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Identification and optimization of unbalance parameters in rotor-bearing systems

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Abstract: The paper describes the identification and optimization of unbalance parameters in rotor-bearing systems. Two methods are proposed for the identification of the unbalance characteristics: the first is based on modal expansion combined with the use of optimization algorithms, while the second relates to the use of modal expansion technique applied to the inverse problem. In this work, the modal expansion technique is used to overcome the issue related to the use of a reduced number of measuring points. The equivalent unbalance forces can then be estimated by expanding the modal displacements into generalized coordinates of the equations of motion of the rotor system. An error due to the use of a modal expansion is however inevitable, and to solve the issue we propose the adoption of an inverse problem formulation to avoid the computation of the displacements at each measurement point. The axial location of the unbalance must be however known in advance, if the inverse problem approach is used to identify the unbalance parameters. We therefore propose in this work an integrated modal expansion/inverse problem methodology combined with an optimization procedure. The technique allows to identify the axial location of the unbalance, its magnitude and phase. Simulation and experimental investigations are carried out to verify the validity of the proposed methods in a double-disk rotor-bearing system. The results show that identification and optimization procedure for the integrated modal expansion/inverse problem approach provides more accurate predictions than the ones given by the pure modal expansion method.

Key words: Unbalance identification, modal expansion, inverse problem, optimization, rotor-bearing system

1. Introduction

Rotor unbalance is recognized as the major factor leading to malfunction and potential catastrophic failure in rotating machines. Dynamic balancing is necessary to reduce the vibration of the rotor, and therefore the identification of the unbalance parameters plays an important role in this activity.

Model-based methods are widely used to identify faults in rotor-bearing system. Jalan and Mohanty[1, 2] have used the finite elements method (FEM) to model rotor system with unbalance and misalignment, whose conditions were successfully identified by using the model-based approach.

Bachschmid et al[3] have proposed a method to identify multiple faults by minimizing the difference between the measured response and the calculated oneusing frequency-domain least-squares from FEM simulations. Deepthikumar et al[4] have described a polynomial curve approach to identify the distribution of unbalance in the rotor system based on FEM data. Arias-Montiel[5] has presented an on-line estimation schemeof the unbalance forces in an asymmetrical rotor-bearing system with two unbalance disks using asymptotic state and force observers. Lees et al.[6] also gave an overview of the recent developments in model based identification of rotating machines.

Dalmazzo Sanches and Pederiva[7] discussed the theoretical and experimental identification of the unbalance and the residual shaft bow in a Laval rotor based on mathematical modeling and correlation analysis of the rotor responses in the time domain. Pennacchi[8] has proposed a method to identify the rotor unbalance parameters by using least square and the M-estimation. Nauclér and Söderström[9] have considered the identification of unbalance as a reformulated linear estimation procedure using a closed form solution, for which the unknowns of the original non-linearized problem are part of a nonlinear regressive model. Shrivastava and Mohanty[10] have proposed a model-based method to estimate the unbalance parameters (amplitude and phase angle) in the plane of a rotor using a Kalman filter and recursive least square-based input force estimation technique. De Oliveira et al.[11] have also described two methodologies based on Fourier series and Legendre polynomials to identify the unbalance.

Sen et al.[12] have presented a detailed analysis of polar and orbit plots of a pristine shaft, and of those related to a system made from an unbalance mass attached to rotor. Ocampo et al.[13] have proposed a novel methodology based on a two degree of freedom (dof) mathematical simplified model of a rotor with different moments of inertia of the transverse section of the shaft to identify the angular position of the unbalance force. The methodology required the analysis of the polar plots of the rotor response, as well as the information from the vibration response of at least four points in the rotor polar plot. Tiwari and Chougale[14] have presented an identification algorithm to estimate the dynamic parameters of Active Magnetic Bearings (AMBs) and their residual unbalances based on the measured AMB controlling currents and the rotor unbalance responses. Lal and Tiwari[15] have formulated an identification algorithm based on a least-squares fit in frequency domain to estimate parameters of multiple faults in a turbine generator model based on the use of forced response.

The dynamic displacements of rotor are essential to identify the unbalance parameters. In general, the number of measurements is considerably lower than the total number of dofs of the rotor-bearing system. A modal technique is normally used to expand the displacements to the full dofs of the model. Sudhakar and Sekhar[16] have used the minimization of equivalent loads and the vibration response to identify the unbalance within on the context of a modal expansion technique. Sekhar[17] has proposed a model-based method for the on-line identification when an unbalance and a crack act simultaneously. Modal expansion methods and reduced basis dynamic expansion techniques have been compared in fault identification analyses. Chatzisavvas and Dohnal[18] have proposed a robust procedure to identify unbalance by using a modal expansion in the time domain and a Least Angle Regression in the frequency domain.

The equation of motion of a rotor-bearing system can be used in an inverse problem for unbalance identification by back-substituting the measured unbalance responses. Torres Cedillo and Bonello[19] have applied an invasive inverse problem approach for unbalance identification and also the balancing, which required some prior knowledge of the structure. The application of the method was related to the high pressure (HP) compressor stage of a rotor in an aero-engine. Menshikov[20] have investigated Krylov inverse problem, the early diagnostics of a rotor unbalance and the probablistic solutions of an inverse problem, whose steady solutions are obtained by algorithms based on Tikhonov regularization. Menshikov[21] considered the identification of the characteristics of the unbalance in a rotor with two supports as the basis for the measurements inverse problem. Specific assumptions were made to extract the exact solutions and obtain estimates of real unbalance characteristics. The development of intelligent identification methodologies provides the possibility for fast and accurate unbalance identification in rotor systems. Fioride Castro et al.[22] have used meta heuristic search methods to identify the unbalance parameters in different rotary systems.

This paper focuses on the identification of unbalance parameters in rotor-bearing system based on two approaches. At first, a method based on modal expansion and optimization is proposed to identify unbalance parameters in a single-disk rotor-bearing system. Another unbalance parameter identification and optimization technique for integrated modal expansion and inverse problem approach is then proposed to identify unbalance parameters in a double-disk rotor-bearing system. Simulations and experiments are carried out to verify the effectiveness of the proposed methods. A dynamic balance experiment is also carried out to assess the reliability of the identification results.

2. Model-based rotor unbalance identification theory

2.1 Equivalent unbalance force identification based on modal expansion

Here the rotor-bearing systems are modeled using finite elements with the continuous rotor-bearing system being discretized in lumped masses and supports. The shaft is modeled with beam elements with two translational (vertical and horizontal) and two rotational (around the vertical and horizontal axes) degrees of freedom per node. The disc is modeled as a rigid mass with gyroscopic moments added to the damping matrix. The bearings are simulated as concentrated translational springs and viscous damper at the corresponding nodes. The equations of motion for the rotor-bearing system can be expressed as:

$$[M]\ddot{r}_0(t)+[D]\dot{r}_0(t)+[K]r_0(t)=\{F_0(t)\} \tag{1}$$

Where [M] and [K] are the mass and stiffness matrices of undamaged rotor-bearing system. The matrix [D] also includes bearing damping, gyroscopic forces. The vector $\{F_0(t)\}$ consists of the initial operating load, while $r_0(t)$ is undamaged rotor vibration displacement.

After adding the unbalance to the undamaged (pristine) rotor system, the motion equation of the rotor system can be expressed as:

$$[M]\ddot{r}(t)+[D]\dot{r}(t)+[K]r(t)=\{F_0(t)\}+\{\Delta F(\beta,t)\} \tag{2}$$

Where $\{\Delta F(\beta,t)\}$ represents the theoretical equivalent loads generated in the system by the fault, and $r(t)$ is the generalized coordinates vector of the damaged rotor. The residual vibrations due to fault are equal to differences of the generalized coordinates between the damaged and pristine configurations:

$$\Delta r(t)=r(t)-r_0(t),\ \Delta\dot{r}(t)=\dot{r}(t)-\dot{r}_0(t),\ \Delta\ddot{r}(t)=\ddot{r}(t)-\ddot{r}_0(t) \tag{3}$$

The rotor system can be regarded as a linear, but at the same time it is assumed that the bearing stiffness and damping remain constant before and after the unbalance excitation at the same speed[23]. Only the effect of the unbalance force is therefore considered here. Subtracting Eq.(1) from Eq. (2), and substituting in Eq. (3) we obtain:

$$[M]\Delta\ddot{r}(t)+[D]\Delta\dot{r}(t)+[K]\Delta r(t)=\{\Delta F(\beta,t)\} \tag{4}$$

The introduction of an unbalance fault leads to the generation of an equivalent unbalance force. The minimization of the difference of the equivalent loads generated computed from the theoretical fault model allows the identification of the fault parameters. The FE model of the rotor bearing system has four dofs per node. The actual measurements in a single point can only be carried out for the translational dofs, and this is not sufficient to identify the unbalance. A modal expansion technique is therefore used to obtain the full dofs responses. The equations of modal expansion are expressed as:

$$\{\Delta r_M(t)\}=[A]\{\Delta r(t)\} \tag{5}$$

These measured residual vibrations $\{\Delta r_M(t)\}$ are related to residual vibrations at full dof $\{\Delta r(t)\}$ by the measurement matrix [A] padded with zeros, except in those rows or columns corresponding to the measurement points. The undamaged (pristine) model matrix Φ is used to extend the displacements of the measuring points. The full residual vibrations are estimated as:

$$\{\Delta r(t)\}=\Phi\left([A\Phi]^T[A\Phi]\right)^{-1}(A\Phi)^T\{\Delta r_M(t)\} \tag{6}$$

The equivalent loads due to the unbalance are obtained as:

$$\{\Delta F(\mathrm{t})\}=[M]\Delta\ddot{r}(t)+[D]\Delta\dot{r}(t)+[K]\Delta r(t) \tag{7}$$

The unbalance force due to a single unbalance with eccentricity u at a phase angle of φ acting on the rotor system at a speed of Ω is replaced by equivalent forces along the horizontal and vertical directions. The equivalent unbalance forces related to at all other nodes are ignored. The mathematical model of the unbalance forces can be expressed as in time domain as:

$$\{\Delta F(\beta,t)\}=u\Omega^2\begin{bmatrix}\cos(\Omega t+\varphi)\\ \sin(\Omega t+\varphi)\end{bmatrix} \tag{8}$$

The objective function for the optimization is established by minimizing the difference between the theoretical unbalance force and the estimated equivalent one. Specific optimization methods are then used to minimize the objective function and identify the unbalance parameters.

2.2 Rotor unbalance parameters identification method for integrated modal expansion and inverse problem

Standard inverse problem techniques need the prior knowledge of the position of the unbalance. We propose here an unbalance identification method for integrated modal expansion applied to the inverse problem to overcome this issue. The modal expansion is used to here first to identify the location of unbalance, followed by the application of an inverse problem with an optimization algorithm to identify unbalance parameters. The system of equation of motion equation of the rotor has the same form as Eq.(1):

$$[M]\ddot{r}(t)+[D]\dot{r}(t)+[K]r(t)=\{F_0(t)\} \tag{9}$$

In general, the steady-state response of the rotor can be expressed as a complex form using Euler's formula ($R = R_0 \exp^{i\Omega t}$,in which R_0 is the amplitude of the vibration displacement, and R is the steady-state displacement at the speed of operation). Substituting R into Eq.(9) solves the amplitude of the rotor vibration displacement R_0:

$$R_0 = \Omega^2 \left[-M\Omega^2 + i\Omega C + G\Omega + K \right]^{-1} F \tag{10}$$

In Eq.(10), C and G are the damping matrix and gyroscopic matrix respectively. Assuming $H = \Omega^2 \left[-M\Omega^2 + i\Omega C + G\Omega + K \right]^{-1}$, the expression of R_0 can be reduced as:

$$R_0 = HF \tag{11}$$

Let N be total number of dofs in the model of the rotor-bearing system. the matrix H will then have dimensions equal to 4N×4N, which makes the calculation of R_0 not appealing from a computational perspective. We therefore establish a matrix B that extracts the rows corresponding to the measuring points, and the columns where the unbalances are located. In addition, the unbalance forces values are contained in by the vector Q. R_0 will therefore obey the following transformation:

$$R_0 = B \times Q \tag{12}$$

Substituting the measured vibration into the Eq.(12),the unbalance forces can be estimated by

$$Q = B(\Omega)^{-1} \times R_0 \tag{13}$$

2.3 Optimization for unbalance parameters

The theoretical load is a function of the unbalance parameters, and the estimated unbalance forces can be expressed by the measured displacements. The objective function for the minimization problem can be expressed as follows:

$$\min \left| \{\Delta F(\beta,t)\} - \{\Delta F(t)\} \right| \tag{14}$$

In Eq.(14), $\{\Delta F(\beta,t)\}$ is the theoretical unbalance force, and $\{\Delta F(t)\}$ is the estimated equivalent unbalance force. These vectors can be converted into complex exponential form:

$$\{\Delta F(\beta,t)\} = u\Omega^2 e^{(i(\Omega t+\varphi))} \tag{15}$$

$$\{\Delta F(t)\} = |F| e^{(i(\Omega t+\alpha))} \tag{16}$$

The vector $\{\Delta F(t)\}$ contains information about the magnitude $|F|$ and phase angle α of unbalance force. To reduce the number of variables that are independent of the unbalance parameters, time is eliminated by dividing Eq.(15) with Eq.(16) and transforming the result in a natural logarithm form, so that Eq.(14) becomes:

$$\min\left|\ln\frac{u\Omega^2}{|F|}+\mathrm{i}(\varphi-\alpha)\right| \tag{17}$$

Unbalance masses are distributed on different disks in double-disk rotor-bearing system. The objective function is therefore established as:

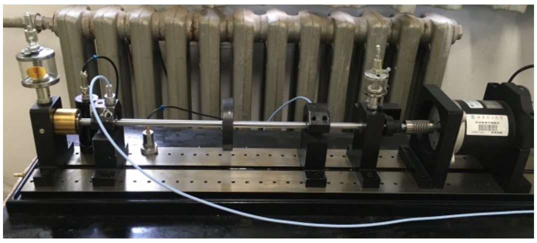
$$\min\left(\left|u_1\Omega^2e^{\mathrm{i}(\Omega t+\varphi_1)}-F_1e^{\mathrm{i}(\Omega t+\gamma_1)}\right|+\left|u_2\Omega^2e^{\mathrm{i}(\Omega t+\varphi_2)}-F_2e^{\mathrm{i}(\Omega t+\gamma_2)}\right|\right) \tag{18}$$

In Eq.(18), u_1, φ_1, u_2, φ_2 are unbalance magnitude and phase in the double-disk rotor-bearing system, while F_1, γ_1, F_2, γ_2 are equivalent unbalance force amplitudes and phases estimated by the application of the inverse problem. To identify the unbalance parameters, different optimization algorithms are used to optimize the objective function (Eq.(17), Eq.(18)). The *Antlion*[24] (ALO), *Simulated annealing*[25] (SA) and *fly fruit algorithm*[26] (FOA) are used in this work, and they are codes into a Matlab(R2014a) platform.

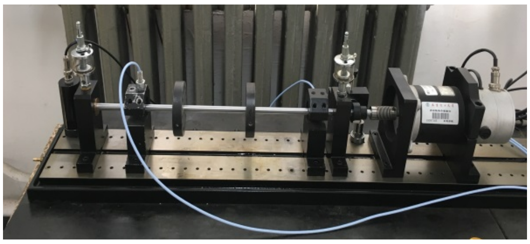
3. Description and modeling of the test rig

3.1 Description of the test rig

Simulations and experiments are carried out by reproducing the test rig shown in Fig.1. The single-disk and double-disk rotors are driven by a DC motor rated at 0.3KW with a maximum speed reaches 12000rev/min. A bellows coupling is used to connect the motor to the shaft. The photoelectric sensor, fixed on the right bearing support records the rotor speed by receiving the reflected pulse. The dynamic displacements of the rotor are measured by three eddy current displacement proximity probes. The main parameters of the test rig are shown in Table 1.



(a)single-disk rotor-bearing system



(b)double-disk rotor-bearing system

Fig. 1 Test rig of rotor-bearing system

Table 1 Parameters of test rig

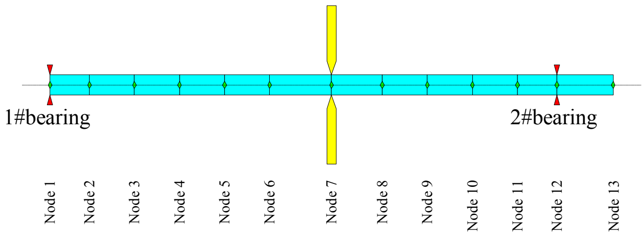
Shaft parameters	
Diameter	10mm
Length	500mm
Density	7810Kg/m^3
Modulus of elasticity	2.08x10^{11}N/m^2
Poisson ratio	0.3
Disc parameters	
Internal diameter	10mm
External diameter	78mm
Thickness	15mm
Mass	0.5Kg

Bearing parameters	
1# Bearing journal diameter	24mm
Clearance	0.05mm
Width	20mm
2#Bearing journal diameter	10mm
Clearance	0.01mm
Width	15mm
Lubricating oil number	32

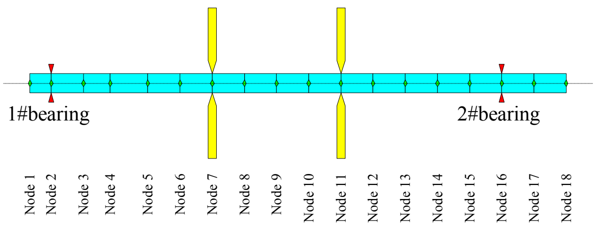
3.2 Modeling and analysis of the test rig

Fig.2 shows the FE models of a single-disk and a double-disk rotor-bearing system. The shaft is discretized into finite beam elements with two translational and two rotational dofs per node according to Timoshenko beam theory. The shafts are discretized into 12 2-nodes elements for the single-disk rotor, and 17 similar beam elements for the double-disk rotor-bearing system. Disc and bearings are modeled as concentrated parameters according to paragraph 2.1.

The Bode diagrams of the two rotor-bearing systems are shown in Fig. 3. The first critical speed is 2800rev/min for the single-disk rotor-bearing system and 2550rev/min in the double-disk one. The measured first critical speeds of the two rotor-bearing systems are 2820rev/min and 2580rev/min respectively. The relative errors are 0.7% and 1.2%, and demonstrate the accuracy of the model. The first three modes of the rotor-bearing system with natural frequencies are shown in Fig.4. In this work, the first mode in the single-disk and the second mode in the double-disk rotor bearing system are used for modal expansion, respectively.

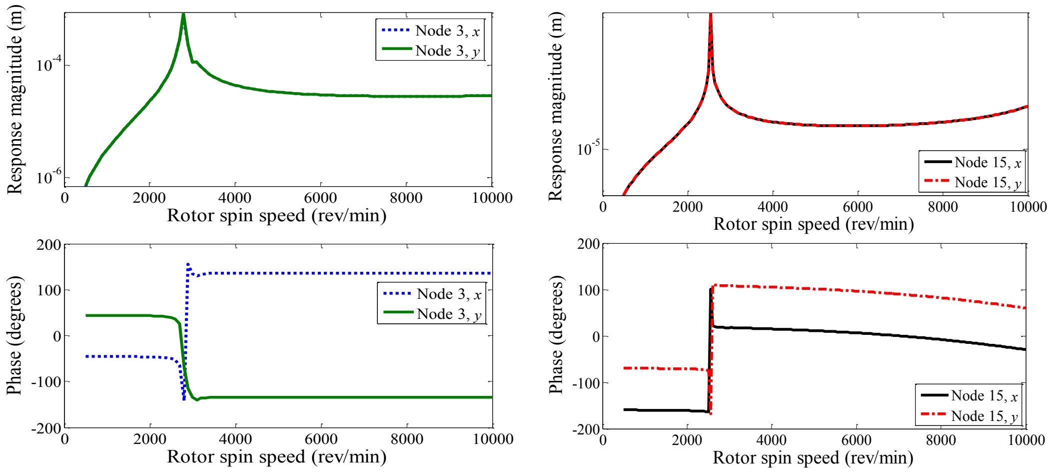


(a) Single-disk rotor-bearing system



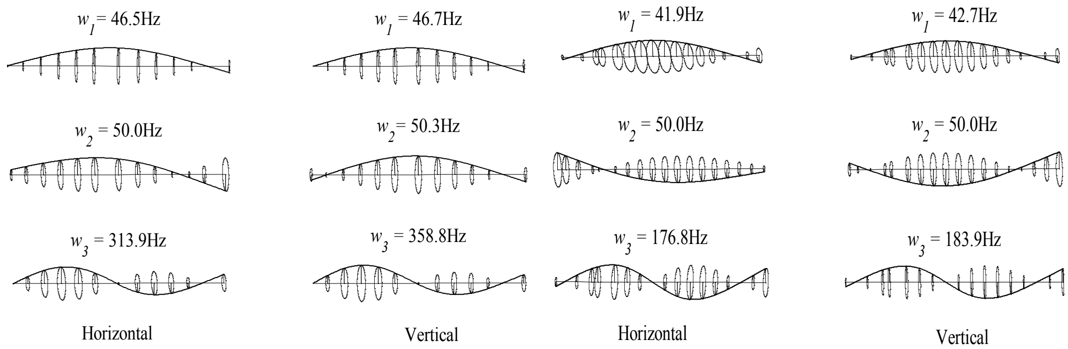
(b) Double-disk rotor-bearing system

Fig. 2 Model of the test rig



(a)Single-disk rotor-bearing system (b) Double-disk rotor-bearing system

Fig. 3 Steady-state response of the rotor system



(a) Single-disk rotor-bearing system (b) Double-disk rotor-bearing system

Fig.4 Mode shapes of the rotor-bearing system related to the first three natural frequencies

4. Numerical simulation

4.1 Identification of the unbalance parameters in the single-disk rotor-bearing system

The disk has 16 holes placed at a radial distance of 30.0mm (Fig.5). The unbalance masses added to the disk are shown in Table 2. The unbalance forces and the vibration responses of the system are calculated at 6000 rev/min.

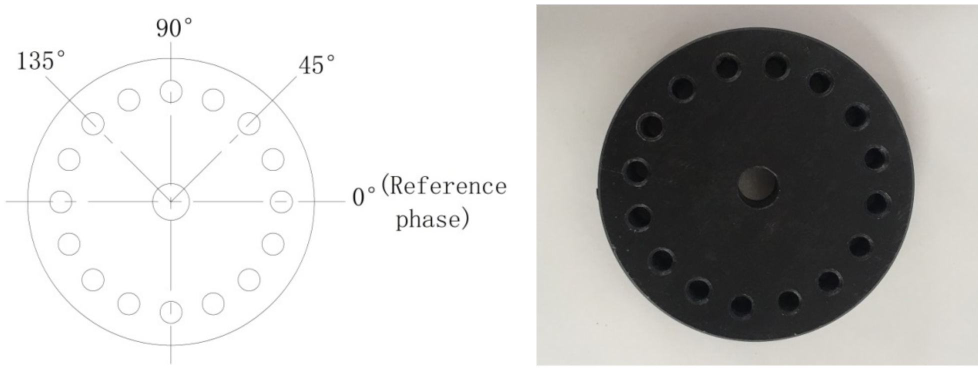


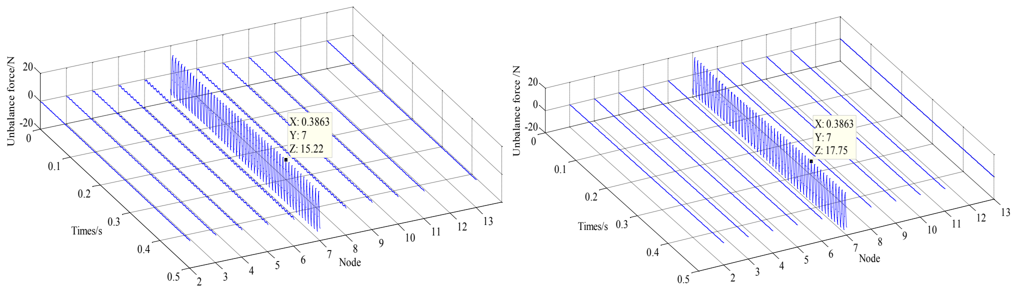
Fig.5 Front view of the disc

Table 2 Unbalance induced in the disc

Unbalance type	Unbalance magnitude(g mm)	Unbalance phase(degree)
1	29.4	45

2	39.3	180
3	45.0	135
4	49.2	225

The displacement of the reduced number of measuring points can be extended to all dofs by modal expansion to estimate the equivalent unbalance forces by using Eq.(7)). Taking the 45g mm at 135 degree case as an example, the equivalent vertical unbalance forces are estimated by modal expansion of 3dofs and all 52 dofs (Fig.6). The carpet plot shows that the magnitude of the equivalent vertical unbalance force calculated expanding 3 dofs is 14.2% smaller than the force calculated when all the 52 dofs are considered. The location of the unbalance is however clearly identified at node 7, which corresponds to the position of the disk. The unbalance parameters are identified by using different optimization algorithms. It is apparent that the FOA performs better the identification of the magnitude of unbalance (Table 3). The iterative process of the three algorithms is shown in Fig 7. ALO makes use of the lowest number of iterations, with the SA requiring the highest number of steps to identify the unbalance. FOA also provides the best performance both in terms of magnitude and phase angle % difference.

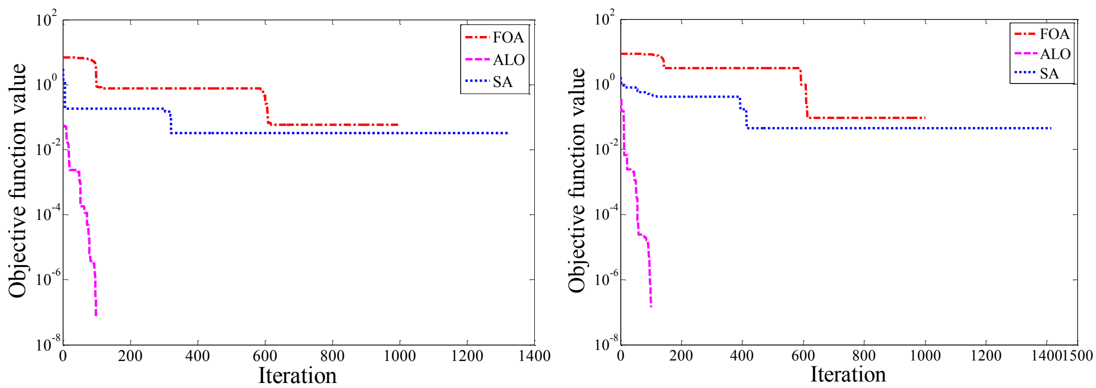


(a) 3dof (b) 52dof

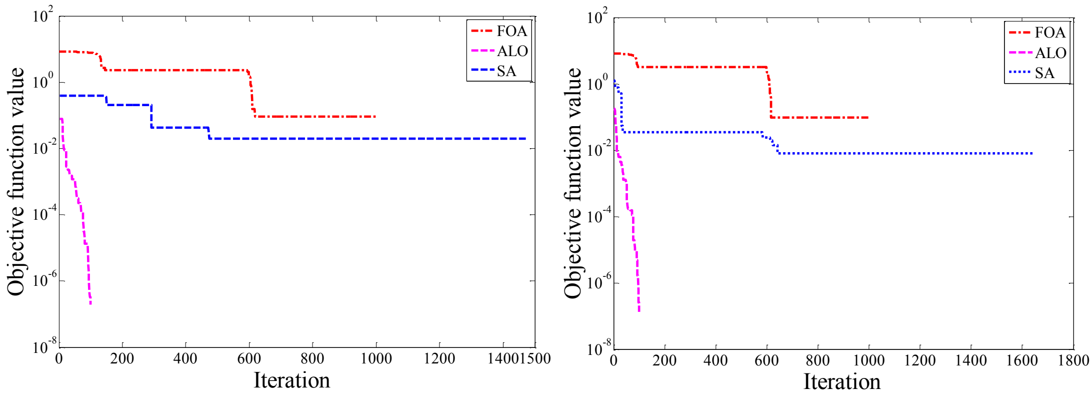
Fig.6 Equivalent vertical forces estimated in the system of an unbalance 45 g mm at 135degree in case of measured vibrations at (a) 3dof and (b) 52dof

Table 3 Results of unbalance identification

Unbalance	Unbalance magnitude (g mm)			Magnitude error (%)			Unbalance phase (degree)			Phase error (%)		
	ALO	SA	FOA	ALO	SA	FOA	ALO	SA	FOA	ALO	SA	FOA
1	25.05	25.88	26.54	-14.80	-11.97	-9.73	45.27	45.00	45.07	0.60	0.00	0.16
2	33.48	33.58	36.74	-14.81	-14.55	-6.51	180.25	180.31	180.15	-0.36	0.17	0.08
3	38.34	39.07	42.00	-14.80	-13.18	-6.67	135.27	134.94	135.21	0.20	-0.04	0.16
4	41.92	42.25	44.24	-14.80	-14.13	-10.08	225.25	225.20	226.88	0.11	0.09	0.84



(a)29.5g mm at 45 degree (b)39.3 g mm at 180 degree



(c) 45g mm at 135 degree (d)49.2 g mm at 225 degree

Fig.7 Iterative process of different unbalance optimization algorithms

4.2 Identification of the unbalance parameters in a double-disk rotor-bearing system

To overcome the inherent limitations of inverse problem, the modal expansion is used to identify the location of unbalance. This integrated modal expansion/inverse problem approach is used to identify unbalance parameters in a double-disk rotor-bearing system. The distribution of the unbalance parameters is tabulated in Table 4. The equivalent unbalance forces at each node are calculated by modal expansion(Fig.8). The largest unbalance forces are located at node 7 and 11, i.e. the locations of the unbalance are correctly identified by using the modal expansion technique. Substituting those unbalance forces into the objective function (Eq.(18)) can then perform the optimization process. In this case, the FOA fails to identify the unbalance due to the presence of a local minimum. Only ALO and SA are used to identify the unbalance in this double-disk rotor-bearing system.

The results of the unbalance parameters identified through modal expansion with the ALO and the SA algorithm are shown in Table 5, while the ones related to the integrated modal expansion/inverse problem approach are tabulated in Table 6. It is quite evident that the unbalance parameters identified by using the integrated approach are more accurate. The iterative process of the unbalance identification is illustrated in Fig.9, showing that the ALO algorithm requires a lower number of iterations.

Table 4 Distribution of the unbalance in the double-disk system

Disk	unbalance(g mm)	Phase(degree)
1	69.9	180
2	49.2	135

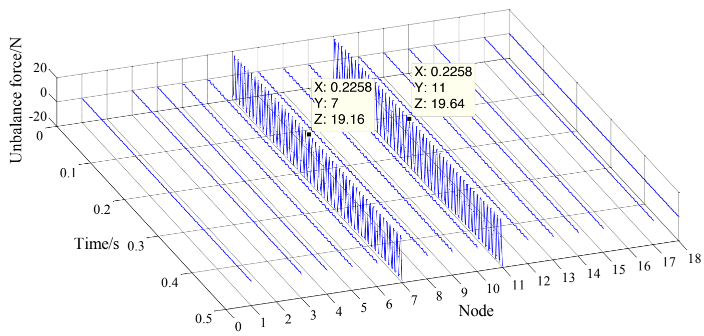


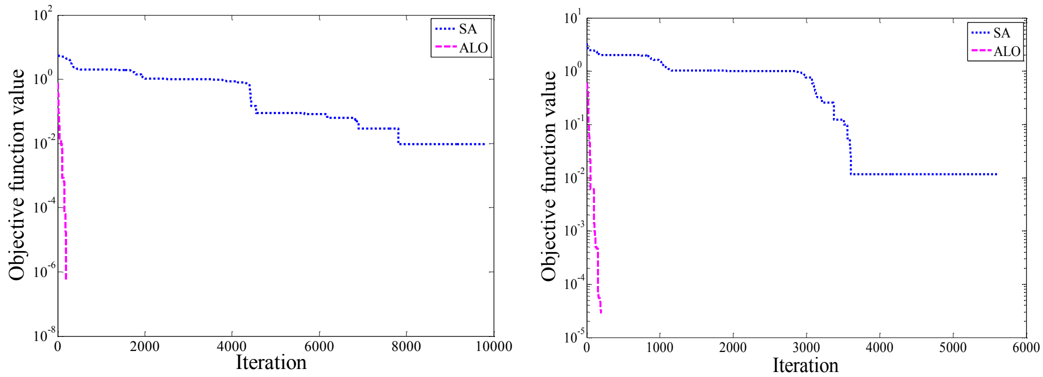
Fig.8 Unbalance forces of at each nodeof the double-disk rotor-bearing system

Table 5 Identified results of unbalance by modal expansion with optimization algorithms

Unbalance disk	Unbalance magnitude(g mm)		Magnitude error (%)		Unbalance phase (degree)		Phase error (%)	
	ALO	SA	ALO	SA	ALO	SA	ALO	SA
1	48.65	48.54	-30.40	-30.56	157.07	157.05	-12.74	-12.75
2	50.22	50.21	2.07	2.05	160.77	161.19	19.09	19.40

Table 6 Identified results of the unbalance by combing the modal expansion and the inverse problem approach with the optimization algorithms

Unbalance disk	Unbalance magnitude(g mm)		Magnitude error (%)		Unbalance phase (degree)		Phase error (%)	
	ALO	SA	ALO	SA	ALO	SA	ALO	SA
1	70.19	70.60	0.41	1.00	179.32	179.51	-0.38	-0.27
2	50.25	50.48	2.13	2.60	136.51	136.41	1.12	1.04



(a) modal expansion (b) integrated modal expansion and inverse problem

Fig.9 Iterative process of the unbalance identification in double-disk rotor-bearing system

5. Experimental unbalance identification

5.1 Identification of the unbalance in a single-disk rotor-bearing system

The single-disk rotor-bearing system shown in Fig.1(a) is used for the experimental unbalance identification. The vibration responses of the rotor-bearing system at 6000rev/min are firstly measured without adding any unbalance masses (Fig.10). The unbalance distribution shown in Table 2 is then applied to the disc, and the corresponding vibration responses are measured (Fig.11). The equivalent unbalance force estimated by modal expansion is plotted in a carpet plot shown in Fig.12, which demonstrates the accuracy of the identification of location of unbalance by the modal expansion technique. The results of the unbalance identification through the modal expansion with the optimization algorithms are tabulated in Table 7. The errors of the unbalance parameters identified by the FOA method are the lowest, while the ALO-derived ones show the largest discrepancies. The iterative processes of the unbalance identification are shown in Fig.13, which indicates the ALO algorithm requires the lowest number of iterations, and the SA the highest.

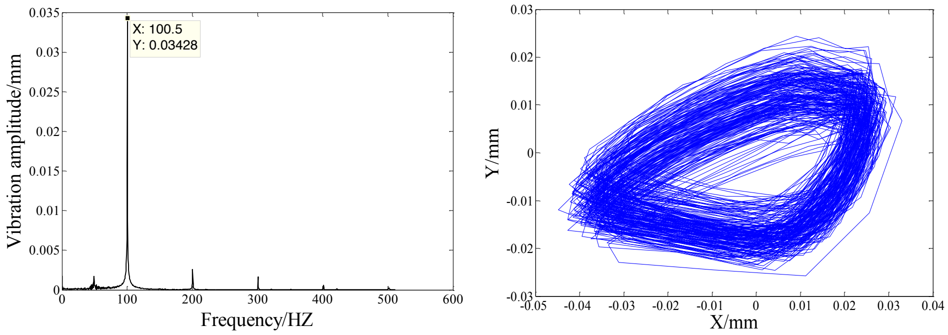
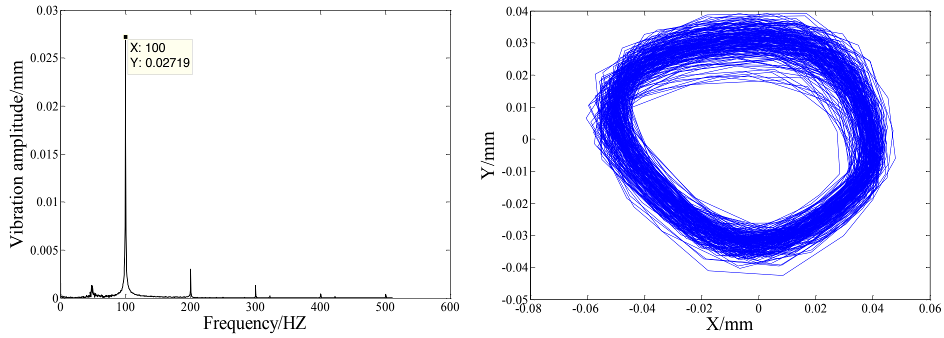
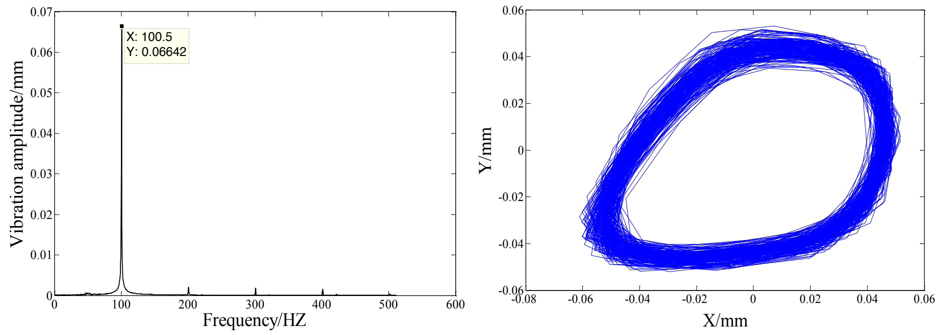


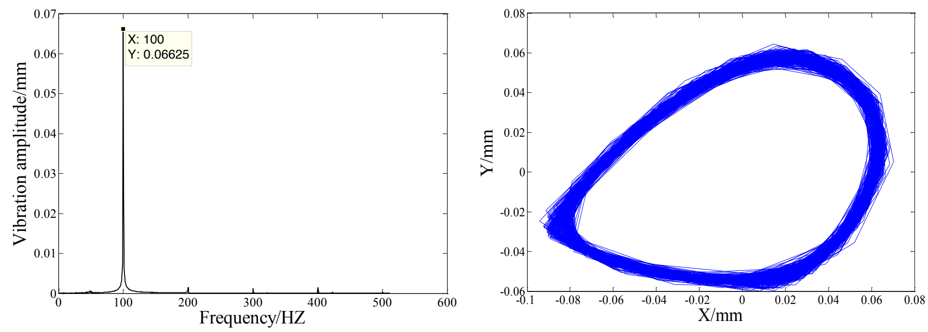
Fig.10 Initial vibration response and orbit without unbalance at 6000rev/min



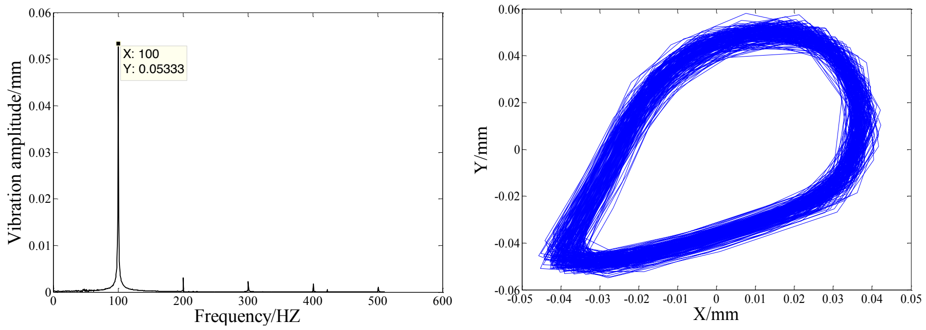
(a)Vibration response and orbit with 29.4g mm at 45 degree



(b) Vibration response and orbit with 39.3g mm at 180 degree

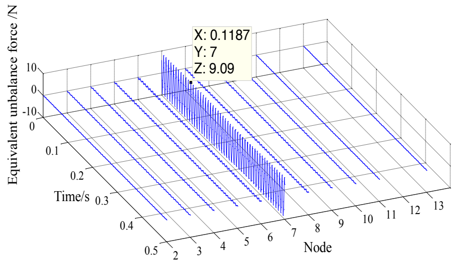


(c) Vibration response and orbit with 45g mm at 135 degree

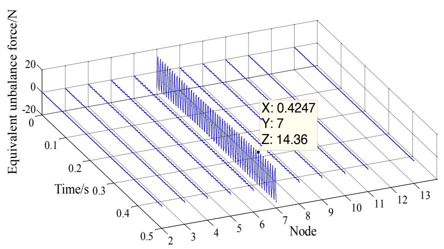


(d) Vibration response and orbit with 49.2g mm at 225 degree

Fig.11 Vibration response and orbit with different unbalance at 6000rev/min

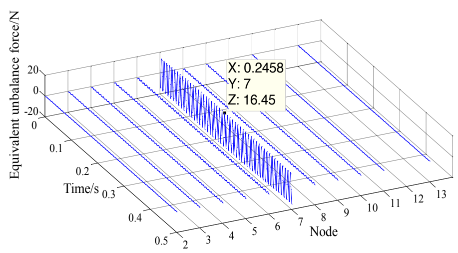


(a)29.4g mm at 45 degree

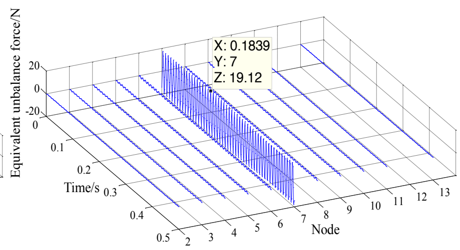


(b) 39.3g mm at 180 degree

5



(c) 45g mm at135 degree

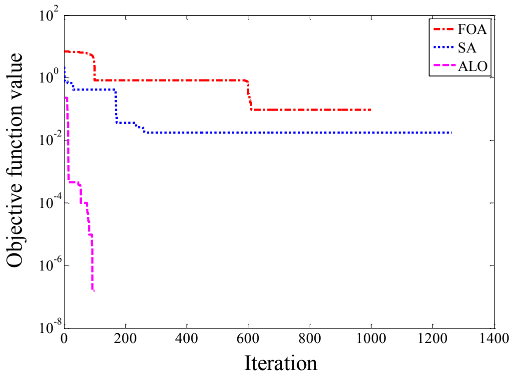


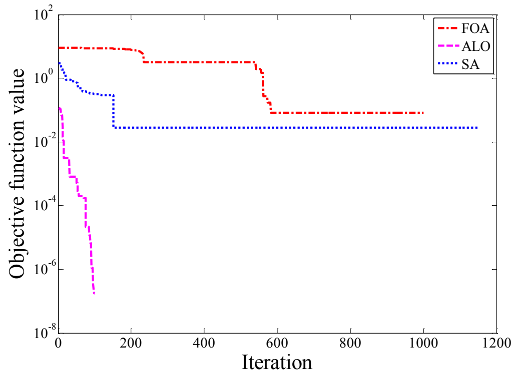
(d) 49.2g mm at 225 degree

Fig.12 Equivalent unbalance force calculated by modal expansion

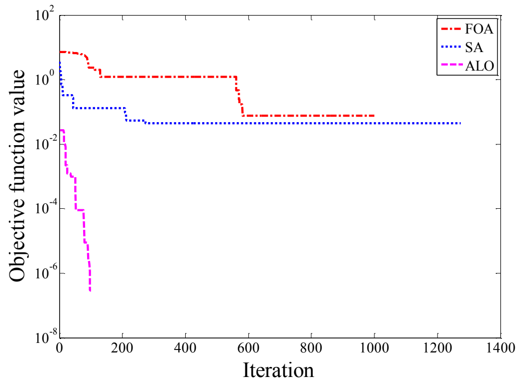
Table 7 Results of unbalance identification by modal expansion with different algorithms

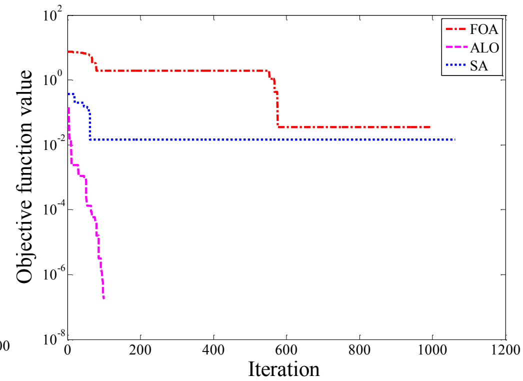
Unbalance	Unbalance magnitude (g mm)			Magnitude error (%)			Unbalance phase (degree)			Phase error (%)		
	ALO	SA	FOA	ALO	SA	FOA	ALO	SA	FOA	ALO	SA	FOA
1	23.08	23.24	24.57	-21.50	-20.95	-16.43	49.79	48.87	45.74	10.64	8.60	1.64
2	36.40	37.19	39.12	-7.38	-5.37	-0.46	187.54	186.50	185.30	4.19	3.61	2.94
3	41.71	43.59	44.51	-7.31	-3.13	-1.09	153.33	152.94	151.12	13.58	13.29	11.94
4	48.50	49.03	49.27	-1.42	-0.35	0.14	220.61	221.17	222.39	-1.95	-1.70	-1.16





(a) 29.4g mm at 45 degree (b) 39.3g mm at 180 degree



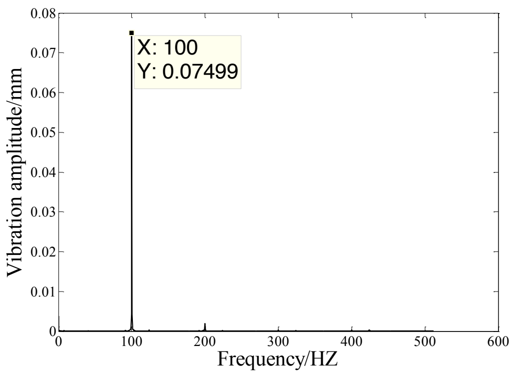


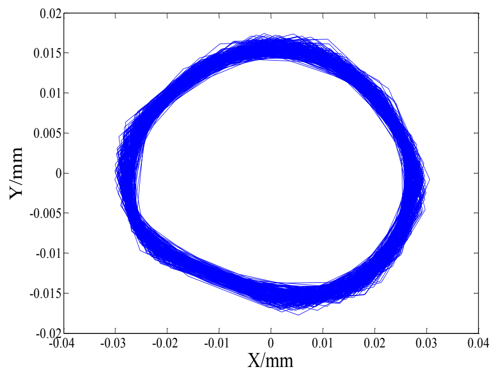
(c) 45g mm at135 degree (d) 49.2g mm at 225 degree

Fig.13 Iterative process of unbalance identification by modal expansion with optimization algorithm

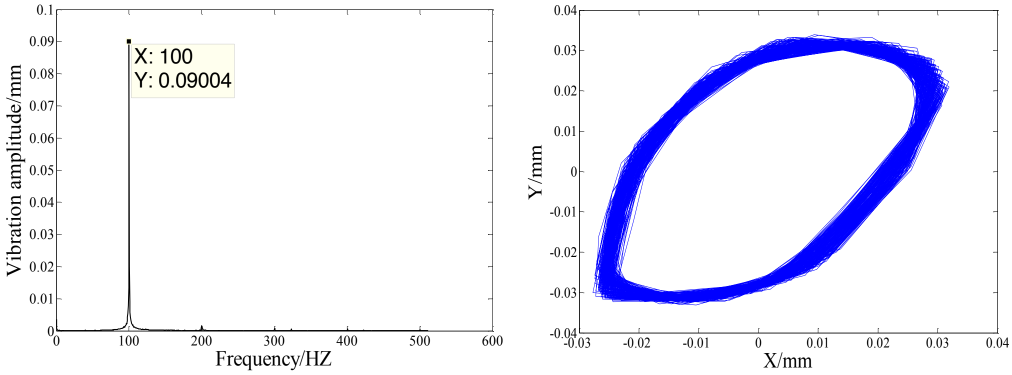
5.2 Identification of the unbalance in a double-disk rotor-bearing system

The double-disk rotor-bearing system of Fig.1(b) is used for this particular experiment. The unbalance masses of Table 4 are now introduced in the two disks. The vibration responses (including the pristine and the fault response) are measured at 6000rev/min (Fig.14). Equivalent unbalance forces are estimated by modal expansion (Fig.15). The identified unbalances through modal expansion with optimization are tabulated in Table 8. The unbalance parameters identified by integrated modal expansion/inverse problem with different optimization algorithms are shown in Table 9.





(a)Initial vibration response and orbit



(b)Unbalance vibration response and orbit with unbalance

Fig.14 Measured vibration response at 6000rev/min

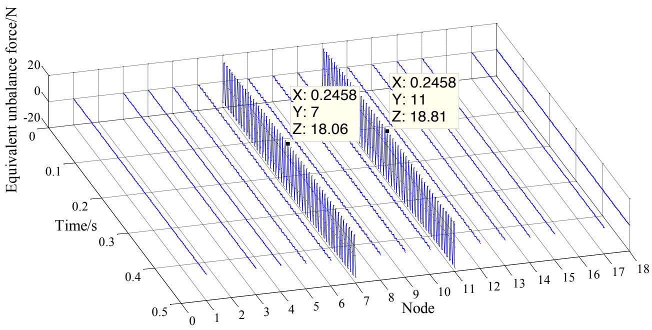


Fig.15 Equivalent unbalance forces estimated by modal expansion in the double-disk system

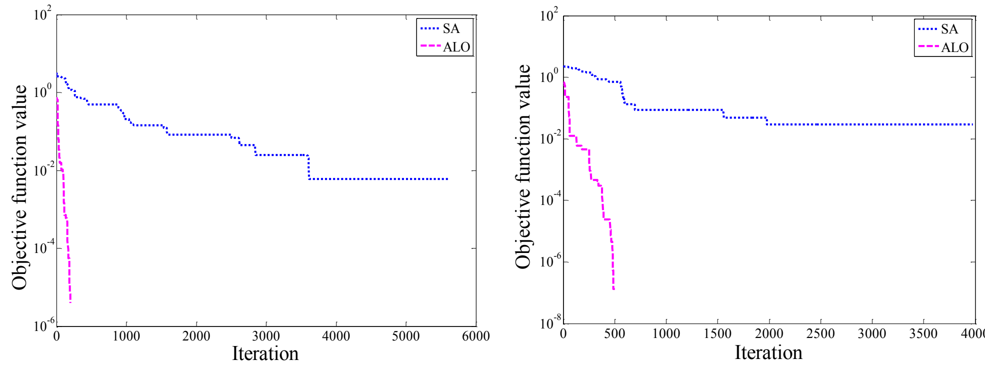
Table 8 Identified results of the unbalance by combing modal expansion with optimization algorithms

Unbalance disk	Unbalance magnitude(g mm)		Magnitude error (%)		Unbalance phase (degree)		Phase error (%)	
	ALO	SA	ALO	SA	ALO	SA	ALO	SA
1	44.23	44.16	-36.72	-36.82	141.36	141.43	-21.47	- 24.43
2	44.25	44.09	-10.06	-10.39	141.36	141.43	4.71	4.76

Table 9 Identified results of the unbalance by the integrated modal expansion/inverse problem approach with optimization algorithms

Unbalance disk	Unbalance magnitude (g mm)		Magnitude error (%)		Unbalance phase (degree)		Phase error (%)	
	ALO	SA	ALO	SA	ALO	SA	ALO	SA
1	78.30	78.16	12.02	11.82	184.20	184.60	2.33	2.56
2	52.15	53.00	6.00	7.72	113.03	112.19	-16.27	- 16.90

From Fig.15, one can easily not that the amplitudes of the equivalent unbalance at nodes 7 and 11 are significantly higher than in other nodes, and this demonstrates the accuracy of the location identification by using modal expansion technique. By comparing the unbalance identification results from Tables 8 and 9 it is clearly that the identification performed by using the integrated modal expansion/inverse problem with optimization provides the most accurate results. The ALO also provides the lower number of iteration steps to identify the parameters (Fig.16).

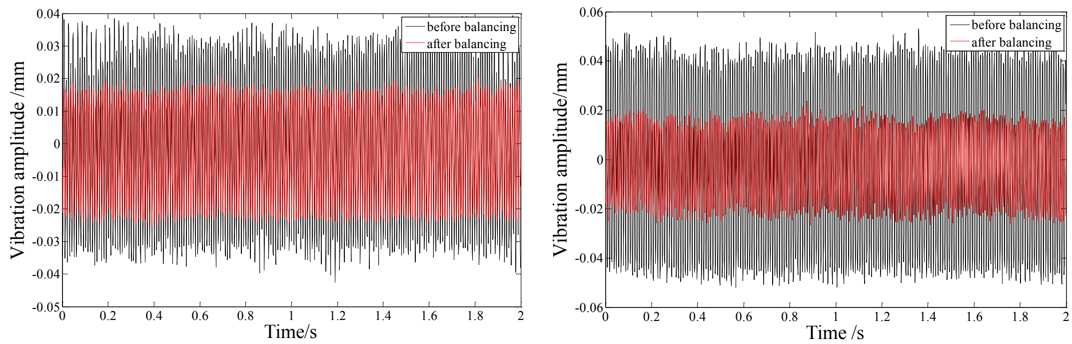


(a) modal expansion (b) integrated modal expansion and inverse problem

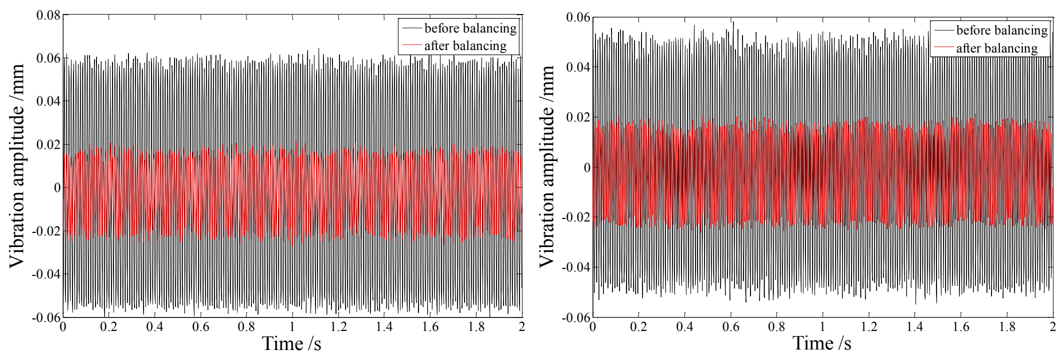
Fig.16 Iterative process for unbalance identification in double disk rotor system

5.3 Comparison of the response before and after balancing

The unbalance parameters have been identified in a single-disk and a double-disk rotor-bearing system, each made by a rotor balanced at 6000rev/min. The comparison of the experimental vibration amplitudes before and after balancing is shown in Figs. 17 and 18. It is quite clear that the vibration amplitudes have significantly decreased after balancing in the two experimental cases. For the single-disk system, the vibration amplitudes have reduced by 43%, 63%, 67% and 62% after balancing; the same responses have decreased by 40% (horizontal) and 53% (vertical) in the double-disk rotor-bearing system case. These results give further evidence that the identification methods proposed in this work are valid.

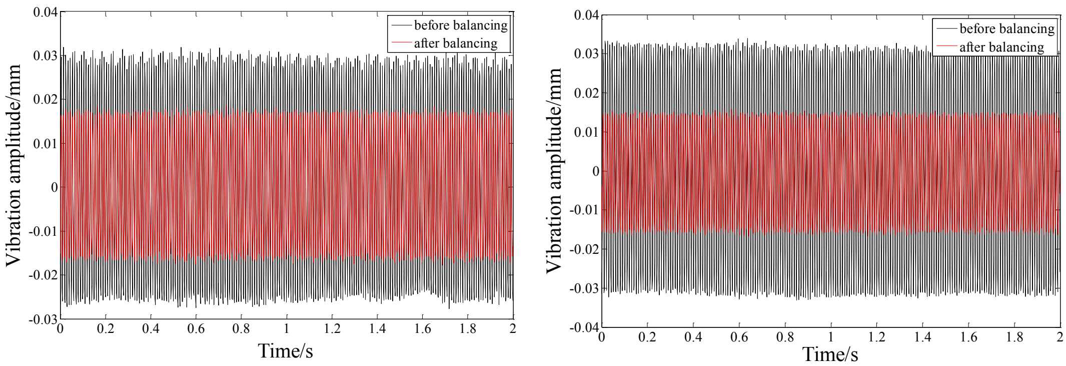


(a)29.4 g mm at 45 degree (b)39.3 g mm at180 degree



(c)45g mm at 135 degree (d)49.2 g mm at 225 degree

Fig.17 Vibration amplitudes along the vertical direction before and after balancing in the single-disk rotor-bearing system



(a)Horizontal vibration (b)Vertical vibration

Fig.18 Vibration amplitude before and after balancing in the double-disk rotor-bearing system

6 Conclusions

This paper has presented a new methodology to simulate and predict unbalances in rotor-bearing systems. A method that combines modal expansion and optimization algorithms is here proposed to identify the unbalance in a single-disk rotor- bearing system. Both simulations and experimental results demonstrate the effectiveness of the proposed identification method, which not only permits to detect the location of the unbalance, but also its defining parameters. Another method that involves an integrated modal expansion /inverse problem approach combined with optimization is also proposed, this time to identify the unbalance parameters in a double-disk rotor-bearing system. This method overcomes the limitation of the state-of-the-art inverse problem applications in the field, which rely upon the prior knowledge of the axial location of the unbalance. Simulation and experimental results demonstrate the validity of these methods. Moreover, the comparison between the method combining modal expansion and optimization, and the one with the integrated modal expansion/inverse problem/optimization shows that the results identified using the latter methodology are more accurate in the case of a double-disk rotor-bearing system. The dynamic balance experiments also demonstrate the validity of the proposed methods, which could provide effective aid to the rotordynamics designer.

In practical terms, the methods here proposed could be successfully used in rotor fault detection techniques, especially in view of the absence of the requirement of prior knowledge of the type of unbalance. This feature is particularly appealing in the case of complex and multiple shaft configurations typical of modern aeroengines or high-speed rotating machines. Although we have used in this work meta-heuristic algorithms, other more standard and industrially mainstream optimization techniques could also be used in case. The main disadvantages of the techniques here presented are the complex calculations involved in the methods, and the difficulty to obtain in some cases a very accurate modal representation of the pristine (undamaged and balanced) rotor-bearing system. Once these aspects are however taken into account in future developments, these modal expansion methods could provide a useful tool to the rotordynamics analyst to increase the reliability and operational readiness of rotating machines elements.

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