Improving Preschoolers' Number Foundations

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About the authors

Jo Van Herwegen is an associate professor in developmental psychology at Kingston University London. Her research focuses on language and number development in both typical development and developmental disorders in order to examine what cognitive abilities and strategies relate to successful performance in typical populations and how these differ in atypical populations so to aid the development of economically valid training programmes.

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Executive summary

Mathematical competence is crucial for educational and financial success in modern societies. There is currently debate whether mathematical abilities later on in life depend on symbolic knowledge, such as counting abilities and digit recognition, or whether they rely upon non-symbolic knowledge, such as the ability to discriminate between large magnitudes that rely upon the approximate number sense (ANS). However, it is unclear whether symbolic abilities rely on non-symbolic ones (one-representation view) or whether symbolic and non-symbolic abilities are distinct systems (dual-representation view). Knowing what abilities predict mathematical success later on in life is important for the development of economically valid and efficient educational programmes, especially for those children who perform low on mathematical ability tasks or low achievers (LA).

Our previous studies had shown that specially developed PLUS games, which target ANS abilities and require children to guess and see where is “more” or “less” very quickly, improved typically developing preschooler’s ANS abilities. However, it was unclear how the PLUS games compared to other training programmes, for example those that target symbolic knowledge, and whether the PLUS programme would benefit children who perform low on mathematical ability tasks.

In this study we first examined which children performed low on mathematical ability tasks. Next, we compared the impact of two different training programmes on LA children’s ANS knowledge, their symbolic knowledge, and their mathematical abilities in general. One of the training programmes focused on non-symbolic abilities using PLUS games, which targeted children’s ANS abilities, and the other programme included DIGIT games that targeted symbolic knowledge and focused on children’s counting abilities and digit knowledge. We included preschoolers as they would have received little formal education so far and thus have limited symbolic knowledge. In addition, we targeted those preschoolers who were performing below average on mathematical ability tasks and who had low ANS abilities. The inclusion of children who had both little symbolic and non-symbolic abilities allowed us to examine the foundations of mathematical abilities and to observe which training programme would benefit children’s general mathematical outcomes most.

We predicted that if ANS abilities form the basis of mathematical abilities then children in the PLUS group would improve more than those in the DIGIT group. However, if non-symbolic and symbolic knowledge are two distinct systems then children in the DIGIT group should show improved mathematical abilities.

Our results showed that, although there are a number of reasons why preschoolers perform low on mathematical ability tasks, most children identified as LA had low ANS abilities as well. This confirms results in previous studies that have found that ANS abilities are important for children's mathematical abilities. The results from the training study showed that both training groups improved equally on a number of mathematical ability tasks that assess symbolic knowledge, including counting abilities, digit recognition, and understanding of counting as well as those that require non-symbolic knowledge, including ANS abilities. Finally, both groups showed improved general mathematical abilities and over 50% of LA children were no longer considered as low achievers on mathematical ability tasks six months later.

Therefore, the current results suggest that LA preschoolers benefit from playing daily mathematical games that target both non-symbolic abilities, the PLUS games, as well as symbolic ones, the DIGIT games. In addition, there is a complex interaction between symbolic, non-symbolic abilities, and mathematical cognition in preschoolers in that children who played DIGIT games also showed improved non-symbolic abilities. Future studies should examine longitudinal outcomes and assesses which LA children continue to show mathematical difficulties or whether the training programmes benefit LA children long term. In addition, larger controlled trials ae needed to verify the current findings. Based upon the current results we would recommend that all preschool children engage in daily games that support mathematical development, including both PLUS and DIGIT games, as this will allow LA children to reach their full potential.

Introduction

Predictors of mathematical success

In our daily life we are surrounded by numbers and good mathematical abilities have been shown to provide better educational and financial outcomes later on in life (Rivera-Batiz, 1992). The development of mathematical abilities has been found to depend on a range of domain general abilities, such as working memory, visuo-spatial abilities, processing speed, as well as domain specific abilities. Domain-specific abilities that relate to mathematical abilities include both verbal and non-verbal number-specific cognitive processes. Counting ability, and in particular the knowledge of the number word sequence, seems to be one of the most discriminating and efficient precursors of early mathematics learning (Passolunghi, Vercelloni, & Schadee, 2007; Krajewski & Schneider, 2009).

Another building block for the development of mathematics includes the non-verbal ability to perceive and discriminate approximate large numerosities without counting or numerical symbols, supported by the Approximate Number System (ANS). This system is shared between species (Feigenson, Dehaene, & Spelke, 2004) and relies upon the ratios presented, known as Weber's fraction (w). Studies have shown that the ANS is present from infancy onwards and that the precision of the ANS develops throughout early childhood, in that with increased chronological age children can discriminate between numerosities that are closer in size (e.g., 8 versus 10 dots versus 8 versus 12 dots).

The ANS is another domain-specific ability that has been shown to relate to mathematical skills (Halberda, Mazzocco, Feigenson, 2008). Indeed, some authors suggest that the acquisition of the meaning of symbolic numerals is done by mapping number words and Arabic digits onto the pre-existing ANS representations (Dehaene, 2001; Piazza, 2010). Research has found a relationship between ANS abilities and mathematical abilities in primary school children and in preschool years (Libertus, Feigenson, & Halberda, 2011). Moreover, longitudinal studies have shown that ANS abilities assessed at 3 years old predict general mathematical achievement at 6 years old (Mazzocco, Feigenson, & Halberda, 2011a). However, not all studies have been able to find a relationship between ANS abilities and mathematical abilities (for a review see De Smedt et al., 2013; Schneider et al., 2016) and it has been argued that performance on symbolic magnitude tasks rather than non-symbolic tasks correlates with mathematical achievement (See Lyons et al. 2014; Vanbinst, Ghesquière & De Smedt, 2015). In addition, it has been claimed that as children progress through the school years counting abilities become an important prerequisite for higher-order arithmetical achievement (e.g., Aunio & Niemivirta, 2010; Krajewski & Schneider, 2009).

Children at risk for Mathematical Learning Difficulties

While most children have a functioning ANS system and manage to develop skills in early numeracy at a young age, there are those who for various reasons fail to acquire early number concepts before they enter formal education. The number of preschool children at risk is still strongly debated but studies that have investigated mathematical difficulties in school-age children and adolescents estimate that between 4 to 14% of children score in the bottom 25% on mathematical achievement tasks across two consecutive years and that these low achieving (LA) children are at risk of developing mathematical learning difficulties (MLD) across their life span (Geary, Hoard, Nugent & Bailey, 2012; Morgan, Farkas, & Wu, 2009). In addition, clinical studies have shown that children who are most likely to develop mathematical difficulties later on in life can be identified one year prior to entering formal education by assessing their number concept development (Aunio, & Niemvirta, 2010). This indicates that LA children, who are at risk for MLD, can be diagnosed early and that early interventions prior to entering school will allow them to develop a basis for number development and skills necessary for formal mathematical knowledge later on in life.

Although children who perform low on mathematical ability tasks form a heterogeneous group (Bartelet, Ansari, Vaessen, & Blomert, 2014; Costa, Nicholson, Donlan, & Van Herwegen, under review), it has been argued that LA children show stronger relationships between ANS precision and mathematical abilities later on in life compared to those with higher mathematical scores (Bonny, & Lourenco, 2012) and that the acuity of the ANS has a direct impact on preschool children's mathematical learning disability outcome (Chu, vanMerle, & Geary, 2013; Mazzocco, Feigenson, & Halberda, 2011b). Still, others have failed to find any differences for ANS scores between LA and typically developing (TD) children (De Smedt &

Gilmore, 2009). Alternatively, it has been proposed that LA children have difficulties with accessing the magnitude information from symbolic knowledge, rather than process magnitudes per se (Rouselle & Noel, 2007). Seeing the different tasks that have been used to assess ANS abilities and the fact that symbolic and non-symbolic abilities are likely to depend on one-another, it is not surprising that there currently is no consensus about the ANS abilities in LA children.

Training studies allow for further insight into the numerical deficits in LA children. If it is the case that ANS abilities form the foundations of mathematical difficulties in LA children then training studies focusing on ANS abilities can serve as a first approach to improve LA's children long-term mathematical outcomes.

Improving number foundations in preschoolers

Seeing the importance of ANS abilities for mathematical abilities later on in life, a number of studies have examined whether ANS abilities are malleable and whether preschoolers' ANS abilities can be improved. For example, a recent study by Van Herwegen and colleagues (2017) showed that playing PLUS games, which are specially developed games that target children's ANS abilities by making the children estimate which quantity has more or less using ratios of different difficulty levels (see page 16-17 for a full description), for 5 weeks improved typically developing preschoolers' ANS abilities. However, this study did not examine whether improving ANS abilities impacted on general mathematical abilities as well (Van Herwegen, Costa, & Passolungi, 2017). In addition, Wang and colleagues (2016) showed that 4- and 5-years-olds had better ANS abilities after a five minute training task and that these improvements impacted on their symbolic mathematical abilities (Wang, Odic, Halberda, & Feigenson, 2016). These studies suggest that ANS abilities can be improved in typically developing preschoolers and that these improvements impact on mathematical achievements.

Although there have been a number of studies that have examined the effect of training programmes in LA children, most of these training studies have focused on mixed training programmes that include both symbolic and non-symbolic abilities or have focused on small numbers that do not fall within the ANS range (i.e. *Number Worlds, Building Blocks, Right Start, Pre-K Mathematics*). There are currently no training studies that focus on ANS abilities alone in LA preschoolers (see Mononen et al., 2014 for a review). A better understanding of how ANS abilities can be improved in LA preschoolers will provide better outcomes for LA children long term but also allow further insight into the importance of ANS abilities for mathematical difficulties.

Aims

The current study aimed to build on previous research and targeted preschool children who have not yet started formal education. It examined 1) which preschool children are low achievers on mathematical tasks, 2) whether games that target the ANS system by allowing children to estimate and match quantities (PLUS games) can improve their ANS abilities as well as DIGIT games that target symbolic knowledge such as digit recognition and counting, 3) whether improving ANS or symbolic abilities in LA preschoolers has an effect on their mathematical abilities, and 4) how improved ANS or symbolic abilities relate to working memory abilities and how working memory abilities relate to general mathematical abilities, both short-term as well as six months later.

Study 1 Low achieving preschool children

This study focused on the first aim of the project and examined which preschoolers are low achievers on mathematical ability tasks. Full details of design, methods, instruments, participants and statistical analyses can be found in the key articles referred to below.

Costa, H.M., Nicholson, B., Donlan, C. & Van Herwegen, J. (2018). Low performance on mathematical tasks in preschoolers: the importance of domain-general and domain-specific abilities. *Journal of Intellectual Disability Research 62*(4), 292-302.

Methods

Participants

Fourteen[1] preschool settings (seven private nurseries and seven free local authority settings) from Greater London agreed to take part in the study. In total 539 children between the ages of 3 and 5 years old attended these nurseries.

All children came from a variety of Socio-Economic Status (SES) backgrounds. SES was measured using mother's highest level of education as parental education is considered to be one of the most stable aspects of SES (Sirin, 2005). Mothers reported in a background questionnaire whether they had no qualifications, finished secondary school (with either O- or A-levels), vocational qualifications, an undergraduate degree, or a post-graduate degree.

Children were included in the study and screened for mathematical difficulties if 1) parental consent and verbal assent from the child was obtained, 2) children spoke English at home, 3) children did not have any developmental issues reported by parents in a parental questionnaire, and 4) children performed within the typical range on the intelligence task, British Ability Scales. An overview of the children excluded from the study is provided below in Table 1.

Reason for Exclusion	Number of children
No parental consent	154
Limited English	33
Diagnosis of developmental issues/ not in typical range of BAS	19
No child assent	24
Child partly completed the assessments but dropped out due to illness, long absence, or moving nursery	26
Total Number of children included	283

Table 1. Overview of the children excluded from the study

Materials

British Ability Scales (BAS3). The BAS*3* is a standardised assessment battery for children aged 3 to 17 years old and measures verbal, non-verbal and general

[1] Originally it was planned to include 480 children across 16 settings but 2 settings had over 100 children in the required age range.

reasoning abilities. We carried out 6 core scales of the Early Years cognitive battery which were used to derive a General Cognitive Ability (GCA) score (*BAS3*; Eliot & Smith, 2011). This summary score has a mean of 100 and a standard deviation of 15. The technical manual reports an average test-retest reliability coefficient for the composite GCA scale of .93 (range= .91-.94) for the early year age range.

Test of Early Mathematical Abilities-3 (TEMA-3). TEMA-3 is a norm-referenced measure that can be used as a diagnostic instrument to determine specific strengths and weaknesses in children's mathematics skills for those aged 3 to 8 years old. Administration takes about 40 minutes with each child and includes the child completing a wide range of mathematical tasks either verbally or on paper, some with or without the use of counting aids (i.e., counters or fingers). The test includes A and B forms of 72 items that can be used interchangeably to measure progress or evaluate training programmes. Internal consistency reliabilities are all above .92; immediate and delayed alternative form reliabilities are in the .80s and .90s. Percentile scores were a measure of interest to include children in the training programmes.

ANS Abilities. In this computer task children were presented with a set of dot presentations on the left and right of the screen. The dot presentations included between 5 and 28 dots and the dot presentations in each trial included either ratios 0.5, 0.6, 0.7 or 0.8. The task included 48 trials. In half of the trials dot size correlated with the amount of dots (i.e., congruent trials) and in the other half of the trials dot size did not correlate with the amount of dots (i.e., incongruent trials). The presentation with 'more' dots was counterbalanced and appeared on either the left or right side of the screen. Children were asked to select the dot presentation that had 'more' using a touch screen. Participants received a score of 1 for each correct trial and the maximum score was 48. Cronbach's α based on average inter-item correlation = .867 (see Gilmore et al, 2013 for more details).

Prior to the actual ANS task, participants were administered a practice task in which it was assessed whether children understood the concept of 'more' in a numerical sense rather than base their decisions on a different variable. In this training task children received up to 24 training trials (or until they have 8 consecutive trials correct). Each training trial showed two dot presentations that had a ratio difference of 1/3 between them. In half of the trials area correlated with number while in the other trials area did not correlate with number. Children received feedback when they picked the incorrect answer (see Negen & Sarnecka, 2015 for a similar approach). Cronbach's α based on average inter-item correlation = .808.

Results

Examination of TEMA scores from all of the children assessed showed that Test or Early Mathematical Abilities-3 (TEMA-3; Ginsburg & Baroody, 2003) is insensitive at the lower range and that even a high percentile score on TEMA is based on getting just a few items correct (or even none correct at age 3). Thirty five percent of the total sample of children in the current study scored a raw score of 6 or less on the TEMA, where the score range for the age group is 0-32 (see Figure 1). This means that TEMA alone does not allow discrimination between those children who are at risk for MLD and those who perform within the typical range. Based upon this knowledge we chose to use a higher cut-off percentile similar to previous studies (see Murphy, Mazzocco, Hanich, & Early, 2007 for a discussion). Eighty-one preschool children (37 males) obtained a raw score of 6 or below on TEMA-3 (mean age in months: 44.38, SD= 5.47).

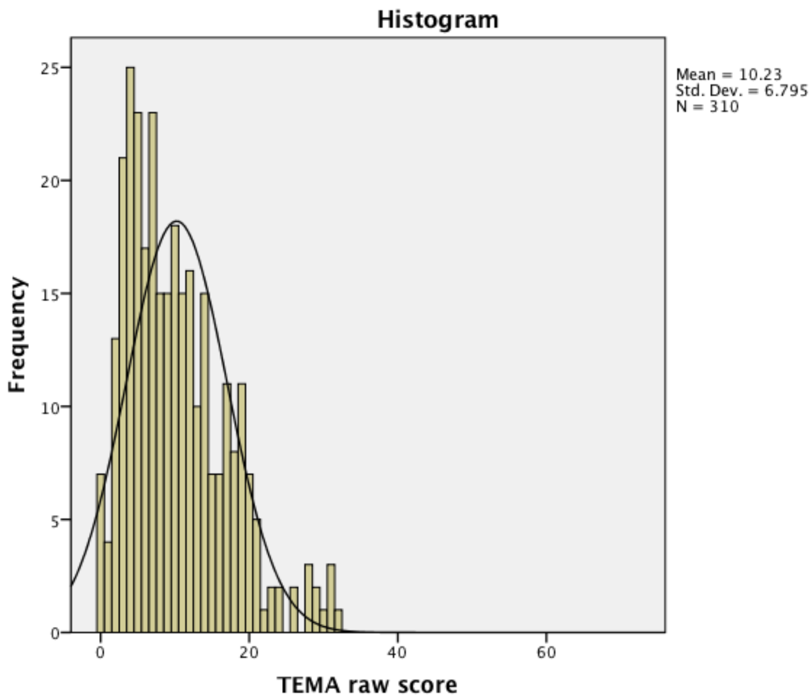
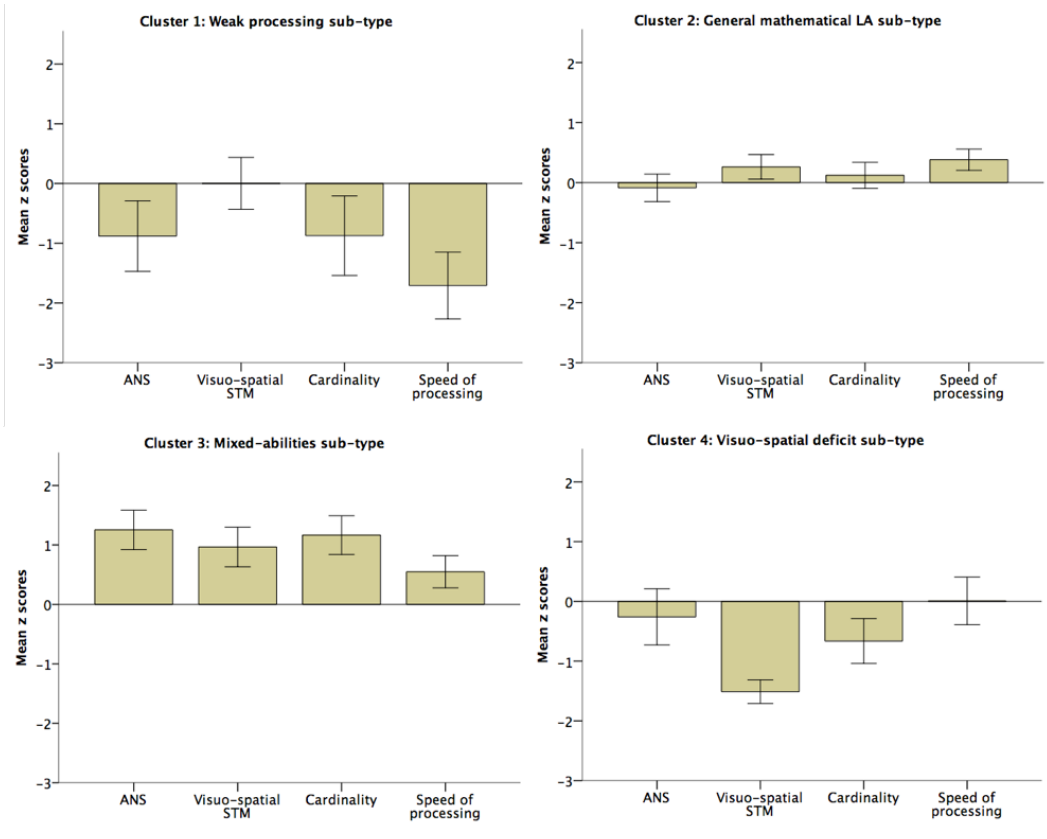


Figure 1. Distribution of TEMA raw scores for all children in Study 1.

We examined the cognitive profiles of these LA children as well as the heterogeneity of the causes of their mathematical difficulties using a number of domain specific and domain general abilities, including ANS performance, Cardinality, speed of processing, and visuo-spatial short term memory (see descriptions of these tasks below). Cluster analysis showed that LA children can be grouped in four sub-types: 1) *a weak processing sub-type* including 13 children who had significant difficulties on the speed of processing task, *2) a general mathematical LA subtype:* this group included 37 children who performed low on both domain specific tasks: ANS and give-a-number task, 3) a group of 15 children did not have any specific deficits on the domain specific and domain general abilities that we included in the analysis, despite low performance on the TEMA, 4) *a visual-spatial deficit sub-type:* this group included 16 children who showed visuo-spatial short term memory difficulties (see Figure 2).

Figure 2. Mean z scores on the domain-general and domain-specific precursors included in the cluster analyses for each of the 4 LAs sub-types.



Even though this study shows that low mathematical achievement scores on TEMA can be caused by a range of domain specific and domain general abilities, 65% of the LA children had low ANS scores (≤ 30/48) and this suggests that providing training to improve ANS abilities might be a step towards improving these children's mathematical outcomes.

Study 2 Improving number foundations in pre-schoolers

This study focused on the remaining three aims of the project and examined whether PLUS games can improve ANS abilities in LA preschoolers as well as whether improved ANS abilities affect mathematical abilities both short-term (directly after training), as well as six months later (follow-up). This study compared the benefits of the non-symbolic training programme using PLUS games to a training programme that focused on symbolic abilities, called DIGIT games as well as performance of a TD group of children who did not meet the criteria for LA. Finally, it examined whether the training programmes had any effects on working memory abilities. Full details of design, methods, instruments, participants and statistical analyses can be found in the key article referred to below.

Van Herwegen, J., Costa, H.M., Nicholson, B., & Donlan, C. (2018). Improving number abilities in low achieving preschoolers: symbolic versus non-symbolic training programs. *Research in Developmental Disabilities [advance online]*.

Methods

Participants

Based upon the outcomes of study 1, we included those children who had a raw score of 6 or lower on TEMA-3. In addition, as the TEMA-3 assesses for a wide range of mathematical abilities and LA children form a heterogeneous group, the low mathematical scores from LA children could be caused by a wide range of domain general and domain specific difficulties. As a result, LA children were only included in study 2 if they achieved a raw score of 6 or below on the TEMA-3 and scored lower than 30 out of 48 trials correct on the Approximate Number System (ANS) task. This ensured that the LA preschool children formed a more homogeneous group, in that they all had low symbolic and non-symbolic abilities. In addition, only children who attended the preschool setting for at least three days per week were included, in order to ensure that all children would be able to have a minimum of 20 sessions for each training programme. Forty-nine LA children (mean age: 44.39 months, SD= 4.97) met the inclusion criteria (See Figure 3).

A control group of twenty-four children (mean age: 45.50 months, SD= 3.73) who did attend the preschool settings as usual and did not participate in any training programmes was selected from those children who performed at or above the 50th centile on the TEMA and had scores above 30 out of 48 trails correct. The inclusion of control group allowed examination whether the training programmes allowed LA children to close the gap in performance with their peers.

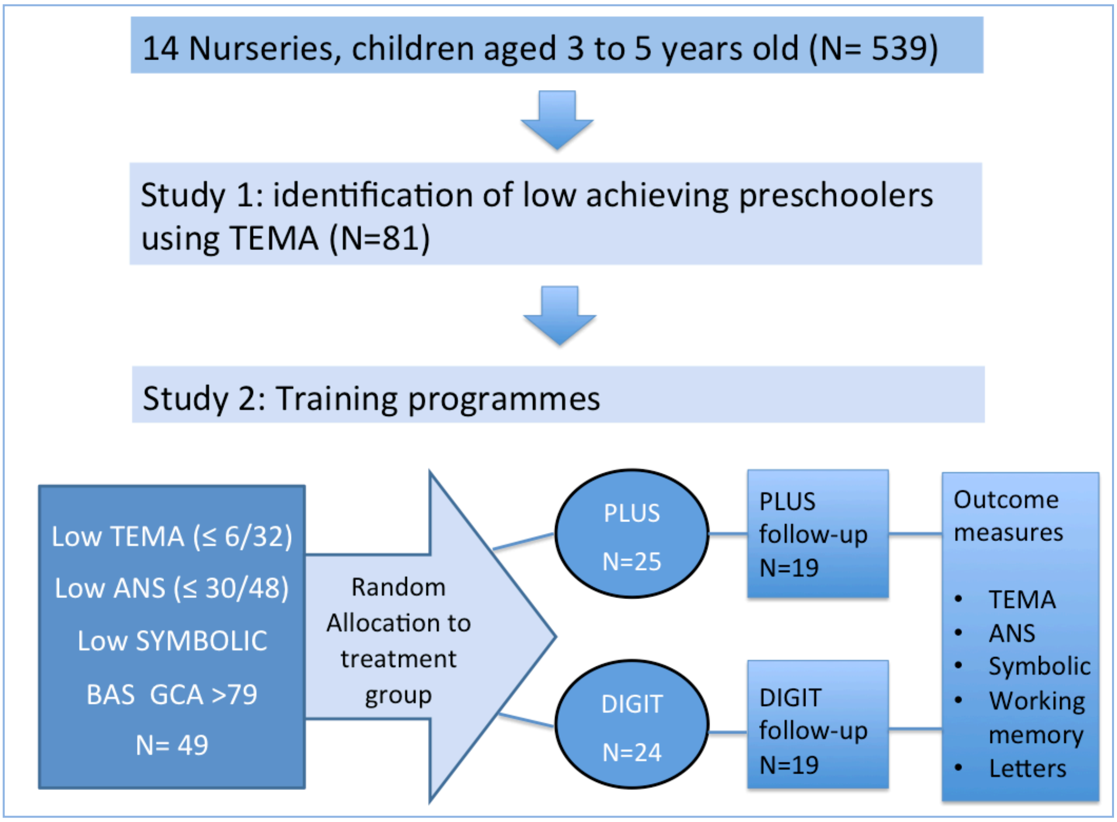


Figure 3. Overview of the children selected for Study 2 and allocation of the training programmes.

Materials

Training Programmes

Using an online random number generator, the LA children were randomly allocated to a non-symbolic training programme, called PLUS, or a symbolic one, called DIGIT. Each training programme took five weeks. During these 5 weeks we aimed to administer 20 sessions of approximately 10 minutes with each child.

Improving non-symbolic ANS abilities: PLUS games. This programme was designed by Dr Jo Van Herwegen and has been used in previous studies in the UK and in Italy. The 8 games included in this programme aim to improve children's confidence with numerosities (large versus small sets) as well as the ANS' acuity. Therefore, the games do not require children to provide an exact answer (e.g., there are ten fish), as long as the answer is approximately correct (e.g., there are lots of fish). During the PLUS games children were prevented from counting the quantities presented by showing stimuli very briefly, and children had to guess which amount showed more or less. The difficulty of the ratios used in the games increased once children were successful with easier ratios. Our past research has shown that by improving children's confidence with guessing and the understanding of numerosities (e.g., more and less), children's counting abilities indirectly improved as well. For a full description of the games see Van Herwegen and colleagues (2017).

Improving symbolic abilities: DIGIT games. The second programme included 8 games that focused on counting skills and digit recognition. These games are more akin to the math activities that are typically used in preschool settings in the UK. Half of the games required children to recognise digits and to put the digits in the correct order whilst the other games focused on counting abilities, including arranging and reciting the number line forward and backwards. These games were designed to teach these skills in a more structured and specific way, and similarly to the PLUS programme, the difficulty level was gradually increased during the course of the programme according to the abilities of the individual child. See Van Herwegen, Costa, Nicholson and Donlan (2018) for a full description of these games.

Pre-, post-, and follow-up assessments

Mathematical Achievement. The TEMA-3 was also administered immediately post-training and 6 months after the first session in order to evaluate whether the training programmes improved the participants' overall mathematical knowledge either short-term or long-term. As two lists (A and B counterbalanced) were used for TEMA to avoid practice effects, not all participants completed the exact same set of items and thus, ability scores were also compared between the three groups across the different time points.

ANS Abilities. The same ANS task as described in study 1 was administered, pre-, immediately post-intervention and at follow-up.

Counting Abilities. To assess counting abilities a *Counting Task* and an *Enumeration Task* was carried out. In the Counting Task children were asked to count out loud, as high as they can, starting from 1. The highest number correctly counted was recorded. In the *Enumeration Task* children were shown a line with 20 equally spaced dots and were asked to count the dots out loud pointing at each of the dots counted. The highest number counted correctly was recorded. Cronbach's alpha for reliability was .875.

Cardinal Principle. To assess cardinality abilities children were assessed on a *Give A Number* task (Wynn, 1992). In this task children were asked over fifteen trials to give the experimenter exactly 1, 2, 3, 4, and 5 beads from a clear bag with different sized beads. The children were then classified depending on the highest number they could correctly give on at least two trials out of three, i.e. 1-knower, 2-knower, 3-knower, 4-knower or 5-knower. Cronbach's alpha for reliability was .865.

Digit knowledge. Recognition of digits or numerical recognition was assessed by showing children flashcards with numbers 1 to 20 in random order. Children were asked to name the digit. The total number of digits correctly named was recorded. Cronbach's alpha for reliability was .938.

Letter Knowledge. This task was added as a control task, in that neither training programmes included any letters, and thus it was predicted that children should not improve on letter knowledge abilities. In this task children were shown all 26 lower-case letters of the alphabet in randomised order and children were asked to either name the letter or sound. The total number of correctly named letters was reported. Cronbach's alpha for reliability was .960.

Working Memory tasks. Working memory were assessed by a verbal and visual *Digit Span Forwards and Digit Span Backwards task* in which pre-schoolers were presented with a string of words or digits and asked to repeat the string back in the same or the reversed order as well as a *Verbal Dual Task* and a *Visual Dual Task* (Lanfranchi, Comoldi, & Vianello, 2004) in which the children had to clap every time they heard the word "ball" or saw a red square. In addition they had to remember the first word of a verbal string or the first position of a visual string.

Results

Repeated measures ANOVAs showed that both training groups had improved TEMA ability scores post-intervention as well as at follow-up, whilst the control group did not show increased scores (see Figure 4). This confirms that the training programmes impacted on children's mathematical abilities and that this difference could not be allocated to the general classroom activities that all children took part in. In addition, half of the children in the PLUS group and 32% of those in the DIGIT group were no longer classified as low performers six months after the start of the programmes.

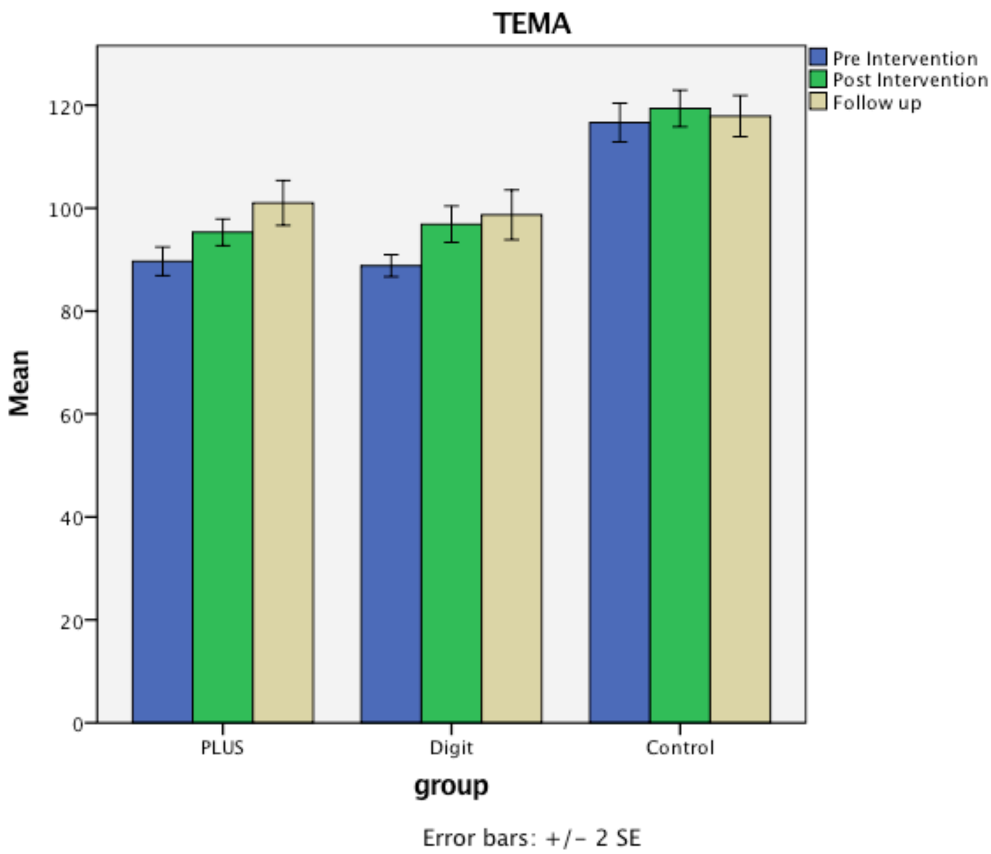


Figure 4. TEMA ability scores for the three groups at each time point.

Also for the ANS task both training groups improved immediately after the programmes, in contrast to children in the control group, and the PLUS group continued to improve six months later. At follow-up there were no more differences between the three groups. While before the training programmes all children in the training programmes performed at or below chance level on the ANS task, about 60% of the children in both training groups performed better than chance level on the ANS task (see Figure 5).

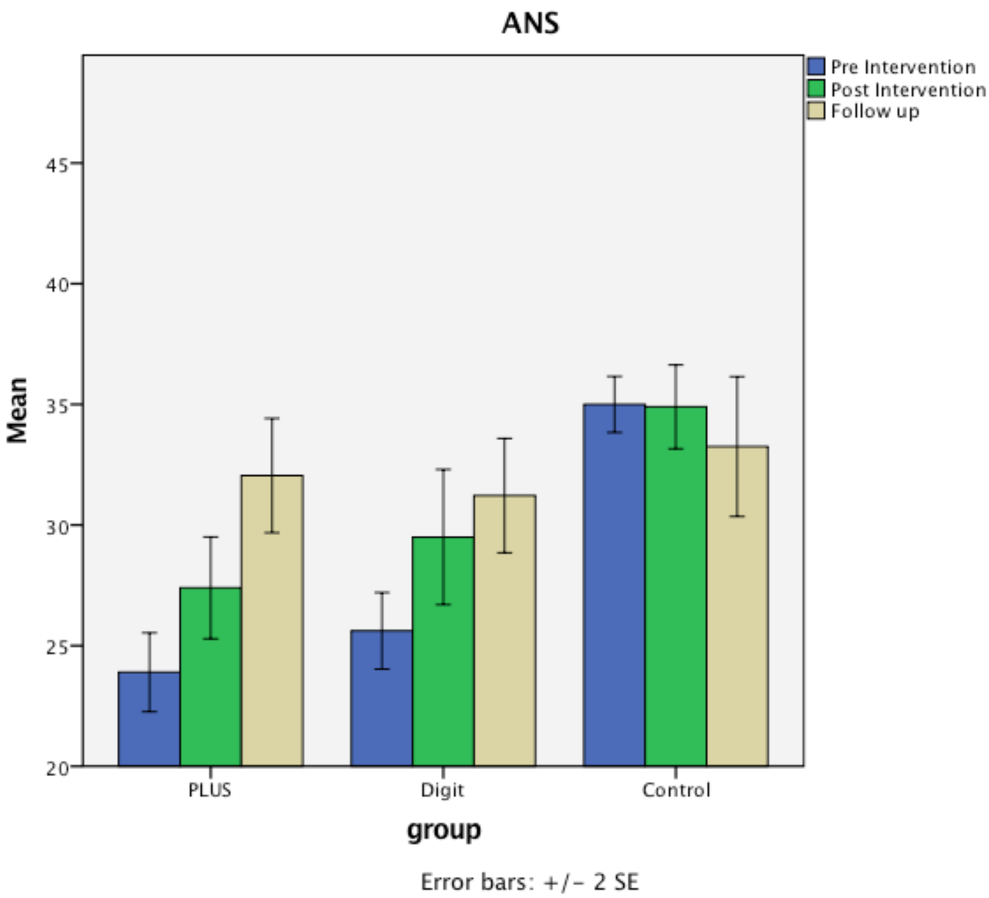


Figure 5. ANS raw scores for the three groups at each time point.

All the children in the three groups showed significantly improved scores for counting, enumeration, cardinality, and digit recognition immediately after the training as well as six months later (see Tables 2 and 3). Although the children in both training programmes performed below the control group, they did not differ from one another.

None of the groups showed significantly improved letter-recognition immediately after the training programmes. However, six months after the start all three groups did recognize more letters, possibly as a result of the education they had received in the meantime.

		Group					
		PLUS		DIGIT		Control	
Task	Time	Mean	SD	Mean	SD	Mean	SD
Counting	Pre intervention	9.00	4.49	10.63	5.45	20.65	9.12
	Post intervention	10.61	4.49	14.00	5.45	21.80	9.12
	Follow-up	15.16	6.54	20.16	10.00	31.20	9.46
Digit Recognition	Pre intervention	4.47	2.83	3.58	3.00	9.33	2.81
	Post intervention	5.92	3.15	5.16	4.75	10.20	4.34
	Follow-up	8.38	2.02	7.32	4.51	13.15	3.90
Enumeration	Pre intervention	10.63	4.83	11.92	1.61	16.35	3.46
	Post intervention	12.87	1.63	13.72	2.95	17.60	3.33
	Follow-up	13.59	2.63	14.89	4.34	20.00	0.00
Letter Recognition	Pre intervention	1.27	1.33	1.81	3.19	10.65	7.18
	Post intervention	1. 40	1.45	1,69	2.44	12.20	7.64
	Follow-up	3.00	2.14	2.94	2.74	14.55	6.96

Table 2. Performance scores for each group at each time point

		Group		
		PLUS	DIGIT	Control
Time	Category	Count	Count	Count
Pre intervention	0-knower	1	1	0
	1-knower	3	5	0
	2-knower	6	5	1
	3-knower	4	3	0
	4-knower	4	3	2
	5-knower	1	2	17
Post intervention	0-knower	0	0	0
	1-knower	4	3	0
	2-knower	2	4	0
	3-knower	5	4	0
	4-knower	5	3	2
	5-knower	2	5	18
Follow-up	0-knower	1	0	0

1-knower	0	1	0
2-knower	0	5	0
3-knower	3	1	0
4-knower	2	1	0
5-knower	12	11	18

Table 3. Number of children per category for the Cardinality task for each group

For all of the three visuo-spatial working memory tasks, all the groups showed improved performance scores at post- and at follow-up assessments (see Table 4). The training groups scored lower compared to the control group across all of the time points but they did not differ from one another. Similarly, for the verbal backwards span and verbal dual working memory tasks, performance in all three groups increased with time but the training groups, although not different from one another, performed lower than the control children. For the verbal forward memory span there were no differences between the three groups and all groups improved equally over time. This suggests that the training programmes improved children's number abilities but not their working memory abilities.

		Group					
		PLUS		DIGIT		Control	
Task	Time	Mean	SD	Mean	SD	Mean	SD
Visual STM forward span	Pre intervention	2.58	1.76	2.68	1.70	4.65	1.48
	Post intervention	3.05	1.36	2.95	.86	5.10	1.40
	Follow-up	3.53	1.46	2.89	1.56	5.30	1.30
Visual WM Backwards span	Pre intervention	.37	1.02	.37	.73	2.30	1.72
	Post intervention	.89	1.23	.95	1.42	3.15	1.55
	Follow-up	1.11	1.48	1.39	1.79	3.55	1.64
Visual WM dual task	Pre intervention	.89	1.29	.79	1.42	3.05	2.15
	Post intervention	.74	.99	1.42	1.78	3.85	2.38
	Follow-up	2.16	2.27	2.53	2.40	5.00	2.15
Verbal STM forward span	Pre intervention	4.16	1.08	4.21	.71	4.55	.77
	Post intervention	4.21	1.12	4.21	1.03	4.75	.97
	Follow-up	4.37	1.00	4.89	1.20	5.25	1.16
Verbal WM Backwards span	Pre intervention	.00	.00	.00	.00	1.10	1.23
	Post intervention	.00	.00	.16	.66	1.35	1.28
	Follow-up	.11	.45	.61	1.09	2.10	1.65

Verbal WM dual task	Pre intervention	.68	.98	1.16	1.09	2.65	2.03
	Post intervention	1.16	1.01	.74	.85	3.05	1.82
	Follow-up	1.74	1.52	1.89	1.49	3.55	1.85

Table 4. Overview of working memory scores for each group at each time point

Key Findings

Traditionally, preschool instruction in the UK is informal and happens during play or in games with LA children receiving very little additional support. Our findings from study 1 show that most preschoolers who perform low on mathematical ability tasks have impaired ANS abilities. Our results from study 2 show that both PLUS and DIGIT games improve preschoolers symbolic and non-symbolic abilities both short term (immediately after the training) as well as six months later. Although we were not able to follow-up these children longitudinally to examine which children continued to receive a formal diagnosis of MLD later on, around half of the children were no longer considered to be low performers at six months after the start of the study. This suggests that playing the PLUS as well as DIGIT games on a regular basis for just 5 weeks during preschool years allows children who perform low on mathematical ability tasks to have an optimal start to their schooling career and might prevent some children from receiving a formal diagnosis of MLD later on.

The fact that improving ANS abilities through PLUS games improved symbolic knowledge, and that improving symbolic knowledge in DIGIT games improved ANS abilities, suggests a complex interaction between symbolic and non-symbolic abilities and mathematical improvements during the preschool years.

Recommendations

We know from previous research that numerical abilities in preschool children predict long-term mathematical abilities and that mathematical abilities in childhood predict financial success in adulthood. Currently mathematics teaching in preschool settings focuses mainly on digit recognition and counting. However, mathematical abilities depend on a number of domain general and domain specific abilities, including ANS abilities. Our results show that ANS abilities are low in children who are at risk for mathematical difficulties but that ten-minute daily programmes that target number foundations in these preschoolers improved their symbolic and non-symbolic abilities after just 5 weeks. Based on the improvements of LA children in our study, playing the PLUS and DIGIT games on a daily basis at preschool would allow for some of the LA children to catch up before they even obtain an official diagnosis. However, the LA children did not catch up with the control group on all of the mathematical tasks and thus longer training programmes that include daily sessions of ten minutes across entire terms or preschool years might be needed in order to raise all LA children across all mathematical tasks.

In addition, many people in the UK are not confident in their mathematical abilities and even have mathematical anxiety. Yet, mathematics is part of daily life and numbers are embedded in almost every activity we do as adults. Thus, number foundation training should be part of everyday routines in the preschool settings (including lunch time, outdoor activities, when waiting in a line etc.) and should include a wide variety of activities that children enjoy so that they obtain a positive affinity with number foundations from preschool age onwards. As LA children struggle with these number foundations sessions should be kept short and allow the child to build on his existing knowledge and experience achievement and confidence with number foundations. The PLUS and DIGIT games are short games that meet these criteria and target different modalities (touch, vision, auditory) so that they can be used in a wide variety of environments and activities within the preschool setting.

In sum, preschool education should include short daily games, embedded into everyday routines, that target both symbolic as well as non-symbolic abilities, such as the PLUS and DIGIT games, in order to provide all children the best chance to perform at their potential and allow successful development of number abilities long term.

Further research is required, using larger sample sizes, to understand the complex relationship between non-symbolic and symbolic foundations for successful mathematical abilities and how these relationships differ in LA children. Using larger controlled trials will allow further examination of whether both the PLUS and DIGIT games are equally beneficial to children in the short term. Moreover, the use of larger sample sizes will allow for the examination of individual differences, including which

LA children benefit most from the interventions and how early years provision and capacity of the work force (e.g., education of the staff, confidence of the staff about their own mathematical abilities etc.) contribute to mathematical abilities. Understanding these individual differences may provide further insight into the underlying mechanisms of mathematical improvements.

Future research should also include a waiting control group, so that it can be clear that the PLUS and DIGIT games improve children's mathematical abilities above and beyond what can be achieved through natural developmental progression.

In addition to large-scale studies, longitudinal studies are required to provide further insight into how the relationships between mathematical outcomes and number foundations change over development and whether PLUS or DIGIT games are equally beneficial long term. Although the majority of the LA children in both training programmes no longer met the criteria of having mathematical difficulties, these results should be confirmed by longitudinal studies as well.

Seeing that ANS abilities develop from infancy onwards, more research is required to examine the relationship between ANS and symbolic number abilities in much younger typically developing children (aged 1.5 to 3 years old). This will require the development of new types of ANS tasks to assess such relationships.

Finally, a number of studies have shown that children with neurodevelopmental disorders, such as Williams syndrome and Down syndrome, have mathematical difficulties as well, including impaired ANS abilities from infancy onwards. Future studies should examine whether these populations benefit from the PLUS and DIGIT training programmes as well.

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