Wireless Information and Power Transfer: Spectral Efficiency Optimization for Asymmetric Full-Duplex Relay Systems

Zhongxiang Wei, *Student Member, IEEE,* Sumei Sun, *Fellow, IEEE,* Xu Zhu, *Senior Member, IEEE,* Yi Huang, *Senior Member, IEEE,* Linhao Dong, *Student Member, IEEE,* and Dong In Kim, *Senoir Member, IEEE.*

***Abstract*—To address the problem of imbalanced received signal-to-interference-and-noise ratio (SINR) at relay and destination nodes in wireless power transfer (WPT)-supported relay system, we propose a novel asymmetric full-duplex (FD) decode-and-forward (DF) WPT relay strategy, where the transmission time slots are not necessarily identical. By introducing asymmetric time slots, higher degree of freedom is obtained than the conventional symmetric WPT relay system. Furthermore, based on the asymmetric strategy, we develop a spectral efficiency (SE)-oriented resource allocation algorithm by jointly designing time slots, transmission power at source and relay. Simulation results show that the proposed asymmetric system demonstrates higher SE than the symmetric WPT-powered FD and the time-switching based FD relay systems. Besides, more energy can be harvested at the relay node by the proposed system benefiting from the enhanced degree of freedom, showing its applicability in WPT-powered relay systems.**

***Index Terms*—Full duplex, asymmetric relay, wireless power transfer, optimization**

I. INTRODUCTION

Wireless power transfer (WPT) technique refers to the transmission of electrical energy from a power source to an electrical node through wireless channels, which is an appealing solution to prolong the lifetime of wireless nodes [1] [2]. Based on the WPT technique, WPT-powered relay is an important application in the scenario where regular battery replacement is inconvenient or even impossible [3]. By harvesting wireless energy from the source, WPT-powered relay is able to continuously assist the communication between source and destination to maintain the network connectivity [4].

The early study in WPT relay system develops from half-duplex (HD) [5] [6] [7], which can be classified into two categories. The first one is based on time-splitting architecture, which needs one time slot for energy harvesting and two more time slots for a two-hop information transmission from source to destination [5]. The other category is based on power-splitting architecture [6] [7], where relay receives signal from source and harvests energy from a part of the signal in the first time slot, and forwards the remnant signals to destination in the second time slot. Both HD WPT relay systems suffer from spectral efficiency (SE) loss either due to the extra time slot or due to the partial relaying of the signal.

To recover the SE loss, [8] proposes to use full duplex (FD) technique. Different from traditional HD, FD operation allows simultaneous transmission and reception over the same frequency, and achieves approximately twice higher SE than HD. Inspired by high SE potential of FD, there has been growing interest in adopting FD in WPT relay system [9]. SE maximization in FD WPT relay system has been investigated in [10] [11], where the energy harvesting operates in the first time slot and information transmission is conducted in the second time slot. During the information transmission phase, self-interference, from relay's transmitter to its receiver, is treated as noise and thus self-interference cancellation (SIC) is required at FD relay. Joint optimization design of source and relay beamformer has been given in [12] for FD multi-input multi-output (MIMO) amplify-and-forward (AF) relay system, where FD relay receives signal and forwards it to users and WPT-powered multiuser resolve the received signal for decoding and energy harvesting.

As discussed in the aforementioned FD WPT relay systems in [10] [11] [12], however, self-interference is treated as noise and needs to be canceled as much as possible by SIC schemes in analog/digital domain [13]. To this end, additional power consumption is required, and complexity of hardware design at relay node is increased. Self-interference is the signal forwarded to the destination and is known by relay node, and hence can be utilized for energy harvesting by relay node [14]. In [15], a symmetric FD relay system has been investigated, where the relay receives information-bearing signal in the first time slot. In the second time slot, the received information signal is forwarded to destination, and concurrently, dedicated energy signals are sent from source to the receiver of relay for energy harvesting. As a result, the relay can harvest energy from both its transmitter and the source node. However, the authors in [15] ignore the fact that the signal-to-interference-and-noise ratio (SINR) of source-relay and relay-destination links can hardly be equivalent in a WPT relay system, which is because transmission power at relay could be much lower than that at source taking into account channel attenuation and low

---

Zhongxiang Wei and Yi Huang are with the Department of Electrical Engineering and Electronics, the University of Liverpool, Liverpool, L69 3GJ, UK, (email: {hszwei,yihuang}@liverpool.ac.uk).

Sumei Sun is with the Institute for Infocom Research, A*STAR, Singapore, 138632, Singapore, (email:sunsm@i2r.a-star.edu.sg).

Xu Zhu is with the Department of Electrical Engineering and Electronics, the University of Liverpool, Liverpool, L69 3GJ, UK. (email: xuzhu@liverpool.ac.uk).

Linhao Dong and Dong In Kim are with the College of Information and Communication Engineering, Sungkyunkwan University, 110745, Suwon, Korea, (email:linhaodon@ssku.edu, dikimg@skku.ac.kr).

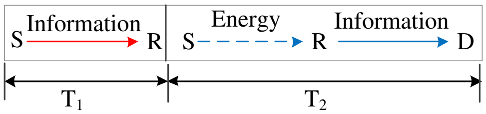


Fig. 1. Illustration of the asymmetric FD WPT-powered DF relay system, where red and blue lines denote the transmission in the first and second time slots, and full line and dashed lines denote energy-bearing signal and information signal transmission, respectively.

harvesting efficiency. As a result, the symmetric transmission system with uniform time slots inevitably reduces the resource utilization efficiency, and the end-to-end SE is always bounded by the relay-destination link.

To address the aforementioned outstanding issues, we propose an asymmetric FD WPT-powered relay system, which is the first work to the best of our knowledge. Our contributions are summarized as follows:

1) We propose a novel asymmetric FD decode-and-forward (DF) WPT-powered relay system to balance the SE of source-relay and relay-destination links, where the transmission time slots from the source and the relay are not necessarily identical. This leads to higher SE than its symmetric counterpart [15]. Also, the duration of two time slots is adaptively adjusted based on the relative SINR, making the proposed system aware of relay's position and allowing more energy harvested. Besides, our proposed system utilizes self-interference for energy harvesting, different from existing FD WPT-powered relay communication systems [10] [11], where self-interference is canceled as noise by SIC operation. As a result, no SIC is required at relay node, leading to simple hardware circuit design at relay node.

2) We maximize the end-to-end SE by jointly allocating time slots T_1 and T_2, transmission power p_s at source and transmission power p_r at relay for the proposed asymmetric system. A SE-oriented algorithm is proposed to this end, which demonstrates higher SE than the symmetric FD WPT-powered relay system [15] and the time-switching based FD WPT-powered relay system [10]. Furthermore, benefiting from the asymmetry of time slots, the proposed system can harvest more energy than the two benchmark systems.

II. SYSTEM MODEL

We consider an FD WPT-powered DF relay system, where the communication between source and destination is assisted by a WPT-powered relay and destination can not hear source directly [10] [15]. We assume that the relay relies on WPT from the source [5]. The relay is equipped with two antennas, one for receiving information and harvesting energy and the other for transmitting information to destination. The source and the destination are both equipped with one antenna. The channel state information is available at the source and relay node, obtained by the pilot assisted reciprocal channel estimation [16]. In the first time T_1, the source sends information signal to the relay, the received signal at the relay is expressed as

$$y_{r,1}[k] = \sqrt{p_s}h_{sr}x_{s,1}[k] + n_r[k], \tag{1}$$

where $x_{s,1}[k]$ is the information-bearing symbol sent by the source with normalized energy $\mathbb{E}\{|x_{s,1}[k]|^2\} = 1$. k denotes the symbol index. p_s is the transmission power at the source. h_{sr} denotes the channel response of source-relay link, which captures large-scale and small-scale fading. $n_r[k]$ is the noise introduced at the relay with variance σ^2. Therefore, the SE of source-relay link in the first time slot T_1 is calculated as

$$C_{SR} = \frac{T_1}{T}\log_2(1 + \frac{p_s|h_{sr}|^2}{\sigma^2}). \tag{2}$$

In the second time T_2, the received signal at the relay is forwarded to the destination, and the received signal by the destination is given as

$$y_d[k] = \sqrt{p_r}h_{rd}x_{s,1}[k-\tau] + n_d[k], \tag{3}$$

where τ denotes the symbol delay at the relay node. p_r denotes the transmission power at the relay. h_{rd} is channel response of relay-destination link and captures the effects of large/small-scale fading. $n_d[k]$ is the noise introduced at the destination with variance σ^2. Therefore, the SE of the relay-destination link is calculated as

$$C_{RD} = \frac{T_2}{T}\log_2(1 + \frac{p_r|h_{rd}|^2}{\sigma^2}). \tag{4}$$

Obviously, the end-to-end SE in the whole time $T = T_1 + T_2$ is expressed as

$$C = \min\{\frac{T_1}{T}\log_2(1 + \frac{p_s|h_{sr}|^2}{\sigma^2}), \frac{T_2}{T}\log_2(1 + \frac{p_r|h_{rd}|^2}{\sigma^2})\}. \tag{5}$$

Concurrently with information transmission by the relay in the second time slot T_2, wireless energy is sent from the source to the relay's receiver for energy harvesting. Since the relay's transmitter is transmitting information signal at the same time, the relay can also receive the self-interference from its transmitter, which can be collected by WPT. Therefore, the received signal at the relay's receiver during the second time slot T_2 is

$$y_{r,2}[k] = \sqrt{p_s}h_{sr}x_{s,2}[k] + \sqrt{p_r}h_{rr}x_{s,1}[k-\tau] + n_r[k], \tag{6}$$

where $x_{s,2}[k]$ is the energy-bearing symbol sent from the source to the relay in time duration T_2 with normalized energy $\mathbb{E}\{|x_{s,2}[k]|^2\} = 1$. h_{rr} is the self-interference channel gain from the relay's transmitter to its receiver capturing large/small-scale fading. We should notice that since the self-interference is collected for energy harvesting, thus no SIC is required. In summary, the total harvested energy at the relay in the second time slot T_2 is

$$E_r = \frac{T_2}{T}\omega E\{|\sqrt{p_s}h_{sr}x_{s,2}[k] + \sqrt{p_r}h_{rr}x_{s,1}[k-\tau]|^2\}, \tag{7}$$

where ω is the energy harvesting efficiency.

Since the transmitted signal $x_{s,1}[k-\tau]$ and $x_{s,2}[k]$ are known by the system, it is natural that we can set the state of $x_{s,2}[k]$ that $x_{s,2}[k]e^{j\angle h_{sr}} = x_{s,1}[k-\tau]e^{j\angle h_{rr}}$, ($\angle h$ denotes the phase of the complex number h) to let harvested energy maximized [15], which is given as $E_r = \frac{T_2}{T}\omega(p_s|h_{sr}|^2 + p_r|h_{rr}|^2)$. Accordingly, the maximum harvested power is $p_r = \omega(p_s|h_{sr}|^2 + p_r|h_{rr}|^2)$.

III. Problem Formulation

Define $C(T_1, T_2, p_s, p_r)$ as the SE (in bit/s/Hz) [1]. Accordingly, the SE-oriented resource allocation optimization problem is formulated as

$$
\begin{aligned}
P1: \ & \underset{T_1,T_2,p_s,p_r}{\operatorname{argmax}} \ C(T_1,T_2,p_s,p_r) = \\
& \min\{\frac{T_1}{T}\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2}), \frac{T_2}{T}\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})\},
\end{aligned} \tag{8}
$$

$s.t.$ $(C1): 0 \le p_s$, $(C2): p_s \le p_{max}$, $(C3): 0 \le p_r, (C4): p_r \le \omega(p_s|_{sr}|^2 + p_r|h_{rr}|^2)$, $(C5): T_1 + T_2 = T$. $(C1)$ and $(C2)$ imply the transmission power at the source should be non-negative and lower than a threshold p_{max}. $(C3)$ and $(C4)$ are the constraints imposed on transmission power at the relay, which should be non-negative and lower than the overall harvested power. $(C5)$ indicates that the summation of two time slots T_1 and T_2 should be equal to T.

It can be deduced that $P1$ is maximized only when $C_{SR} = C_{RD}$, which is easy to prove by a counter example. Therefore, the optimization problem $P1$ can be rewritten as

$$
\begin{aligned}
P2: \ & \underset{T_1,T_2,p_s,p_r}{\operatorname{argmax}} \ C(T_1,T_2,p_s,p_r) \\
& = \frac{T_1}{T}\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2}) + \frac{T_2}{T}\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2}),
\end{aligned} \tag{9}
$$

$s.t.$ $(C1), (C2), (C3), (C4), (C5)$, and $(C6)$: $\frac{T_1}{T}\log_2(1 + \frac{p_s|h_{sr}|^2}{\sigma^2}) = \frac{T_2}{T}\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})$.

The transformation from $P1$ to $P2$ is straightforward. Now we give Proposition 1 to help us solve the optimization problem $P2$.

Proposition 1: The transformed end-to-end SE $\frac{T_1}{T}\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2}) + \frac{T_2}{T}\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})$ is jointly-concave with respect to all the variables T_1, T_2, p_s, p_r in the feasible domain.

The proof of Proposition 1 is provided in APPENDIX A.

Because $P2$ is a standard concave optimization problem, as proved by Proposition 1, it can be solved by using the Karush-Kuhn-Tucker (KKT) conditions, and the Lagrangian function is given by

$$
\begin{aligned}
L = & \frac{T_1}{T}\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2}) + \frac{T_2}{T}\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2}) - \\
& \mu(p_s - p_{max}) - \phi\big(p_r - \omega(p_s|h_{sr}|^2 + p_r|h_{rr}|^2)\big) - \\
& \lambda\big(\frac{T_1}{T}\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2}) - \frac{T_2}{T}\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})\big) - \\
& \varphi(T_1 + T_2 - T),
\end{aligned} \tag{10}
$$

where μ, ϕ, φ and λ are Lagrange multipliers. It can be derived from the KKT conditions that $\mu \ge 0, \phi \ge 0, \varphi \ge 0$ and $\lambda \in (-1, 1)$. Let μ^*, ϕ^*, φ^* and λ^* denote the optimal Lagrange multipliers, it has

$$
\frac{\partial L}{\partial T_1} = \frac{(1-\lambda^*)}{T}\log_2(1+p_s^*\frac{|h_{sr}|^2}{\sigma^2}) - \varphi^*\{ \begin{matrix} <0, T_1^* = 0 \\ =0, T_1^* > 0 \end{matrix} \tag{11}
$$

$$
\frac{\partial L}{\partial T_2} = \frac{(1+\lambda^*)}{T}\log_2(1+p_r^*\frac{|h_{rd}|^2}{\sigma^2}) - \varphi^*\{ \begin{matrix} <0, T_2^* = 0 \\ =0, T_2^* > 0 \end{matrix} \tag{12}
$$

$$
\frac{\partial L}{\partial p_s} = \frac{T_1^*\frac{|h_{sr}|^2}{\sigma^2}(1-\lambda^*)}{T(1+p_s^*\frac{|h_{sr}|^2}{\sigma^2})} - \mu^* + \frac{\omega|h_{sr}|^2\phi^*}{1-\omega|h_{rr}|^2}\{ \begin{matrix} <0, p_s^* = 0 \\ =0, p_s^* > 0 \end{matrix} \tag{13}
$$

$$
\frac{\partial L}{\partial p_r} = \frac{T_2^*\frac{|h_{rd}|^2}{\sigma^2}(1+\lambda^*)}{T(1+p_r^*\frac{|h_{rd}|^2}{\sigma^2})} - \phi^*\{ \begin{matrix} <0, p_r^* = 0 \\ =0, p_r^* > 0 \end{matrix} \tag{14}
$$

From (11) and (12), we have the equality of

$$
\frac{T_1^*}{1-\lambda^*} = \frac{T_2^*}{1+\lambda^*}. \tag{15}
$$

According to (15) and constraint $(C5)$, it can be derived that the optimal time slot T_1 is

$$
T_1^* = \frac{1-\lambda^*}{2}T, \tag{16}
$$

and the optimal time slot T_2^* is

$$
T_2^* = \frac{1+\lambda^*}{2}T. \tag{17}
$$

By substituting (16) and (17) into (13) and (14), the optimal power allocation at the source and the relay are given by

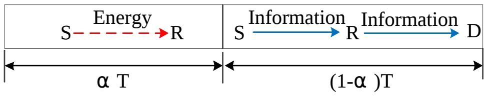
$$
p_s^* = [\frac{(1-\lambda^*)^2}{2(\mu^* - \frac{\omega|h_{sr}|^2\phi^*}{1-\omega|h_{rr}|^2})} - \frac{1}{\frac{|h_{sr}|^2}{\sigma^2}}]^+, \tag{18}
$$

$$
p_r^* = [\frac{(1+\lambda^*)^2}{2\phi^*} - \frac{1}{\frac{|h_{rd}|^2}{\sigma^2}}]^+, \tag{19}
$$

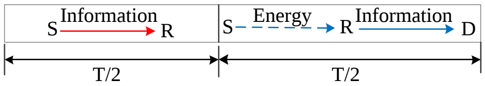
where $[x]^+ = \max\{0, x\}$. After obtaining the optimal results $T_1^*, T_2^*, p_s^*, p_r^*$, the next aim is to determine the optimal Lagrange multipliers. The sub-gradient method is used to update the multipliers μ, ϕ and λ, while multiplier φ has

---

[1] We have assumed that the transmission power at the relay relies on WPT from the source [5] [10], while the circuit power consumed by the relay is negligible as compared to the radiation power, which could be the case for low-complexity wireless devices (such as wireless sensors, tags, etc.) with simple electronics and low signal processing requirement [15] [17].



(a) Time switching-based FD WTP-enabled relay (TS-F) system



(b) Symmetric FD WTP-enabled relay (SYm-F) system

Fig. 2. (a) Illustration of the T-SF system. (b) Illustration of the Sym-F system.

been omitted naturally in deriving (15). According to the subgradient method, the updating rules can be given as

$$\begin{aligned} &\mu(j+1) = \mu(j) + \epsilon_1(p_s - p_{max}); \\ &\phi(j+1) = \phi(j) + \epsilon_2(p_r - \frac{\omega|h_{sr}|^2 p_s}{1-\omega|h_{rr}|^2}); \\ &\lambda(j+1) = \lambda(j) + \\ &\qquad \epsilon_3\big(\log_2(1 + p_s^* \frac{|h_{sr}|^2}{\sigma^2}) - \log_2(1 + p_r^* \frac{|h_{rd}|^2}{\sigma^2})\big); \end{aligned} \tag{20}$$

where ϵ_1, ϵ_2 and ϵ_3 are the corresponding step size [18]. Now the optimization problem $P2$ is readily solved. Based on our theoretic analysis, an asymmetric FD WPT-powered relay (Asym-F) algorithm is proposed, as summarized in Algorithm 1.

Algorithm 1 Asymmetric FD WPT-powered relay (Asym-F) Algorithm

```
Input: Initial multipliers $\lambda(0)$, $\phi(0)$ and $\mu(0)$, channel condition $h_{sr}, h_{rd}, h_{rr}$, and power related parameters, $p_{max}, \omega$.
Output: Optimal Lagrange multipliers $\lambda^*$, $\phi^*$ and $\mu^*$, optimal results $T_1^*$, $T_2^*$, $p_s^*$ and $p_r^*$.
repeat
    Allocate $T_1, T_2, p_s, p_r$ as
    $T_1^* = \frac{1-\lambda^*}{2}T$;
    $T_2^* = \frac{1+\lambda^*}{2}T$;
    $p_s^* = [\frac{(1-\lambda^*)^2}{2(\mu^* - \frac{\omega|h_{sr}|^2\phi^*}{1-\omega|h_{rr}|^2})} - \frac{1}{\frac{|h_{sr}|^2}{\sigma^2}}]^+$;
    $p_r^* = [\frac{(1+\lambda^*)^2}{2\phi^*} - \frac{1}{\frac{|h_{rd}|^2}{\sigma^2}}]^+$;
    Update multipliers
    $\mu(j+1) = \mu(j) + \epsilon_1(p_s - p_{max})$;
    $\phi(j+1) = \phi(j) + \epsilon_2(p_r - \frac{\omega|h_{sr}|^2 p_s}{1-\omega|h_{rr}|^2})$;
    $\lambda(j+1) = \lambda(j) + \epsilon_3(\log_2(1 + p_s^* \frac{|h_{sr}|^2}{\sigma^2}) - \log_2(1 + p_r^* \frac{|h_{rd}|^2}{\sigma^2}))$;
until Convergence
```

IV. SIMULATION RESULTS

The performance of the proposed asymmetric system with the Asym-F algorithm is demonstrated in this section, compared with the performance of the following two benchmarking systems:

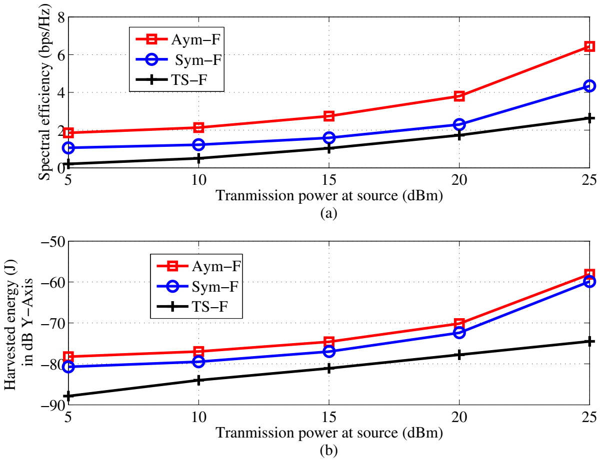


Fig. 3. SE performance vs. transmission power at source, where $d_{SR} = d_{RD} = 10$ m.

1) Time switching-based FD WPT-powered relay (TS-F) system [10], where the relay only harvests energy from the source in the first time slot T_1. In the second time slot T_2, the source node transmits information signal to the destination with the help of a FD relay. The self-interference is mitigated as noise by the relay node, and hence SIC amount is set to 80 dB in [10]. T_1 and T_2 are calculated according to ([10], Eq.(16)).

2) Symmetric FD WPT-powered relay (Sym-F) system [15], where the value of two time slots are identical. The illustration of the two benchmarking systems are depicted by Fig. 2.

The total bandwidth is $B = 1$ MHz. The PL model is adopted as PL $= 31.7 + 10\theta\log_{10}(d/d_0)$, where $\theta = 2.76$ is the PL exponent, d is the distance between two nodes, and $d_0 = 1$ m is the reference distance. The PL model corresponds to a practical scenario at 900 MHz [17]. Without loss of generality, we set that $d_{SD} = d_{SR} + d_{RD} = 20$ m and $d_{RR} = 0.1$ m, where d_{SD}, d_{SR}, d_{RD} and d_{RR} are the distances of source-destination, source-relay, relay-destination links and relay' transmitter to relay's receiver. The harvest efficiency ω is set to 0.8. The AWGN power spectral density is -174 dBm/Hz. We assume that the small-scale fading follows Rayleigh distribution.

Fig. 3 (a) shows the SE performance of three systems under different transmission power at source node, ranging from 5-25 dBm. It can be seen that the proposed Asym-F shows the highest SE among three systems, which is around 1.5 times as high as that of Sym-F and twice as high as that of TS-F system. It is because the Asym-F can adjust the value of time slots according to the relative SINR at relay and destination. Therefore, the SE of source-relay and relay-destination links can be well balanced. As a comparison, uniform time slots are allocated by the Sym-F system. Since the received SINR of relay-destination link could be much lower than that of source-relay link, the end-to-end SE is bounded by the second link, and a large portion of SE of source-relay link is wasted. Also, the TS-F shows the poorest SE performance among the three systems. It is because its relay's transmission power can be only harvested from the source node in the first time slot, while the self-interference is canceled as noise. Therefore, the SE of the TS-F is limited by its weak available transmission power.

Fig. 3 (b) demonstrates the harvested energy under different transmission power at source node. It can be observed that the proposed Asym-F system is superior to the two benchmarks. It is because the asymmetric system may allocate more time for the second time slot, helping relay harvest more energy from source and relay's transmitter. Besides, the TS-F system only harvests energy from source in the first time slot, therefore it shows the lowest harvested energy. At last, it is obvious that the harvested energy keeps slowly increasing with respect to the transmission power at source node by the TS-F system, while the harvested energy of Asym-F and Sym-F systems increases more rapidly than that of the TS-F system, which is beneficial from utilizing self-interference for energy harvesting.

V. CONCLUSION

In this paper, we have proposed a novel asymmetric full duplex (FD) wireless power transfer (WPT)-powered relay system, which addresses the problem of imbalanced signal-to-interference-and-noise ratio (SINR) at relay and destination nodes and obtains higher degrees of freedom than the symmetric counterpart [15]. Then an asymmetric FD WPT-powered relay (Asym-F) algorithm is developed to maximize overall spectral efficiency (SE) of the proposed system by jointly allocating time slots, transmission power at source and at relay. Compared to the symmetric FD relay system (Sym-F) [15] and time-switching based FD relay (TS-F) system [10], the proposed system achieves almost twice as high as SE. Furthermore, more energy can be harvested by our system than the two benchmarking systems in [10] and [15], showing its applicability in WPT-powered relay system.

APPENDIX A
PROOF OF PROPOSITION 1

By substituting $(C5)$ into $(C6)$, T_1 and T_2 can be calculated as

$$T_1 = \frac{T\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2})}{\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2})+\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})}, \tag{21}$$

$$T_2 = \frac{T\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})}{\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2})+\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})}. \tag{22}$$

By substituting (21) and (22) into (9), we get an equivalent objective function as

$$\underset{T_1,T_2,p_s,p_r}{\text{argmin}} \quad \frac{1}{2}\Bigg(\frac{1}{\log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2})}+\frac{1}{\log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})}\Bigg). \tag{23}$$

For simplicity, we consider two inter functions $y_1 = \log_2(1+\frac{p_s|h_{sr}|^2}{\sigma^2})$ and $y_2 = \log_2(1+\frac{p_r|h_{rd}|^2}{\sigma^2})$. It is easy to prove that y_1 and y_2 are both concave and positive functions in terms of variables. Now we define a function $u(x) = \frac{1}{x}$, whose first and second derivatives are calculated as $u'(x) = -\frac{1}{x^2}$ and $u''(x) = \frac{2}{x^3}$, respectively. It can be derived that $u(x)$ is a convex function and keeps decreasing when $x > 0$. Since we have proven that y_1 and y_2 are both concave and positive functions, it is readily to derive that $\frac{1}{y_1}$ and $\frac{1}{y_2}$ are both convex functions. At last, the convexity of (23) confirms the concavity of the objective function in (9).

REFERENCES

[1] K. Tomita, R. Shinoda, T. Kuroda, and H. Ishikuro, "1-W 3.3-16.3-V boosting wireless power transfer circuits with vector summing power controller," *IEEE J. Solid-State Circuit.*, vol. 47, no. 11, pp. 2576-2585, Nov. 2012.

[2] J. Wang, C. Jiang, Z. Han, Y. Ren, and L. Hanzo, "Network association strategies for an energy harvesting aided super-WiFi network relying on measured solar activity, *IEEE J. Sel. Areas Commun.*, vol. 34, no. 12, pp. 3785-3797, Dec. 2016.

[3] G. Amarasuriya, E. G. Larsson, and H. V. Poor, "Wireless information and power transfer in multiway massive MIMO relay networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 6, pp. 3837-3855, Jun. 2016.

[4] K. Liu, "Performance analysis of relay selection for cooperative relays based on wireless power transfer with finite energy storage," *IEEE Trans. Veh. Technol.*, vol. 65, no. 7, pp. 5110-5121, Jul. 2016.

[5] A. A. Nasir, X. Zhou, S. Durrani, and R. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," *IEEE Trans Wireless Commun.*, vol. 12, no. 7, pp. 3633-3636, July. 2013.

[6] R. Tao, A. Salem, and K. A. Hamdi," Adaptive relaying protocol for wireless power transfer and information processing," *IEEE Commun. Lett.*, vol. 20, no. 10, pp. 2027-2030, Oct. 2016.

[7] Z. Zhou, M. Peng, Z. Zhao, and Y. Li," Joint power splitting and antenna selection in energy harvesting relay channels," *IEEE Process. Lett.*, vol. 22, no. 7, pp. 823-827, Jul. 2015.

[8] Z. Wei, X. Zhu, S. Sun, Y. Huang, L. Dong, and Y. Jiang, "Full-duplex vs. half-duplex amplify-and-forward relaying: which is more energy efficient in 60 GHz dual-hop indoor wireless systems?" *IEEE J. Sel. Areas Commun.*, vol. 33, no. 12, pp. 2936-2947, Dec. 2015.

[9] H. Ju and R. Zhang, "Optimal resource allocation in full-duplex wireless-powered communication network," *IEEE Trans. Commun.*, vol. 62, no. 10, pp. 3528-3540, Oct. 2014.

[10] C. Zhong, H. A. Suraweera, G. Zheng, I. Krikidis, and Z. Zhang, "Wireless information and power transfer with full duplex relaying," *IEEE Trans. Commun.*, vol. 62, no. 10, pp. 3447-3461, Oct. 2014.

[11] M. Mohammadi, B. K. Chalise, H. A. Suraweera, C. Zhong, G. Zheng, and I. Krikidis, "Throughput analysis and optimization of wireless-powered multiple antenna full-duplex relay systems ," *IEEE Trans. Commun.*, vol. 64, no. 4, pp. 1769-1785, Apr. 2016.

[12] Z. Wen, X. Liu, N. C. Beaulieu, R. Wang, and S. Wang, "Joint source and relay beamforming design for full-duplex MIMO AF relay SWIPT systems," *IEEE Commun. Lett.*, vol. 20, no. 2, pp. 320-323, Feb. 2016.

[13] Z. Wei, X. Zhu, S. Sun, and Y. Huang, "Energy-efficiency oriented cross-layer optimization for multiuser full-duplex decode-and-forward indoor relay networks at 60 GHz," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 12, pp. 3366-3379, Dec. 2016.

[14] D. Hwang, K. C. Hwang, D. I. Kim, and T. J. Lee, "Self-energy recycling for RF powered multi-antenna relay channels," *IEEE Trans Wireless Commun.*, vol. 16, no. 2, pp. 812-824, Feb. 2017.

[15] Y. Zeng and R. Zhang, " Full-duplex wireless-powered relay with self-energy recycling," *IEEE Wireless Commun. Lett.*, vol. 4, no. 2, pp. 201-203, Apr. 2015.

[16] Y. Zeng, and R. Zhang, "Optimized training design for wireless energy transfer," *IEEE Trans. Commun.*, vol. 63, no. 2, pp. 536-550, Feb. 2015.

[17] Y. Zeng H. Chen, and R. Zhang, " Bidirectional wireless information and power transfer with a helping relay," *IEEE Wireless Commun. Lett.*, vol. 20, no. 5, pp. 862-865, May 2016.

[18] D. P. Bertsekas, *Nonlinear Programming*, 2nd edition. Athena Scientific, 1999.