Application of Inductive Heating to Granular Media: Modelling of Electrical Phenomena

P. DUQUENNE, A. DELTOUR‡ and G. LACOSTE*

I.N.P.T./G.S.I., 6, allée Emile Monso, B.P. 4038, 31029 Toulouse Cedex, France

A model is examined in order to predict the behaviour of a granular bed made with conductive particles subjected to an inductive electromagnetic field. The model shows how heat generated in the bed can be described by its electric impedance for the high-frequency generator required for inductive heating. Once the relationship between electrical characteristics and power dissipation has been established, comparisons between experimental and theoretical results are presented and the validity of the model is discussed.

Un modèle est proposé qui doit permettre de prévoir le comportement d'un lit granulaire électriquement conducteur chauffé par induction. Ce modèle exprime l'échauffement au sein du lit à partir de l'impédance électrique que ce lit représente pour le générateur Haute Fréquence utilisé. Ce travail justifie dans un premier temps l'équivalence entre les caractéristiques électriques d'un lit et la dissipation énergétique dont il est le lieu. Des résultats expérimentaux sont ensuite confrontés à ceux de la simulation et permettent de préciser le domaine de validité du modèle.

Keywords: inductive heating, electromagnetism, granular media.

Although the application of inductive heating to dispersed media seems to be of great interest in process engineering (Duquenne et al., 1993), there have been few publications regarding this subject. The characteristics of the application of inductive heating were studied and commented upon (Mioduszewski, 1982; Seghrouchni, 1989), or simulated (Delage and Ernst, 1983), or briefly examined as part of studies of broader scope. (Catton and Jacobson, 1987; Hardee and Nilson, 1977).

The widespread use of dispersed media in chemical engineering depends on the contact area produced and the turbulent fluid flow. The transport rate is usually expressed as the product of a contact surface area, a transfer coefficient and the difference in the potential of exchange (difference of temperature for heat transfer; difference of concentration for mass transfer). Granular media involve large contact areas and if the fluid flow is turbulent, high values of transfer coefficients are achieved, allowing large transfer rates. In this regard, the poor thermal conductivity of the granular solid phase often limits applications requiring heat transfer. This explains why the use of inductive heating is usually limited to the heating of homogeneous metallic media, such as reactor walls (Leclercq and Zampaolo, 1991) or hot-plates for food industries (Durosset, 1991). Uniformity of heat generation in granular media has already been demonstrated (Ul'yanov et al., 1982), investigated (Seghrouchni et al., 1991); and applied at the lab scale (Catton and Jakobson, 1987; Somerton et al., 1984). An example of this uniformity is displayed in Figure 1 (Duquenne et al., 1993); the case is of a 192 mm-I.D. reactor loaded with a 8 mm-diameter steel ball bed percolated by water (56.6 L/h). Power input was 3600 W at a frequency of 3.75 kHz (further description in Table 1). The fluid phase temperature was measured along the reactor at three different radial positions and no significant change was observed. Other experiments were carried out to make sure that the temperature stability was not a consequence of radial dispersion. However, uniformity of temperature was also observed in the solid phase. The fact that a stable temperature can be maintained suggests that the application of inductive heating to dispersed or granular media is likely to undergo a significant development in the next few years. Applications may concern improvement of existing apparatus such as fluid heaters (Seghrouchni et al., 1991), or new process devices: for example, *in situ* regeneration of spent activated carbon (Duquenne, 1992).

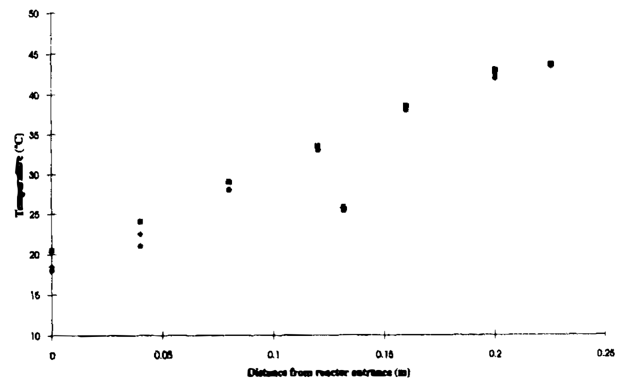

Figure 1 — Uniformity of heat generation by inductive heating in a steel ball bed percolated by a fluid; measurements at 3 different radial positions. ● : r = 0; ◆ : r = 48 mm; ■ : r = 96 mm.

The present work is a continuation of an experimental study (Duquenne et al., 1993) of the energy distribution in graular beds placed in an inductive field and aims more precisely at a good understanding of the phenomena. We have developed a model which predicts the heating of a bed comprised of conductive particles.

In this paper, we will briefly present the equations governing inductive heating phenomena and their application to a given shape of load in the reactor; in the following, "load" stands for whatever is put inside the inductor coil and inductively heated. After discussing the different resolution methods, a simplified solution is given. The solution is tested by comparison with experimental data, and the results of this comparison are commented upon.

Equations

Eddy currents generating so-called inductive heating are ruled by Maxwell's equations:

$$\mathrm{rot}(H) = i, \text{ with div } (D) = \Sigma \qquad (1)$$

*Author to whom correspondence should be addressed.

‡Present address: Institut de Mécanique des Fluides, E.N.S.E.E.I.H.T., 2 rue Camichel, 31071 Toulouse Cedex, France.

TABLE 1
Main Characteristics of Reactors and Loads

	Dimensions	Material and Conductivity	Reactor	Inductor	Frequency range
cylinders	diameter: 18 mm height: 195 mm				
tubes	diameters: 161 and 101 mm height: 195 mm	stainless steel $\sigma = 1.39 \times 10^6\ (\Omega.m)^{-1}$	192 mm I.D. 225 mm high	63 turns Ro = 0.8 Ω	4 to 20 kHz
	diameter: 8 mm				
ball beds	diameters: 1.8,3, 3.8,5 and 7.65 mm	lead $\sigma = 4 \times 10^6\ (\Omega.m)^{-1}$	82 mm I.D. 100 mm high	52 turns Ro = 0.33Ω	

$$\mathrm{rot}(E) = -\partial B/\partial t, \text{ with div } (B) = 0 \quad (2)$$

where $D = \epsilon E$, $B = \mu H$, and, if electrical displacement is neglected, $i = \sigma E$. It gives, for a steady-state regime:

$$\mathrm{rot}(H) = \sigma E, \text{ with div } (D) = 0 \quad (3)$$

$$\mathrm{rot}(E) = -\partial B/\partial t, \text{ with div } (B) = 0 \quad (4)$$

One can understand that, in general, resolution of such a system leads to extremely tedious calculations. When the inductively heated medium is a homogeneous solid, some geometric simplifications may be introduced in order to carry out these calculations — as, for example, in the cases of cylinders, or infinite or thin plates (Fournet, 1985). Moreover, if geometric simplifications are not applicable, the behaviour of a homogeneous solid may be deduced from the behaviour of a cylinder (for example) by the introduction of an approximate and empirical 'shape coefficient'. This saves a great deal of time by eliminating the need to perform direct calculations using Maxwell's equations (Duperrier, 1952).

CASE OF A CYLINDER: EQUIVALENT ELECTRICAL RESISTANCE

The case of a long cylinder plunged into a given uniform induction field oriented parallel to the cylinder axis will be examined. We adopt polar coordinates (r,θ,z), neglect displacement currents before conduction ones, and assume that the cylinder is long enough compared to its diameter so that phenomena do not depend on z.

In the following, we shall assume that a load, whatever it may be, has the same height L as the inductor coil.

Maxwell's equations are thus written as:

$\mathrm{rot}(H) = \sigma E$:

$$\sigma E_r = 0 \quad (5)$$

$$\sigma E_\theta = -\partial H_z/\partial r \quad (6)$$

$$\sigma E_z = 0 \quad (7)$$

$$\mathrm{rot}(E) = -\mu \partial H/\partial z = -\mu\frac{\partial H}{\partial t} - \partial E_\theta/\partial z = 0 \quad (8)$$

$$0 = 0 \quad (9)$$

$$1/r\partial(rE_\theta)/\partial r = -\mu\partial H_z/\partial t \quad (10)$$

These equations indicate that only E_θ and H_z are not uniformly zero. The system description then can be shortened to:

$$\partial^2 H_z/\partial r^2 + 1/r\partial H_z/\partial r = \sigma\mu\partial H_z/\partial t \quad (11)$$

$$\sigma E_\theta = -\partial H_z/\partial r \quad (12)$$

Resolution is usually performed by the application of Bessel and Kelvin functions (Fournet, 1985; Orfeuil, 1981) to Equation (11), assuming that the inductive field amplitude outside the cylinder, H_o, is deduced from inductor geometry, and that H_z has a finite value at r = 0; the electric field is then calculated using Equation (12). Since the profile of E_θ is determined inside the load, one can calculate energy dissipation by the Joule effect, or determine it with the help of the Poynting's vector flux through the cylinder surface.

Power generation inside the load (represented here by the cylinder) is generally expressed by the product of two terms, one of which incorporates the working parmeters (such as the cylinder's radius R and length L, its electrical conductivity σ and the amplitude of the magnetic field, H_0). The other term — F— is called the "power transmission factor". Thus, the power generation is described by:

$$W_e = 2^{0.5}\,(\pi RL/\sigma\delta)\,H_o^2\,F \quad (13)$$

where δ is the penetration depth of eddy currents in the material constituting the load, for a given frequency f of the alternating magnetic field (Duquenne et al., 1993; Seghrouchni, 1989; Delage and Ernst, 1983; Fournet, 1985; Duperrier, 1952; Orfeuil, 1981):

$$\delta = (\pi\mu\sigma f)^{-0.5} \quad (14)$$

In Equation (13), F is the power transmission factor (Fournet, 1985; Orfeuil, 1981), expressed as a combination of Bessel functions of the parameter $\times = 2^{0.5} R/\delta$:

$$F = 2^{0.5}\,\frac{ber\,(x)\,ber'\,(x) + bei\,(x)\,bei'\,(x)}{ber^2\,(x) + bei^2\,(x)} \quad (15)$$

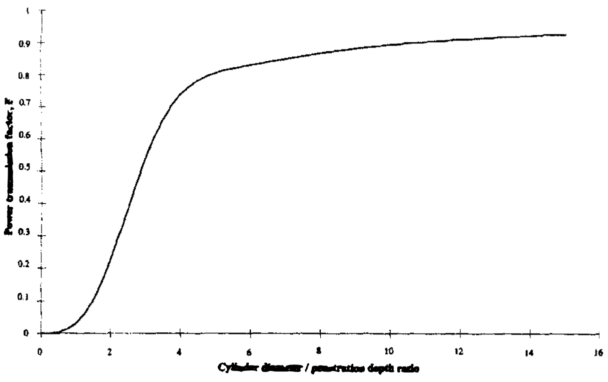

Figure 2 — Power transmission factor versus D_b/δ.

The evolution of the power transmission factor F versus D_b/δ is given in Figure 2. The main result concerning this factor is that, when the frequency increases, the penetration depth δ becomes negligible compared to characteristic dimensions of the material constituting the load, and power transferred to this load can be easily calcualted since $F = 1$. In this case, eddy currents are assumed to be uniform inside a zone of thickness δ near the periphery of the heated piece, and zero everywhere else. This simplification is no longer valid when δ and D_b are of comparable magnitude, and one must then take into consideration the non-uniform repartition of these eddy currents: this repartition is expressed by the power transmission factor F. However, Figure 2 clearly shows that heating efficiency for a given load increases with frequency, although it is not worth increasing this frequency further once 90% of the limit value is reached.

The magnetic field H_0, for an ideal inductor (which means that its length L is infinite compared to its diameter D_c), comprising N/L turns per unit length is known to be uniform inside the volume the coil delimits, and zero outside (Fournet, 1985). Inside the inductor coil, it keeps the same direction, and its value is given by:

$$H_0 = 2^{0.5}\ (N/L)\ I_e \qquad (16)$$

where H_0 is the peak value of the magnetic field, while I_e is the RMS value of current in the inductor coil (hence the factor $2^{0.5}$). If the inductor cannot be considered as ideal (i.e. if D_c is not negligible compared to L), the field generated will still be assumed to be uniform inside, its geometry only altering its value by Nagaoka's coefficient K_i (Delage and Ernst, 1983; Duperrier, 1952):

$$K_i = 1 + 0.44\ D_c/L;\ \text{hence, } H_0 = 2^{0.5}\ \frac{N}{LK_i}\ I_e\ . \qquad (17)$$

This last definition of H_0 is more reliable that that given by Equation (16) since real inductors seldom show infinite length-to-diameter ratios: further calculations will be performed using this definition.

Remembering that magnetic permittivity μ can be written as $\mu = \mu_0 \times \mu_r$ (where $\mu_0 = 4\pi \times 10^{-7}$ Henry/m and μ_r is the dimensionless relative magnetic permittivity of the material constituting the load), power dissipation in the load composed of a single cylinder can be expressed as:

$$W_e = 4(\pi^2\ R\ N^2/L)(10^{-7}\ \mu_r,\ f/\sigma)^{0.5}\ F\ I_e^2 \qquad (18)$$

Equation (18) shows that power transferred to the load is consumed by an equivalent electrical resistance R_{eq} directly connected with the generator, this resistance being defined by:

$$R_{eq} = 4(\pi^2\ R\ N^2/L)(10^{-7}\ \mu_r,\ f/\sigma)^{0.5}\ F \qquad (19)$$

THERMODYNAMIC DETERMINATION OF R_{eq}

Power dissipation can also be deduced from the load temperature evolution with time, if we assume that this temperature is uniform in the material. For a given mass m of material having a heat capacity C_p, the power transferred is:

$$W_e = m\ C_p\ \mathrm{d}T/dt \qquad (2)$$

which leads to a second determination of the load's apparent resistance "viewed" by the generator:

$$R_{eq} = m\ C_p\ (\mathrm{d}T/\mathrm{d}t)/I_e^2 \qquad (21)$$

If the inductor coil has a given electrical resistance R_o, we can define a value of heating efficiency: useful power is defined by $R_{eq}\ I_e^2$, and losses in the coil by $R_o\ I_e^2$. Thus efficiency is given by:

$$\eta = \frac{R_{eq}}{R_{eq} + R_o} \qquad (22)$$

CASE OF A GROUP OF CYLINDERS

The preceding developments remain valid for a group of cylinders, assuming that:

— the inductive field is the same for all cylinders. This means that it is uniform in the whole reactor (volume delimited by the inductor coil). In fact, the height-to-diameter ratio of the inductor must be as large as possible.

— the cylinders are all identical, and their diameter is small in relation to their length. In the case of a single cylinder, this ensures that z-dimension had no influence on any phenomena. Here, this hypothesis is also dictated by the fact that the cylinders must be electromagnetically independent from each other. The main reason is that calculating the mutual influences between cylinders would be feasible for a small number of them, but could hardly be carried out in any other case (Durand, 1968). In other words, it is important that, when swept by eddy currents, each cylinder can be considered as an ideal inductor, generating no electromagnetic field outside of the volume it occupies, thus introducing no mutual inductances with the other cylinders. In the case we chose, in which the inductor and the load always have the same length L, this condition is achieved when the preceding hypothesis is fulfilled.

In this case, if the load is constituted of N_c identical cylinders, the heat generation in them can be characterized as above by the equivalent electrical resistance they represent for the generator:

$$R_{eq} = 4\ N_c\ (\pi^2\ R\ N^2/L)(10^{-7}\ \mu_r,\ f/\sigma)^{0.5}\ F \qquad (23)$$

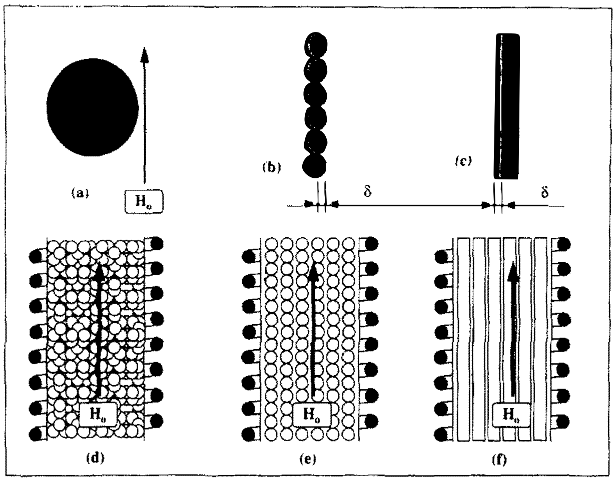

Figure 3 — (a) Path of the eddy currents at the surface of a sphere; (b): sectional view of eddy currents in a well-ordered pile of spheres; (c): sectional view of eddy currents in a cylinder; (d): analogy between a randomly-packed bed; (e): regular piles of particles; (f): a bundle of cylinders.

Case of a granular bed

The problem is totally different for a granular bed, since the shape of the particles constituting the load does not allow the geometric simplifications that made the resolution of Maxwell's equations feasible for a cylinder, even if we imagine that the particles are spherical.

In fact, a method exists that permits the prediction of a sphere's behaviour when in an inductive field, by considering this sphere as a pile of infinitely thin cylinders of different diameters (Horoskozo, 1978). However, this method does not solve the problem of mutual inductances between each sphere in the bed.

Another way of performing this calculation is illustrated in Figure 3. At a first glance, the behaviour of a sphere subjected to electromagnetic induction can be compared to that of a whorl (Horoskozo, 1978; Durand, 1968) as far as the generation of eddy currents is concerned; these currents develop themselves at the surface of the sphere, following planes perpendicular to the direction of the inductive field (Figure 3a), within a zone of thickness δ at the periphery of the sphere. In this case, a regular pile of spheres parallel to the field (Figure 3b) may be seen as equivalent to a cylinder made of the same material as the spheres, and whose surface is swept by eddy currents of dimension δ (Figure 3c). The problem is then the determination of its diameter, assuming that its length is the same as that of the pile. Since eddy currents are easier to calculate for cylinders than for spheres (Durand, 1968), the model we propose aims at considering the granular bed (Figure 3d) as a bundle of juxtaposed piles (Figure 3e), each pile being made of well aligned superposed balls, and thus comparable to a cylinder (Figure 3f) (Duquenne, 1992). Previous experiments (Seghrouchni, 1989) have shown that, in the case of metallic beds, contact between balls in the bed had no influence on the creation of eddy currents inside them; everything happens as if these balls were electrically insulated from one another.

For the model, we adopt the following hypothesis:

— the bed is comprised of identical and spherical particles;

— the functional cylinders are all identical, and their length is equal to the reactor height;

— the cylinders and particles have the same diameter D_b, and are made from the same material. The choice of an identical diameter is arbitrary, and will have to be reconsidered in case of divergence between theoretical and experimental results,

— the total mass of fictional cylinders is equal to that of the granular bed, so that the calculated equivalent resistance keeps the same meaning for both terms of heat generation (identical mass and heat capacity),

— granular bed porosity (leading to load mass) is uniform and is not influenced by the reactor walls. In fact, this is true if the ratio of the diameters of the reactor and the balls is greater than 10 (Haughey and Beveridge, 1969; Benenati and Brosilow, 1962).

According to these hypotheses, conservation of mass gives the number of fictional cylinders N_c:

$$N_c \; \pi(D_b{}^2/4) \; L = (1 - \psi) \; \pi(D_c{}^2/4)L \qquad (24)$$

where ψ is the bed void fraction; hence:

$$N_c = (1 - \psi) \; (D_c/D_b)^2 \qquad (25)$$

The equivalent resistance of a granular bed can thus be expressed as follows:

$$R_{eq} = 2 \; (1 - \psi) \; \frac{(\pi D_c \; N)^2}{(L \; D_b)} \; (10^{-7} \; \mu_r \, f/\sigma)^{0.5} \; F \ldots \qquad (26)$$

Experimental validation

Experiments have been performed with two different reactors encircled with Litz wire inductor coils, and three types of loads; Table 1 shows the main characteristics of these elements. A generator provides frequencies varying from 0 to 20 kHz, and an oscilloscope gives access to tension, current, frequency, and the difference in phase between tension and current. A set of Luxtron optic fiber temperature probes allows the acquisition of up to four temperature (Duquenne et al., 1993). The choice of optic fiber is governed by the fact that any metallic device inside the reactor would be inductively heated, altering the temperature readings.

Checking with cylinders

In the first series of measurements, the load consisted of various numbers of 18 mm-diameter, 195 mm-high stainless steel cylinders, the electrical conductivity of which was $\sigma = 1.39 \times 10^6$ $(\Omega.\text{m})^{-1}$. Stainless steel was chosen for its non-magnetic properties ($\mu_r = 1$), yet a well known property of inductive heating is that it becomes more efficient as the value of μ_r increases. Unfortunately, this value is seldom known precisely; this is why, in the case of a model validation, it is better to use a less efficient material with a more reliable determination of this parameter. These cylinders were insulated, both thermally and electrically, with glasswool that prevented any electrical contact.

A first and rapid investigation of temperature evolution at the surface of different rods showed that all experienced the same power dissipation when inductively heated together, which means that the inductive field can be taken as being constant in the reactor and confirms the results previously obtained (Duquenne et al., 1993). This investigation does not lead to the thermodynamical calculation of the load's equilvalent electrical resistance (Equation (21)), since we can not assume the temperature to be uniform inside the rods

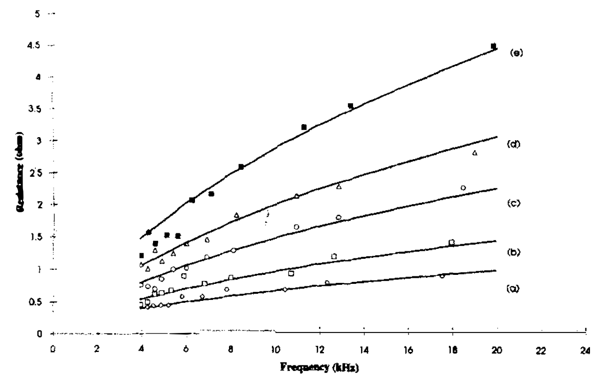

Figure 4 — Electrical equivalent resistances: load composed of different numbers of 18 mm-diameter, 195 mm-high stainless steel rods. Comparison between experimental data points and predicted lines. Number of rods: (a): 8; (b): 12; (c): 19; (d): 28; (e): 38.

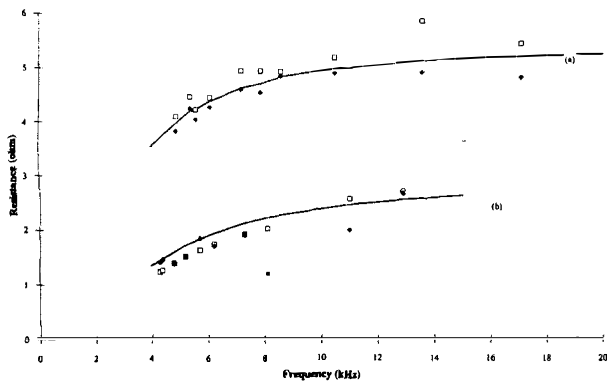

Figure 5 — Comparison between electrical (□) and thermodynamical (◆) determinations of load equivalent resistance (lines: calculated values) for 1 mm-thin stainless steel tubes. Tube diameters: (a): 161-mm O.D.; (b): 101-mm O.D.

at any given time. However, the electrical determination of this resistance, deduced from oscilloscope readings allowing calculation of load impedance, was compared to the model's predictions, as shown in Figure 4. These measurements were performed for a 4-20 kHz frequency range. Apart from a higher error for small values of resistance due to fluctuations in the tension supply at "low" frequencies, it can be noted that the calculations gave a reliable idea of the load's electrical behaviour. The observed agreement between the measurements and the predictions indicate that resistance — and thus efficiency — increases with frequency for a given load, and also with the reactor filling rate at a fixed frequency, accordingly to classical observations (Duperrier, 1952; Orfeuil, 1981) and calculations (Delage and Ernst, 1983) about inductive heating.

Electric and thermodynamic measurements

We must now justify that the resistance of a given load can be linked to a rise of temperature as theory would suggest. For this purpose, we have measured a load's electrical characteristics and the temperature evolution with time (Figure 5). So that we can assume that the temperature is uniform in the load at a fixed time, this load is a 1 mm-thick stainless steel tube insulated with glasswool. Two tubes

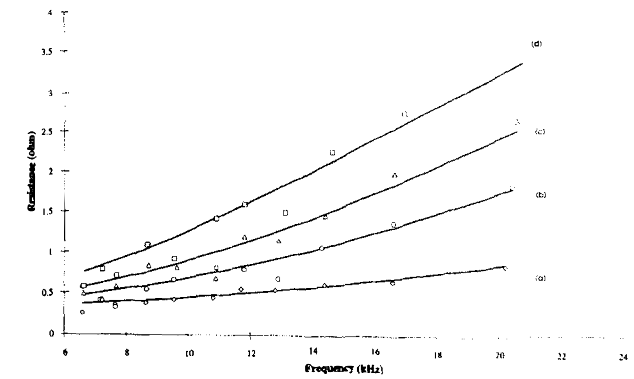

Figure 6 — Comparison between experimental and predicted resistances for granular beds: (a): 1.8 mm; (b): 3 mm; (c): 3.8 mm; (d): 5 mm.

were tested, one of 161 mm-outer diameter, having a mass of 0.949 kg, and the other of 101 mm O.D. with a mass of 0.588 kg. The graph shows good agreement between theoretical prediction and experimental result for both tubes. We can also observe that the model's predictions are still reliable, provided that a slight change is made in the definition of the power transmission factor F: the one given by Equation (15) is for a cylinder, whereas in the case of a tube, air takes the place of conductor material at $r = 0$, inducing modifications in the writing of boundary conditions (Orfeuil, 1981; Fournet, 1985; Duquenne, 1992).

Granular beds

We have shown that the model is adequate for the prediction of the electrical behaviour of a load composed of cylinders and that this electrical behaviour is representative of the thermal evolution of this load. We now have to check the hypothesis according to which a bed of spherical conductor particles could be seen as a group of juxtaposed cylinders.

In the following, the particles constituting the bed will be lead balls, having $\mu_r = 1$ and $\sigma = 4 \times 10^6$ $(\Omega.\text{m})^{-1}$. Due to weight problems the reactor has been changed for a smaller one having the same length-to-diameter ratio: 82 mm-I.D., 100 mm-high, the coil comprising 52 turns. Five different beds have been tested, with ball diameters of 1.8, 3, 3.8, 5 and 7.65 mm. Void fractions varied between 0.395 and 0.402.

Calculations of equivalent electrical resistances (Equation (26)) with the four smaller ball sizes gave satisfactory results when compared to experimental data (Figure 6), which allows us to state that a granular bed can be viewed as a bundle of cylinders of the same diameter as the particles and having the same total mass. Classical properties of induction heating of homogeneous pieces are suitable for dispersed media: energy transfer from the inductor coil to the load is improved by a rise of frequency, and efficiency is improved by an increase in particle size, even if the global dimensions of the bed are defined by those of the reactor, and thus constant.

Results concerning the 7.65 mm diameter balls reveal a divergence between experimental and predicted values (Figure 7), the difference increasing with frequency. Slight signs of this phenomenon can be interpreted from Figure 6 for the two greater diameters at high frequencies (greater

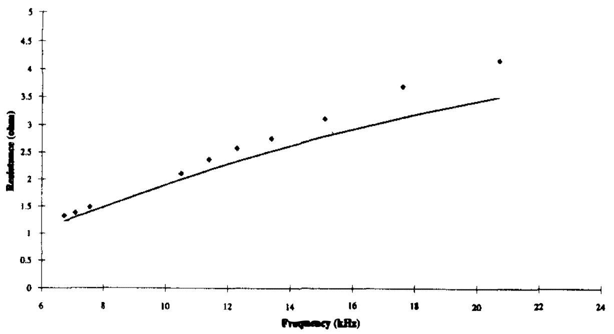

Figure 7 — Comparison between experimental and predicted resistances for granular beds: 7.65 mm.

values of resistance). Good agreement between theoretical and practical results indicates that the model is valid below a given diameter for spheres, a diameter which is estimated to be slightly greater than 5 mm.

The diameter of the balls has an important influence on one of the hypotheses we made: in our theory, fictional cylinders constituting the bed are supposed to be electromagnetically indpendent from one another, and thus must be of a very elongated shape so that they can be considered as ideal coils (infinite length). In this case, when swept by eddy currents, they neither modify the magnetic field outside the volume they define, nor perturb the currents passing on the surface of the surrounding cylinders. As our cylinders are all of the same length, equal to the reactor height, a critical diameter exists above which they can no longer be considered as being infinitely long (i.e. electromagnetically independent), and mutual inductances between them must be taken into account. All the currents imposed on any one given cylinder by its neighbours are added to the currents generated in that cylinder by the inductor. These additional currents all alternate with the same frequency, the result being another alternating current of the same frequency, but with a higher peak value than if all these cylinders were independent. This explanation can justify that a divergence appears above a given diameter, and that theoretical predictions are lower than experimental values (Duquenne, 1992).

Conclusion

A model has been proposed in order to predict the heat generation in a granular bed subjected to an inductive field. This model, based on the interpretation of the electrical characteristics of an inductor with its load, allows a fast and easy determination of the load's behaviour, since it avoids the tedious calculations usually linked to Maxwell's equations.

The validity of the electrical interpretation of phenomena has been tested and found to be reliable: as a consequence, it eliminates the necessity of placing temperature probes inside the load, permitting the data to be acquired externally.

The model has been tested with beds of varying granulometry and provided quite good predictions of their behaviour. However, it must always be kept in mind that calculated values are reliable only if the bed granulometry does not exceed a critical value. If this value depends only on the shape of fictional cylinders, the limiting factor would be a reactor height-to-particle diameter ratio greater than about 20, corresponding to a upper limit of 5 mm diameter for balls inside a 100 mm-high inductor coil.

Acknowledgement

The authors are grateful to the Conseil Régional Midi-Pyrénées and to the Agence Française pour la Maîtrise de l'Energie who financially supported this work, and especially want to express their gratitute to Mr. Michel Molinier for his far-seeing and competent advice.

Nomenclature

bei	= imaginary part of the Bessel function of the first type, zero order
bei'	= imaginary part of the Bessel function of the first type, first order
ber	= real part of the Bessel function of the first type, zero order
ber'	= real part of the Bessel function of the first type, first order
B	= magnetic induction, Tesla
C_p	= load heat capacity, J/kg · K
D	= electrical displacement field, A·s/m^2
D_b, D_c	= cylinder (or ball) and coil diameter, m
E, E_θ	= electric field, and its θ-component, V/m
f	= frequency, Hz
F	= power transmission factor –
H,H_0,H_z	= magnetic field — outside the load, and z-component, A·tr/m
i	= current density A/m^2
I_e	= effective current in the inductor coil, A
K_i	= Nagaoka coefficient of the inductor coil –
L	= cylinder length, m
m	= load mass, kg
N	= number of turns on the coil –
N_c	= number of cylinders constituting the load –
R	= cylinder radius, m
R_o	= inductor resistance, W
R_{eq}	= load equivalent resistance, W
T	= load temperature, °C
t	= time, s
W_e	= power dissipation in the load, W
x	= $2^{0.5}$ R/δ –

Greek letters

δ	= penetration depth m
ϵ	= electrical permittivity, F/m
η	= efficiency –
μ, μ_0	= magnetic permeability, and vacuum magnetic permeability, H/m
μ_r	= relative magnetic permeability –
σ	= electrical conductivity, $(\Omega.m)^{-1}$
Σ	= electrical charge density, C/m^3
ψ	= bed void fraction –

References

Benenati, R. F. and C. B. Brosilow, "Void Fraction Distribution in Beds of Sphers", AIChE J. **8** 3, 359-361 (1962)

Catton, I and J. O. Jakobsson, "The Effect of Pressure on Dryout of a Saturated Bed of Heat-Generating Particles", J. Heat Trans. ASME 109, 185-195 (1987).

Delage, D. and R. Ernst, "Modélisation Electrique d'un Système de Fusion par Induction en Creuset Froid", Rev. Gén. d'Electrothermie, **4**, 262-272 (1983).

Duperrier, S., "Pratique du Chauffage Electronique", Chiron, Paris (1952).

Duquenne, P., "Application du Chauffage par Induction Electromagnétique à des Milieux Granulaires", I.N.P.T. thesis, Toulouse, France (1992).

Duquenne, P., A. Deltour, and G. Lacoste, ''Application of Inductive Heating to Granular Media, Temperature Distribution in a Granular Bed'', Int. J. Heat Mass Trans. **36**, 2473-2478 (1993).

Durand, E., ''Magnétostatique'', Masson, Paris (1968).

Durosset, P., ''Applications du Chauffage par Induction dans les Industries Agro-Alimentaires'', in Proc. European Congress on Induction and its Industrial Applications, Strasbourg, France, March 20-22 (1991).

Fournet, G., ''Electromagnétisme à partir des Equations Locales'', Masson, Paris (1985).

Hardee, H. C. and R. H. Nilson, ''Natural Convection in Porous Media with Heat Generation'', Nuclear Sci. Eng. **63**, 119-132 (1977).

Haughey, D. P. and G. S. G. Beveridge, ''Structural Properties of Packed Beds — a Review'', Can. J. Chem. Eng. **47**, 130-144 (1969).

Horoskozo, E., ''Induction Heating of Rotating Bodies'', Electrowärme Int. **36**, B2, 89-93 (1978).

Leclerq, J. and M. Zampaolo, ''Application de l'Induction Electromagnétique au Chauffage des Réacteurs Chimiques'', in Proc. European Congress on Induction and its Industrial Applications, Strasbourg, France, (March 20-22 (1991).

Mioduszewski, D., ''Inductive Heating of Spent Granular Activated Carbon'', Q.E.D. Corporation, Ann Arbor, MI (1982).

Orfeuil, M., ''Electrothermie Industrielle'', Dunod, Paris (1981).

Seghrouchni, M., ''Contribution à l'Etude d'un Echangeur Granulaire à Termes Sources Inductifs Distribués'', I.N.P.T. thesis, Toulouse, France (1989).

Seghrouchni, M., G. Ferry, A. Deltour and G. Lacoste, ''Nouvelle Conception d'un Echangeur Granulaire à Termes Sources Inductifs Distribués: Caratère Volumique du Chauffage'', Rev. Gén. Therm. Fr. **349**, 17-23 (1991).

Somerton, C. W., J. M. McDonough and I. Catton, ''Natural Convection in a Volumetrically Heated Porous Layer'', J. Heat Trans. **106**, 241-244 (1984).

Ul'yanov, S. V., I. O. Protod'yakonov and P. G. Romankov, ''Model of Heat and Mass Transport in a Capillary-Porous Solid Heated by a High-Frequency Electromagnetic Field'', Teoreticheskie Osnovy Khimicheskoi Tekhnologii **16**(14), 468-473 (1982).

Manuscript received August 11, 1993; revised manuscript received June 22,1994; accepted for publication July 4, 1994.