restrict attention to the additively separable case $U = \ln \tilde{U}_1 \tilde{U}_2 = \ln \tilde{U}_1 + \ln \tilde{U}_2 = U_1 + U_2$. To sum up, the case of commodity specific consumption capital is represented by the accumulation equations $\dot{S}_i(t) = c_i(t) - \delta_i S_i(t)$ and the utility function $U = U_1(c_1, S_1) + U_2(c_2, S_2)$.

In the case with no commodity-specific consumption capital we restrict attention to the simplest case possible and assume that a single good, $c(t)$, accumulates two distinct consumption capitals – we call it *eating capital*, S_1, and *weight*, S_2, – and that preferences are of the general form represented by (1). Algebraically this amounts to

$$\begin{aligned} \dot{S}_1(t) &= c(t) - \delta_1 S_1(t), \\ \dot{S}_2(t) &= c(t) - \delta_2 S_2(t) \end{aligned} \tag{5}$$

with preferences given by $U(c, S_1, S_2)$.

The optimal paths of $c(t)$ (the case of no commodity-specific consumption capital) or $(c_1(t), c_2(t))$ (commodity-specific consumption capital) and $(S_1(t), S_2(t))$ are determined by the first order conditions

$$\begin{aligned} c \text{ or } (c_1, c_2) &= \arg\max\{U + \lambda_1 \dot{S}_1 + \lambda_2 \dot{S}_2\}, &(6)\\ \dot{\lambda}_1 &= (r + \delta_1)\lambda_1 - U_{S_1} &(7)\\ \dot{\lambda}_2 &= (r + \delta_2)\lambda_2 - U_{S_2} &(8)\end{aligned}$$

and the transversality condition

$$\lim_{t\to\infty} e^{-rt}[\lambda_1(t)(\tilde{S}_1(t) - S_1(t)) + \lambda_2(t)(\tilde{S}_2(t) - S_2(t))] = 0 \qquad (9)$$

for all feasible states $\tilde{S}_1(.), \tilde{S}_2(.)$.

λ_i are the shadow prices of the consumption stocks S_i. They measure the value of the future benefits or costs of consumption. Through the price system λ_1, λ_2 rational consumers take into account the future consequences of current actions. Depending on whether consumption capital has beneficial or harmful effects we expect the prices to be positive or negative.

II. Addiction and Cyclical Consumption Paths

A commodity, c, is called *addictive* if its current consumption increases as habits derived from its previous consumption accumulate. Hence, in the case of commodity-specific consumption capital a consumer displays addictive behavior if $c_i(t)$ increases with an increase in its corresponding $S_i(t)$. This occurs if preferences are characterized by *adjacent complementarity*.[1]

The case of non commodity-specific habits requires a separate treatment. Firstly, we have to present a formal definition of addiction. Secondly, we need to explore if the relationship between addiction and adjacent complementarity carries over to this case. This is not a trivial issue because past consumption affects current utility and hence current consumption through several channels and therefore the interaction of the stocks will be of importance. In our example it is the current level of both *eating capital* and *weight* that influence $c(t)$.

To shed some light on these open questions we concentrate on the optimal path for the simplest case with non-specificity as summarized through (5). Introducing a transformation of variables such that the steady state capital stocks and the steady state level of the single consumption good are zero and allowing for linearizations, the dynamics of the optimal solution is governed by

$$\begin{aligned} S_1(t) &= a_{11}e^{\mu_1 t} + a_{12}e^{\mu_2 t}, &(10)\\ S_2(t) &= a_{21}e^{\mu_1 t} + a_{22}e^{\mu_2 t}, &(11)\end{aligned}$$

[1] The relationship between addiction and adjacent complementarity in the case of commodity-specific consumption capital first appeared explicitly in Marcel Boyer (1983) and was discussed at length by Iannaccone (1986) and Becker and Murphy (1988). It appeared implicitly but was not explicitly recognized in Harl E. Ryder and Geoffrey M. Heal (1973).

where μ_1 and μ_2 are the stable roots (or the two smaller roots in the case of instability) of the linearized modified Hamiltonian system. These dynamics imply that locally around the steady state the optimal $c(t)$ is a linear function of both $S_1(t)$ and $S_2(t)$, i.e.,

$$\begin{aligned} c(t) &= \alpha S_1(t) + \beta S_2(t) \\ &= \frac{(\mu_1+\delta_1)(\mu_2+\delta_1)}{\delta_1-\delta_2} S_1(t) - \frac{(\mu_2+\delta_2)(\mu_2+\delta_1)}{\delta_1-\delta_2} S_2(t) \end{aligned} \tag{12}$$

(see the Appendix for derivations). This relationship allows for a general discussion of addictive and other behavior in the case of non commodity-specific consumption capital. We call a consumer *fully addicted* to c if both stocks of consumption capital are positively correlated to c, i.e., if α and β are positive. In terms of our example, a consumer is fully addicted to c if both an increase in eating capital S_1 and an increase in weight S_2 cause him to consume more of c. This behavior will lead to a monotonic consumption profile and is consistent with convergence to (stability) or divergence from the steady state (instability). A consumer need not be fully addicted to c, but may instead display addictive and satiating behavior at the same time. We call this behavior *partially addicted.* This occurs if one of the stocks is positively correlated to c and the other one negatively. As we will see this behavior most likely results in consumption cycles. But before discussing cyclical consumption patterns let us answer the question what causes a consumer to become fully addicted if habits are not commodity-specific. The next proposition gives an affirmative answer to this question.

PROPOSITION 1 *If consumption capital S_1 and S_2 are not commodity-specific then a consumer is addicted to c (i.e. α and β are both positive) if and only if his behavior displays adjacent complementarity with respect to both stocks.*

Proof: See Appendix.

Proposition 1 generalizes the relationship between addiction and adjacent complementarity to non commodity-specific habits. It also reveals that addiction implies that past consumption as measured in S_1 or S_2 raises the marginal utility of current consumption, i.e., $U_{cS_i} > 0$.

The proof of Proposition 1 also shows that addiction with respect to both habits implies monotonic consumption paths and that it is consistent with convergence to the steady state: As long as we have $-\delta_1 < \mu_1 < -\delta_2$ and $0 > \mu_2 > -\delta_2$ the consumer displays addictive behavior that results in monotonic convergence to the steady state.

As is clear from equation (12) cyclical consumption paths require that one of the stocks is positively and the other one is negatively correlated with c. These cycles can be expressed as damped/explosive oscillations or limit cycles depending on whether the eigenvalues μ_1 and μ_2 are complex with negative (damped oscillations), positive (explosive oscillations), or zero (limit cycles) real parts. As has been demonstrated by Becker and Murphy (1988) the roots μ_1 and μ_2 are conjugate complex if preferences display adjacent complementarity with respect to the stock with the higher depreciation rate and distant complementarity otherwise. In any case, conjugate complex roots imply that consumption is positively correlated with S_1, the stock with the higher depriciation rate, and negatively correlated with S_2. In light of equation (12) this allows for an interesting economic interpretation. Cyclical consumption patterns require two counterbalancing forces: An addictive force that causes current consumption to rise as past consumption accumulates and a satiating force that causes it to fall. The higher depreciation rate of the addictive stock then generates that there are periods with increasing as well as decreasing consumption.

This discussion summarizes the existence of cyclical consumption paths in a model without commodity-specific consumption capital. What remains to be shown is if consumption cycles are possible with commodity-specific habits.

PROPOSITION 2 *If consumption capital is commodity-specific optimal consumption paths are monotonic.*

Proof: See Appendix.

This is an interesting result and underlines the importance of non-specificity for cyclical consumption profiles. Only if a single consumption good accumulates two stocks of consumption capital, one of which generates addictive and the other one satiating behavior, consumption cycles are possible. These cycles, however, are the result of forward looking behavior. Only a consumer with a desire to eat and a dislike for weight who anticipates the future consequences of his current actions can end up in eating and dieting cycles.

III. Persistent Consumption Cycles: A Numerical Example

The general discussion carried out in the preceeding section has revealed that different configurations for optimal policies are possible depending on the nature of the consumption capital (whether it is commodity-specific or not) as well as on complementarity in consumption. In particular, we have seen that the theory of rational addiction is capable of explaining cyclical consumption paths expressed as damped/explosive waves or limit cycles. In this Section we present a numerical example that establishes a stable limit cycle as optimal policy.[2] Our interest in limit cycles as optimal consumption paths is motivated by the fact that a *stable* limit cycle can be identified as persistent oscillatory behavior and hence is capable of explaining binges that continue to cycle throughout much of a person's lifetime.

We have to restrict our analysis to the case without commodity-specific consumption capital. We specify the utility function as

$$U(c, S_1, S_2) = c^{\eta} S_1^{\epsilon} + \frac{\theta}{2}c^2 + \frac{\nu}{2}S_1^2 + \frac{\xi}{2}S_2^2 + \pi c S_1. \tag{13}$$

[2] It should be noted that Marcel Boyer (1978) does mention the possibility of a limit cycle as optimal consumption path but does not characterize it. Becker and Murphy characterize cyclical consumption in terms of damped (explosive) waves but do not discuss limit cycles.

The utility function (13) is strictly concave jointly in (c, S_1, S_2) as long as

$$0 < \eta,\ \epsilon < 1,\ \ \eta + \epsilon < 1,\ \ \theta < 0,\ \nu < 0,\ \xi < 0,\ \ \theta\nu - \pi^2 > 0$$

As the inequality for the marginal utility of consumption ($U_c > 0$) is not globally satisfied for (13) we restrict our numerical calculations to the region of the (c, S_1, S_2) - space where it holds.

For the numerical example we make use of the following parameter values:

$$\begin{aligned} \delta_1 = 0.45,\ \delta_2 = 0.14,\ \eta = 0.50,\ \epsilon = 0.49 \\ \theta = -2,\ \nu = -16,\ \xi = -10,\ \pi = 5.5 \end{aligned} \tag{14}$$

To prove the existence and stability of the limit cycle we apply the Hopf Bifurcation Theorem (see John Guckenheimer and Philip Holmes, 1990, for details). Among other things this requires the existence of a pair of imaginary roots μ_1 and μ_2. As noted, pure imaginary roots result in a positive correlation between c and S_1 and a negative one between c and S_2.

PROPOSITION 3 *If the parameters are specified as in (14) and the time preference rate assumes the critical value* $r_0 = 5.8218875$ *then there exists a pair of imaginary roots that give rise to the local existence of limit cycles. Those cycles are stable and occur for time preference rates* r *slightly greater than* r_0.

Proof: A detailed numerical analysis of this result can be found in Dockner and Feichtinger (1989).

The cyclical trajectories corresponding to the stable limit cycle are depicted in Figure 1. These paths show the following characteristics. Initially, both eating capital and weight are low. Given the positive correlation between S_1 and c consumption rises. While c increases eating capital rises more rapidly than weight. Eventually, consumption levels off while weight continues to increase. With declining consumption and a high rate of depreciation eating capital declines. Dieting also causes weight to decline so that after some time both eating capital and weight are at their initial values and the cycle starts all over again.

IV. Concluding Remarks

This paper presents two new propositions in the theory of rational addiction. Firstly, we show that the notion of commodity-specific consumption capital is crucial for the understanding of cyclical consumption paths. Only a model of rational addiction without commodity-specific consumption capital is capable of explaining consumption cycles. Secondly, we demonstrate that consumption behavior may very well end up in persistent oscillations, i.e., a limit cycle. We derive this result in terms of a numerical example and use it to explain binges that continue to cycle much throughout a person's lifetime.

APPENDIX

Derivation of Equation (12)

Differentiation of (10) and (11) with respect to time t and substituting equation (5) yields

$$c(t) = (a_{11}\mu_1 + a_{11}\delta_1)e^{\mu_1 t} + (a_{12}\mu_2 + a_{12}\delta_1)e^{\mu_2 t}$$
$$c(t) = (a_{21}\mu_1 + a_{21}\delta_2)e^{\mu_1 t} + (a_{22}\mu_2 + a_{22}\delta_2)e^{\mu_2 t}$$

Comparing coefficients gives equation (12).

Proof of Proposition 1

From Dockner and Feichtinger (1991) we know that the stable roots are given by

$$\mu_1 = \frac{r}{2} - \sqrt{(\frac{r}{2})^2 - \frac{K}{2} + \frac{1}{2}\sqrt{K^2 - 4 \det J}}$$
$$\mu_2 = \frac{r}{2} - \sqrt{(\frac{r}{2})^2 - \frac{K}{2} - \frac{1}{2}\sqrt{K^2 - 4 \det J}}$$

where

$$K = -\gamma_1 - \gamma_2 + A_1 + A_2; \text{ and } \det J = \gamma_1\gamma_2 - \gamma_1 A_2 - \gamma_2 A_1$$

with $\gamma_i \equiv \delta_i(\delta_i + r)$ and $A_i \equiv -\frac{1}{U_{cc}}[(r + 2\delta_i)U_{cS_i} + U_{S_iS_i}]$ There is said to be *adjacent complementarity* with respect to stock S_i if $A_i > 0$ and *distant complementarity* if $A_i < 0$ (see Ryder and Heal, 1973).

According to equation (12) and keeping in mind that $\delta_1 > \delta_2$, α and β are both positive if and only if $\mu_1 + \delta_1 > 0$ and $\mu_2 + \delta_2 > 0$ and $\mu_1 + \delta_2 < 0$ hold simultaneously. It is easily shown that this, however, is equivalent with $A_1 > 0$ and $A_2 > 0$. Q.E.D.

Proof of Proposition 2

Iannaccone (1986) has shown that with commodity-specific consumption capital the optimal path of S_i near the steady state is approximated by $S_i(t) = S_i^* + k_i e^{\mu_i t}$ where S_i^* is the steady state level, k_i is a constant and μ_i is a stable eigenvalue given by

$$\mu_i = \frac{r}{2} - \sqrt{(r/2 + \delta_i)^2 - A_i}.$$

As is shown in Dockner and Feichtinger (1989) strict concavity of preferences implies $(r/2 + \delta_i)^2 - A_i > 0$. Hence $S_i(t)$ can never be cyclical. Noting that in the neighborhood of the steady state $\text{sgn}(\dot{c}_i) = \text{sgn}(A_i)\,\text{sgn}(\dot{S}_i)$ implies that consumption can never be cyclical, too. Q.E.D.

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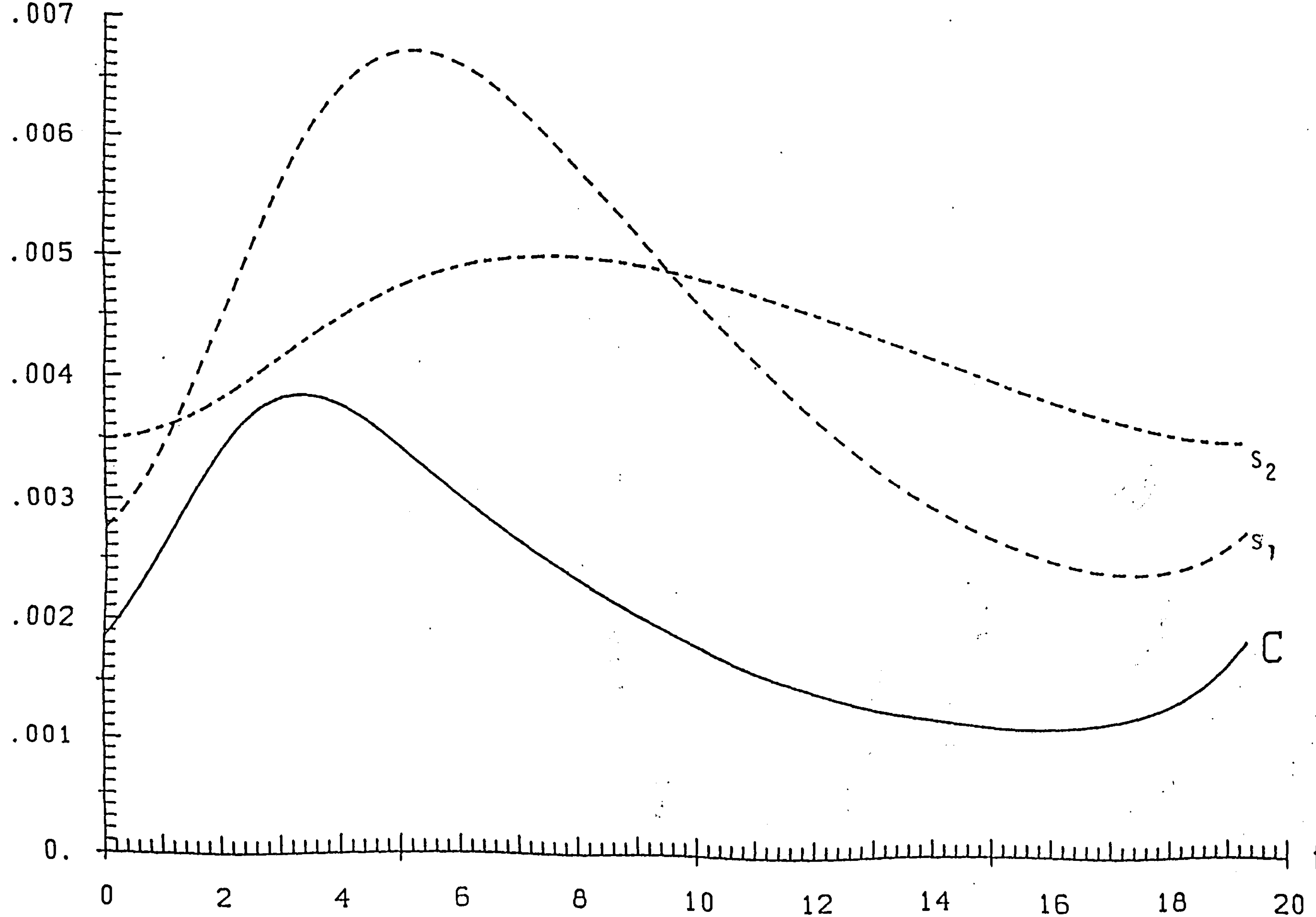

FIGURE 1: OPTIMAL PATHS

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