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# Real-Time Scheduling in Wireless Networks

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We study a canonical real-time scheduling problem for time-slotted collocated wireless networks serving users with diverse real-time requirements and wireless loss patterns arising from fading. We study two traffic patterns: periodic arrivals with deadline equal to period, and renewal arrivals, in which a new packet from a user arrives when the current one leaves. Wireless channel conditions are modelled as Bernoulli losses.

For periodic arrivals, we prove that the optimal policy that minimizes the expected number of deadline misses has a strong property: it only switches between users on arrivals, successful completions or deadline expiry. When users have similar periods, this optimal policy is a linear switching curve characterized by a single number. Our result explicitly captures the trade-off between two competing aspects of the problem: the real-time tendency to schedule users in earliest-deadline-first (EDF) order, and the wireless tendency to exploit multi-user diversity by scheduling users with good channel conditions first. When users have similar channels, a common occurrence, we establish the optimality of EDF. For renewal arrivals, the optimal policy continues to have a switching structure, although not necessarily characterized by a single parameter. It schedules the user most likely to complete when it also has the earlier deadline. When the ``better'' user has a later deadline, it is scheduled till a worse user's deadline gets ``close'', and then the worse user is scheduled till expiry.

Our results for periodic arrivals are strong and significant. They reduce the search space for optimal wireless real-time scheduling policies by an exponential order of magnitude. They establish optimality of ``virtual-deadline-first'' policies, where each user's deadline is modified to take channel quality into account. Policies in this class are easy to implement in a distributed manner.

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# Real-time scheduling in wireless networks

Vivek Raghunathan, Min Cao and P. R. Kumar

**Abstract**

We study a canonical real-time scheduling problem for time-slotted collocated wireless networks serving users with diverse real-time requirements and wireless loss patterns arising from fading. We study two traffic patterns: periodic arrivals with deadline equal to period, and renewal arrivals, in which a new packet from a user arrives when the current one leaves. Wireless channel conditions are modelled as Bernoulli losses.

For periodic arrivals, we prove that the optimal policy that minimizes the expected number of deadline misses has a strong property: it only switches between users on arrivals, successful completions or deadline expiry. When users have similar periods, *this optimal policy is a linear switching curve characterized by a single number*. Our result *explicitly captures the trade-off between two competing aspects of the problem: the real-time tendency to schedule users in earliest-deadline-first (EDF) order, and the wireless tendency to exploit multi-user diversity by scheduling users with good channel conditions first.* When users have similar channels, a common occurrence, we establish the optimality of EDF. For renewal arrivals, the optimal policy continues to have a switching structure, although not necessarily characterized by a single parameter. It schedules the user most likely to complete when it also has the earlier deadline. When the "better" user has a later deadline, it is scheduled till a worse user's deadline gets "close", and then the worse user is scheduled till expiry.

Our results for periodic arrivals are strong and significant. *They reduce the search space for optimal wireless real-time scheduling policies by an exponential order of magnitude.* They establish optimality of "virtual-deadline-first" policies, where each user's deadline is modified to take channel quality into account. Policies in this class are easy to implement in a distributed manner.

**Index Terms**

Real-time, wireless networks, loss, opportunistic scheduling, dynamic programming.

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# I. Introduction

We study a canonical scheduling problem for wireless systems with real-time application latency requirements. Traditionally, such real-time guarantees are provided using 802.11 contention window modulation and frame spacing. The impact of wireless losses and fading diversity on real-time scheduler design have largely been ignored. This paper focuses on developing a firm theory for real-time wireless scheduling.

There are two orthogonal issues in such wireless real-time scheduler design: (a) design of scheduling *policies*, and (b) design of distributed *protocol mechanisms*. We focus on the policy aspect, noting that the optimal policy structure fundamentally influences mechanism design. (Arguably, the breakthrough in Liu-Layland's seminal work on hard real-time scheduling [1] was the proof of sufficiency of static priority CPU scheduling mechanisms; today, real-time systems mainly use static priority scheduling.)

We now detail how the wireless version of this problem differs from classical hard real-time scheduling. Schedulability analysis for hard real-time systems is typically based on the classical Liu-Layland framework [1]. A model of user arrivals and deadlines is assumed. It is known that rate-monotonic scheduling (RMS) and earliest-deadline-first (EDF) are respectively optimal static and dynamic priority policies respectively in the sense of maximizing the schedulable region. The context for such analysis is CPU scheduling and is completely deterministic. It does not account for the possibility that a user scheduled in a slot may not complete in that slot, due to, for example, the loss of a packet.

On the other hand, wireless systems are rife with such losses, caused by large scale signal attenuation, small scale multipath fading and wireless interference effects. Thus, unreliable intermediate quality links are common in practice [2], specially with IEEE 802.11 and 802.15.4 networks. A popular technique for combating link quality fluctuation is "opportunistic scheduling" [3], in which a sender preferentially transmits to receivers with better wireless channel quality.

We consider a canonical wireless real-time scheduling problem with two real-time users, each associated with a deadline. One user is less urgent than the other and has a longer deadline. Suppose the less urgent user has a "good" wireless channel, and the more urgent user has a "poor" wireless channel, as shown in Fig. 1. We pose the following question: how does one schedule these users so as to minimize the expected number of deadline misses? Real-time theory suggests the use of earliest deadline first (EDF). On the other hand, wireless networking theory suggests the use of opportunistic scheduling. The wireless real-time scheduling problem is an inter-play between these two competing aspects.

We study two different arrival models. The first is periodic arrivals [1], where jobs of user $i$ arrive every $t_i$ slots, with deadline equal to period. The second is renewal arrivals, in which user $i$ is associated with a deadline $t_i$ and generates a new packet only when its previous packet has left the system. The probability structure of the wireless channels of both users is assumed Bernoulli. We wish to find the policy that minimizes the expected number of deadline misses. This wireless real-time scheduling problem is an example of the restless bandits problem in dynamic programming; the general restless bandits problem [4] is still open.

Our main results are as follows:

1. For the periodic model, *we prove that the optimal policy only switches between users on deadline expiry, successful completions or new arrivals.* We further show that this policy always schedules the better user when it has the earlier deadline. When both users have similar periods, we prove that *the optimal policy has a linear switching structure specified by a single number*, which explicitly characterizes the inter-play between the real-time and wireless aspects of the problem. When both users have similar channel quality, we show that EDF is optimal.

2. For the renewal model, we prove a switching structure for the optimal policy. When both users are identical and have similar wireless channel quality, EDF is optimal.

This optimal scheduler is an inter-play between two competing tendencies: the need to schedule EDF to meet real-time requirements, and the need to schedule "good" users to maximize multi-user diversity gain. Our results are strong, especially for the periodic model, where *they characterize the optimal policy by a single threshold and thus, reduce the search space for optimal policies by an exponential order of magnitude.* For users with similar periods, *the optimal policy for users is characterized by a single number*. Such linear switching structure is unusually strong from a stochastic control point of view. Our results suggest the optimality of "virtual-deadline-first" policies, where each user's deadline is modified to take link quality into account. Such policies are easy to implement in a distributed manner.

Our results are also significant from a practical viewpoint. Our simulation studies (excluded here) show that in terms of total deadline misses, the optimal policies discussed in this paper are almost always 300% better than a pure real-time approach like EDF, or a pure wireless approach like prioritizing the better channel.

We describe our model, problem formulation and main results in Sections II and III. The periodic model is described in detail in Sections IV and V. The renewal model is described in detail in Sections VI and VII. Finally, we discuss extensions of our model in Section VIII and related work in Section IX.

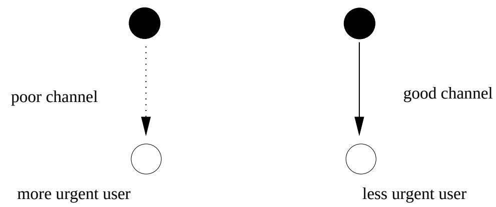


Fig. 1. Two wireless real-time users.

## II. Model and Problem Formulation

### A. Model

We consider a system with equal length time slots. Consider a canonical scenario with two collocated real-time users sharing a wireless channel, as in Fig. 1. Each job of user $i$ has a deadline $t_i$, and a service time requirement of exactly one slot. A job leaves the system if it is scheduled in a slot and completes successfully in that slot, or if its deadline expires. We consider two traffic patterns:

1. *Periodic arrivals*: user $i$'s jobs arrive $t_i$ slots apart.

2. *Renewal arrivals*: if a job of user $i$ with deadline $s$ is successfully serviced at $t \leq s$, then it leaves the system and is replaced at $t$ by a new job of that user with deadline $t + t_i$. Instead, if it could not be serviced by $s$, then it expires, and is replaced by a new job with deadline $s + t_i$.

Periodic arrivals are practically realistic, and are the most frequently used model in the real time systems literature [5]. Renewal arrivals are less common, and are sometimes used to model video data. Renewal arrivals also simplify the problem formulation and suggest a tractable analysis [1].

The collocated nature of the shared wireless medium is modelled by assuming that exactly one user can be scheduled in any slot. We model the wireless channel for user $i$ using a Bernoulli i.i.d. loss model. Given that user $i$ is scheduled in slot $k$, there is an independent and equal probability $p_i$ of the job not completing service ("packet lost"). The motivation for this loss model is threefold: (i) it corresponds to a popular metric used to characterize wireless links in practice, viz., the ETX metric [2], (ii) it models the well known fact that wireless links are not homogeneous and vary in quality from user to user depending on the location of the sender and the receiver [2], and (iii) its memoryless nature forces us to not rely on instantaneous knowledge of channel conditions, which is important when such conditions cannot be reliably inferred, e.g., in 802.11.

Traditional hard real-time schedulability guarantees are of the form: "If arrivals satisfy property P, then a particular policy $\pi$ ensures that no deadlines are missed". Random losses make this objective hard to attain. Instead, *we focus on finding a policy that minimizes the expected number of deadline misses*. We

[1] As we shall see later, this intuition is misguided.

formulate this as a stochastic control problem. Define the age of a user as the duration in slots since that user's last job arrived.

### B. Periodic Model: Problem Formulation

In the periodic model, we keep track of the ages of the two users, as well as which jobs are currently in the system. Define the state of the system by the four tuple $(i,j)^{xy}$, where $i$ is the age of the first user, $j$ is the age of the second user, and $x$ and $y$ are $0-1$ variables, where $1$ represents the completion of the job corresponding to the first (second) user. Given the state $(i(n),j(n))^{x(n)y(n)}$ of the system, and the user $u(n)$ scheduled at time $n$, the system evolves as a controlled Markov chain. Let $q_k[(i,j)^{xy} \to (i',j')^{x'y'}]$ be the probability of transition from state $(i,j)^{xy}$ to state $(i',j')^{x'y'}$, given that user $k$ is scheduled. The Markovian transition matrix for the periodic model is specified under both control actions $k \in \{1,2\}$ in Table I. We model the cost of deadline misses by associating a one-step cost function $c_k(i,j)^{xy}$ with

TABLE I

TRANSITION MATRIX FOR PERIODIC MODEL

| | | |
|---|---|---|
| $i < t_1-1,\ j < t_2-1$ | $q_1[(i,j)^{00} \to (i+1,j+1)^{00}] = p_1$ | $q_2[(i,j)^{00} \to (i+1,j+1)^{00}] = p_2$ |
| | $q_1[(i,j)^{00} \to (i+1,j+1)^{10}] = (1-p_1)$ | $q_2[(i,j)^{00} \to (i+1,j+1)^{01}] = (1-p_2)$ |
| $i = t_1-1,\ j < t_2-1$ | $q_1[(t_1-1,j)^{00} \to (0,j+1)^{00}] = 1$ | $q_2[(t_1-1,j)^{00} \to (0,j+1)^{00}] = p_2$ |
| | | $q_2[(t_1-1,j)^{00} \to (0,j+1)^{01}] = 1-p_2$ |
| $i < t_1-1,\ j = t_2-1$ | $q_1[(i,t_2-1)^{00} \to (i+1,0)^{00}] = p_1$ | $q_2[(i,t_2-1)^{00} \to (i+1,0)^{00}] = 1$ |
| | $q_1[(i,t_2-1)^{00} \to (i+1,0)^{10}] = 1-p_1$ | |
| $i = t_1-1,\ j = t_2-1$ | $q_1[(t_1-1,t_2-1)^{00} \to (0,0)^{00}] = 1$ | $q_2[(t_1-1,t_2-1)^{00} \to (0,0)^{00}] = 1$ |
| $i < t_1-1,\ j < t_2-1$ | $q_1[(i,j)^{10} \to (i+1,j+1)^{10}] = 1$ | $q_2[(i,j)^{10} \to (i+1,j+1)^{10}] = p_2$ |
| | | $q_2[(i,j)^{10} \to (i+1,j+1)^{11}] = (1-p_2)$ |
| $i = t_1-1,\ j < t_2-1$ | $q_1[(t_1-1,j)^{10} \to (0,j+1)^{00}] = 1$ | $q_2[(t_1-1,j)^{10} \to (0,j+1)^{00}] = p_2$ |
| | | $q_2[(t_1-1,j)^{10} \to (0,j+1)^{01}] = 1-p_2$ |
| $i < t_1-1,\ j = t_2-1$ | $q_1[(i,t_2-1)^{10} \to (i+1,0)^{10}] = 1$ | $q_2[(i,t_2-1)^{10} \to (i+1,0)^{10}] = 1$ |
| $i = t_1-1,\ j = t_2-1$ | $q_1[(t_1-1,t_2-1)^{10} \to (0,0)^{00}] = 1$ | $q_2[(t_1-1,t_2-1)^{10} \to (0,0)^{00}] = 1$ |
| $i < t_1-1,\ j < t_2-1$ | $q_1[(i,j)^{01} \to (i+1,j+1)^{01}] = p_1$ | $q_2[(i,j)^{01} \to (i+1,j+1)^{01}] = 1$ |
| | $q_1[(i,j)^{01} \to (i+1,j+1)^{11}] = (1-p_1)$ | |
| $i = t_1-1,\ j < t_2-1$ | $q_1[(t_1-1,j)^{01} \to (0,j+1)^{01}] = 1$ | $q_2[(t_1-1,j)^{01} \to (0,j+1)^{01}] = 1$ |
| $i < t_1-1,\ j = t_2-1$ | $q_1[(i,t_2-1)^{01} \to (i+1,0)^{00}] = p_1$ | $q_2[(i,t_2-1)^{01} \to (i+1,0)^{00}] = 1$ |
| | $q_1[(i,t_2-1)^{01} \to (i+1,0)^{10}] = 1-p_1$ | |
| $i = t_1-1,\ j = t_2-1$ | $q_1[(t_1-1,t_2-1)^{01} \to (0,0)^{00}] = 1$ | $q_2[(t_1-1,t_2-1)^{01} \to (0,0)^{00}] = 1$ |
| $i < t_1-1,\ j < t_2-1$ | $q_1[(i,j)^{11} \to (i+1,j+1)^{11}] = 1$ | $q_2[(i,j)^{11} \to (i+1,j+1)^{11}] = 1$ |
| $i = t_1-1,\ j < t_2-1$ | $q_1[(t_1-1,j)^{11} \to (0,j+1)^{01}] = 1$ | $q_2[(t_1-1,j)^{11} \to (0,j+1)^{01}] = 1$ |
| $i < t_1-1,\ j = t_2-1$ | $q_1[(i,t_2-1)^{11} \to (i+1,0)^{10}] = 1$ | $q_2[(i,t_2-1)^{11} \to (i+1,0)^{10}] = 1$ |
| $i = t_1-1,\ j = t_2-1$ | $q_1[(t_1-1,t_2-1)^{11} \to (0,0,0,0)] = 1$ | $q_2[(t_1-1,t_2-1)^{11} \to (0,0)^{00}] = 1$ |

every state $(i,j)^{xy}$ when user $k$ is scheduled, as shown in Table II.

Define $V^{\pi}(i,j)^{xy}$ as the expected number of average deadline misses over the infinite horizon when policy $\pi$ is used, given that we start in state $(i,j)^{xy}$, where the expectation is over the probability measure

TABLE II

ONE-STEP COST FUNCTION FOR PERIODIC MODEL

| | | |
|---|---|---|
| $i < t_1-1, j < t_2-1$ | $u_1(i,j)^{00}=0$ | $u_2(i,j)^{00}=0$ |
| $i = t_1-1, j < t_2-1$ | $u_1(i,j)^{00}=p_1$ | $u_2(i,j)^{00}=1$ |
| $i < t_1-1, j = t_2-1$ | $u_1(i,j)^{00}=1$ | $u_2(i,j)^{00}=p_2$ |
| $i = t_1-1, j = t_2-1$ | $u_1(i,j)^{00}$ $=1+p_1$ | $u_2(i,j)^{00}$ $=1+p_2$ |
| $i < t_1-1,\ j < t_2-1$ | $u_1(i,j)^{10}=0$ | $u_2(i,j)^{10}=0$ |
| $i = t_1-1,\ j < t_2-1$ | $u_1(i,j)^{10}=0$ | $u_2(i,j)^{10}=0$ |
| $i < t_1-1,\ j = t_2-1$ | $u_1(i,j)^{10}=1$ | $u_2(i,j)^{10}=p_2$ |
| $i = t_1-1,\ j = t_2-1$ | $u_1(i,j)^{10}=1$ | $u_2(i,j)^{00}=p_2$ |
| $i < t_1-1,\ j < t_2-1$ | $u_1(i,j)^{01}=0$ | $u_2(i,j)^{01}=0$ |
| $i = t_1-1,\ j < t_2-1$ | $u_1(i,j)^{01}=p_1$ | $u_2(i,j)^{01}=1$ |
| $i < t_1-1,\ j = t_2-1$ | $u_1(i,j)^{01}=0$ | $u_2(i,j)^{01}=0$ |
| $i = t_1-1,\ j = t_2-1$ | $u_1(i,j)^{01}=p_1$ | $u_2(i,j)^{01}=1$ |
| $i < t_1-1,\ j < t_2-1$ | $u_1(i,j)^{11}=0$ | $u_2(i,j)^{11}=0$ |
| $i = t_1-1,\ j < t_2-1$ | $u_1(i,j)^{11}=0$ | $u_2(i,j)^{11}=0$ |
| $i < t_1-1,\ j = t_2-1$ | $u_1(i,j)^{11}=0$ | $u_2(i,j)^{11}=0$ |
| $i = t_1-1,\ j = t_2-1$ | $u_1(i,j)^{11}=0$ | $u_2(i,j)^{11}=0$ |

induced by $\pi$. Then, we would like to find the optimal $\pi^*$ satisfying:

$$\pi^* = \text{argmin}_\pi V^\pi(i,j)^{xy}$$
$$V^\pi(i,j)^{xy} = \lim_{n\to\infty} E_\pi[\frac{1}{n}\sum_{n=1}^{\infty} c_{\pi_n}(i_n,j_n)^{x_n y_n}]. \tag{1}$$

### C. Renewal Model: Problem Formulation

The system state is the two-tuple $(i,j)$ describing the ages of the two users. Given the state $(i(n),j(n))$ and the user $u(n)$ scheduled at time $n$, the system evolves as a controlled Markov chain. Let $q_k[(i,j)\to(i',j')]$ be the probability of transition from state $(i,j)$ to state $(i',j')$, given that user $k$ is scheduled. The Markovian transition matrix is specified for the renewal model under both control actions $k\in\{1,2\}$ in Table III. $c_k(i,j)$ models the cost of deadline misses, with $c_k(i,j)$ representing the one-step cost in state $(i,j)$ when user $k$ is scheduled, as shown in Table IV.

TABLE III

TRANSITION MATRIX FOR RENEWAL MODEL

| | | |
|---|---|---|
| $i < t_1-1,\ j < t_2-1$ | $q_1[(i,j)\to(i+1,j+1)]=p_1$ | $q_2[(i,j)\to(i+1,j+1)]=p_2$ |
| | $q_1[(i,j)\to(0,j+1)]=(1-p_1)$ | $q_2[(i,j)\to(i+1,0)]=(1-p_2)$ |
| $i = t_1-1,\ j < t_2-1$ | $q_1[(t_1-1,j)\to(0,j+1)]=1$ | $q_2[(t_1-1,j)\to(0,j+1)]=p_2$ |
| | | $q_2[(t_1-1,j)\to(0,0)]=1-p_2$ |
| $i < t_1-1,\ j = t_2-1$ | $q_1[(i,t_2-1)\to(i+1,0)]=p_1$ | $q_2[(i,t_2-1)\to(i+1,0)]=1$ |
| | $q_1[(i,t_2-1)\to(0,0)]=1-p_1$ | |
| $i = t_1-1,\ j = t_2-1$ | $q_1[(t_1-1,t_2-1)\to(0,0)]=1$ | $q_2[(t_1-1,t_2-1)\to(0,0)]=1$ |

Defining the value function $V^\pi(i,j)$ as the expected number of average deadline misses over the infinite horizon when policy $\pi$ is used, given that we start in state $(i,j)$, our goal is to find the optimal

TABLE IV

ONE-STEP COST FUNCTION FOR RENEWAL MODEL

| | | |
|---|---|---|
| $i < t_1 - 1, j < t_2 - 1$ | $u_1(i,j) = 0$ | $u_2(i,j) = 0$ |
| $i = t_1 - 1, j < t_2 - 1$ | $u_1(i,j) = p_1$ | $u_2(i,j) = 1$ |
| $i < t_1 - 1, j = t_2 - 1$ | $u_1(i,j) = 1$ | $u_2(i,j) = p_2$ |
| $i = t_1 - 1, j = t_2 - 1$ | $u_1(i,j) = 1 + p_1$ | $u_2(i,j) = 1 + p_2$ |

$\pi^*$ satisfying:

$$\pi^* = \text{argmin}_\pi V^\pi(i,j) = \lim_{n\to\infty} E_\pi[\frac{1}{n}\sum_{n=1}^{\infty} c_{\pi_n}(i_n, j_n)]$$

*Note 1: The renewal model description is more compact than the periodic model, because the renewal model always ensures that there is exactly one job corresponding to each user in the system. With periodic arrivals, we need to keep track of which jobs are in the system, increasing the state space dimensionality.*

*On the other hand, periodic arrivals have completely deterministic job arrival sequences, decided by the start state. In contrast, the renewal model's arrival sequence in the future is policy-dependent, and is influenced by actions taken in the present. Renewal arrivals introduce a perverse incentive to "postpone work", since completing jobs early generates new jobs faster, and thus "a policy that completes work early is forced to do more work." As we shall see later, this leads to weaker results for the renewal model.*

## III. MAIN RESULTS

### A. Periodic Model

*Theorem 1:* The optimal policy $\pi^*$ for the wireless real-time stochastic scheduling problem (1) for the periodic model has the following strong property: along any sample path, it switches between users only on a new arrival, successful completion, or deadline expiry. Equivalently, when both jobs are in the system, the optimal policy is constant along diagonals in the state space, as illustrated in Fig. 2. Furthermore, it has the "weak EDF" property, i.e., it always schedules the better user, i.e., the user with smaller $p_i$ and thus, the one more likely to complete, whenever it has an earlier deadline. Thus, assuming $p_2 \le p_1$,

$$\pi^*(i,j)^{00} = \pi^*(i+1, j+1)^{00}, \tag{2}$$

$$i \le j \Rightarrow \pi^*(i,j)^{00} = 2. \tag{3}$$

*Theorem 2:* The optimal policy $\pi^*$ has a linear switching structure when $t_1 = t_2$. This structure is

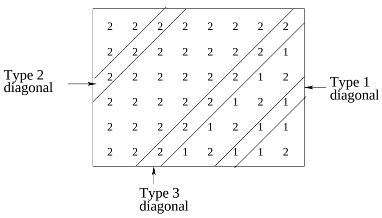


Fig. 2. Optimal policy for periodic model is constant along the diagonals.

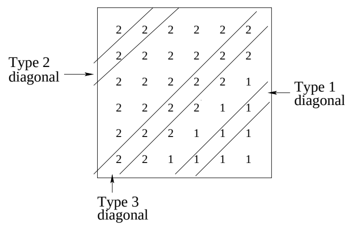


Fig. 3. Optimal policy for periodic model is a linear switching curve when $t_1 = t_2$.

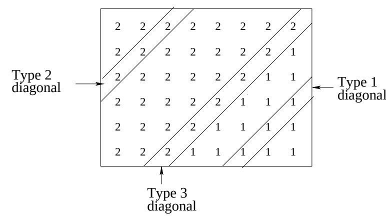


Fig. 4. EDF is optimal for periodic model when channels are symmetric

characterized by a single parameter. In addition to (2, 3), we have:

$$\pi^*(i,j)^{00} = 1 \Rightarrow \pi^*(i',j)^{00} = 1 \;\forall\; i' \geq i, \tag{4}$$

$$\pi^*(i,j)^{00} = 1 \Rightarrow \pi^*(i,j')^{00} = 1 \;\forall\; j' \leq j, \tag{5}$$

$$\pi^*(i,j)^{00} = 2 \Rightarrow \pi^*(i,j')^{00} = 2 \;\forall\; j' \geq j, \tag{6}$$

$$\pi^*(i,j)^{00} = 2 \Rightarrow \pi^*(i',j)^{00} = 2 \;\forall\; i' \leq i, \tag{7}$$

An example of linear switching structure is shown in Fig. 3. We conjecture that linear switching structure for the optimal $\pi^*$ is also true for the general case.

Finally, if there is only one job left in the system at any instant, $\pi^*$ schedules that job:

$$\pi^*(i,j)^{10} = 2, \tag{8}$$

$$\pi^*(i,j)^{01} = 1. \tag{9}$$

In classical real-time theory, it is known that earliest-deadline-first (EDF) is optimal for the periodic model [5]. That theory does not consider settings in which a job scheduled in a slot does not complete in that slot. In other words, EDF is optimal when $p_1 = p_2 = 0$. EDF has the following properties: (i) it plays the same action along diagonals, and (ii) the switching curve is along the principal diagonal starting from the top-right corner.

In wireless environments, multipath fading results in varying channel conditions to different receivers. This is combated by opportunistic scheduling [6], which schedules the "best" link in every slot. For a Bernoulli loss model where the multipath fading is time-invariant, assuming $p_2 \leq p_1$, opportunistic scheduling would provide strict priority to user 2 in every slot. (If fairness is taken into account, user $1$ may also be scheduled.)

The optimal wireless real-time policy for the periodic model inter-plays two competing tendencies:

1. *The real-time tendency to schedule in EDF order.*
2. *The wireless tendency to schedule opportunistically to take advantage of better channel quality if*

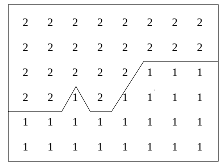


Fig. 5. Optimal policy for renewal model has switching structure.

*possible.*

Like EDF in lossless real-time environments, the optimal policy's action is constant along diagonals of the state space even in wireless lossy environments. Like opportunistic schedulers, it favors the stronger user, i.e., the one more likely to complete, and sometimes schedules it even if it has a longer deadline. Linear switching structure suggests that the optimal policy is in the class of virtual-deadline-first policies, where the lossy nature of the wireless medium provides an opportunistic scheduling correction term to pure EDF.

When both users have similar channels, a common occurrence, we can exactly derive the optimal policy.

*Theorem 3:* Let both users have similar channel statistics, i.e., $p_1 = p_2$. Then, the earliest deadline first (EDF) policy, illustrated in Fig. 4 is optimal:

$$\pi^*(i,j)^{00} = \text{argmin}(t_1 - i, t_2 - j). \tag{10}$$

### B. Renewal Model

*Theorem 4:* The optimal policy $\pi^*$ for the wireless real-time stochastic scheduling problem for the renewal model described in Section II has a switching structure, shown in Fig. 5. Assuming $p_2 \leq p_1$,

$$\pi^*(i,j) = 1 \Rightarrow \pi^*(i,j') = 1 \ \forall \ j' \leq j, \tag{11}$$

$$\pi^*(i,j) = 2 \Rightarrow \pi^*(i,j') = 2 \ \forall \ j' \geq j. \tag{12}$$

When both users are identical and have similar channel statistics, we can exactly derive the optimal policy:

*Theorem 5:* Let both users be identical, i.e., $t_1 = t_2$ and have similar channels, i.e., $p_1 = p_2$. Then, the earliest deadline first (EDF) policy is optimal:

$$\pi^*(i,j) = \text{argmin}(t_1 - i, t_2 - j). \tag{13}$$

### C. A brief note on proof techniques

Our proofs for both the periodic and renewal models will use value and policy iteration induction techniques from dynamic programming for the $\beta-$discounted cost version of the stochastic control problem [7]. By passing to the limit as the discount factor $\beta \to 1$, these results carry over to the average cost formulation described in Section II. The tricky part in these techniques consists of the setup of the induction hypothesis. Typically, the property $P$ that we wish to establish is too weak to be directly used as the induction hypothesis. Instead, we need to "strengthen the induction hypothesis" in order to obtain a statement that we can prove using policy or value induction. There is no systematic way of doing this, and the process of setting up the induction hypothesis is largely an art. We attempt to provide intuition on our choice of hypothesis whenever possible.

## IV. Periodic Model: Proof of Theorem 1 and Theorem 2

For any real-valued function $V(i,j)^{xy}$ and $\beta < 1$, define the dynamic programming operator $T$ as follows:

$$(TV)(i,j)^{xy} = \text{min}_{k\in\{1,2\}}[c_k(i,j)^{xy} + \beta \sum_{(i',j')^{x'y'}} q_k[(i,j)^{xy} \to (i',j')^{x'y'}]V(i',j')^{x'y'}].$$

Define $V^*(i,j)^{xy}$ as the optimal expected infinite horizon discounted cost, given that we start in state $(i,j)^{xy}$. Then, there exists a stationary optimal policy $\pi^*(i,j)^{xy}$ with $V^* = TV^*$, where $\pi^*(i,j)^{xy}$ satisfies:

$$\pi^*(i,j)^{xy} = \text{argmin}_{k\in\{1,2\}}[c_k(i,j)^{xy} + \beta \sum_{(i',j')^{x'y'}} q_k[(i,j)^{xy} \to (i',j')^{x'y'}]V^*(i',j')^{x'y'}].$$

We start off by establishing (8, 9).

### A. *Proof of (8, 9) using value induction*

One method to establish structural results in dynamic programming is the technique of value iteration [7]. To establish property $P$ on $V^*$, where $P$ denotes a closed set in the space of value functions, we start off with some $V^0$ where the property holds, and then show that the dynamic programming operator $T$ preserves the property. Then, since $T$ is a contraction mapping [7], we can pass to the limit using $V^* = \lim_{n\to\infty} T^n V^0$, to establish $P$ for $V^*$. Consider the following hypotheses $H_n^1$:

1. *Monotonicity of value function*

$$V^n(i,j)^{00} \geq V^n(i,j)^{10}, \tag{14}$$

$$V^n(i,j)^{00} \geq V^n(i,j)^{01}, \tag{15}$$

$$V^n(i,j)^{10} \geq V^n(i,j)^{11}, \tag{16}$$

$$V^n(i,j)^{01} \geq V^n(i,j)^{11}. \tag{17}$$

2. *Emulation property*

$$V^n(i,j)^{00} \leq V^n(i,j)^{10} + 1, \tag{18}$$

$$V^n(i,j)^{00} \leq V^n(i,j)^{01} + 1, \tag{19}$$

$$V^n(i,j)^{10} \leq V^n(i,j)^{11} + 1, \tag{20}$$

$$V^n(i,j)^{01} \leq V^n(i,j)^{11} + 1. \tag{21}$$

3. *Policy structure*

$$u^n(i,j)^{10} = 2, \tag{22}$$

$$u^n(i,j)^{01} = 1, \tag{23}$$

Remember that $V^*(i,j)^{00}$ counts the optimal discounted number of deadline misses starting from state $(i,j)^{00}$. The number of jobs in the system at $(i,j)^{01}$ is dominated by the number of jobs in the system at $(i,j)^{00}$, and the ages of the jobs in the system are identical; thus, the monotonicity condition is intuitive. To see why the emulation property is true, observe that starting from $(i,j)^{00}$, we can emulate what the optimal policy would do starting from $(i,j)^{01}$ and would be no further off from the optimal policy in the latter case than by the possible additional deadline miss due to the single job of the $2^{nd}$ user initially present in the former case. This argument can be formalized using stochastic coupling.

The induction proof follows by starting with $V^0(i,j)^{xy} = 0$ and $u^0(i,j)^{xy}$ of the form in (8, 9) with $u^0(i,j)^{00} = 2$ satisfying the hypothesis. Then, assuming that $H^1_{n'}$ holds till $n-1$, we can show that the dynamic programming operator $T$ preserves the property across the $n^{th}$ iteration; thus $H^1_n$ is true for $V^n = TV^{n-1}$. The inductive step itself is tedious, and consists of enumerating $4$ different cases depending on whether the ages $(i,j)$ are in the interior, the right boundary, the top boundary or the top right corner of the state space; these cases are repeated for the proofs of (14, 15, 16, 17, 18, 19, 20, 21, 22, 23); a total of $40$ cases in all. We skip the details.

### B. Diagonal Constancy

We now prove Theorem 1, partly recapitulated here, using the technique of policy iteration [7].

*The Diagonal Constancy Theorem*: The optimal policy $\pi^*(i,j)^{00}$ is constant along diagonals. In other words, $\pi^*(i,j)^{00} = \pi^*(i+1,j+1)^{00}$.

*Proof:* Given a stationary control policy $u$ and a real-valued function $V(i,j)^{xy}$, define $T^u$ as follows:

$$(T^u V)(i,j)^{xy} = c(i,j)^{xy} + \beta \sum_{(i',j')^{x'y'}} q_k[(i,j)^{xy} \to (i',j')^{x'y'}] V(i',j')^{x'y'}.$$

To establish a property $P$ on $\pi^*$ using policy iteration, one starts off with some $u^0$ where $P$ holds. Then, one shows iteratively that if $u^n$ satisfies $P$, then the following iteration generates $u^{n+1}$ that also satisfies $P$:

$$T^{u^n} V^{u^n} = V^{u^n}, u^{n+1} = \text{argmin } TV^{u^n}.$$

Policy iteration monotonically improves the policy [7], and converges to the optimal policy $\pi^*$ in a finite number of iterations; this establishes that $\pi^*$ also satisfies $P$. Consider the following induction hypothesis $H_n^2$:

1) $u^n(i,j)^{00}$ is constant along diagonals of the state space, i.e., $u^n(i+1,j+1)^{00} = u^n(i,j)^{00}$.
2) If both users have the same deadline, i.e., they are on a "type 3" diagonal, as in Fig. 2, it is optimal to schedule the user more likely to complete, i.e., the one with smaller $p_i$. Assuming $p_2 \le p_1$, $u^n(t_1 - i, t_2 - i)^{00} = 2 \; \forall \; i$.

We start off with $u^0(i,j)^{00} = 2$ for all $i,j$, which satisfies the hypothesis. We also assume that $u^0(i,j)^{01} = 1$ and $u^0(i,j)^{10} = 2$ for all $i,j$. It is easily established that policy iteration does not ever change these latter values; thus $u^n(i,j)^{01} = 1$ and $u^n(i,j)^{10} = 2$ for all $i,j$ and all $n$ till policy iteration terminates.

We first consider a "type 1" diagonal where the deadline of user 1 is closer than the deadline of user 2. Such a "type 1" diagonal hits the boundary at $i = t_1 - 1$, as shown in Fig. 2. We assume that $u^n(i,j)^{00} = 1$ along this diagonal, and show that if $u^{n+1}(i,j)^{00} = 2$ for some $(i,j)$ on a "type 1" diagonal, then $u^{n+1}(i,j)^{00} = 2$ for all points on such a diagonal. Depending on the value of $(i,j)$, we enumerate four cases:

1. $i < t_1 - 2$, $j < t_2 - 2$: Observe that:

$$\begin{aligned}
& u^{n+1}(i,j)^{00} = 2 \\
\Leftrightarrow \; & 0 \le (p_1 - p_2) V^{u^n}(i+1,j+1)^{00} + (1-p_1) \cdot \\
& V^{u^n}(i+1,j+1)^{10} - (1-p_2) V^{u^n}(i+1,j+1)^{01} \\
\Leftrightarrow \; & 0 \le (p_1 - p_2)(p_1 V^{u^n}(i+2,j+2)^{00} + (1-p_1) \cdot \\
& V^{u}(i+2,j+2)^{10}) + (1-p_1)(p_2 V^{u}(i+2,j+2)^{10} \\
& +(1-p_2) V^{u^n}(i+2,j+2)^{11}) - (1-p_2)(p_1 \cdot \\
& V^{u}(i+2,j+2)^{01} + (1-p_1) V^{u}(i+2,j+2)^{11}) \\
\Leftrightarrow \; & 0 \le p_1((p_1 - p_2) V^{u^n}(i+2,j+2)^{00} + (1-p_1) \cdot \\
& V^{u^n}(i+2,j+2)^{10} - (1-p_2) V^{u}(i+2,j+2)^{01}) \\
\Leftrightarrow \; & u^{n+1}(i+1,j+1)^{00} = 2.
\end{aligned}$$

2. $i = t_1 - 2$, $j < t_2 - 2$: The above proof also ensures that $u^{n+1}(i,j)^{00} = 2 \Rightarrow u^{n+1}(i-1,j-1)^{00} = 2$ in this case. We now show that $u^{n+1}(i,j)^{00} = 2 \Rightarrow u^{n+1}(i+1,j+1)^{00} = 2$, where $i+1 = t_1 - 1$ is on the boundary. Observe that:

$$\begin{aligned}
& u^{n+1}(t_1-2,j)^{00} = 2 \\
\Rightarrow\ & 0 \le (p_1 - p_2)V^{u^n}(t_1-1,j+1)^{00} + (1-p_1)\times \\
& V^u(t_1-1,j+1)^{10} - (1-p_2)V^u(t_1-1,j+1)^{01} \\
\Rightarrow\ & 0 \le (p_1-p_2)(p_1 + \beta V^{u^n}(0,j+2)^{00}) + (1-p_1) \\
& \times\beta(p_2 V^{u^n}(0,j+2)^{00} + (1-p_2)V^{u^n}(0,j+2)^{01}) \\
& -(1-p_2)(p_1 + \beta V^{u^n}(0,j+2)^{01}) \\
\Rightarrow\ & 0 \le p_1[(p_1-1) + \beta(1-p_2)\times \\
& \quad (V^u(0,j+2)^{00} - V^u(0,j+2)^{01})] \\
\Rightarrow\ & u^{n+1}(t_1-1,j+1,0,0) = 2.
\end{aligned}$$

3. $i = t_1 - 1$, $j < t_2 - 1$: In this case, we need to show that $u^{n+1}(i,j)^{00} = 2 \Rightarrow u^{n+1}(i-1,j-1)^{00} = 2$. Now,

$$\begin{aligned}
u^{n+1}(t_1-1,j)^{00} = 2 \Rightarrow (p_1-1) + \beta(1-p_2)\times \\
(V^{u^n}(0,j+1)^{00} - V^{u^n}(0,j+1)^{01}) \ge 0.
\end{aligned}$$

We will now show that the RHS above is the same as:

$$\begin{aligned}
& (p_1-p_2)V^{u^n}(t_1-1,j)^{00} + (1-p_1)\times \\
& V^{u^n}(t_1-1,j)^{10} - (1-p_2)V^{u^n}(t_1-1,j)^{01} \ge 0,
\end{aligned}$$

which is the same as $u^{n+1}(t_1-2,j-1)^{00} = 2$. Observe:

$$\begin{aligned}
& (p_1-p_2)V^{u^n}(t_1-1,j)^{00} + (1-p_1)\times \\
& \quad V^{u^n}(t_1-1,j)^{10} - (1-p_2)V^{u^n}(t_1-1,j)^{01} \\
=\ & (p_1-p_2)(p_1 + \beta V^{u^n}(0,j+1)^{00}) + (1-p_1)\times \\
& \quad \beta(p_2 V^{u^n}(0,j+1)^{00} + (1-p_2)V^{u^n}(0,j+1)^{01}) \\
& \quad -(1-p_2)(p_1 + \beta V^{u^n}(0,j+1)^{01}) \\
=\ & p_1(p_1-1) + \beta p_1(1-p_2)\times \\
& \quad (V^{u^n}(0,j+1)^{00} - V^{u^n}(0,j+1)^{01}) \ge 0. \\
\Rightarrow\ & u^{n+1}(t_1-2,j-1)^{00} = 2.
\end{aligned}$$

Suppose instead that $u^n(i,j)^{00} = 2$ for all $(i,j)$ along a "type 1" diagonal. We now show that if $u^{n+1}(i,j)^{00} = 1$ for some $(i,j)$ on this diagonal, then $u^{n+1}(i,j)^{00} = 1$ for all points $(i,j)$ on the diagonal. Depending on $(i,j)$, we enumerate four cases:

1. $i < t_1 - 2$, $j < t_2 - 2$: Observe that:

$$\begin{aligned}
& u^{n+1}(i,j)^{00} = 1 \\
\Leftrightarrow\ & 0 \geq (p_1 - p_2)V^{u^n}(i+1,j+1)^{00} + (1-p_1) \times \\
& V^{u^n}(i+1,j+1)^{10} - (1-p_2)V^{u^n}(i+1,j+1)^{01} \\
\Leftrightarrow\ & 0 \geq (p_1 - p_2)(p_2 V^{u^n}(i+2,j+2)^{00} + (1-p_2) \times \\
& V^{u^n}(i+2,j+2)^{01}) + (1-p_1)(p_2 V^{u^n}(i+2,j+2)^{10} \\
& +(1-p_2)V^{u^n}(i+2,j+2)^{11}) - (1-p_2) \times \\
& (p_1 V^{u^n}(i+2,j+2)^{01} + (1-p_1)V^{u^n}(i+2,j+2)^{11}) \\
\Leftrightarrow\ & 0 \geq p_2((p_1 - p_2)V^{u^n}(i+2,j+2)^{00} + (1-p_1) \times \\
& V^{u^n}(i+2,j+2)^{10} - (1-p_2)V^{u^n}(i+2,j+2)^{01}) \leq 0 \\
\Leftrightarrow\ & u^{n+1}(i+1,j+1)^{00} = 1.
\end{aligned}$$

2. $i = t_1 - 2$, $j < t_2 - 2$: In this case, the above proof ensures that $u^{n+1}(i,j)^{00} = 1 \Rightarrow u^{n+1}(i-1,j-1)^{00} = 1$. We still need to show that $u^{n+1}(i,j)^{00} = 1 \Rightarrow u^{n+1}(i+1,j+1)^{00} = 1$, where $i+1 = t_1 - 1$ is on the boundary. Observe that:

$$\begin{aligned}
& u^{n+1}(t_1 - 2, j)^{00} = 1 \\
\Rightarrow\ & 0 \geq (p_1 - p_2)V^{u^n}(t_1 - 1, j+1)^{00} + (1-p_1) \times \\
& V^{u^n}(t_1 - 1, j+1)^{10} - (1-p_2)V^{u^n}(t_1 - 1, j+1)^{01} \\
\Rightarrow\ & 0 \geq (p_1 - p_2)(1 + \beta(p_2 V^{u^n}(0,j+2)^{00} \\
& +(1-p_2)V^{u^n}(0,j+2)^{01})) + (1-p_1) \times \\
& \beta(p_2 V^{u^n}(0,j+2)^{00} + (1-p_2)V^{u^n}(0,j+2)^{01}) \\
& -(1-p_2)(p_1 + \beta V^{u^n}(0,j+2)^{01}) \\
\Rightarrow\ & 0 \geq p_2[(p_1 - 1) + \beta(1-p_2)(V^{u^n}(0,j+2)^{00} \\
& \quad - V^{u^n}(0,j+2)^{01})] \\
\Rightarrow\ & u^{n+1}(t_1 - 1, j+1)^{00} = 1.
\end{aligned}$$

3. $i = t_1 - 1,\ j < t_2 - 1$: Observe that:

$$\begin{aligned} &u^{n+1}(t_1-1,j)^{00} = 1 \Rightarrow (p_1 - 1)\\ &\quad + \beta(1-p_2)(V^{u^n}(0,j+1)^{00} - V^{u^n}(0,j+1)^{01}) \leq 0. \end{aligned}$$

We will now show that the RHS above is the same as:

$$\begin{aligned} &(p_1-p_2)V^{u^n}(t_1-1,j)^{00} + (1-p_1)V^{u^n}(t_1-1,j)^{10}\\ &\quad - (1-p_2)V^{u^n}(t_1-1,j)^{01} \leq 0, \end{aligned}$$

which means $u^{n+1}(t_1-2,j-1)^{00} = 1$. This is true, because:

$$\begin{aligned} &(p_1-p_2)V^{u^n}(t_1-1,j)^{00} + (1-p_1)V^{u^n}(t_1-1,j)^{10}\\ &\quad - (1-p_2)V^{u^n}(t_1-1,j)^{01}\\ &= (p_1-p_2)(1+\beta(p_2V^{u^n}(0,j+1)^{00} + (1-p_2) \times\\ &V^{u^n}(0,j+1)^{01})) + (1-p_1)\beta(p_2V^{u^n}(0,j+1)^{00} +\\ &(1-p_2)V^{u^n}(0,j+1)^{01}) - (1-p_2) \times\\ &(p_1 + \beta V^{u^n}(0,j+1)^{01})\\ &= p_2(p_1-1) + \beta p_2(1-p_2)(V^{u^n}(0,j+1)^{00}\\ &\quad - V^{u^n}(0,j+1)^{01}) \leq 0\\ &\Rightarrow u^{n+1}(t_1-2,j-1)^{00} = 1. \end{aligned}$$

This establishes that all points on a “type 1” diagonal form an equivalence class with respect to the optimal control during each step of policy iteration. In other words, they are either all $1$ or all $2$.

Next, consider “type 2” diagonals, intersecting the boundary at $j = t_2 - 1$, as shown in Fig. 2. Here, the deadline of user 2 is lower than the deadline of user 1. By symmetry considerations, the Diagonal Constancy Theorem is true for “type 2” diagonals too, and the proof is identical to that for “type 1” diagonals.

To complete the proof of Theorem 1, we establish that if both users are on a “type 3” diagonal and have the same relative deadline, as in Fig. 2, then it is optimal to schedule the better user. To show this, suppose user $2$ is better, i.e., $p_2 \leq p_1$, and start with $u^0(t_1-i,t_2-i)^{00} = 2$ on the “type 3” diagonal. Now, suppose it is first optimal to schedule $1$ at some $(i,j)$ on this diagonal during step $n$ of policy iteration. The proofs for the corresponding “type 1” diagonal case ensure that for $i < t_1 - 2, j < t_2 - 2$ on this diagonal, $u^n(i,j)^{00} = 1 \Leftrightarrow u^n(i+1,j+1)^{00} = 1$. On the other hand, we can establish by direct computation (omitted here) that $u^n(t_1-1,t_2-1)^{00} = u^n(t_1-2,t_2-2)^{00} = 2$ for all $n$. This provides a contradiction and thus, $u^n(i,j) = 2$ for all $i,j$ and $n$ on a “type 3” diagonal. ■

### C. Weak EDF structure

*Corollary 1:* It is optimal to schedule the better user whenever it has a lower (or equal) deadline. Thus, assuming $p_2 \leq p_1$, $\pi^*(i,j)^{00} = 2$ for $i, j$ such that $t_2 - j \leq t_1 - i$.

*Proof:* We first establish that $\pi^*(i, t_2 - 1)^{00} = 2$, i.e., it is optimal to schedule the better user 2 when its deadline is 1. We prove this by value iteration starting from $V^0(i,j,0,0) = 0$. Define $V^n(i,j)^{xy}_{|k}$ as the value function at time step $n$ when the user $k$ is scheduled. For the inductive step, observe that: 1. For $i < t_1 - 1$:

$$V^n(i, t_2 - 1)^{00}_{|1} = 1 + \beta(p_1 V^{n-1}(i+1, 0)^{00} + (1-p_1) V^{n-1}(i+1, 0)^{10}).$$

$$V^n(i, t_2 - 1)^{00}_{|2} = p_2 + \beta V^{n-1}(i+1, 0)^{00}.$$

$$\text{Hence, } V^n(i, t_2 - 1)^{00}_{|1} - V^n(i, t_2 - 1)^{00}_{|2} \geq 0,$$

where the last line comes from $p_2 \leq p_1$ and the emulation property.
2. $V^n(t_1 - 1, t_2 - 1)^{00}_{|1} - V^n(t_1 - 1, t_2 - 1)^{00}_{|2} = p_1 - p_2 \geq 0$.

Now, using Diagonal Constancy, we can conclude that it is optimal to schedule the better user 2 along "type 2" and "type 3" diagonals, i.e., whenever it has the non-greater deadline, as shown in Fig. 2. ■

### D. Linear switching structure when $t_1 = t_2$

Diagonal Constancy ensures that we need to simply establish switching structure along one of the axes to prove that the optimal policy has linear switching structure. We start off by analyzing the sample paths followed by the system. In every slot, both jobs age by 1 respectively, unless the system is at one of the boundaries $(i, t-1)^{xy}$ or $(t-1, j)^{xy}$, in which case, the expired job is replaced by a new job from the same user with age 0. Thus, system sample paths evolve in $(i,j)$-space along diagonals, with appropriate wrapping around on the boundaries, as shown in Fig. 6.

We now use multi-step dynamic programming and Diagonal Constancy to establish linear switching structure for the optimal policy when users have similar periods, i.e., $t_1 = t_2$, thereby proving Theorem 1. In this case, system evolution occurs along any one of $t$ independent "communicating classes", as shown in Fig. 7. Consider a policy $u(i,j)^{xy}$ satisfying Diagonal Constancy, and the weak EDF property. We study a multiple time slot evolution of the system to relate $V^u(i,j)^{xy}$ at $(i,0)^{00}$, $(i,0)^{10}$, $(0,j)^{00}$ and $(0,j)^{01}$.

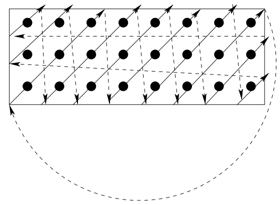

Fig. 6. Sample paths evolve along diagonals of $(i, j)$-space

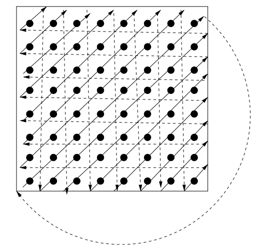

Fig. 7. When $t_1 = t_2 = t$, sample paths evolve along $t$ independent classes in $(i, j)$-space.

1. Since $u(i,j)^{01} = 1$, we have, for $j > 0$, that:

$$V^u(0,j)^{01} = \beta[p_1 V^u(1,j+1)^{01} + (1-p_1)V^u(1,j+1)^{11}]$$
$$= ...$$
$$= \beta^{t-j}[p_1^{t-j} V^u(t-j,0)^{00} + (1-p_1^{t-j})V^u(t-j,0)^{10}] \tag{24}$$

2. Since $u(i,j)^{10} = 2$, we can analogously express $V^u(i,0)^{10}$ in terms of $V^u(0,t-i)^{00}$ and $V^u(0,t-i)^{10}$:

$$V^u(i,0)^{10} = \beta^{t-i}[p_2^{t-i} V^u(0,t-i)^{00} + (1-p_2^{t-i}) \times V^u(0,t-i)^{01}] \tag{25}$$

3. By the weak EDF property, $u(i,j)^{00} = 2$ for $i < j$. Thus, for $j > 0$, we can derive an expression for $V^u(0,j)^{00}$ in terms of $V^u(t-i,0)^{00}$ and $V^u(t-i,0)^{10}$:

$$V^u(0,j)^{00} = \beta[p_2 V^u(1,j+1)^{00} + (1-p_2)V^u(1,j+1)^{01}]$$
$$= ... = \beta^{t-j-1} p_2^{t-j} + \beta^{t-j} h_{t-j} V^u(t-j,0)^{00}$$
$$+ \beta^{t-j}(1-h_{t-j})V^u(t-j,0)^{10}, \tag{26}$$

where we define $h_i$ as:

$$h_i \triangleq p_1^{i-1} + p_1^{i-2}p_2 + ...p_2^{i-1}$$
$$- (p_1^{i-1}p_2 + p_1^{i-2}p_2^2 + ...p_1 p_2^{i-1})$$

Two properties of $h_i$, easily proven from the definition, will be useful later in the proof: here:

$$h_i - p_2^i = (1-p_2)\frac{p_1^i - p_2^i}{p_1 - p_2} \tag{27}$$
$$h_i - p_1^i = (1-p_1)\frac{p_1^i - p_2^i}{p_1 - p_2} \tag{28}$$

By the Diagonal Constancy Theorem, type 1-diagonals (where $t-i < t-j$) can either have $u(i,j)^{00} = 1$ along the entire diagonal or $u(i,j)^{00} = 2$ along the entire diagonal. Suppose $u(i,0)^{00} = 1$. Then, we can derive an expression for $V^u(i,0)^{00}$, labeled here as $V^u(i,0)^{00}|_1$, in terms of $V^u(0,t-i)^{00}$ and

$V^u(0,t-i)^{01}$:

$$V^u(i,0)^{00}_{|1} = \beta^{t-i-1}p_1^{t-i} + \beta^{t-i}h_{t-i}V^u(0,t-i)^{00}$$
$$+\beta^{t-i}(1-h_{t-i})V^u(0,t-i)^{01} \quad (29)$$

On the other hand, if $u(i,0)^{00} = 2$, the corresponding expression for $V^u(i,0)^{00} = V^u(i,0)^{00}_{|2}$ in terms of $V^u(0,t-i)^{00}$ and $V^u(0,t-i)^{01}$ is:

$$V^u(i,0)^{00}_{|2} = \beta[p_2V^u(i,0)^{00} + (1-p_2)V^u(i,0)^{01}]$$
$$= ...$$
$$= \beta^{t-i-1}h_{t-i} + \beta^{t-i}p_2^{t-i}V^u(0,t-i)^{00}$$
$$+ \beta^{t-i}(1-p_2^{t-i})V^u(0,t-i)^{01} \quad (30)$$

We will use (24, 25, 26, 29, 30) to show that the optimal control policy $u^*$ has switching structure along the horizontal axis , i.e., $u^*(i,0)^{00} = 1 \Rightarrow u^*(i+1,0)^{01} = 1$. While it is possible to use value induction on the contraction mapping $T^u$ [7], policy induction or stochastic coupling, we instead exploit the structure of the multi-slot description to directly compute $V^u(i,0)^{00}$, $V^u(i,0)^{10}$ given a possibly optimal $u(i,j)^{xy}$. In doing so, we use four observations:

1. The optimal $u^*(i,j)^{xy}$ and $V^*(i,j)^{xy}$ satisfy $V^*(i,j)^{xy} = TV^*(i,j)^{xy} = T^{u^*}V^*(i,j)^{xy}$.
2. Diagonal Constancy and weak EDF ensure that $u^*(i,j)^{xy}$ is specified by $u^*(i,0)^{00}$ for $i > 0$.
3. Since $t_1 = t_2 = t$, we can decide whether $u^*(i,0)^{00} = 1$ or 2 from the sign of $V^u(i,0)^{00}_{|1} - V^u(i,0)^{00}_{|2}$. This is because any sample path beginning at $(i,0)^{00}$ only passes through states $(i+k,k)^{xy}$ and states $(k',t-i+k')^{xy}$ with $k \leq t-i-1$, $k' \leq i-1$ and $x,y = 0,1$.
4. Finally, given $u(i,0)^{00}$, we can solve for $V^u(i,0)^{00}, V^u(i,0)^{10}, V^u(0,t-i)^{00}, V^u(0,t-i)^{01}$ by jointly solving (24, 25, 26, 29) if $u(i,0)^{00} = 1$, or (24, 25, 26, 30) if $u(i,0)^{00} = 2$.

If $u(i,0)^{00} = 1$, we can use (24, 25, 26, 29) to derive:

$$V^u(0,j)^{00} - V^u(0,j)^{01} = \beta^{t-j-1}p_2^{t-j} + \beta^{t-j} \times$$
$$(h_{t-j} - p_1^{t-j})(V^u(t-j,0)^{00} - V^u(t-j,0)^{10}) \quad (31)$$
$$V^u(i,0)^{00}|_1 - V^u(i,0)^{10} = \beta^{t-i-1}p_1^{t-i} + \beta^{t-i} \times$$
$$(h_{t-i} - p_2^{t-i})(V^u(0,t-i)^{00} - V^u(0,t-i)^{01}) \quad (32)$$

We can solve for $L_1 = V^u(i,0)^{00}_{|1} - V^u(i,0)^{10}$ by substituting (31) in (32) to get:

$$L_1 = V^u(i,0)^{00}_{|1} - V^u(i,0)^{10}$$
$$= \beta^{t-i-1}p_1^{t-i} + \beta^{t-i}(h_{t-i} - p_2^{t-i})$$
$$\times (\beta^{i-1}p_2^i + \beta^i(h_i - p_1^i)(V^u(i,0)^{00}_{|1} - V^u(i,0)^{10}))$$

$$\Rightarrow L_1 = \frac{\beta^{t-i-1}p_1^{t-i} + \beta^{t-1}(h_{t-i} - p_2^{t-i})p_2^i}{1 - \beta^t(h_{t-i} - p_2^{t-i})(h_i - p_1^i)}. \tag{33}$$

Note that the decision to schedule user 1 at state $(i,0)^{00}$ affects not only $V^u(i,0)^{00}_{|1}$, but also $V^u(i,0)^{10}, V^u(0,t-i)^{00}$ and $V^u(0,t-i)^{01}$. For brevity, our notation does not capture this explicitly, and we will be careful to clarify any ambiguity. We now explicitly solve for $V^u(i,0)^{00}_{|1}$ in terms of $p_1, p_2, t, i$ using (24, 25, 26, 29). Observe that in this case, we have:

$$\begin{aligned}
V^u(i,0)^{10} &= \beta^{t-i}(p_2^{t-i}V^u(0,t-i)^{00} + (1-p_2^{t-i}) \times \\
&\quad V^u(0,t-i)^{01}) \\
&= \beta^{t-i}p_2^{t-i}[\beta^i h_i V^u(i,0)^{00}_{|1} + \beta^i(1-h_i)V^u(i,0)^{10} \\
&\quad + \beta^{i-1}p_2^i] + \beta^{t-i}(1-p_2^{t-i})[\beta^i p_1^i V^u(i,0)^{00}_{|1} \\
&\quad + \beta^i(1-p_1^i)V^i(i,0)^{10}]
\end{aligned}$$

Substituting (33) and rearranging , we get:

$$\begin{aligned}
V^u(i,0)^{00}_{|1} &= L_1 + \beta^t V^u(i,0)^{00}_{|1} + \beta^{t-1}p_2^t \\
&\quad - \beta^t L_1(p_2^{t-i}(1-h_i) + (1-p_2^{t-i})(1-p_1^i)) \\
&= \frac{L_1(1-\beta^t(p_2^{t-i}(1-h_i) + (1-p_2^{t-i})(1-p_1^i)) + \beta^{t-1}p_2^t}{1-\beta^t},
\end{aligned} \tag{34}$$

where $L_1$ is given by (33).

Suppose, instead, that $u(i,0)^{00} = 2$. In this case, we use (24, 25, 26, 30) to derive:

$$L_2 = V^u(i,0)^{00}|_2 - V^u(i,0)^{10} = \beta^{t-i-1}h_{t-i}. \tag{35}$$

As noted above, the $V^u(i,0)^{10}$ in this case with $u(i,0)^{00} = 2$ differs from $V^u(i,0)^{10}$ in the previous case where $u(i,0)^{00} = 1$. Again, we can explicitly solve (24, 25, 26, 30) to derive a closed form expression for $V^u(i,0)^{00}_{|2}$, and obtain:

$$\begin{aligned}
&V^u(i,0)^{00}_{|2} \\
&= \frac{L_2(1-\beta^t(p_2^{t-i}(1-h_i) + (1-p_2^{t-i})(1-p_1^i)) + \beta^{t-1}p_2^t}{1-\beta^t},
\end{aligned} \tag{36}$$

where $L_2$ is given by (35). Finally, we combine (34) and (36) to obtain:

$$\begin{aligned}
V^u(i,0)^{00}_{|1} - V^u(i,0)^{00}_{|2} &= (L_1 - L_2) \times \\
\frac{(1-\beta^t(p_2^{t-i}(1-h_i) + (1-p_2^{t-i})(1-p_1^i))}{1-\beta^t}&,
\end{aligned} \tag{37}$$

It is easily shown that the terms in the numerator and denominator multiplying $L_1 - L_2$ in the above expression are always positive. Thus, the sign of $V^u(i,0)^{00}_{|1} - V^u(i,0)^{00}_{|2}$ is the same as the sign of $L_1 - L_2$. Further, we are concerned with the average cost case where $\beta \to 1$. ( This is the only step in the proof where we use $\beta \to 1$.) The following lemma completes the proof of linear switching structure:

*Lemma 1:* Let $f(i,\beta) = L_1(p_1,p_2,t,i,\beta) - L_2(p_1,p_2,t,i,\beta)$. Suppose $\lim_{\beta\to 1} f(i,\beta) \le 0$. Then, $\lim_{\beta\to 1} f(i+1,\beta) \le 0$.

Before proving the lemma, let us see why this implies the result for the average cost case. Observe that:

$$\begin{aligned}
& u^*(i,0)^{00} = 1 \\
& \Rightarrow \lim_{\beta\to 1}(1-\beta)[V^{u^*}(i,0)^{00}_{|1} - V^{u^*}(i,0)^{00}_{|2}] \le 0 \\
& \Rightarrow \lim_{\beta\to 1} f(i,\beta) = (L_1 - L_2) \le 0 \\
& \Rightarrow \lim_{\beta\to 1} f(i+1,\beta) \le 0, \text{ by Lemma 1} \\
& \Rightarrow \lim_{\beta\to 1}(1-\beta)[V^{u^*}(i+1,0)^{00}_{|1} - V^{u^*}(i+1,0)^{00}_{|2}] \\
& \qquad \le 0 \quad \Rightarrow u^*(i+1,0,0,0) = 1,
\end{aligned} \tag{38}$$

establishing linear switching structure along the bottom row $j = 0$ of the horizontal axis. Combined with the Diagonal Constancy Theorem, this establishes a linear switching structure for the optimal policy $u^*(i,j)^{00}$ when both users are in the system.

*Proof:* (Lemma 1) We first use (33, 35) to obtain:

$$\begin{aligned}
& \lim_{\beta\to 1} f(i,\beta) = \lim_{\beta\to 1}(L_1 - L_2) \\
& = ... = \frac{(h_{t-i} - p_2^{t-i})(p_2^i - \frac{1-p_1}{1-p_2} + h_{t-i}(h_i - p_1^i))}{1 - (h_{t-i} - p_2^{t-i})(h_i - p_1^i)},
\end{aligned}$$

where the last line comes from (27, 28). These properties of $h_i$ also ensure that the first term in the numerator, and the denominator are both always positive. Thus, the sign of $\lim_{\beta\to 1} f(i,\beta)$ is the same as the sign of $g(i) = p_2^i - \frac{1-p_1}{1-p_2} + h_{t-i}(h_i - p_1^i)$. Further, we can establish that $g(i)$ decreases with $i$:

$$\begin{aligned}
& g(i+1) - g(i) = p_2^i(p_2 - 1) + h_{t-i-1} \times \\
& (h_{i+1} - p_1^{i+1}) - h_{t-i}(h_i - p_1^i) \\
& = p_2^i(1-p_2)(p_1^{t-i-1} - 1) + p_1^i p_2^{t-i-1}(p_1 - 1) \le 0
\end{aligned}$$

To complete the proof of the lemma, we combine the above observations to get:

$$\begin{aligned}
& \lim_{\beta\to 1} f(i,\beta) \le 0 \Rightarrow g(i) \le 0 \\
& \Rightarrow g(i+1) \le 0 \text{ since } g(i) \text{ decreases with } i \\
& \Rightarrow \lim_{\beta\to 1} f(i+1,\beta) \le 0.
\end{aligned}$$

■

## V. Periodic Model: Optimality of EDF when channels are similar

If both channels have similar statistics, i.e., $p_1 = p_2$, we can establish Theorem 3, that the optimal policy is EDF, directly using value iteration, making use of Diagonal Constancy and the emulation property. A shorter proof uses symmetry. The weak EDF property implies that it is optimal to schedule the better user whenever it has the lower deadline. Thus, $p_2 \geq p_1$ implies that $\pi^*(i,j)^{00} = 2$ whenever $t_2 - j \leq t_1 - i$. Since $p_1 \geq p_2$, we also have $\pi^*(i,j)^{00} = 1$ whenever $t_1 - i \leq t_2 - j$. Thus, the optimal policy is EDF.

## VI. Renewal Model: Proof of switching structure

We now prove Theorem 4 for renewal arrivals. Given any real valued $V(i,j)$, define the dynamic programming operator $T$, operating on $V(i,j)$ as:

$$(TV)(i,j) = \text{min}_{k\in\{1,2\}} c_k(i,j) + \beta \sum_{(i',j')} q_k[(i,j) \to (i',j')] V(i',j'),$$

Define $V^*(i,j)$ for the renewal model problem as the optimal expected infinite horizon discounted cost, given that we start in state $(i,j)$. As before, there exists a stationary optimal policy $\pi*(i,j)$ that satisfies:

$$\pi^*(i,j) = \text{argmin}_{k\in\{1,2\}} c_k(i,j) + \beta \sum_{(i',j')} q_k[(i,j) \to (i',j')] V^*(i',j').$$

Starting from $V^0(i,j)$ appropriately chosen, we establish Theorem 4 using induction on the number of iterations $n$ of the dynamic programming operator $T$. Define $V^n(i,j) = T^n V^0(i,j)$, and the corresponding optimal control to be $\pi^n(i,j)$ Consider the hypothesis $H_n^3$:

1. *Switching structure of policy*

$$\pi^n(i,j) = 1 \Rightarrow \pi^n(i,j') = 1 \ \forall \ j' \leq j, \tag{39}$$

$$\pi^n(i,j) = 2 \Rightarrow \pi^n(i,j') = 2 \ \forall \ j' \geq j. \tag{40}$$

2. *Quasi submodularity*: Define $f^n(i,j)$ as:

$$f^n(i,j) = \beta[(p_1 - p_2)V^n(i+1,j+1) + (1-p_1)V^n(0,j+1) - (1-p_2)V^n(i+1,0)]$$

$$f^n(t_1-1,j) = (p_1 - 1) + \beta(1-p_2)(V^n(0,j+1) - V^n(0,0))$$

$$f^n(i,t_2-1) = (1-p_2) + \beta(1-p_1)(V^n(0,0) - V^n(i+1,0))$$

$$f^n(t_1-1,t_2-1) = p_1 - p_2$$

$$\text{Then, } f^n(i,j) \text{ is non-decreasing with } j \text{ from j = } 0 \text{ to } t_2 - 2. \tag{41}$$

3. *Monotonicity of value function*

$$i' \geq i \Rightarrow V^n(i',j) \geq V^n(i,j), \tag{42}$$

$$j' \geq j \Rightarrow V^n(i,j') \geq V^n(i,j). \tag{43}$$

4. *Emulation property*

$$i' \geq i \Rightarrow V^n(i,j) + 1 \geq V^n(i',j), \tag{44}$$

$$j' \geq j \Rightarrow V^n(i,j) + 1 \geq V^n(i,j'). \tag{45}$$

We start with $V^0(i,j) = 0$ and $u^0(i,j) = 2$ satisfying the hypothesis. Assuming that $H^3_{n'}$ holds upto time step $n-1$, we can establish each of the sub-conditions for $H^3_n$ by successively enumerating cases depending on whether $(i,j)$ are in the interior, the right boundary, the top boundary or the top right corner of the state space for (39, 40, 42, 43, 44, 45). We skip the details.

## VII. Renewal Model: Optimality of EDF when users are identical and channels are similar

For the special case of identical users, i.e., $t_1 = t_2$, and similar channels, i.e., $p_1 = p_2$, we can establish the optimality of EDF using value induction. Consider the following value induction hypothesis $H^4_n$:

1. *EDF structure of policy*

$$i \leq j \Rightarrow u^n(i,j) = 2, \tag{46}$$

$$i > j \Rightarrow u^n(i,j) = 1. \tag{47}$$

2. *Symmetry of value function*

$$V^n(i,j) = V^n(j,i). \tag{48}$$

The symmetric case is a special version of the more general asymmetric case ($t_1 \neq t_2, p_1 \neq p_2$), for which we have already established that the value function satisfies the monotonicity and emulation hypothesis (42, 43, 44, 45) in Section VI. Starting from $V^0(i,j) = 0 \; \forall \; i,j$ and $u(i,j)$ specified in (46) and (47) satisfying the hypothesis, we can use these properties to establish that the dynamic programming operator $T$ in this case preserves the induction hypothesis $H^4_n$. by enumerating four cases depending on whether $(i,j)$ is in the interior, the right boundary, the top boundary, or the top-right corner of the state space, and proving (46, 47, 48) for each of these cases (totally 12 cases in all), establishing the optimality of EDF.

## VIII. EXTENSIONS

### A. Differentiating between deadline misses of different users

Our definition of optimality counts the total number of deadline misses, and treats deadline misses from both users identically. Suppose instead, that a deadline miss of user $i$'s packet is weighted as $w_i$ and we wish to find the policy that minimizes the total weighted number of deadline misses, allowing for differentiation between the two users' real-time requirements. Our earlier results and proofs generalize to this case.

### B. Impact of imperfect scheduling

Our model implicitly assumes that the scheduling mechanism can perfectly execute the decisions of the scheduling policy. In practice, wireless real-time schedulers will possibly be implemented using 802.11 contention window modulation; this may lead to priority inversion resulting from the fact that 802.11 contention window modulation algorithms are randomized and cannot guarantee that the user to be scheduled in a slot is actually scheduled in that slot. We can extend our model to probabilistically capture the impact of such imperfect scheduling mechanisms using a parameter $q$ to capture the probability of scheduling mechanism error. Our results hold for such a model so long as $q \neq \frac{1}{2}$; the proofs are analogous to the proofs described earlier.

### C. Future Work

We are currently studying a broader range of underlying models. These include models which allow jobs to require multiple units of communication time to complete, models for the $N$ task periodic arrival problem, and Gilbert-Eliot channel fading models. These can be formulated in our stochastic control framework; what complicates the analysis is that wireless scheduling problem formulations with hard real-time deadlines intrinsically lead to exponentially large state spaces and the “curse of dimensionality”.

## IX. RELATED WORK

The seminal Liu and Layland paper [1] established the optimality of RMS and EDF. Opportunistic scheduling is a technique to improve throughput performance in wireless systems [3], [6]. The exponential rule scheduler [8] produces small packet delays and achieves fairness with respect to user delay tails. In [9], the authors observe that EDF may not be optimal for deadline constrained users in the presence of losses.

[10] considers a two state Markov model for channels, and shows that even with perfect channel state information, the “feasible-EDF” policy is not necessarily optimal. We have recently discovered that [11]

has independently shown a switching structure for an inter-packet deadline (IPD) arrival model similar to renewal arrivals. However, they do not consider the more practical periodic model. As we show in the paper, the periodic model has much stronger structural properties than the renewal model and is critical to the strong results we obtain, although the periodic analysis is considerably more involved. As described in Note 1, renewal arrival models have weaker structural results because they introduce a "perverse incentive for the optimal policy to postpone doing work till it can be postponed no more".

We use stochastic control techniques to derive our structural results. Our problem is an example of the restless bandits problem in dynamic programming; the general restless bandits problem [4] is still open. Our Diagonal Constancy and linear switching structure results are unusually strong in stochastic control. Several stochastic control problems in the queuing system theory literature exhibit (weaker) switching structure [12], [13]. Such structural results are often related to submodularity of the associated value function [14].

## X. Conclusion

We have introduced a first principles approach for wireless hard real-time scheduling in lossy environments, and studied optimal scheduling policies for a canonical problem in the area. The optimal policy has a very strong structure, and captures the trade-off between the real-time tendency to schedule users in EDF order, and the wireless tendency to schedule users opportunistically. Our results characterize the optimal policy by a single number, reducing the search space for optimal wireless real-time scheduling policies by an exponential order of magnitude, and allow protocol designers to restrict their attention to the class of "virtual-deadline-first" policies, which are easily implemented in a distributed manner.

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