

PVM-based intelligent predictive control of HVAC systems

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Abstract: This paper describes the application of a complete MBPC solution for existing HVAC systems, with a focus on the implementation of the objective function employed. Real-time results obtained with this solution, in terms of economical savings and thermal comfort, are compared with standard, temperature regulated control.¹

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1. INTRODUCTION

Model Based Predictive Control (MBPC) is perhaps the most proposed technique for HVAC control (Ruano et al., 2006, Ma et al., 2012, Castilla et al., 2014, Chen et al., 2015, Huang et al., 2015), since it offers an enormous potential for energy savings. Despite the large number of papers in this topic during the last years, there are only a few reported applications of the use of MBPC for existing buildings, under normal occupancy conditions, one of the first them being a previous work by the authors (Ferreira et al., 2012a). To the best of our knowledge, there is not yet a commercial application of MBPC for HVAC control. This paper is a step in this direction. Based on the approach proposed in (Ferreira et al., 2012a), researchers from the University of Algarve, together with the spin-off company EasySensing, Intelligent Systems, and an installation and maintenance company of HVAC systems, Rolear, Ltd, improved the existing MBPC approach (Ferreira et al., 2012a) and installed a complete solution, coined Intelligent MBPC (IMBPC) HVAC, in one building of the University. The current paper discusses the improvements proposed to the existing MBPC approach, focusing on the MBPC objective function, and the results obtained by the IMBPC HVAC system, in terms of energy and economical costs, and thermal comfort.

Section 2 describes the experimental setup. Section 3 introduces the Intelligent MBPC (IMBPC) system, and Section 4 discusses the MBPC objective function. Section 5 addresses the system installation, and Section 6 the results obtained. Conclusions are drawn in Section 7.

2. EXPERIMENTAL SETUP

The experiments were conducted in three lecture rooms, in the second floor of building 7 of the Gambelas campus of the

University of Algarve, in the south of Portugal. Rooms 2.13 and 2.12, are adjacent with walls facing west and north (room 2.12). Room 2.11 shares the same corridor with 2.12 and 2.13 and has walls exposed to the north and east. Room 2.11 has a capacity of 71 occupants, and an area of 253.13 m². The other two rooms have an area of 131.25 m² and a capacity of 31 occupants.

The HVAC system used in the experiments is composed of one Mitsubishi Variable Refrigerant Flow (VRF) system, with an outdoor air cooled inverter compressor PUHY-250YMF-C unit (denoted as outdoor unit), located on the building roof, connected to ceiling concealed ducted EFY-P63VMM indoor units (denoted as interior units). Rooms 2.12 and 2.13 have one internal unit, denoted UI 2.1.2 and UI 2.1.1, respectively, while room 2.11 has two (UI 2.1.5 and UI 2.1.6). The system can be centrally managed by a Task Vista 4 Building Management System (BMS), executing in a dedicated PC, to which all the units are connected via a LonWorks communication bus.

3. THE IMBPC HVAC SYSTEM

3.1 Hardware

IMBPC needs information on the weather variables. An intelligent, energy autonomous weather station provides atmospheric air temperature (TA), air relative humidity (HR), and global solar radiation (SR) measurements, as well as their forecasts over a user-defined Prediction Horizon (PH). All this data is wirelessly transmitted to a receptor, which in turn is connected to a TCP/IP network. Further details on the design and operation of the intelligent weather station can be found in (Mestre et al., 2015).

IMBPC needs also information about the climate of the rooms it will control. For this purpose, it uses small, cheap, self-

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powered wireless sensors that were designed and built specifically for building automation applications, by the authors. Three different types of devices were created (Receptor, Repeater and Transmitter). The following inside variables are measured by these Self-Powered Wireless Sensors (SPWS): air temperature (TA_i) and relative humidity (RH_i), movement (M), state of windows/doors, wall temperature and light.

3.2 Software

IMBPC assumes, for the moment, the existence of a BMS, able to measure and control the HVAC equipment. Three major software components exist: an interface to the BMS, a data acquisition module, which is responsible to communicate with the intelligent weather station, the SPWS, and with the BMS interface module, and a control module, which is responsible to execute model predictions, the MBPC algorithm, and the communication of the control actions to the BMS interface.

As the predictive models are Radial Basis Function (RBF) Neural Networks (NN), there is additionally the need to design them. In this work, the design is performed by a Multi-Objective Genetic Algorithm (MOGA). The RBF NNs are used as dynamic models, in NAR (Nonlinear AutoRegressive) or NARX (NAR with eXogenous inputs) configurations. Denoting as y the modelled variable, and considering only one exogenous input, v , the estimation (\hat{y}), at instant k , can be given as:

$$\hat{y}[k] = f \left(\begin{matrix} y[k-d_{o_1}], \dots & d_{o_n} \\ v[k-d_{i_1}], \dots & d_{i_n} \end{matrix} \right) = f(\{y[k]\}, \{v[k]\}) \quad (1)$$

As the objective is to determine the evolution of the forecasts over PH , (1) must be iterated over the horizon.

The MOGA based model design framework is an hybrid of an evolutionary algorithm and a derivative-based algorithm. The evolutionary part searches the admissible space of the number of neurons and the number of inputs (lags for the modelled and exogenous variables) for the RBF models. Before being evaluated in MOGA, each model has its parameters determined by a Levenberg-Marquardt algorithm (Levenberg, 1944, Marquardt, 1963) minimizing an error criterion that exploits the linear-nonlinear relationship of the RBF NN model parameters (Ruano et al., 1991, Ferreira et al., 2002). For more details on MOGA, please see, for instance (Ferreira and Ruano, 2011).

The IMPBC HVAC approach assumes the existence of schedules for each room under control. Denoting the occupation period by $t_{oc} = [t_{os} \ t_{oe}]$, t_{os} being the start of occupation, and t_{oe} its end, and by t_{op} a time (to be defined) before t_{os} , and by k_{os} , k_{oe} and k_{op} the corresponding sample indices, the approach can be formulated as:

$$\min_{U[k] \in v_{PH}} \left(\sum_{i=k+1}^{k+PH} J[i] \right)_{U_k} \Big|_{s.t. |\Theta[j]| < \Theta_T, j \in [k_{os} \ k_{oe}]} \quad (2)$$

In (2), Θ denotes the Predicted Mean Vote (PMV) index (Fanger, 1972), which will be used to measure thermal comfort, $\Theta_T = 0.5$ is the thermal comfort limit, and $U[k]$ represents a sequence of control actions, at time k , out of all the allowable sequences of control actions (v_{PH}) within the prediction horizon PH . $J[i]$ represents an estimate of the economic cost incurred in applying the control action $u[i]$.

The restriction in (2) needs a model to determine the evolution of Θ . It will be obtained using a RBF static model that approximates the mapping:

$$\Theta = f(TA_i, HR_i, \bar{T}_r) \quad (3)$$

where \bar{T}_r denotes the mean radiant temperature which will be, in this work, estimated by the temperature of the ceiling of the room. Model (3) will be selected from a data base of existing models, parameterized by a context vector $C = \{I_{cl}, M_r, V_{ai}\}$, where I_{cl} denotes the clothing insulation, M_r is the metabolic rate, and V_{ai} is the air velocity in the room. For an explanation of the use of context vectors and RBF NNs to estimate thermal comfort, please see (Ferreira et al., 2012d). The evolution of (3) over PH is obtained by the evolution of its arguments. As the objective function (2) needs to be computed, the next section discusses the way to do it.

4. MBPC OBJECTIVE FUNCTION

In (Ferreira et al., 2012a), the energy spent in the k^{th} interval was estimated as:

$$J[k] = \hat{E}[k] = \begin{cases} 1 + \frac{|TR[k] - \hat{TA}_i[k]|}{\lambda}, & TR[k] \neq 0 \\ 0, & TR[k] = 0 \end{cases} \quad (4)$$

In (4) TR denotes the air conditioning reference temperature (a value of 0 meaning that it is off, and λ a scaling factor).

Two alternatives were considered to estimate the energy: the use of a dynamic model and of a constant model. Noting that in the pilot installation a split system is used (one external unit is shared between up to four internal units), four models had to be determined: (A) internal unit is operating in the sampling interval; (B) internal units operating in the sampling interval; (c) internal units operating in the sampling interval; (D) internal units operating in the sampling interval.

Using historical data, it has been found that models achieve, on average, better results than the old method. The performance of the dynamic and static models is nearly equivalent. As the estimation procedure of the static model is simpler, and its application in real-time is less time-consuming, the static model, with constants show in Table 1, will be used.

Table 1. Constants used for energy estimation

Case	Constant
A	0.0603
B	0.0650
C	0.0805
D	0.1041

Employing the static model, and the conditions of the electricity contract of the University, the energy spent within each sampling interval can be easily formulated into an economical cost. For more details, please see (Ruano et al., 2015b).

As the previous MBPC algorithm (Ferreira et al., 2012c, Ferreira et al., 2012b) did not take into account occupation periods, the objective here is to apply it in accordance to the schedules, or even modify the algorithm itself. At each sampling interval, the following objective function is minimized:

$$\min_{U^{(k)} \in v_{PH}} J_{1:PH}[k] = \left(\sum_{i=k+1}^{k+PH} J[i] \right)_{U_k} \quad (5)$$

$$s.t. |\hat{\Theta}[i]| < \Theta_T, \quad i = 1:PH$$

The MBPC uses predictive models that forecast the climate inside the room throughout PH . A PH of 48 steps is used and, with a sampling interval of 5 minutes, this is translated in a 4 hours horizon. The simplest possibility is to start MBPC 4 hours before t_{os} , i.e., at $t_{op} = t_{os} - 4h$, and ending its execution at t_{oe} . A wiser solution is, before executing MBPC at any time $t \in [t_{op} \ t_{oe}]$, to verify, with the use of the room models, if within the whole interval $]t \ t + PH[\cap t_{oc}$ the room will be in thermal comfort without the need of using the air conditioning. If that is the case, there is no need to execute the MBPC. This solution is denoted as *Scheduled MBPC 4 hours*.

After analyzing historical data, it was verified that it is possible to assume that thermal comfort may be ensured during the occupation period, by starting MBPC closer to the start of the occupation period. We shall try a value of 2 hours ($t_{op} = t_{os} - 2h$), and denote this solution as *Scheduled MBPC 2 hours*.

The two previous solutions do not involve changing the previous MBPC algorithm. A full implementation of schedules needs, however, a reformulation of the algorithm. At a time t before the start of occupation, MBPC aims to achieve thermal comfort during the whole PH , i.e., for $]t \ t + PH[$. But it should only be concerned with the interval $]t \ t + PH[\cap t_{oc}$. On the other hand, at a time t near the end of the occupancy, i.e. $t + PH > t_{oe}$, we aim to guarantee thermal comfort only in the period $[t \ t_{oe}]$ and not in the full period $]t_{oe} \ t + PH[$. This can be done by changing the objective function of (5) to:

1. In case $t < t_{os}$:

$$\min_{U^{(k)} \in v_{PH}} \left(\sum_{i=k+1}^{k_{os}} J[i] \right)_{U_k} + \min_{U^{(k)} \in v_{PH}} \left(\sum_{i=k_{os}+1}^{k+PH} J[i] \right)_{U_k} \quad (6)$$

$$s.t. |\hat{\Theta}[i]| < \Theta_T$$

2. In case $t + PH > t_{oe}$:

$$\min_{U^{(k)} \in v_{PH}} \left(\sum_{i=k_{os}+1}^{k_{oe}} J[i] \right)_{U_k} \quad (7)$$

$$s.t. |\hat{\Theta}[i]| < \Theta_T$$

The reference temperature supplied by the MBPC has discrete, integer values. As such, it is possible to employ search techniques such as the Branch-and-Bound (BaB) method in order to find an optimal sequence of control actions that minimises the cost function. BaB methods are structured search techniques commonly used to solve complex discrete optimisation and combinatorial programming problems by dividing them into smaller sub-problems using a tree structure.

A discussion of the BaB algorithm is outside the scope of this paper. The reader may find information about the BaB algorithm in (Sousa et al., 1997) and its application for HVAC MBPC control in (Ferreira et al., 2012b, Ferreira et al., 2012a). Essentially, the BaB algorithm was modified in such a way that, for each node of the tree, constraints could be present or absent. This approach is denoted as *Modified MBPC*.

These 3 methods will be compared between themselves, and with the usual operation of the original algorithm, starting its execution in t_{os} and ending at t_{oe} . This will be denoted as *Standard MBPC*. The four methods will be compared in terms of the energy spent, the economic cost, and the percentage of PMV violation, given as:

$$V_{PMV} = \frac{\sum_{i=k_{os}}^{k_{oe}} \begin{cases} 1, & |\Theta[i]| > \Theta_T \\ 0, & |\Theta[i]| < \Theta_T \end{cases}}{k_{oe} - k_{os}} * 100\% \quad (8)$$

Several simulation scenarios, taken from days in January 2014, were considered. In all simulations, the values of the weather variables were the measured ones; the climate inside room 2.12 was obtained using models; for rooms 2.11 and 2.13 the measured values were employed. The static model was used for energy estimation, and the MBPC algorithm employed the economic cost function.

One scenario will be presented. It assumes that the room would be occupied for 145 minutes, starting at 21h30m. Figure 3 shows the state of the air conditioning of room 2.12 (please note that different y-values were used for the 4 methods, for the sake of clarity).

Scheduled MBPC 4 hours is operating for 72% of the time before the occupancy while this value is equal to 30% and 21% for Scheduled MBPC 2 hours and Modified MBPC, respectively. The standard MBPC is off for this whole period, as the control algorithm only starts in the beginning of the occupancy; however, in order to compensate for the discomfort in the room, the HVAC has to operate for 75%

within this period, whereas scheduled MBPC 4 hours, scheduled MBPC 2 hours and modified MBPC are running for only 36%, 14% and 35%, respectively.

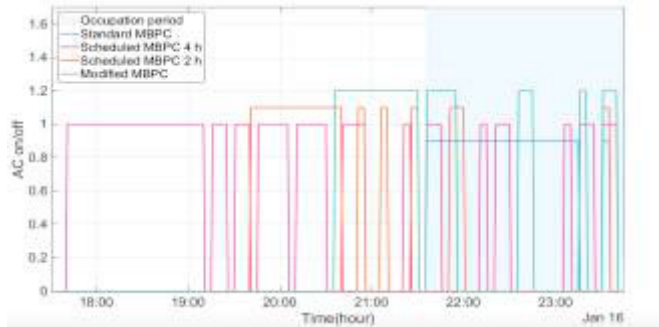


Figure 3 – Air conditioning states, with the four approaches

For visual representation of the thermal comfort criterion, the PMV evolutions for all approaches are plotted in figure 4. Standard MBPC is out of the comfort zone for 54% of the first occupation period, although the HVAC system is operating for 75% inside the occupation period. All the other MBPC methodologies lead the system to lie in thermal comfort zone for the whole considered period. It is also noticeable that with scheduled MBPC 4 hours and scheduled MBPC 2 hours the room enters the thermal comfort zone well before the start of the occupation period, which does not happen with modified MBPC. This, as it can be seen in Table 3, is translated in considerable savings for this last approach.

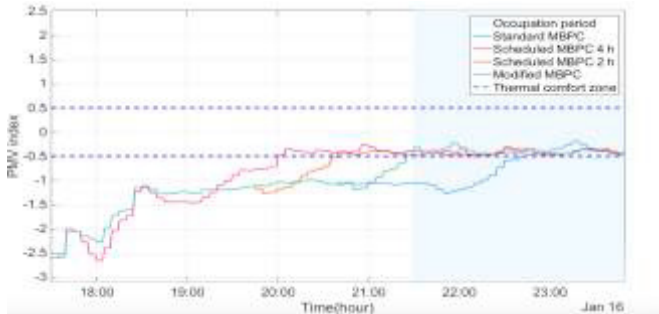


Figure 4 – PMV evolution with the 4 MBPC approaches

Table 3. Energy, cost and percentage of PMV violation for the 4 MBPC approaches

Method	4 hours before		Occupation		V_{PMV}
	Energy (kWh)	Cost (€)	Energy (kWh)	Cost (€)	
Standard MBPC	0.00	0.00	3.15	0.25	54%
Scheduled MBPC 4h	8.47	1.19	1.64	0.13	0%
Scheduled MBPC 2h	4.45	0.90	0.45	0.04	0%
Modified MBPC	2.98	0.51	1.48	0.13	0%

Analysing the results it can be concluded that the three new approaches achieve the goal of maintaining thermal comfort within scheduled periods of occupation. The Modified MBPC has the potential of obtaining higher savings of energy and economical costs, as it employs thermal comfort restrictions only during occupation periods, and not before or after.

4. SYSTEM INSTALLATION

Rolear technicians installed, in one week of June, the intelligent weather station, the SPWS in the three rooms, and

the interface to the BMS, the acquisition and control platforms. The data acquisition process started, taking place in the period between 2015-05-14 00:05:00+01:00 and 2015-06-06 12:00:00+01:00 (23 days and a half, corresponding to 6768 samples). The atmospheric data was collected by the intelligent weather station, the room data was obtained using the SPWS devices, and the HVAC data using the BMS interface software. During the acquisition period, two PRBS (Pseudo Random Binary Signals) sequences were included in the reference temperature of the 2.12 room air conditioning, in order to conveniently excite the system.

After the data has been acquired, the predictive models were designed. The first 300 samples could not be used for model design as atmospheric models use lags up to 300. The samples after 5001 (prediction set) will be used to assess the prediction performance of the models. As a prediction horizon of 48 steps will be used, the data available to build the models had $5000 - 300 - 4 = 4652$ samples. They were automatically divided into 60% (2790) for training, and 20% (931) for testing and validation. *ApproxHull* (please see (Ruano et al., 2015a) for details), was used to ensure that the vertices of the approximated convex hull of the whole design data were incorporated in the training set. After this step, the application of MOGA was divided in two phases: in the first one, standard parameterizations are used; the results obtained from the first design are analyzed, the search space is reduced and goals are set to the RMSE of the training sets and complexity for the second iteration.

4.1 Atmospheric Models

For this set of predictive models, the NAR formulation is used. The search space of the lags incorporates samples up to one day and one hour before, i.e., 300 samples.

4.2 Room Models

For this set of predictive models, the NARX architecture is used. The search space of the lags, for the modelled variable as well as the exogenous variables, incorporates samples up to 2 hours before, i.e., 24 samples. The air temperature, relative humidity and mean radiant temperature models have the form:

$$TA_i[k] = f\left(\left\{\begin{array}{l} TA_i[k], \{HR_i[k]\}, \{TA[k]\}, \\ \{SR[k]\}, \{TR[k]\}, \{M[k]\} \end{array}\right.\right) \quad (9)$$

$$HR_i[k] = f\left(\left\{\begin{array}{l} \{HR_i[k]\}, \{TA_i[k]\}, \{HR[k]\}, \\ \{SR[k]\}, \{TR[k]\} \end{array}\right.\right) \quad (10)$$

$$\bar{T}_r[k] = f\left(\left\{\begin{array}{l} \{\bar{T}_r[k]\}, \{TA_i[k]\}, \{TR[k]\}, \{M[k]\} \end{array}\right.\right) \quad (11)$$

5. REAL-TIME CONTROL RESULTS

Several real-time experiments were conducted in the two last weeks of June, 2015. During this period there are no lectures in the University, only exams. A list of exams scheduled for room 2.12 was obtained, in order to define MBPC control schedules.

To compare the performance of the MBPC system operating in room 2.12 against standard, temperature-regulated proprietary control, rooms, 2.11 and 2.13 were used. During

the periods that 2.12 was under MBPC scheduling, the indoor units of rooms 2.11 and 2.13 were turned on, with a constant reference temperature of 24°C.

One experiment, which took place in the 26th of June, will be described here. In this day, there were two exams scheduled for room 2.12: the first, between 9h to 11h and the other between 11h to 13h. As it was aimed to test the MBPC with multiple schedules, two scheduling periods were considered: between 9h to 13h05m, and between 16h to 18h05m.

Figures 5 to 7 show the evolution of different variables for this day, considering the three rooms. The first graph

illustrates the evolution of TA_i and of \bar{T}_r . The next graph shows the evolution of TA . The third graph depicts the evolution of RH and RH_i . The fourth and fifth graphs show the evolution of M and Θ , respectively. The last graph illustrates the evolution of TR . For the case of room 2.12 (figure 5), besides the measured variables the 1-step-ahead predictions are also shown. As it can be observed, these are very good. In the last line, the cost values are also shown for each instant.

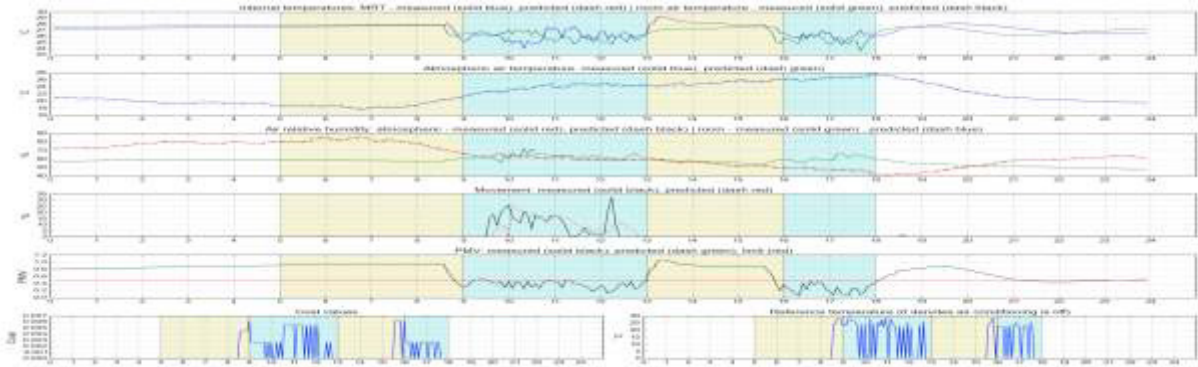


Figure 5 – Room 2.12 variables



Figure 6 – Room 2.11 variables

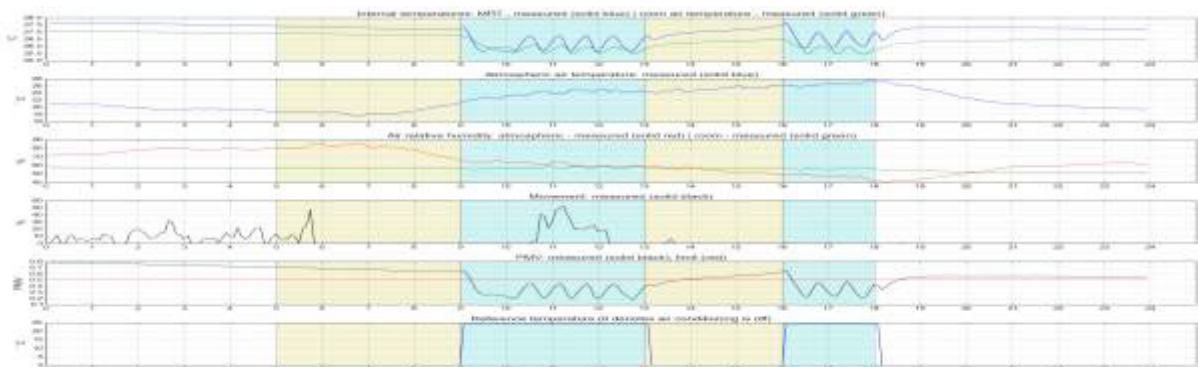


Figure 7 – Room 2.13 variables

Analysing figure 5, it can be concluded that there was some coincidence (although not perfect) between the scheduled occupation and the real one. Room 2.12 began to be used at around 9h30m (and not at 9h) and ended its occupation at around 12h30m (not at 13h). It was not occupied, as expected, in the afternoon period. During these periods, room

2.11 was empty and room 2.13 was occupied between 10h30m and 12h.

During the scheduled periods, room 2.12 was in thermal comfort (apart a few instants in the morning period); the same did not happen to rooms 2.11 and 2.13. In the sequel, the values of energy, economical cost, and percentage of PMV violation are show, for each room/unit, and for 5

different periods: 4 hours between the first scheduled period (*Prior 1*), within the first scheduled period (*Sch 1*), between the end of the first scheduled period and the start of the second scheduled period (*Prior 2*), within the second scheduled period (*Sch 2*), and during the union of the four last periods (*Total*).

Table 4 shows the energy (in kWh) consumed by the four units, for the 5 periods.

Table 4. Energy (in kWh) for each room and period.

Period /Room	Prior 1	Sch 1	Prior 2	Sch 2	Total
2.12/UI 1.1.2	1.50	4.35	0.94	2.17	8.97
2.11/UI 1.1.5	0.03	1.34	0.02	0.81	2.21
2.11/UI 1.1.6	0.03	10.23	0.02	5.13	15.42
2.13/UI 1.1.1	0.03	6.08	0.02	3.17	9.31

Unit 1.1.2 in room 2.12 consumes, during occupation periods, 42% and 70% of the energy spent by unit 1.1.6 and unit 1.1.1, respectively. The unit under MBPC control spends actually less energy considering the total period. This happens because, due to the outside weather, the unit does not need to cool the room so much time during the prior periods. The behaviour of unit 1.1.5 is totally different of unit 1.6. We have identified this problem in other experiments, which suggest that this unit might not be operating correctly.

Table 5 illustrates the economic costs (in €) for each unit and for all periods.

Table 5. Economical cost (in €) for each room and period.

Period /Room	Prior 1	Sch 1	Prior 2	Sch 2	Total
2.12/UI 1.1.2	0.12	0.86	0.09	0.22	1.30
2.11/UI 1.1.5	0.00	0.50	0.00	0.08	0.59
2.11/UI 1.1.6	0.00	1.88	0.00	0.53	2.42
2.13/UI 1.1.1	0.00	1.25	0.00	0.33	1.58

As it can be seen, MBPC achieves significant savings, in terms of economic costs. Finally, Table 6 depicts the percentage of PMV violation, for the three rooms. Although the objective is to maintain thermal comfort only during occupation periods, this value is shown for all periods, and also computed for the union of both occupation periods.

As expected, MBPC guarantees thermal comfort during the occupation periods. Due to the outside weather during that day, the differences to the other rooms are not so significant as verified in other experiments.

Table 6. Percentage of PMV violation for each room and period.

Period /Room	Prior 1	Sch 1	Prior 2	Sch 2	Both	Total
2.12	96%	2%	95%	0%	1%	51%
2.11	83%	17%	46%	9%	14%	43%
2.13	100%	6%	75%	13%	9%	51%

As expected, MBPC guarantees thermal comfort during the occupation periods. Due to the outside weather during that day, the differences to the other rooms are not so significant as verified in other experiments.

6. CONCLUSIONS

The proposed MBPC HVAC solution achieves important savings in the electricity bill, compared with scheduled standard temperature regulation control, while maintaining thermal comfort during the whole period of occupation,

which is not achieved with standard control. The expected savings will depend obviously on the outside weather, but also on the ratio between the prior time and the occupation time of the rooms. If the occupation time is large, savings in the order of 50% are to be expected.

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