A Simple and Visually-orientated Approach for Type Synthesis of Overconstrained 1T2R Parallel Mechanisms

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Abstract: This paper presents a simple and highly visual approach for the type synthesis of a family of overconstrained parallel mechanisms that have one translational and two rotational movement capabilities. It considers, especially, mechanisms offering the accuracy and dynamic response needed for machining applications. This family features a spatial limb plus a member of a class of planar symmetrical linkages, the latter connected by a revolute joint either to the machine frame at its base link or to the platform at its output link. Criteria for selecting suitable structures from among numerous candidates are proposed by considering the realistic practical requirements for reconfigurability, movement capability, rational component design and so on. It concludes that few can simultaneously fulfill the proposed criteria, even though a variety of structures have been presented in the literature. Exploitation of the proposed structures and evaluation criteria then leads to a novel 5-DOF hybrid module named TriMule. A significant potential advantage of the TriMule over the Tricept arises because all the joints connecting the base link and the machine frame can be integrated into one single, compact part, leading to a lightweight, cost effective and flexible design particularly suitable for configuring various robotized manufacturing cells.

Keywords: Parallel mechanisms, type synthesis, conceptual design

1. Introduction

There is continuing academic and industrial interest in the application of parallel mechanisms having three degrees of freedom (DOF) that provide one translation and two rotations at the movable platform, that is they have 1T2R movement capability where T and R respectively denote a translational and a rotational DOF [1-3]. Several successful 5-DOF hybrid parallel kinematic machines (PKM) intended for application as machine tools are built upon 1T2R parallel mechanisms that offer desirable overall performance characteristics in terms of accuracy, rigidity, high workspace/footprint ratio, dynamic responses and flexibility compared to articulated industrial robots with serial architectures. They include the Ecospeed with the Sprint Z3 head used for aerospace manufacturing of large aluminum alloy components, and the Tricept and the Exechon used for high-speed milling, drilling, deburring and other manipulations, either as a stand-alone machine or as part of a movable robotized cell.

Overconstrained 1T2R parallel mechanisms can have several advantages over their counterparts without overconstraints, such as higher stiffness and greater cost-effectiveness. It is easier to obtain explicit expressions for the inverse/forward kinematics, for example, because the geometric conditions of joint axes allow fewer passive (idle) joints to be used. A typical example is the Exechon [4], where two 4-DOF UPR limbs lying always in a common plane are combined with one 5-DOF SPR spatial limb to form an overconstrained 1T2R parallel mechanism. Here, R, P, U and S represent a revolute, prismatic, universal and spherical joint, respectively. In this special case, two linearly independent constraint wrenches that must be imposed by the plane can be allocated to the two 4-DOF UPR limbs lying in that plane. Meanwhile, overconstraints are often used in the design of 1T2R compliant mechanisms for the improvement of static and dynamic performances [5-7]. Although the overconstrained architecture enables the 1T2R parallel mechanisms to achieve rather high stiffness and accuracy, type synthesis of this kind of

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parallel mechanisms is by no means an easy task. The Grübler–Kutzbach formula is no longer valid for determining the number of DOFs (mobility) in the structure because it cannot identify the common and/or virtual constraints induced by the special geometrical arrangements of the joint axes [8].

There have been major efforts over recent decades towards finding effective methods for the type synthesis of lower mobility overconstrained parallel mechanisms in general [9-14] and those having 1T2R movement capability in particular [12-14]. Methods currently available include those based mainly upon screw theory [15-17] and upon group theory [18-20]. For example, Kong and Gosselin [12] proposed a family of 1T2R overconstrained parallel mechanisms having a motion pattern equivalent to a UP or a PU serial kinematic chain. This idea was then extended by Li and Hervé [13] to synthesize a family known as RPR-equivalent parallel mechanisms because they have the motion pattern of an RPR serial kinematic chain. This led to numerous new architectures comprising either two 4-DOF limbs plus one 5-DOF limb, or three 4-DOF limbs. By exploring the underlying relationship between the parasitic motion and the intended 1T2R motion, Li and Hervé [14] proposed a method for the type synthesis of 1T2R parallel mechanisms free from parasitic motion by detecting and removing idle pairs. Building upon Grassmann line geometry and the atlas method, Xie and Liu [21] recently presented a graphical approach for type synthesis of 1T2R parallel mechanisms, where the Blanding rules were adopted to convert permitted motions into constraint wrenches and *vice versa* in a visually understandable manner. Based upon the similar idea, Hopkins *et al*. [22-24] proposed a general and systematic approach known as Freedom and Constraint Topology (FACT) for type synthesis of parallel precision flexure systems by creating a visible mapping between freedom and constraint spaces. Other methods such as linear transformations [25], single-open-chain units [26-27] and the G_F set [28] have also become available. Although the abovementioned methods are developed from various mathematical backgrounds, they use a common approach of first creating individual limbs and then setting assembly conditions in the final step. The use of these methods requires both considerable knowledge and skill in mathematics and also a deep understanding of the underlying theory. It would, therefore, be highly beneficial to develop a simpler, more visual, and effective approach that can be understood by a wider range of researchers and is also, even more importantly, accessible to design engineers. Moreover, another challenging issue arising from type synthesis is that, although a large number of feasible topological architectures can be synthesized, there is a lack of criteria for evaluating which are the better practical choices even among those having the same 1T2R movement capability [3]. The development of appropriate criteria to select structures suitable for specific applications remains an open issue.

This paper addresses the aforementioned issues by presenting a simple, easily visualized yet effective approach for the type synthesis of a family of overconstrained 1T2R parallel mechanisms. It then establishes criteria to select structures best suited to building 5-DOF hybrid PKMs for machining applications. As an extension to reference [29], this paper presents the explanations and discussions on the type synthesis approach and the criteria for selecting suitable structures in a detailed manner. Having now established the nature of the problem at hand, the remainder of the paper is organized as follows. By fully exploiting the common constraints imposed by the motion of a plane, Section 2 presents a methodology for synthesizing overconstrained 1T2R parallel mechanisms featuring a spatial limb plus a member of a class of planar symmetrical linkages. It examines two subclasses, those with and those lacking a properly constrained non-actuated limb. Focusing on practically realistic mechanism designs, Section 3 investigates the criteria for selecting suitable structures from among numerous candidates. Exploiting the approach, structures and criteria proposed here, Section 4 presents a novel 5-DOF hybrid PKM module that may offer potential advantages over the well-established Tricept design. Finally, conclusions are drawn in Section 5.

2. **Methodology**

While Section 3 will explore in some detail the issues of which mechanism structures have the most practical potential, some general, widely-seen design guidelines will be invoked directly while setting up the strategy. Cost-effective designs will tend to involve few different types of joints and as many near-identical or interchangeable sub-systems as possible. Symmetry tends to aid this interchangeability and often reduces overall vulnerability to thermal and mechanical disturbances in the major structural loops of machines. Prismatic actuated joints tend to provide better drive stiffness, which is important in applications to machine tools, whereas revolute passive joints tend to provide more compact designs, especially as they can combine into universal or spherical joints.

2.1. Strategy for type synthesis

Viewed in terms of kinematic inversion [30], Fig. 1(a) demonstrates the generic structure of the proposed family of overconstrained 1T2R parallel mechanisms. Members of this family feature a spatial limb plus a planar linkage lying within the plane denoted by $\prod$. The two end links of the spatial limb are represented by Body I and Body III, whilst those of the planar linkage are denoted by Body II and Body III. In this arrangement, Body I and Body II are connected by a revolute joint, denoted by R, with its joint axis *n*-*n* parallel to $\prod$. In addition, actuated prismatic joints (denoted by P) are employed, in the spatial limb and in each of two identical limbs of the planar linkage in order to achieve high stiffness. Note that the planar linkage may also have one properly constrained non-actuated limb. The term 'properly constrained' here means that the number and type of DOF of the limb is exactly the same as those of the output link of the planar linkage.

Using these descriptions, two subfamilies of overconstrained 1T2R parallel mechanisms can be synthesized by giving Body I (or Body III) either of two roles in the system. In the first subfamily, Body I is taken as the machine frame, and Body III thereby becomes the output link of the planar linkage to which the mechanism platform is attached, with Body II being the base

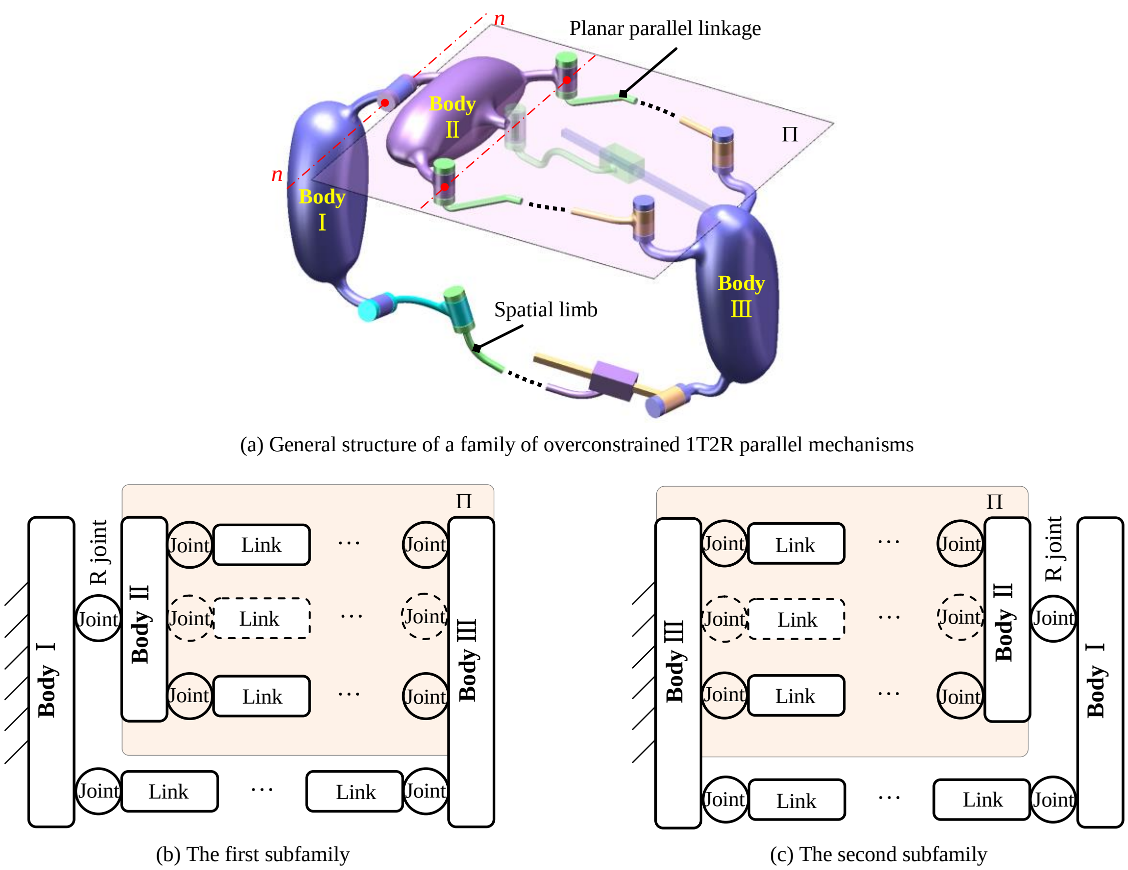


Fig. 1. A family of overconstrained 1T2R parallel mechanisms

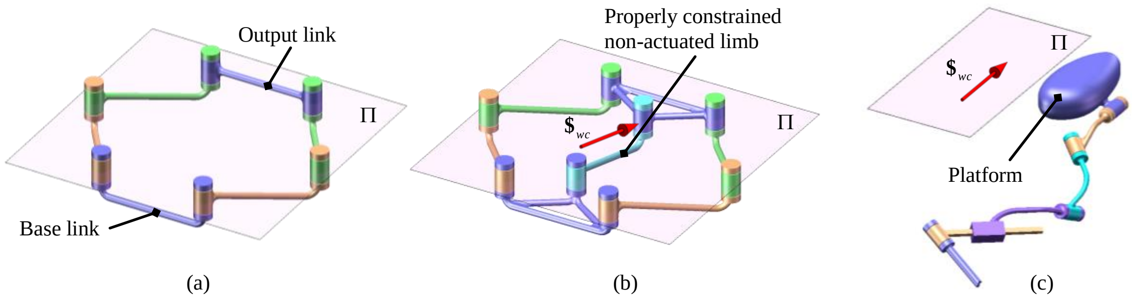


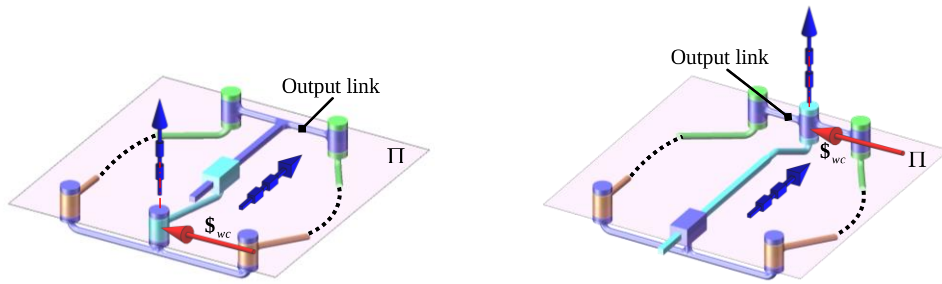
Fig. 2. (a) A six bar planar linkage, (b) A seven bar planar linkage, and (c) An 5-DOF spatial limb.

link of the planar linkage. For this case, the platform (Body III) undergoes a 1T1R motion within plane Π and an additional 1R motion about the axis *n*-*n* with respect to the base (Body I), resulting a 1T2R positioning parallel mechanism (see Fig. 1(b)). Kinematic inversion produces the second subfamily where Body I is taken as the platform, Body II as the output link of the planar linkage, and Body III as the machine frame to which the base link is attached. Hence, the motions of platform (Body I) arise from a 1T1R internal motion of the output link (Body II) within plane Π and a 1R relative rotation about the axis *n*-*n* with respect to the output link, leading to a 1T2R orientating parallel mechanism (see Fig. 1(c)).

Compared to existing methods [12-14], this strategy significantly simplifies the problem at hand by fully and directly exploiting the common constraints imposed by Π. In addition, the major difference of this approach from the FACT method [22-24] lies in that the common constraints provided by a plane Π are considered first and then attributed to the limbs involved. Hence, it carries out the type synthesis of the overconstrained 1T2R parallel mechanisms in a simple manner though the principle behind these two methods is the same. For illustration purposes only, consider a basic form of the planar linkage with a single-loop closure composed of six revolute joints as shown in Fig. 2(a); planar linkages having other types of joints and even multi-loops can also be considered in a similar manner. For this specific case, the axis *n*-*n* is confined to be parallel to the common normal to the two R joints at either the base link or the output link (see Fig. 1). Since the common constraints of Π must comprise one constraint force and two constraint couples, the output link in this basic form has 2T1R internal mobility. Hence, a 1T internal motion must be restricted by imposing one additional constraint wrench $\mathbf{\$}_{wc}$ (actually, a pure force) with its axis parallel to Π. Then, 1T2R movement capability of the platform can be achieved by simultaneously adding a 1R motion about the axis *n*-*n*. The required constraint wrench can be generated by either of two ways: (i) by using a properly constrained non-actuated limb (an RR limb, for example) embedded between two actuated limbs to achieve structural symmetry as shown in Fig. 2(b), in which case the complete mechanism will incorporate a 6-DOF spatial limb; or (ii) by using a 5-DOF spatial limb (an RPS limb for example) as shown in Fig. 2(c). Screws are shown throughout this paper by the arrow notations developed in [11] and summarized in Table I. Consequently, two classes of overconstrained 1T2R parallel mechanisms in each subfamily can be generated with ease using the procedures introduced below.

Table I Screw notations

Screw Type	Notation	Physical meaning	Pitch
$\mathbf{\$}_{ta}$		Rotation	0
		Translation	∞
$\mathbf{\$}_{wc}$		Constraint force	0
		Constraint couple	∞



(a) RP limb (b) PR limb

Fig. 3. Constraint wrench imposed by a properly constrained non-actuated limb

Table II The overconstrained 1T2R parallel mechanisms simultaneously actuated by three internal or external $\underline{\mathrm{P}}$ joints

Class	Type			
Using a properly constrained non-actuated limb	R(2R$\underline{\mathrm{P}}$R-RP)-U$\underline{\mathrm{P}}$S	R(2R$\underline{\mathrm{P}}$R-PR)-U$\underline{\mathrm{P}}$S	(2R$\underline{\mathrm{P}}$R-RP)R-U$\underline{\mathrm{P}}$S	(2R$\underline{\mathrm{P}}$R-PR)R-U$\underline{\mathrm{P}}$S
	R(2$\underline{\mathrm{P}}$RR-RP)-$\underline{\mathrm{P}}$US	R(2$\underline{\mathrm{P}}$RR-PR)-$\underline{\mathrm{P}}$US	(2$\underline{\mathrm{P}}$RR-RP)R-$\underline{\mathrm{P}}$US	(2$\underline{\mathrm{P}}$RR-PR)R-$\underline{\mathrm{P}}$US
Not using a properly constrained non-actuated limb	R(2R$\underline{\mathrm{P}}$R)-S$\underline{\mathrm{P}}$$\mathrm{R}_1$	(2R$\underline{\mathrm{P}}$R)R-S$\underline{\mathrm{P}}$$\mathrm{R}_1$	R(2R$\underline{\mathrm{P}}$R)-$\mathrm{R}_1$$\underline{\mathrm{P}}$S	(2R$\underline{\mathrm{P}}$R)R-$\mathrm{R}_1$$\underline{\mathrm{P}}$S
	R(2$\underline{\mathrm{P}}$RR)-$\underline{\mathrm{P}}$$\mathrm{R}_1$S	(2$\underline{\mathrm{P}}$RR)R-$\underline{\mathrm{P}}$$\mathrm{R}_1$S	R(2R$\underline{\mathrm{P}}$R)-U$\underline{\mathrm{P}}$$\mathrm{R}_2\mathrm{R}_1$	(2R$\underline{\mathrm{P}}$R)R-U$\underline{\mathrm{P}}$$\mathrm{R}_2\mathrm{R}_1$

R---revolute joint; P---prismatic joint; U---universal joint; S---spherical joint.

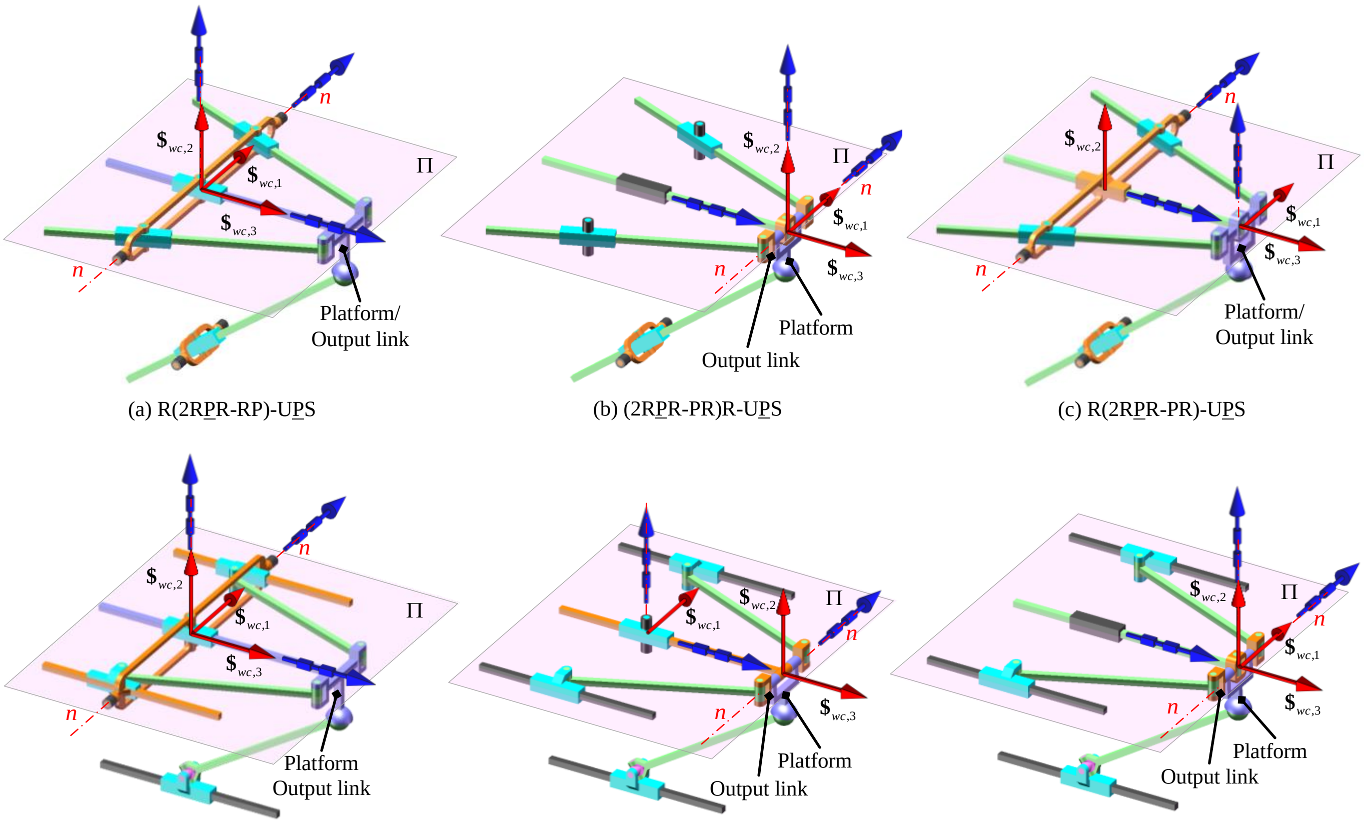
2.2. The class using a properly constrained non-actuated limb

Members belonging to this class feature a 6-DOF spatial limb plus a stand-alone 1T1R planar parallel mechanism containing a properly constrained non-actuated limb that is connected by an R joint to the machine frame at its base link in the first subfamily, or to the platform at its output link in the second. Two types of such properly constrained limbs, denoted by RP and PR (P indicating a passive prismatic joint), are available for achieving structural symmetry. Both impose on the output link a constraint wrench (a force) $\mathbf{\$}_{wc}$ lying in $\prod$, normal to the P joint direction and passing through the R joint axis, as depicted in Fig. 3. As a result, the 1T internal motion of the output link parallel to the wrench axis is restricted. The 1T1R planar linkages considered here are none other than those with Assur groups of the third class [31].

This class uses a 6-DOF spatial limb to provide the platform with an actuation to generate 1R motion about the axis *n*-*n*. Considering only those structures simultaneously actuated by three $\underline{\mathrm{P}}$ joints (two internal to the planar linkage and one external), the overconstrained 1T2R parallel mechanisms that can be synthesized are listed in the upper part of Table II, with six typical architectures illustrated in Fig. 4. Taking the mechanism shown in Fig. 4(a) as an example, it is clear that the constraint wrench system imposed upon the platform by the limbs can be decomposed into: (i) a force $\mathbf{\$}_{wc,1}$ imposed by the properly constrained non-actuated limb, and (ii) a force $\mathbf{\$}_{wc,2}$ and a couple $\mathbf{\$}_{wc,3}$ imposed jointly by all the limbs of the planar linkage. A similar decomposition is applicable to the other mechanisms. Fig. 4 shows clearly that the platform rotates about the axis *n*-*n* and about the R joint axis of the properly constrained non-actuated limb.

2.3. The class not using a properly constrained non-actuated limb

Members belonging to this class generally feature a 5-DOF spatial limb plus a 2T1R six-bar planar linkage such as that shown in Fig. 2(a), connected by an R joint just as discussed in Section 2.2. This planar linkage, having mobility 3, is under-constrained in terms of the overall design freedoms. Hence, the 5-DOF spatial limb is employed not only to provide the



(a) R(2RPR-RP)-UPS (b) (2RPR-PR)R-UPS (c) R(2RPR-PR)-UPS

(d) R(2PRR-RP)-PUS (e) (2PRR-RP)R-PUS (f) (2PRR-PR)R-PUS

Fig. 4. Typical overconstrained 1T2R parallel mechanisms using a properly constraint non-actuated limb

platform with an actuation to generate 1R motion about the axis *n*-*n*, but also to impose a constraint wrench $\$_{wc}$ (a pure force) upon the output link that restricts a 1T motion parallel to $\prod$.

Numerous 5-DOF spatial limbs are available to do this [10], but we here consider, for reasons of practical application, only those having four R joints and one P joint. Within this category, the location and direction of the constraint wrench can uniquely be determined by the conditions: (i) the axis of one R joint, denoted by R_1, is parallel to the wrench axis; (ii) the axes of the other three R joints, each denoted by R_2, intersect the wrench axis at a common point; and (iii) the axis of the P joint is normal to the wrench axis, that is, normal to the R_1 joint axis. On sequentially ordering all joints from the machine frame to the platform and utilizing joint substitutions, these conditions reveal the four possible limb structures to be SPR_1, UPR_2R_1, R_1PS and PR_1S. In order to achieve an appropriate workspace, assembly of the 5-DOF spatial limb to the planar linkage must correctly place the direction of the R_1 joint axis with respect to a specified reference line parallel to $\prod$. The direction of the R_1 joint axis can be determined by one of following ways according to which 5-DOF limb structure is used:

(i) For an SPR_1 limb or a UPR_2R_1 limb, where the R_1 joint connects the spatial limb with the platform, the R_1 joint axis should be parallel to the common normal of the axes of the two R joints connecting the output link of the planar linkage. As a result, the 1T permitted motion confined within $\prod$ is normal to the R_1 joint axis as shown in Fig. 5(a).

(ii) For an R_1PS (PR_1S) limb where the R_1 (P) joint connects the spatial limb with the machine frame, the direction of the 1T permitted motion must be prescribed first. It can be normal to the base link of the planar mechanism (more precisely, normal to the common normal of the axes of the two R joints connecting that base link), or it can be parallel to the angular bisector of the two P joint directions of the planar linkage. Then, the R_1 joint axis should be placed normal to the prescribed direction of the 1T permitted motion, as shown in Fig. 5(b) and (c).

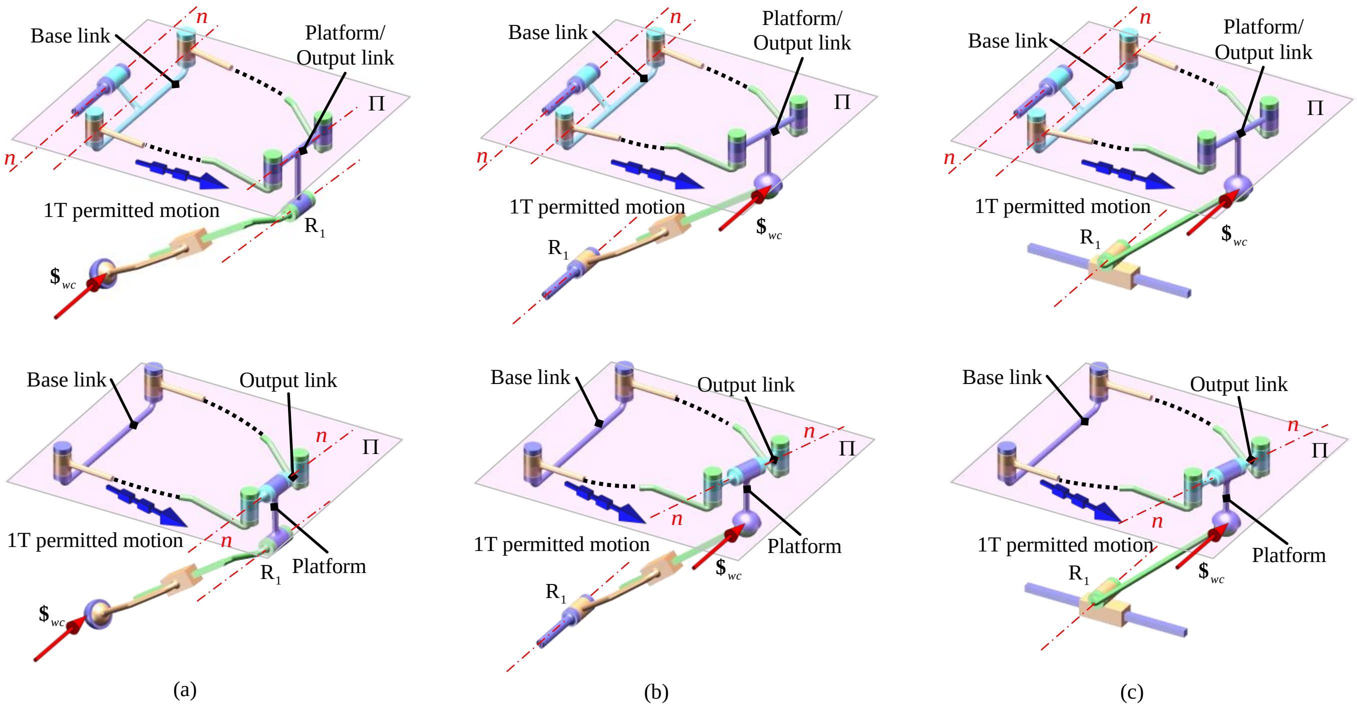


Fig. 5. Location and direction of the constraint wrench and the R_1 joint axis.

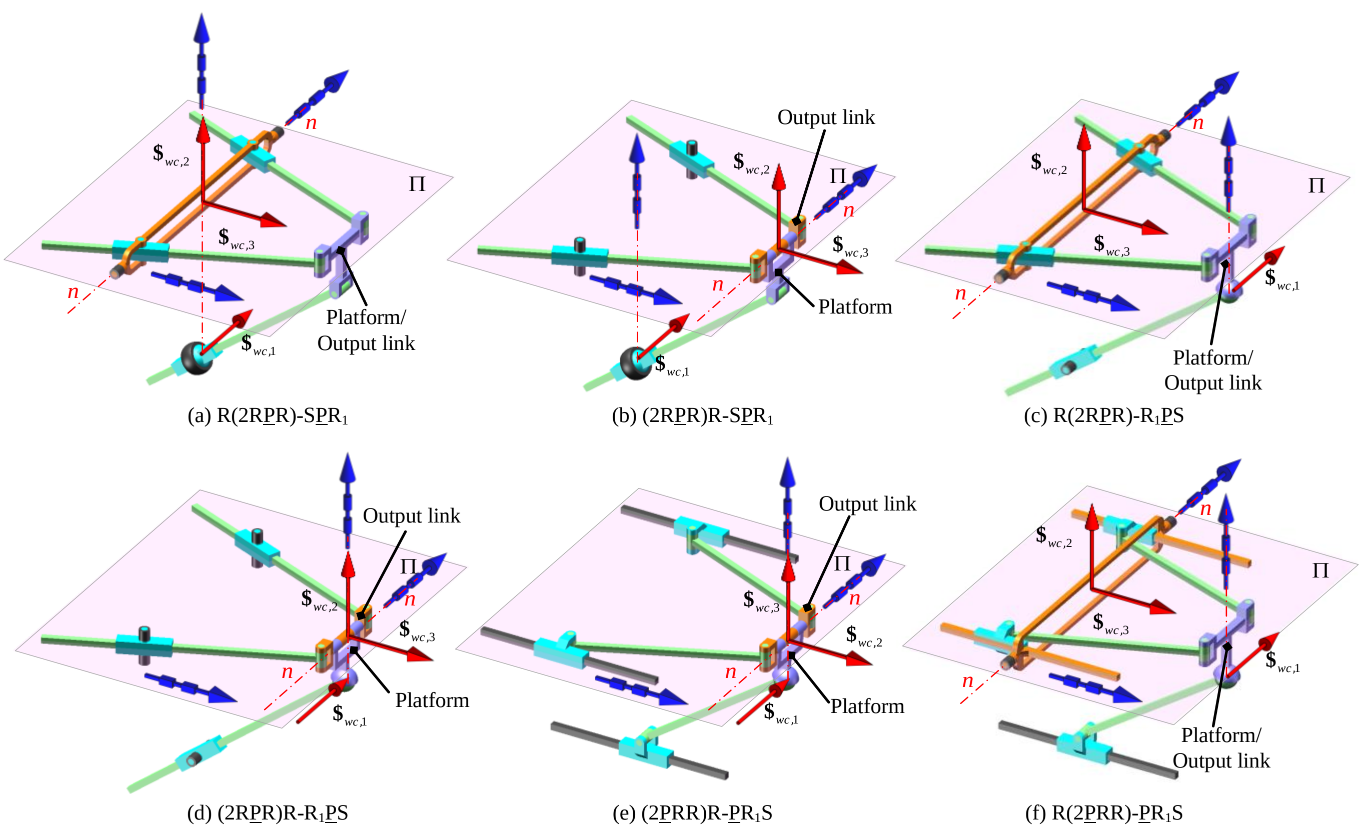
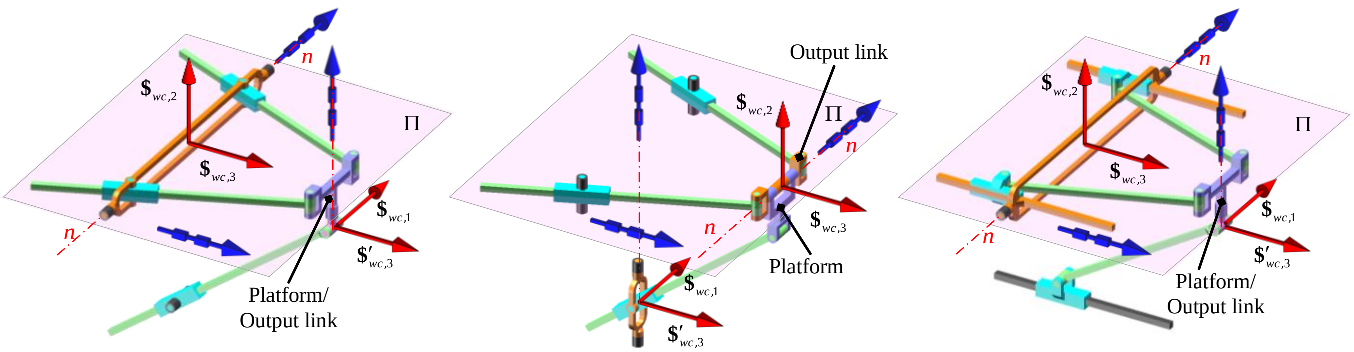


Fig. 6. Typical overconstrained 1T2R parallel mechanisms using two 4-DOF limbs plus one 5-DOF limb.

The orientation of the axis of the constraint wrench $\$_{wc}$ provided by the spatial limb clearly varies with the system



(a) R(2R$\underline{\mathrm{P}}$R)-R$_1\underline{\mathrm{P}}$U (b) (2R$\underline{\mathrm{P}}$R)R-U$\underline{\mathrm{P}}$R$_1$ (c) R(2$\underline{\mathrm{P}}$RR)-$\underline{\mathrm{P}}$R$_1$U

Fig. 7. Typical overconstrained 1T2R parallel mechanisms using three 4-DOF limbs.

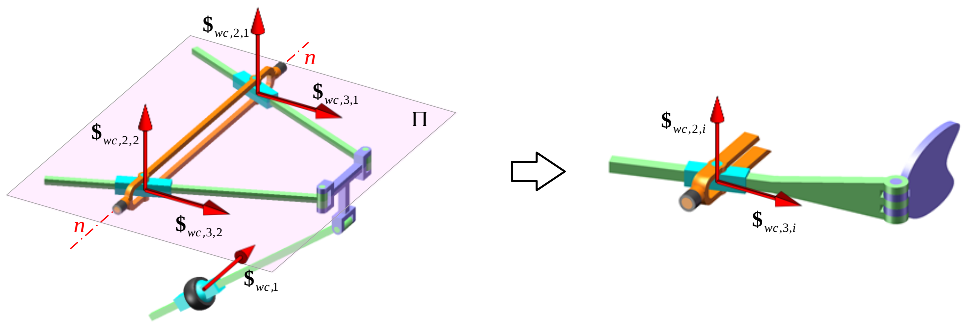
configuration when using limb arrangements covered by category (i) but it remains constant when using those from category (ii). The lower part of Table II lists only the overconstrained 1T2R parallel mechanisms of this class that are simultaneously actuated by three internal and external $\underline{\mathrm{P}}$ joints (although many other possible mechanisms exist). All of them have three R_2 joints integrated into an S joint to achieve a two-link limb design, or, occasionally, two R_2 joints integrated into a U joint. Fig. 6 shows typical examples that use a two-link limb. As an example, Fig. 6(a) shows readily that the constraint wrench system imposed upon the platform by the limbs can be decomposed into: (i) a force $\$_{wc,1}$ imposed by the spatial limb; and (ii) a force $\$_{wc,2}$ and a couple $\$_{wc,3}$ imposed jointly by the two actuated limbs of the planar linkage. This decomposition is also applicable to the other mechanisms. Fig. 6 demonstrates that the platform rotates about the axis *n*-*n* and about the axis normal to $\prod$ that passes though the centre of the S joint. Finally, note that one of three R_2 joints in the 5-DOF spatial limbs shown in Fig. 6(b), 6(c), and 6(f) could in principle be saved if the other two R_2 joints are arranged as shown in Fig. 7, such that the presumed additional constraint couple $\$'_{wc,3}$ (which has its screw axis determined by the cross product of those two R_2 joint axes) is exactly aligned with $\$_{wc,3}$.

3. Criteria for selecting suitable structures

Given the huge number of candidate 1T2R parallel mechanisms, it is a challenging issue to identify those best suited for developing 5-DOF hybrid robotized modules where high rigidity and high dynamic responses are essential requirements. Indeed, the justification for the developments in Section 2 has already drawn on general design principles and guidelines. Now, four important criteria will be proposed by examining the real-world implications of practical requirements for movement capability, the rationalization of component design, module reconfigurability, *etc*.

- *Criterion 1*: A suitable structure should have a movement capability, appropriate to the application, that favours either the positioning or orientating of its platform.

To further explore this criterion, we allocate the 1T2R parallel mechanisms shown in Fig. 6 into three classes. Members belonging to the first class have both of their rotational axes proximal to the machine frame. A typical structure is shown in Fig. 6(a) where the platform rotates about the axis *n*-*n* attached to the machine frame and about the axis normal to $\prod$ and passing through the centre of the S joint connecting the spatial limb to the machine frame. This arrangement allows a relatively large positioning capability to be achieved at the platform and an A/C wrist must then be attached to it to achieve full 5-DOF movement capability. Members of the second class are exactly kinematic inversions of those in the first. A typical structure is shown in Fig. 6(e) where the platform rotates about the axis *n*-*n* attached to the output link and about the axis normal to $\prod$ that passes through the centre of the S joint connecting the spatial limb to the platform. This arrangement allows a relatively large



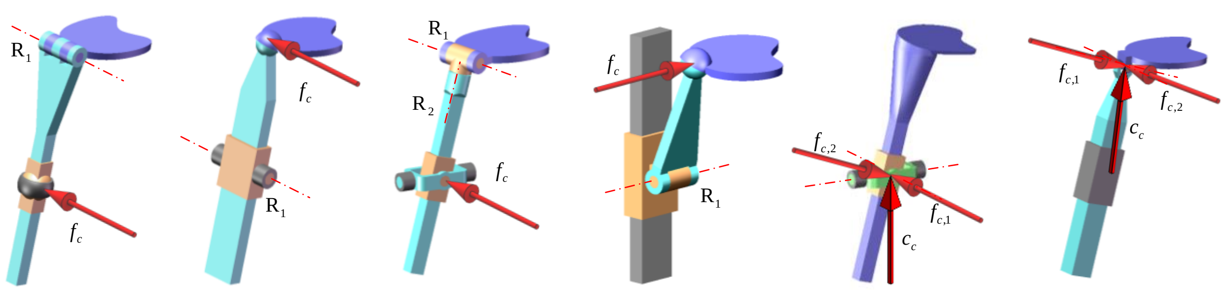
(a) R(2RPR)-SPR$_1$ (b) R(RPR) limb

Fig. 8. Constraint wrenches of the R(2RPR)-SPR$_1$ parallel mechanism

orientating capability to be obtained directly and then an X-Y gantry must be attached to the machine frame (for example) to achieve the full 5-DOF movement capability. Members of the third class have a mix of the features of those in the first and second classes, i.e. one rotation axis is proximal to the platform and the other to the machine frame. A typical structure is shown in Fig. 6(c), which differs from that in Fig. 6(a) only by reversing the order of joints in the 5-DOF spatial limb. This is an inadequate arrangement because it leads to a significant loss of positioning capability at the platform even though this should be a priority when an A/C wrist will inevitably be needed anyway to give full orientating capability of the end-effector. Hence, we conclude that the structures shown in Fig. 6(b), 6(c), 6(f) and Fig. 7 are not suitable for use. This criterion also applies with very similar arguments to the structures shown in Fig. 4. A common feature of members of the first and second classes is that their structures should have identical or nearly identical actuated limbs (due to the overconstraints).

- *Criterion 2*: A suitable structure must allow (through the type and arrangement of its joints) the main body of all lower mobility limbs to be given shapes having high bending and/or torsional stiffness-to-mass ratios.

This criterion is of crucial importance for the mechanical realization of a lightweight yet rigid limb design, which is a key factor in delivering both accuracy and dynamic response. We take the parallel mechanism shown in Fig. 6(a) as an example to explain this criterion. Utilizing the method developed in [32], the two linearly independent constraint wrenches, i.e. $\$_{wc,2}$ (a force) and $\$_{wc,3}$ (a couple), imposed upon the platform can further be decomposed into two groups, each containing a force $\$_{wc,2,i}$ and a couple $\$_{wc,3,i}$ allocated to the ith ($i=1,2$) RPR limb lying in $\prod$ as shown in Fig. 8(a). The axis of $\$_{wc,2,i}$ is aligned with the axis of the R joint connecting the ith limb with the base (end) link of the planar linkage and the direction of $\$_{wc,3,i}$ is parallel to that of $\$_{wc,3}$. Hence, the main body of the RPR limb should be shaped generally as shown in Fig. 8(b), with a larger second moment of area of its cross-section in regions carrying larger bending moments, in order to achieve a high bending stiffness-to-mass ratio. This shape matches naturally to the needs for attaching the P and R joints, or, conversely, this is a rational joint arrangement for maximizing performance. Analogously, Fig. 9 shows the constraint wrenches imposed upon



(a) SPR$_1$ (b) R$_1$PS (c) UPR$_2$R$_1$ (d) PR$_1$S (e) UP (f) PU

Fig. 9. Constraint wrenches imposed upon commonly-used 5-DOF actuated and 3-DOF non-actuated limbs

f_c --- a constraint force; c_c ---- a constraint couple several commonly-used lower mobility limbs. It is clear that the S$\underline{P}$R$_1$, $\underline{P}$R$_1$S and UP limbs are more suitable than the others because they can be rationally shaped, as shown, for resisting against bending and torsional moments. Furthermore, designs in which an R joint itself effectively acts as a cantilever should be completely avoided whenever a constraint wrench is transversely applied to the joint axis as illustrated in Fig. 7(a) and 7(c), even though an R$_2$ joint can be saved by doing so.

- *Criterion 3*: A suitable structure should have a relatively large ratio of workspace volume against footprint so that it can be integrated into a rigid yet compact module for configuring various manufacturing cells (exactly as exhibited by the Sprint Z3 head, the Tricept and the Exechon).

This criterion is an extension of Criterion 1. A 1T2R parallel mechanism intended to provide positioning capability should be internally actuated by $\underline{P}$ joints, where the active $\underline{P}$ joints are connected with the machine frame *via* other types of joints, so as to achieve a relatively large workspace volume/footprint ratio without mechanical interference. Structures satisfying this criterion are shown in Fig. 4(a) and Fig. 6(a), for example. Conversely, those intended for orientating capability should be externally actuated by $\underline{P}$ joints, where the active $\underline{P}$ joints are directly attached to the machine frame, in order to better achieve a lightweight yet rigid limb design.

- *Criterion 4*: A suitable structure should have an explicit solution to its inverse kinematics, an important issue for CNC control. An explicit solution to the forward kinematics is not essential, but is very useful for rapid online monitoring of the platform poses and so could be taken as a second-level criterion.

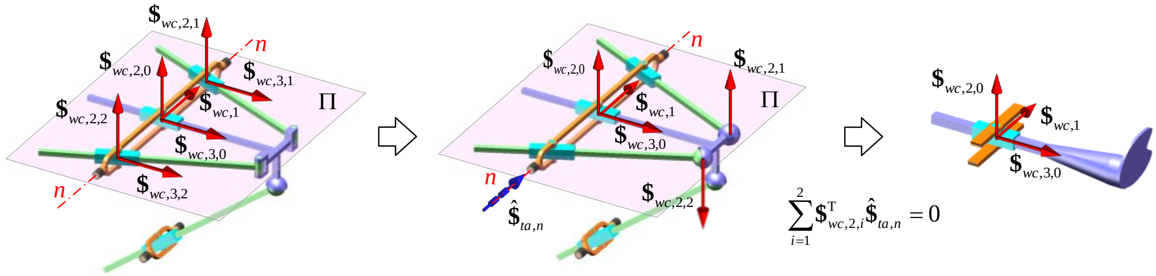
It is easily shown that all the constrained 1T2R parallel mechanisms given in Table II have explicit solutions to the inverse kinematics. However, only those belonging to the class containing a properly constrained non-actuated limb may have explicit forward kinematics because the internal motion within $\prod$ is then completely dominated by planar parallel mechanisms with Assur groups of the third class.

The evaluations against these four proposed criteria of all the overconstrained 1T2R parallel mechanisms shown in Table II

Table III Examination of the 1T2R parallel mechanisms by four criteria

Type	Figure	Criterion 1	Criterion 2	Criterion 3	Criterion 4
R(2R$\underline{P}$R-RP)-U$\underline{P}$S	4(a)	●	●	●	●*
(2R$\underline{P}$R-PR)R-U$\underline{P}$S	4(b)	●	○	●	●*
(2R$\underline{P}$R-RP)R-U$\underline{P}$S	-	○	○	○	●*
R(2R$\underline{P}$R-PR)-U$\underline{P}$S	4(c)	○	○	○	●*
R(2$\underline{P}$RR-RP)-$\underline{P}$US	4(d)	●	●	○	●*
R(2$\underline{P}$RR-PR)-$\underline{P}$US	-	○	○	○	●*
(2$\underline{P}$RR-RP)R-$\underline{P}$US	4(e)	○	○	○	●*
(2$\underline{P}$RR-PR)R-$\underline{P}$US	4(f)	●	○	●	●*
R(2R$\underline{P}$R)-S$\underline{P}$R$_1$	6(a)	●	●	●	●
(2R$\underline{P}$R)R-S$\underline{P}$R$_1$	6(b)	○	○	○	●
R(2R$\underline{P}$R)-R$_1$$\underline{P}$S	6(c)	○	○	○	●
(2R$\underline{P}$R)R-R$_1$$\underline{P}$S	6(d)	●	○	●	●
(2$\underline{P}$RR)R-$\underline{P}$R$_1$S	6(e)	●	●	●	●
R(2$\underline{P}$RR)-$\underline{P}$R$_1$S	6(f)	○	○	○	●
R(2R$\underline{P}$R)-U$\underline{P}$R$_2$R$_1$	-	●	○	●	●
(2R$\underline{P}$R)R-U$\underline{P}$R$_2$R$_1$	-	○	○	○	●

●---Satisfied; ○---Not satisfied; *---Have explicit forward kinematics



(a) R(2RPR-RP)-UPS (b) R(2RPS-RP)-UPS (c) RP limb

Fig. 10. Constraint wrenches of the R(2RPR-RP)-UPS and R(2RPS-RP)-UPS parallel mechanisms

are summarized in Table III. It can be seen that only the structures shown in Fig. 4(a), Fig. 6(a) and Fig. 6(e) have the potential to fulfill all these criteria simultaneously. It should be pointed out that the structure shown in Fig. 6(a) is exactly the one already used to build the Exechon.

4. An Example

Exploiting the structures obtained and evaluated as superior in Sections 2 and 3, we propose a novel 5-DOF hybrid PKM module using the overconstrained 1T2R parallel mechanism shown in Fig. 4(a). The mechanism essentially comprises a 6-DOF UPS spatial limb plus a stand-alone 1T1R planar linkage containing a properly constrained non-actuated RP limb. The planar linkage is connected by a pair of R joints to the machine frame at either side of its base link. Note that two linearly independent constraint wrenches, i.e. $\$_{wc,2}$ and $\$_{wc,3}$, jointly imposed by all the limbs of the planar linkage can be decomposed into three groups, each containing a force $\$_{wc,2,i}$ and a couple $\$_{wc,3,i}$ allocated to the ith ($i = 0,1,2$) limb lying in plane $\prod$ as shown in Fig. 10(a). By the criteria proposed in Section 3, it is preferable in practice to use two RPS limbs to replace the two RPR limbs. This arrangement not only avoids torsional moments imposed by the constraint on the two RPR limbs, but also allows the three actuated limbs to be identical, leading to the very cost effective design shown in Fig. 10(b). However, note that this arrangement remains a constraint forces $\$_{wc,2,i}$ ($i = 1,2$) imposed on the RPS limb at the centre of the S joint, satisfying $\sum_{i=1}^{2} \$_{wc,2,i}^{\mathrm{T}} \hat{\$}_{ta,n} = 0$ where $\hat{\$}_{ta,n}$ denotes the unit twist of the axis n-n as shown in Fig.10(b). Therefore, in order to prevent the RPS limbs from bending deformation, the properly constrained non-actuated limb should now be rationally shaped as shown in Fig. 10 (c) for fully resisting against bending and torsional moments imposed upon the platform. These considerations result in the mechanical realization of a newly patented 5-DOF hybrid PKM module known as the TriMule [33] and shown in Fig. 11(a). Its interchangeable limbs match those of the existing Tricept (see Fig. 11(b)). However, a critical new feature is that the use of the planar linkage with a properly constrained non-actuated limb enables the use of a compact and quite rigid sub-frame to connect it to the main frame at the n-n axis. Thus, the base link of the planar linkage is elaborately designed into a single ('three-in-one') part that locates the rear R joints of the two actuated RPS limbs, the R and P joints of the RP limb and the two R joints to the machine frame, so allowing the module weight to be dramatically reduced since only a small outer frame is required. This novelty offers a lightweight, cost effective and flexible design for a 5-DOF hybrid PKM module particularly suitable for configuring various robotized manufacturing cells as illustrated by Fig. 12.

(a) TriMule (b) Tricept

Fig. 11. 3D views of the TriMule and the Tricept

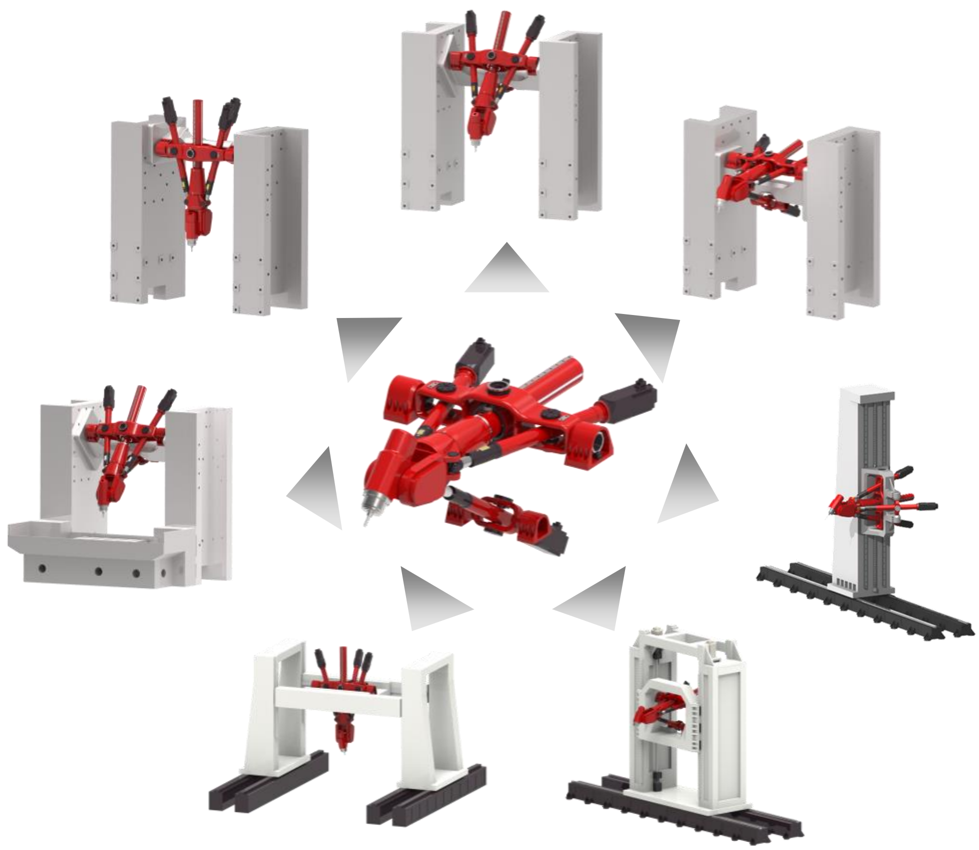


Fig. 12. Conceptual design of the manufacturing cells configured by the TriMule

5. Conclusions

This paper presents a simple, illustrative and easily understandable approach for the type synthesis of overconstrained 1T2R parallel mechanisms. The following conclusions are drawn.

(1) Overconstrained 1T2R parallel mechanisms can be visualized as being composed of a spatial limb plus a member of a class of planar symmetrical linkages. The whole planar linkage is provided with an out-of-plane freedom by a revolute joint connected either between its base link and the machine frame or between its output link and the platform.

(2) By considering realistic, practical requirements for movement capability, for the rationality of component design, *etc.*, four important criteria are proposed for selecting suitable mechanism structures. They lead to the conclusion that few of

the numerous candidate 1T2R parallel mechanisms are of practical usefulness.

(3) Drawing directly on the structures and evaluation criteria proposed here, a novel 5-DOF hybrid PKM module named TriMule is presented. A significant potential advantage of the new design over the established Tricept design arises because its geometry enables the integration of all the joints connecting to the base link of its planar linkage into a single compact part, thereby allowing the module weight to be dramatically reduced.

(4) Having determined with confidence that the TriMule design is competitive with the established Tricept design, its kinematic, static, and dynamic analyses will now be fully investigated and reported in future papers.

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