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ROBUST ADAPTIVE MODEL PREDICTIVE CONTROL FOR INTELLIGENT DRINKING WATER DISTRIBUTION SYSTEMS

by

AYODEJI OPEOLUWA AJIBULU

A thesis submitted to

The University of Birmingham

for the degree of

DOCTOR OF PHILOSOPHY

School of Electronic, Electrical and Systems Engineering

The University of Birmingham

January 2018

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Abstract

Large-scale complex systems have large numbers of variables, network structure of interconnected subsystems, nonlinearity, spatial distribution with several time scales in its dynamics, uncertainties and constrained. Decomposition of large-scale complex systems into smaller more manageable subsystems allowed for implementing distributed control and coordinations mechanisms.

This thesis proposed the use of distributed softly switched robustly feasible model predictive controllers (DSSRFMPC) for the control of large-scale complex systems. Each DSSRFMPC is made up of reconfigurable robustly feasible model predictive controllers (RRFMPC) to adapt to different operational states or fault scenarios of the plant. RRFMPC reconfiguration to adapt to different operational states of the plant is achieved using the soft switching method between the RRFMPC controllers.

The RRFMPC is designed by utilizing the off-line safety zones and the robustly feasible invariant sets in the state space which are established off-line using Karush Kuhn Tucker conditions. This is used to achieve robust feasibility and recursive feasibility for the RRFMPC under different operational states of the plant. The feasible adaptive cooperation among DSSRFMPC agents under different operational states are proposed.

The proposed methodology is verified by applying it to a simulated benchmark drinking water distribution systems (DWDS) water quality control.

To the Almighty GOD and my Father, the creator of the universe

Acknowledgments

First, I would thank my God and Father who gave me life and all that was required to complete this research work. I would like to appreciate my first supervisor Professor Mietek Brdys for his advice, assistance, counsel, constructive criticisms and encouragement. I enjoyed his continuous support, academic discussions and technical guidance until his untimely death at the crucial stage of my research.

I am most grateful to my second supervisor Professor Xiao-Ping Zhang for taking up the supervision of my research work after the death of my first supervisor. Without his continuous guidance, support, counsel, assistance and encouragement, this work would not have been presented here. You made me finish this research work excellently despite the impact of the death of my first supervisor.

My special appreciation goes to my lovely wife Toyin and wonderful sons, Joshua and David for their invaluable support, love and encouragement. I am also thankful to my Sister Funmi Favour, my wonderful brother and his wife, Dr. Jide Benson, my covenant family friends, Pastor Temidayo and Bunmi Olasunkanmi, my mum, my sisters and all my friends for all your invaluable support and encouragement. I specially appreciate my Pastor, Nathaniel Odin for his spiritual support. I appreciate all the members of the Winners family for their love and spiritual support.

I appreciate TETFUND Nigeria for the scholarship for this program and to everyone who has contributed positively to the success of this research work, I am grateful.

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Abbreviations

ACDMPC	Adaptive cooperative distributed model predictive control
CIS	Critical Infrastructure Systems
CDMPC	Cooperative distributed model predictive control
CDRFMPC	Cooperative distributed robustly feasible model predictive control
DRFMPC	Distributed Robustly Feasible Model Predictive Control
DSSRFMPC	Distributed Softly Switched Robustly Feasible Model Predictive Control
DWDS	Drinking water distribution systems
KKT	Karush Kuhn Tucker
MPC	Model predictive control
RFMPC	Robustly feasible model predictive control
RRFMPC	Reconfigurable Robustly feasible model predictive control
RCRFMPC	Recursively Robustly feasible model predictive control
SSRFMPC	Softly switched robustly feasible model predictive control
SSRCRFMPC	Softly switched recursively robustly feasible model predictive control

Chapter 1

Introduction

In this chapter, the motivation for this research work is presented in Section 1.1. The Research objectives and contributions are explained in Section 1.2. The organization of the thesis is outlined in Section 1.3.

1.1 Motivation

Control is the process of making a given physical plant or process behave in some ways so that the response of the plant is in conformity with the desired specifications. Control may be automatic or manual, open loop or closed loop. The physical plants are classified as large-scale systems or small-scale systems, linear or nonlinear systems, constrained or unconstrained systems. Large-scale complex systems have multivariable parameters with large dimensions, nonlinearities, spatial distribution with several time scales in its dynamics, uncertainties, constrained and has network structure [1] [2].

Our rapidly growing modern society relies heavily on and is constantly being driven by the increasing number of the large-scale complex interconnected systems such as telecommunication networks, social networks, power networks and drinking water distribution systems. These systems are described as Critical Infrastructure Systems (CIS) due to their direct impact on the day to day running of the society [3] [4].

For the CIS to meet the required operational and service delivery objectives under different operational conditions, advanced control structure and control technology are needed. The control structure and the control technology must ensure the reliable and sustainable operation of the CIS under different operational conditions such as sensor faults, actuator faults, CIS components faults, failures of communication links or anomalies occurring in the technological operation of the CIS physical processes [2]

The control structure is made up of control agents and it receives measurements from the CIS, computes its control actions and executes the control actions on the CIS to achieve the specified control objectives as illustrated in Figure 1.1.

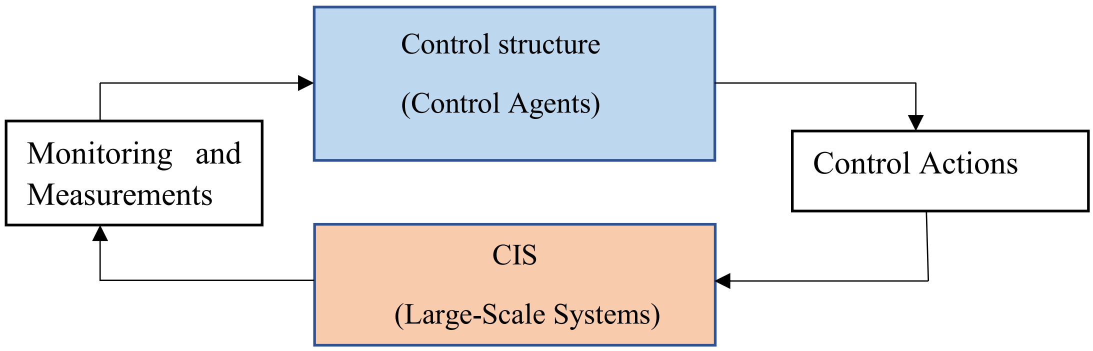

Figure 1.1 The relationship between CIS (Large-Scale systems) and the control structure that controls the system

The control structure in the CIS determines the following [5] [6]:

- the number of control agents that executes control actions on the system and its subsystems
- the arrangement or architecture of the control agents

- the computation method the control agents will use to process the measurements to generate the control actions
- the communication method and the coordination mechanisms that the control agents have among each other
- the hierarchy or level of authority between the control agents

Model Predictive control (MPC) is an advanced control technology in the Industry with the capabilities for the control of highly complex multivariable processes and handling of constrained linear systems or nonlinear systems [7]. This makes MPC an obvious choice as the control agent for the CIS. The MPC is a control technique which repetitively solves on-line over the plant output prediction horizon the open loop MPC optimization task and applies the first part of the generated control sequence into the plant. At every time step, the initial output or state of the MPC is updated by measuring the output or state and using it as information feedback [8] [9] [10] [11]. Central to the implementation of MPC is the explicit use of the model of the plant to be controlled for prediction of optimal control inputs which influences the future behavior of the plant. It is therefore important that the model of the system to be used for MPC design be simple and accurate. The model must be accurate enough to capture the process dynamics, disturbance inputs and account for uncertainties in its parameter estimation.

In the application of MPC, there is always a model-reality mismatch which must be considered and incorporated in the MPC design to ensure that the MPC is robustly feasible while satisfying the plant input, state or output constraints. Robust feasibility is defined at any time instant such that at the current state of the plant, the control input that satisfies the plant input state or output constraints can be determined by the designed MPC. The designed Robustly Feasible MPC (RFMPC) must achieve robust feasibility or generate feasible control inputs under all allowable uncertainty scenarios of the plant over the whole control time horizon.

CIS are typically distributed over a large geographical and are made up of several interconnected subnetworks controlled by a set of controllers or control agents organized in a multiagent architecture. The multiagent architecture can be hierarchical, decentralized or distributed. Each subnetwork is controlled by a controller or control agent or a set of controllers and may work cooperatively or in conflict with other control agents to achieve the system overall control objectives. It thus calls for the application of suitable control agent architectures for the given control task.

Reliable operation of the CIS [2] under different range of operational conditions usually requires the following functions:

- the use of fault detection and isolation (FDI) or fault detection and diagnosis (FDD) system [12] [13]
- distributed state observer mechanisms to predict or robustly estimate the operational state of the CIS [14] [15]
- Fault-tolerant system or control agent adaptive or self-reconfiguration mechanisms in [16] [17] [18] [19] [20] [21]

The current operational state (OS) of a CIS is determined by the states of the CIS processes, states of CIS components, states of all sensors, states of all actuators, disturbance inputs, states of the communication channels and current operating ranges of the processes [2]. The knowledge of the current operational state of the CIS enables the control agents to use the most suitable control strategy to achieve the control objectives at this operational state.

The operational state of CIS may change due to the following:

- Faults in the CIS components
- Faults in the sensors or actuators
- Disturbance inputs not captured in the robust controller design
- Faults in the communication channels used in the CIS

The typical operational states [2] [22] are normal, disturbed and emergency operational states. In RFMPC design, the objective function and the prediction model is usually mapped to a specific control strategy that will achieve specific control objectives at a specific operational state of the CIS. This makes a single objective and single model RFMPC unsuitable for achieving a full range of control objectives under different operation states. It thus calls for the design of multiple RFMPCs for a variety of control objectives each suited for a specific operational state of the CIS.

Given a control task, a suitable control strategy is chosen for the design of the RFMPC to accomplish this task. If there are no faults in the CIS, the RFMPC will continue to achieve the control tasks at this normal operational state. If there are faults in the CIS and there is a possibility of achieving the given control task at this disturbed operational state, there is a need to change the controller strategy to adapt to the disturbed operational state to achieve the control task. Being in the disturbed operational state, the controller receives the monitoring information on the current operational state to determine whether to move to the normal operational state or continue in the disturbed operational state. If there are faults in the system and there is no possibility of achieving the control task at this emergency operational state, there is a need to change the controller strategy to adapt to the emergency operational state. Being in the emergency operational state, the controller receives the monitoring information on the current operational state to determine whether to move to the normal operational state or continue in the emergency operational state.

Change of controller strategy may involve a change of constraints, change of objective function and change of model used for the design of the RFMPC [23] . Each operational state of the system may be described by different mathematical models and these models will be used to design multiple RFMPCs to achieve specific control strategy and control task at the operational states. The use of multiple models of the plant to achieve the same or different sets of control objectives is one of the techniques of fault tolerant control system design [16].

This calls for the application of multiple RFMPCs that can be switched to each other and adapted to fit into the real time operational condition of the plant. The method of switching between the RFMPCs need to be done in a soft way to avoid infeasibility of the control actions, damage of the actuators and the impulsive or spike phenomenon associated with hard switching. The soft switching of RFMPC is a form of adaptive control or MPC reconfiguration technique [16].

Earlier work [23] [24] [25] on soft switching of MPC are for nonlinear and linear systems respectively. In [26] , soft switching of MPC was used as a robust adaptive strategy for switching MPC with different models rather than the continuous parameter update.

In [27] [28] [29] soft switching of MPC was used to switch multiple linear MPC at different operational states of the plant instead of using a single nonlinear MPC. In all these applications, the soft switching of MPC was not done in a distributed MPC (DMPC) framework where coordination of the DMPCs to achieve the control objectives is very important and the effects of soft switching of MPC in one subnetwork or subsystem may affect the states of other interconnected subnetworks or subsystems.

The aim of this thesis is to develop a control approach utilizing multiple RFMPCs arranged in distributed architecture that incorporates distributed soft switching mechanisms between the RFMPCs in each subnetwork of the CIS for achieving the given control task for the CIS. Coordination of the distributed softly switched RFMPC (DSSRFMPC) and recursive feasibility of the control actions will be addressed in this research work.

1.2 Research Objectives and Contributions

1. Multiagent RFMPC architecture and control structure capable of handling all operational objectives of the controlled plant and the network is studied. In this case, one RFMPC represents one control task with a unique model and control strategy, and it is suitable for one operational state of the controlled plant and the network. DWDS water quality control is studied. A two-layer hierarchical structure of quantity and quality control in DWDS used in [30] [31] is modified. In Chapter 2 of the thesis, the lower level water quality controllers in [30] [32] are proposed to be implemented with Softly switched RFMPC (SSRFMPC) to handle pipe breaks, pipe leakages and valve faults. Pipe breaks, pipe leakages and valve faults are selected disturbance scenarios that can change the operational state of the DWDS. The designed RFMPCs are configured to operate in hierarchical distributed architecture for the control of each subnetwork in the DWDS. The hierarchical distributed multiagent architecture is proposed to achieve the water quality control tasks over the DWDS. The hierarchical distributed multiagent architecture are designed to work

cooperatively. The adaptive cooperation schemes are proposed in Chapter 4 and Chapter 5 of the thesis.

2. RFMPC with off-line safety zone is to be designed such that at any control time instant, the initial state of the plant is within the robustly feasible state and the recursive feasibility for the whole control duration is guaranteed for on-line applications. The off-line safety zone technique is a form of constraint restriction technique to account for the model-reality mismatch. The off-line safety zone is to be calculated off-line and applied such that the constraint satisfaction is guaranteed for the whole control duration time. In Chapter 4 of the thesis, the RRFMPC is designed by utilizing the off-line safety zones and the robustly feasible invariant sets in the state space which are calculated off-line using Karush Kuhn Tucker conditions. This is used to calculate the robustly feasible initial states over the prediction horizon under different operational states of the plant to achieve robust feasibility and recursive feasibility for the RFMPC.

3. The distributed soft-switching systems for on-line reconfiguration of the multiagent RFMPCs to respond to failures or faults or operational conditions occurring in the controlled system and network, and adapting the control strategy to the new operational conditions is to be designed. Each RFMPC has its own robustly feasible invariant set. These robustly feasible invariant sets are used by the soft switching mechanism to softly switch from one RFMPC to another in each subnetwork. Distributed Soft switching system suitable for switching RFMPC based on the operational state of the system is proposed in Chapter 4 and Chapter 5 of the thesis. Each RFMPC is designed to fit a specific operational state and can be softly switched to another RFMPC if the current operational state changes. In Chapter 4 and Chapter

5 of the thesis, the soft switching is operated in a distributed architecture and the analysis of distributed soft switching for each operational state on other distributed agents is presented and discussed. Adaptive cooperative coordination strategies for softly switched distributed RFMPC are proposed in Chapter 4 and Chapter 5 of the thesis.

4. DWDS water quality control is studied. In Chapter 3 of the thesis, the existing path analysis in [33] [34] with the backward tracking algorithm used for model structure acquisition in the DWDS is further modified to the proposed forward tracking algorithm to improve modeling accuracy. The proposed forward tracking algorithm is used for partitioning of the DWDS based on the principle of superposition and controllability of the monitored nodes by the injection nodes. The forward tracking algorithm is suitable for application to any flow-based delayed- input, reactive carrier load systems to acquire the model structure. Time-varying parameter models of chlorine residual are obtained via the proposed node-to-node analysis of the DWDS. The node-to-node analysis model is based on determining the model of each monitored node as a resultant impact of all the connecting nodes on the monitored node. In Chapter 3 of the thesis, the time-varying parameter models of chlorine residual control for a drinking water distribution system are proposed and are used to design the Robustly Feasible Model predictive control (RFMPC) for each operational state of the plant.

5. Multi-monitored node output control using single injection node input control technique is proposed and applied on the DWDS water control in Chapter 6 of the thesis. This is a form of Single Input Multiple Output Control. This technique used

one RFMPC to generate robustly feasible control input for more than one monitored node at every time step.

6. The proposed methodology is applied for water quality control of a benchmark drinking water distribution system. Simulation results are discussed in Chapter 6 of the thesis.

1.3 Organization of the Thesis

In Chapter 2, multi-agent MPC control structure design for large-scale systems is presented. DWDS operational control and control tasks are explained. A smart control structure for DWDS quantity and quality control is proposed. The switching of RFMPC under different operational states is presented.

In Chapter 3, mathematical modeling of chlorine residuals in drinking water distribution system and the model parameter estimation is presented. The models of the drinking water distribution system components are explained and the physical laws in DWDS are presented. The hydraulic and quality simulation using EPANET is explained and the models used discussed. Control-based approach modeling of chlorine residuals in DWDS is explained. The operational faults and disturbance scenarios in DWDS water quality control are presented. Model parameter estimation approach for all considered operational states of the DWDS water quality control and the simulation experiment design are presented.

In Chapter4, the basic MPC structure is explained. Robustly feasible MPC (RFMPC) design is discussed and the method for achieving robust feasibility and recursive feasibility using Karush Kuhn Tucker conditions and set invariance theory is proposed. Distributed RFMPC design is presented. The limitations of the existing cooperative strategies for distributed MPCs are identified and the adaptive feasible cooperative strategies for DRFMPCs are proposed.

In Chapter5, the soft switching of RFMPC is presented. Soft switching system, and the functionalities of the components are proposed. Soft switching scenarios for DSSRFMPC with the proposed adaptive cooperative strategies are presented. The soft switching analysis for the RFMPC under different operational states was proposed and the algorithm for its implementation presented.

In Chapter6, the proposed control approach and methodologies are applied to the DWDS water quality control. The benchmark DWDS is used for the simulation and implementation of the proposed methodologies. The results are presented and discussed.

In Chapter7, the conclusions of the research work are presented and recommendations for future research work proposed.

Chapter 2

Multi-agent Model Predictive Control for DWDS

In this chapter, model predictive control structure design for large-scale systems is presented in Section 2.1. The DWDS operational control and the water quality control tasks are explained in Section 2.2. A smart control structure for DWDS water quality control is proposed and discussed in Section 2.3. The summary of Chapter 2 is presented in Section 2.4

2.1 Model predictive control structure design for large-scale complex systems

Control of large-scale systems is usually addressed in a multi-agent control framework. A single or centralized control agent-based control approach of large-scale systems may be difficult due to inherent computational complexity, communication bandwidth limitations, and reliability problems. For these reasons, many distributed control structures have been developed and applied for the control of large-scale systems [6]. The use of multiple MPC as control agents for a large-scale system is referred to as multi-agent model predictive control [5] [35]. The control design approach for these large-scale systems usually starts by specifying the control tasks to be executed on the system which may involve accomplishing a certain number of control objectives. The control tasks may be specified by a human or some artificial entity or they may follow from some behavioural characteristics of the system [5].

The control tasks execute actions on the large-scale system such that the control tasks are accomplished while satisfying the constraints of the system. The control task may be tracking, regulation and economic. The control structure executes the control tasks for the large-scale system by determining the method of communication and the communication protocols between the control agents controlling the different subnetworks in the large-scale system. The way the control agents are arranged to communicate with each other in achieving the control tasks is referred to as the control agent architecture. This is illustrated in Figure 2.1. The control architectures are centralized MPC, decentralized multi-agent MPC, distributed multi-agent MPC, and hierarchical multi-agent MPC. In Figure 2.1 (a) the centralized MPC consisting a single controller receives information about the system state $X_1 \; . . . \; X_N$, computes the control actions and execute the control actions $u_1 \; . . . \; u_K$ on the system to generate the desired system outputs $Y_1 \; . . . \; Y_M$. In Figure 2.1 (b) the decentralized multi-agent MPC consist of many local controllers that control each subnetwork by receiving information about their local states $X_1 \; . . . \; X_{N,}$, computes control actions $u_1 . . . \; u_K$ without exchanging control information with other local controllers to generate the desired system outputs $Y_1 \; . . . \; Y_M$. In Figure 2.1 (c) the distributed multi-agent MPC consist of many local controllers controlling each subnetwork by receiving information about their local states $X_1 \; . . . \; X_N$, computes control actions $u_1 \; . . . \; u_K$ by exchanging control information S with other local controllers to generate the desired system outputs $Y_1 \; . . . \; Y_M$. In Figure 2.1 (d) the hierarchical distributed multi-agent MPC has an upper control layer that receives system-wide information about the systems states and provides local control goals for the lower layer consisting of a set of local controllers that may exchange information to coordinate their control tasks

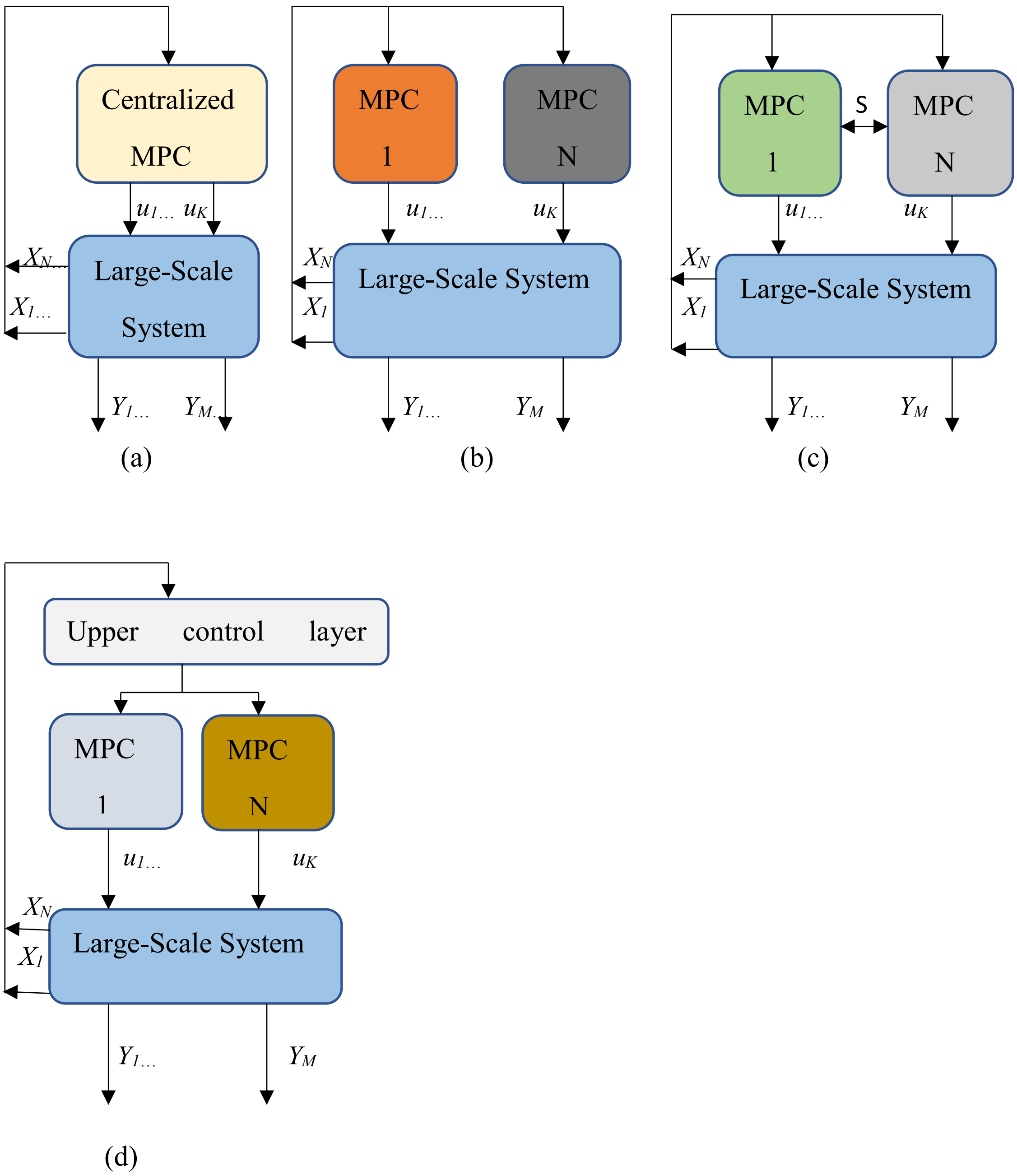

Figure 2.1 Control architectures (a) Centralized MPC architecture (b) Decentralized multi-agent MPC architecture (c) Distributed multi-agent MPC architecture (d) Hierarchical multi-agent MPC architecture [36]

Most Industrial large-scale systems are still being controlled by decentralized or distributed multi-agent control structures. The decentralized multi-agent MPC has an advantage over the distributed multi-agent MPC because there is no exchange of control information between the local MPC controllers and this results in lower communication costs and faster control. The decentralized multi-agent MPC has reduced overall performance compared to distributed multi-agent MPC because subnetwork interactions are neglected and there is no coordination between the local controllers in computing the control actions. There are few published decentralized MPC algorithms with guaranteed properties. In [37], it was reported that research ideas in the industrial automation in the 70's focused mostly on decentralized linear control of interconnected systems. In [38], overview of the concepts of decentralized control is presented. Decentralized receding horizon control for large-scale dynamically coupled systems is in [39]. Decentralized adaptive control in [40]. In [41], Almost decentralized Lyapunov-based nonlinear MPC that uses the state information of neighboring systems for feedback only was presented. In [42], Partitioning approach oriented to the decentralized MPC of large-scale systems with application to water distribution networks was presented. In [43], Plug-and-Play decentralized MPC with guaranteed robustness and applied to power networks was presented.

Pioneering works in hierarchical control and coordination has been described in [44]. The last two decades has witnessed several applications of distributed and hierarchical multi-agent MPC to large-scale systems and CIS because of rapid advancement in ICT and development of smart sensors, actuators, and onboard communication technology. In [36] , it was reported that over 30 research groups worldwide are active in the development of distributed MPC approaches. In [45], The contributions of the research group are listed.

In [36] the distributed MPC approaches of the research groups was categorized into three groups of features: 1) Process features, 2) Control architecture features, and 3) theoretical property features. The process features include the type of system, the type of process, the type of model, the type of control goal and the type of coupling. The control architecture features include the type of architecture, the type of controller knowledge about the overall system, the type of control action computation, the type of controller's attitude in achieving the control objectives, the type of communication, the type of communication protocol and the type of optimization variables considered. The theoretical features include optimality as compared to a centralized approach, guaranteed stability and robustness. In [46] [47], Comparison of distributed MPC schemes is described. In [48], A tutorial review and future research directions in distributed MPC is presented. Coordination of multiple DMPC is addressed in [49] [50]. Hierarchical MPC in [51] [52]. In [53], Comparison of centralized, distributed and hierarchical model predictive control schemes for electromechanical oscillations damping in large scale power systems is presented.

2.2 Operational control of DWDS

DWDS is a large-scale complex network system comprising water pipes, water pumps, valves, water storage tanks, chlorine booster stations and water reservoirs connected to transport safe and clean water to the user nodes. The water transported to and received at the user nodes must: meet the water quality requirements, satisfy the time-varying water demands, minimize the operating costs and maintain prescribed water flows, maintain the prescribed water pressure and water head in the whole DWDS.

The physical structure of drinking water supply system as described in [54] consists of three main parts as illustrated in Figure 2.2:

- Treatment works
- Supply network of trunk mains and main reservoirs
- Distribution networks of small diameter pipes and local reservoirs

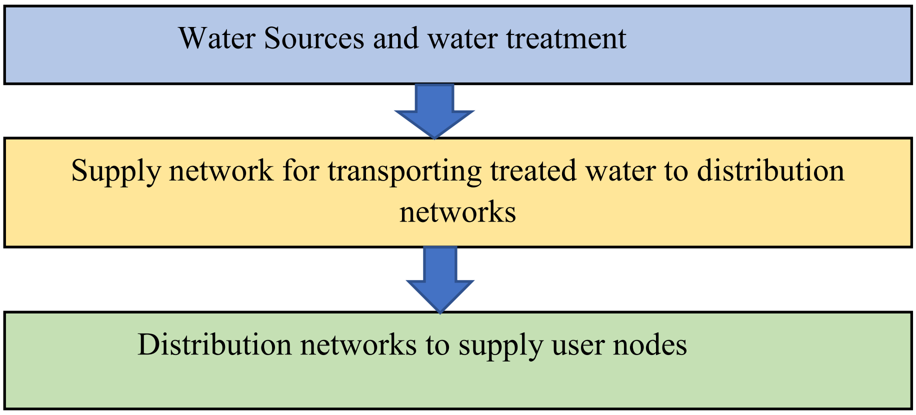

Figure 2.2 Physical structure of drinking water supply system

Drinking water sources are usually taken from underground sources such as wells and springs or from ground sources such as lakes and rivers. The water sources are fed to the water treatment plants to filter out the unwanted materials and substances using physical and chemical methods to make the water clean, safe and healthy for users' consumption. The treated water is then transported to the supply network of trunk mains and main reservoirs where it is pumped to the distribution networks of small diameter pipes and local reservoirs to service the needs of the users. This is illustrated in Figure 2.3 and Figure 2.4.

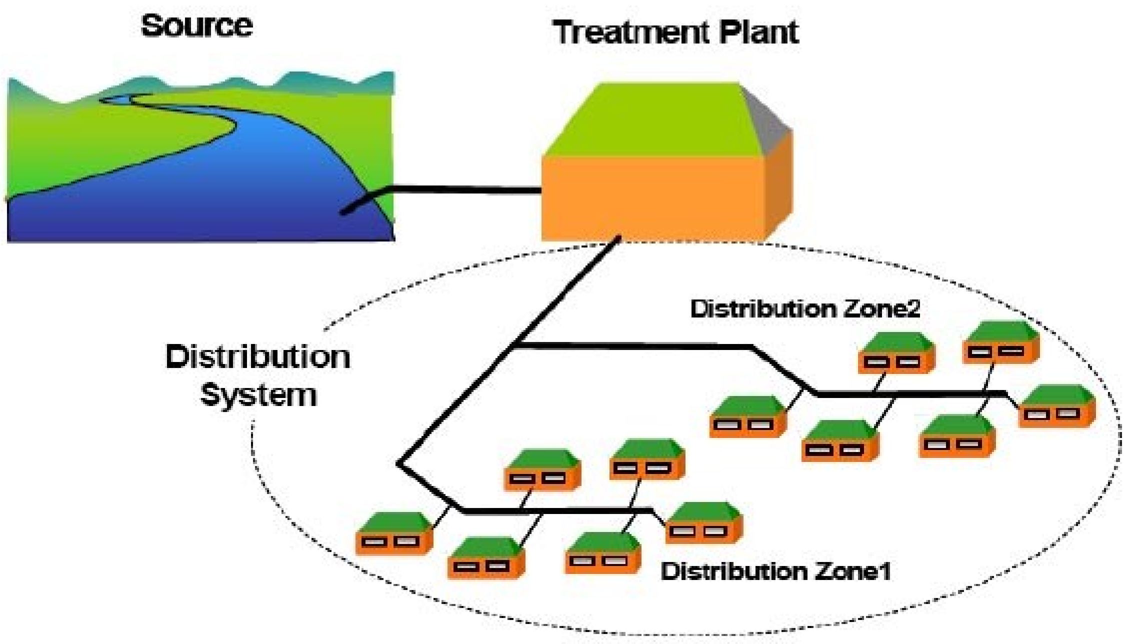

Figure 2.3 Typical drinking water supply system [55]

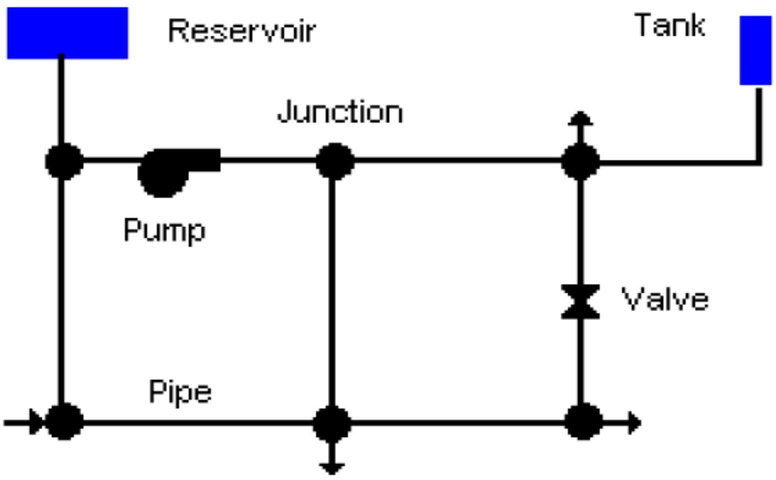

Figure 2.4 Schematic of drinking water distribution system [56]

2.2.1 Control tasks in DWDS

There are two major control tasks in DWDS: quantity and quality. The quantity control tasks deal with control of water flows, water pressure, electrical pump schedules, valve schedules and minimizing the operating costs. The active and controllable components in quantity control are the electrical pumps and valves. The constraints to be considered in quantity control include the operating limits of the pumps and valves, the control objectives, the physical laws governing the hydraulics and relationships between water flows and water heads. Minimizing the electricity charges due to pumping of water is the main operational cost to be minimized in quantity control. The operational control of DWDS was addressed in [54] [32]

In DWDS, chlorine is usually used as a disinfectant to kill the micro-organisms that cause water-borne diseases and it is usually applied at the last stage of water treatment in the water treatment plant. During the transportation of water in the DWDS, the chlorine residual decays and these micro-organisms can grow up on the pipe walls and tank walls in the DWDS as not all them are killed at the water treatment plant. It thus calls for Chlorine injection boosting to be carried out at certain nodes called chlorine injection nodes in the DWDS to maintain the chlorine residual within the specified bounds to reduce these water-borne diseases causing micro-organisms in the DWDS to a harmless level for human consumption [31]

The quality control tasks deal with maintaining the chlorine residual at the user nodes within prescribed lower and upper limits and maintaining other water quality standards. In this thesis, maintaining the chlorine residual at the user nodes within the prescribed lower and upper limits is the control task to be achieved. This is achieved by controlling the chlorine booster stations in the network. The chlorine residuals at the user nodes are the mixed outputs of chlorine from the treatment plants and the chlorine booster station and are the control variables for water quality control. The designed controllers will control the chlorine residual values at the user nodes or monitored nodes within the output constraints described by:

$$y^{min}(k) \leq y(k) \leq y^{max}(k) \tag{2.1}$$

Over the time horizon $k \in [k_0, k_0 + T_{control}]$, where $T_{control}$ is the control time horizon, $y^{min}(k)$ is the minimum chlorine residual requirement at the user node and $y^{max}(k)$ is the maximum chlorine residual requirement at the user node. For this design, $y^{min}(k) = 0.25mg/l$ and $y^{max}(k) = 0.35mg/l$.

2.3 Multi-agent model predictive control of DWDS

2.3.1 Features of DWDS water quality control problem

The water quantity control has an impact on the water quality control and the formulation of control problem for the water quality control. The quality control depends on water flows and flow velocity for its modeling [57] [58]. The equations relating the water flow in the pipe and the chlorine concentration are presented in Chapter 3 of this Thesis. Water quality is determined by water transportation from one node to another, mixing of water of different chlorine residual values at junction nodes and tanks and chlorine residual decay. There are large time delays due to the transportation of chlorine from the chlorine injection nodes to the monitored nodes and long residence time of stored water in the tanks.

The features of the drinking water distribution system water quality control problem to be considered for the control structure design are:

1) The quantity and quality control have different time-scales. For quantity control, the control problem is formulated based on a water demand prediction over the control horizon of typically 24 hours. The water demand varies hourly, daily and seasonally. The hydraulics is modeled and solved in discrete time steps called quantity steps which are typically 1 hour or 2 hours. The quality control is modeled and solved in discrete time steps called quality steps which are typically 5 minutes or 10 minutes.
2) There are uncertainties in water demand predictions, chlorine reaction kinetics, hydraulic coefficients such as water flows, water flow velocity, pump characteristics and valve characteristics
3) Faults such as pipe breaks, valve, pump, and chlorine booster malfunctions constitute disturbance scenarios that can change the operation state of the DWDS from normal to disturbed or emergency operational state.
4) Conflict of operational goals of quantity and quality control. There are multi-objectives to be achieved in DWDS quantity and quality control and this includes the minimization of electricity costs of pumping, the minimization of cost of chlorine injection, the minimization of cost of raw water taken from the water sources and minimization of the cost of water treatment at the treatment plants. For example, one of the main goals of the water quality control can be achieved by reducing the retention of water from the treatment plant (the source of DWDS) to the user nodes. This means that more time will be allocated to pumping of water in the DWDS and this will increase the operational costs of the DWDS. It is therefore necessary to

solve the quantity and quality control in an integrated way to avoid conflict of control goals.

These DWDS water quality control problem features are considered in the control structure design.

2.3.2 Water quality control of DWDS

DWDS is a large- scale complex system and it is spatially distributed. A centralized Control architecture for the DWDS will not be a good choice because of computational complexity, centralized communication complexity, and maintenance flexibility. It therefore needs to be partitioned or decomposed to several interacting subnetworks for achieving the water quality control tasks.

The Chlorine booster stations are used to inject chlorine at certain nodes to achieve the water quality control at the monitored nodes and the Chlorine sensors are used at the monitored nodes to measure the chlorine residual. The different on-line water quality monitoring and early warning systems in DWDS are described in [59]. The Chlorine booster station represents the actuator of the subnetwork and the monitored nodes controlled by the chlorine booster station represents the outputs of the subnetwork. The location of the Chlorine booster stations and Chlorine sensors is important to the achievement of the water quality control tasks. The partitioning of the DWDS for water quality control is usually based on the selection of chlorine booster or injection nodes and the associated monitored nodes controlled by the selected chlorine injection nodes. In [60], Chlorine booster location model which minimizes the number of chlorine booster stations installed was presented. Optimal location of chlorine booster stations was addressed in [61] [62] [63] [64]. In general, the selection of the chlorine booster or injection nodes and the monitored nodes is based on the system controllability and observability respectively [65]. The value of the chlorine residual at the monitored node that is controllable by a given chlorine injection node is given by:

$$y(k) = F(u(k), k - i) \tag{2.2}$$

Where $y(k)$ is the value of chlorine residual at the monitored node controlled by the chlorine injected $u(k)$ at the injection node and operator F is a function of the water flow velocity, rate of chlorine decay and chlorine transportation time. The monitored node is controllable by the injection node if the value of the chlorine residual at the monitored node $y(k)$ can be changed by the injected chlorine $u(k)$ at time instant $k - i$, where i is the delay time or chlorine transportation time to the monitored node.

In the DWDS partitioning for water quality control, a selected chlorine booster station node may affect or have couplings or interactions with other monitored nodes outside its assigned subnetwork. These couplings or interactions are to be considered in the control design for water quality control. In this thesis, the selection of the chlorine booster nodes and monitoring nodes is selected based on the controllability and the observability of the nodes respectively using the forward tracking algorithm developed to track the impact of the injected chlorine from each node to other nodes. This is achieved by using superposition principle to determine the controllability of each node used as injection node and the observability of the nodes used as monitored nodes.

The forward tracking algorithm is further discussed in Chapter 3 of this thesis. The number of partitions of the DWDS is the number of Chlorine booster stations that controls the monitored nodes. The benchmark DWDS used in this thesis is partitioned using the forward tracking algorithm into distributed subnetworks or zones.

2.3.3 Smart Control structure with soft switching capabilities for water quality control of DWDS

The hierarchical control structure is usually used for the real-time control of drinking water supply network [30] [32]. Figure 2.5 illustrate the proposed hierarchical structure of a nation-wide real-time control of drinking water supply network.

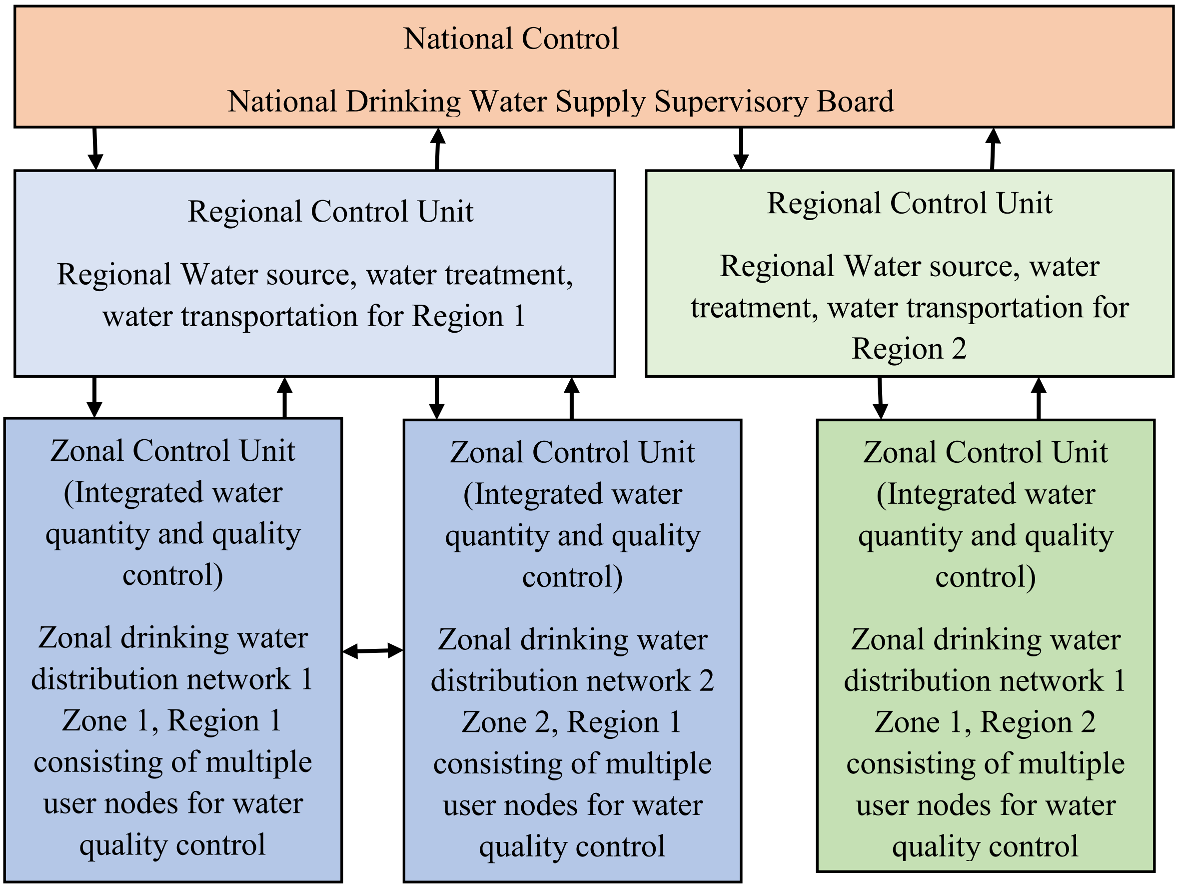

Figure 2.5 Proposed Hierarchical structure of a nation-wide real-time control of drinking water supply network. [44] [30]

The hierarchical control structure is proposed for the integrated control of quantity and quality control of the drinking water supply network. The higher layer corresponds to the drinking water supply network control characterized by slower dynamics and the lower layer corresponds to the drinking water supply network control characterized by faster dynamics. In Figure 2.5, the functions of each level of the hierarchy are as follows:

- National Drinking Water Supply Supervisory Board (National Control): this is the highest level in the hierarchy. Its functions are to provide supervisory controls, set water production targets based on predicted water demands for each region to all the regional water production and transport units in the nation. It also ensures optimal and sustainable utilization of the water resources in the nation. The output prediction and control time step may be monthly or yearly. The information feedback to the National Drinking Water Supply Supervisory Board (National Control) is the real-time information on the water production quantity, actual water demands and fault management information from each of the Regional Control Unit. The functions of the National Control may be implemented with MPC. The model is given as:

$$y_N(k) = F_N(u_N(k), k) \tag{2.3}$$

  where $y_N(k)$ is the National water production target at time instant k, operator F_N is a function of water demands and water production costs, $u_N(k)$ is the incentive such as increase or decrease in budget for National water production or water production regulatory policies to facilitate or reduce National water production.

- Regional water source, water treatment and transport unit (Regional Control Unit): Each region has its own water source, water treatment and transport unit to transport the treated water to its region. The water pumps and large reservoirs are in the Regional water source, water treatment, and transport unit. The water production at each region follows the set-point or water production target set by the National Drinking Water Supply Supervisory Board. A region is made up of several Zonal distribution networks and each Zonal distribution network has its own total water demand. Each Regional water source, water treatment, and transport unit, transports water with the required water flow, pressure and water quality to match the water demand of each Zonal distribution network it supplies. The information feedback to the Regional water source, water treatment, and transport unit include real-time actual water demand in each Zonal distribution network, water flow, pressure and fault management information. Optimization of the water treatment costs and cost of transport of treated water are considered in this level. The control and output prediction horizon may be daily or weekly. The functions of the Regional Control may be implemented with MPC. The model is given as:

$$y_R(k) = F_R(u_R(k), k) \tag{2.4}$$

where $y_R(k)$ is the Regional water production target with the required quality, pressure and flow at time instant k, operator F_R is a function of water demands and water treatment costs, electricity costs for pumping, and water transportation costs, $u_R(k)$ is the valve schedules, pump schedules and chlorine injection schedules.

- Zonal distribution networks (Zonal Control Unit): The Zonal distribution networks are to serve multiple water users in different areas. The water user grouping into zones may be done by partitioning the Regional distribution networks for water quality control. Tanks, pumps, and different valves are in the zonal distribution networks. Integrated drinking water quantity and quality control of the water supplied to each zonal distribution network is executed at this level [30]. The quantity control and output prediction horizon may be 1 hour or 2 hours. Tank operations, pump schedules, valve schedules, water flow, flow velocity, pressure and chlorine injection schedules are optimized at this level. Feedback water quality control using the chlorine booster stations is executed at this level. The control prediction horizon of the water quality control may be 5 minutes or 10 minutes. Chlorine injection dosage optimization is considered at this level. Information feedback to the Zonal distribution networks includes actual real-time water demand for the users, water flow, flow velocity, water heads, pressure, valves status and fault management information. Interactions through pipe connections to other zonal networks are to be considered in water quality control at this level. The functions of the Zonal Control may be implemented with MPC. The coordination of the distributed MPC controllers for water quality control is executed at this level. The model is given as:

$$y_Z(k) = F_Z(u_Z(k), k) \tag{2.5}$$

where $y_Z(k)$ is the water quality at the monitored nodes at time instant k, operator F_Z is a function of water demands, water flow velocity and rate of chlorine decay $u_R(k)$ is the chlorine injection at the injection nodes.

In [30], a two-level hierarchical integrated quantity and quality control structure for DWDS was presented. In this thesis, a hierarchical multi-layer distributed integrated quantity and quality control structure for DWDS with additional capabilities of softly switching the controllers to adapt to the current operational conditions of the DWDS is proposed. The soft switching of MPC will be implemented for DWDS water quality control at the Zonal control level in the proposed hierarchical structure in Figure 2.6.

Real-time modeling and operations of the drinking water distribution system using a smart system was addressed in [66]. Drawing from the concept in [66], the smart system with integrated soft switching capabilities is proposed in Figure 2.6. The smart system functions can be divided into monitoring, control and fault management. The monitoring unit consists of on-line sensors and smart meters and Geographic Information system (GIS). The control unit consists of Supervisory control and data acquisition (SCADA) system, real-time network modeling system and real-time operations optimization system. The fault management unit consists of real-time event detection and early warning system, real-time network anomaly detection system and asset integrity management system.

In Figure 2.6, the supervisory control unit at the Zonal distribution network (Zonal Control) collects real-time DWDS network parameters data from the monitoring unit and fault management unit to assess the current operational state of the DWDS.

The Supervisory control unit softly switches to the most suitable quantity controller at the upper-level integrated quantity and quality control unit to generate optimized pump schedules, valve schedules and injection schedules for the Zonal drinking water distribution network after determining the current operational state of the DWDS.

The Supervisory control unit also softly switch to the most suitable controller at the lower level quality control unit for the current operational state of the DWDS. The quantity and quality controllers are implemented using MPC controllers to execute its control actions.

The operation of the proposed Hierarchical multi-layer Structure for Optimizing Integrated Quantity-Quality Control with soft switching capabilities of RMPC Controllers in Figure 2.7 is as follows:

- At the beginning of a 24 hour-prediction horizon, the DWDS quantity and quality states are measured by the on-line sensors in the monitoring unit or estimated and sent to the supervisory control unit at the Zonal control level, integrated quantity and quality optimizer, lower level quality control optimizer and the fault management unit.

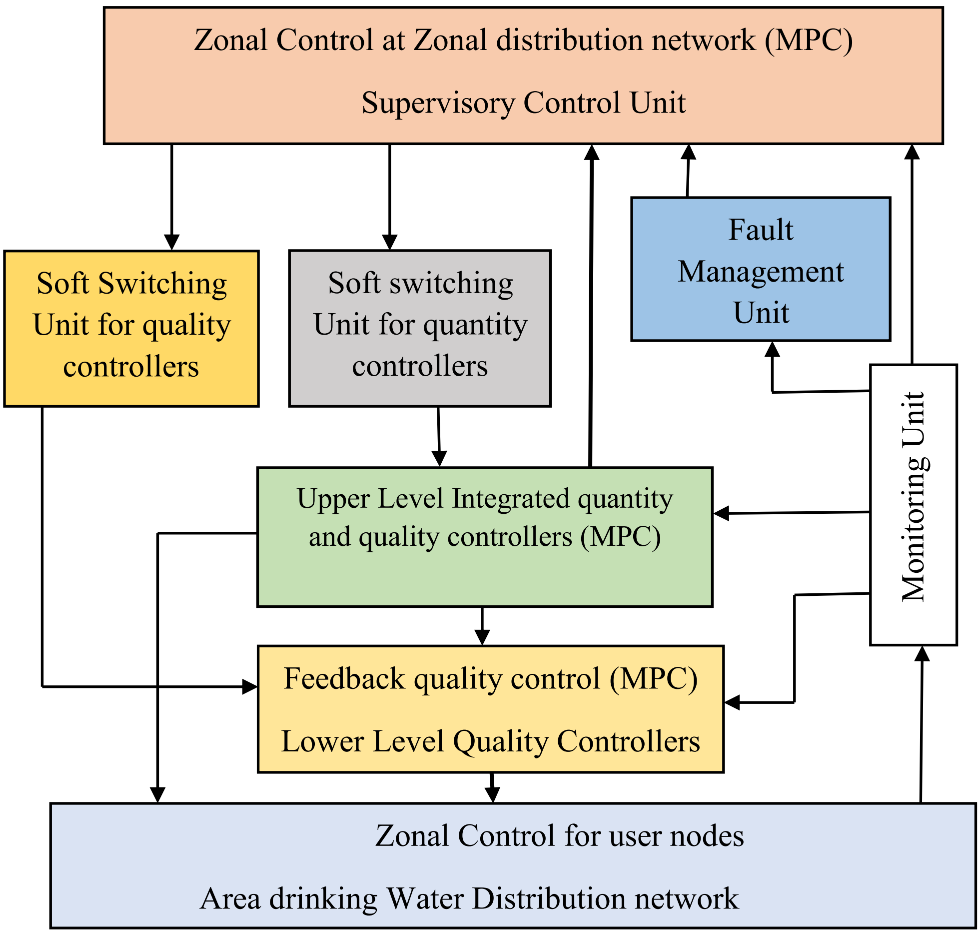

Figure 2.6 Proposed Smart Control structure for operational control of DWDS

- The area user water demand prediction and the electricity tariff are also sent to the integrated quantity and quality optimizer
- The Integrated quantity-quality optimization problem is solved to produce optimized pump schedules, valve schedules, and chlorine injection schedules (for the chlorine booster stations) over the next 24 hour-prediction horizon. The chlorine injection schedules produced at the integrated quantity-quality controller are not accurate due to many uncertainties in the system. The correction is provided by the lower level quality controller [31] [30].
- The pump and valve schedules are applied to the DWDS and maintained during the hydraulic time step of 1 hour or 2 hours. The quantity and quality states are measured, the user demand prediction and electricity tariff are sent to the integrated quantity and quality optimizer at the end of each hydraulic time step and new pump and valve schedules and chlorine injection schedules are produced and applied to the DWDS. The sequence is repeated at the end of the control time horizon.
- The Optimized flow schedules and hydraulic information needed for quality control is sent from the Integrated quantity and quality optimizer to the Zonal Control Unit. The Zonal Control unit consist of the Supervisory control and Data Acquisition Unit (SCADA), real-time network modeling system and real-time operations optimization system. The fault management unit consists of fault detection and diagnosis (FDD) unit, state estimator unit for both quantity and quality operational states, real-time event detection and early warning system, real-time network anomaly detection system and asset integrity management system. The monitoring unit consists of on-line sensors and smart meters and Geographic Information system (GIS).

- The Zonal Control unit receives the monitoring information and fault management information from the DWDS at quality time steps and compares it with the hydraulic information and Optimized flow schedules from the Integrated quantity and quality optimizer to assess the operational state of the DWDS to determine whether there is a fault in the DWDS or not. With the new operational state determined by the Zonal Control unit, the possibility of fulfilling the present control task is assessed based on the model used for the design of the operating MPC. If the present operating MPC cannot fulfill the control task, a new MPC which can fulfill the control task for the new operational state is identified by the Zonal Control unit and softly switched to by the soft switching unit. The soft switching unit softly switches the new MPC at the Lower level quality control of the structure.

The detailed design and operation of the soft switching unit are presented in Chapter 5. The soft switching for the water quantity control is not addressed in this thesis. The Lower level control structure now consists of multiple MPC water quality controllers that can be softly switched depending on the assessed operational states by the Zonal Control unit. The operational states can be normal, disturbed and emergency as described in [2] .

In this thesis, the normal operational state is a state of no fault in the DWDS. The disturbed operational state is a state when there is the possibility of still fulfilling the water quality control task despite the disturbance in the DWDS. The emergency operational state is a state when there is no possibility of fulfilling the water quality control task due to the disturbance in the DWDS.

Under normal operational states of the DWDS, the lower level normal RFMPC controllers are engaged for the water quality control. This is illustrated in the flowchart Figure 2.7

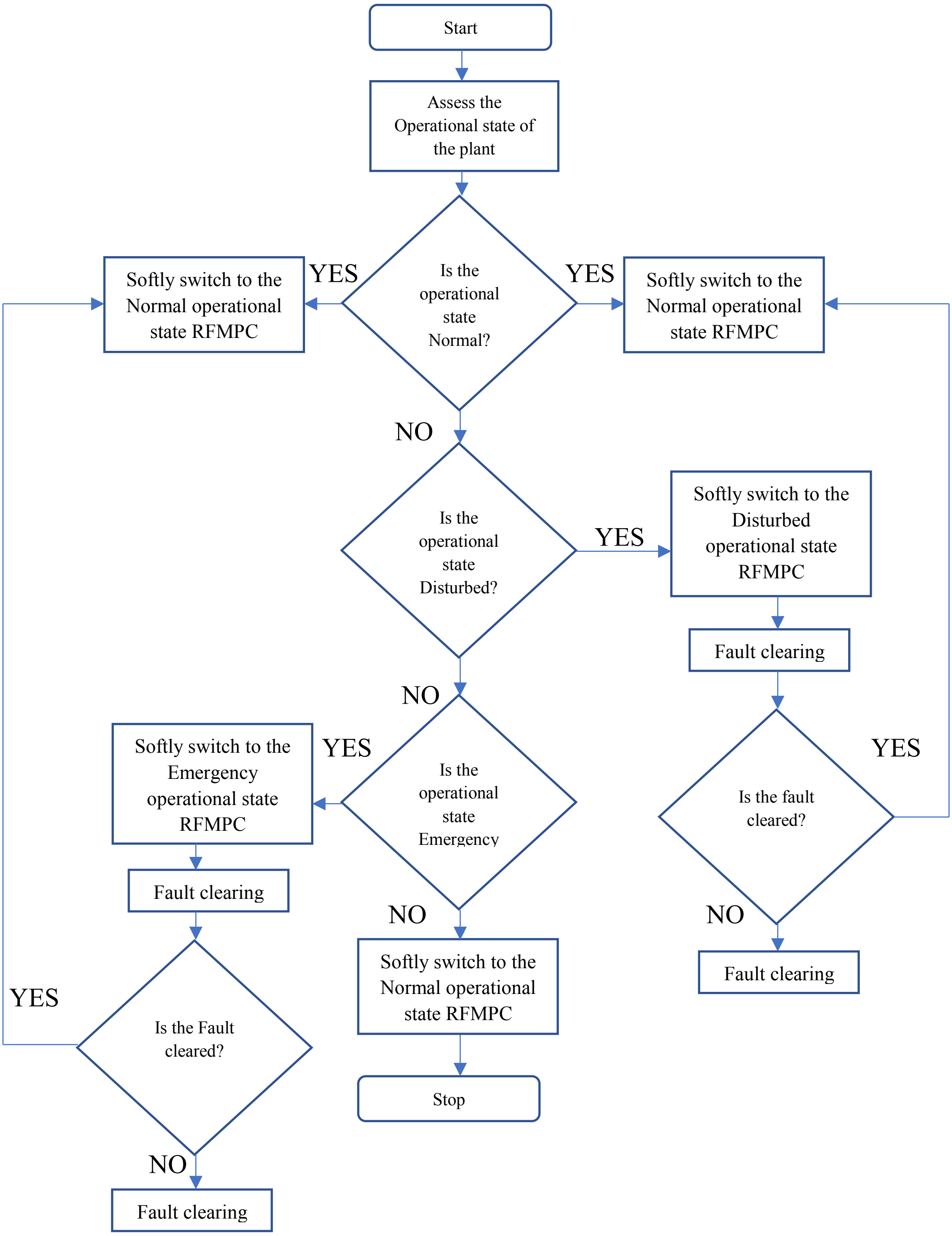

Figure 2.7 Soft switching among the operational states flow chart

At each quality time step, the monitored node chlorine residual level information is compared with the reference chlorine bounds. The correction is provided by the RFMPC to produce the accurate chlorine injection for the required monitored nodes chlorine residual level.

Under the disturbed operational states, the lower level disturbed RFMPC are engaged for the water quality control. At each quality time step from the instance of softly switching from normal operational state to the disturbed operational state, the disturbed MPC uses a new model and modified constraints to produce accurate chlorine injections for the monitored nodes that are controllable at this operational state.

Under the emergency operational states, the lower level emergency RFMPC controllers are engaged. At each quality time step from the instance of softly switching from normal operational state or disturbed operational state to the emergency operational state, the emergency RFMPC uses a new model and modified constraints to produce no chlorine injections. The soft switching among the operational states is shown in Figure 2.8. The fault-clearing process brings the DWDS back to the normal operational state from disturbed or emergency operational state. When the fault in the DWDS is cleared, the Zonal Control unit assesses the operational state for no-fault and softly switches to the normal operational state MPC.

The soft switching of RFMPC at the lower level RFMPC for the DWDS is proposed as a form of adaptive control to handle the disturbances that may affect the quantity control which has a direct impact on quality control. The quantity control operates in time steps of 1 hour or 2 hours compared to the quality control time step of 5 minutes or 10 minutes; therefore, any disturbance scenario that affects the quantity control will impact the quality control if an adaptive control approach is not put in the design to guaranty the quality control under these disturbance scenarios.

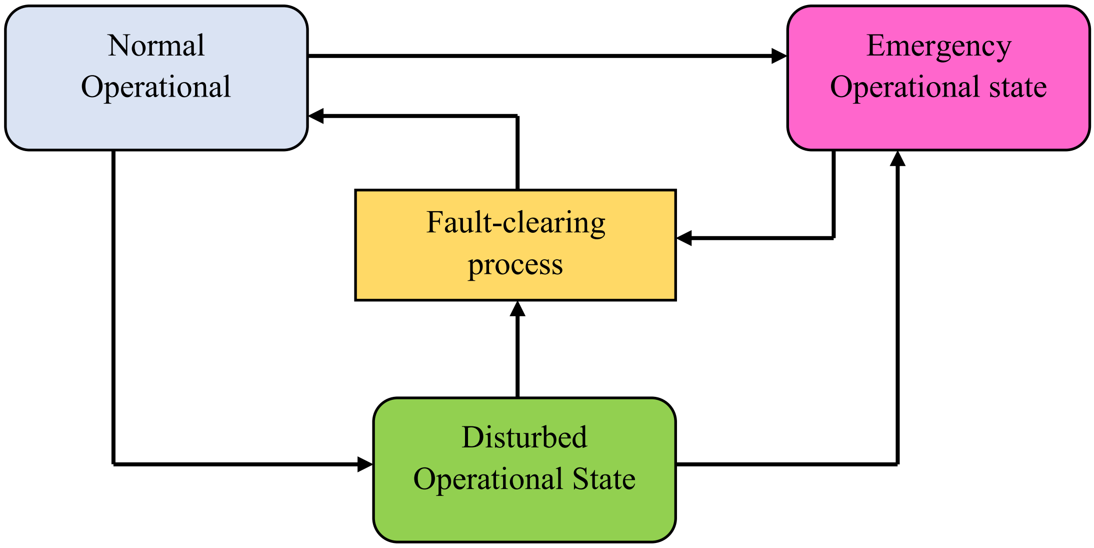

Figure 2.8 Soft switching among the operational states

It is the author's opinion that no single MPC can guaranty the achievement of the quality control under different disturbance scenarios. It thus calls for the application of multiple MPC controllers that can be softly switched to adapt to the current operational state of the plant. The control input which is the injected chlorine from the chlorine booster station may impact the monitored nodes in other zonal distribution networks if the water flow paths exist from this injection node to the monitored nodes. This is referred to as zonal network interactions. This is accounted for in each subnetwork modeling and by using a suitable communication protocol or coordination scheme between interacting area networks during the control input generation at each time step. The communication between control agents in different zonal networks is coordinated by the coordinator for communication. The coordinator determines the protocols for information exchange between agents. Information exchange between interacting control agents includes control input sequence generated by the interacting agents and the operational state of the interacting area networks.

In this thesis, the design of the RFMPC, the soft switching unit, and coordination schemes is carried out under different disturbance scenarios that represent disturbed and emergency operational states. The disturbance scenarios considered in this thesis for simulation of change of operational states include pipe breaks or leakages and valve faults. The design of the fault detection and diagnosis mechanism and operational state estimator is not carried out in this thesis, which is assumed to be contained in the fault management unit.

The mathematical models of Chlorine residual for DWDS under different disturbance scenarios, the model parameter estimation, the design of RFMPCs, the design of Soft switching mechanism and coordination schemes is presented in Chapter 3, chapter 4 and Chapter 5 of the thesis.

2.4 Summary

Multi-agent MPC Control structure design for DWDS has been presented. The hierarchical and distributed multi-agent MPC structure has been applied for the control of many large-scale systems. Operational control of drinking water distribution systems (DWDS) has been presented and the control tasks identified. A hierarchical distributed multi-agent RFMPC structure was proposed for DWDS quantity and quality control. A smart and adaptive control structure for DWDS water quality control was also proposed and the functionalities of the components discussed. The soft switching system for RFMPC for different operational states of the DWDS was proposed.

Chapter 3

Mathematical model of chlorine residuals in DWDS and model parameter estimation

In this chapter, the physical models in DWDS is presented in Section 3.1. The physical laws in DWDS are explained in Section 3.2. In Section 3.3, control-design based chlorine residual modeling in DWDS is explained. Model structure determination for chlorine residual modeling is presented in Section 3.4. Uncertainties in chlorine residual modeling is presented in Section 3.5. Chlorine residual modeling in DWDS under different disturbance scenarios is presented in Section 3.6. Model parameter estimation is presented in Section 3.7. The simulation experiment design is explained in Section 3.8. Time-varying model parameter estimation is presented in Section 3.9. The summary is presented in Section 3.10

3.1 Physical models in DWDS

DWDS consists of Reservoirs, Tanks, pumps, Pipes, Valves, Chlorine booster stations and the user nodes connected to transport safe and clean water to the users for domestic, industrial and economic purposes.

There are two aspects of control in the DWDS; Quantity and Quality control. In this thesis, water quality control is considered. The water quality control task considered in this thesis is maintaining the chlorine residual at the user nodes within the specified lower and upper bounds. To achieve this water quality control task, a suitable mathematical model for chlorine residual in the DWDS must be derived. This model is to be used for RFMPC design for water quality control in the DWDS.

Chlorine is transported by water in the DWDS. The Chlorine residual modeling in the DWDS is governed by:

- water flow
- flow velocity
- mixing of water at the junction nodes and tanks
- chlorine transport times in the pipes
- water detention times in tanks
- the chemical reaction of chlorine in water and with the pipe walls

The hydraulics, therefore, has a significant impact on chlorine residual modeling. Each component of the DWDS is to be modeled and the governing laws presented. The following general assumptions are made in the mathematical modeling for the DWDS [54] [67]:

1) The inertial effect of the water in the pipe is neglected
2) Water is treated as incompressible fluid
3) Constant temperature and air pressure within the DWDS. In conditions of constant temperature and pressure, a liquid will assume the shape of its container and fill a portion equal to the liquid volume [68]. Water is considered incompressible; that is, their volume does not change appreciably under pressure or change in temperature [68].
4) Constant density and viscosity. Changes caused by chlorine injections are neglected

5) Instantaneous dynamics of the water network components are neglected. For example, pump and valve on or off and water pressure propagation dynamics in long pipes.

These assumptions clearly introduce modeling errors, but these errors are considered small and it is incorporated in the controller design. Systems approach to modeling [67] and operational control of DWDS is addressed in [54].

3.1.1 Pipes

Pipes are links through which water travels from one point to another in the DWDS. The water in a pipe flows from the higher node to the lower node of the pipe. This water flow is driven primarily by a water pressure difference or head difference in the pipe nodes. This is shown in Figure 3.1. The hydraulic input parameters for pipes are pipe length, pipe diameter and Roughness coefficient for headloss calculations.

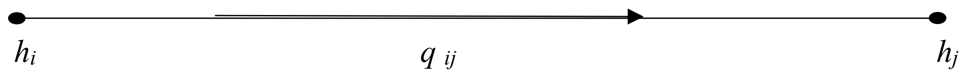

Figure 3.1 Model of a pipe

The water quality input parameters for pipes are wall reaction coefficient and bulk reaction coefficient.

The outputs that can be computed for pipes includes [56]:

- Flow rate

- Flow Velocity
- Headloss
- Average water quality over the pipe length

The Hydraulic headloss by water flowing in a pipe is due to friction with the pipe walls and can be computed by using:

- Hazen -Williams formula
- Darcy-Weisbach formula
- Chezy-Manning formula

Each formula uses the following equation to calculate the headloss between the start and end node of the pipe [56]:

$$h_L = Aq^B \tag{3.1}$$

Where h_L = headloss (length), q = flow rate (Volume / Time), A = Resistance coefficient, and B = flow exponent.

The head-flow relationship in [54] of water flowing in a pipe can be written as:

$$q_{ij} = \emptyset_{ij}\left(\Delta h_{ij}\right) = \emptyset_{ij}\left(h_i - h_j\right) \tag{3.2}$$

Where q_{ij} is the flow from node i to node j. $i \rightarrow j$ is defined as the positive direction of the flow and $q_{ij} \leq 0$ is when the flow is from $j \rightarrow i$.

$$\Delta h_{ij} = h_i - h_j = g_{ij}\left(q_{ij}\right) = R_{ij} q_{ij} \left|q_{ij}\right|^{\alpha - 1} \tag{3.3}$$

Where R_{ij} is the pipe resistance and α is the flow exponent for calculating headloss due to friction losses. Hazen-Williams equation is most frequently used in modeling the DWDS because of its calculating simplicity and accuracy that satisfies operational control purposes. Applying Hazen-Williams equation, equation 3.3 can now be written as:

$$\Delta h_{ij} = h_i - h_j = g_{ij}\left(q_{ij}\right) = R_{ij} q_{ij} \left|q_{ij}\right|^{0.852} \tag{3.4}$$

$$R_{ij} = \left(1.21216 \times 10^{10} \times L_{ij}\right) / \left({C_{ij}}^{1.852} \times {D_{ij}}^{4.87}\right)$$

Where L_{ij} , D_{ij} and C_{ij} denote the pipe length, pipe diameter and Hazen-Williams roughness coefficients respectively. If the pipe length is in *m* and the diameter in *mm* and the heads are in *m*, then the flow is in *litre/ sec*. The Hazen-Williams roughness coefficient is determined by the material of the pipe and it changes considerably with age and manufacturer. The roughness coefficient must be calibrated regularly to establish an accurate dynamic hydraulic model [31]

The roughness coefficient is a source of uncertainty in the hydraulic modeling of the DWDS and pipes are classified as passive elements of the DWDS.

3.1.2 Pumps

Pumps are active elements of the DWDS. Pumps impart energy into the DWDS by increasing the hydraulic head of the water. Pumps are driven by electrical motors and it is the main electrical energy consumption in the DWDS. Pumps are also used as pressure booster devices to maintain the water head in the DWDS. The input parameters for a pump are its pump curve (the combination of heads and flows that the pump can produce) and its node in the DWDS. The output parameters are flow and head gain. The following types of pumps are used in the DWDS:

- Fixed speed pumps
- Variable speed pumps
- Variable throttle pumps

The models for each of the pump is in [54]. In this thesis, the fixed speed pump is used for the simulation of the DWDS.

The pump stations usually consist of all the types of pumps arranged in parallel as presented in. Pumps are controlled actuators characterised by operating limits. The pump operating limits include the operating speed of the pump, and the water head limits deliverable by the pump. These limits are considered as model constraints.

The head-flow relationship for a fixed speed pump connecting nodes " i " and " j " or the pump characteristic curve, is a nonlinear function which is written as:

$$\Delta h_{ij} = g^{f}(q_{ij}) \tag{3.5}$$

Where headloss Δh_{ij} is defined as

$$\Delta h_{ij} = h_i - h_j \quad \text{where } h_i \geq h_j \tag{3.6}$$

The nonlinear function $g^f(q_{ij})$ can be approximated by quadratic function:

$$g^f(q_{ij}) = A_{ij}\, q_{ij}{}^2 + B_{ij}\, q_{ij} + h_{0,ij} \tag{3.7}$$

where A_{ij} is the resistance coefficient and $A_{ij} < 0$, $B_{ij} \leq 0$ and $h_{0,ij}$ is the shut-off head.

3.1.3 Valves

Valves are used to limit the pressure or flow at a specific point in the DWDS. Valves are pipe fittings, and they may be operated manually, either by using a hand wheel, lever or pedal. Valves may also be automatic, driven by changes in pressure or flow. More complex control systems using valves requiring automatic control based on an external input (i.e., regulating flow through a pipe to a changing set point) require an actuator [69]. The input parameters for the valve are [56]:

- Diameter of the valve
- Setting of the valve
- Status of the valve
- The node in the DWDS

The outputs for a valve are flow rate and headloss. There are different types of valves used for different functions and applications in the DWDS:

- Flow control valves (FCV): Limit the flow rate at a specified value
- Pressure sustaining valves (PSV): Maintain the pressure to some value
- Pressure Reducing valves (PRV): Reduce water pressure
- Pressure breaker valves (PBV): Create a specified headloss across the valve
- Check valves (CV): Control the flow in one direction only
- Throttle Control Valve (TCV): Headloss characteristics change with time

Valves are controlled actuators and are characterised by operating limits. These limits are considered as model constraints. The model for each type of valve is in [54] [56]

A continuous description for control valves is given by:

$$\Delta h_{ij} = R_{ij}\, q_{ij}|q_{ij}| \tag{3.8}$$

where R_{ij} is the resistance of the valve and

$$R_{ij} = \frac{K_{v,ij}}{2g\,(A_{max,ij}{}^{2})} \tag{3.9}$$

Where g is the gravitational acceleration, $K_{v,ij}$ is the minor headloss coefficient and $A_{max,ij}$ is the maximum valve cross-section area.

3.1.4 Tanks

Tanks stores the DWDS drinking water. They are the dynamic components in the DWDS because of their storage capabilities and properties. The input parameters for the tanks are [56]:

- Bottom elevation (where water level is zero)
- Initial, minimum and maximum water levels
- Shape of the tank with its dimensions
- Initial water quality

The outputs of the tanks are water quality and hydraulic head. Tanks are also water quality source points. Reservoirs represent an infinite external water source to the DWDS. The input parameters for the reservoirs are [56]:

- Its hydraulic head
- Initial water quality

The outputs of the reservoirs are its hydraulic head and water quality. The model of a tank is shown in Figure 3.2.

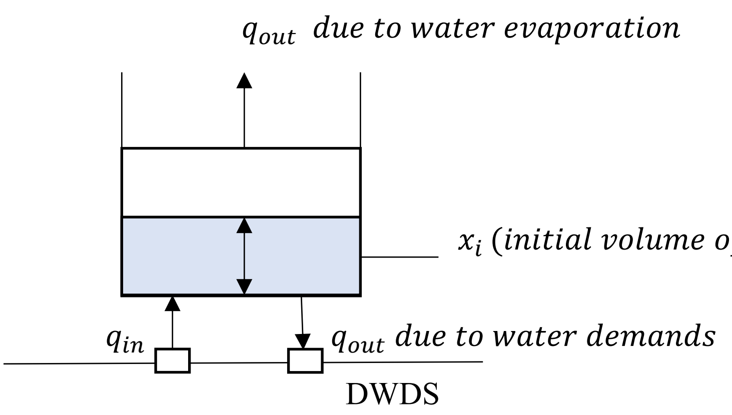

Figure 3.2 Model of a Tank

For the tank in Figure 3.2, the mass balance expression relating the initial volume of water stored, the manipulated inflows and outflows and the demands as presented in can be written as:

$$x_i\ (k+1) = x_i\ (k) + \Delta t\ (\sum_i q_{in,i}\ (k) - \sum_j q_{out,j}\ (k)) \tag{3.10}$$

Where $q_{in,i}\ (k)$ and $q_{out,j}\ (k)$ correspond to the *i-th* inflow and the *j-th* outflow respectively, x_i is the stored volume of water in the tank and Δt is the sampling time.

The maximum and the minimum tank volume are the model constraints.

3.2 Physical laws in the DWDS

3.2.1 Conservation of energy law

The principle states that the difference in energy between two points must be the same regardless of the path that is taken [67]. Therefore, for any two points connected in a network, the difference in energy is equal to the energy gains from pumps and energy losses in pipes and fittings that occur in the path between them. The following holds:

$$\sum h_{ij} = \partial E_r \tag{3.11}$$

Where $h_{ij} = h_i - h_j$ denotes the head change across a pipe in a path and ∂E_r the head difference between the start node and end node of a path. If the start node and the end node in a path are the same node, then the path is a loop. For a loop:

$$\sum h_{ij} = 0 \tag{3.12}$$

3.2.2 Conservation of Mass

The principle of conservation of mass states that the fluid mass that enters any pipe will be equal to the mass leaving the pipe [67]. This can be written as:

$$\sum_{Pipes} Q_i - W_{d,i} - \frac{dX}{dt} = 0 \tag{3.13}$$

Where $\sum_{Pipes} Q_i$ is the sum of all pipe inflows to the node i, $W_{d,i}$ is the water demand at node *i* and $\frac{dX}{dt}$ is the change in the water storage in the tank.

The conservation of mass equation is applied to all junction nodes and tanks in the DWDS and one equation is written for each of them [67].

3.2.3 Flow continuity law

The flow continuity law states that the sum inflows and outflows is equal to zero for every non-reservoir node and holds for reservoir nodes that are in steady state [67]. For every junction node *j*, the following holds:

$$\sum_{i \in J_j} q_{ij} = W_{d_j} \tag{3.14}$$

Where J_j denotes set of nodes linked to node j, W_{d_j} is the water demand allocated to the *j-th* node.

3.2.4 Simulation model of DWDS

In this chapter, EPANET software package is used for hydraulic and water quality model simulation for the DWDS. EPANET is widely used in the simulation of the water network. EPANET is a computer program that performs an extended period simulation of hydraulic and water quality behaviour within pressurized pipe networks [56]. EPANET tracks the flow of water in each pipe, the pressure at each node, the height of water in each tank, and the concentration of a chemical species throughout the network during a simulation period comprised of multiple time steps. EPANET is designed to be a research tool for improving our understanding of the movement and fate of drinking water constituents within distribution systems. It can be used for many kinds of applications in distribution systems analysis.

In using the EPANET, the user draws the DWDS network, edit the properties for pipes, valves, tanks, reservoirs, chlorine boosters, pumps, hydraulic time steps, quality time steps, period of simulation, roughness coefficients, chemical reaction kinetics and energy usage, and set the rules that determines how the system should be operated. The hydraulic / water quality analysis is run, and the results of the analysis viewed .

EPANET's water quality solver are based on the principles of conservation of mass and reaction kinetics. The following approaches were used in EPANET:

- Advective transport in pipes
- Mixing at pipe junctions
- Mixing in storage facilities
- Bulk flow reactions
- Pipe wall reactions
- Lagrangian time-based solver for water quality

These approaches used by EPANET are discussed in section 3.2.4.1 to section 3.2.4.6

3.2.4.1 Advective transport in pipes

A dissolved substance will travel down the length of a pipe with the same average velocity as the carrier fluid while at the same time reacting at some given rate. The mass transportation of a single chemical is described by the advection-diffusion equation (ADE) in [31] [57] [58]. The ADE is derived from the law of mass conservation and Fick's Law of diffusion, which states that the mass of a solute crossing a unit area per unit time in a given direction is proportional to the gradient of solute concentration in that direction [31]. This is given as:

$$\frac{\partial C_i}{\partial t} + v_i \frac{\partial C_i}{\partial x} = r(C_i) + D_i \frac{\partial^2 C_i}{\partial t^2} + S(C_i) \tag{3.16}$$

Where C_i = concentration (mass/volume) in pipe i as a function of distance x and time t, v_i = flow velocity in pipe i and r = rate of reaction as a function of concentration, D_i is the diffusion matrix and S is a function of sources of substances or sinks within the pipe.

Longitudinal dispersion is usually not an important transport mechanism under most operating conditions, $S = 0$. Neglecting the diffusion effects, $D_i = 0$; the advective transport within a pipe in (3.16) can be represented by the following equation:

$$\frac{\partial C_i}{\partial t} + v_i \frac{\partial C_i}{\partial x} = r(C_i) \tag{3.17}$$

The neglected longitudinal dispersion is considered as modeling error especially during low water consumption period when the flow velocity is low. The flow velocity v_i in (3.17) is assumed constant during one hydraulic time step [56].

3.2.4.2 Mixing at Pipe Junctions

A junction node is a point where two or more pipes are connected. At junction nodes receiving inflow from two or more pipes, the mixing of fluid is taken to be complete and instantaneous. Thus, the concentration of a substance in water leaving the junction is the flow-weighted sum of the concentrations from the inflowing pipes. For a node k, we can write:

$$C_{i\backslash x=0} = \frac{\sum_{j \in I_k} Q_j \, C_{j\backslash x=L_j} + Q_{k,ext} \, C_{k,ext}}{\sum_{j \in I_k} Q_j + Q_{k,ext}} \tag{3.18}$$

Where i = link with flow leaving node k, I_k = set of links with flow into k, L_j = length of link j, Q_j = flow in link j, $Q_{k,ext}$ = external source flow entering the network at node k, $C_{k,ext}$ = concentration of external flow entering at node k. $C_{i\backslash x=0}$ represents the concentration at the start of link i, and $C_{j\backslash x=L_j}$ is the concentration of the end link [56].

3.2.4.3 Mixing in storage facilities

It is assumed that the contents of storage facilities (tanks and reservoirs) are completely mixed. This is a reasonable assumption for many tanks operating under fill-and-draw conditions if sufficient momentum flux is imparted to the inflow. Under completely mixed conditions the concentration throughout the tank is a blend of the current contents and that of any entering water. At the same time, the internal concentration could be changing due to reactions [56]. The following equation expresses these phenomena:

$$\frac{\partial(V_s C_s)}{\partial t} = \sum_{i \in I_s} Q_i C_{i\backslash x=L_i} - \sum_{j \in O_s} Q_j C_s + r(C_s) \qquad (3.19)$$

Where V_s = volume in storage at time t, C_s = concentration within the storage facility, I_s = set of links providing flow into the facility, and O_s = set of links withdrawing flow from the facility.

3.2.4.4 Bulk flow reactions

As the dissolved substance in water moves down in a pipe or resides in the storage tanks, it can undergo reaction with certain components in the water. The rate of reaction can generally be described as a power function of concentration [56]:

$$r = kC^n \tag{3.20}$$

Where k = a reaction constant and n = the reaction order.

The decay of many substances such as chlorine can be modeled adequately as a simple first-order reaction. The details of all equations used for bulk reactions are in [56]

3.2.4.5 Pipe wall reactions

While water is flowing through pipes, dissolved substances can be transported to the pipe wall and react with material such as corrosion products or biofilm that are on or close to the wall. The amount of wall area available for reaction and the rate of mass transfer between the bulk fluid and the wall will also influence the overall rate of this reaction [56]

The surface area per unit volume, which for a pipe equals 2 divided by the radius, determines the former factor. The latter factor can be represented by a mass transfer coefficient whose value depends on the molecular diffusivity of the reactive species and on the Reynolds number of the flow [56]. For first-order kinetics, the rate of a pipe wall reaction can be expressed as:

$$r = \frac{2k_w k_f C}{R(k_w + k_f)} \tag{3.21}$$

Where k_w = wall reaction rate constant, k_f = mass transfer coefficient, and R = pipe radius

3.2.4.6 Lagrangian time-based solver for water quality

The EPANET's water quality simulator uses a Lagrangian time-based approach to tracking the rate of discrete parcels of water as they move along pipes and mix together at junctions between fixed-length time steps. The Lagrangian approach update the conditions in variable sized segments of water at either uniform time increments or only at times when a new segment reaches a downstream pipe junction [70].

In, water quality modeling can be divided into two general categories: Eulerian and Lagrangian. Eulerian techniques divide the network into equally sized segments or volumes and track the chlorine residuals at the boundaries and within each segment [70]. The Lagrangian methods record and track each mass of chlorine that enters the DWDS by tracking parcels of water as they travel through the network. The techniques can be further classified as a time-driven method (TDM) and event-driven method (EDM). The time-driven method updates the position and age of each slug of chlorine at fixed intervals of time. The event-driven method only updates the position and age of the chlorine at certain hydraulic event, such as the end of the hydraulic step, reach a junction node and flow velocity changes [70].

Other research publications on water quality analysis, design, and operations are [71] [72] [73].

3.3 Control-design based Chlorine Residual Modeling in DWDS

The modeling and algorithms used in EPANET for simulations as presented in previous sections are adequate, useful and accurate for DWDS simulations and water network design. However, all the nodes and storage tanks in the DWDS are involved in the model and result in distributed models. These distributed models provide only an implicit input-output relationship that is not suitable for controller design.

In control design-based approach to modeling, we need an explicit input-output relationship between the chlorine injected at the injection nodes and the chlorine residual concentration at the monitored nodes. Chlorine can be injected or added to the DWDS from many sources or nodes and transported to multiple user nodes through many pipe paths. The physical distance between the chlorine injection input nodes and the output nodes may be measured in feet or kilometres or miles. Associated with the transport of chlorine from the point of injection to the user node is the variable time transportation time or delays.

As Chlorine is transported in the DWDS, it reacts with substances in water, with the pipe walls and mixed with flows of different quality at the junction nodes. This is described by the impact coefficients in the obtained input-output model in [33]. The inputs for chlorine residual concentration modeling are the chlorine injected and added at the chlorine source nodes while the output is the chlorine residual concentration at the monitored node. The following are considered in the control design approach to modeling the chlorine residual concentration at the monitored nodes in the DWDS:

- A suitable method to determine the chlorine transportation paths from the injection nodes to the monitored nodes in the DWDS.

- A suitable method to calculate the transportation time of the injected chlorine from the injection node to the monitored node. Calculating the transportation time is challenging due to the time varying user demands in the DWDS and the topology of the DWDS. The transportation time in the input-output model is continuous in time and time varying. The transportation time is affected by the tank dynamics and must be incorporated in the final model.
- A suitable method to handle the continuous and time-varying transportation time.
- A suitable method to calculate the impact factors or impact coefficients on the injected chlorine due to chlorine decay and mixing at junction nodes.

In the next section, the Input-output model for chlorine residual modeling in [31] [33] [34] is further developed using the proposed flow-Path dependent forward tracking algorithm to acquire the model structure.

3.3.1 Input-Output Model Formulation for Chlorine Residual in DWDS without Tanks

3.3.1.1 Chlorine Residual Modeling in Pipes

The model of chlorine residual in water through a pipe [33] [34]is given by:

$$y(k) \ = \ \beta(k)u(k-d(k)) \tag{3.22}$$

where $y(k)$ is the chlorine residual of water exiting the pipe at the output node, $u(k)$ is the chlorine concentration of water enter entering the pipe at the input node, $d(k)$ is the transport sampled time of chlorine from the input to the output node (the actual transport time is $d(k)T$), T is the sampling interval time, $\beta(k) = e^{(-\alpha d(k)T)}$ is the decay factor for the pipe, $\alpha \gg 0$ is the reaction rate coefficient and characterizes how quickly the chlorine decays in water. Different types of pipes and water qualities results in different rates of decay of the chlorine in the water.

The transport sampled time $d(k)$ is continuously time varying due to time varying water demands and should be discretized to eliminate this time variation. The range of the variation of transport time or time delay denoted by $\overline{d}$ (maximum time delay) and $\underline{d}$ (minimum time delay) over the modeling horizon $[0, T_h]$ is considered and calculated numerically by [31]:

$$I_{ij} \triangleq \{n_{min}, n_{min} + 1, \ldots + n_{max} - 1, n_{max}\} \tag{3.23}$$

$$n_{min} = round\left(\frac{\underline{d}}{\Delta T_d}\right)$$

$$n_{max} = round\left(\frac{\overline{d}}{\Delta T_d}\right)$$

where $round\ (\cdot)$ is the function that takes the integer that is closest to a real numberΔT_d is the discretization time step and I_{ij} is the time delay number in pipe *ij* over the modeling horizon.

The chlorine residual of water exiting the pipe in equation (3.22) is thus approximated by:

$$y(k) = \sum_{i(k) \in I_{ij}} a_i(k) u(k - i(k)) + \varepsilon(k)\ for\ k \in [0, T_h] \tag{3.24}$$

Where $a_i(k)$ is the impact coefficient associated with the transport time or time delay numbers, I_{ij} is the time delay number in pipe ij over the modeling horizon, $\varepsilon(k)$ is the model error due discretization of the time delays, $u(k - i(k))$ is the injected chlorine at the input node of the pipe and $t = \ k\Delta T_d \ , k = 0,1,2, \ldots$

The model equation in (3.24) now has time-varying parameters and no time varying time delays.

In the DWDS, pipes are connected in series and in parallel. The model of pipes connected in series can be obtained by summing the transport time in each pipe and multiplying the decay factor in each pipe.

For a network with any number of pipes in parallel, it is assumed that the mixing at the junction node is complete and instantaneous. Each pipe or set of pipes connected to a junction node (which is a monitored node) from the chlorine injection node forms a path that chlorine travels through to arrive at the junction node. Thus, the model is given by:

$$y(k) = \ \sum_{i \in I_{(k)}} a_i\,(k)u(t - i(k)) \ + \ \varepsilon(k) \ \ for \ k \ \in \ [0, T_h] \tag{3.25}$$

Where $I_{(k)}$ is the set of all the discrete time delay number of each path over the modeling time horizon. A path can be active if there is a flow path through it or inactive over certain time due to hydraulic dynamics in the DWDS. The model equation in (3.24) represents a single-input and single output situation.

In DWDS, a single chlorine injection input may control multiple monitored nodes and a monitored node may be controlled by multiple chlorine injection nodes. Thus, we have single-input, single-output systems (SISO), single-input, multiple-output systems (SIMO) and multiple-input, multiple-output systems (MIMO) in DWDS water quality control. The MIMO model for one output [31] is given by:

$$y(k) = \sum_{m=1}^{n_M} \sum_{i \in I_{1,m}(k)} a_{1,m,i}(k) u_m(k - i(k)) + \varepsilon(k) \quad for \ k \in [0, T_h] \qquad (3.26)$$

Where there are n_M inputs, $a_{1,m,i}$ is the *i^th^* impact coefficient of the first output under the *m^th^* input.

3.3.1.2 Chlorine Residual Modeling in DWDS with Tanks

Tanks are used to store water in the DWDS. They are used primarily to satisfy demand fluctuations and equalize operating pressures. Tanks add dynamics to chlorine residual modeling because of the long residence times of water in the tanks. There are two types of tanks in the DWDS and they are switching tank and continuous tank. In switching tank, it is operated in fill and drain cycles while the continuous tank is operated is fill and draw cycles simultaneously.

In this thesis, the switching tank is considered in the modeling. The following general assumptions are used in chlorine residual modeling for tanks in the DWDS [56] [67]:

- Tank contents are completely mixed instantaneously
- The kinetics of chlorine reactions to substance in tank water is first-order with respect to the chlorine concentrations in tanks

These assumptions contribute to model-reality mismatch and it is treated as modeling errors which are incorporated in the controller design.

In this thesis, it is assumed that the chlorine residual in the switching tank is measurable at the quality time steps using suitable sensors connected to the monitoring unit of the DWDS. It is also assumed that the chlorine booster station can be put in the tank node. The switching tank is therefore considered as chlorine source during the draining cycle in the chlorine residual modeling in this thesis. This is shown in Figure 3.3.

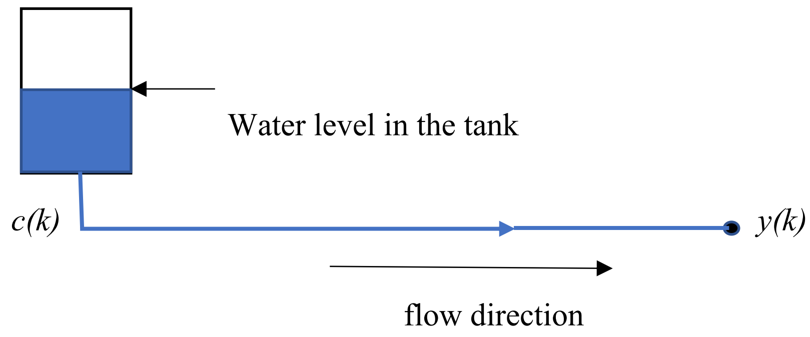

Figure 3.3 Model of chlorine residual in DWDS with tank during the draining cycle of the tank

The model of chlorine residual in the DWDS with a tank during the draining cycle is given by:

$$y(k) = \sum_{i(k) \in I_{ij}} a_i(k)c(k - i(k)) + \varepsilon(k) \quad for \;\; k \in [0, T_h] \tag{3.27}$$

Where $y(k)$ is the chlorine residual at the exit node of the pipe, $a_i(k)$ is the impact coefficient associated with the transport time or time delay numbers, I_{ij} is the time delay number in pipe ij over the modeling horizon, $\varepsilon(k)$ is the model error due discretization of the time delays $c(k - i(k))$ is the injected chlorine or chlorine residual value from the tank at the input node of the pipe during the draining cycle of the tank and $k = k\Delta T_d, k = 0,1,2, \ldots$

3.4 Model structure determination for chlorine residual modeling in DWDS

3.4.1 Path Analysis Algorithm

In [31] [33] [34] [74] the model structure information for chlorine residual modeling in DWDS is obtained using the Path analysis algorithm. The Path analysis algorithm provides the following information:

- The chlorine transportation path from the chlorine injection nodes or chlorine source nodes to the monitored nodes
- The transportation time of chlorine in each path
- Chlorine injection inputs at certain nodes and their impacts at other nodes

In the application of the Path analysis algorithm, it is assumed that the hydraulic information such as water flows, flow velocity, and pipe lengths are available. The Path analysis algorithm uses a backward tracking algorithm recursively through time and the topological space of the DWDS to track the path of the chlorine at the monitored node to the injection node. The questions answered by the backward tracking algorithm are: for the chlorine arriving now at a given junction node, which input did the chlorine originate at, what path did the water take through the pipes to get to the output junction node and how long did the water spend in each of those pipes.

In this thesis, the Path analysis algorithm developed by [33] is further modified to use a forward tracking algorithm. The forward tracking algorithm tracks the path of injected chlorine from any node used as injection node through the network to the monitored node using the hydraulic information provided at the upper level of the hierarchical integrated quantity and quantity control structure presented in Chapter 2 of this thesis.

The forward tracking algorithm works in a predictive manner using the predicted hydraulic information at the upper level of the hierarchical integrated quantity and quantity control structure. The forward tracking algorithm is suitable for online identification in chlorine residual modeling in the DWDS.

3.4.1.1 Forward tracking Algorithm

The proposed modified Path Analysis Algorithm is a forward tracking algorithm based on the concept of water head difference and time of arrival or time of impact of the injected chlorine. The concept of water head difference is based on the physical law that water in a pipe flows from the higher head node to the lower head node. The application of this concept is that at every quality time step, the water head difference is assessed, and the path of flow is determined for each pipe and for each path (set of pipes) that the injected chlorine travels through to the monitored node. This is shown in Figure 3.4

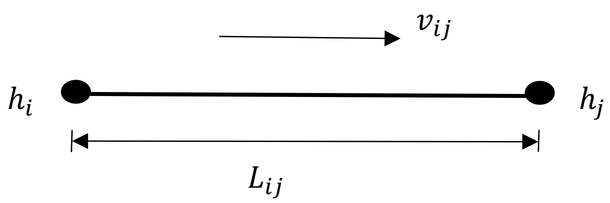

Figure 3.4 Water flow in a pipe from node *i* to node *j*

The time of arrival or the time of impact of the injected chlorine at the monitored node is based on a calculation of the detention or transportation time of the injected chlorine through the paths to the monitored node and assessing whether the injected chlorine will arrive at the monitored node at the time of monitoring. This is illustrated in Figure 3.5 by the flow chart. The transportation time is calculated by using a tracking time τ step given by the following relationship:

$$\tau \leq \Delta T_c \tag{3.28}$$

$$\tau \; v_{ij}(k) \leq \; L_{ij}, \;\; any\, k, \; L_{ij} \tag{3.29}$$

For computational efficiency, the tracking time can be selected to be as:

$$\frac{\Delta T_c}{5} \leq \tau \leq \frac{\Delta T_c}{2} \tag{3.30}$$

Where ΔT_c is the quality time step, L_{ij} is the length of pipe or pipes in cascade, and v_{ij} is the flow velocity.

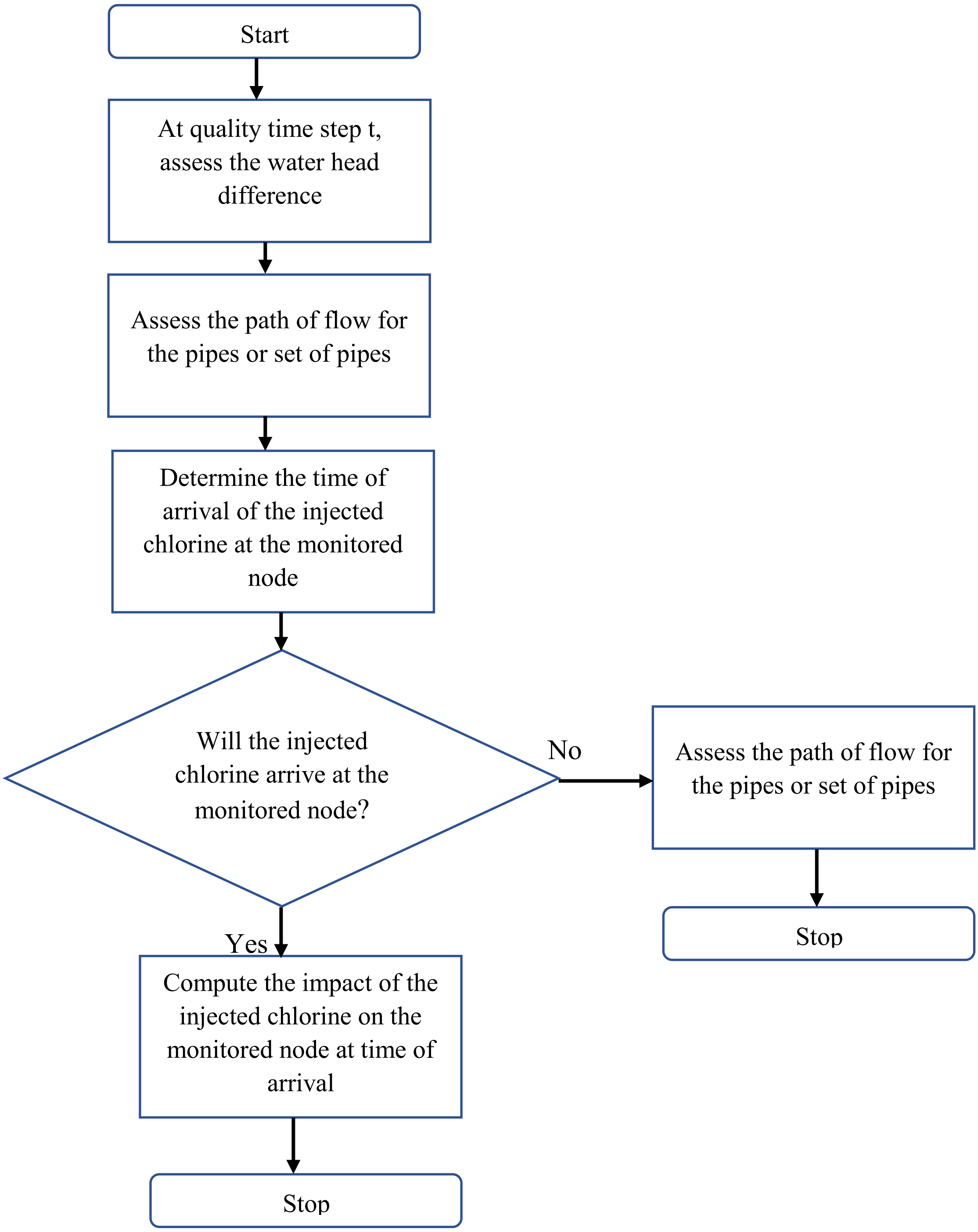

Figure 3.5 Modified path analysis algorithm flow chart

The developed modified Path analysis forward tracking algorithm is as follows:

Algorithm 3.0: Modified Path analysis forward tracking Algorithm as illustrated in Figure 3.5

Obtain the predicted hydraulic information from the upper level of the integrated quantity and quality control structure at the hydraulic time step

i. For all quality time step, ΔT_c at monitored node j in the DWDS, obtain the water head difference $h_i - h_j$ and find the direction of flow between the injection node and the monitored node.

ii. If there is a flow from the chlorine injection node to the monitored node under consideration at any quality time step, ΔT_c over the modeling horizon, the flow counter for this Path is set to 1, else it is set to 0.

iii. Calculate the transportation or detention time for this path using as follows:

The transportation time is a relationship between the water flow velocity and the length of the pipe or the path (set of pipes in cascade). The transportation time or detention time, Td of the injected chlorine is the time it will take for the injected chlorine to arrive at the monitored node.

$$Td = \frac{L_{ij}}{v_{ij}} \tag{3.31}$$

Where Td is the detention time, L_{ij} is the length of pipe or pipes in cascade and v_{ij} is the flow velocity for the hydraulic time step.

iv. Using the flow counter obtained in (iii) and the detention time in (iv), determine whether a Path of flow exist at each quality time step from the injection node to the considered monitored node. The detention or transportation time at each quality time step and the flow counter is used to determine the arrival time or time of impact of the injected chlorine at the considered monitored node. The injected chlorine will arrive if there exist a flow-path from the time of injection to the time it arrives (detention time) at the monitored node or else it will not arrive at the monitored node.

v. Repeat step (i) to (v) for each injection node or chlorine source node to each monitored node in the DWDS to obtain the nodes controlled by each injection nodes and the time step the monitored node is impacted by the injected chlorine from the injection node.

Performing the modified Path analysis forward tracking algorithm from the injection node to the monitored nodes produced the model structure information which are chlorine inputs to the monitored nodes, the Paths and the detention time in each path. The tanks in the DWDS are treated as chlorine sources during their draining periods and the extended Path analysis forward tracking algorithm was applied to determine the monitored nodes impacted by the tank outputs during the draining periods. This improved the modeling accuracy and speed of modeling.

The developed modified Path analysis forward tracking algorithm is used to partition the DWDS into subnetworks for distributed water quality control. Each node in the DWDS is used as chlorine injection node while other nodes are used as monitored nodes to test the controllability and observability of the nodes (this is superposition principle approach). This is illustrated in Figure 3.6 with a benchmark DWDS. The node circled with red is the chlorine injection node and the nodes circled with blue are the monitored nodes. The modified Path analysis forward tracking algorithm is used to find the flow paths (indicated by red arrows) and determine the controllable nodes. From Figure 3.6, only six nodes are controllable by this injection node acting alone.

Each node in the DWDS is used as chlorine injection node acting alone while other nodes are used as monitored nodes to determine their controllability by the injection node. This procedure is repeated until the DWDS is partitioned into areas based on the controllability and observability of the nodes.

The water flows used for the DWDS partitioning is the average water flows under a normal operational state of the DWDS. In this thesis, the choice of the number of partitioned areas in the DWDS is based on analysis and design of RFMPC controllers.

The modified Path analysis forward tracking algorithm can be used for more complex DWDS network and can be adapted for application to any flow-based systems modeling.

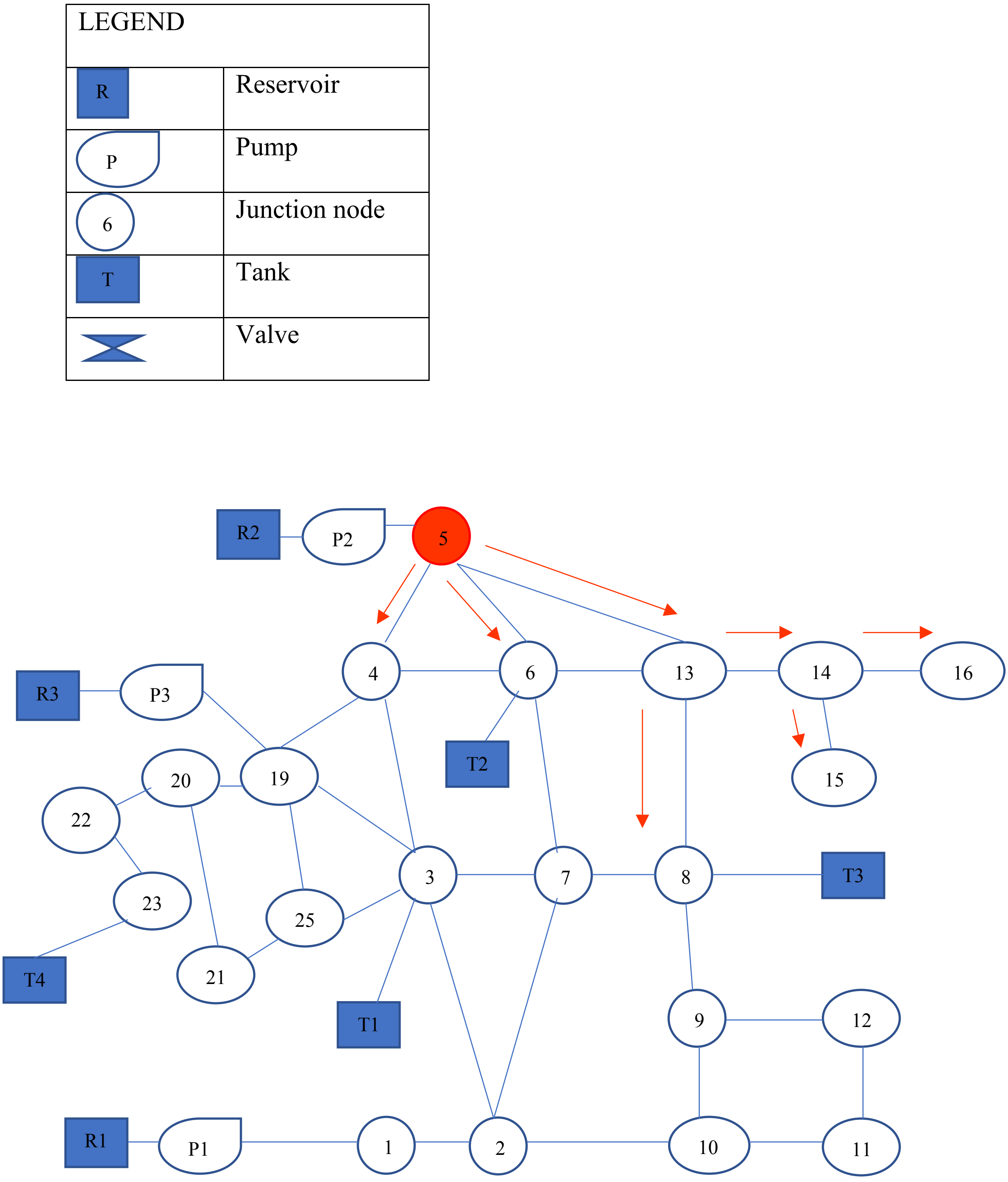

Figure 3.6 DWDS partitioning using the modified Path analysis algorithm and superposition principle

3.4.1.2 Chlorine Residual Modeling by node-to-node analysis

The Path analysis algorithm and the modified Path analysis forward tracking algorithm is suitable for model structure and transport time of chlorine determination from chlorine injection node to the monitored nodes. In this thesis, a node-to-node analysis method for calculating the chlorine residual model of the injected chlorine from the injection node to the monitored node is proposed. Each node is considered as an active or passive chlorine source to the nearest connecting node. A chlorine booster node is an active chlorine source node while other nodes are passive chlorine source nodes. The chlorine injection node is the source node and the nearest connecting nodes are the monitored nodes. For example, in Figure 3.6, node 5 is selected as the chlorine injection node, the nearest connecting nodes are node 4, node 6 and node 13 are selected as the monitored nodes. The transport time of chlorine in the pipes connecting node 5 to node 4, node 5 and node 6, node 5 and node 13 are calculated and the impact coefficients estimated to obtain the chlorine residual model for injected chlorine from node 5 to the nearest connecting node. The model for the chlorine injected at node 5 and monitored at node 13 is given as:

$$y_{13}(k) = \sum_{i_{5\_13} \in I_{ij_{5\_13}}} a_{i_{5\_13}}(k) u_5\big(k - i_{5_{13}}(k)\big) + u_{13}(k) + \varepsilon(k) \qquad (3.32)$$

Where $y_{13}(k)$ is the chlorine residual value at node 13 at discrete time $k, I_{ij_{5\_13}}$ is the set of the transport time for chlorine between node 5 and node 13, $i_{5_{13}}(k)$ is the transport time of chlorine at discrete time k from node 5 to node 13 at discrete time k, $a_{i_{5\_13}}(k)$ is the impact coefficient on the injected chlorine from node 5 to node 13, $u_5\big(k - i_{5_{13}}(k)\ \big)$ is the injected chlorine from node 5 at time $\left(k - i_{5_{13}}(k)\right)$, $u_{13}(k)$ is the initial chlorine residual value at node 13 at discrete time k, and $\varepsilon(k)$ is the model error.

To calculate the impact of the injected chlorine from node 13 to the nearest connected node to node 13 is node 14. The model is given as:

$$y_{14}(k) = \sum_{i_{13\_14} \in I_{ij_{13\_14}}} a_{i_{13\_14}}(k) u_{13}\left(k - i_{13_{14}}(k)\right) + u_{14}(k) + \varepsilon(k) \qquad (3.33)$$

$$u_{13}\left(k - i_{13_{14}}(k)\right) = y_{13}\left(k - i_{13_{14}}(k)\right) \qquad (3.34)$$

Substituting Equation (3.32) and (3.34) in (3.33) gives:

$$y_{14}(k) = \sum_{i_{13\_14} \in I_{ij_{13\_14}}} a_{i_{13\_14}}(k) y_{13}\left(k - i_{13_{14}}(k)\right) + u_{14}(k) + \varepsilon(k) \qquad (3.35)$$

Where $y_{14}(k)$ is the chlorine residual value of node14 at discrete time k, $I_{ij_{13\_14}}$ is the set of the transport time for chlorine between node 13 and node 14, $i_{13_{14}}(k)$ is the transport time of chlorine at discrete time k, $a_{i_{13\_14}}(k)$ is the impact coefficient on the injected chlorine from node 13 to node 14, $y_{13}\left(k - i_{13_{14}}(k)\right)$ is the chlorine flowing from node 13 at time $\left(k - i_{13_{14}}(k)\right)$ to node 14, $u_{14}(k)$ is the initial chlorine residual value at node 14 at discrete time k, and $\varepsilon(k)$ is the model error. The impact of the injected chlorine from node 5 on node 14 is in $y_{13}\left(k - i_{13_{14}}(k)\right)$ which is given in equation (3.32) and (3.35).

The node-to-node analysis for chlorine residual modeling is proposed. The procedure for application is as follows:

- Determine the chlorine injection nodes and monitored nodes using the Path analysis forward tracking algorithm and superposition principle
- Determine the paths from the injected node to the monitored node
- Identify the nearest connecting nodes in each path, starting from the injection node and through all the nodes to the monitored node
- Derive the chlorine residual model for each connecting node
- Get the overall model and estimate the parameters

The application of the node-to-node chlorine residual modeling is presented in detail in chapter 6 of this thesis.

3.5 Uncertainties in Chlorine Residual Modeling in DWDS

The uncertainties in DWDS from viewpoint of chlorine residual modeling for control has been addressed in [31]. These uncertainties must be incorporated in the models of chlorine residuals in DWDS. The uncertainties are caused by the following:

1) Time-varying water demand: the operational control of DWDS uses nominal water demand prediction to generate optimized pump and valve schedules. The optimized pump and valve schedules are applied to the EPANET or numerical simulator to generate the hydraulic information that is used for chlorine residual modeling. The difference between the real-time water demand and the predicted water demand will

generate inaccurate water flow, flow velocity, heads and pressure in the EPANET or numerical simulator. The uncertainties in the hydraulic information caused by uncertain water demand will affect the chlorine transportation time calculated and the mixing at the junction nodes in the final chlorine residual model.

2) Pump hydraulic characteristic curve: the pump characteristic curve is approximated by a nonlinear function as shown in (3.7). The actual speed of the pump motor is different from the nominal speed and this generates inaccurate hydraulic characteristic curve, inaccurate head-flow relationship, and inaccurate flow velocity.
3) Roughness Coefficient of the pipes: the roughness coefficient of the pipe depends on the material of the pipe, the manufacturer of the pipe and varies with the age of the pipe. The difference between the actual roughness coefficient of each pipe and the one used for modeling will result in inaccurate water flows, flow velocity, and water heads.
4) Chlorine reaction kinetics: the first-order chlorine reaction kinetics has been used in EPANET for describing the chlorine decay in DWDS. The chlorine reaction depends on the temperature of the water, the quality of the water, pipe age and pipe material. The difference between the actual reaction kinetics and the assumed first-order reaction kinetics will result in simulator error and this will cause the actual chlorine residual measured to be different from the simulation result.

All these uncertainties must be accounted for in the robust controller design.

3.6 Chlorine residual modeling in DWDS under different disturbance scenarios

The chlorine residual modeling in [31] [33] [34] used the hydraulic information already available and did not consider the disturbance scenarios that could change the operational state of the hydraulics of the DWDS in its daily operation. This change in operational state may affect the model structure for the chlorine residual modeling for the DWDS.

In this thesis, these disturbance scenarios are considered and incorporated in the controller design. These disturbance scenarios in the DWDS can be caused by the following:

- Pipe breaks or leakages in the DWDS
- Valve faults
- Pump faults
- Sensor faults
- Deliberate attacks on the DWDS or accidental damage of the DWDS components

In this thesis, pipe breaks and valve faults are considered as disturbance scenarios. The pipe breaks are simulated by breaking or removing the links between two junction nodes and modelled. These disturbance scenarios create three types of operational states [2] which are Normal, Disturbed and Emergency operational states. The normal operational state is a state of no fault in the DWDS and the water quality control task can be achieved by the designed controller. The disturbed operational state is a state such that there are pipe breaks and valve faults, but some monitored nodes are still controllable by the injection nodes.

The emergency operational state is a state such there are pipe breaks and pipe faults and no monitored node is controllable at all by the injection node.

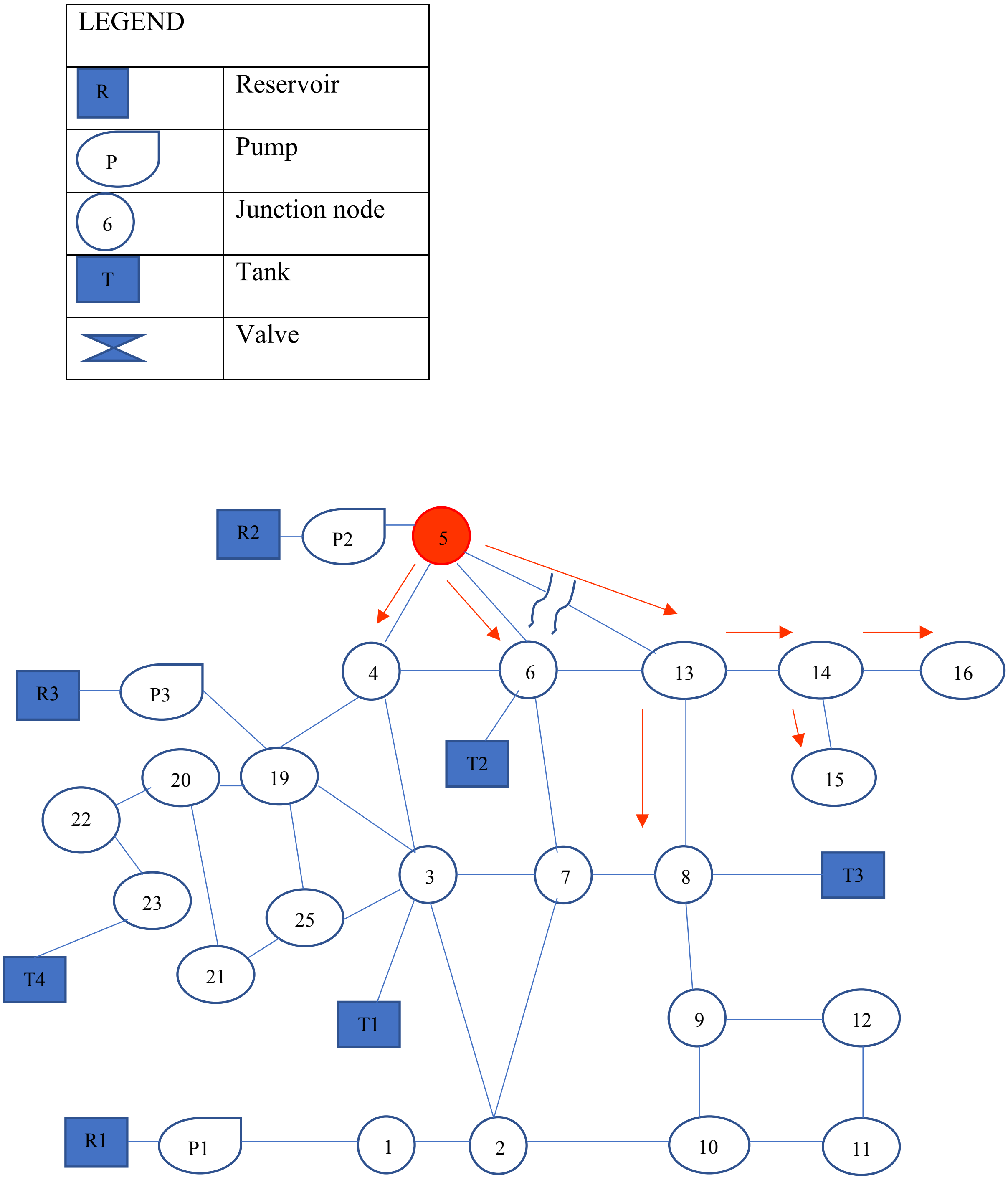

Figure 3.7 Simulation of pipe breaks by removing the link between node 5 and node 13

LEGEND	
R	Reservoir
P	Pump
6	Junction node
T	Tank
	Valve

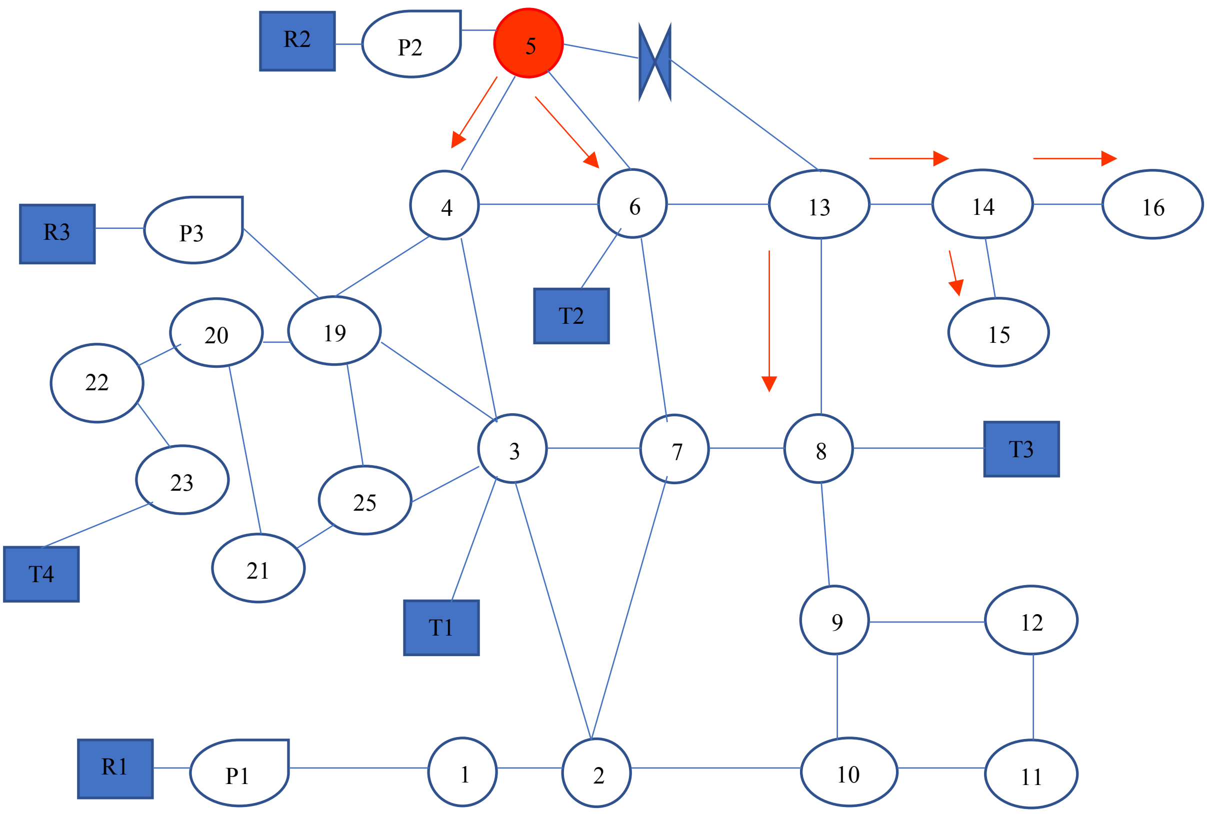

Figure 3.8 Simulation of pipe leakage and valve faults using flow control valve between node 5 and node 13

For simulation using EPANET, the pipe breaks are simulated by removing the links between the nodes to be modelled as illustrated in Figure 3.7. The flow control valve (FCV) and emitter are used to simulate pipe leakage and valve faults as illustrated in Figure 3.8.

Different settings of the flow control valve are used to simulate pipe breaks, pipe leakages, and valve faults. The model of pipe break in Figure 3.7 is given as:

$$y_{13}(k) = \sum_{i_{6\_13} \in I_{ij_{6\_13}}} a_{i_{6\_13}}(k)u_6\big(k - i_{6_{13}}(k)\big) + u_{13}(k) + \varepsilon(k) \tag{3.36}$$

The impact on node 13 is from node 6 as given in (3.36)

The model of valve faults and pipe leakage in Figure 3.8 is given as:

$$y_{13}(k) = \sum_{i_{5\_13} \in I_{ij_{5\_13}}} a_{i_{5\_13}}(k)u_5\big(k - i_{5_{13}}(k)\big) + u_{13}(k) + \varepsilon(k) \tag{3.37}$$

The impact on node 13 is from node 5 as given in (3.37)

3.6.1 Chlorine residual modeling in DWDS under Normal Operational State

The normal operational state is the most desirable operational state for the DWDS. It is a state of no fault in the DWDS (this is applicable to each partitioned Region in the DWDS) which includes:

- No pipe breaks
- No pipe leakage
- No valve faults (the flow rate is per the settings)

The control task of water quality control for all the monitored nodes in the DWDS can be achieved by the designed RFMPC controller. The chlorine residual model derived for the normal operational state of the DWDS is to be used to design the normal operational state RFMPC controller.

3.6.2 Chlorine residual modeling in DWDS under Disturbed Operational State

The disturbed operational state is a state of faults in the DWDS but not in the critical pipes such that the control task of water quality control can be achieved for some monitored nodes in the DWDS or partitioned areas in the DWDS. The critical pipes are the pipes that supply water to the chlorine injection nodes in the DWDS.

This is illustrated in Figure 3.9. In Figure 3.9, the red circled nodes are the chlorine injection nodes, the blue circled nodes are the monitored nodes and the red arrows shows the critical pipes. The normal operational state model structure and model for chlorine residual modeling is affected and may not be suitable for the disturbed operational state.

A new model structure and model for chlorine residual modeling is to be derived and used to design the disturbed operational state RFMPC controller. The simulations and analysis of the disturbed operational state of the DWDS are presented in detail in chapter 6 of this thesis.

LEGEND	
R	Reservoir
P	Pump
6	Junction node
T	Tank
	Valve

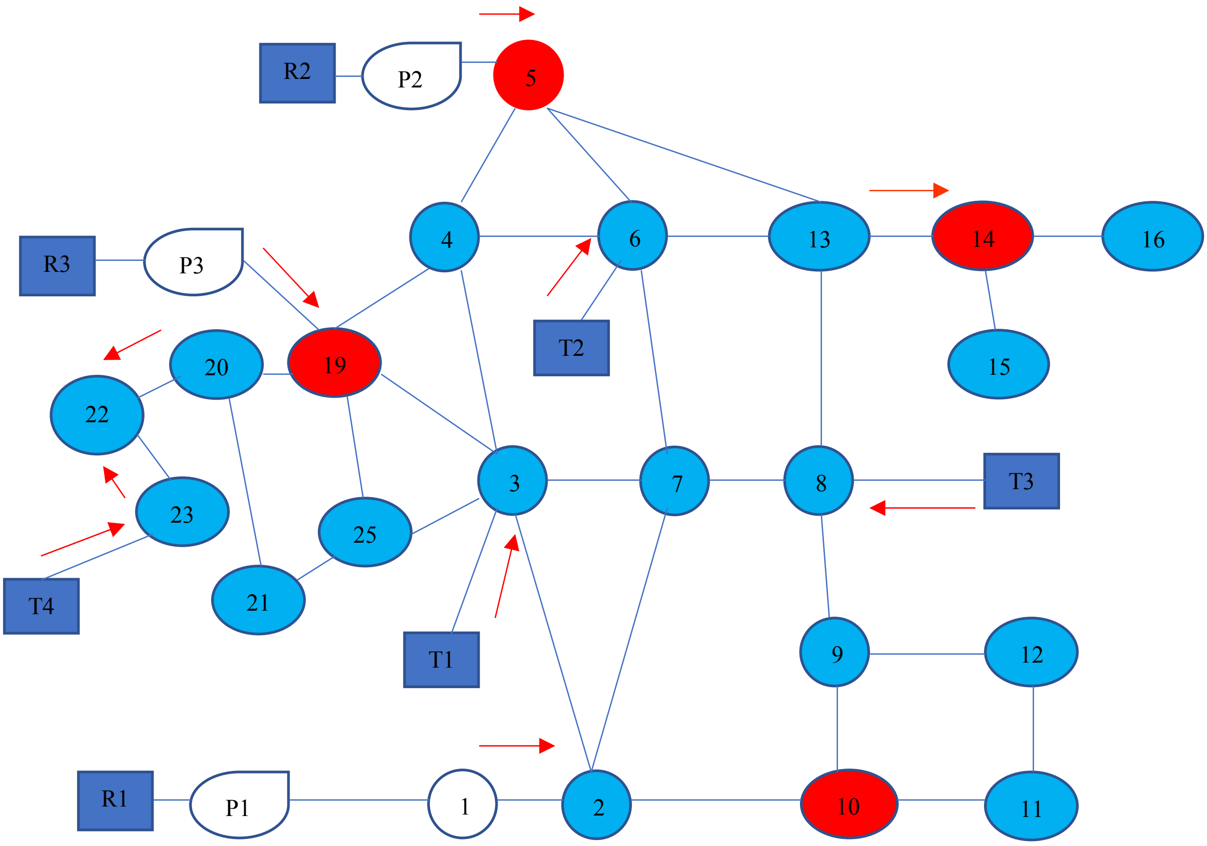

Figure 3.9 Critical pipes in the benchmark DWDS

3.6.3 Chlorine residual modeling in DWDS under Emergency Operational State

The emergency operational state of faults in the critical pipes of the DWDS such that the control task of water quality control cannot be achieved for all the monitored nodes in the DWDS or partitioned zones in the DWDS. A new model structure and model for chlorine residual modeling is to be derived and used to design the emergency operational state RFMPC controller. The control strategy may need to be changed to adapt to the emergency operational state. In Figure 3.9, a pipe break in the critical pipe 25 (red arrow) feeding the chlorine injection node 14 (red circled node) affects water flow to node 15 and node 16 monitored nodes; thus, the controllability of the monitored nodes 15 and 16, cannot be achieved by the chlorine injection from node14.

3.7 Model Parameter Estimation

The chlorine residuals input-output models in section 3.3.1 are suitable for MPC controller design and the parameters need to be estimated. The chlorine residuals at the monitored nodes are governed by water flows, flow velocity, mixing at junction nodes, chlorine decay injected chlorine from injection nodes and detention times of water in tanks. The hydraulic information needed for chlorine residual modeling is generated at the upper level of the hierarchical integrated quantity and quality control structure presented in chapter 2 of this thesis.

Water flows, flow velocity, water heads, and pressure are driven by water demands. Water demands are time-varying and can change hourly, daily, monthly and seasonally. The water demand needs to be predicted to obtain the water flow, flow velocity, heads and pressure for chlorine residual modeling. There are different types of parameters to consider for chlorine residual modeling and they are constant parameters, slowly varying parameters, varying parameters and fast varying parameters [31] For example, the length of the pipes is a constant parameter, the roughness coefficient of the pipe is a slowly varying parameter and needs to be calibrated at certain periods, the chlorine reaction constant is a varying parameter and needs to be calibrated regularly and water demand is a fast-varying parameter and needs to be predicted. The procedure for chlorine residual model parameter estimation is illustrated in Figure 3.10.

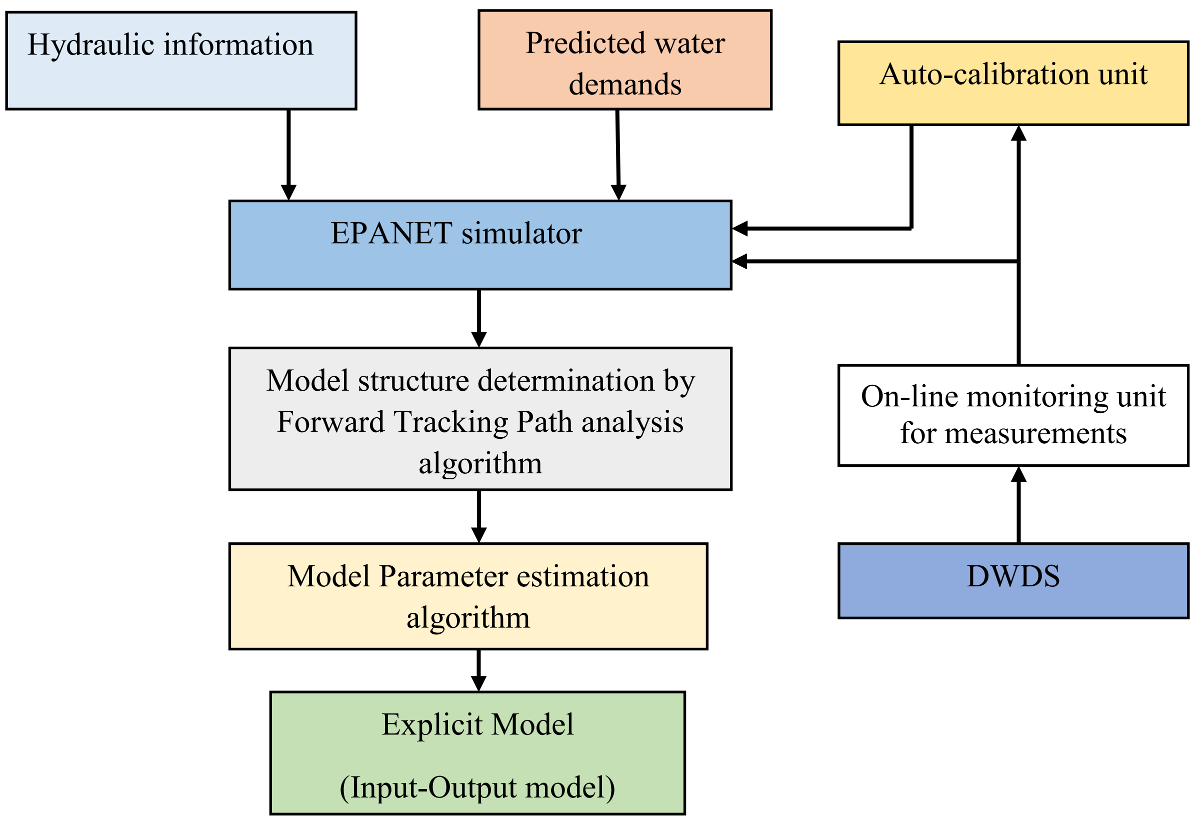

Figure 3.10 Procedure for chlorine residual model parameter estimation

In Figure 3.10, the hydraulic information for setting up the EPANET simulator is supplied. The hydraulic information includes the length and diameter of each pipe in the DWDS, the pump characteristic curve, the pump and valve schedules, the elevation of the storage tanks, the diameter and the volume of each storage tank, and the elevation at each node. The predicted water demand and water usage pattern for each node are supplied to EPANET simulator. The chlorine reaction constant and the roughness coefficient is supplied to the EPANET and it is obtained from the auto-calibration unit. The auto-calibration unit calibrates the measurements from the online monitoring unit. The chlorine injection nodes and the monitored nodes are selected in the EPANET. The initial quality of water in each storage tank is selected in the EPANET.

The EPANET simulates the water network to generate flows, flow velocity, heads, pressure, chlorine residual values at each node, the volume of water in each tank and other hydraulic information. The model structure is determined by the modified Path analysis forward tracking algorithm presented in section 3.4.1.1. The parameters of the models obtained by the model structure are estimated by the model parameter estimation algorithms. Finally, the explicit input-output model suitable for MPC design is generated.

Remarks 3.1:

- Water demands vary hourly, daily, monthly and seasonally, therefore the water flow patterns for each day of the week are not the same. The chlorine residual models will also vary with the water demands
- There is need for real-time online modeling of the chlorine residual to update the chlorine residual models used by the MPC to improve the performance of the MPC

- The chlorine residual models obtained in the normal operational state of the DWDS may not be suitable for Model Predictive Control of the disturbed or emergency operational state of the DWDS.

3.7.1 Parameter estimation

Consider the Equation (3.24) given as:

$$y(k) = \sum_{i\,(k) \in I_{ij}} a_i\,(k) u(k - i(k)) \; + \; \varepsilon(k) \;\; for \;\; k \in [0, T_h]$$

It can be written in a compact form as:

$$y_n(k) = \emptyset_{y_n}(t)^T \theta_{y_n}(k) \; + \; \varepsilon_{y_n}(k) \quad for \;\; k \in [0, T_h] \qquad (3.38)$$

Where $y(t)$ is the chlorine residual output observations at the monitored node over the modeling time horizon,

$$\emptyset_{y_n}(k)^T = [u_n\left(k - i_{n,1,lower}\,(k)\right) . . . u_n\left(t - i_{n,1,upper}\,(k)\right)] \qquad (3.39)$$

$$\theta_{y_n}(k) = [\, a_{n,1,l}\,(k) . . . a_{n,1,u}\,(k)\,] \qquad (3.40)$$

$\varepsilon_{y_n}(k)$ is the time-varying model error, $\emptyset_{y_n}(k)^T$ is the regressor vector and $\theta_{y_n}(k)$ is the parameter vector.

Equation (3.38) can be written as:

$$y(k) = \emptyset(k)^T\theta(k) + \varepsilon(k) \quad for \ k \in [0, T_h] \tag{3.41}$$

3.7.2 Set-membership and Point-parametric approach to model parameter estimation

Parameters map the input to the output space to make the model output equal to the real plant output. There is always a model-reality mismatch as no model is exactly equal to the real plant model.

In equation (3.41), the time-invariant system can be written as:

$$y(k) = \emptyset(k)^T\theta + \varepsilon(k) \quad for \ k \in [0, T_h] \tag{3.42}$$

From equations (3.41) and (3.42),

Parameter set $\theta(k)$ and θ for which the equations (3.41) and (3.42) holds at every time instant over the modeling horizon is the feasible parameter set [75] [76] and it is bounded over each time instant over the modeling horizon.

In this thesis, the time- varying system (3.41) is used. In [31] [77],it has been shown that there is an internal link between the input and the uncertainty in parameters and the model errors under the point-parametric model concept. The point-parametric model concept for time- varying parameter vector including if necessary, dedicated parameter representing model error is defined; for any plant input $u(k)$,under predicted disturbance d(k) , there exists $\{\emptyset(k), \boldsymbol{\theta}_p(k)\}$ that the model output for the plant equals the real plant output.

Different inputs $u(k)$ require different trajectories of $\boldsymbol{\theta}_p(k)$ in order to produce the real plant output for all time instants over the considered time horizon.

Consider a time-varying system:

$$y(k) = \boldsymbol{E}\big(u, \boldsymbol{\theta}_p(u, k), k\big) + \ \mathcal{E}_p(u, k) \tag{3.43}$$

where: $\boldsymbol{E}$ is the model of the system (linear or nonlinear); u represents the input function of time; $\boldsymbol{\theta}_p$ and $\mathcal{E}_p$ are input generated model parameter and input generated model error. The uncertainties in the system may be in the parameters $\boldsymbol{\theta}_p$ and or model error $\mathcal{E}_p$.

We consider the type of model in which all uncertainties and model errors are allocated to the model parameters yielding:

$$y(k) = \boldsymbol{E}\big(u, \boldsymbol{\theta}_p(u, k), k\big) \tag{3.44}$$

where the model parameters $\boldsymbol{\theta}_p$ must be time-varying for the model to be suitable. For any plant input $u(t)$, there exists $\{\boldsymbol{\theta}_p(k), \emptyset(k)\}$ associated with this input that yields the system output $y(k)$ for all $k \in [k_0, k_0 + T_m]$ over the considered modeling time horizon. There exist trajectories of $\boldsymbol{\theta}_p$ that yields the system output $y(k)$ in (3.44). where T_m is the modeling time horizon.

3.8 Simulation Experiment Design

The goal of the experiment design is to design the special inputs to generate the required data for parameter estimation. System inputs are hard constraints due to actuator operational limits which can be expressed at each time instant $k \in [k_0, k_0 + T_m]$ as:

$$0 \leq u_{min}(k) \leq u(k) \leq u_{max}(k) \quad (3.45)$$

The input sequence over the modeling time horizon is given by:

$$u_{T_m} = [u(k_0), u(k_0 + 1), \ldots, u(k_0 + T_m)] \quad (3.46)$$

The observed output sequence over the modeling time horizon is given by:

$$y_{T_m} = [y(k_0), y(k_0 + 1), \ldots, y(k_0 + T_m)] \quad (3.47)$$

under the predicted disturbance (water demands) given by:

$$d_{T_m} = [d(k_0), d(k_0 + 1), \ldots, d(k_0 + T_m)] \quad (3.48)$$

The representation of inputs for the experiment design and the theorem guiding the experiment design is in [31]. The experiment design process is guided by the theorem in [31]. The number of experiments, N_E that are needed to solve the robust parameter estimation task is given by $N_E = 2^{T_m}$ therefore the numerical complexity of the task scales exponentially with the length of prediction horizon. It is possible to excite the plant with a finite number of inputs for the experiment using the minimum and the maximum input limits [31]. For each experiment under predicted disturbance, excite the plant with u^j

where: $u^j = [u^j(1), \ldots, u^j(T_m)]$ is the j-th experiment input sequence and $u^j(k)$ have either a value of u_{min} or u_{max}

Each experiment with u^j produces an output sequence y^j based on (3.47) and (3.48) yielding:

$$y^j = \emptyset^j(k)^T \boldsymbol{\theta}_p{}^j(k) \tag{3.49}$$

where:

$y^j = [y^j(1),. . ., y^j(T_m)]$ is the j-th experiment output sequence;

$\emptyset^j(k) = [\emptyset^j(k)(1),. . ., \emptyset^j(k)(T_m)]$ is the j-th experiment regressor sequence,

$\boldsymbol{\theta}_p{}^j = [\boldsymbol{\theta}_p{}^j(1),. . ., \boldsymbol{\theta}_p{}^j(T_m)]$ is the j-th experiment parameter vector sequence

Let $\{u(k), y(k)\}$ be input-output pairs corresponding to a system in (3.46) and (3.49), there exists point-parametric model parameter $\boldsymbol{\theta}_p(k)$ such that $y(k) = \emptyset(k)^T \boldsymbol{\theta}_p(k)$ for each time instant $k \in [k_0, k_0 + T_m]$ where all uncertainties and model error are allocated to the model parameter $\boldsymbol{\theta}_p$. The point-parametric approach is to obtain the parameter bounds for each operating point of the plant in the modeling time horizon. The simulation experiment using EPANET was carried out using different combinations of $u_{min}(k)$ and $u_{max}(k)$ as in (3.46) to obtain (3.49) on the benchmark DWDS. The details of the experiment are explained in Chapter 6 of this thesis.

3.9 Time-varying Model Parameter Estimation

The parameter estimation process is performed based on the observed input-output data pairs and the overall number of the pairs is determined by the total number of experiments.

A feasible parameter set due to the *E* experimental inputs over *N* time modeling horizon in [31] [77] [78] can be defined as:

$$\boldsymbol{\theta}_p{}^j(k) \in \Omega\,(\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k))$$

$$\Omega\,(\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k)) \triangleq \left\{ \begin{matrix} \boldsymbol{\theta}_p{}^j(k) \in \mathbb{R}^M : y^j = \emptyset^j(k)^T\, \boldsymbol{\theta}_p{}^j(k) \\ \boldsymbol{\theta}_p{}^l(k) \le \boldsymbol{\theta}_p{}^j(k) \le \boldsymbol{\theta}_p{}^u(k) \\ j = 1, \ldots, E\,, k = 1, \ldots, N \end{matrix} \right\} \tag{3.50}$$

where $\Omega\,(\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k))$ is the union of the parameter sets that corresponds to the experimental inputs; $\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k)$ are the union bounds at *k*; *M* is the dimension of the parameter vector and *E* is the experiment number.

The least conservative estimation for set $\Omega\,(\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k))$ is carried out by solving the following optimization task:

$$\left[\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k)\right] = argmin\{\, J\big(\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k)\big)\} \tag{3.51}$$

$$[\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k), \boldsymbol{\theta}_p{}^j(k)\ ]$$

Subject to $\boldsymbol{\theta}_p{}^j(k) \in \Omega\,(\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k))$

where:

$$J\big(\boldsymbol{\theta}_p{}^l(k), \boldsymbol{\theta}_p{}^u(k)\big) = (\boldsymbol{\theta}_p{}^u(k) - \boldsymbol{\theta}_p{}^l(k))^T\, P\, (\boldsymbol{\theta}_p{}^u(k) - \boldsymbol{\theta}_p{}^l(k)) \tag{3.52}$$

The choice of *P* could be an identity matrix of dimension of the parameter vector. From (3.46) we considered the type of model where all model uncertainties are allocated to the parameter and the parameter $\boldsymbol{\theta}_p(k)$ is time-varying for the model to work.

This modeling approach is used in this thesis to estimate the model parameters for the DWDS under different operational states. The simulation is discussed in detail in chapter 6 of this thesis.

3.10 Summary

The drinking water distribution systems (DWDS) hydraulic laws and model of the DWDS components is presented. The control-design based approach to chlorine residual control is explained. The existing Path analysis backward tracking algorithm for model structure acquisition is further modified to Path analysis forward tracking algorithm to account for changing operational states of the DWDS and improve the modeling accuracy. The node-to-node analysis for chlorine residual modeling is proposed and discussed. Model parameter estimation using set-membership and point parametric approach is presented. The simulation experiment design and model parameter estimation for time-varying parameter system is presented and discussed. The model parameter bounding at each time instant over the modeling horizon is presented. The time-varying parameter model developed is suitable for RFMPC design for DWDS water quality control.

Chapter 4

Robustly Feasible Model Predictive Control Design

In this Chapter, basic MPC structure is presented in Section 4.1. RFMPC design with recursive properties using the Karush Kuhn Tucker conditions was proposed in Section 4.2. Distributed robustly feasible MPC (DRFMPC) with the proposed adaptive feasible cooperation is presented in section 4.3. The summary is presented in section 4.4.

4.1 Model Predictive Control (MPC)

4.1.1 Introduction

MPC is an advanced control technology in the Industry. It is also referred to as receding horizon control (RHC). MPC uses an explicit process model to predict the future response of a plant [79]. At each control time step or sampling interval, the MPC algorithm optimizes the future plant behaviour by computing a control sequence of future manipulation of the plant using the current state of the plant as the initial state. The first input in the control sequence is applied to the plant, and the prediction horizon moves forward, and the entire calculation is repeated at the next control time steps.

The pioneering ideas of MPC started in the 1960s [80]. The applications of MPC began to increase rapidly in the 1980s after publication of the papers of [81] on IDCOM, Dynamic Matrix Control (DMC) by [82] and Generalized Predictive Control (GPC) by [83] [84]. The survey of industrial MPC is in [85] [86] . The major selling points of MPC technology [10] includes:

- Ability to handle multivariable parameters for system control
- Constraint handling and management capabilities
- Its ability to control unstable processes
- Flexibility in online computation
- Easy to tune
- Ability to handle structural changes in the plant

The MPC technology has been developed to maturity and many types of MPC and applications has been developed. The basic components of MPC technique are still the same. The components of MPC are prediction model, objective function, receding horizon, constraints handling, optimization and manipulated variables which are the degrees of freedom of control. The robustness, stability and performance properties of MPC has been investigated and published in the literature over the years. Robustness is the ability of the MPC to satisfy the system constraints and achieve the control objectives under uncertainty scenarios in the system.

MPC have been developed according to the certain process features [87], control features [88]and with the guaranty of robustness and stability [89] [90]. MPC technology is now mature but still faces a major challenge of guaranteeing stability and robustness against uncertainty in the control of uncertain systems [79]. The uncertainty includes model-reality mismatch, disturbances, and state estimation error. Guaranteeing stability, robustness, and performance in the systems while satisfying the system constraints is important in MPC design.

With the increasing use of MPC as a control agent in many applications [91], many published papers have been presented with stability and robust properties on decentralized MPC, distributed MPC and hierarchical decentralized and hierarchical distributed MPC [92] [93] [94] [95]. One of the research trends now is the development of MPC with self-reconfiguration feature [96] [97].

4.1.2 Model Predictive Control Architecture

Model Predictive Control (MPC) is a model-based controller design concept. The model of the plant is used to predict the future response of the plant. The optimal control actions for the plant is determined by minimizing a user-defined objective function which penalizes the difference between the predicted output trajectories and the reference trajectories over a finite prediction and control horizon. At each control time step, the initial state of the plant is measured, a finite-horizon open-loop optimization problem is solved to calculate the current control action. A control action sequence over the control horizon is generated and the first part of the control action sequence is applied to the plant. The prediction horizon moves forward, and the same procedure is repeated at each control time step. Figure 4.1 illustrates the idea of MPC in [8] Figure 4.1 is adapted from [98].

H_p and H_c denotes the finite time prediction horizon and control horizon. The current time instant is k and the present output is y_k. $u_{.|k}$ denote the control input and $y_{.|k}$ denote the predicted output. The set- point trajectory may be fixed or time varying depending on the operation of the plant process. A reference trajectory considering an ideal or desired tracking trajectory from the current output to the set-point trajectory can be defined over the prediction horizon H_p before running the MPC. In a receding horizon operation, only the first control action $u_{k|k}$ from the control sequence is applied to the plant process over the control step. Next, the process output or state variables are measured, and the optimization problem is solved again over the prediction horizon with the initial conditions updated from the measurements [8] [98].

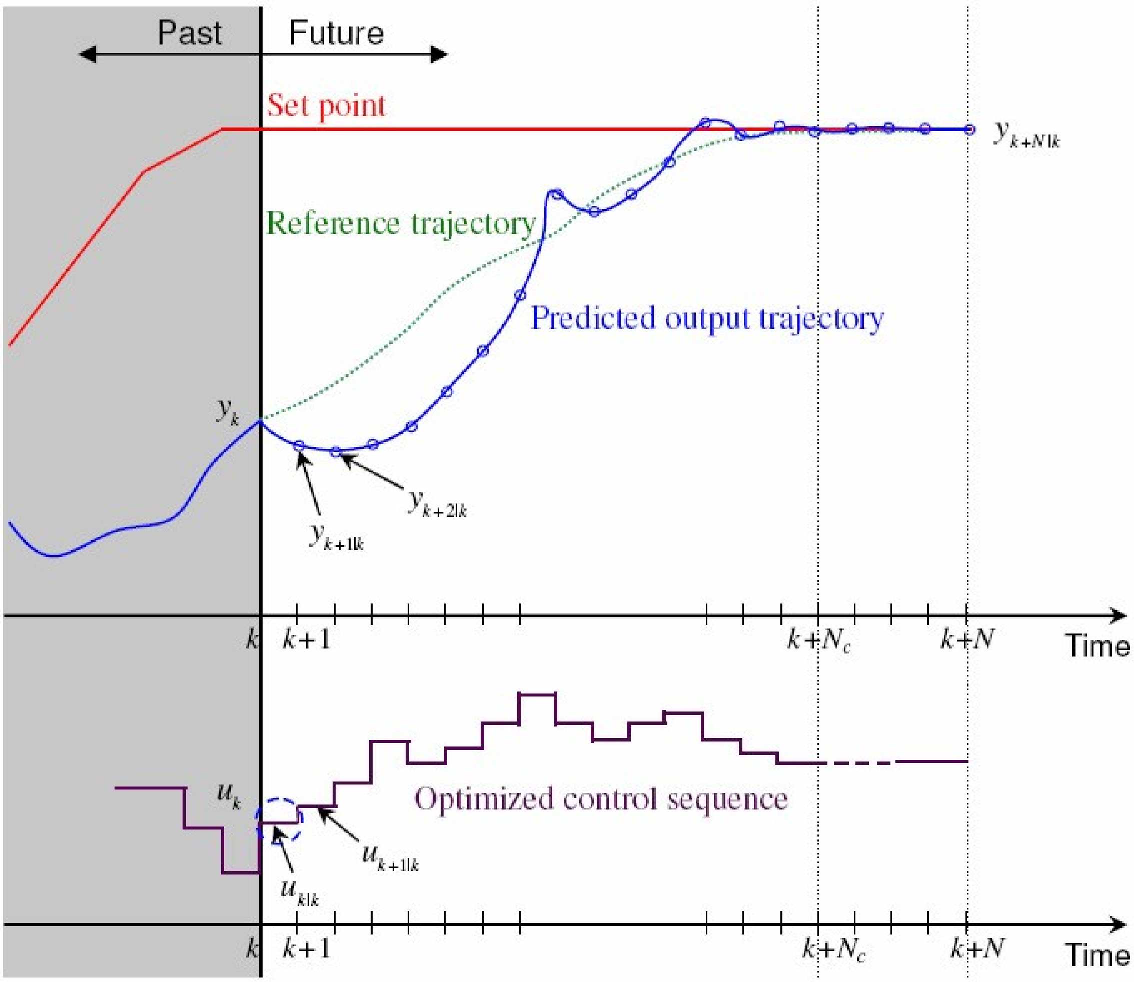

Figure 4.1 Basic Model Predictive Control: operational concept [98]

The generalized basic MPC algorithm for the operation of a basic MPC controller is as follows:

Basic Algorithm of MPC

1. At time k, obtain or measure the current state x_k of the plant;

2. Determine the control sequence $u_{.|k}$ by solving a finite horizon optimal control problem;

3. Apply the first element in the control sequence, $u_k = u_{k|k}$ to the plant;

4. $k \leftarrow k + 1$. Go to step 1.

The basic structure of MPC is illustrated in Figure 4.2.

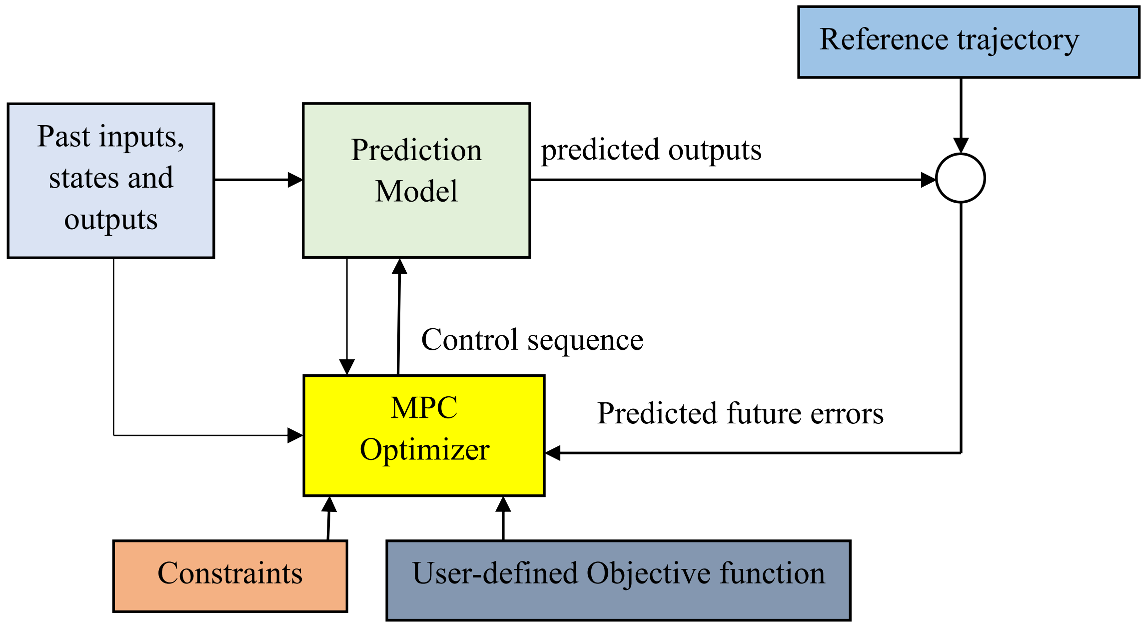

Figure 4.2 MPC basic structure

We assume that the output prediction and control horizon are equal such that:

$$H_p = H_c$$

The vector of predicted outputs over the prediction horizon is defined as:

$$y_{.|k} = [y(k|k) \ . \ . \ . \ \ y(k + H_p - 1|k)\ ]^T \qquad (4.1)$$

Where $y(k+i|k)$ is the predicted model output at $k+i$, $i = 0 \ldots H_p$ and the predicted model output is executed at time instant k.

The vectors of predicted control input, state, and disturbance over the prediction are respectively defined as:

$$u_{.|k} = [u(k|k) \ \ldots \ \ u(k+\ H_p - 1|k)\ ]^T \tag{4.2}$$

$$x_{.|k} = [x(k|k) \ \ldots \ \ x(k+\ H_p - 1|k)\ ]^T \tag{4.3}$$

$$d_{.|k} = [d(k|k) \ \ldots \ \ d(k+\ H_p - 1|k)\ ]^T \tag{4.4}$$

The reference values over the prediction horizon are given as:

$$r_{.|k} = [r(k|k) \ \ldots \ \ r(k+\ H_p - 1|k)\ ]^T \tag{4.5}$$

In dynamic network systems, there are vectors of inputs u, states x and outputs y and they are constrained as follows:

$$u \ \in \mathbb{U} = [\,u^{min}, u^{max}] \ \subset \ \mathbb{R}^l \tag{4.6}$$

$$y \ \in \mathbb{Y} = [\,y^{min}, y^{max}] \ \subset \ \mathbb{R}^m \tag{4.7}$$

$$x \ \in \mathbb{X} = [\,x^{min}, x^{max}] \ \subset \ \mathbb{R}^n \tag{4.8}$$

The disturbance w is unknown but is assumed bounded in a closed set $\mathbb{W}$ such that

$$w \ \in \ \mathbb{W} \subset \mathbb{R}^p$$

By using the model of the plant explicitly, the predicted output over the prediction horizon can be calculated with the information of the current and past states $x(k)$, past inputs $u(k)$ and the disturbance $w(k)$ is given as:

$$y_{.|k} = G\ (x(k), u(k), u_{.|k}, \theta, w_{.|k}) \tag{4.9}$$

Where $G\ (\cdot)$ denotes the plant model and θ denotes the plant model parameters. The model could be linear or non-linear, time-varying or time invariant.

The user-defined objective function, $J\ (\cdot)$ which can be defined using L_1, L_2, or L_∞ norms is used to measure the distance between the reference trajectories and the predicted output trajectories. If the model $G\ (\cdot)$ is linear and L_2 norm is used, the optimization problem is a quadratic programming problem [10] given as:

$$u_{.|k} = argmin_{u_{.|k}} J(u_{.|k}) = (r_{.|k} - y_{.|k})^T\ P\left(r_{.|k} - y_{.|k}\right) + (\Delta u_{.|k}{}^T\ Q\ u_{.|k}) \tag{4.10}$$

$$\text{Subject to: } C_y(y_{.|k}) \leq 0$$

$$C_u(u_{.|k}) \leq 0$$

Where $C_y(\cdot)$ denotes constraints on the outputs and $C_u(\cdot)$ denotes constraints on the inputs.

In (4.10), at time instant k , the minimum control input sequence that achieves the minimum error between the reference output and the actual plant output over the prediction horizon and satisfies the input and output constraints is determined. This approach can be used in many reference tracking applications [10] .

4.2 Robustly Feasible Model Predictive Control (RFMPC)

There are different approaches to designing the robustly feasible MPC. The safe feasibility tubes in the state space were used to design the RFMPC in [99] [100]. Min-Max approach where the worst-case scenario of the system uncertainty is used to generate the control actions and design the RFMPC is in [101]. Reference governor approach is used in [102] to generate a reference trajectory over the prediction horizon and the control inputs generated under the reference trajectory steers the system to the desired state without violating the constraints under uncertain scenarios in the system. The constraint restriction approach is used in [103] to restrain the input constraints and design RFMPC. Iterative safety zone method is used in [104] and was applied for robust predictive control of chlorine residuals in DWDS. Non-iterative safety zone that utilized Lipschitz constants of the nonlinear network mappings for the design of RFMPC was proposed in [105]. RFMPC based on calculated robust invariant set is used in [106] [107] [108]. Off-line formulation of RMPC is used in [109].

In this thesis, the off-line or non-iterative safety zone approach that utilizes robustly feasible invariant sets is proposed for the design of the RFMPC. The recursive feasibility is guaranteed with this approach. There are three operational states for the DWDS considered in this thesis and the feasibility of control actions must be guaranteed in any of the operational states. The RFMPC controllers used in this thesis are arranged in a distributed architecture; it therefore needs proper coordination schemes to ensure recursive feasibility in all the RFMPC controllers.

4.2.1 Robust Feasibility of a Nominal Model

The main source of uncertainty in MPC design is the model uncertainty due to model-reality mismatch. The MPC uses the prediction model for predicting the future behaviour of the plant and if the predicted state or output is different from the real plant state or output, the MPC optimizer may not be able to generate control actions that will satisfy the system constraints. With model uncertainty, the plant state or output may be driven to a dangerous region where feasible control actions cannot be generated by the MPC. In safety-critical applications, infeasibility of control actions must never happen and recursive feasibility under uncertainty scenarios must be guaranteed [106]

Consider this MPC optimization task:

$$u_{.|k} = argmin_{u_{.|k}} J(u_{.|k}, x_{k|k}, w_{.|k}) \tag{4.11}$$

Subject to: $x_{k|k} = x(k)$

$$C(x_{.|k}, u_{.|k}, d_{.|k}) \leq 0$$

$$C_u(u_{.|k}) \leq 0$$

$$C_y(y_{.|k}) \leq 0$$

The solution of equation (4.11) is determined by the initial state $x_{k|k}$. The MPC uses explicitly the model of the plant and the initial state of the plant at every control time step to generate the control sequence. For some values of initial states over the prediction horizon the MPC optimizer will not be able to generate control actions $u_{.|k}$ and this causes infeasibility [98]. Due to model-reality mismatch, the control actions generated in (4.11) may satisfy the model constraints but may violate the real plant constraints [98]. To avoid infeasibility at any time instant k over the prediction horizon, the plant is steered away from the infeasible initial states or steered into feasible initial states. The feasible initial states are an invariant set. We shall distinguish between the various types of feasible states. Using the nominal model of the plant, we shall determine the sets of feasible initial states over the prediction horizon. Earlier work by [98] used Karush Kuhn Tucker (KKT) optimality conditions to determine the feasible initial states.

Using KKT optimality conditions, $u_{.|k}$ is a local minimizer of (4.11) and must satisfy the KKT conditions such that:

- $C\left(x_{.|k}, u_{.|k}, w_{.|k}\right) \leq 0$
- There exists a vector of Lagrangian multipliers μ_i such that

$$\nabla J\left(u_{.|k}, x_{k|k}, w_{.|k}\right) - \sum_{i=1}^{NM} \mu_i \nabla C_i = 0 \tag{4.12}$$

- $\mu_i C_i = 0 \ for\ 1 \ll i \ll M$ (4.13)
- $\mu_i \geq 0 \ for\ 1 \ll i \ll M$ (4.14)

The set of feasible initial states $x_{k|k}$ that $u_{.|k}$ exists that satisfies the KKT conditions is denoted by $\mathbb{X}_f\ (k) \triangleq \left\{x_{k|k} \in \mathbb{X} : \exists\, u_{.|k} \in \mathbb{U}\right\}$. [98]

Calculating $\mathbb{X}_f\ (k)$ is computationally demanding and a box approximation was proposed in [98]

$\mathbb{X}_f\ (k) = \left[\mathbb{X}_f{}^{min}\ (k), \mathbb{X}_f{}^{max}\ (k)\right] \subset \mathbb{X}_f\ (k)$ over each time instant in the prediction horizon.

$$\mathbb{X}_f{}^{min}\ (k) = [\mathrm{x}_f{}^{min}\ (k) \ldots \mathrm{x}_f{}^{min}\left(k + H_p - 1\right)] \tag{4.15}$$

$$\mathbb{X}_f{}^{max}(k) = [\mathrm{x}_f{}^{max}(k)\ .\ .\ .\ \mathrm{x}_f{}^{max}\left(k + H_p - 1\right)] \tag{4.16}$$

In this thesis, the output or state constraints are tightened by safety zones. The safety zones $\mathcal{E} \triangleq \left[\mathcal{E}^{min}, \mathcal{E}^{max}\right]$ are chosen to tighten the output or state constraints such that if the tightened constraints are violated, in real plant the real plant output or state constraints are not violated. With safety zones used to tighten the feasible initial states and using the nominal model, the KKT optimality conditions is used as in (4.15) and (4.16), we calculate a set of robustly feasible tightened initial states denoted by:

$$\mathbb{X}_{Rf,S}(k) \triangleq \left\{x_{k|k} \in \mathbb{X}_{Rf,S} : \exists\, u_{.|k} \in \mathbb{U}\right\}$$

$$x_{k|k} + \mathcal{E}^{min} \ll x_{k+i|k} \ll x_{k|k} - \mathcal{E}^{max}$$

$\mathbb{X}_{Rf,S}(k) = \left[\mathbb{X}_{Rf,S}{}^{min}(k), \mathbb{X}_{Rf,S}{}^{max}(k)\right] \subset \mathbb{X}_{Rf,S}(k)$ over each time instant in the prediction horizon.

$$\mathbb{X}_{Rf,S}{}^{min}(k) = [\mathrm{x}_{Rf,S}{}^{min}(k)\ .\ .\ .\ \mathrm{x}_{Rf,S}{}^{min}\left(k + H_p - 1\right)] \tag{4.17}$$

$$\mathbb{X}_{Rf,S}{}^{max}(k) = [\mathrm{x}_{Rf,S}{}^{max}(k)\ .\ .\ .\ \mathrm{x}_{Rf,S}{}^{max}\left(k + H_p - 1\right)] \tag{4.18}$$

The choice of the value of the safety zone is done offline via simulations and it is used explicitly in the RFMPC design. Robust feasibility is achieved by steering the system state to the tightened robustly feasible state at every control time step. This is called one step robust feasibility. For different operational states of the plant, the robustly feasible states must be calculated.

The set of normal operational state robustly feasible states is denoted by:

$$\mathbb{X}_{Rf,S,N}\,(k)\ \triangleq\ \left\{x_{k|k}\ \in\ \mathbb{X}_{Rf,S,N}\ :\ \exists\, u_{.|k}\ \in\ \mathbb{U}\right\}$$

$\mathbb{X}_{Rf,S,N}\,(k) = \left[\,\mathbb{X}_{Rf,S,N}{}^{min}\,(k), \mathbb{X}_{Rf,S,N}{}^{max}\,(k)\right]\ \subset\ \mathbb{X}_{Rf,S,N}\,(k)$ over each time instant in the prediction horizon.

$$\mathbb{X}_{Rf,S,N}{}^{min}\,(k) = [\mathrm{x}_{Rf,S,N}{}^{min}\,(k)\ .\ .\ .\ \mathrm{x}_{Rf,S,N}{}^{min}\left(k+\ H_p-1\right)] \tag{4.19}$$

$$\mathbb{X}_{Rf,S,N}{}^{max}\,(k) = [\mathrm{x}_{Rf,S,N}{}^{max}\,(k)\ .\ .\ .\ \mathrm{x}_{Rf,S,N}{}^{max}\left(k+\ H_p-1\right)] \tag{4.20}$$

This is the set of robustly feasible initial states in the normal operational state of the plant.

The set of disturbed operational state robustly feasible states is denoted by:

$$\mathbb{X}_{Rf,S,D}\,(k)\ \triangleq\ \left\{x_{k|k}\ \in\ \mathbb{X}_{Rf,S,D}\ :\ \exists\, u_{.|k}\ \in\ \mathbb{U}\right\}$$

$\mathbb{X}_{Rf,S,D}\,(k) = \left[\,\mathbb{X}_{Rf,S,D}{}^{min}\,(k), \mathbb{X}_{Rf,S,D}{}^{max}\,(k)\right]\ \subset\ \mathbb{X}_{Rf,S,D}\,(k)$ over each time instant in the prediction horizon.

$$\mathbb{X}_{Rf,S,D}{}^{min}\,(k) = [\mathrm{x}_{Rf,S,D}{}^{min}\,(k)\ .\ .\ .\ \mathrm{x}_{Rf,S,D}{}^{min}\left(k+\ H_p-1\right)] \tag{4.21}$$

$$\mathbb{X}_{Rf,S,D}{}^{max}\,(k) = [\mathrm{x}_{Rf,S,D}{}^{max}\,(k)\ .\ .\ .\ \mathrm{x}_{Rf,S,D}{}^{max}\left(k+\ H_p-1\right)] \tag{4.22}$$

This is the set of robustly feasible initial states in the disturbed operational state of the plant.

The set of emergency operational state robustly feasible states is denoted by:

$$\mathbb{X}_{Rf,S,E}\ (k) \triangleq \{x_{k|k} \in \mathbb{X}_{Rf,S,E} : \exists\ u_{.|k} \in \mathbb{U}\}$$

$\mathbb{X}_{Rf,S,E}\ (k) = \left[\ \mathbb{X}_{Rf,S,E}{}^{min}\ (k), \mathbb{X}_{Rf,S,E}{}^{max}\ (k)\right] \subset \mathbb{X}_{Rf,S,E}\ (k)$ over each time instant in the prediction horizon.

$$\mathbb{X}_{Rf,S,E}{}^{min}\ (k) = [\mathrm{x}_{Rf,S,E}{}^{min}\ (k) \ldots \mathrm{x}_{Rf,S,E}{}^{min}\ (k + H_p - 1)] \tag{4.23}$$

$$\mathbb{X}_{Rf,S,E}{}^{max}\ (k) = [\mathrm{x}_{Rf,S,E}{}^{max}\ (k) \ldots \mathrm{x}_{Rf,S,E}{}^{max}\ (k + H_p - 1)] \tag{4.24}$$

This is the set of robustly feasible initial states in the emergency operational state of the plant.

It is usually desired that a controlled plant operates reliably under wide ranges of operational states. To guaranty, the feasibility of control actions when there is a change of operational state in the plant, the initial state of the plant in the new operational state must be within the robustly feasible states of the operational state. We shall establish the following to determine the appropriate switching method for the RFMPC controllers:

- $\mathbb{X}_{Rf,S,N}\ (k) \cap \mathbb{X}_{Rf,S,D}\ (k)$; if it is nonempty at time instant k for the switching, then hard switching of the RFMPC controllers can be done otherwise soft switching of the RFMPC controllers will be done

- $\mathbb{X}_{Rf,S,N}\ (k)\ \cap\ \ \mathbb{X}_{Rf,S,E}\ (k)$; if it is nonempty at time instant k for the switching, then hard switching of the RFMPC controllers can be done otherwise soft switching of the RFMPC controllers will be done
- $\mathbb{X}_{Rf,S,D}\ (k)\ \cap\ \ \mathbb{X}_{Rf,S,E}\ (k)$; if it is nonempty at time instant k for the switching, then hard switching of the RFMPC controllers can be done otherwise soft switching of the RFMPC controllers will be done.

4.2.2 Invariant sets

Set invariance plays a fundamental role in the design of control systems for constrained systems since the constraints can be satisfied for all time if and only if the initial state is contained inside an invariant set [106] [110]. The invariant set theory will be used to proof the concept of recursive feasibility and recursive robust feasibility. The following definitions from [98] [106] [110] are useful for understanding the concept of recursive feasibility.

Definition 1.0 (***Positively Invariant set)***: The set $\Omega\ \in\ \mathbb{R}^n$ is a *positively invariant* set for the autonomous system $x_{k+1}\ = f(x_k)$ *if* and only if $\forall\ x_0\ \in\ \Omega$ the system state evolution satisfies $x_k\ \in\ \Omega\ ,\ \forall\, k\ \in [1,\infty]$. The set Ω is *invariant* if and only if $x_0\ \in\ \Omega$ implies $x_k\ \in\ \Omega\ ,\ \forall\, k\ \in [0,\infty]$

Definition 1.1 (***Maximal Positively Invariant set)***: The set $\mathcal{O}_\infty\ (\Omega)$ is the *maximal positively invariant set* contained in Ω for the autonomous system $x_{k+1}\ = f(x_k)$ if and only if $\mathcal{O}_\infty\ (\Omega)$ is positively invariant and contains all positively invariant sets contained in Ω, that is Υ is positively invariant only if $\Upsilon \subseteq \mathcal{O}_\infty\ (\Omega) \subseteq \Omega$.

Definition 1.2 (***Control Invariant set)***: The set $\Theta \in \mathbb{R}^n$ is a *control invariant set* for the system $x_{k+1} = f(x_k, u_k)$ if and only if there exists a feedback control law $u_k = g(x_k)$ such that Θ is a positively invariant set for the closed-loop system $x_{k+1} = f(x_k, g(x_k))$ and u_k is an admissible control input for $\forall\, x_k \in \Theta$.

Definition 1.3 (***Maximal Control Invariant set)***: The set $\mathcal{C}_\infty(\Theta)$ is the *maximal control invariant set* contained in Θ for the system $x_{k+1} = f(x_k, u_k)$ if and only if $\mathcal{C}_\infty(\Theta)$ is control invariant and contains all control invariant sets contained in Θ, that is, γ_∞ is control invariant only if $\gamma_\infty \subseteq \mathcal{C}_\infty(\Theta) \subseteq \Theta$.

Definition 1.4 (***Robust Positively Invariant set)***: The set $\Gamma \in \mathbb{R}^n$ is *robust positively invariant* for the system $x_{k+1} = f(x_k, w_k)$ if and only if $\forall\, x_0 \in \Gamma$ and $\forall\, w_k \in \mathbb{W}$, the system evolution satisfies $x_k \in \Gamma$, $\forall\, k \in [1, \infty]$. Where $w_k \in \mathbb{W}$ is the disturbance set.

Definition 1.5 (***Maximal Robust Positively Invariant set)***: The set $\mathrm{V}_\infty(\Gamma)$ is the *maximal robust positively invariant set* contained in Γ for the system $x_{k+1} = f(x_k, w_k)$ if and only if $\mathrm{V}_\infty(\Gamma)$ is robust positively invariant and contains all the robust positively invariant sets contained in Γ.

Definition 1.6 (***Robust control Invariant set)***: The set $\Omega \in \mathbb{R}^n$ is a *robust control invariant* set for the system $x_{k+1} = f(x_k, u_k, w_k)$ if and only if there exists a feedback control law $u_k = g(x_k)$ such that Ω is a robust positively invariant set for the closed-loop system $x_{k+1} = f(x_k, g(x_k), w_k)$ and $u_k \in \mathbb{U}$, $\forall\, x_k \in \Omega$

Definition 1.7 (***Maximal Robust Control Invariant set)***: The set $\Gamma_\infty\ (\Omega)$ is the *maximal robust control invariant set* contained in Ω for the system $x_{k+1} = f(x_k, u_k, w_k)$ if and only if $\Gamma_\infty\ (\Omega)$ is robust control invariant and contains all the robust control invariant sets contained in Ω.

In [111], the concept of operability index was presented. The operability index describes the input-output relationships rather than the system dynamic states. Input and output values are defined by spaces in $\mathbb{R}^m$ and $\mathbb{R}^n$. These spaces are the feasible regions which are bounded by the inequalities describing the ranges of the inputs or outputs. The following definition of spaces in [111] are of relevance to RFMPC design:

Definition 1.8 (***Available Input Space (AIS)*****):** the set of attainable values of the process inputs. These are the values of the constraints on the inputs.

Definition 1.9 ***(Achievable Output Space (AOS)):*** this is the set of values which the process outputs can obtain given the available input space (AIS).

Definition 1.10 (***Desired Output Space (DOS)):*** this represents the desired output values of a process.

Definition 1.11 (***Expected Disturbance Space (EDS)):*** all the values of the expected disturbances to the system.

The recursive robust feasibility is achieved based on the set invariance and operability index. The complete theory is in [106] [110] [111].

4.2.3 Recursive Robust Feasibility for Robustly Feasible MPC

It has been established that at every control time step, the initial state of the plant must be inside the robustly feasible or robust control invariant set to guaranty the feasibility of the control actions at the control time step. In this thesis, the off-line safety zones have been used to tighten the state and output constraints and the robust feasible initial states calculated. The achievable output space is now shrunk to the tightened desired output space. Other recursive feasibility approaches are in [112] which uses a finite number of possible values for the uncertainties and model their combinations in a scenario tree. Recursive feasibility guarantees in move-blocking MPC is addressed in [113]. Recursive feasibility testing in [114] In [115], design and implementation of recursive MPC have been presented. Extended recursively feasible MPC using two-stage online optimization is addressed in [116]. Methods for computation of invariant sets using interval arithmetic approach is in [117].

In this thesis, robust feasibility must be guaranteed for different operational states of the plant to be controlled at all time steps. Recursive robust feasibility implies that at all time steps, the system state is inside the robustly feasible control set.

For the normal operational state of the plant, the recursive robust feasibility is defined as:

$\forall\, x_k \in \mathbb{X}_{Rf,S,N}\,(k)$ for a system $x_{k+1} = f\left(x_k, u_k\,, w_{k,N}\right)$, $\forall\, k \in [1, \infty]$ and

$u_k \in \mathbb{U}\,, w_{k,N} \in \mathbb{W}_N$,where $\mathbb{W}_N$ is the expected disturbance set for the normal operational state of the plant.

For disturbed operational state of the plant, the recursive robust feasibility is defined as:

$\forall\, x_k \in \mathbb{X}_{Rf,S,D}\,(k)$ for a system $x_{k+1} = f\big(x_k, u_k\,, w_{k,D}\,\big)\,, \forall\, k \in [1, \infty]$ and

$u_k \in \mathbb{U}\,, w_{k,D} \in \mathbb{W}_D$,where $\mathbb{W}_D$ is the expected disturbance set for the disturbed operational state of the plant.

For the emergency operational state of the plant, the recursive robust feasibility is defined as:

$\forall\, x_k \in \mathbb{X}_{Rf,S,E}\,(k)$ for a system $x_{k+1} = f\big(x_k, u_k\,, w_{k,E}\,\big)\,, \forall\, k \in [1, \infty]$ and

$u_k \in \mathbb{U}\,, w_{k,E} \in \mathbb{W}_E$,where $\mathbb{W}_N$ is the expected disturbance set for the emergency operational state of the plant.

We assume the output prediction horizon is the same as the control horizon

$$H_p = H_c = N$$

The algorithm is given as follows:

Algorithm 4.1

For $k = \overline{1{:}N}$

1. Use the KKT optimality conditions to calculate the set of all feasible initial states at all time instants over the prediction horizon [98] $\mathbb{X}_f\,(k) \triangleq \{x_{k|k} \in \mathbb{X} : \exists\, u_{.|k} \in \mathbb{U}\}$

   Let the vector of all decision variables be denoted by

   $$\Gamma = [u_{.|k}{}^T\,, x_{k|k}\,, \mu_1, \;...\; \mu_{NM}]^T \tag{4.25}$$

   $\mathrm{X}_f{}^{min}\,(k) = argmin_\Gamma x(k)$

Subject to:

$$x_{k|k} \in \mathbb{X}$$

$$C\left(x_{.|k}, u_{.|k}, w_{.|k}\right) \leq 0$$

$$\nabla J\left(u_{.|k}, x_{k|k}, w_{.|k}\right) - \textstyle\sum_{i=1}^{NM} \mu_i \nabla C_i = 0$$

$$\mu_i C_i = 0 \;\; for \; 1 \ll i \ll M$$

$$\mu_i \geq 0 \;\; for \; 1 \ll i \ll M$$

This is done for all the operational states of the plant, that is normal, disturbed and emergency operational states.

$$\mathrm{X}_f{}^{max}(k) = argmax_{\Gamma} x(k)$$

Subject to:

$$x_{k|k} \in \mathbb{X}$$

$$C\left(x_{.|k}, u_{.|k}, w_{.|k}\right) \leq 0$$

$$\nabla J\left(u_{.|k}, x_{k|k}, w_{.|k}\right) - \textstyle\sum_{i=1}^{NM} \mu_i \nabla C_i = 0$$

$$\mu_i C_i = 0 \;\; for \; 1 \ll i \ll M$$

$$\mu_i \geq 0 \;\; for \; 1 \ll i \ll M$$

This is done for all the operational states of the plant that is normal, disturbed and emergency operational states.

2. Use the KKT optimality conditions to calculate the set of all robustly feasible initial states obtained by tightening the state or output constraints with the safety zones $\mathcal{E} \triangleq \left[\mathcal{E}^{min}, \mathcal{E}^{max}\right]$ at all-time instants over the prediction horizon. $\mathbb{X}_{Rf}\,(k) \triangleq \left\{x_{k|k} \in \mathbb{X}_{Rf} : \exists\, u_{.|k} \in \mathbb{U}\right\}$

   The tightened state constraints are now added to the set of constraints as follows:

$$\mathrm{X}_{Rf}{}^{min}\,(k) = argmin_{\Gamma} x(k)$$

Subject to:

$$x_{k|k} \in \mathbb{X}$$

$$x_f{}^{min}(k) + \mathcal{E}^{min} \ll x(k+1) \ll x_f{}^{max}(k) - \mathcal{E}^{min}$$

$$C\left(x_{.|k}, u_{.|k}, w_{.|k}\right) \leq 0$$

$$\nabla J\left(u_{.|k}, x_{k|k}, w_{.|k}\right) - \sum_{i=1}^{NM} \mu_i\, \nabla C_i = 0$$

$$\mu_i C_i = 0 \;\; for\; 1 \ll i \ll M$$

$$\mu_i \geq 0 \;\; for\; 1 \ll i \ll M$$

This done for all the operational states of the plant

$$\mathrm{X}_{Rf}{}^{max}\,(k) = argmax_{\Gamma} x(k)$$

Subject to:

$$x_{k|k} \in \mathbb{X}$$

$$x_f{}^{max}(k) + \mathcal{E}^{max} \ll x(k+1) \ll x_f{}^{max}(k) - \mathcal{E}^{max}$$

$$C\left(x_{.|k}, u_{.|k}, w_{.|k}\right) \leq 0$$

$$\nabla J\left(u_{.|k}, x_{k|k}, w_{.|k}\right) - \sum_{i=1}^{NM} \mu_i\, \nabla C_i = 0$$

$$\mu_i C_i = 0 \;\; for\; 1 \ll i \ll M$$

$$\mu_i \geq 0 \;\; for\; 1 \ll i \ll M$$

This is done for all the operational states of the plant.

3. The Recursively Robustly Feasible MPC (RCRFMPC) optimization formulation is given as: $u_{.|k} = argmin_{u_{.|k}} J(u_{.|k}, x_{k|k}, w_{.|k})$

   Subject to:

   $x_{k|k} \in \mathbb{X}_{Rf}$

   $C(x_{.|k}, u_{.|k}, w_{.|k}) \leq 0$

The RCRFMPC controllers are designed for all the operational states of the plant. The requirements for selecting the type of switching between the RRFMPC controllers is already discussed. The softly switched RCRFMPC is described as softly switched recursively feasible MPC (SSRCRFMPC).

The simulation of the RCRFMPC is presented in chapter 6 of this thesis.

4.3 Distributed Robustly Feasible MPC (DRFMPC) Design

Model Predictive Control is typically implemented in a centralized architecture where all the system measurements are sent a central MPC controller and the control outputs are used to drive the actuators in the system. In large-scale systems, centralized control by MPC is not suitable due to computation complexity, communication limitations, and flexibility in maintenance and management. For these reasons, many decentralized, distributed and hierarchical MPC architectures have been proposed and applied in the industry [6] . The decentralized, distributed and hierarchical MPC structure is designed to achieve the control goals for the system as a centralized MPC structure with reduced communication costs, reduced computation complexity and provide flexibility of maintenance or management of the system.

The decentralized MPC control structure has lower communication costs and faster control but has overall reduced performance compared to the distributed MPC control structure. Interactions between the subnetworks are neglected in the computation of control actions as each decentralized MPC computes its own local network control actions. In distributed MPC structure, there is communication between interacting MPC controllers to compute their control actions. This results in better performance than the decentralized MPC control approach. Coordination is very important in distributed MPC architecture.

Different distributed MPC control approaches are compared in [118]. Coordination of distributed MPC is in [119] [120] [121] . Robust distributed MPC has been addressed in [122]. An iterative scheme for distributed MPC is used in [123] [124] . Distributed economic MPC is addressed in [125]. Distributed MPC based on agent negotiation is presented in [126]. Distributed MPC for Fault-tolerant cooperative control is presented in [127]. Non-iterative distributed MPC is presented in [128].

In this thesis, the designed recursively robustly feasible MPC (RCRFMPC) is used to implement the distributed robustly feasible MPC. One of the aims of this thesis is to formulate coordination approach to distributed RFMPC working cooperatively to achieve a set of control objectives under a wide range of operational states of the plant.

4.3.1 Distributed Model Predictive Control Problem

In distributed MPC [129], each MPC controls a subnetwork of the overall network. There are three types of decision variables to be considered: local variables, interaction variables, and remote variables. The local variables are strictly variables within the local subnetwork allocated to the local controller. The interaction variables are the variables allocated to the controller neighbours for information exchange. The remote variables are other variables in the overall network [129].

The standard MPC formulation [129] can be written as:

$$\min_S J\ (S) \tag{4.26}$$

$$\text{Subject to: } G\ (S) \ll 0$$

$$H\ (S) = 0$$

Where S is the vector of the decision variables including control variables U and state variables X over the prediction horizon, $G\ (S)$ are the inequality constraints in the system and $H\ (S)$ are the equality constraints that must be satisfied.

For *i*th MPC controller, where $i = 1,2,\ldots,M$,

$$S = S_i^{local} \cup S_i^{interaction} \cup S_i^{remote}$$

The MPC formulation for each MPC [129] is written as:

$$\min_{S_i} J_i\ (S_i^{local},\ S_i^{interaction}) \tag{4.27}$$

$$\text{Subject to: } G_i\left(S_i^{local},\ S_i^{interaction}\right) \ll 0$$

$$H_i\left(S_i^{local},\ S_i^{interaction}\right) = 0$$

4.3.2 Cooperative Distributed RFMPC

Cooperative Distributed MPC (DMPC) was first proposed in [130] and later developed in [49]. Cooperative Distributed MPC (DMPC) is presented in [48] [131] [132] [133] [134] .

In cooperative DMPC (CDMPC) or cooperative distributed RFMPC (CDRFMPC) there is a communication network for exchange of information among the MPC. In CDMPC approach, each controller considers the effects of its inputs on the entire system using the global objective function or centralized objective function. Iteratively, each controller optimizes its own set of inputs if the rest of the inputs of its neighbours are fixed to the last agreed value and share the resulting optimal trajectories and a final optimal trajectory is computed at each sampling time as a weighted sum of the most recent optimal trajectories with the optimal trajectories computed at the last sampling time [48]

The CDMPC controllers use the following implementation strategy [48]:

1. At k, all the DMPC controllers receive the local full state measurement $x_i(k)$ from the local monitoring units.

2. Each controller evaluates its own future input trajectory based on $x_i(k)$ and the latest received input trajectories of all the other controllers at iteration c = 1. Iteration number is denoted by c.

3. At iteration (c ≥1): the controllers exchange their future input trajectories $u_{i,f}(k+j|k)$.

Each controller calculates the current set of inputs trajectories $u_{i,c}(k+j|k)$ based on the received future input trajectories from other controllers.

4. If a convergence condition is satisfied, each controller sends its entire future input trajectory to control its local network or if the convergence condition is not satisfied, go to Step 3 at iteration (c←c + 1).

4. When a new measurement is received, go to Step 1 (k←k + 1).

At each iteration, each controller solves the following optimization problem in (4.27).

The CDMPC implementation strategy presented above [48] has some limitations and issues to be addressed. In this thesis, the limitations of the cooperative implementation strategy are identified, and solutions are proposed for implementation.

A typical plant is subjected to different disturbance scenarios and faults that may change its operational state. Normal, disturbed and emergency operational states were proposed in [2] A change of the operational state of the plant may imply the following:

- change of control strategy to adapt to the operational state
- change of model structure and a need to use a suitable model for the current operational state
- change of constraints considered in the MPC design

In a distributed RFMPC-controlled network, each subnetwork may be subjected to different disturbance scenarios and faults. The disturbance scenarios or faults in one subnetwork may propagate to other subnetworks and eventually degrade the performance of the overall network. It thus calls for a cooperative DRFMPC implementation strategy that incorporates the changing operational state of the plant in its operation and iteration process.

Changing the control strategy of the local RFMPC for a subnetwork to adapt to its current operational state may make this local RFMPC operate non-cooperatively with other DRFMPC in the network. It thus calls for techniques to achieve cooperation among the DRFMPC when any of the controllers is changing its control strategy.

The question of how to guaranty feasibility, stability, performance and optimality of each subnetwork during the change of operational state in any of the subnetwork needs to be addressed in the CDMPC implementation.

In this thesis, conditions that must be established and the proposed solutions for this CDMPC strategy is presented.

4.3.2.1 Conditions for adaptive cooperative DMPC implementation

The idea for the adaptive cooperation of DMPC presented in [135]. Fault-tolerant cooperative control using DMPC was proposed in [14]. In this thesis, the CDMPC implementation that adapts to the changing operational state of the subnetworks is described as adaptive cooperative DMPC. The conditions that must be put in place for adaptive cooperative DMPC implementation are:

1) Models of each operational state of the subnetworks must be available. This is to be acquired through comprehensive system identification process which simulates all possible disturbance scenarios and faults and the effects on other subnetworks. The models must be available or known to all interacting subnetworks.
2) The sets of robustly feasible states for each operational state of the subnetworks must be available. The intersection of the sets should be determined if it is nonempty.
3) A local monitoring unit with fault detection and diagnosis mechanisms for each subnetwork to identify the operational state of each subnetwork must be available.
4) The most suitable Control strategy to be used at each operational state of each subnetwork must be available

5) A mechanism for switching from one control strategy to another one in each subnetwork without degrading the guaranteed properties such as robust feasibility, stability, performance, and optimality must be available.
6) A supervisory control to initiate the change of control strategy during the change of operational state in each subnetwork must be available.
7) A communication protocol to indicate the specific operational state of each subnetwork must be available.

The conditions listed above are used to develop the adaptive CDMPC (ACDMPC) implementation strategy.

4.3.2.2 Adaptive Cooperative DMPC implementation strategy

The proposed ACDMPC used the following implementation strategy:

1. At k, each DMPC controller receives the local full state measurement with the type of operational state from its local monitoring unit and the interacting subnetwork local monitoring unit. The state information received at k is

$$X_i(k) = (\,X_i{}^{local}, X_i{}^{oper}, X_{m,inter}{}^{local}, X_{m,inter}{}^{oper})$$

Where $X_i(k)$ is the full state measurements received by ith local MPC at time instant k, $X_i{}^{local}$ is the local state measurements of ith local MPC, $X_i{}^{oper}$ is the type of operational state of the ith local network which may be normal, disturbed or emergency, $X_{m,inter}{}^{local}$ is the local state measurements of the interacting mth subnetwork and $X_{m,inter}{}^{oper}$ is the type of operational state of the mth interacting local network which may be normal, disturbed or emergency.

2. Each controller evaluates its own future input trajectory based on $X_i(k)$ and:

(i) the latest received input trajectories of all the other controllers at iteration c = 1, if the operational state of the local network and the interacting subnetworks has not changed from the previous time step or

(ii) the predicted input trajectories of all the other controllers at iteration c = 1, if the operational state of the local network has not changed but the operational state of the interacting subnetworks has changed from the previous time step or

(iii) the predicted input trajectories of all the other controllers at iteration c = 1, if the operational state of the local network has changed but the operational state of the interacting subnetworks has not changed from the previous time step

(iv) the control strategy that is most suitable to adapt current operational state. Soft switching of the MPC controllers to change to the new control strategy implies the old controller strategy is gradually reduced while the new controller strategy is increased

3. At iteration (c ≥1): the controllers exchange their future input trajectories $u_{i,f}(k+j|k)$.

Each controller calculates the current set of inputs trajectories $u_{i,c}(k+j|k)$ based on the received future input trajectories from other controllers.

4. If a convergence condition is satisfied, each controller sends its entire future input trajectory to control its local network or if the convergence condition is not satisfied, go to Step 3 at iteration (c←c + 1).

5. When a new measurement is received, go to Step 1 (k←k + 1).

The new adaptive cooperative implementation strategy has been proposed. It is validated by simulation in chapter 6 of this thesis. The recursively robustly feasible MPC (RRFMPC) already designed is used to implement the distributed Recursively Robustly Feasible MPC utilizing the adaptive cooperative MPC implementation strategy presented.

The soft switching mechanism is designed in chapter 5 of this thesis to achieve soft switching between two recursively robustly feasible MPC controllers. The arrangement is described as Distributed Softly Switched Robustly Feasible MPC (DSSRFMPC). The design of DSSRFMPC is presented in Chapter 5 of this thesis.

4.4 Summary

Robustly Feasible Model Predictive Control (RFMPC) design was presented in this chapter. The architecture and basic control structure of a basic MPC was presented. Different approaches to RFMPC design was discussed. Robust feasibility of a nominal model was presented, robust feasibility and recursive feasibility of a nominal model were established by using KKT conditions to calculate robustly feasible and recursive feasible initial states for all operational states of the plant over the prediction horizon. Robustly feasible invariant sets were presented based on set invariance theory. DRFMPC design was presented. The existing CDMPC coordination strategy was discussed and the limitations identified. The proposed implementation strategy for adaptive cooperative MPC that addressed the limitations of the existing CDMPC coordination strategy was presented.

Chapter 5

Soft Switching for Distributed Robustly Feasible Model Predictive Control

In this Chapter, Soft switching for Distributed Robustly Feasible Model Predictive Control design is presented in section 5.1. The distributed softly switched robustly feasible MPC (DSSRFMPC) components and the soft switching operation scenarios are presented in section 5.2. The soft switching design and analysis is presented in section 5.3. The summary is presented in section 5.4

5.1 Soft Switching of Robustly Feasible Model Predictive Controllers

The reliable and sustainable operation of the CIS or large-scale systems under different operational conditions such as sensor faults, actuator faults, CIS components faults, failures of communication links or anomalies occurring in the technological operation of the CIS physical processes is desirable [2]. The CIS operational state may change due to the faults in the CIS components, faults in the sensors or actuators, disturbance inputs not captured in the robust controller design and faults in the communication channels used in the CIS.

A given plant may be required to meet different sets of control objectives under different disturbance scenarios. This calls for the application of multiple MPC controllers each with a specific control strategy and model to achieve different control objectives for a specific operational state of the plant. The MPC with the most suitable control strategy and components for the current operational state of the plant is now switched to from the MPC that is been used before.

The current operational state (OS) of a CIS is determined by the states of the CIS processes, states of CIS components, states of all sensors, states of all actuators, disturbance inputs, states of the communication channels and current operating ranges of the processes [2]. The knowledge of the current operational state of the CIS enables the controllers to use the most suitable control strategy to achieve the control objectives at this operational state.

The switching of MPC from the old to the new one can be done in two ways: hard or soft. Hard switching implies engaging the new MPC instantly and putting it to use. The effects of hard switching are a generation of impulsive transients and spikes that may damage the actuators. In hard switching, the new MPC may not generate feasible control actions if the initial state of the plant at the time of switching is not inside the robustly feasible set of the new MPC [24].

Soft switching is a gradual process of engaging the new MPC while disengaging the old MPC. The effects of soft switching are smoothening of impulsive transients and achievement of robust feasibility [98].

The idea of using soft switching for MPC is presented in [22] [24] [98]. Three operational states were proposed, and they are normal, disturbed and emergency operational states. In [24], supervised soft switching of RFMPC is presented using the linear time-invariant system with bounded additive disturbances. In [23], the earlier work by [24] was further improved to soft switching for nonlinear systems.

In this thesis, the approach to the soft switching of RFMPCs is different from the earlier work done by [23] [24]. This thesis considers soft switching of RFMPCs in a distributed RFMPC framework working cooperatively and utilizing iterative coordination algorithms for computation of its control actions. The following factors are considered in the soft switching design for distributed robustly feasible model predictive controllers:

- The effects of disturbances and faults on a local network and the interacting subnetworks. The effects of faults or disturbances may propagate from one local network to the interacting local networks
- The coordination strategy to be used during the soft switching process
- The effect of change of control strategy by a distributed RFMPC controller
- The duration of the soft switching process and how robust feasibility is guaranteed in the local network other interacting local networks
- What initiates the soft switching process and terminates it

5.1.1 Soft Switching system components

The soft switching system is made up of different components and they are:

- fault management unit
- supervisory control unit
- switching box
- Softly switched RFMPC controllers

The soft switching system is illustrated in Figure 5.1.

In this thesis, the soft switching system is distributed, that is each subnetwork of the overall network has its own soft switching system.

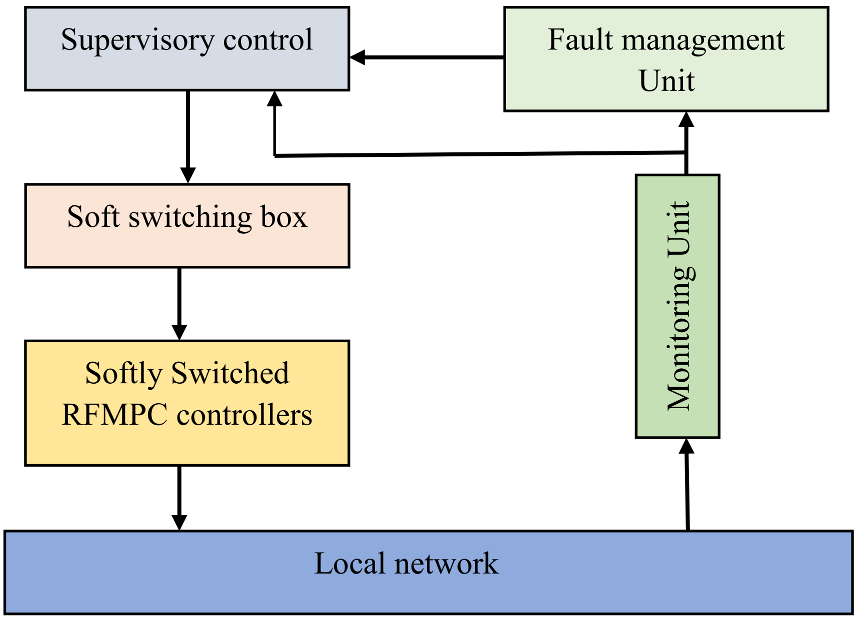

Figure 5.1 Soft switching system components

In Figure 5.1, the monitoring unit delivers all measurement information to the fault management unit and the supervisory control. The fault management unit consists of the fault detection and isolation system which access the faults and disturbances in the local network based on the monitoring information. The fault management unit delivers the information on the faults in the network to the supervisory control. The supervisory control uses the information from the fault management unit and the monitoring unit to determine the operational state of the local network. The supervisory control selects the most suitable RFMPC controllers and activates the soft switching box to execute the soft switching process.

The question to be answered is what makes a specific RFMPC controller suitable for a specific operational state.

1) The RFMPC must have the suitable model for the RFMPC to use for prediction of the future behaviour of the plant at this operational state. This implies that different models must be used for different operational states of the plant.
2) The RFMPC must have the suitable control strategy which is defined by the objective function of the RFMPC controller to achieve the possible control objective at this operational state. It is worth noting that some operational states may require a new control strategy different from the old RFMPC to achieve new control objectives if there is no possibility of achieving the old control objectives.
3) The RFMPC must have the suitable constraints formulation for the operational state and satisfy the system's constraints in computing its control actions.
4) The RFMPC must guaranty certain properties such as robust feasibility and stability of the plant at this operational state.

5.2 Soft Switching of Distributed Robustly Feasible Model Predictive Controllers

Large-scale systems control is usually addressed in multi-agent distributed MPC framework. The entire large-scale system is decomposed or partitioned into interacting subnetworks and each subnetwork or local network is controlled by an MPC or RFMPC. For large-scale systems incorporating distributed soft switching of RFMPC controllers, different scenarios of soft switching procedures are proposed in this thesis.

In presenting these soft switching scenarios for distributed RFMPC controllers, two interacting local networks are used as examples. The interaction between the local networks may be through the flow of materials and energy or through inputs or states. Strong interactions between the local networks imply there are interaction or shared variables between the local networks and cannot be neglected when the control actions for the local networks is computed by the RFMPC controllers. The scenarios present soft switching of distributed RFMPC controllers at different operational states of each of the local network. We consider Local network 1 and Local network 2 as interacting subnetworks of an overall network.

Soft switching of distributed RFMPC controllers Scenario 1: Local network 1 is in normal operational state and Local network 2 is in the normal operational state. There is no soft switching because there is no change of operational state in the local networks. The coordination strategy is:

1. At k, Local network 1 and Local network 2 DRFMPC controllers receive the local full state measurements. The state information received at k is

$$X_1(k) = ({X_1}^{local}, {X_1}^{normal}, {X_2}^{local}, {X_2}^{normal})$$

$$X_2(k) = ({X_2}^{local}, {X_2}^{normal}, {X_1}^{local}, {X_1}^{normal})$$

Where $X_1(k)$ is the full state measurements received by local network 1 RFMPC at time instant k, and $X_2(k)$ is the full state measurements received by local network 2 RFMPC at time instant k, $X_1{}^{local}$ is the local state measurements of local network 1, $X_1{}^{normal}$ is the normal operational state of the local network 1, $X_2{}^{local}$ is the local state measurements of local network 2, $X_2{}^{normal}$ is the normal operational state of the local network 2.

2. Local network 1 RFMPC controller evaluates its own future input trajectory based on $X_1(k)$, $G_{1,n}(k)$, $J_{1,n}(k)$ and the latest received input trajectories of Local network 2 RFMPC controller at iteration c = 1; Local network 2 RFMPC controller evaluates its own future input trajectory based on $X_2(k)$, $G_{2,n}(k)$, $J_{2,n}(k)$ and the latest received input trajectories of Local network 1 RFMPC controller at iteration c = 1, where c is the iteration number, $G_{1,n}(k)$ is the normal operational state model of local network 1, $J_{1,n}(k)$ is the control strategy and objective function for local network 1 normal operational state, $G_{2,n}(k)$ is the normal operational state model of local network 2 and $J_{2,n}(k)$ is the control strategy and objective function for local network 2 normal operational state

3. At iteration (c ≥1): the local network controllers RFMPC 1 and RFMPC 2 exchange their future input trajectories $u_{1,f}(k+j|k)$ and $u_{2,f}(k+j|k)$. RFMPC 1 controller calculates the current set of inputs trajectories $u_{1,c}(k+j|k)$ based on the received future input trajectories $u_{2,f}(k+j|k)$ from RFMPC 2 controller and RFMPC 2 controller calculates the current set of inputs trajectories $u_{2,c}(k+j|k)$ based on the received future input trajectories $u_{1,f}(k+j|k)$ from RFMPC 1 controller.

4. If a convergence condition is satisfied, the RFMPC1 controller sends its entire future input trajectory to control local network 1 actuators and the RFMPC2 controller sends its entire future input trajectory to control local network 2 actuators or if the convergence condition is not satisfied, go to Step 3 at iteration (c←c + 1).

5. When a new measurement is received, go to Step 1 (k←k + 1).

Soft switching of distributed RFMPC controllers Scenario 2: Local network 1 is in normal operational state and Local network 2 is in disturbed operational state. There is soft switching in local network 2 because there is a change of operational state in the local network 2 from normal to disturbed operational state. The coordination strategy is:

1. At k, Local network 1 and Local network 2 DRFMPC controllers receive the local full state measurements. The state information received at k is

$$X_1(k) = (\, {X_1}^{local}, {X_1}^{normal}, {X_2}^{local}, {X_2}^{disturbed})$$

$$X_2(k) = (\, {X_2}^{local}, {X_2}^{disturbed}, {X_1}^{local}, {X_1}^{normal})$$

Where $X_1(k)$ is the full state measurements received by local network 1 RFMPC at time instant k, and $X_2(k)$ is the full state measurements received by local network 2 RFMPC at time instant k, ${X_1}^{local}$ is the local state measurements of local network 1, ${X_1}^{normal}$ is the normal operational state of the local network 1, ${X_2}^{local}$ is the local state measurements of local network 2, ${X_2}^{disturbed}$ is the disturbed operational state of the local network 2.

2. Local network 1 RFMPC controller evaluates its own future input trajectory based on $X_1(k)$, $G_{1,n}(k)$, $J_{1,n}(k)$ and the predicted input trajectories of Local network 2 RFMPC controller based on available disturbed operational state model and objective function $G_{2,d}(k)$,$J_{2,d}(k)$ at iteration c = 1; Local network 2 RFMPC controller evaluates its own future input trajectory based on $X_2(k)$, $G_{2,d}(k)$, $J_{2,d}(k)$ and the latest received input trajectories of Local network 1 RFMPC controller at iteration c = 1, where c is the iteration number, $G_{1,n}(k)$ is the normal operational state model of local network 1, $J_{1,n}(k)$ is the control strategy and objective function for local network 1 normal operational state, $G_{2,d}(k)$ is the disturbed operational state model of local network 2 and $J_{2,d}(k)$ is the control strategy and objective function for local network 2 for the disturbed operational state

3. At iteration (c ≥ 1): the local network controllers RFMPC 1 and RFMPC 2 exchange their future input trajectories $u_{1,f}(k+j|k)$ and $u_{2,f}(k+j|k)$. RFMPC 1 controller calculates the current set of inputs trajectories $u_{1,c}(k+j|k)$ based on the received future input trajectories $u_{2,f}(k+j|k)$ from RFMPC 2 controller and RFMPC 2 controller calculates the current set of inputs trajectories $u_{2,c}(k+j|k)$ based on the received future input trajectories $u_{1,f}(k+j|k)$ from RFMPC 1 controller.

4. If a convergence condition is satisfied, the RFMPC1 controller sends its entire future input trajectory to control local network 1 actuators and the RFMPC2 controller sends its entire

future input trajectory to control local network 2 actuators or if the convergence condition is not satisfied, go to Step 3 at iteration (c←c + 1).

5. When a new measurement is received, go to Step 1 (k←k + 1).

Soft switching of distributed RFMPC controllers Scenario 3: Local network 1 is in normal operational state and Local network 2 is in the emergency operational state. There is soft switching in local network 2 because there is a change of operational state in the local network 2 from normal to emergency operational state. The coordination strategy is:

1. At *k*, Local network 1 and Local network 2 DRFMPC controllers receive the local full state measurements. The state information received at *k* is

$$X_1(k) = ({X_1}^{local}, {X_1}^{normal}, {X_2}^{local}, {X_2}^{emergency})$$

$$X_2(k) = ({X_2}^{local}, {X_2}^{emergency}, {X_1}^{local}, {X_1}^{normal})$$

Where $X_1(k)$ is the full state measurements received by local network 1 RFMPC at time instant *k*, and $X_2(k)$ is the full state measurements received by local network 2 RFMPC at time instant *k*, ${X_1}^{local}$ is the local state measurements of local network 1, ${X_1}^{normal}$ is the normal operational state of the local network 1, ${X_2}^{local}$ is the local state measurements of local network 2, ${X_2}^{emergency}$ is the disturbed operational state of the local network 2.

2. Local network 1 RFMPC controller evaluates its own future input trajectory based on $X_1(k)$, $G_{1,n}(k)$, $J_{1,n}(k)$ and the predicted input trajectories of Local network 2 RFMPC controller based on available emergency operational state model and objective function $G_{2,e}(k)$,$J_{2,e}(k)$ at iteration c = 1; Local network 2 RFMPC controller evaluates its own future input trajectory based on $X_2(k)$, $G_{2,e}(k)$, $J_{2,e}(k)$ and the latest received input trajectories of Local network 1 RFMPC controller at iteration c = 1, where c is the iteration number, $G_{1,n}(k)$ is the normal operational state model of local network 1, $J_{1,n}(k)$ is the control strategy and objective function for local network 1 normal operational state, $G_{2,e}(k)$ is the emergency operational state model of local network 2 and $J_{2,e}(k)$ is the control strategy and objective function for local network 2 for the emergency operational state

3. At iteration (c ≥1): the local network controllers RFMPC 1 and RFMPC 2 exchange their future input trajectories $u_{1,f}(k+j|k)$ and $u_{2,f}(k+j|k)$. RFMPC 1 controller calculates the current set of inputs trajectories $u_{1,c}(k+j|k)$ based on the received future input trajectories $u_{2,f}(k+j|k)$ from RFMPC 2 controller and RFMPC 2 controller calculates the current set of inputs trajectories $u_{2,c}(k+j|k)$ based on the received future input trajectories $u_{1,f}(k+j|k)$ from RFMPC 1 controller.

4. If a convergence condition is satisfied, the RFMPC1 controller sends its entire future input trajectory to control local network 1 actuators and the RFMPC2 controller sends its entire future input trajectory to control local network 2 actuators or if the convergence condition is not satisfied, go to Step 3 at iteration (c←c + 1).

5. When a new measurement is received, go to Step 1 (k←k + 1).

Soft switching of distributed RFMPC controllers Scenario 4: Local network 1 is in disturbed operational state and Local network 2 is in disturbed operational state. There is soft switching in local network 1 and local network 2 because there is a change of operational state in the local network 1 and local network 2 from normal to disturbed operational state. The coordination strategy is:

1. At k, Local network 1 and Local network 2 DRFMPC controllers receive the local full state measurements. The state information received at k is

$$X_1(k) = (X_1{}^{local}, X_1{}^{disturbed}, X_2{}^{local}, X_2{}^{disturbed})$$

$$X_2(k) = (X_2{}^{local}, X_2{}^{disturbed}, X_1{}^{local}, X_1{}^{disturbed})$$

Where $X_1(k)$ is the full state measurements received by local network 1 RFMPC at time instant k, and $X_2(k)$ is the full state measurements received by local network 2 RFMPC at time instant k, $X_1{}^{local}$ is the local state measurements of local network 1, $X_1{}^{disturbed}$ is the disturbed operational state of the local network 1, $X_2{}^{local}$ is the local state measurements of local network 2, $X_2{}^{disturbed}$ is the disturbed operational state of the local network 2.

2. Local network 1 RFMPC controller evaluates its own future input trajectory based on $X_1(k)$, $G_{1,d}(k)$, $J_{1,d}(k)$ and the predicted input trajectories of Local network 2 RFMPC controller based on available disturbed operational state model and objective function

$G_{2,d}(k), J_{2,d}(k)$ at iteration c = 1; Local network 2 RFMPC controller evaluates its own future input trajectory based on $X_2(k)$, $G_{2,d}(k)$, $J_{2,d}(k)$ and the predicted input trajectories of Local network 1 RFMPC controller based on $G_{1,d}(k)$, $J_{1,d}(k)$ at iteration c = 1, where c is the iteration number, $G_{1,d}(k)$ is the disturbed operational state model of local network 1, $J_{1,d}(k)$ is the control strategy and objective function for local network 1 disturbed operational state, $G_{2,d}(k)$ is the disturbed operational state model of local network 2 and $J_{2,d}(k)$ is the control strategy and objective function for local network 2 for the disturbed operational state

3. At iteration (c ≥1): the local network controllers RFMPC 1 and RFMPC 2 exchange their future input trajectories $u_{1,f}(k+j|k)$ and $u_{2,f}(k+j|k)$. RFMPC 1 controller calculates the current set of inputs trajectories $u_{1,c}(k+j|k)$ based on the received future input trajectories $u_{2,f}(k+j|k)$ from RFMPC 2 controller and RFMPC 2 controller calculates the current set of inputs trajectories $u_{2,c}(k+j|k)$ based on the received future input trajectories $u_{1,f}(k+j|k)$ from RFMPC 1 controller.

4. If a convergence condition is satisfied, the RFMPC1 controller sends its entire future input trajectory to control local network 1 actuators and the RFMPC2 controller sends its entire future input trajectory to control local network 2 actuators or if the convergence condition is not satisfied, go to Step 3 at iteration (c←c + 1).

5. When a new measurement is received, go to Step 1 (k←k + 1).

Soft switching of distributed RFMPC controllers Scenario 5: Local network 1 is in disturbed operational state and Local network 2 is in the emergency operational state. There is soft switching in local network 1 and local network 2 because there is a change of operational state in the local network 1 from normal to disturbed operational state and local network 2 from normal to emergency operational state. The coordination strategy is:

1. At k, Local network 1 and Local network 2 DRFMPC controllers receive the local full state measurements. The state information received at k is

$$X_1(k) = ({X_1}^{local}, {X_1}^{disturbed}, {X_2}^{local}, {X_2}^{emergency})$$

$$X_2(k) = ({X_2}^{local}, {X_2}^{emergency}, {X_1}^{local}, {X_1}^{disturbed})$$

Where $X_1(k)$ is the full state measurements received by local network 1 RFMPC at time instant k, and $X_2(k)$ is the full state measurements received by local network 2 RFMPC at time instant k, ${X_1}^{local}$ is the local state measurements of local network 1, ${X_1}^{disturbed}$ is the disturbed operational state of the local network 1, ${X_2}^{local}$ is the local state measurements of local network 2, ${X_2}^{emergency}$ is the emergency operational state of the local network 2.

2. Local network 1 RFMPC controller evaluates its own future input trajectory based on $X_1(k)$, $G_{1,d}(k)$, $J_{1,d}(k)$ and the predicted input trajectories of Local network 2 RFMPC controller based on available emergency operational state model and objective function $G_{2,e}(k)$,$J_{2,e}(k)$ at iteration c = 1; Local network 2 RFMPC controller evaluates its own future input trajectory based on $X_2(k)$, $G_{2,e}(k)$, $J_{2,e}(k)$ and the predicted input trajectories

of Local network 1 RFMPC controller based on $G_{1,d}(k)$, $J_{1,d}(k)$ at iteration c = 1, where c is the iteration number, $G_{1,d}(k)$ is the disturbed operational state model of local network 1, $J_{1,d}(k)$ is the control strategy and objective function for local network 1 disturbed operational state, $G_{2,e}(k)$ is the disturbed operational state model of local network 2 and $J_{2,e}(k)$ is the control strategy and objective function for local network 2 for the emergency operational state

3. At iteration (c ≥1): the local network controllers RFMPC 1 and RFMPC 2 exchange their future input trajectories $u_{1,f}(k+j|k)$ and $u_{2,f}(k+j|k)$. RFMPC 1 controller calculates the current set of inputs trajectories $u_{1,c}(k+j|k)$ based on the received future input trajectories $u_{2,f}(k+j|k)$ from RFMPC 2 controller and RFMPC 2 controller calculates the current set of inputs trajectories $u_{2,c}(k+j|k)$ based on the received future input trajectories $u_{1,f}(k+j|k)$ from RFMPC 1 controller.

4. If a convergence condition is satisfied, the RFMPC1 controller sends its entire future input trajectory to control local network 1 actuators and the RFMPC2 controller sends its entire future input trajectory to control local network 2 actuators or if the convergence condition is not satisfied, go to Step 3 at iteration (c←c + 1).

5. When a new measurement is received, go to Step 1 (k←k + 1).

Soft switching of distributed RFMPC controllers Scenario 6: Local network 1 is in emergency operational state and Local network 2 is in the emergency operational state. There is soft switching in local network 1 and local network 2 because there is a change of operational state in the local network 1 from normal to emergency operational state and local network 2 from normal to emergency operational state. The coordination strategy is:

1. At k, Local network 1 and Local network 2 DRFMPC controllers receive the local full state measurements. The state information received at k is

$$X_1(k) = ({X_1}^{local}, {X_1}^{emergency}, {X_2}^{local}, {X_2}^{emergency})$$

$$X_2(k) = ({X_2}^{local}, {X_2}^{emergency}, {X_1}^{local}, {X_1}^{emergency})$$

Where $X_1(k)$ is the full state measurements received by local network 1 RFMPC at time instant k, and $X_2(k)$ is the full state measurements received by local network 2 RFMPC at time instant k, ${X_1}^{local}$ is the local state measurements of local network 1, ${X_1}^{emergency}$ is the emergency operational state of the local network 1, ${X_2}^{local}$ is the local state measurements of local network 2, ${X_2}^{emergency}$ is the emergency operational state of the local network 2.

2. Local network 1 RFMPC controller evaluates its own future input trajectory based on $X_1(k)$, $G_{1,e}(k)$, $J_{1,e}(k)$ and the predicted input trajectories of Local network 2 RFMPC controller based on available emergency operational state model and objective function $G_{2,e}(k)$,$J_{2,e}(k)$ at iteration c = 1; Local network 2 RFMPC controller evaluates its own

future input trajectory based on $X_2(k)$, $G_{2,e}(k)$, $J_{2,e}(k)$ and the predicted input trajectories of Local network 1 RFMPC controller based on $G_{1,e}(k)$, $J_{1,e}(k)$ at iteration c = 1, where c is the iteration number, $G_{1,e}(k)$ is the emergency operational state model of local network 1, $J_{1,e}(k)$ is the control strategy and objective function for local network 1 emergency operational state, $G_{2,e}(k)$ is the emergency operational state model of local network 2 and $J_{2,e}(k)$ is the control strategy and objective function for local network 2 for the emergency operational state

3. At iteration (c ≥ 1): the local network controllers RFMPC 1 and RFMPC 2 exchange their future input trajectories $u_{1,f}(k+j|k)$ and $u_{2,f}(k+j|k)$. RFMPC 1 controller calculates the current set of inputs trajectories $u_{1,c}(k+j|k)$ based on the received future input trajectories $u_{2,f}(k+j|k)$ from RFMPC 2 controller and RFMPC 2 controller calculates the current set of inputs trajectories $u_{2,c}(k+j|k)$ based on the received future input trajectories $u_{1,f}(k+j|k)$ from RFMPC 1 controller.

4. If a convergence condition is satisfied, the RFMPC1 controller sends its entire future input trajectory to control local network 1 actuators and the RFMPC2 controller sends its entire future input trajectory to control local network 2 actuators or if the convergence condition is not satisfied, go to Step 3 at iteration (c←c + 1).

5. When a new measurement is received, go to Step 1 (k←k + 1).

The coordination strategy for all the distributed RFMPC soft switching scenarios is verified through simulation study in chapter 6 of this thesis.

5.3 Soft switching analysis

Consider a system

$$x(k+1) = f\big(x(k), u(k), w(k)\big) \tag{5.1}$$

$$y(k) = h\,(x(k))$$

With $f : \mathbb{R}^n \times \mathbb{R}^l \times \mathbb{R}^p \to \mathbb{R}^q$, $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^l$, $y(k) \in \mathbb{R}^m$, $w(k) \in \mathbb{R}^p$, $x(k)$ denotes the state of the system, $u(k)$ denotes the control input, $y(k)$ denotes the plant output and $w(k)$ is an unknown disturbance. The system constraints are convex sets containing the origin in their interior:

$x(k) \in \mathbb{X}$, $u(k) \in \mathbb{U}$, $y(k) \in \mathbb{Y}$, $w(k) \in \mathbb{W}$; Assuming the output and control prediction horizon is the same

$$H_p = H_c$$

Let $\Gamma_{.|k} = [\, u_{.|k}{}^T, x_{.|k}{}^T, y_{.|k}{}^T \,]$ be the set of all decision variables where

$$u_{.|k} = [\, u(k) \,.\,.\,.\, u\big(k + H_p - 1\big)]$$

$$x_{.|k} = [\, x(k) \,.\,.\,.\, x\big(k + H_p - 1\big)]$$

$$y_{.|k} = [\, y(k) \,.\,.\,.\, y\big(k + H_p - 1\big)]$$

Let the objective function be:

$$J(\Gamma_{.|k}, x(k+H_p|k)) \tag{5.2}$$

$\mathbb{X}_{Rf,S}(k)$ is the robustly feasible initial states at each time instant k

Consider the finite horizon RFMPC optimization task at time instant k:

$$min_{\Gamma_{.|k}, x(k+H_p|k)} J(\Gamma_{.|k}, x(k+H_p|k)) \tag{5.3}$$

Subject to: $x(k|k) = x(k) \in \mathbb{X}_{Rf,S}$

$$F(\Gamma(k+i|k),\ w(k+i|k)) = 0$$

$$x(k+1+i|k) = f(x(k+i|k), \Gamma(k+i|k), w(k+i|k))$$

$$x(k+i|k) + \mathcal{E}^{min} \ll x(k+1+i|k) \ll x(k+i|k) - \mathcal{E}^{max}$$

$$x(k+1+i|k) \in \mathbb{X}_{Rf,S}, i = \overline{0{:}H_p}$$

$$u(k+1+i|k) \in \mathbb{U}, i = \overline{0{:}H_p}$$

$$y(k+1+i|k) \in \mathbb{Y}, i = \overline{0{:}H_p}$$

Where $\mathcal{E} = [\mathcal{E}^{min}, \mathcal{E}^{max}]$ denotes the safety zones.

Let $J(\Gamma_{.|k}, x(k+H_p|k)) = (Y^{ref} - Y^{out})^T(Y^{ref} - Y^{out}) + \Delta u^T \Delta u$ (5.4)

Where Y^{ref} = Reference output; Y^{out} is the real plant output and it is predicted using the model of the plant; Δu is the incremental input that is used to steer the system to the desired state.

For a plant where the state is the output of the system:

Let $u(k|k) = u(k) + \Delta u(k|k)$ (5.5)

Where $u(k|k)$ is the input value at time instant *k*; $u(k)$ is the initial value of input at time instant *k*; and $\Delta u(k|k)$ is the calculated incremental input at time instant *k*.

Let $Y^{out}(k|k) = y(k) + \Delta y(k|k)$ (5.6)

Where $Y^{out}(k|k)$ is the predicted plant output at time instant *k*; $y(k)$ is the initial value of output at time instant *k*; and $\Delta y(k|k)$ is the calculated incremental output at time instant *k*.

$$\Delta y(k|k) = A_{model}(k) \times \Delta u(k|k) \quad (5.7)$$

$$y(k) = A_{model}(k) \times u(k|k) \quad (5.8)$$

$$Y^{out}(k|k) = A_{model}(k) \times u(k) + A_{model}(k) \times \Delta u(k|k) \quad (5.9)$$

Where $A_{model}(k)$ is the time-varying model of the plant.

Equation (5.4) can now be written as:

$$J\left(\Gamma_{.|k}, y\left(k + H_p|k\right)\right) = (Y^{ref} - Y^{out})^T(Y^{ref} - Y^{out}) + \Delta u^T \Delta u \quad (5.10)$$

$$min_{\Gamma_{.|k}, y(k+H_p|k)} J\left(\Gamma_{.|k}, y\left(k + H_p|k\right)\right)$$

$$= \Delta u^T(A_{model}(k)^T A_{model}(k) + I)\,\Delta u + 2A_{model}(k)^T \Delta u^T(Y^{out}(k|k) - Y^{ref})$$

Where *I* am the identity matrix; $A_{model}(k)$ is the time-varying prediction model of the plant and is determined by the operational state of the plant and Y^{ref} is the reference output.

Let the time instant of change of operational state of the plant be *k*, the approach in the adaptive cooperative strategy is to use the last values of the agreed control trajectories of the other RFMPC controllers at this time instant while the fault management system and the supervisory control of the subnetwork determines the type of operational state that the plant is in and the type of switching required (hard or soft). At time instant $(k+1)$, for soft switching process, a convex combination of the old RFMPC and the new RFMPC is performed [24] [98] over a switching time from $\overline{(k+1)\colon(k+1+T_s)}$ and the generated control trajectories are used by the other DRFMPC controllers all through the soft switching duration. Where T_s is the soft switching time duration.

Let the old RFMPC Objective function be $J_{1,Normal}$ for normal operational state and the new RFMPC Objective function be $J_{2,Disturbed}$ for disturbed operational state.

The convex combination of $J_{1,Normal}$ RFMPC and $J_{2,Disturbed}$ RFMPC is given as:

$$J_{combined} = (1 - \omega(k))\, J_{1,Normal} \quad + \quad (\omega\,(k))\; J_{2,Disturbed} \tag{5.11}$$

Where $\omega(k)$ is a time varying scalar and a weighting factor used in the convex combination of the RFMPC during the soft switching process. It changes from 0 to 1 as time increases. At $(k+1+T_s)$ the soft switching stops and the new RFMPC is now fully engaged.

In this thesis, two scenarios of soft-switching are considered as follows:

Scenario 1: Let the objective function $J_{1,Normal}$ and $J_{2,Disturbed}$, the constraints and the reference output be the same, but the prediction models used are different:

$$Y_N{}^{out}(k|k) = A_{model,N}(k) \times u(k) + A_{model,N}(k) \times \Delta u(k|k) \tag{5.12}$$

Where $Y_N{}^{out}(k|k)$ is the Normal operational state plant output and $A_{model,N}(k)$ is the Normal operational state prediction model

$$Y_D{}^{out}(k|k) = A_{model,D}(k) \times u(k) + A_{model,D}(k) \times \Delta u(k|k) \tag{5.13}$$

Where ${Y_D}^{out}(k|k)$ is the Disturbed operational state plant output and $A_{model,D}(k)$ is the Disturbed operational state prediction model

$$min_{\Gamma_{.|k}, y(k+H_p|k)} J_{combined}\left(\Gamma_{.|k}, y\left(k + H_p|k\right) \right) \tag{5.14}$$

$$= \Delta u^T\big(\omega(k)A_{model,D}(k)^T A_{model,D}(k) + A_{model,N}(k)^T A_{model,N}(k) - \omega(k)A_{model,N}(k)^T A_{model,N}(k) + I\big)\,\Delta u + 2\Delta u^T(\omega(k)A_{model,D}(k)^T {Y_D}^{out}(k|k) - \omega(k)A_{model,D}(k)^T Y^{ref} - \omega(k)A_{model,N}(k)^T {Y_N}^{out}(k|k) - \omega(k)A_{model,N}(k)^T Y^{ref} + A_{model,N}(k)^T {Y_N}^{out}(k|k) - A_{model,N}(k)^T Y^{ref})$$

Using quadratic programming;

$$min_{\Gamma_{.|k}, y(k+H_p|k)} J_{combined}\left(\Gamma_{.|k}, y\left(k + H_p|k\right) \right) = 0.5\, X^T H\, X + f^T X$$

$$\mathrm{H} = 2\big(\omega(k)A_{model,D}(k)^T A_{model,D}(k) + A_{model,N}(k)^T A_{model,N}(k) - \omega(k)A_{model,N}(k)^T A_{model,N}(k) + I\big)$$

$$\begin{aligned} f^T = 2\Delta u^T(\omega(k)A_{model,D}(k)^T {Y_D}^{out}(k|k) &- \omega(k)A_{model,D}(k)^T Y^{ref} \\ &- \omega(k)A_{model,N}(k)^T {Y_N}^{out}(k|k) - \omega(k)A_{model,N}(k)^T Y^{ref} \\ &+ A_{model,N}(k)^T {Y_N}^{out}(k|k) - A_{model,N}(k)^T Y^{ref}) \end{aligned}$$

Where $A_{model,N}(k)$ is the time varying prediction model for the Normal operational state of the plant; $A_{model,D}(k)$ is the time varying prediction model for the Disturbed operational state of the plant, ${Y_N}^{out}(k|k)$ is the predicted output at time instant k of the plant at Normal operational state and ${Y_D}^{out}(k|k)$ is the predicted output at time instant k of the plant at Disturbed operational state.

The approach in Equation (5.11) and (5.14) can be used for soft switching from disturbed operational state to emergency operational state, disturbed to normal operational state and emergency to normal operational state.

Scenario 2: Let the objective function $J_{1,Normal}$ and $J_{2,Disturbed}$, the reference output be the same, but the prediction models used, and the constraints are different:

(5.14) still holds but the constraints are modified by:

$$C_{combined} = (1 - \omega(k))\, C_{1,Normal} \quad + \quad (\omega\,(k))\;\; C_{2,Disturbed} \tag{5.15}$$

Where $C_{combined}$ is the combined constraints of the Normal and disturbed operational states, $C_{1,Normal}$ is the constraints expression for the Normal operational state of the plant and $C_{2,Disturbed}$ is the constraints expression for the Disturbed operational state of the plant.

The algorithm for the soft switching is as follows:

Algorithm 5.1: Soft switching

1) At time instant k when there is a change of operational state; the monitoring unit sends the information to the supervisory control and the fault management unit. The new operational state is determined. The last agreed values of the optimal control trajectories are exchanged with other interacting DRFMPC controllers
2) At time instant $(k+1)$; the type of switching is determined by the supervisory control. For hard switching, the new RFMPC for the operational state is engaged immediately $\omega(k)$ is set to 1 in Equation (5.11). For soft switching, different values of $\omega(k)$ is used where $0 \le \omega(k) \le 1$ in Equation (5.10) and (5.11)
3) For smooth and fast soft switching; T_s is chosen to be 2 and 4 control time steps and

$\omega(k)$ values are evenly increased over the switching duration from $(k+1):(k+1+T_s)$

Equations (5.10) and (5.11) is adapted to be used to calculate the feasible control inputs over the switching duration from $\overline{(k+1):(k+1+T_s)}$ and the optimal control trajectories generated is exchanged with other DRMPCs during the soft switching duration

4) At $(k+1+T_s)$; the new RFMPC for the new operational state is now fully engaged.
5) If any of the interacting DRFMPC is changing its operational state during the soft switching process of the DRFMPC under consideration, steps 1 to step 4 is also repeated for the interacting DRFMPC.

The soft switching presented here is part of the proposed adaptive cooperative strategy for DRFMPC earlier presented in Chapter 4 and section 5.3 of this chapter. The soft switching application is illustrated in Chapter 6 of this thesis.

5.4 Summary

The soft switching for RFMPC and DRFMPC has been presented. The soft switching system components and functionalities have been presented. Adaptive cooperative strategy for DRFMPC under different operational states and reconfiguration has been presented. Different scenarios of adaptive cooperation during soft switching of DRFMPC controllers was proposed. Soft switching analysis was presented for switching from one RFMPC to another and the conditions for the operations presented and discussed. The reconfiguration of the RFMPC and the adaptive cooperation of the DRFMPC was proposed and the algorithm for its implementation presented.

Chapter 6

Application to Water quality control of DWDS

In this Chapter, Distributed Softly Switched Robustly Feasible Model Predictive Control application to drinking water distribution system water quality control is presented. In section 6.1, the simulation environment is presented and discussed. The benchmark DWDS water quality control by the distributed softly switched robustly feasible MPC (DSSRFMPC) is presented and the simulation results discussed in section 6.2 In section 6.3, the RFMPC design is presented. In section 6.4, the DSSRFMPC and soft switching applications is presented. In section 6.5, the summary of the chapter is presented.

6.1 Simulation Environment Setup

6.1.1 EPANET

EPANET software package was published in the year 2000 by the National Risk Management Research Laboratory of United States Environment Protection Agency (USEPA) [56].

EPANET is widely used in the simulation of the water network. EPANET is a computer program that performs an extended period simulation of hydraulic and water quality behaviour within pressurized pipe networks. EPANET tracks the flow of water in each pipe, the pressure at each node, the height of water in each tank, and the concentration of a chemical species throughout the network during a simulation period comprised of multiple time steps. EPANET is designed to be a research tool for improving our understanding of the movement and fate of drinking water constituents within distribution systems. It is packaged in two parts: as graphic interface and as algorithm part. The algorithm part is embedded into other applications by inserting the source codes of the simulation algorithms of hydraulics and quality directly [31]. The graphic interface is used to construct the water distribution network, calibrate and input the coefficients of the network, run the simulation and obtain the result data in Microsoft windows platform. The network files are stored as flat files and can be accessed by other applications such as MATLAB.

6.1.2 EPANET-MATLAB Toolkit

The EPANET-Matlab Toolkit is an open-source software, originally developed by the KIOS Research Centre for Intelligent Systems and Networks of the University of Cyprus which operates within the Matlab environment, for providing a programming interface for the latest version of EPANET, a hydraulic and quality modeling software created by the US EPA, with Matlab, a high-level technical computing software. The goal of the EPANET Matlab Toolkit is to serve as a common programming framework for research and development in the growing field of smart water networks. The EPANET-Matlab Toolkit features easy to use commands/wrappers for viewing, modifying, simulating and plotting results produced by the EPANET libraries [136]. The EPANET-Matlab Toolkit is used in this thesis as the EPANET-MATLAB interface to provide the hydraulic and quality data from the benchmark DWDS used in this thesis.

6.1.3 MATLAB and Quadratic Programming

In this thesis, MATLAB codes were written to calculate the chlorine transportation time, the chlorine path analysis, forward tracking, execute experiments, implement parameter estimation and the DRFMPC design. The quadratic programming tool is used for parameter estimation and RFMPC design. The RFMPC, SSRFMPC and DSSRFMPC were designed using MATLAB Codes written to implement the controller actions.

6.1.4 Hardware and Software Specification

The hardware and software specification of the simulation platform are listed as follows:

Microsoft Windows:		Windows 7 Professional
CPU	:	Intel ® core ™ 2 Duo CPU 2.93GHZ, 2.93GHZ
Memory	:	4GB
System type	:	64-bit OS
Manufacturer	:	Driver Pack Solution
Model	:	Dell Optiplex780
MATLAB	:	R2014a
EPANET	:	version 2.0

6.2 Benchmark Drinking Water Distribution System

6.2.1 Benchmark DWDS

The benchmark DWDS used in this thesis is presented in Figure 6.1.

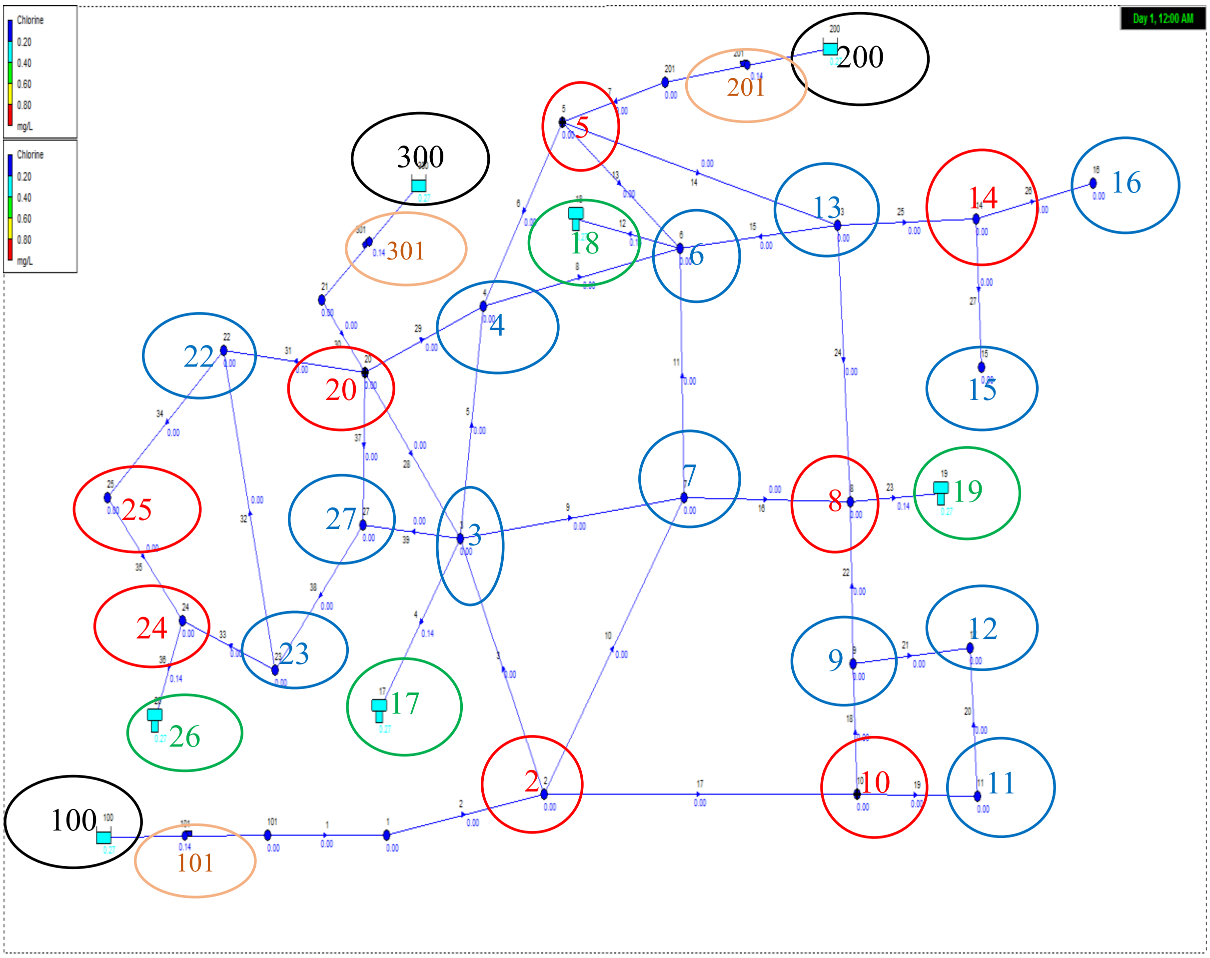

Figure 6.1 Benchmark DWDS used in this thesis

A benchmark drinking water distribution system was considered in this thesis for verifying the proposed methodologies. The network structure is illustrated in Figure 6.1 using EPANET. The benchmark DWDS is shown clearly in Figure 6.2.

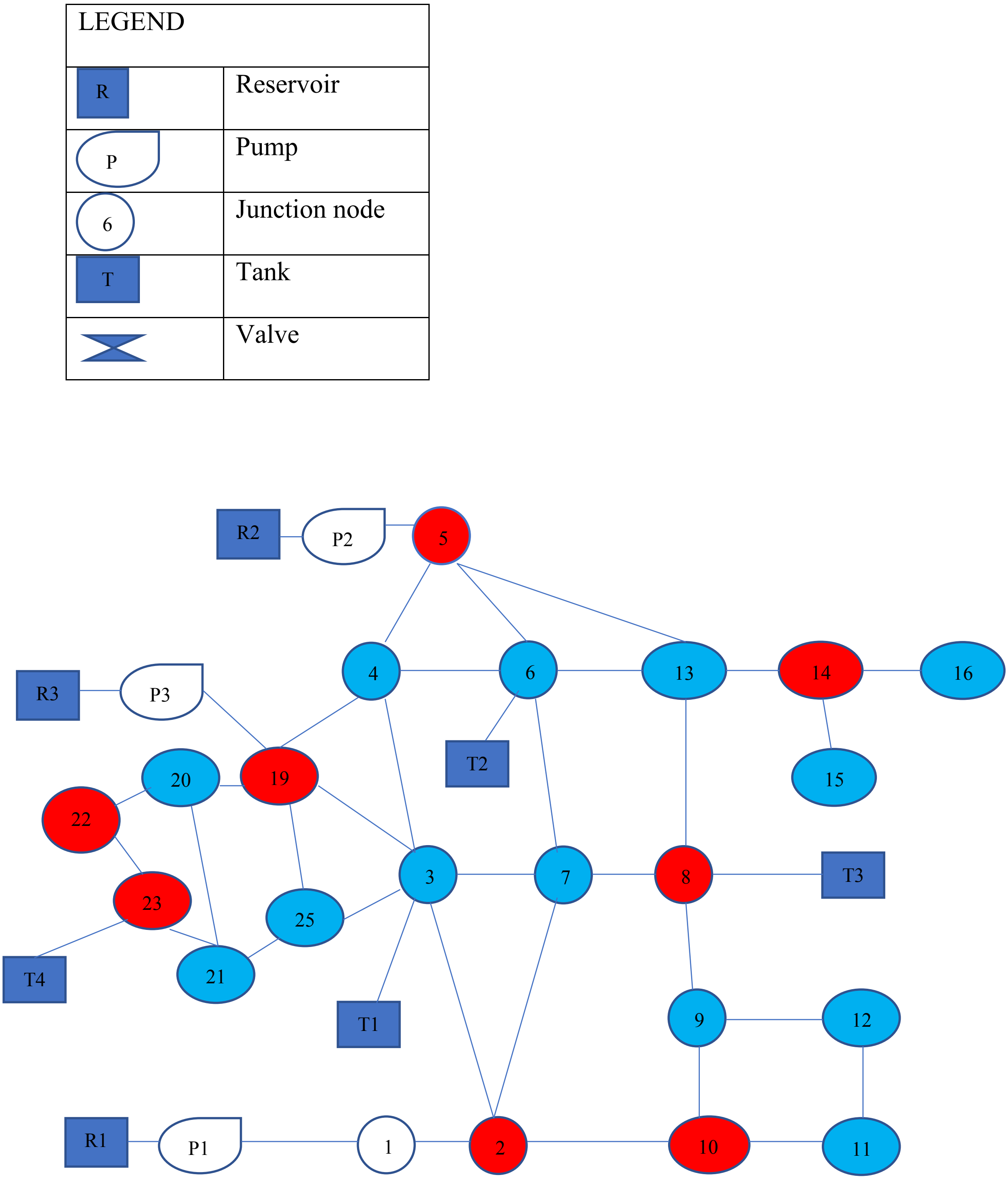

Figure 6.2 Benchmark DWDS with the injection nodes and monitored nodes

The network file is attached in Appendix *B*. There are 27 network nodes, 39 pipes, and 4 storage tanks in the system in Figure 6.1. Figure 6.2 is equivalent to Figure 6.1. The nodes are compared below:

Table 1.1 Comparing the nodes in Figure6.1 with nodes in Figure 6.2

Nodes in Figure 6.1	Nodes in Figure 6.2	Remarks
100	R1- Reservoir1	
200	R2- Reservoir2	
300	R3- Reservoir3	
101	P1-Pump1	
201	P2-Pump2	
301	P3-Pump3	
17	T1-Tank1	
18	T2-Tank2	
19	T3-Tank3	
26	T4-Tank4	
20	19	
22	20	
25	22	
24	23	
23	21	
27	25	

In Figure 6.2, the water is pumped from the water sources at node R1, node R2 and node R3 and pumped by three variable speed pumps; pump 1, pump 2 and pump 3. It is also supplied by the switching tanks, tank1, tank2, tank3 and tank4. Node 2, node 5, node 8, node 10, node 14, node 19, node 22 and node 23 are selected as chlorine injection nodes (with red circles). Nodes 3, 4, 6, 7, 9, 11, 12, 13, 15, 16, 20, 21 and 25 are selected as the chlorine monitored nodes (with blue circles). The Benchmark DWDS has 3 Regions. Each Region is partitioned into zones using the normal operational state of the DWDS and the chlorine controllability and observability presented in section 2.3.2.

Region 1 consists of three chlorine injection nodes (node 5, node 14 and node 8) as illustrated in Figure 6.3. Node 5 controls monitored nodes 4, 6 and 13. Node 14 controls monitored nodes 15 and 16. Node 8 controls node 7.

Region 2 consists of two chlorine injection nodes (node2 and node 10) as illustrated in Figure 6.4. Node 2 controls the monitored node 3 and node 7.

Node 10 controls the monitored node 11, node 9 and node 12. The monitored nodes in Region 2 are node 7, node 9, node 11 and node 12.

Region 3 consists of three chlorine injection nodes (node19, node 22 and node 23) as illustrated in Figure 6.5. Node 19 controls the monitored node 3, node 4, node 20 and node 25.

Node 22 controls the monitored node 22. Node 23 controls the monitored node 23 and node 23. The monitored nodes in Zone 3 are node 3, node 4, node 20, node 21 and node 25.

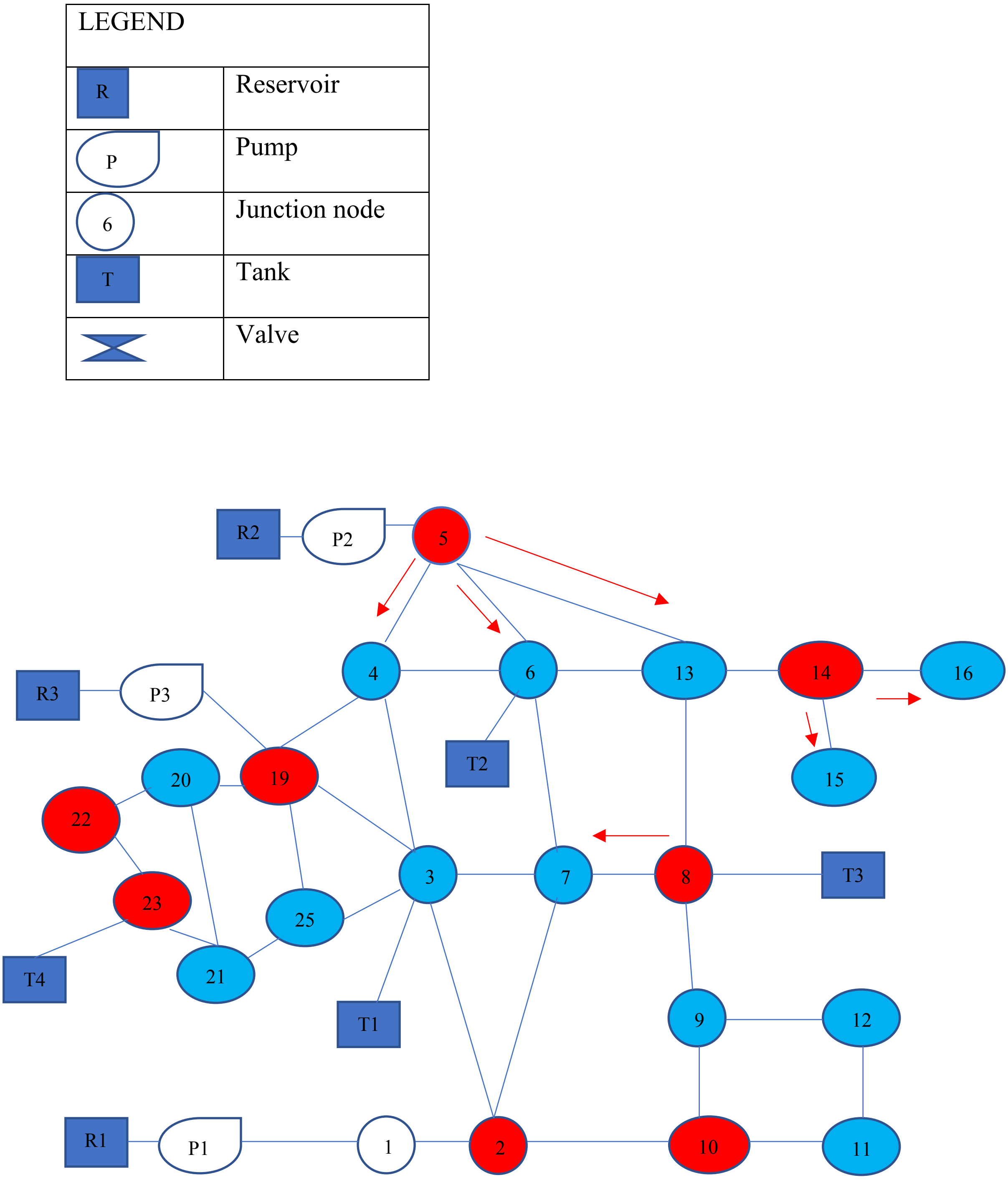

Figure 6.3 Benchmark DWDS Zone 1 consisting of node 5, node 8 and node 14 as chlorine injection nodes

LEGEND	
R	Reservoir
P	Pump
6	Junction node
T	Tank
	Valve

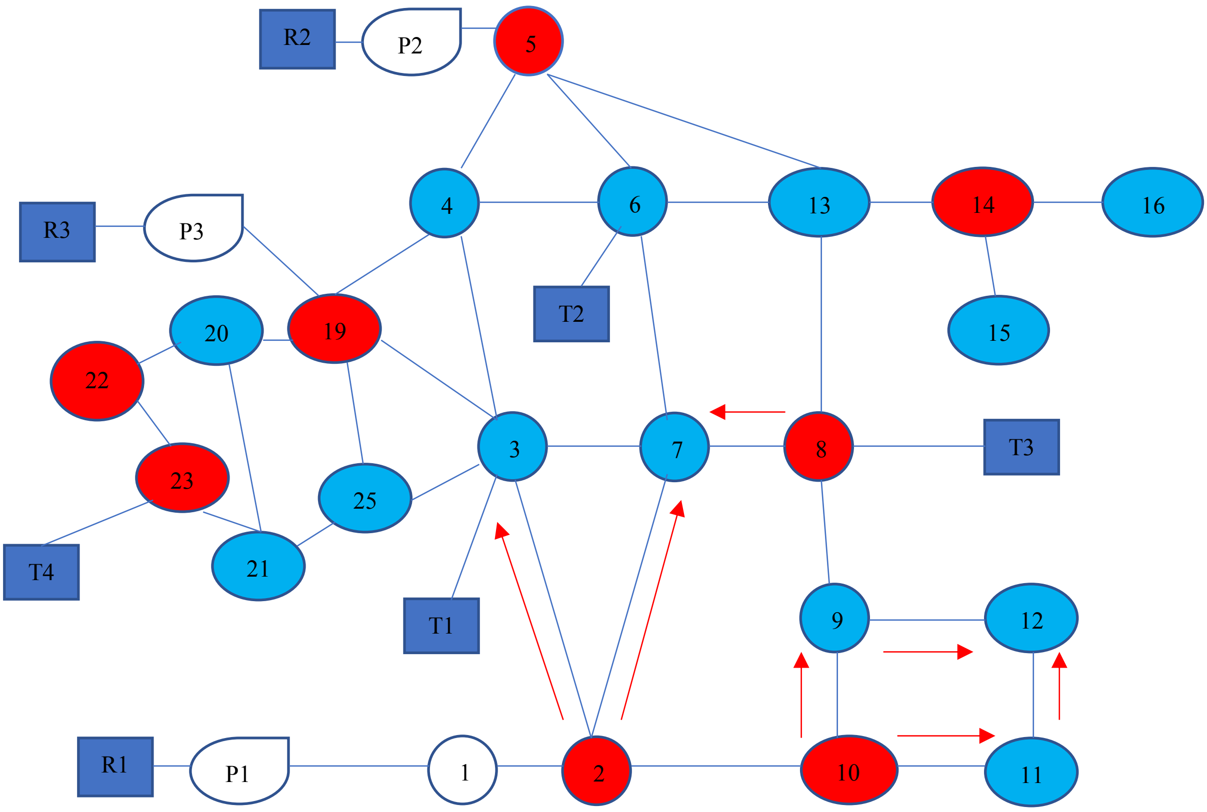

Figure 6.4 Benchmark DWDS Zone 2 consisting of node 2, node 8 and node 10 as chlorine injection nodes

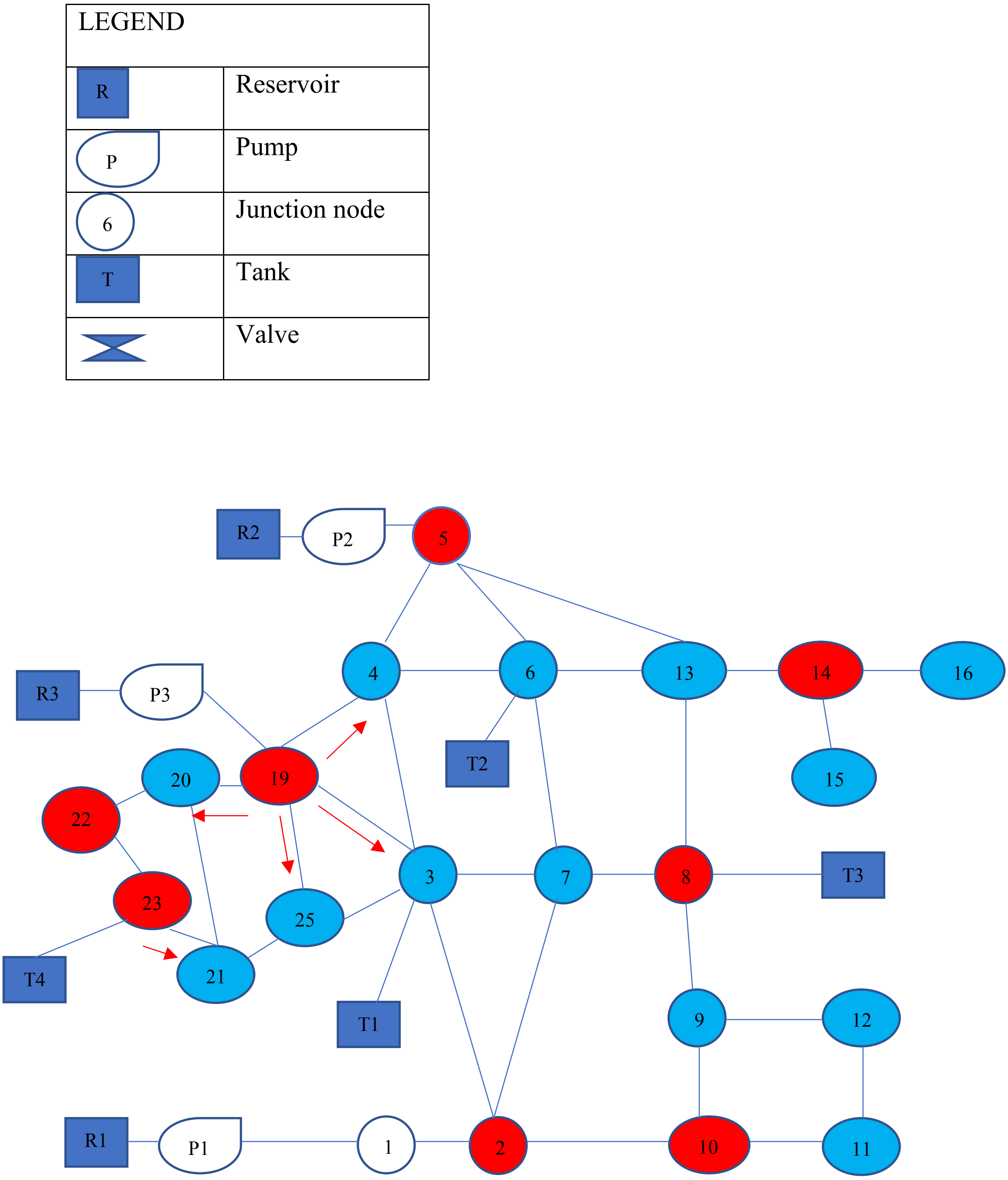

Figure 6.5 Benchmark DWDS Zone 3 consisting of node 19, node 22 and node 23 as chlorine injection nodes

6.2.2 DWDS hydraulic and quality time steps

The DWDS hydraulic time step used in this thesis is one hour (1 hour) and the quality time step is 5minutes. The DWDS hydraulics control is usually done on a 24hours prediction horizon. In this thesis, the hydraulics control at the upper level of the hierarchical control structure is done on a 24 hydraulic time steps and the quality control at the lower level of the hierarchical control structure is done with a 288 quality time steps.

The simulation of the DWDS quantity and quality control is done on a time horizon of 48hours or 576 quality time steps. The water head at node 5 and node 13 in the normal operational state is illustrated in Figure 6.6 and Figure 6.7 respectively.

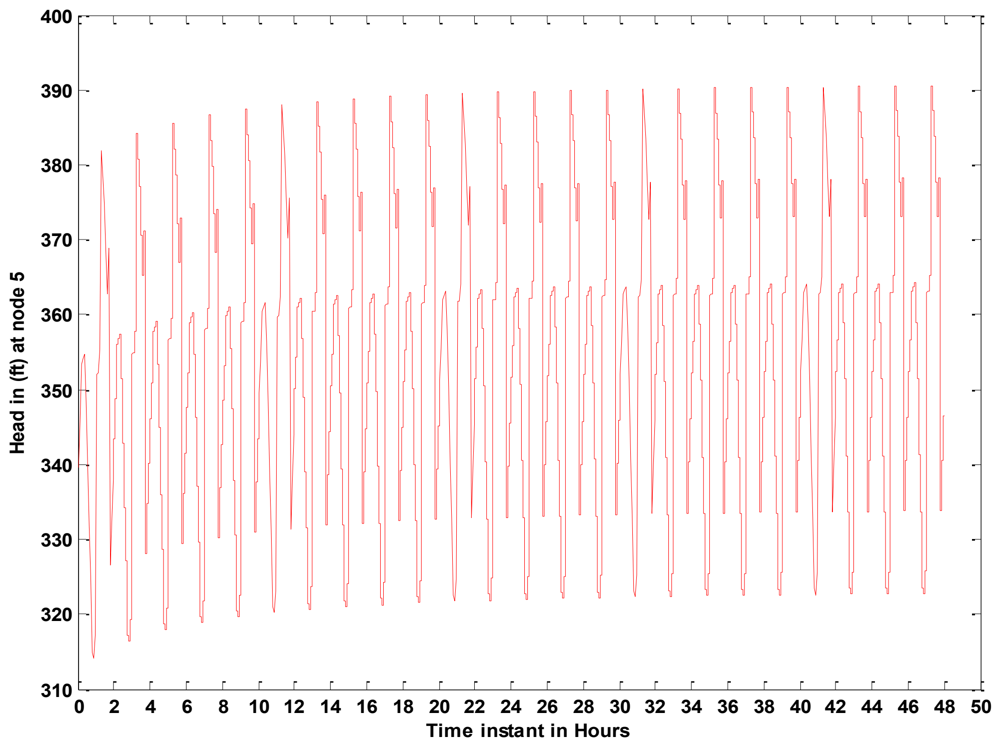

Figure 6.6 The water head (in feet) at node 5 in the normal operational state

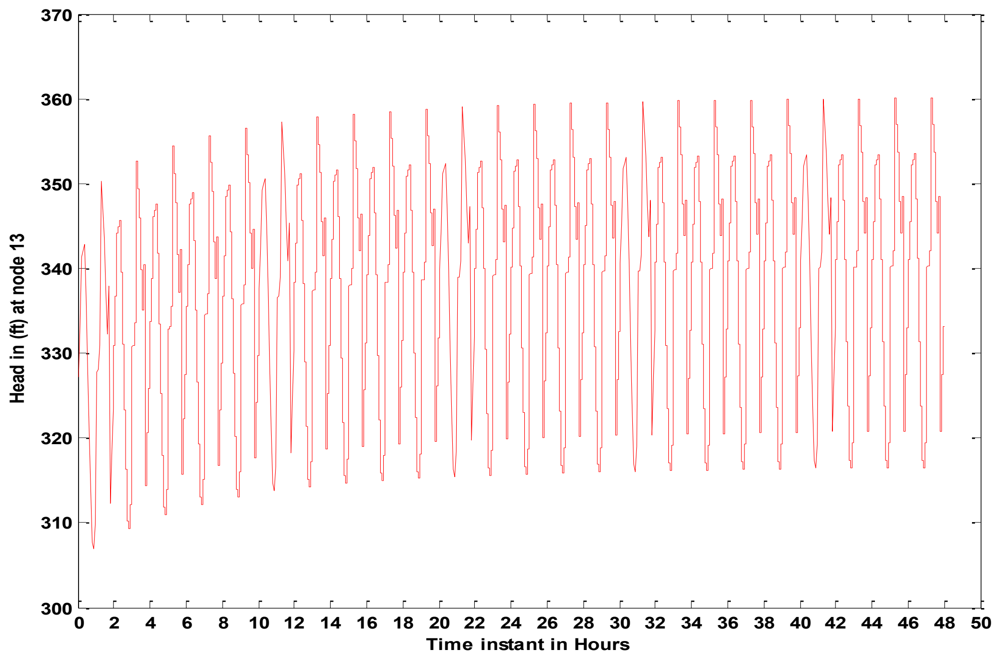

Figure 6.7 The water head (in feet) at node 13 in the normal operational state

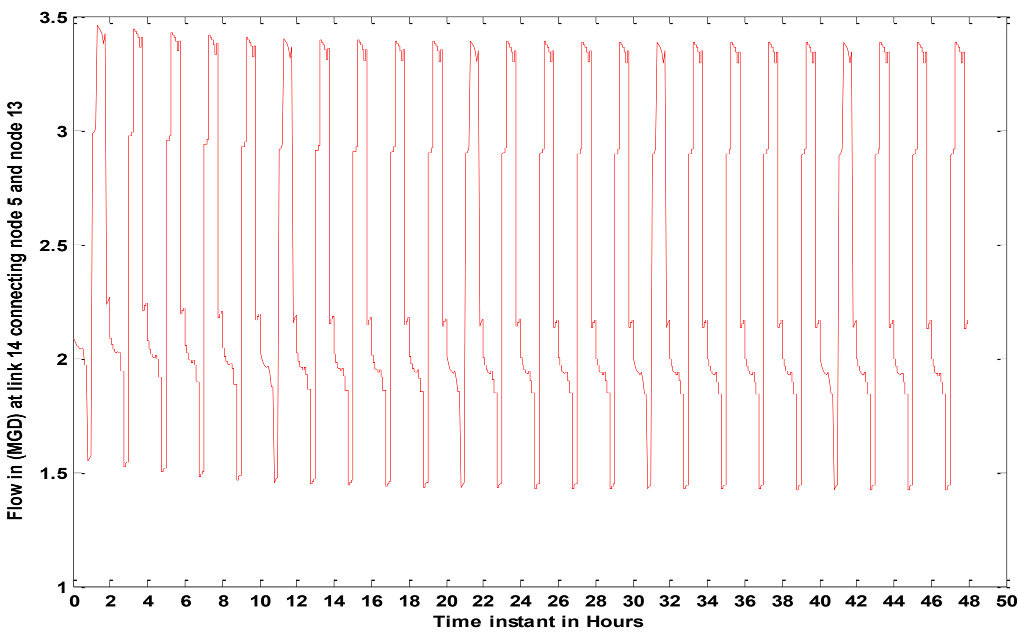

Figure 6.8 Flow in million gallons per day (MGD) at link 14 connecting node 5 and node 13

The water flow between node 5 and node 13 at hydraulic time step of I hour is illustrated in Figure 6.8.

The chlorine injections at node 5 at quality time steps of 5 minutes over the modeling horizon of 48 hours is illustrated in Figure 6.9

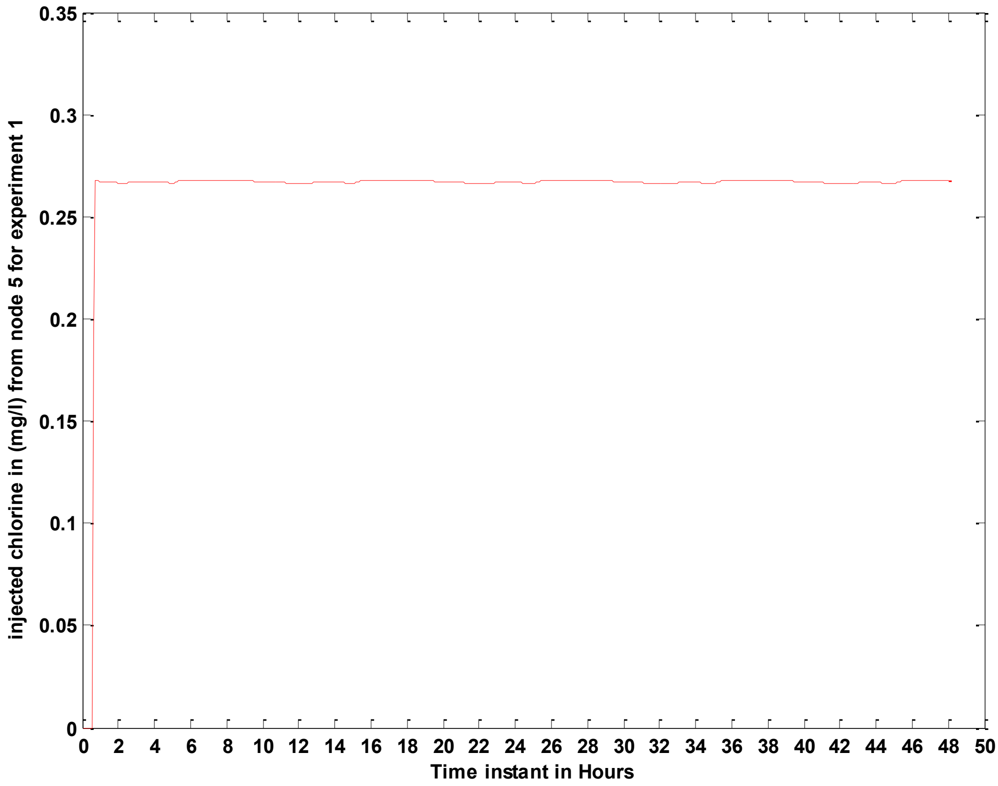

Figure 6.9 Chlorine injections at node 5 at quality time steps of 5 minutes over the modeling horizon of 48 hours

In Figure 6.8 and Figure 6.9, there is water flow from node 5 to node 13 at the normal operational state of the DWDS in Region 1. Node 5 is an injection node and can control node 13. If there is a pipe break of the link between node5 and node 13, node 13 may not be controllable by node 5. Different chlorine injection patterns as explained in Chapter 3 of the thesis were used for the simulation experiments and the chlorine residual values monitored at the monitored nodes. Figure 6.9 also illustrates one of the experiments at node5. The chlorine values monitored at node 13 is illustrated in Figure 6.10. The experiments were carried out for modeling and parameter estimation as presented in Chapter 3 of the thesis. The modeling horizon is 48 hours to ensure all the transient effects are out from [0,3] [hour] before the parameter estimation is carried out. The output prediction is carried out between [5,31][hour] and the control prediction is carried out between [4,31][hour].

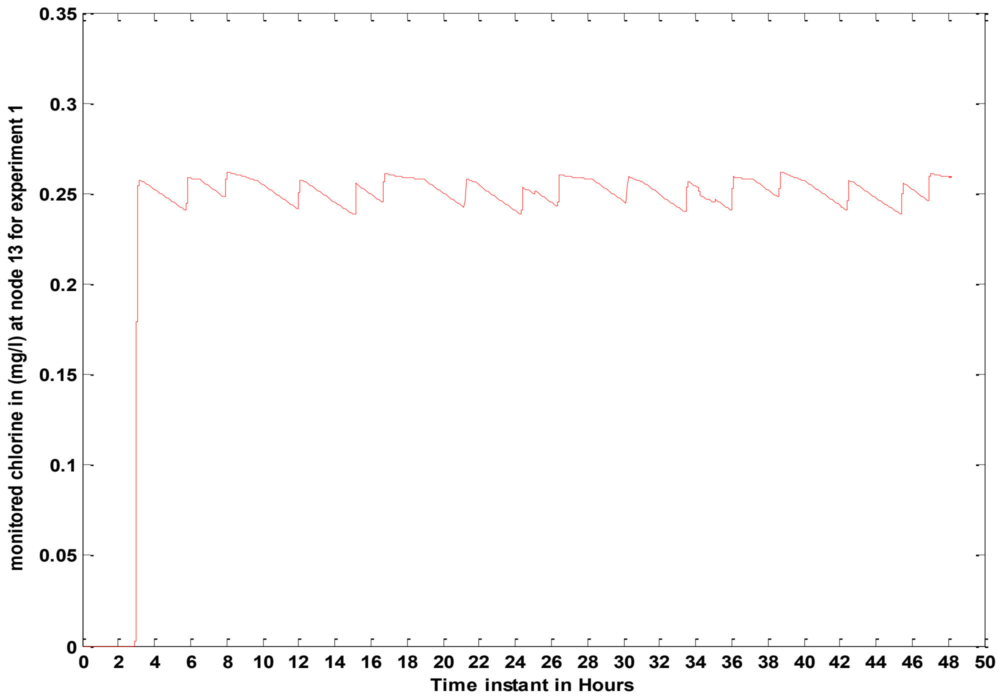

Figure 6.10 Chlorine residual values at node 13

6.2.3 Chlorine transport time in pipes

The calculation of chlorine transport time has been presented in Chapter 3, section 3.1.1 using the forward tracking algorithm. The modified Path analysis forward tracking algorithm described in Algorithm 3.0 is illustrated in Figure 6.11, Figure 6.12, Figure 6.13, Figure 6.14 and Figure 6.15 respectively. The calculation is done for the normal operational state of the plant. The length L, of pipe 14 connecting node 5 and node 13 is 4500ft and the tracking time τ is 1 minute. The water head at node 5 at [10,11] [hour] is shown in Figure 6.10, the water head at node 13 at [10,11] [hour] is shown in Figure 6.11. The time of impact of the chlorine injected at node 13 at [10,11] [hour] is shown in Figure 6.12. The flow velocity at pipe 14 at [10,11] [hour] is shown in Figure 6. 13 and the chlorine transport time from node 5 to node 13 at [10,11] [hour] is shown in Figure 6.14.

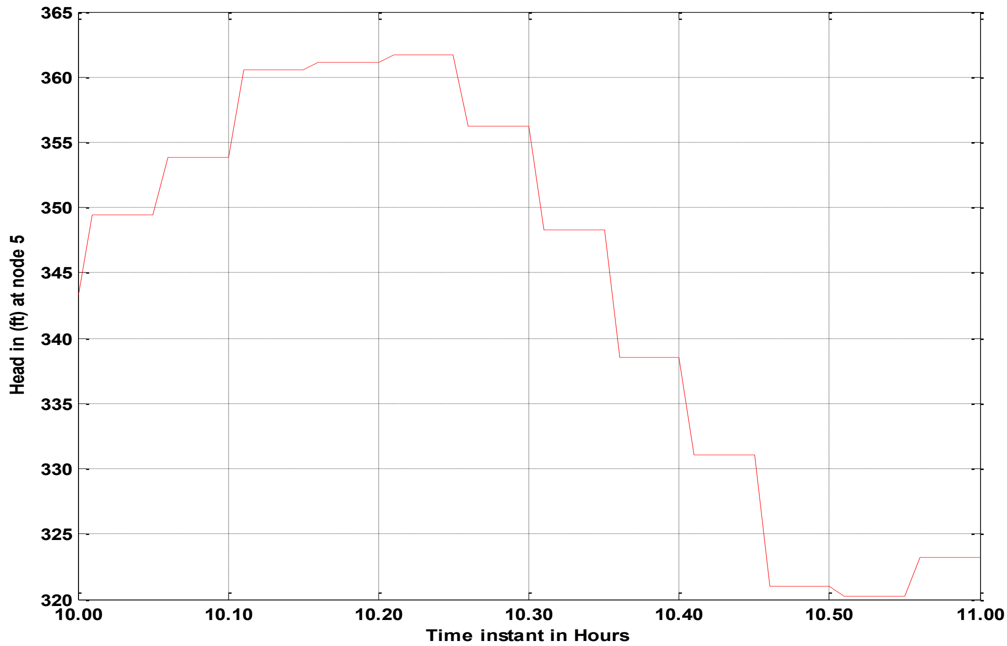

Figure 6.11 Water head at node 5

The water head dropped at node 5 and node 13 in Figure 6.11 and Figure 6.12 resulting in the drop in water flow velocity in Figure 6.14. There is continuous water flow from node 5 to node 13 and the injected chlorine from node 5 will impact node 13 as illustrated in Figure 6.13. The chlorine transportation time increased when the water flow velocity decreased as illustrated in Figure 6.15.

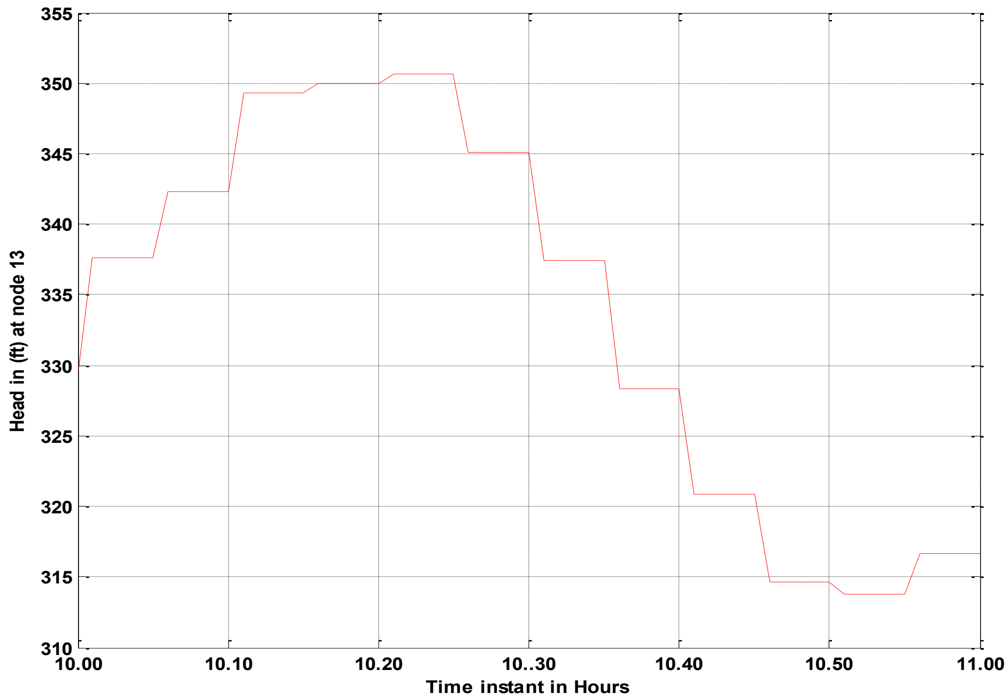

Figure 6.12 Water head at node 13

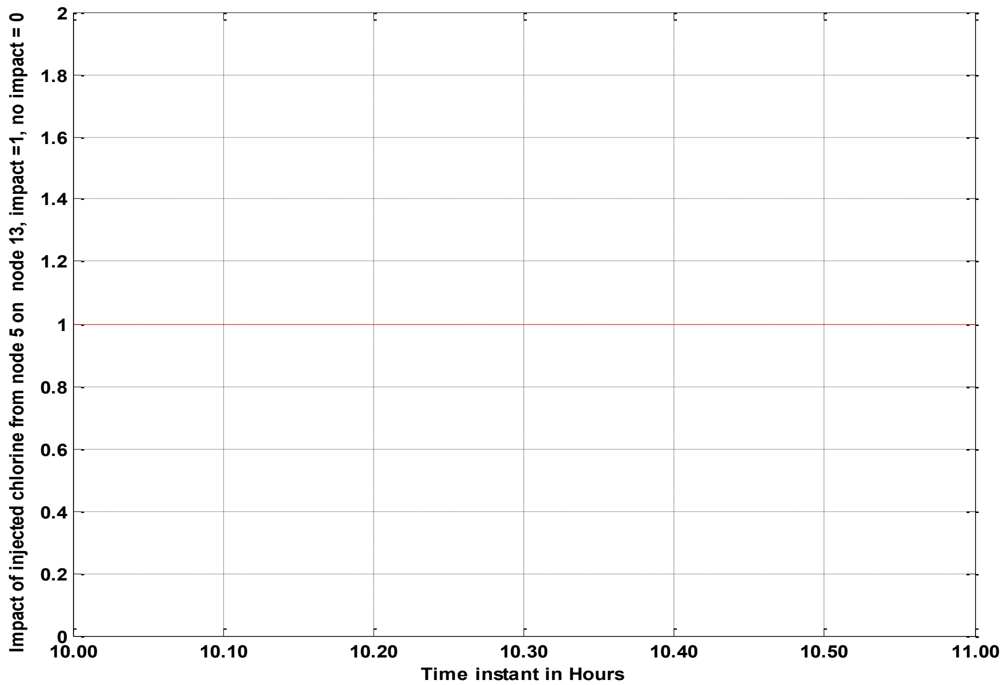

Figure 6.13 The time of impact of the chlorine injected at node 13 at [10,11] [hour]

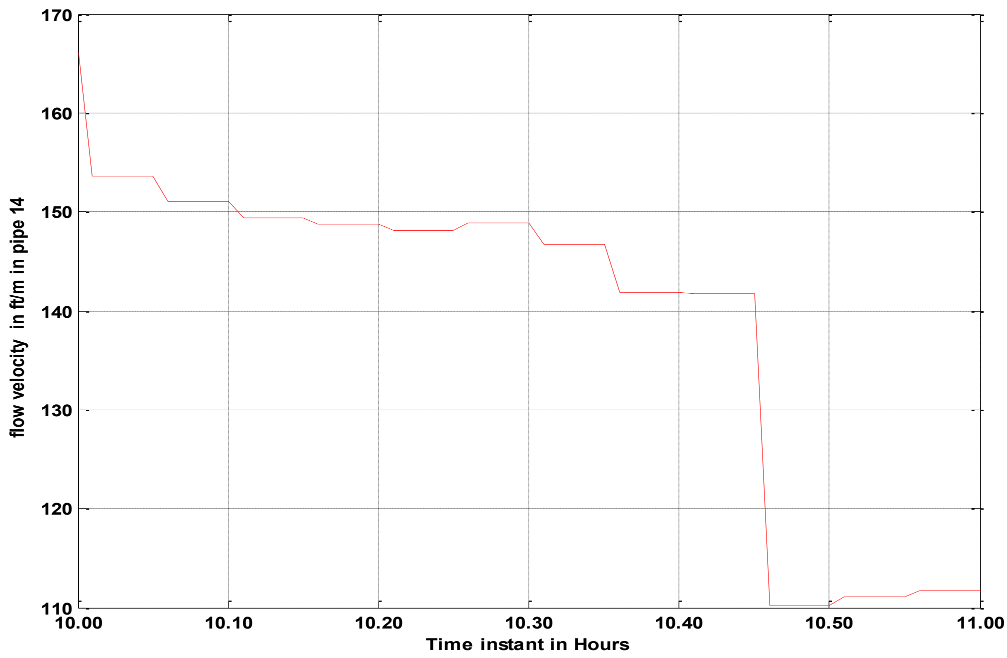

Figure 6.14 Water flow velocity at pipe 14

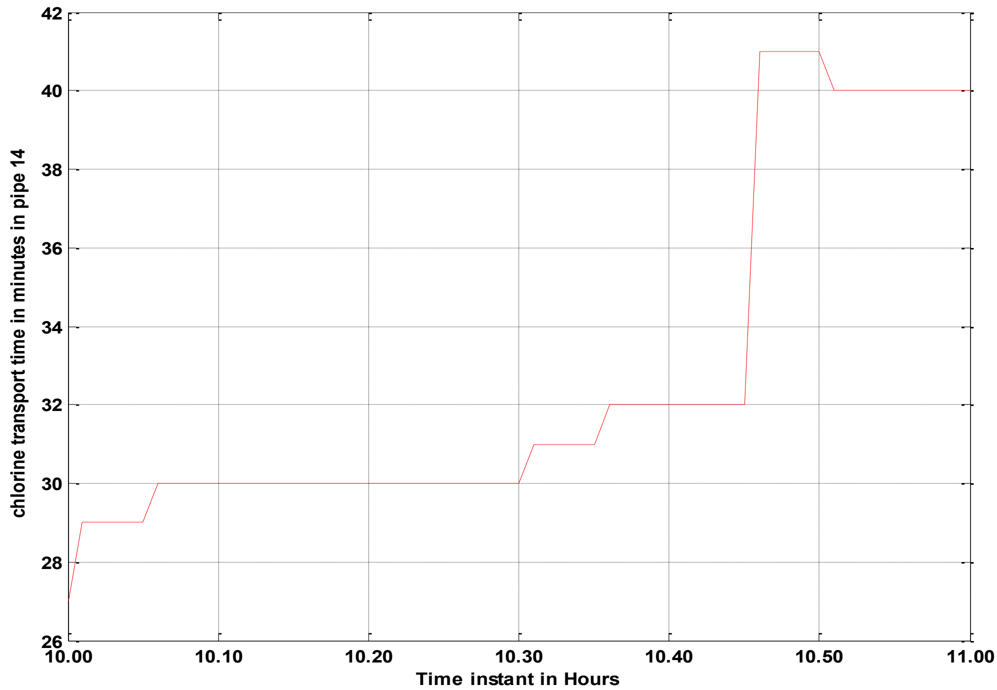

Figure 6.15 Chlorine transport time in pipe 14 at [10,11] [hour]

6.2.4 Model Parameter estimation

The model parameter estimation has been presented in chapter 3 section 3.6. and section 3.7. The parameters to be estimated are the impact coefficients on the injected chlorine from the injection nodes to the monitored nodes. The set bounded parameters as presented in section 3.7 are illustrated in Figure 6.16, Figure 6.17, Figure 6.18 and Figure 6.19 for normal operational state of the plant. Figure 6.16 illustrates the parameter bounds for the impact coefficients a_i for injected chlorine from node 5 to node 13 at [10,11] [hour].

Figure 6.17 illustrates the parameter bounds for the impact coefficients a_i for injected chlorine from node 19 to node 20 at [10,11] [hour].

Figure 6.18 illustrates the parameter bounds for the impact coefficients a_i for injected chlorine from node 9 to node 12 at [10,11] [hour]. Figure 6.19 illustrates the parameter bounds for the impact coefficients a_i for injected chlorine from node 10 to node 11 at [10,11] [hour]. The Chebyshev parameter is the half or bound centre of the parameter bound and it is used for RFMPC design.

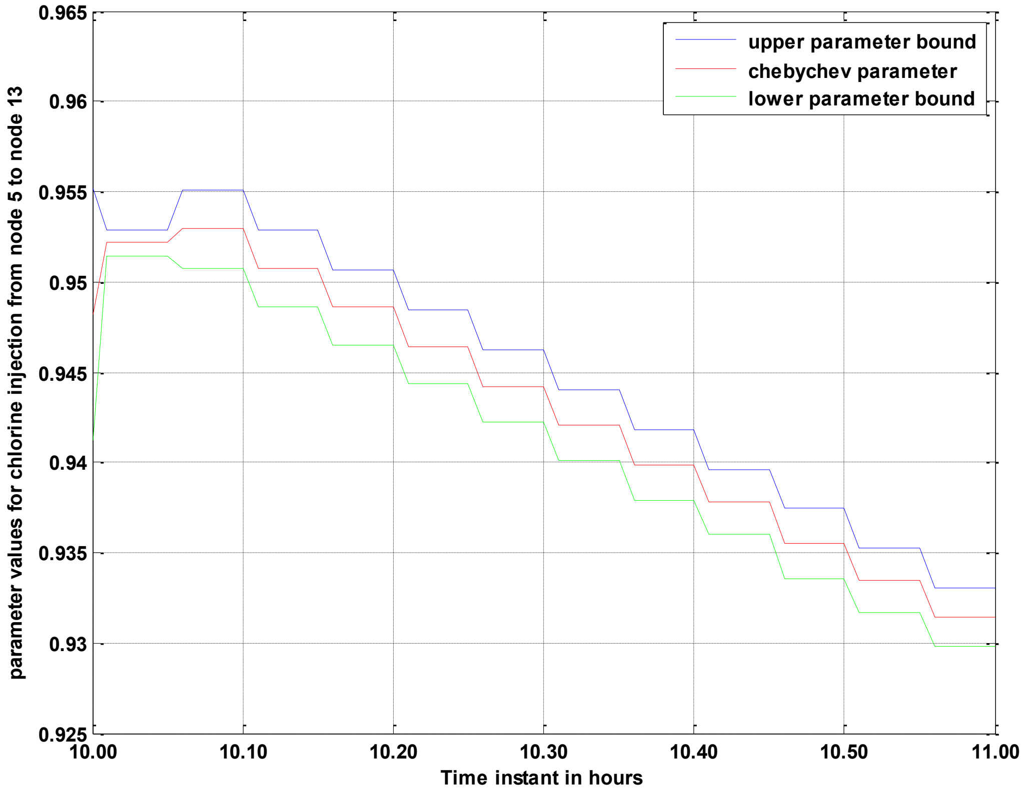

Figure 6.16 Parameter bounds for the impact coefficients a_i for injected chlorine from node 5 to node 13 at [10,11] [hour].

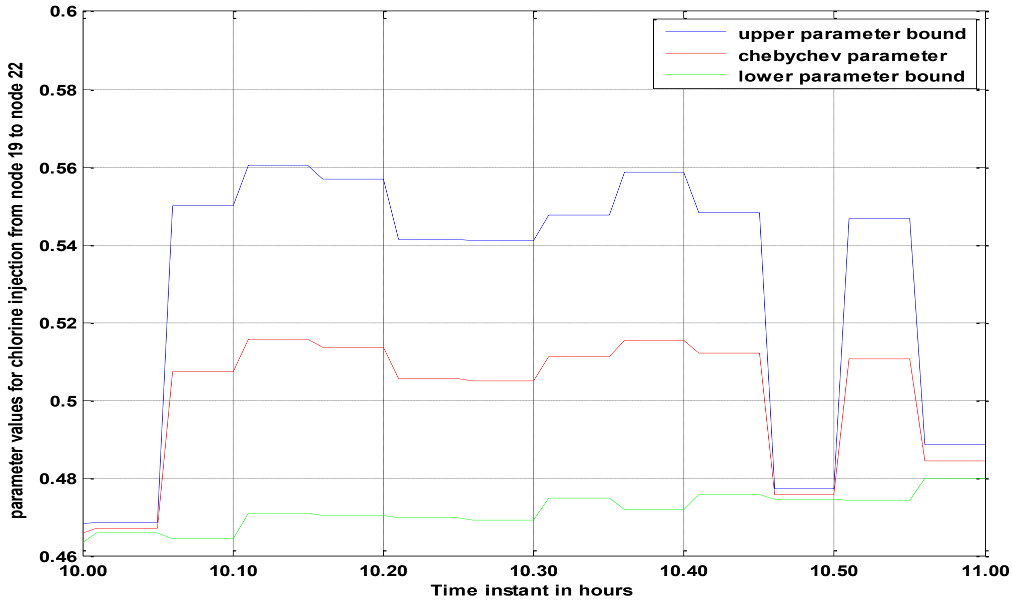

Figure 6.17 Parameter bounds for the impact coefficients a_i for injected chlorine from node 19 to node 20 at [10,11] [hour].

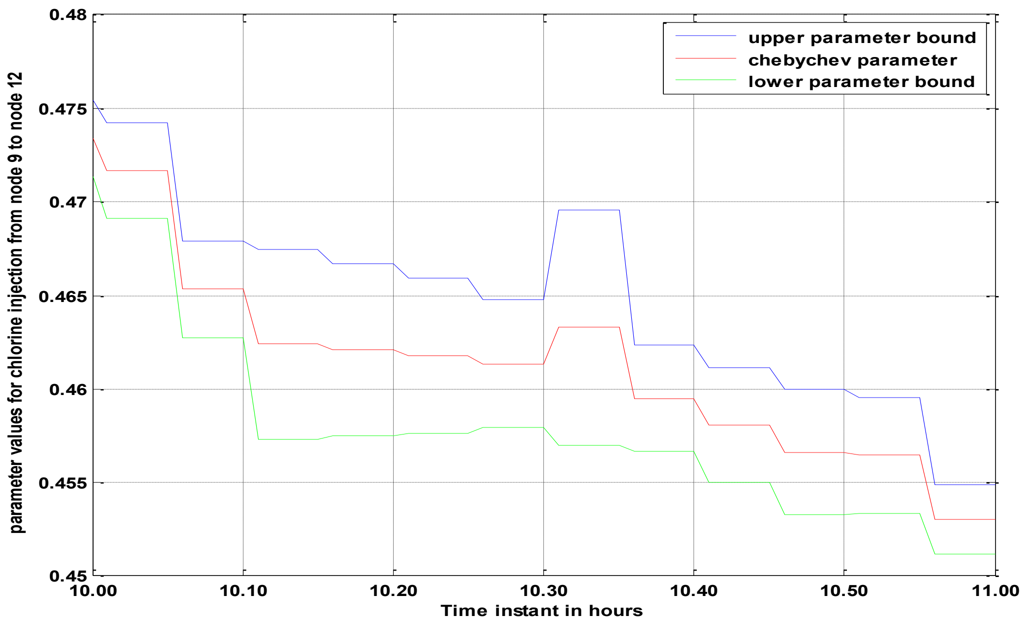

Figure 6.18 Parameter bounds for the impact coefficients a_i for injected chlorine from node 9 to node 12 at [10,11] [hour].

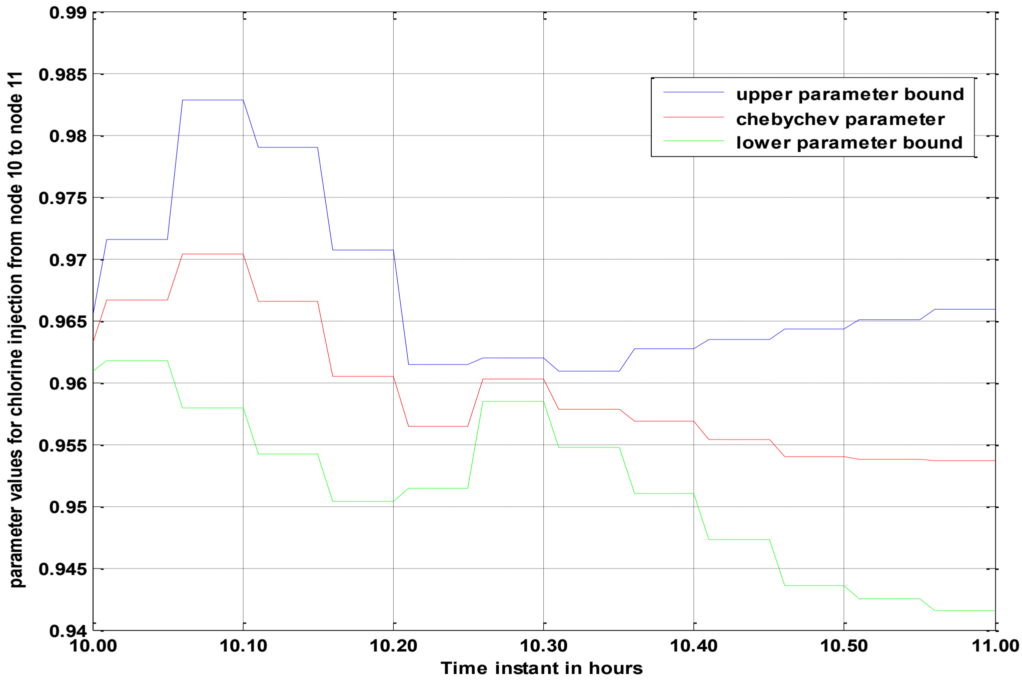

Figure 6.19 Parameter bounds for the impact coefficients a_i for injected chlorine from node 19 to node 20 at [10,11] [hour].

6.2.4.1 Disturbed operational state of the Benchmark DWDS

The disturbed operational state of the Benchmark DWDS is simulated by the pipe break of the pipe 17 connecting node 2 and node 10. A water flow path now exists between node 8 and node 9. Node 10 no longer controls node 9, it only controls node 11. Node 8 controls node 9 and node 12. Node 11 is now controlled by node 10 and node 8.

The Parameter bounds for the impact coefficients a_i for injected chlorine from node 9 to node 12 at [10,11] [hour] under the disturbed operational state is illustrated in Figure 6.20. The model parameters are different from the model parameters presented in Figure 6.18 for node 9 to node 12 under normal operational state of the plant.

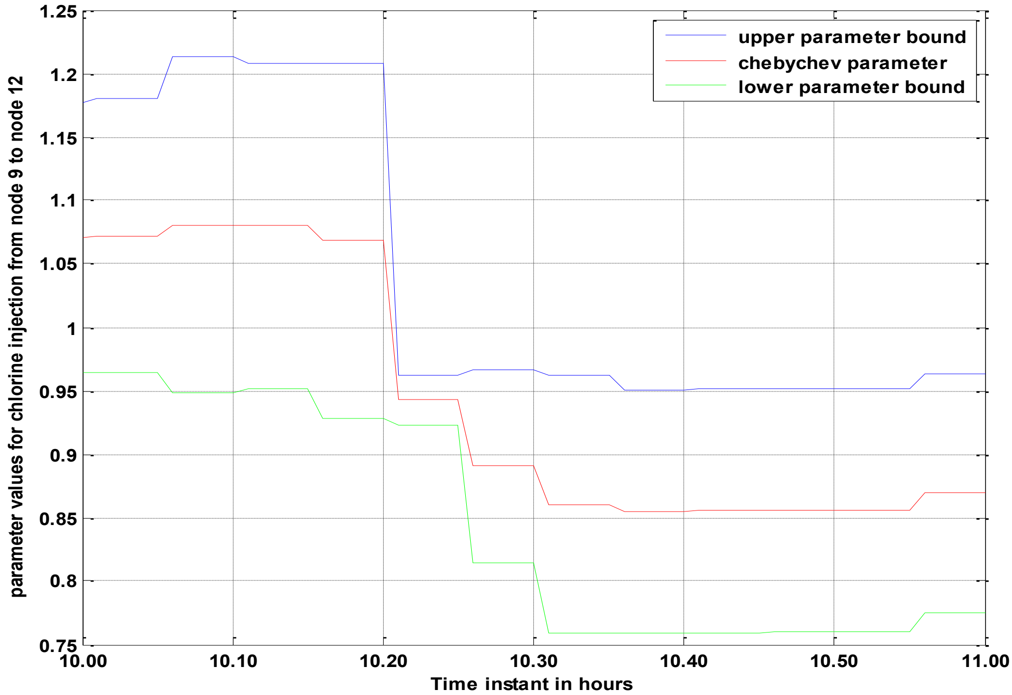

Figure 6.20 Parameter bounds for the impact coefficients a_i for injected chlorine from node 9 to node 12 at [10,11] [hour] under the disturbed operational state of the plant

6.3 Robustly Feasible MPC (RFMPC) Design

Robustly feasible Model Predictive Control (RFMPC) design has been presented in chapter 4 of this thesis. The prediction models, the reference zone for water quality control, the constraints for RFMPC design and the safety zones for RFMPC design are presented in the following sections.

6.3.1 Prediction Models

Prediction models are needed to predict the plant outputs and for RFMPC design.

$$y(k) = \sum_{i(k) \in I_{ij}} a_i(k) u(k - i(k)) \tag{6.1}$$

Where $y(k)$ is the monitored chlorine residual value at the monitored node, $a_i(k)$ is the time-varying impact coefficient parameter, $u(k)$ is the injected chlorine at the injection node and $i(k)$ is the chlorine transportation time number or discretized delay number. The output prediction over the prediction horizon at time instant *k* is written as:

$$Y(k|k) = \begin{bmatrix} y(k|k) \\ y(k+1|k) \\ y(k+2|k) \\ . \\ . \\ . \\ y(k+H_p-1|k) \end{bmatrix}_{H_p \times 1} \tag{6.2}$$

The control inputs over the control horizon at time instant *k* is written as:

$$U(k|k) = \begin{bmatrix} u(k|k) \\ u(k+1|k) \\ u(k+2|k) \\ . \\ . \\ . \\ u(k+H_c-1|k) \end{bmatrix}_{H_c \times 1} =$$

$$\begin{bmatrix} u(k - i(k)\,|k) + \Delta u(k|k) \\ u(k - i(k+1) + 1|k) + \Delta u(k+1|k) \\ u(k - i(k+2) + 2|k) + \Delta u(k+2|k) \\ . \\ . \\ . \\ u(k - i(k + H_c - 1) + H_c - 1|k) + \Delta u(k + H_c - 1|k) \end{bmatrix}_{H_c \times 1} \tag{6.3}$$

Where $u(k - i(k)\,|k)$ is the initial chlorine residual value at the injection node at time instant $(k - i(k)\,|k)$ and $\Delta u(k|k)$ is the calculated incremental input by the RFMPC to steer the output at the monitored node to the desired reference zone.

The output prediction and the control prediction horizon is equal to 24 hours as used in the DWDS.

$$H_p = H_c = 24\ hours$$

The time-varying impact coefficients parameters over the prediction horizon at time instant k is written as:

$$A_{model}(k|k) = \begin{bmatrix} a_i\,(k|k) & 0\,0\,0\,.\,.\,.\,0 \\ 0\ \ a_i\,(k+1|k) & 0\,0\,.\,.\,.\,0 \\ & . \\ & . \\ & . \\ 0\ \ 0\,0\,0\,.\,.\,. & a_i\,(k+H_p-1|k) \end{bmatrix}_{H_p \times H_p} \tag{6.4}$$

The output prediction matrix is now written as:

$$Y(k|k) = A_{model}(k|k) \times U(k|k) \tag{6.5}$$

$$Y(k|k) = A_{model}(k|k) \times u(k - i(k)\,|k) + A_{model}(k|k) \times \Delta u(k|k) \tag{6.6}$$

6.3.2 Reference Zone for water quality control

The designed controllers will control the chlorine residual values at the user nodes or monitored nodes within the output constraints described by:

$$Y^{min}(k|k) \leq Y(k|k) \leq Y^{max}(k|k) \tag{6.7}$$

Over the time horizon $k \in [k_0, k_0 + T_{control}]$, where $T_{control}$ is the control time horizon, $Y^{min}(k)$ is the minimum chlorine residual requirement at the user node and $Y^{max}(k)$ is the maximum chlorine residual requirement at the user node. For this design, $Y^{min}(k) = 0.25mg/l$ and $Y^{max}(k) = 0.35mg/l$.

6.3.3 Input Constraints

The input constraints are due to the limitations of the chlorine injection actuators. The input constraints are written as;

$$U^{min}(k|k) \leq U(k|k) \leq U^{max}(k|k) \tag{6.8}$$

For this design, $U^{min}(k) = 0mg/l$ and $U^{max}(k) = 1.0mg/l$.

6.3.4 Output Constraints

The output constraints are the bounds or reference zone presented in section 6.3.2 but now tightened by safety zones as presented in chapter 4 section 4.2 for robust feasibility. The safety zones are chosen arbitrarily offline such that if the tightened output constraints are violated, the real plant output constraints are not violated. For this design, the offline safety zones used is written as;

$$\mathcal{E} \triangleq [\mathcal{E}^{min}, \mathcal{E}^{max}] = [0.03, 0.03] \tag{6.9}$$

The safety zones are chosen to be equal for minimum and maximum value of the safety zones. Different values could be chosen for $\mathcal{E}^{min}, \mathcal{E}^{max}$

The output constraints tightened with the safety zones is written as:

$$Y^{min}(k|k) + \mathcal{E}^{min} \leq Y(k|k) \leq Y^{max}(k|k) - \mathcal{E}^{max} \qquad (6.10)$$

For monitored nodes controlled by an injection node, the predicted outputs of the monitored nodes tightened by the safety zones are used as output constraints in the RFMPC optimization task.

6.3.5 Performance Index

The performance index for the RFMPC design for water quality control is written as:

$$J\left(\Gamma_{.|k}, y\left(k + H_p|k\right) \right) = (Y^{ref} - Y^{out})^T(Y^{ref} - Y^{out}) + \Delta u(k)^T \Delta u(k) \qquad (6.10)$$

$$min_{\Gamma_{.|k}, y(k+H_p|k)} J\left(\Gamma_{.|k}, y\left(k + H_p|k\right) \right)$$

Subject to: $U^{min}(k|k) \leq U(k|k) \leq U^{max}(k|k)$

$$Y^{min}(k|k) + \mathcal{E}^{min} \leq Y(k|k) \leq Y^{max}(k|k) - \mathcal{E}^{max}$$

$$= \Delta u^T(A_{model}(k)^T A_{model}(k) + I)\, \Delta u(k) + 2A_{model}(k)^T \Delta u(k)^T (Y^{out}(k|k) - Y^{ref})$$

Where Y^{ref} is the reference chlorine residual value, Y^{out} is the predicted output over the prediction horizon, $A_{model}(k)$ is the prediction model and $\Delta u(k)$ is the incremental input matrix calculated by the RFMPC.

Equation (6.10) is a quadratic programming task to minimize the error between the reference chlorine output and the predicted chlorine outputs over the prediction horizon.

The Recursive feasibility is achieved by steering the output to the robustly feasible output states as presented in chapter 4 of this thesis. The RFMPC at node 5 controlling monitored node 13 under normal operational state is illustrated in Figure 6.21 and Figure 6.22.

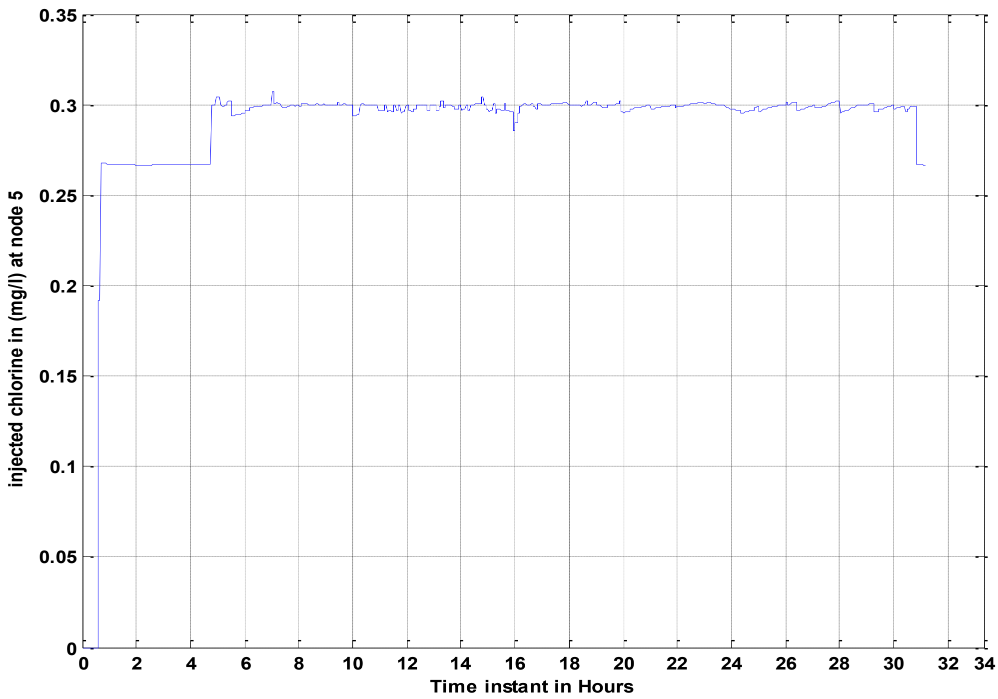

Figure 6.21 RFMPC at node 5 for chlorine residual control under the normal operational state of the plant

The injections at node 5 starts at 4.8 hours in Figure 6.21 because injections earlier than 4.8 hours will not have impact on the monitored node 13 due to transients and chlorine transportation time at this time of injection. Nothing can be observed at monitored node 13 for injections from node 5 before 4.8 hours as illustrated in Figure 6.22 from [0, 3] [hour]

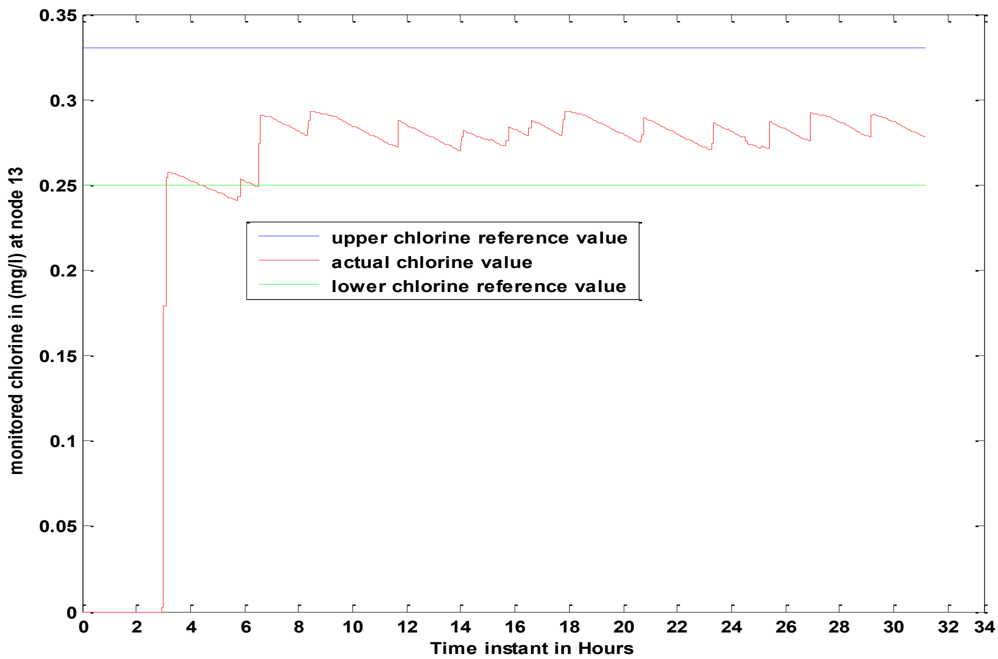

Figure 6.22 With RFMPC at node5: Chlorine residual values monitored at node 13 under the normal operational state of the plant at [7,31] [hour]

6.3.6 RFMPC for multiple output node control

The RFMPC at node 10 controlling monitored node 9, node 11 and node 12 under normal operational state is illustrated in Figure 6.23, Figure 6.24, Figure 6.25 and Figure 6.26. The multiple output node control was achieved by using the tightened predicted outputs of node 9, node 11 and node 12 as constraints in the optimization task of the RFMPC for node 12 control.

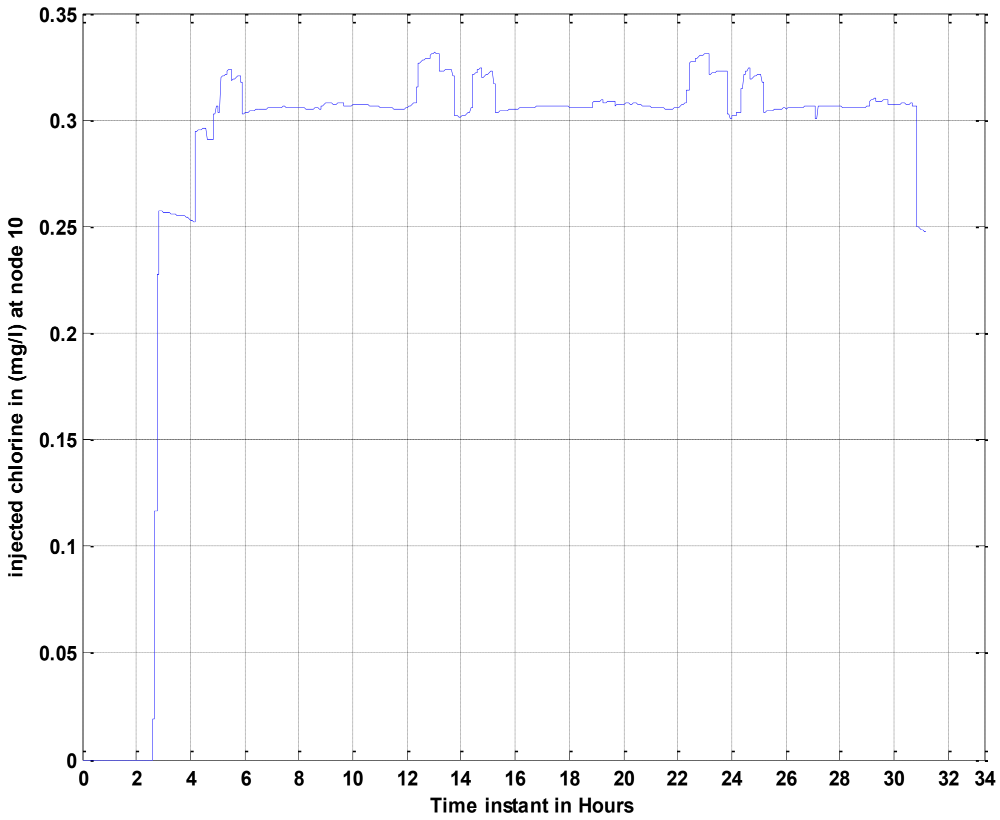

Figure 6.23 RFMPC at node 10 for chlorine residual control of multiple monitored node 9, node 11 and node 12 under the normal operational state of the plant

The variations of injected chlorine at node 10 in Figure 6.23 are due to varying demands and varying water flow velocity which demand more chlorine to be injected by the RFMPC to keep the chlorine level at the monitored nodes within the designed chlorine bounds.

The monitored nodes for node 10 chlorine injections are illustrated in Figure 6.24, Figure 6.25 and Figure 6.26.

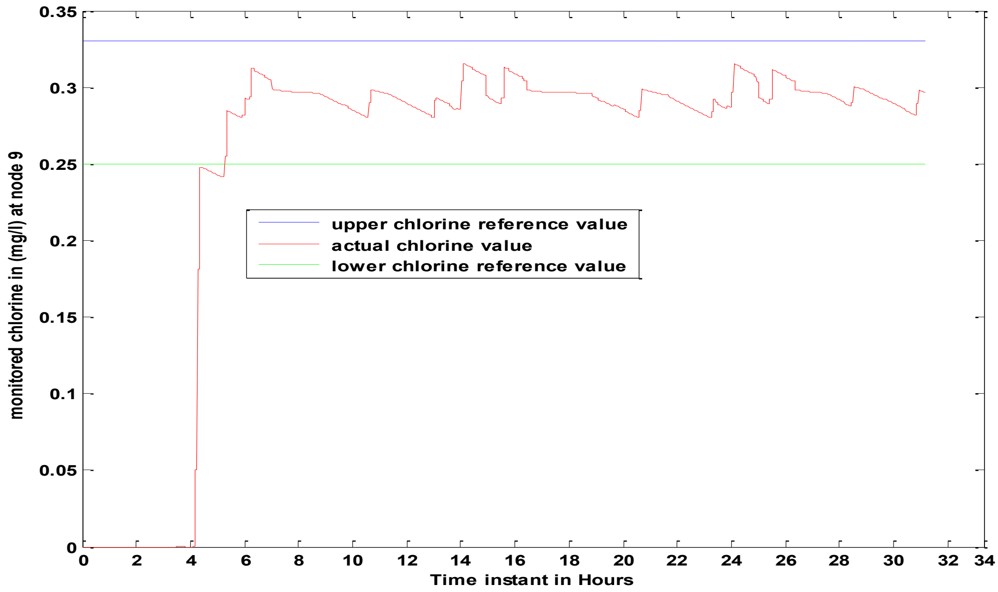

Figure 6.24 Chlorine residual values monitored at node 9 under the normal operational state of the plant at [6,30] [hour]

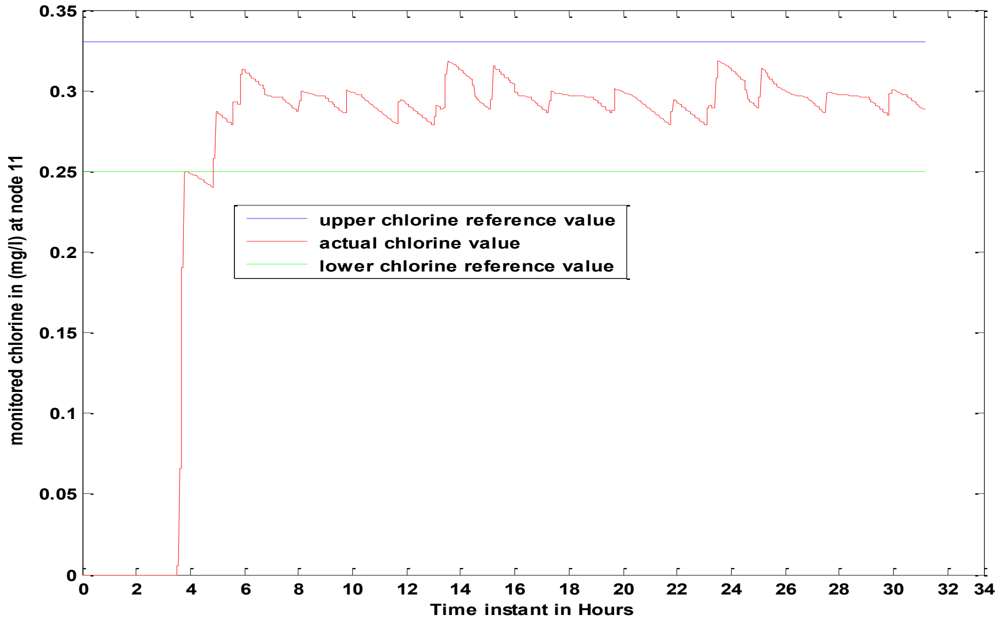

Figure 6.25 Chlorine residual values monitored at node 9 under the normal operational state of the plant at [6,30] [hour]

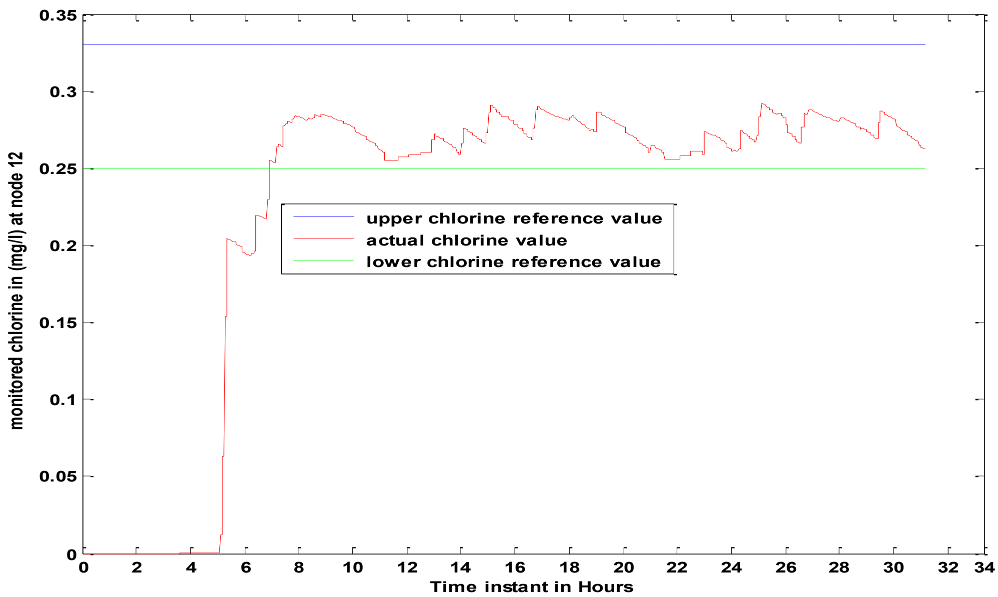

Figure 6.26 Chlorine residual values monitored at node 9 under the normal operational state of the plant at [8,32] [hour]

6.3.7 Distributed Robustly Feasible MPC (DRFMPC) Design

The DRFMPC has been presented in Chapter 4 of this thesis. The CDMPC has been presented in Chapter 4 of this thesis. The implementation is applied for the chlorine residual control of node 3 and node 4 of the benchmark DWDS. Node 3 is in the Region 3 of the benchmark DWDS and it is controlled by injections from node 19, node 2 and tank 1. The output prediction for node 3 is written as:

$$Y_3(k|k) = \begin{bmatrix} A_{20_3 Model(k|k)} & 0 & 0 \\ 0 & A_{2_3 Model}(k|k) & 0 \\ 0 & 0 & A_{29_3 Model}(k|k) \end{bmatrix} \times \begin{bmatrix} U_{20}\ (k|k) \\ U_2\ (k|k) \\ U_{29}\ (k|k) \end{bmatrix}$$

The output prediction for node 4 is written as:

$Y_4(k|k)$

$$= \begin{bmatrix} A_{20_4Model(k|k)} & 0 & 0 & 0 \\ 0 & A_{5_4Model}(k|k) & 0 & 0 \\ 0 & 0 & A_{2_4Model}(k|k) & 0 \\ 0 & 0 & & A_{29_4Model}(k|k) \end{bmatrix}$$

$$\times \begin{bmatrix} U_{20}\ (k|k) \\ U_5\ (k|k) \\ U_2\ (k|k) \\ U_{29}\ (k|k) \end{bmatrix}$$

The RFMPC at node 19 and node 2 for chlorine residual control of node 3, node 4 and node 20 under normal operational state are illustrated in Figure 6.27, Figure 6.28, Figure 6.29, Figure 6.30 and Figure 6.31

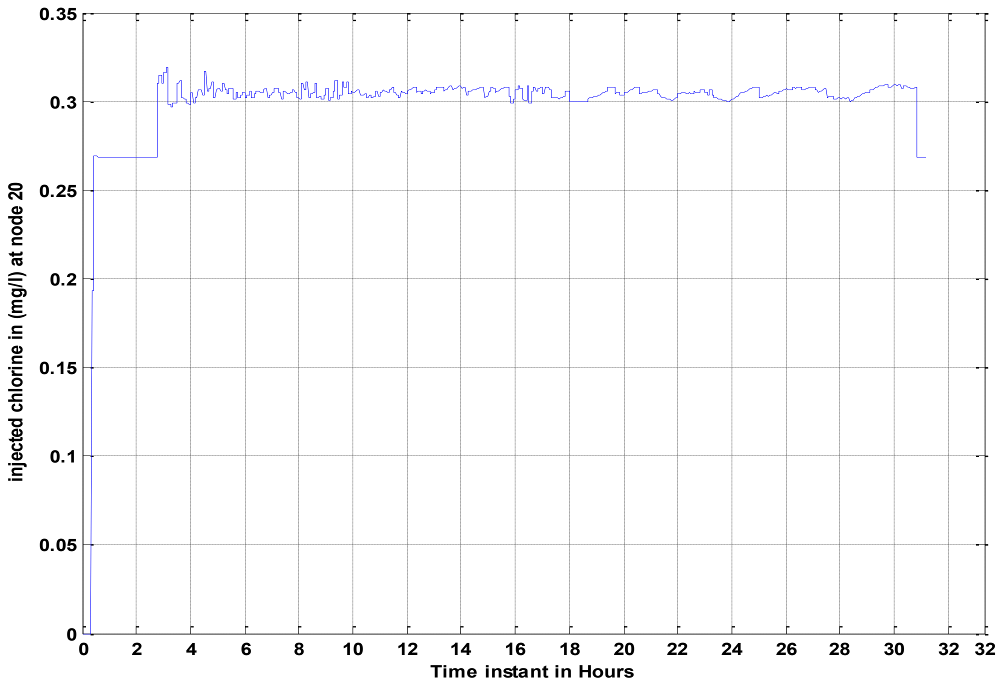

Figure 6.27 RFMPC at node 19 for chlorine residual control of monitored node 22, node 25, node 3 and node 4 under the normal operational state of the plant

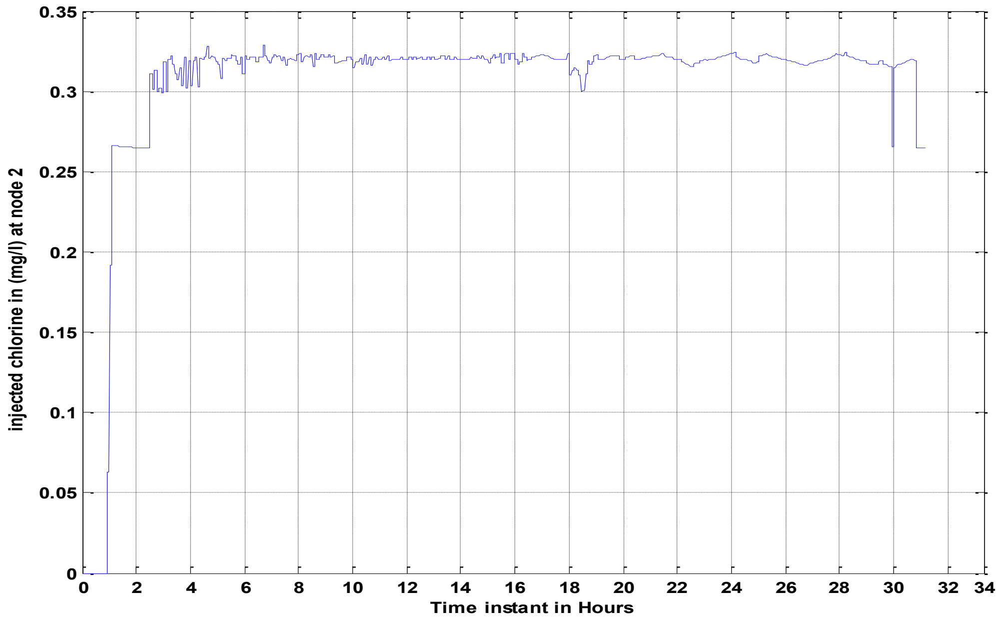

Figure 6.28 RFMPC at node 2 for chlorine residual control of monitored node 7, node 3 and node 4 under the normal operational state of the plant

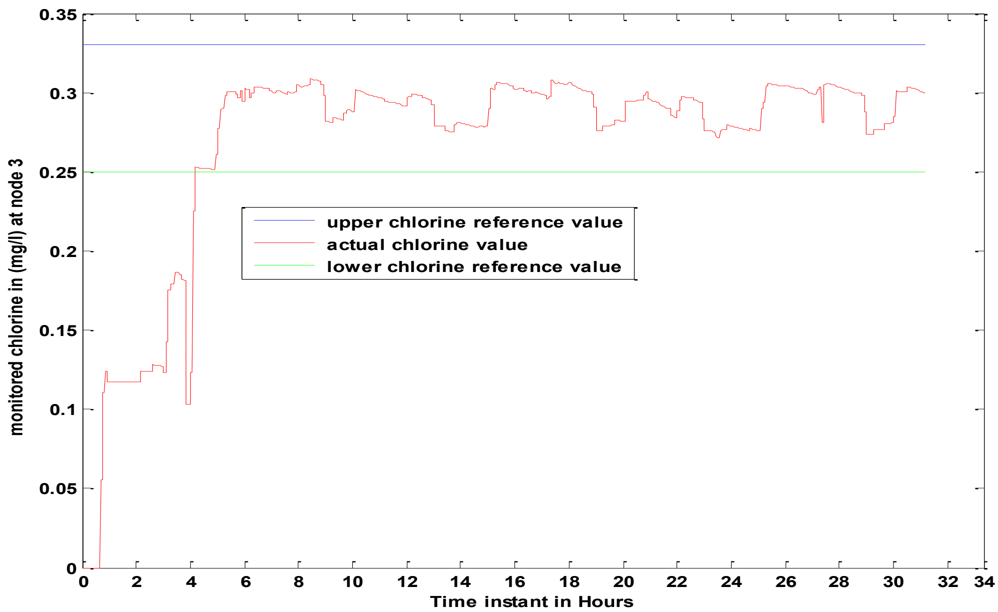

Figure 6.29 Chlorine residual values monitored at node 3 under the normal operational state of the plant at [6,30] [hour]

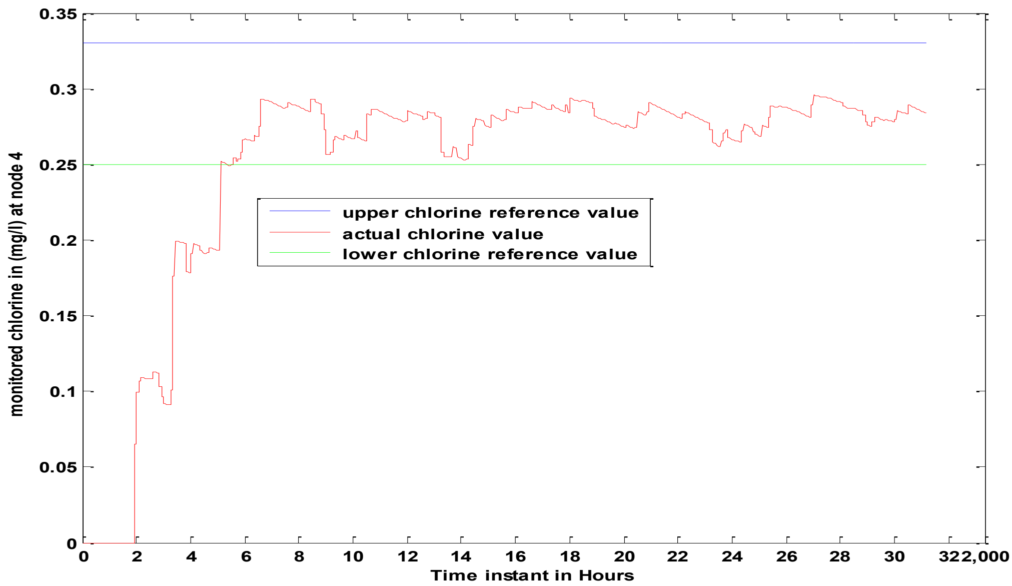

Figure 6.30 Chlorine residual values monitored at node 4 under the normal operational state of the plant at [6,30] [hour]

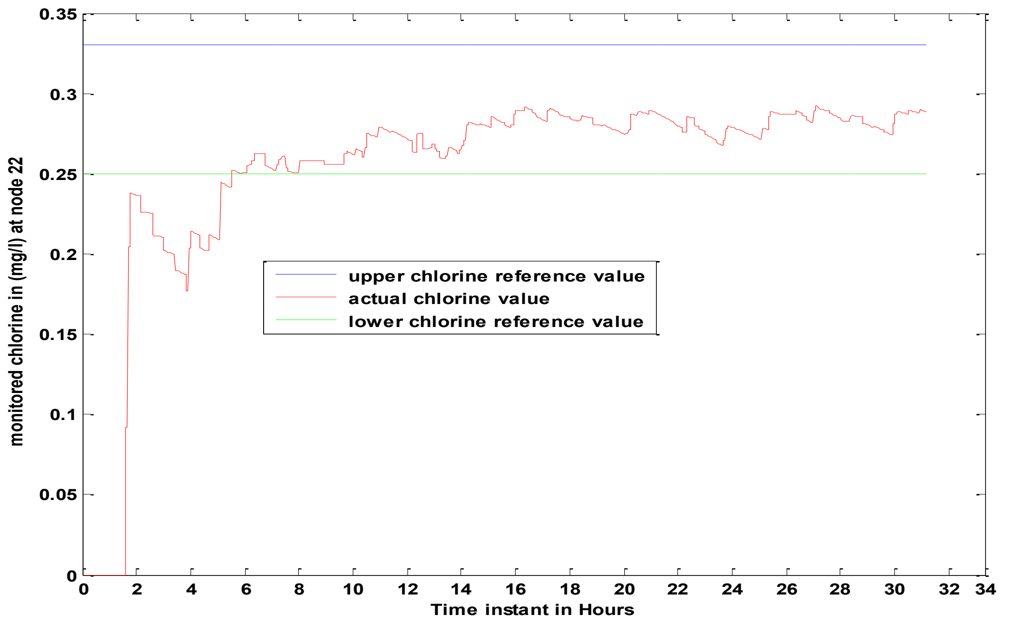

Figure 6.31 Chlorine residual values monitored at node 22 under the normal operational state of the plant at [6,30] [hour]

6.4 Distributed Softly Switched RFMPC (DSSRFMPC)

The need for soft switching of RFMPCs has been presented in all the chapters of this thesis. The soft switching may involve the change of prediction models used by the RFMPCs, change of constraints and change of control strategy. In this application, the change of prediction model is considered, while the control strategy is the same for the RFMPCs.

In simulating this, the pipe 17 connecting node 2 and node in the benchmark DWDS is assumed to be broken such that no water flow exists between node 2 and node 10. The time instant k that this pipe break occurred is at [10] [hour] on the output and control prediction horizon. At the next quality time step [10.05] [hour] the operational state of the plant is fully assessed by the fault management unit and the need for soft switching determined by the supervisory controller unit. At the next quality time step [10.10] [hour] the soft switching process starts and at [10.20] [hour] the new RFMPC is fully engaged.

The DWDS is operating at normal operational state up till [10] [hour] when the pipe break occurred. The new operational state is the disturbed operational state as no water flow exists between node 2 and node 10. Node 9 and node 12 are no longer controlled by node 10 but by node 8 from Region 1. The soft switching is used to engage node 8 RFMPC. Node 11 is now controlled by node RFMPC and node 8 RFMPC.

At [12.00] [hour] the pipe break is repaired, and the fault is cleared and the DWDS is restored back to the normal operational state from the disturbed operational state. Soft switching process is engaged according to algorithm 5.1 in chapter 5 of this thesis. Figure 6.31 and Figure 6.32 illustrates the two scenarios.

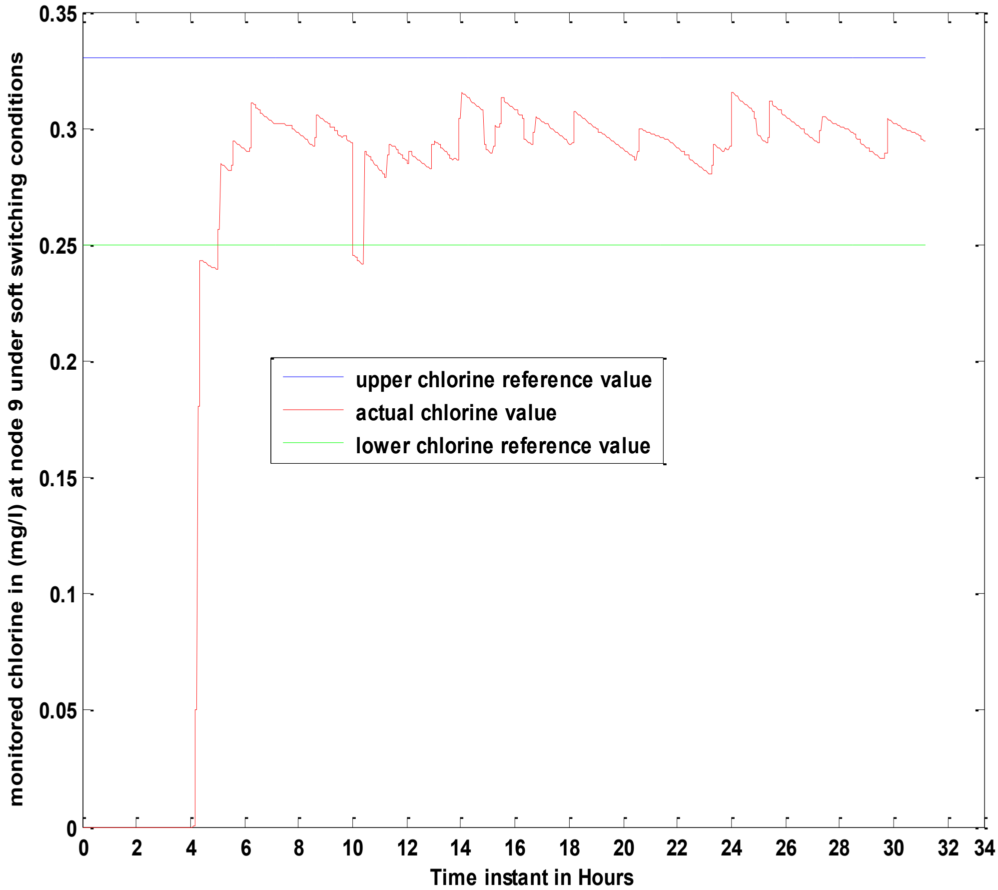

Figure 6.32 Chlorine residual values monitored at node 9 (Figure 6.26) under the soft switching conditions from normal operational to disturbed operational state of the plant and from disturbed to normal operational state

The Chlorine injections at node 8 under the soft switching conditions is illustrated in Figure 6.32

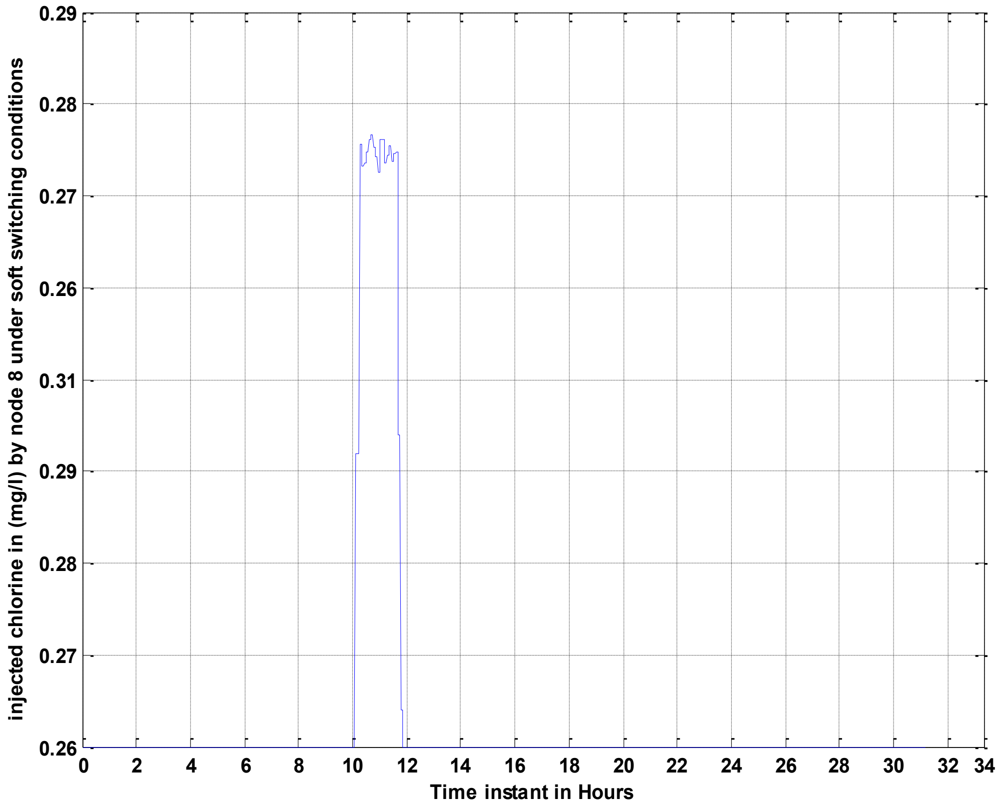

Figure 6.33 Chlorine injections at node 8 under the soft switching conditions from normal operational to disturbed operational state of the plant and from disturbed to normal operational state

6.5 Summary

The application of the control approach presented in this thesis has been applied to water quality control in this chapter. The existing path analysis algorithm has been extended by the proposed forward tracking algorithm and validated. The multiple nodes control using one RFMPC is validated. The proposed adaptive cooperation scheme is implemented and validated. The soft switching is also implemented and validated.

Chapter 7

Conclusions and recommendations for future works

7.1 Conclusions

Control of large-scale complex systems is the focus of this research work. CIS are large-scale complex systems. The reliable and sustainable operations of the CIS under different disturbance scenarios or operational states is desirable. The proposed methods of achieving the reliable and sustainable operations of the CIS under different operational states in this thesis includes:

1. The choice of a suitable control structure and control agents for the CIS. MPC was chosen as the control agent suitable for the CIS
2. The MPC is designed to be robustly feasible as RFMPC. This robust feasibility capabilities of the MPC is to ensure that the control inputs that satisfies the input, state and output constraints of the CIS are determined by the optimizer in the MPC under uncertainties in the CIS.
3. The RFMPC is designed to be recursively feasible over the whole control time steps.
4. The recursively RFMPC (RCRFMPC) is designed to be reconfigurable to adapt to the different operational states of the CIS.
5. The reconfiguration of the RCRFMPC is achieved by softly switching from one RCRFMPC used for one operational state of the CIS to the RCRFMPC used for another operational state. This is the softly switched RRFMPC (SSRRFMPC). CIS can be

decomposed into smaller and manageable subsystems. The decomposition of the CIS into smaller manageable subsystems allows the use of distributed control structures and coordination mechanisms between the distributed controllers for the CIS. The distributed arrangement of the designed SSRRFMPC is the distributed SSRRFMPC (DSSRRFMPC).

6. The change of operational state in one subsystem in the CIS may impact the operational state of the other subsystem in the CIS. Adaptive cooperative strategies for the DSSRRFMPC under different operational states of the CIS is proposed to achieve a coordinated control.

The CIS studied in this thesis was the DWDS. The operational control of DWDS was studied. A smart control structure for the reliable and sustainable operation DWDS was proposed in this thesis. A modified two-layer hierarchical structure with the lower level implemented with SSRRFMPC was proposed for DWDS water quality control. DWDS water quality control modeling involved chlorine residual modeling. This thesis proposed path analysis with forward tracking algorithm and node-to-node analysis method for improved modeling accuracy for chlorine residual in the DWDS. Time-varying models of chlorine residual control for DWDS which are suitable for designing RFMPC was developed in this thesis.

KKT conditions and set invariance theory were used to calculate the robustly feasible and recursive feasible initial states for all operational states of the CIS over the prediction horizon. The design of RFMPC and RRFMPC was achieved. Reconfiguration of the RRFMPC through soft switching under different operational states of the CIS was achieved. The soft switching was achieved using the convex combination of the RRFMPC strategies. The soft switching system components, functionalities and analysis were presented. The limitations of the existing Cooperative DRFMPC coordination strategy were identified. Adaptive cooperative strategy for DSSRFMPC under different operational states and reconfiguration was proposed. Different scenarios of adaptive cooperation during soft switching of DSSRFMPC were proposed. The application of the control approach and methods proposed in this thesis was applied to DWDS water quality control. The results verified the proposed control approach and methods. The proposed control approach and methods in this thesis is recommended for reliable and sustainable operation of the CIS.

7.2 Future works

Based on the research work presented in this thesis, the proposed future research directions and future works are as follows:

1. Fault and risk analysis is proposed to be included in the system modeling for Softly Switched RFMPC design. A comprehensive modeling or models should be done for every possible operational state of the system based on faults and risk analysis for the system
2. Improved DWDS simulator for fault detections and simulations. The existing DWDS simulator should be upgraded to simulate all possible faults in the DWDS.
3. Uncertainty handling in DWDS quality control. Further research work is recommended in modeling the uncertainties associated with DWDS water quality control
4. The use of more zones and multiple DSSRFMPC for DWDS quality control
5. Application of DSSRFMPC to other CIS such as power system networks, traffic networks and smart grid systems.

List of published paper

Published Paper

A. Ajibulu and M. Brdys, “Point-Parametric modeling for Model Predictive Control in Dynamic networks,” in Methods and Models in Automation and Robotics, Miedzyzdroje, Poland, 2015.

APPENDIX A

Network file of the Benchmark DWDS

[JUNCTIONS]

;ID	Elevation	Demand	Pattern	
1	90	0	1	;
2	110	1	1	;

3	95	1	1	;
4	110	3	1	;
5	100	1	1	;
6	103	3.5	1	;
7	97	3	1	;
8	103	1.5	1	;
9	107	0.25	1	;
10	112	0.25	1	;
11	115	0.5	1	;
12	112	0.5	1	;
13	110	0.25	1	;
14	120	0.25	1	;
15	135	0.25	1	;
16	130	0.25	1	;
101	90	0	1	;
201	100	0	1	;
20	100	1	1	;
22	95	2	1	;
23	80	0.5	1	;
25	90	0.5	1	;
24	87	1	1	;
21	100	0	1	;
27	100	2	1	;

[RESERVOIRS]

;ID	Head	Pattern	
100	90		;
200	90		;

300 90 ;

[TANKS]

;ID	Elevation	InitLevel	MinLevel	MaxLevel	Diameter	MinVol	VolCurve
17	240	90	50	150	30	0	
18	240	78	50	150	30	0	
19	240	78	40	150	30	0	
26	240	78	40	150	50	0	

;

[PIPES]

;ID	Node1	Node2	Length	Diameter	Roughness	Minor Loss	Status	
1	101	1	2000	20	100	0	Open	
2	1	2	800	18	100	0	Open	
3	2	3	5000	16	100	0	Open	
4	3	17	700	10	100	0	Open	
5	3	4	3700	12	100	0	Open	
6	4	5	3900	15	100	0	Open	;
7	5	201	2100	16	100	0	Open	
8	4	6	2500	10	100	0	Open	
9	3	7	3100	10	100	0	Open	
10	2	7	5500	15	100	0	Open	
11	7	6	3700	12	100	0	Open	

12	6	18	900	8	100	0	Open
13	6	5	2900	12	100	0	Open
14	5	13	4500	15	100	0	Open
15	6	13	2500	10	100	0	Open
16	7	8	2700	10	100	0	Open
17	2	10	3100	12	100	0	Open
18	10	9	1900	12	100	0	Open
19	10	11	1600	8	100	0	Open
20	11	12	1500	6	100	0	Open
21	9	12	1650	8	100	0	Open
22	9	8	2900	8	100	0	Open
23	8	19	1900	12	100	0	Open
24	8	13	3100	12	100	0	Open
25	13	14	1600	8	100	0	Open
26	14	16	1750	6	100	0	Open
27	14	15	1500	6	100	0	Open
30	21	20	800	18	100	0	Open

29	20	4	2500	12	100	0	Open
31	20	22	3000	12	100	0	Open
32	22	23	5000	12	100	0	Open
33	23	24	1000	8	100	0	Open
36	24	26	1000	12	100	0	Open
34	22	25	5000	12	100	0	Open
35	25	24	2000	8	100	0	Open
28	20	3	1000	12	100	0	Open
37	20	27	4000	12	100	0	Open
38	27	23	1000	12	100	0	Open
39	27	3	1000	12	100	0	Open

[PUMPS]

;ID	Node1	Node2	Parameters		
101	100	101	HEAD 1	PATTERN 2	;
201	200	201	HEAD 1	PATTERN 3	;
301	300	21	HEAD 1	PATTERN 3	;

[VALVES]

;ID	Node1	Node2	Diameter	Type	Setting	Minor Loss

```
[TAGS]

[DEMANDS]
;Junction        Demand        Pattern        Category

[STATUS]
;ID              Status/Setting

[PATTERNS]
;ID              Multipliers
;
 1     0.7     0.6     0.5     0.5     0.5     0.6
 1     0.8     1       1.1     1.25    1.25    1.2
 1     1.15    1.15    1.1     1       1.1     1.15
 1     1.25    1.4     1.25    1.2     1.1     1
;
 2     1       1       1       1       1       1
 2     1       1       1       1.25    1.25    1.25
 2     1.2     1.2     1.2     1.2     1.3     1.3
 2     1.3     1.4     1.3     0.98    0.98    0.98
;
 3     1       1       1       1       1       1
 3     1       1       1       0.95    0.95    0.95
 3     1.2     1.2     1.2     1.3     1.3     1.3
 3     1.3     1.3     1.3     1.05    1.05    1.05
;
 4     1       1       1       1       1       1
 4     1       1       1       1       1       1
```

4	1	1	1	1	1	1
4	1	1	1	1	1	1
;						
5	1	1	1	1	1	1
5	1	1	1	1	1	1
5	1	1	1	1	1	1
5	1	1	1	1	1	1
;						
6	1	1	1	1	1	1
6	1	1	1	1	1	1
6	1	1	1	1	1	1
6	1	1	1	1	1	1

[CURVES]

;ID	X-Value	Y-Value
;PUMP:		
1	6	280

[CONTROLS]

[RULES]

[ENERGY]

Global Efficiency	75
Global Price	0
Demand Charge	0

[EMITTERS]

;Junction	Coefficient

[QUALITY]

;Node	InitQual
100	0.27
200	0.27
300	0.27
17	0.27
18	0.27
19	0.27
26	0.27

[SOURCES]

;Node	Type	Quality	Pattern
5	SETPOINT	0.3	4
8	SETPOINT	0.27	5
10	SETPOINT	0.27	5
14	SETPOINT	0.27	4
20	SETPOINT	0.27	6
25	SETPOINT	0.27	6
24	SETPOINT	0.26	6
17	SETPOINT	0.29	6
18	SETPOINT	0.27	4

[REACTIONS]

;Type	Pipe/Tank	Coefficient

[REACTIONS]

Order Bulk 1

Order Tank 1

Order Wall 1

Global Bulk -0.5

Global Wall -1

Limiting Potential 0

Roughness Correlation 0

[MIXING]

;Tank Model

[TIMES]

Duration 48

Hydraulic Timestep 1:00

Quality Timestep 0:05

Pattern Timestep 0:05

Pattern Start 0:00

Report Timestep 0:01

Report Start 0:00

Start ClockTime 12 am

Statistic None

[REPORT]

Status No

Summary No

Page 0

[OPTIONS]

Units	MGD
Headloss	H-W
Specific Gravity	1
Viscosity	1
Trials	40
Accuracy	0.001
CHECKFREQ	2
MAXCHECK	10
DAMPLIMIT	0
Unbalanced	Continue 10
Pattern	1
Demand Multiplier	1.0
Emitter Exponent	0.5
Quality	Chlorine mg/L
Diffusivity	1
Tolerance	0.01

[COORDINATES]

;Node	X-Coord	Y-Coord
1	1409.95	1255.92
2	3992.89	1682.46
3	2618.48	4383.89
4	2997.63	6824.64
5	4300.95	8767.77
6	6220.38	7440.76
7	6291.47	4810.43

8	9016.59	4763.03
9	9063.98	3056.87
10	9135.07	1682.46
11	11101.90	1658.77
12	10983.41	3222.75
13	8803.32	7677.73
14	11078.20	7748.82
15	11172.99	6184.83
16	12997.63	8127.96
101	-533.18	1255.92
201	5983.41	9194.31
20	1058.76	6130.82
22	-1258.31	6363.64
23	-415.74	2993.35
25	-3165.19	4811.53
24	-1934.59	3514.41
21	349.22	6895.79
27	1025.50	4523.28
100	-3234.60	1232.23
200	8684.83	9549.76
300	1934.59	8104.21
17	1291.57	2560.98
18	4514.22	7748.82
19	10509.48	4857.82
26	-2389.14	2461.20

[VERTICES]

;Link	X-Coord	Y-Coord

```
[LABELS]
;X-Coord          Y-Coord          Label & Anchor Node

[BACKDROP]
 DIMENSIONS          0.00          0.00          10000.00          10000.00
 UNITS          None
 FILE
 OFFSET          0.00          0.00
[END]
```

APPENDIX B

Sample MATLAB Program for calculating the Transport or Detention time of Chlorine in the DWDS

```
%% Paths delay calculations

%% Path delay for Fp14_15
% Links 27 involved
C_PathFp14_15 = zeros(576,1); % memory location to store the Paths data
for k = 576:-1:1 % This path is active between time 1 to 576 for all
time instants in minutes which starts from the total sum of time when
the flow starts in the last pipe
    P_ath1Fp14_15 = ([W27_14_15(k,1)]  )* Fp14_15(k,1)  ;
    disp(P_ath1Fp14_15)
    C_PathFp14_15(k,1) = P_ath1Fp14_15; % store in memory location for
TDlink

end

% Path delay for Fp14_16
% Links 26  involved
C_PathFp14_16 = zeros(576,1); % memory location to store the Paths data
for k = 576:-1:1 % This path is active between time 1 to 576 for all
time instants in minutes which starts from the total sum of time when
the flow starts in the last pipe
    P_ath1Fp14_16 = ([W26_14_16(k,1)]  )* Fp14_16(k,1)  ;
    disp(P_ath1Fp14_16)
    C_PathFp14_16(k,1) = P_ath1Fp14_16; % store in memory location for
TDlink

end

% Path delay for Fp14_13
% Links 25  involved
C_PathFp14_13 = zeros(576,1); % memory location to store the Paths data
for k = 576:-1:1 % This path is active between time 1 to 576 for all
time instants in minutes which starts from the total sum of time when
the flow starts in the last pipe
    P_ath1Fp14_13 = ([W25_14_13(k,1)]  )* Fp14_13(k,1)  ;
    disp(P_ath1Fp14_13)
    C_PathFp14_13(k,1) = P_ath1Fp14_13; % store in memory location for
TDlink

end

%% Path delay for Fp20_22
% Links 34  involved
C_PathFp20_22 = zeros(576,1); % memory location to store the Paths data
```

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