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## Storing electricity in a country's electrical grid as a key energy problem of the 21st century

Alexander S. Belenky<sup>a</sup>, Richard C. Larson<sup>b</sup>, Leonid A. Roginskiy<sup>c</sup>

<sup>a</sup>*Department of Mathematics, Faculty of Economic Sciences and the International Laboratory of Decision Choice and Analysis, The National Research University Higher School of Economics, Moscow, Russia, and Center for Engineering Systems Fundamentals, Massachusetts Institute of Technology, Cambridge, MA, USA.*

<sup>b</sup>*Center for Engineering Systems Fundamentals; Institute for Data, Systems, and Society; Massachusetts Institute of Technology, Cambridge, MA, USA.*

<sup>c</sup>*Senior Researcher, Russian Academy of Sciences, Moscow, Russia.*

### Abstract

Basic problems of optimizing the structure of a country's electrical grid by incorporating storage facilities and renewable sources of energy into the grid are formulated, and the authors' vision on how to approach some of these problems is offered. A game model for analyzing the potential of an electrical grid with storing facilities to serve its customers and for finding fair (equilibrium) electricity tariffs in it is discussed, and an elementary scheme for estimating advantages of using these fair tariffs by a customer of the grid is proposed.

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### 1. Introduction.

Among the problems of properly developing the national electrical grid in any country, storing electricity in the grid remains critical to an effective and sustainable use of energy, since the grid accounts for a sizable part of the country's energy consumption. For instance, in 2009 in the U.S., 41% of all the energy consumption accounted for its national electrical grid [1].

The ability of the grid:

- a) to make solar, wind, bio, and other alternative, renewable sources of clean energy a vital part of the grid,
- b) to provide a reasonable balance between the availability of electricity in the grid and a thrifty consumption of electricity by the end-customers,
- c) to minimize negative effects of possible outages,
- d) to be adjustable to trends in economic and demographic changes in the country and in the world, and
- e) to be capable of taking advantage of technological breakthroughs in developing new generating systems, transmitting systems, capacitors, accumulators, meters, etc., in the next 50 years

Alexander S. Belenky  
E-mail address: [abelenky@hse.ru](mailto:abelenky@hse.ru).

much depends on how electricity can be stored and released within the grid, in what volumes, and in which parts of the grid.

Effective technologies for storing electricity can contribute greatly to reducing the country's dependence on fossil fuel in general and on imported fossil fuel in particular. A systems approach to analyzing the listed abilities of the grid and other issues relating to storing electricity in it has a high potential to affect the policies of the government of every country in regulating and deregulating the country's electrical grid. These policies in turn affect the investment climate around the grid and may increase the investors' confidence in considering the grid as a whole and its part associated with storing electricity as potential areas of investment.

Particularly, by making investment in the system of storing electricity in the grid at least risk worthy (based upon the expected return on investment in that part of the grid compared with that in the other areas of the energy market), these policies are likely to increase investors' confidence in the outcome of their decisions on investing both in the grid and in systems of storing electricity in the grid. To be trustworthy, this systems approach should be based on mathematical modeling of the grid functioning to provide interested investors with reliable quantitative estimates of their decisions to invest or not to invest in the grid. Both properly adjusted known mathematical models and those to be developed can contribute to forming the foundation of a decision-support system capable of serving the government, the potential investors, and the end-customers with information on strategies of developing and using the grid.

The following two directions of the systems analysis of the grid in any country in the framework of its national strategy of developing this grid seem promising:

1. Developing mathematical models of the grid functioning that would allow one

a) to analyze the potential of the grid to serve the country in the 21st century under possible scenarios of political, economic, technological, and demographic developments both in the country and in the world,

b) to determine the optimal structure of the grid minimizing the effects of potential blackouts on the grid customers, including those caused by natural disasters and cyberattacks,

c) to analyze how the above-mentioned national strategy may affect the future of electricity supply in the country in general and that via the electricity spot markets in particular, and

d) to develop tools for calculating "equilibrium" electricity prices (tariffs) encouraging thrifty consumption of electricity in the country.

2. Developing economic mechanisms encouraging competition among service providers for the right to build-up elements of the electricity storage system and for the right to "wire-up" regions and municipalities of the country to the sources of electricity available in the grid. These sources should include those from the storage system while taking into account regulatory and deregulatory measures, along with tax, wholesale and retail pricing policies that affect investor's confidence in investing in developing a system for storing electricity in the grid.

Advances in both directions will likely constitute a competitive edge in the energy market for any country that undertakes the challenge of applying the systems analysis methodology and techniques in tackling problems of storing electricity as part of the grid development problems. The aim of this paper is to outline the authors' vision on what three particular problems from the above first direction are, and how they can be approached from the systems analysis viewpoint.

## **2. Determining an optimal structure of the grid with storing facilities: available mathematical models and new challenges.**

Making solar, wind, and other alternative sources of energy a sizable part of the grid is often considered a strategic goal by many countries in the 21st century. Yet, achieving this goal presents substantial difficulties, since an immediate consumption and transmission of most of the generated electricity throughout the grid currently underlie a general strategy of using electricity in most countries. However, alternative sources of energy cannot be effectively used for immediate consumption in principle, since, for instance, sun may not shine, and wind may not blow when the electricity demand is high, whereas they may produce plenty of electricity when the demand is low. Thus, rationally storing electricity in the grid and then releasing the accumulated electricity at the requested amounts, even taking into account inevitable losses of energy associated with storing electricity, seems to be a

reasonable approach to encouraging incentives to produce electricity from alternative sources of energy in sizable volumes.

While the conventional wisdom and the existing studies [1] suggest that storing electricity in bulk in the grid is too expensive and technologically difficult, this viewpoint can be challenged [2]-[5]. Indeed, a rational deployment of:

- a) already existing electricity storages such as hydroelectric pump storages,
- b) electricity storages that are build based on new technological ideas such as compressing air in underground reservoirs (for instance, abandoned mines) and underwater reservoirs and releasing the compressed air to generate electricity, and
- c) electricity storages incorporating new large chemical storage systems (the use of which is believed to have the potential to defer for years the need to construct at least some new transmission lines [6]),

along with rationally supplying electricity for both replenishing the storages and the immediate consumption, may turn out to be at least competitive with the current underlying strategy of immediate consumption of electricity in the grid. Even today's chemical systems for storing electricity can accumulate electricity produced by alternative sources of energy [6]-[8]. Moreover, widely anticipated technological breakthroughs in this field may only make the storage of electricity in the grid more effective in the years to come [1].

It is clear that a comparative quantitative analysis of all available electricity storage systems is needed to detect a) where the deployment of particular systems (and which ones) is the most effective now, and b) where it will be such in the future based upon trends in economic, technological, and demographic changes both in the country and in the world.

A comparison of the existing structure of the country's electric grid with possible structures incorporating electricity storage facilities into the grid can identify several prospective configurations of the future country's electric grid, both local and nationwide, and merits and deficiencies of all such configurations should be analyzed. One can expect that, eventually, such an analysis may even provide recommendations on reducing capacities of fossil-fuel generating facilities on account of an effective deployment of storage facilities in the grid.

Problems of multiple facility location in a network—which is the key to finding the above-mentioned configurations of the grid—have extensively been studied by operations researchers [9], [10]. Yet, a comprehensive quantitative analysis of the capability of an electrical grid that incorporates storage facilities to meet customers' demand while reducing chances of blackouts and outages and minimizing the damage that they may cause still presents a challenge. Currently, there are no mathematical tools directly applicable for such an analysis though some inventory control models [11] can be used for obtaining and evaluating possible allocations. Developing such tools in the form of easy-to-operate software will help evaluate perspectives for incorporating alternative sources of energy and electricity storage facilities into the grid, along with those for replacing fossil-fuel facilities producing electricity with alternative sources of energy. In particular, the tools will help evaluate the strategy (and the schedule) of replacing fossil-fuel based generating facilities with alternative sources of energy, as well as the volume of electricity currently produced by fossil-fuel sources that is possible (and reasonable) to replace with electricity produced from these alternative sources of energy. All possible evaluations are critical for establishing confidence in investors considering their potential investment in alternative sources of energy and in electricity storage facilities in electrical grids.

A looming problem of supplying stations for changing batteries in electromobiles can serve as an example of problems relating to storing electricity in the grid that require immediate attention as electricity starts competing with gasoline as a fuel for cars and trucks. That is, recent breakthroughs in developing effective batteries for electromobiles [12], [13] will likely lead to the appearance of recharge stations—similar to gas stations—where a driver will be able to quickly replace batteries for his/her car or truck and continue to travel. Whether each such station should be supplied with electricity on site, or it should be supplied with charged batteries only, collect used batteries, and send collected used batteries to a facility, where all these batteries will be recharged, can be determined by formulating and solving this problem as one of pick-up-delivery ones. Solutions to this type of problems have been proposed over the years [9], [10]. In both cases, developing such a net of stations is, in fact, equivalent to adding a new big customer “distributed over the grid,” since recharge stations are likely to appear throughout the country, and a strategy of their allocation and operation much depends on the strategy of storing electricity in the grid [3].

### 3. Modeling the electricity supply meeting the customers' demand in an electrical grid with electricity storage facilities.

The structure of electricity supply in many countries is currently determined by a) long-term contracts for buying and selling electricity among wholesalers and distributors, and b) so-called spot markets in which the optimization of operational decisions of a supply-demand kind relating to buying and selling electricity beyond amounts produced to fulfill the obligations relating to long-term contracts is conducted. The latter is done through spot market auctions to balance supply and demand in real time, and, for instance, in Texas, up to 5% of all the traded electricity is traded via these auctions [14], [15].

To a large degree, the very existence of electricity spot markets owes to both the nature of the traded product (electricity) and a manner in which the existing grids currently operate, that is, under limited electricity storage capacities. A rapid development of effective electricity storage technologies may substantially affect the necessity for holding electricity auctions (at least in volumes at which it is currently sold via auctions in spot markets throughout the grid). Moreover, it may transform the whole problem of balancing electricity supply and demand in real time to problems of allocating resources and inventory control that can be formulated in either optimization or game forms (though in some countries, long-term contracts are likely to remain the major form of supply for most of electricity produced and consumed there).

The demand for electricity beyond long-term contracts between the distributors and large end-customers can be modeled as a stochastic process with either a finite or infinite population (large number of customers) [16]-[18]. Patterns of arrivals of orders, along with volumes to be supplied within each pattern, can be detected based on available statistical data and quantitatively evaluated with the use of standard operations research techniques developed for inventory control problems [19]. In the case of a finite population, the evaluation of the demanded volume of electricity in both the best and the worst case scenarios can be done by mathematical programming techniques, which was demonstrated for problems of a similar mathematical nature in other service systems [20]. In contrast, in the case of infinite population, for instance, the approximation of the distribution functions of demanded electricity volumes by some standard distribution functions may be a good match. The patterns should reflect both orders for additional amounts of electricity (compared to already ordered amounts under long-term contracts) and refusals to use certain amounts of already ordered electricity.

One can assume that based on these patterns, rational strategies for using electricity storage facilities can be calculated so that providing only certain amounts of electricity representing the unpredictable part of the demand may remain subject to auctions in spot markets. Whether fulfilling even the orders corresponding to this unpredictable part of the demand from replenishable electricity storage facilities in the grid is economically and technologically preferable to fulfilling them by auctions should be quantitatively evaluated. Here, an idea to combine auctions and real time inventory control techniques may be a reasonable alternative to the status quo as well. In any case, such a quantitative evaluation requires a description of the grid functioning, and a variant of this description is presented in [21], [22].

### 4. Finding “equilibrium” prices encouraging thrifty consumption of electricity in a regional electrical grid with storage facilities.

As shown in [21], the interaction of grid customers with a traditional electricity generator, systems transforming the energy of wind and solar into electricity, and systems for storing electricity within a regional electrical grid can be viewed as a three-person game on polyhedra. This game can be written as [21]

$$\begin{aligned}
 \langle \hat{y}, \hat{x} \rangle - \langle \hat{y}, \hat{t} \rangle - f_1(\hat{y}) - f_2(\hat{y}) &\rightarrow \max_{\hat{x} \in \hat{M}} \\
 \langle \hat{y}, \hat{t} \rangle + \langle \hat{y}, \hat{s} \rangle + f_1(\hat{y}) &\rightarrow \max_{(\hat{t}, \hat{s}) \in \hat{T} \times \hat{S}}, \\
 \langle \hat{y}, \hat{x} \rangle + \langle \hat{y}, \hat{s} \rangle + \langle \Delta, \hat{y} \rangle &\rightarrow \min_{\hat{y} \in \hat{\Omega}},
 \end{aligned} \tag{Game 1}$$

where the vectors  $\hat{x} \in \hat{M}$ ,  $\hat{y} \in \hat{\Omega}$ ,  $\hat{t} \in \hat{T}$ , and  $\hat{s} \in \hat{S}$  are those whose components are a) the prices at which the electricity produced by the generator is sold to all the grid customers ( $\hat{x}$ ), b) the volumes of electricity

produced by the electricity generator and by wind and solar sources of energy and those bought by all the grid customers ( $\hat{y}$ ), and c) the tariffs for transmitting electricity that both the generator and the grid customers pay to the transmission company ( $\hat{t}$ ,  $\hat{s}$ , respectively). Here,  $\Delta$  is the vector whose components reflect the expenses associated with operating systems for receiving energy from wind and solar, as well as those for storing electricity. Also, the functions

$$f_1(\hat{y}) = \langle \theta \epsilon, \text{MAX}_{\text{loss}}(\hat{y}) \rangle, f_2(\hat{y}) = \langle \epsilon, \text{MAX}_{\text{expen}}(\hat{y}) \rangle,$$

are such that in the vector functions  $\text{MAX}_{\text{loss}}(\hat{y})$  and  $\text{MAX}_{\text{expen}}(\hat{y})$ , each component is the maximum function of a finite number of linear functions of the vector  $\hat{y}$ . Finally,  $\epsilon \in \mathbb{R}_+^{24}$  is the vector with all the components equalling 1, whereas  $\theta$  is a positive real number.

In this game, the first player is the generator (which may be a group of companies acting as one legal entity), the second player is the transmission company (or a group of such companies acting as one legal entity), and the third one is the “unified customer” (which consists of  $n$  utility companies,  $m$  industrial companies, and  $r$  groups of end users licensed to operate low voltage distribution lines (so-called advanced customers)) that receives electricity from both the generator and renewable sources of energy (both directly and via the storage systems available in the grid). The first player tries to maximize its profit associated with producing and selling electricity, the second player tries to maximize its revenue from transmitting electricity to the customers, and the third player tries to minimize its expenses associated with both buying and transmitting electricity and with operating the storage facilities available in the grid. As shown in [21], [22], this three-person game is an auxiliary one that is connected with an  $m + n + r + 2$ -person game in which  $m$  industrial customers,  $n$  utility companies, and  $r$  groups of advanced customers interact with the generator, renewable sources of energy, and the transmission company within a regional electrical grid. In this game, a thrifty electricity consumption is the “unified customer’s” intent reflected in its payoff function.

As proven in [21], a quadruple of the vectors  $(\hat{y}^*, \hat{x}^*, \hat{t}^*, \hat{s}^*)$  is a Nash equilibrium point in Game 1 if and only if the triple of the vectors  $(\hat{y}^*, \hat{x}^*, \hat{s}^*)$  forms the saddle point  $(\hat{y}^*, (\hat{x}^*, \hat{s}^*))$  in the game

$$(\hat{y}^*, (\hat{x}^*, \hat{s}^*)) \in S p_{(\hat{y}, (\hat{x}, \hat{s})) \in \hat{\Omega} \times (\hat{M} \times \hat{S})}(\langle \hat{y}, \hat{x} + \hat{s} \rangle + \langle \Delta, \hat{y} \rangle), \quad (\text{Game 2})$$

which is a two-person game on the polyhedra  $\hat{\Omega}$  and  $\hat{M} \times \hat{S}$  of the player strategies ( $\hat{\Omega}$  for the first player and  $\hat{M} \times \hat{S}$  for the second one), and the inclusion  $\hat{t}^* \in \text{Argmax}_{\hat{t} \in \hat{T}} \langle \hat{y}^*, \hat{t} \rangle$  holds. Moreover, as shown in [21], any equilibrium point in Game 1 is an equilibrium point in the above-mentioned  $m + n + r + 2$ -person game.

A detailed description of the variables and constraints from both games and the proof of the above assertion are presented in [21]. Though the games are formulated there for a regional electrical grid, games that are based on the same underlying ideas can be formulated for a country’s electrical grid as well.

## 5. Estimating the payback period of an electricity storage system.

Let us show how an electricity storage system can be deployed in the grid by its end customers to reduce their electricity bills. To this end, let us consider a community of grid customers, for instance, that of households, collectively consuming an average of  $W$  kilowatt-hours of electric energy per day (twenty-four hours) from a regional electrical grid, where the electricity tariff depends on the time within the twenty-four hour period. Let us assume that the community plans to acquire (and install) two pieces of equipment that would allow it a) to store electric energy during the night hours, when the tariff is cheap, and b) to use this stored energy during the day hours, without taking any volume of electric energy from the grid. These two pieces of equipment consist of the equipment that accumulates and stores electric energy and the equipment that transforms the stored electric energy into electric energy of industrial parameters, which can be used by each household from the community.

The equipment acquisition requires making an investment, and the community attempts to estimate whether this investment will be beneficial. To this end, the community needs to estimate, in particular, how long would be the payback period for this equipment.

Let us assume that the community is located in the mountains terrain, where the “night” energy can be used to pump water by electric pumps in a reservoir that is located in a place that is several hundred meters higher in the mountains than the community’s location. The equipment that is planned to be acquired will work as follows:

The electric energy obtained during night hours will be transformed into the potential energy of water by the first above-mentioned piece of the equipment. In the daytime, the water from the reservoir will drift via tubes to a turbine, which will rotate the shaft of a generator producing electric energy of industrial parameters (the second piece of the equipment). During the daytime, each household from the community will be plugged off the grid and be supplied with electric energy by the generator.

In both transformations of electric energy, first into the potential energy of water and then back into the electric energy, a part of the electric energy is lost, and both losses are measured by the efficiency coefficients. Let  $W_1$  be the volume of “cheap” electric energy, and let  $W_2$  be the volume of the transformed energy. Then the efficiency coefficient of the first transformation of electric energy is

$$\eta_1 = \frac{W_2}{W_1}.$$

Since after the reverse transformation of energy, the volume of energy  $W_2$  is to provide the volume of energy  $W$  necessary for the daily functioning of the community, the efficiency coefficient of the reverse transformation is

$$\eta_2 = \frac{W}{W_2}$$

so that the cumulative efficiency coefficient of the energy transformation is

$$\eta = \frac{W}{W_1} = \eta_1 \times \eta_2.$$

Let  $\alpha_d$  be a day electricity tariff, and  $\alpha_n$  be a night one, where  $\alpha_d > \alpha_n$ . Then the cost of the electric energy  $W_1 = \frac{W}{\eta}$  that is received from the grid at the low tariff equals  $W_1\alpha_n = \frac{W}{\eta}\alpha_n$  per every twenty-four hours, whereas the cost of the energy for  $t$  days at this tariff equals  $W_1\alpha_n t = \frac{W}{\eta}\alpha_n t$ .

If the necessary electric energy were taken from the grid at the standard (day) tariff, its cost would be equal to  $W\alpha_d t$ , so the difference

$$P = W\alpha_d t - \frac{W}{\eta}\alpha_n t = W\left(\alpha_d - \frac{\alpha_n}{\eta}\right)t$$

represents the community’s profit “generated” as a result of consuming electricity at the “cheap” tariff. It is further assumed that the relations between the numbers  $\alpha_d$ ,  $\alpha_n$ , and  $\eta$  are such that the inequality  $\alpha_d - \frac{\alpha_n}{\eta} > 0$  holds.

Now, let  $Q$  be the total expenses associated with the functioning of the community’s system for transforming and storing electric energy. They include a) fixed expenses  $Q_c$ , determined by those associated with buying and installing the equipment, and b) variable expenses  $Q_t$ , associated with operating the equipment. Thus,

$$Q = Q_c + Q_t.$$

The variable expenses can be represented by the product

$$Q_t = q \times t$$

where  $q$  is the variable daily expenses, and  $t$  is the time (the number of days) of the equipment deployment.

The payback condition for the equipment for transforming and storing electricity within  $t$  days can be written as

$$W\left(\alpha_d - \frac{\alpha_n}{\eta}\right)t \geq Q_c + qt,$$

and, consequently,

$$W\left(\alpha_d - \frac{\alpha_n}{\eta}\right) \geq \frac{Q_c}{t} + q,$$

The payback period for the equipment can be determined from the equation

$$W\left(\alpha_d - \frac{\alpha_n}{\eta}\right) = \frac{Q_c}{t_i} + q,$$

where  $t_l$  is the time required to recover all the fixed expenses  $Q_c$  so that

$$t_l = \frac{Q_c}{W(\alpha_d - \frac{\alpha_n}{\eta}) - q}.$$

It is clear that the fixed expenses  $Q_c$  depend on the expected daily volume of electric energy consumption  $W$ . These expenses decrease as this volume decreases, and they increase as  $W$  increases. Let this regularity be described, for instance, by the function

$$Q_c = a + b \times W^\gamma$$

where  $a$ ,  $b$ , and  $\gamma$  are some positive, real numbers.

Without any loss of generality, proceeding for the existing technology of using the equipment, one can assume that daily expenses  $q$  for operating the equipment do not depend on  $W$  (since under the existing technologies, the work of the equipment is controlled by automatic systems). So for the estimating purposes, one can assume that  $q$  is much smaller than  $W(\alpha_d - \frac{\alpha_n}{\eta})$ , and the payback period can then be calculated by the formula

$$t_l = \frac{a + b \times W^\gamma}{W(\alpha_d - \frac{\alpha_n}{\eta})}.$$

The first derivative of this function over the variable  $W$  equals

$$t'_l = \frac{b(\gamma - 1)W^\gamma - a}{(\alpha_d - \frac{\alpha_n}{\eta})W^2},$$

and it can be used for analyzing the dependency of the payback period on  $W$ .

When  $0 < \gamma \leq 1$ , the inequality  $t'_l < 0$  holds for any  $W > 0$ , so the payback period  $t_l$  is a monotonically decreasing function of  $W$ , and there is no minimum payback period for any feasible values of  $a$ ,  $b$ , and  $\gamma$ . If, however,  $\gamma > 1$ , one can easily be certain that the function  $t_l$  attains its minimum, and this minimum is attained when  $t'_l$  assumes its zero value at the value of the electric energy consumption equalling

$$W_{opt} = \left( \frac{a}{b(\gamma - 1)} \right)^{\frac{1}{\gamma}}.$$

In this case, the minimal value of the payback period  $t_l$  equals

$$t_{lmin} = \frac{\gamma(\gamma - 1)^{\frac{1-\gamma}{\gamma}} a^{\frac{\gamma-1}{\gamma}} b^{\frac{1}{\gamma}}}{\alpha_d - \frac{\alpha_n}{\eta}}.$$

Particular values of the coefficient  $\gamma$  for particular types of the equipment can be determined proceeding from the available statistical data on the cost of the equipment depending on the volume of the electric power that the equipment of each particular type can deliver.

To roughly estimate the payback period for a particular type of the equipment, and taking into account the above remark on the relations between  $q$  and  $W(\alpha_d - \frac{\alpha_n}{\eta})$ , one can use the formula

$$t_l = \frac{Q_c}{W(\alpha_d - \frac{\alpha_n}{\eta})}.$$

Also, assuming that the value of the efficiency coefficient in contemporary systems of transforming energy is close to unit, and to estimate the order of the payback period, let  $\eta = 1$ , which leads to the formula

$$t_l = \frac{Q_c}{W(\alpha_d - \alpha_n)}.$$

Since the night tariff  $\alpha_n$  for electric energy is usually substantially smaller than the day tariff  $\alpha_d$  (for instance, it is almost four times smaller in the city of Moscow), let  $\alpha_d = k\alpha_n$ , where  $k > 1$ . Then

$$t_l = \frac{Q_c}{W(k - 1)\alpha_n}.$$

## 6. Conclusion.

Systems studies of prospective configurations of a country's electrical grid incorporating electricity storage facilities of different types may result in offering solutions that are favorable to changing the way the electricity market currently operates if a) reasonable solutions to establishing the "regulatory status" of electricity storage facilities in the grid are found, and b) "storage economics" recommends reasonable types, capacities, sizes, and locations of storage facilities in the grid.

The "regulatory status" of electricity storage facilities in the grid affects a possible investment in both developing and incorporating these facilities into the grid, since its absence leaves uncertain how any investment in electricity storage facilities will be considered by both the investors and the authorities, which constraints may be imposed on technologies that particular electricity storage facilities will be using, and how all the associated costs will be recovered. However, a dual feature of electricity storage facilities associated with their ability to provide

a) transmitting functions, such as a voltage support at peak hours, and a reduction of the level of the transmission congestion, and

b) generating functions, such as "shaving electricity production peaks" as a result of redistributing the electricity production between the day time and the night time

complicates the matter of adopting fair regulations that would encourage investments relating to storing electricity in the grid.

"Storage economics," which takes into consideration inevitable losses of energy in putting electricity in storage facilities and releasing it from them, should answer the following question: When, where, and at what amount should electricity be stored in the grid, taking into consideration the dynamics and absolute values of wholesale and retail electricity prices in the grid?

Advances in these two directions of systems studies of electrical grids will create incentives for improving the investment climate around electricity markets, which currently does not look encouraging [23].

Mathematical models to be developed should allow one to provide a detailed analysis of the reliability of any electrical grid and its resilience with respect to blackouts and outages. The use of these models should help answer several key questions, in particular:

a) What amount of electricity should a particular country produce on a daily basis under a reasonable use of electricity storage facilities in its electrical grid to "shave all the peaks and fill all the valleys" of the electricity demand?

b) What amount of electricity would be reasonable to store under known or expected distributions of the electricity demand in the grid as a whole and in particular parts of the grid?

c) Which part of the total amount of electricity should be produced by large electricity generators to replenish storage facilities in the grid, and what amount of electricity should be stored on a daily basis and in which parts of the grid?

d) When auctions of electricity are needed, and what volumes of electricity should be auctioned provided electricity can be stored in the grid, and renewable sources of energy are part of the grid?

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