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# OBSERVING AND MODELLING THE LEGLESS JUMPING MECHANISM OF CLICK BEETLES FOR BIO-INSPIRED ROBOTIC DESIGN 

BY<br>OPHELIA BOLMIN

## THESIS

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#### Abstract

Click beetles (Coleoptera: Elateridae) have evolved a unique jumping mechanism to right themselves when on their dorsal side without using their legs or any other appendages. This work describes and analyzes the stages of the click beetle jump using high-speed video recordings and scanning electron micrographs of six beetle species, namely Alaus oculatus, Ampedus linteus, Hemicrepidius sp., Melanactes sp., Melanotus spp. and Parallelosthetus attenuatus. The jump of the click beetle is divided into three consecutive stages: the pre-jump stage (energy storage), and the take-off and airborne stages (energy release). Morphological measurements of the previously mentioned species as well as three additional species, namely Agriotes sp., Athous sp. and Lacon discoideus are taken, and isometric scaling across the species is observed. The body of the click beetle is considered as two masses linked by a hinge. Dynamic and kinematic models of the jump stages are developed. Non-dimensional analysis of the airborne stage is used to analyze the jump and identify the contribution of kinematic and morphological governing parameters. An energetics model is developed to describe the energy exchanges between the three stages of the jump. Kinematic and dynamic models are used to calculate the hinge stiffness and the elastic energy stored in the body during the jump. The derived models provide a framework that will be used for the design of a click beetle inspired self-righting robot.


## Dedication

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## 1 Introduction

### 1.1 Motivation

Robots inspired by animals have been designed to perform specific tasks autonomously in remote places. Current conventional autonomous robots are either wheeled or multipedal $[1,2,3,4,5]$. Some of these robots encounter difficulties when traversing unknown and unstructured environments due to limited robot motion gaits. Their design focuses on improved stability and balance to prevent falls. However, when navigating uneven terrains, falling is inevitable.

Many clever solutions to high mobility (and robustness) can be found in nature, particularly among arthropods: spiders, crabs, centipedes and insects. Insects are relatively small, often very fast and robust creatures that developed fascinating capabilities for navigating through rough environments by optimizing stability and speed, for overcoming obstacles (which are often comparable or larger then their body size) and for recovering from falls. Since the early 2000's, bio-inspired robots have been designed to perform specific tasks autonomously, or demonstrate a particular motion capability, such as either jumping or speed $[2,6,7]$. Examples of such robots are shown in Fig. 1 and include:

- The Miniature $7 g$ jumping robot (Fig.1a), developed at the Ecole Polytechnique Federale de Lausanne (Switzerland), is a very small robot (about 5 cm height) inspired by the desert locust that can overcome obstacles f about 20 times its height [2].
- VelociRoACH (Fig.1b), developed at the University of California at Berkeley (USA), is the fastest hexapedal robot of it's size, with a top speed of 27 times its body length per second [6].
- X-Rhex (Fig.1c), developed at the University of Pennsylvania (USA), is a modular robotic platform inspired by the cockroach. It is designed to maneuver in rough terrain - overcome obstacles and have a great mobility - with various payload [7].


Figure 1: From left to right: a. "A miniature 7 g jumping robot" (Ecole Polytechnique Federale de Lausanne, Switzerland) [2], b. "VelociRoACH" (UC Berkeley, USA) [6] and c. "X-RheX" (University of Pennsylvania, USA) [7].

As many insect-inspired robots, VelociRoACH's and X-RheX's design take inspiration from the cockroach. When cockroaches fall, they use their legs to swing around until finding an object in their surroundings (a rock, a tree, a blade of grass) to use as leverage to flip over. Significant strain and energetic demand is placed on a single leg to overturn the entire body [8]. In addition, solid and immobile objects are not found in all terrains, such as deserts.

Jumping is one of the most effective way to overcome large obstacles, compared to one's body size. Most animals jump by using their legs or other appendages to generate the motion. Hence many robots are designed to generate the jumping motion by using their legs, whereas these are directly actuated or not. Zhang et al. classify jumping robots based on the leg jumping principles found in nature [9], as presented in Table 2.

| Leg design (in nature) | Long legs | Short legs |
| :---: | :---: | :---: |
| Inspiration | Frog, kangoroo, cricket, <br> grasshopper, rabbit | flea, froghopper |
| Actuation type (robot) | Direct actuation | Indirect actuation |
| Energy storage | no | yes |
| Example | Mowgli [10], <br> Jumping frog robot [11] | MSU Jumper [12], <br> Grillo III [4] |

Table 1: Zhang et al. classification of jumping robots [9].

All the robots presented in Table 2 have a self-righting mechanism. Self-righting is
a fundamental component of jumping robot design as robots must be able to recover from landing and stand up to start the next jump. Self-righting techniques fall into two categories: passive or active recovery. Passive recovery is based on the position of the center of mass with respect to the leg or foot of the robot. The EPFL jumper [13] and Jollbot [14] are two examples of jumping robots using passive recovery. Active recovery, i.e. recovery enabled by the presence of actuators, is the one of the most commonly recovery technique found in nature: animals use their muscles to power their legs (such as beetles), spine or even their head like turtles do [15]. The Frogbot [16] and the MSU jumper [12] (Fig. 2) are two examples of jumping robots using active recovery principles. Fig. 2.b shows how the MSU jumper rights itself by using two lateral legs integrated in the design.


Figure 2: a) MSU Jumper prototype. b) Self-righting method based on two legs actuated by a single motor. [12]

However, these jumping robots might encounter difficulties when navigating through uneven and dangerous terrain where a robot can easily have its leg or part of the body stuck in a crack or damage a leg during the mission.

One of nature's solutions to jump and recover from falls can be found in click beetles (Coleoptera: Elateridae). Click beetles (Fig. 3) have evolved a unique self-righting jumping mechanism that does not depend on legs or other appendages. From an initially inverted position, they jack-knife their body to catapult themselves into the air and land back on their feet. This unique solution decouples the self-righting attributes (body and hinge, see Sect.2.2) from any other locomotion features (legs, wings). All the vital organs and muscles are protected within a strong shell, the cuticle. The objective of the work presented here is to understand the fast jumping mechanism of the click beetle and use its governing principles
to advance self-righting techniques for robotic applications.


Figure 3: From left to right: Parallelosthetus attenuatus (Photograph by Eric Rzeszutko, used with permission), Melanotus spp., Ampedus linteus and Alaus oculatus (Photographs by Ophelia Bolmin). Live specimens of these four species were studied among others as part of this work.

### 1.2 Background on Click Beetles

About 10,000 species in 400 genera of click beetles can be found worldwide [17]. While many insects jump to escape predators, click beetles have evolved a unique jumping mechanism to right themselves. Several assumptions have been made to explain the origin and function of this jumping mechanism. The clicking noise characterizing the jump can act as a communication technique or as a deterrent to enemies. Abshagen reports that chicks, which received click beetles as feed, were frightened when click beetles jumped. [19]. Verhoeff, Quelle and Fürsch postulate that the clicking motion acts as a wiggling mechanism which beetles use to liberate themselves from their pupation substrate in decaying wood [20, 21, 22]. Ribak shows that the click beetle's jump is usually vertical [23] and they therefore land within 50 cm of their take-off location. If many animals jump to escape predators, the jump of the click beetle would make a very poor escape method. When feeling threatened, Ribak reports that click beetles rather play dead or run away [24]. The click beetle jumping mechanism is most probably a multi-purpose mechanism. This study considers the jump as a self-righting mechanism.

In 1971, Evans described in detail click beetle morphology and the mechanics of the jump [25]. Kaschek extended Evans' work in 1984 to the study of the jump mechanics of 22 species of various sizes (the body length varied from $8.8 \mathrm{~mm} \pm 0.4$ to $24.0 \mathrm{~mm} \pm 2.0$ ) [26]. Subsequent work by Ribak further explained the morphological constraints and jump characteristics [23,24]. Dissections of click beetles and environmental scanning electron microscopy
(ESEM) micrographs allowed for a better understanding of the beetle's exoskeletal structure [25]. Click beetles perform the jump from an inverted position by arching their body while storing energy in enlarged muscles in their prothorax, cuticle (exoskeleton), tendons and other biomechanical springs, until suddenly releasing this stored energy. The basic body morphology of a click beetle can be divided into three parts: two subunits or masses and the hinge, as shown in Fig. 4. The first mass includes the head and prothorax, the second mass includes the mesothorax, metathorax and abdomen, the hinge is comprised of a peghold and a mesosternal lip.


Figure 4: The beetle body can be divided into two masses and a hinge: the head and the prothorax make up one mass (M1) and the mesothorax, metathorax and abdomen make up second mass (M2). The two masses are linked by a hinge.

### 1.3 Research Goals and Objectives

Previous studies explained how the beetle is able to jump without legs ( [25], [26] [23, 24]) but many of the details are still unclear. These details are important to accurately model the click beetle's jump and emulate the jump in a novel robotic design. For instance, where exactly is the energy for the jump stored? What are the contributions of the muscles and the exoskeleton to this energy storage? What are the friction forces involved in the peghold - mesosternal lip system? How do the total mass and mass ratio between the two body subunits influence the take-off and aerial stages of the jump? What parameters dominate the physics of the jump?

The goal of this bio-inspired research is to create a framework for robot design by looking at the jump from a micro and macro-mechanical point of view. Observing and modeling the
click beetle jump drives the design of a future self-righting autonomous bio-inspired robot while the methods and models developed using mathematical and engineering tools inform the general understanding of the jump and the insect physiology.

The goal of this thesis is to observe, understand and model the click beetle jump, as well as to identify morphological and kinematic governing parameters.

The remainder of this thesis is organized as follows: the methods used and the stages of the jump are detailed in Section 2. Dynamic and kinematic models are derived based on observing the jump of live beetles and are presented in Section 3. Results are presented and discussed in Section 4.

### 1.4 Original Research Contributions

The original research contributions of this work are presented bellow:
[R.C.1] The correlation between the mass and the length of click beetles from various species indicate isometric scaling.
[R.C.2] The three consecutive stages of the jump are identified and analyzed. In particular, the angular velocity and acceleration of the body during the take-off stage are extracted.
[R.C.3] The hinge structure is detailed and assumed to be the source of the clicking sound characteristic to the jump of click beetles.
[R.C.4] It is shown that the trajectory of the body during the airborne stage follows a ballistic motion.
[R.C.5] A wing deployment behavior is observed during the airborne stage and increased the likelihood of the beetle landing on its feet.
[R.C.6] An energetic analysis of the jump stages is developed, as well as kinematic and dynamic models for each stage of the jump. The hinge stiffness, the equivalent body stiffness and a simulation of the airborne trajectory are derived.
[R.C.7] A non-dimensional analysis enables the identification and correlation of the dominant morphological and kinematic parameters during the airborne stage.

These research contributions will be highlighted in the thesis text.

## 2 Materials and Methods: Observing the Click Beetle Jump

### 2.1 Morphological Measurements

Morphological measurements of 9 different click beetles species and kinematic measurements of 6 different click beetle species were obtained. The masses, the lengths and the number of specimens studied are presented in Table 2.

| Genus | Species | n | Average dry mass <br> $(\mathrm{mg}) \pm 3 \mathrm{mg}$ | Average body length <br> $(\mathrm{mm}) \pm 0.1 \mathrm{~mm}$ |
| :---: | :---: | :---: | :---: | :---: |
| Agriotes | sp. | 3 | 1.7 | 4.7 |
| Alaus | oculatus | 2 | 392.2 | 37.3 |
| Ampedus | linteus | 1 | 10.0 | 9.5 |
| Athous | sp. | 2 | 18.6 | 13.0 |
| Hemicrepidius | sp. | 4 | 62.0 | 19.0 |
| Lacon | discoideus | 1 | 13.2 | 11.3 |
| Melanactes | sp. | 1 | 147.7 | 23.7 |
| Melanotus | spp. | 16 | 24.4 | 13.6 |
| Parallelostethus | attenuatus | 4 | 50.4 | 18.3 |

Table 2: Click beetles species studied as part of this work, their average masses and body lengths.

The specimens used for this study were collected in various locations in Illinois, Michigan and Ohio. Agriotes sp. was found in Michigan, Alaus oculatus was found in Allerton Park, IL $\left(39^{\circ} 59^{\prime} 54.9^{\prime \prime} \mathrm{N} 88^{\circ} 39^{\prime} 01.2^{\prime \prime} \mathrm{W}\right)(\mathrm{n}=1)$ and on private land in Champaign County ( $\mathrm{n}=1$ ). Ampedus linteus was found at the Vermillion River Observatory, IL ( $40^{\circ} 04^{\prime} 06.0^{\prime \prime} \mathrm{N}$ $\left.87^{\circ} 33^{\prime} 13.5 " \mathrm{~W}\right)(\mathrm{n}=1)$, Athous sp. was found in Brownfield Woods, IL ( $40{ }^{\circ} 08^{\prime} 56.0^{\prime \prime} \mathrm{N} 88$ $\left.{ }^{\circ} 09^{\prime} 57.1^{\prime \prime} \mathrm{W}\right)(\mathrm{n}=1)$ and Allerton Park ( $\mathrm{n}=1$ ), Hemicrepidius sp. was found in Brownfield Woods ( $\mathrm{n}=2$ ) and Carle Park, IL ( $40^{\circ} 06^{\prime} 18.1^{\prime \prime} \mathrm{N} 88^{\circ} 12^{\prime} 44.1^{\prime \prime W}$ ) ( $\mathrm{n}=2$ ), Lacon discoideus ( $\mathrm{n}=1$ ), Melanactes sp. ( $\mathrm{n}=1$ ) and Parallelostethus attenuatus were found in Brownfield

Woods ( $\mathrm{n}=4$ ), and Melanotus sp. was found in Brownfield ( $\mathrm{n}=5$ ), Allerton Park ( $\mathrm{n}=1$ ), Carle Park ( $\mathrm{n}=1$ ), on private land in Middleville, Michigan ( $\mathrm{n}=6$ ) and in Lima, Ohio ( $\mathrm{n}=2$ ) and at University of Illinois in Urbana-Champaign ( $40^{\circ} 06^{\prime} 40.9^{\prime \prime} \mathrm{N} 88^{\circ} 13^{\prime} 30.1^{\prime \prime} \mathrm{W}$ ) ( $\mathrm{n}=1$ ). Alcohol traps and collection at lights were used. The beetles collected were kept in individual boxes and fed with sugar and water once a week (Appendix A).

The mass presented in Table 2 is the average dry mass of the specimens studied. The mass of live animals varies across days, which is why the dry mass of the specimens is used for this study to have a consistent measurement over time for the mass of all individuals. The mass of a live animal is approximately two times the dry mass. The animals were placed in an oven (Chicago Surgical \& Electrical Co. 300) at $28{ }^{\circ} \mathrm{C}$ for approximately 10 days. The mass was measured every two days until stabilization (Appendix B), using a Fisher Scientific accuSeries ${ }^{\circledR}$ scale. The length of the click beetles was determined by adding the length of the two-subunits, measured from "tip to tip" (Appendix B) using Mitutoyo Absolute calipers. Fig. 5 shows how the body mass relates to the length of the beetle. The mass of the 34 specimens is proportional to the body length cubed. This means that the mass scales with volume i.e. indicates isometric scaling. [R.C.1]


Figure 5: Body length dependance on the mass of 34 click beetle specimens.

The mass ratio between the head+prothorax $\left(m_{1}\right)$ and mesothorax+metathorax+abdomen
$\left(m_{2}\right)$ varies between the different species. Fig. 6 shows the relationship between the mass ratio $\left(m_{1} / m_{2}\right)$ and the mass of the beetle ( $\mathrm{n}=10$ ) (Fig.6.A) and the relationship of the length ratio $\left(l_{1} / l_{2}\right)$ (length of head + prothorax/length of the mesothorax + metathorax + abdomen) and the mass of the beetle ( $\mathrm{n}=11$ ) (Fig.6.B, also see Appendix A).



Figure 6: Relationship between the mass ratio $\left(m_{1} / m_{2}\right)$ and the mass of the beetle ( $\mathrm{n}=10$ ) (left). Relationship between the length ratio $\left(l_{1} / l_{2}\right)$ and the mass of the beetle ( $\mathrm{n}=11$ ) (right).

Further measurements of the mass ratio of all the beetles collected will enable a better understanding of the relationship between the mass ratio, the length ratio and the mass.

Fig. 7 shows the relationship between and the mass ratio $(\mathrm{n}=10)$.


Figure 7: Relationship between the length ratio and the mass ratio. Sample size same as in Fig. 6 A.

For each specimen that was filmed as part of this work, we estimated the mass moment
of inertia. We assume the body of the beetle to be a rigid uniform cylinder, rotating about its center of mass. These assuptions were previously made by Kaschek [26]. Hence:

$$
\begin{equation*}
I=m\left(\frac{l^{2}}{12}+\frac{d^{2}}{16}\right) \tag{1}
\end{equation*}
$$

where m is the dry mass of the beetle, 1 the total body length and $d$ the average radius of the beetle. As a first approximation, d was calculated by averaging the maximum width and the maximum body height for each specimen. The calculated inertia values are presented Fig. 8.


Figure 8: Mass moment of inertia of specimens studied in relation to the body mass.

The radius of the curvature of the elytra R was measured from high resolution images, using the ImageJ software (Fig. 9) [27]. This metric characterizes the elytra shape of each beetle depending on its concavity.


Figure 9: Definition of the radius of the elytra of a Melanotus spp. specimen.

The radii of the elytra of 6 Melanotus spp. and 2 Parallelostethus attenuatus animals are shown Fig. 10. The radius of each beetle was measured considering the body segment going from the middle of the third sternite to the base of the wings (Appendix B).


Figure 10: Radii of the elytra for Melanotus spp. ( $\mathrm{n}=6$ ) and Parallelosthetus attenuatus $(\mathrm{n}=2)$ specimens with respect to the mass.

Further measurements of all the specimens available will enable a better understanding of the correlation between the radius of the elytra and the beetle's morphology (mass, length).

### 2.2 Stages of the Click Beetle Jump

The click beetle jump can be divided into three consecutive stages (Fig.11). During the prejump stage or energy storage stage (Fig. 11.1), the beetle arches its body and stores energy. This position is held by friction for a varying amount of time, ranging from half a second to 2-3 seconds. During the take-off stage (Fig. 11.2), the beetle starts releasing energy while still in contact with the floor and generates the necessary impulse to propel its body into the air. Both the pre-jump and take-off stages are characterized by translation of the beetle's hinge and the rotation of the two masses relative to the hinge (Fig.11.1b-Fig.11.2a). The head of the beetle leaves the ground first, followed by the body. The beetle then somersaults in the air during the airborne stage (Fig. 11.3). [R.C.2]

### 2.2.1 Pre-jump Stage

The hinge is composed of a peghold and a mesosternal lip, which comprise an interface with topography that allows them to be held in place or slip. The anatomy of the hinge is sketched in Fig.12.1. Removal or significant damage to these parts prevents the ability to jump [25]. Once part of the peghold (Fig. 12) is removed, the beetle cannot maintain the arched, energy storage position (Fig. 11.1.c), and therefore cannot perform the jump [25].

The peghold and mesosternal lip of Melanotus spp. and Alaus occulatus were photographed using a Phonom Pro environmental scanning electron microscope (ESEM) (Fig.13). Variations in shapes between species were observed (Appendix E). During the pre-jump stage, the peghold's friction plate, shown in Fig.13.1a, slides over the mesosternal lip. The peghold slides from position $a$ to position $b$ as shown in Fig. 12.2. At the end of the stage, the peghold comes to rest as the friction plate drops into a semi-spherical ledge on the mesosternal lip, as shown in Fig. 13.2b.


Figure 11: The observed jump stages of click beetles


Figure 12: 1: Anatomy of the hinge, based on Evans' morphological sketch [25]. The beetle is shown resting on its dorsal side. The M4 and M2 muscles located in the prothorax store mechanical energy prior to the jump. 2: During the pre-jump stage, the peghold slides over the mesosternal lip and comes to rest as its friction plate drops into a semi-spherical ledge on the mesosternal lip (position $b$ ).

The resting position of the hinge is held by friction while the energy is transferred from the M4 and M2 muscles (Fig. 12) to the cuticle and other biomechanical springs [25, 29]. Once the energy required to perform the jump is stored, the arched overall body position is released: the peghold slips back over the mesosternal lip and inserts into the cavity (i.e. the peghold moves from position $b$ to position $a$ as shown in Fig. 12.2). We postulate that the mechanical slip over the ledge is the source of the audible "click" as the beetle launches [R.C.3]. Given that the hinge's motion is constrained by the junction of the prothorax and mesothorax sections, both peghold and mesosternal lip deform elastically to allow the slip of the friction plate over the lip, as shown by the different deformed states of the peghold shown in Fig.12.2. Two rows of hairs, aligned parallel to the mesosternal cavity, guide the peghold into the cavity after the energy is released from the muscles and into the body's biomechanical springs (Fig. 13.4). [R.C.3]


Figure 13: ESEM micrographs of the side (1) and dorsal (2) views of the peghold and ventral (3) and side (4) views of the mesosternal lip of Alaus oculatus.

### 2.2.2 Take-off Stage



Figure 14: Frames of high speed video footage of the take-off stage. 1.shows the initial resting position and 2 . shows the frame when the abdomen leaves the ground.

The take-off stage is defined by the motion of the click-beetle from the first instance of the energy release (the "click", when the peg frees itself from its resting position on the edge of the lip) to the moment when the beetle leaves the ground. The motion of Melanotus spp. ( $\mathrm{n}=9$ ), Hemicrepidius sp. ( $\mathrm{n}=2$ ), Parallelosthethus attenuatus ( $\mathrm{n}=4$ ), Ampedus linteus $(\mathrm{n}=1)$ and Alaus oculatus $(\mathrm{n}=1)$ was recorded using a Photron Fastcam SA-Z at a rate of 20,000 frames per second. The frames of interest are presented in Fig. 14. The downwards motion of the hinge is characteristic of this stage and causes the rotation of the two masses around the hinge. Angular accelerations and velocities were derived from high-speed footage of six different species are presented Fig. 15. The acceleration is considered to be constant during the take-off stage as a first assumption.


Figure 15: Angular velocity (left) and angular acceleration (right) of Melanotus spp. ( $\mathrm{n}=8$ ), Melanactes sp. ( $\mathrm{n}=1$ ), Hemicrepidius sp. ( $\mathrm{n}=1$ ), Parallelosthethus attenuatus (n=3), Ampedus linteus $(\mathrm{n}=1)$ and Alaus oculatus $(\mathrm{n}=1)$ during the take-off stage.

The angular velocity and acceleration varied from $1.1 * 10^{4} \mathrm{rad} / \mathrm{s}$ to $1.3 * 10^{5} \mathrm{rad} / \mathrm{s}$ and $5.3 * 10^{7} \mathrm{rad} / \mathrm{s}^{2}$ and $6.7 * 10^{8} \mathrm{rad} / s^{2}$ respectively. Prior work done on click beetles does not include observations of the take-off stage and data on angular velocities and accelerations during this stage. [R.C.2]

### 2.2.3 Airborne Stage

The airborne stage of Melanotus spp. ( $\mathrm{n}=8$ ), Hemicrepidius sp. ( $\mathrm{n}=1$ ), Melanactes sp . ( $\mathrm{n}=1$ ), Parallelosthethus attenuatus $(\mathrm{n}=4)$, Ampedus linteus $(\mathrm{n}=1)$ and Alaus oculatus $(\mathrm{n}=1)$ was recorded using a Photron FastCam SA-Z at a rate of 20,000 frames per second. Such a high number of frames per second is not required to observe the airborne stage but is necessary to observe the take-off stage of the jump. The trajectory of the center of mass was reconstructed by tracking the shape of the beetle using Photron FASTCAM Analysis Software [28]. The position of the geometrical center of the beetle was derived and we assimilate the center of mass with the geometrical center assuming constant mass density and width of the specimens. The jumps of sixteen beetles were analyzed and the trajectory of the center of mass for each beetle with respect to time was determined.


Figure 16: Definition of the maximum jump height $h$, landing distance $d$, the take-off angle $\alpha$, and the resultant take-off velocity $V_{0}$

Each beetle was made to jump on 3 days within a 7 days period during Summer 2017. For each beetle, 5 to 7 jumps per day were recorded. The maximum jump height and landing distance, defined in Fig. 16, was measured. The take-off angle, velocity and acceleration of the center of mass at take-off were measured in two different ways. The first method involved calculating the velocity by deriving the position with respect to time and measuring the takeoff angle $\alpha$ from the position of the beetle with respect to time:

$$
\begin{align*}
V_{x} & =\frac{\partial x}{\partial t}, V_{y}=\frac{\partial y}{\partial t}  \tag{2}\\
\alpha & =\tan ^{-1}\left(\frac{y}{x}\right) \tag{3}
\end{align*}
$$

In the second method, the take-off velocity and take-off angle for each jump were calculated by solving the ballistic motion system of equations for $v_{0}$ and $\alpha$ after measuring $h$ and $d$ from the videos:

$$
\begin{align*}
& h=\frac{v_{0}^{2} \sin (\alpha)^{2}}{2 g}  \tag{4}\\
& d=\frac{v_{0}^{2}}{g} \sin (2 \alpha) \tag{5}
\end{align*}
$$

Comparison between results of the two methods show that $97 \%$ of the jumps followed a ballistic motion within $15 \%$ (the total number of jumps analyzed was 159) [R.C.4]. Hence we assume that the trajectory of the center of mass follows a ballistic motion.

The mean values of the maximum jump height, take-off angle, velocity and acceleration were calculated for each individual. The results are presented in Fig. 17.


Figure 17: Average jump height (top left), landing distance (top right), take-off angle (middle right), angular velocity at take-off (middle left), velocity (bottom left) and acceleration at take-off (bottom right) of Melanotus spp., Hemicrepidius sp., Parallelosthethus attenuatus, Ampedus linteus and Alaus oculatus. Each data set is presented with a $95 \%$ interval of confidence.

The values for the take-off velocities and take-off angles are agree with previous work by Ribak et al. [23] and Kaschek [26]: our analysis of the jumps of 16 beetles provided take-off angles ranging from $72.5^{\circ}$ to $87.6^{\circ}$, and take-off velocities ranging from 0.78 to $1.69 \mathrm{~m} / \mathrm{s}$ (Fig. 17). The take-off velocities were calculated by deriving the position of the beetle for each jump with respect to time (Eq.2). The maximum jump height was between 41.7 and 119.7 $\mathrm{mm} \pm 2 \mathrm{~mm}$. The beetles landed at 12.4 mm to $59.1 \mathrm{~mm} \pm 2 \mathrm{~mm}$ away from their take-off location.

The take-off acceleration is defined as the peak acceleration which happens at the beginning of the take-off stage. The upwards acceleration of the center of mass during the take-off stage results in the airborne stage. The peak acceleration calculated ranges between 132 $\mathrm{m} / \mathrm{s}^{2}$ and $287 \mathrm{~m} / \mathrm{s}^{2}$. This definition of the acceleration differs from the definition used by Evans [25] and Kaschek [26]. Evans and Kaschek assumed the acceleration to be constant during take-off and calculated it by dividing the take-off velocity by the take-off time (about $0.64 \mathrm{~ms}[25])$. They defined the take-off time by determining the time interval between the frame where the beetle is still in contact with the ground (just about to become airborne) and the subsequent frame that showed that the beetle had left the ground. When this method was used to calculate the acceleration of one of our Parallelosthetus attenuatus specimen, we also estimated a take-off time of 0.65 ms , which mean an acceleration of $2295 \mathrm{~m} / \mathrm{s}^{2}$.

These values are in the same range of accelerations reported by Evans and Kaschek, however, as we explained above, we will consider the acceleration of the beetles at take-off to be the peek acceleration as presented in Fig.17.

Previous work on click beetles has shown that the animals only land on their feet (ventral side down) $50 \%$ of the time [23]. The beetles studied here also landed on their feet or on their dorsal side randomly, and if needed, would jump again until landing on their feet, ventral side down. However, we observed several individuals which would open their wings before landing. The beetles either flapped their wings or used them to glide presumable attempting to stabilize their body in the air. The wing deployment behavior was also random, as several individuals from the same species would perform consecutive jumps with or without deploying their wings (Fig.18). Beetles which deployed their wings almost always landed on
their feet. [R.C.5]


Figure 18: Wing deployment during the airborne stage for a) Melanotus spp., b) Melanotus spp. and c) Ampedus linteus.

## 3 Modeling Approach

### 3.1 Energetics

An energetic approach was taken to model how click beetles store and release energy in order to jump [R.C.6]. We assume that all energy is stored during the pre-jump stage. The muscles in the prothorax (Fig. 12) contract, extending the ventral length and pulling the hinge away from the substrate and rotating the two masses around the hinge. Gronenberg et al. suggested that the accelerations and velocities observed during the energy release stage cannot be created by muscle contraction alone, because muscles have inherently slow contraction velocity. Small animals and insects have other elastic biomechanical springs, such as the cuticle, tendons and rubbery proteins [29]. For the remainder of this study, we will assume that the beetles' combined biomechanical springs are responsible for the energy release during the take-off and airborne stages. The amount of energy released during the jump can be derived using an energy balance approach.

During the pre-jump stage, we assumed the following:

- The work of the muscles is used to position the body in the stored-energy state, and "spring load" the hinge to its launch-ready configuration. The body's upwards motion is due to the hinge rotation and the deformation of the peghold (snap-fit mechanism, see Sect. 3.2).
- The remainder of the muscle energy is transfered onto the beetle's cuticle, tendons and other biomechanical springs, referred to as $E_{\text {biom,spring }}$.
- Friction with the ground and gravitationnal effects can be neglected.
- There are no observed bulk body deformations; all deformations are local to the constriction between the prothorax and the mesothorax, referred as the hinge (Fig.4).

Based on the aforementioned assumptions, the energy balance during the pre-jump stage is expressed by:

$$
\begin{equation*}
E_{\text {muscles }}=E_{\text {motion }}+E_{\text {snap-fit }}+E_{\text {biom }, \text { spring }} \tag{6}
\end{equation*}
$$

Where $E_{\text {biom,spring }}$ is defined as the energy stored in the beetle's body :

$$
\begin{equation*}
E_{\text {biom }, \text { spring }}=E_{\text {cuticle }}+E_{\text {proteins }}+E_{\text {deflectionSF }}+\ldots \tag{7}
\end{equation*}
$$

Given that the hinge's motion is constrained by the junction of the prothorax and mesothorax sections, both peghold and mesosternal lip deform elastically to allow the slip of the friction plate over the lip, as shown by the different deformed states of the peghold shown in Fig. 12. The strain energy generated by this deformation is referred in Eq (7) as $E_{\text {deflectionSF }}$.

The peghold/lip interaction is modeled as a mechanical "snap-fit" mechanism, which is described in detail in Section 3.2. This purely mechanical model assumes that the energy needed to both load and release the mechanism is equal hence $E_{\text {snap-fit }}$ is similar during the pre-jump and take-off stages. The energy released during the downwards impulse of the hinge is converted to the airborne motion, referred to as $E_{t a k e-o f f}$. Therefore the energy balance for the take-off stage is expressed by:

$$
\begin{equation*}
E_{\text {biom,spring }}=E_{\text {snap-fit }}+E_{\text {take-off }} \tag{8}
\end{equation*}
$$

During the airborne stage, the center of mass of the beetle has an overall ballistic motion. The difference in masses between the two main body sections create angular momentum, which causes somersaulting maneuvers. The energy of the somersaulting motion is referred to as $E_{\text {rotation }}$. The head and prothorax of the beetle oscillate around the hinge during the airborne stage. We neglect head and prothorax oscillations in this study. The energy equilibrium during the airborne stage is expressed by:

$$
\begin{equation*}
E_{\text {take-off }}=E_{\text {ballistic }}+E_{\text {rotation }} \tag{9}
\end{equation*}
$$

Considering the click beetle's body as a biomechanical spring, we can interpret $E_{\text {muscles }}$, $E_{\text {cuticle }}$ and $E_{\text {snap-fit }}$ introduced in Equations 6 through 9 as the energies developed and transfered between the body's subsystems. These energies will derived from high-speed video recordings and friction experimental data (see Sect. 3.1). Muscles actuate the system, creating the motion and the potential energy storage in the cuticle, which acts like a spring. The snap-fit mechanism constitutes the trigger to energy release. We obtain the equivalences presented in Eq. 10, 11 and 12.

$$
\begin{equation*}
E_{\text {muscle }}=E_{\text {actuator }} \quad(10) \quad E_{\text {cuticle }}=E_{\text {spring }} \tag{11}
\end{equation*}
$$

$$
\begin{equation*}
E_{\text {snap-fit }}=E_{\text {trigger }} \tag{12}
\end{equation*}
$$

### 3.2 Pre-jump Model

The hinge mechanism, composed of the peghold and the mesosternal lip (Fig. 12.2) is modeled as a snap-fit mechanism [R.C.6], as shown in Fig.19. Snap-fits geometrically combine the deflection of a cantilevered beam with a friction wedge at the tip of the cantilevered beam to create a discontinuous motion (Fig. 19). As the friction wedge slips over a lip, the elastic deformation of the beam is recovered quickly as it snaps back to a neutral position. In the click beetle, the peghold is the cantilever beam, and the friction plate is the wedge. The energy needed to load and release this trigger (i.e. to reach the resting position in Fig.12) is defined as $E_{\text {snap-fit }}$.

The inputs to this model are the effective peghold cross-


Figure 19: The peghold (top) is modeled as a snap-fit mechanism (bottom) [30]. sectional area, the deflection distance, the angle of the frictional pad with respect to the axis of slip, the effective Young's modulus of the peghold, and the friction coefficient of the peghold/lip interface. We assume that the peghold can be approximated as a hollow cylinder, as an internal cavity has been observed (images not shown). This geometry is measured directly from the ESEM images, and is summarized in Section 4. The equation for the deflection, $\delta$, of a fixed-free beam of modulus $E$, length $L$, and area moment of inertia $I$ due to load $P$ (Eq. 13) is combined with the friction coefficient, $\mu$, and the angle of the friction wedge, $\alpha$, to provide the insertion force $W$ of the snap-fit (Eq. 14). This insertion force over some effective slip distance and angular translation is considered to contribute substantially to the hinge stiffness, $K$ (see Sect.4.1), and therefore the launch energy, $E_{\text {snap-fit }}$.

$$
\begin{equation*}
\delta=\frac{P L^{3}}{3 E I} \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
W=P \frac{\mu+\tan (\alpha)}{1-\mu * \tan (\alpha)} \tag{14}
\end{equation*}
$$

A micro-mechanical experiment on the bending of the hinge was constructed to measure the force required to load/release the snap-fit mechanism. Dry specimens of Melanotus spp. were mounted in foam from the posterior tip of the abdomen up to about 1 mm short of the thorax-abdomen junction (i.e. the hinge). The hinge began in a neutral position, and
was not loaded during drying. A nylon thread was affixed to the dorsal side of the head and a force transducer with a sensitivity on the order of single microNewtons and a maximum force of approximately 200 mN . A linear stage translated the mounted body at $\mathrm{v}=0.25 \mathrm{~mm} / \mathrm{s}$, causing the hinge to rotate and moving the peghold from position $a$ to position $b$ as shown in Fig. 12.2 (Appendix G). When it is loaded and the peghold snaps over the lip, a force drop is observed, and the magnitude of the force drop is reported in Sect. 4.

### 3.3 Take-off Model

The energy release stage is initiated when the beetle is resting on the substrate. When the muscle tension is released, the peghold slips over the mesosternal lip with low frictional resistance, launching the mass into the air through leverage against the ground. This motion is modeled by a crank-slider mechanism as shown in Fig. 20. [R.C.6]


Figure 20: Crank-slider mechanism modelling the dynamics of the takeoff stage.

We assume that the friction with the ground can
be neglected and that there are no body deformations. The angular acceleration at the beginning of the take-off stage is assumed to be constant and is considered as an input to the crank-slider model. The two links of the crank-slider are connected by an ideal pin joint, which is actuated by a torque $(\mathrm{T})$. We assume as a first approximation that $T=K * \gamma$, where $\gamma$ is the body's angle and $K$ is the stiffness of the modeled spring, which is composed of the stiffness of the snap-fit mechanism and the beetle body's biomechanical springs [29]. The equation of motion of the crank-slider is solved for the stiffness coefficient K given each beetle's mass ratio, where the geometric parameters are defined as shown Fig. 21.


Figure 21: Take-off model parameters definition

The equation of motion is:

$$
\begin{gather*}
T=\frac{l_{1} l_{2} \cos (\alpha) \cos (\beta)\left[1+m_{1} a_{G_{1 y}}\right]}{l_{2} \cos (\beta)+l_{1} \cos (\alpha)\left[1+m_{1} a_{G_{1 y}}\right]}\left(\frac{2 I_{G_{1}} \ddot{\alpha}}{l_{1} \cos (\alpha)\left[1+m_{1} a_{G_{1} y}\right]}-\frac{m_{1} a_{G_{1} y}}{1+m_{1} a_{G_{1} y}}\right. \\
\left.-\frac{\tan (\alpha)}{m_{1} a_{G_{1} x}+m_{2} a_{G_{2} x}-\frac{I_{G_{2}} \ddot{\beta}}{l_{2} \cos (\beta)}+l_{2} \tan (\beta) m_{2} a_{G_{2} x}}-\frac{m_{2} a_{G_{2} x}}{\cos (\beta)}+m_{1} a_{G_{1} y}+m_{2} a_{G_{2} x}\right) \tag{15}
\end{gather*}
$$

Where $\dot{\alpha}$ is the angular velocity and $\ddot{\alpha}$ the angular acceleration of link $1, \dot{\beta}$ is the angular velocity and $\ddot{\beta}$ the angular acceleration of link $2, a_{G_{i} x}$ the linear acceleration of the center of mass of link i in the x direction, $a_{G_{i} y}$ the linear acceleration of the center of mass of link i in the y direction and $I_{G_{i}}$ is the mass moment of inertia of the link $\mathrm{i}(\mathrm{i}=1$ or 2 ). The calculations are detailed in Appendix H .

Initial angles, velocities and accelerations are derived from the high-speed video recordings of the take-off stage (Fig. 14, Fig.15). Future work will include replacing the constant acceleration with a time dependent acceleration derived from high speed videos data.

### 3.4 Airborne Model

During the airborne phase, we neglected the oscillations of the head and prothorax around the hinge. The click beetle is modeled as two point masses connected with a rigid massless link, as shown in Fig.22. The dynamics of the system are derived and simulated using MATLAB [31]. Given an initial velocity, take-off angle and angular momentum of the two masses, the overall trajectory of the beetle in the air, i.e. the landing location, maximum height of the center of mass (ballistic motion), and the number of rotations of the two masses around the center of mass is predicted. The model inputs were derived from the high speed
recording of the complete beetle jumps.

### 3.5 Non-dimensional Analysis

### 3.5.1 The Vaschy-Buckingham or " $\pi$ "-Theorem

The Vaschy-Buckingham theorem [32] or " $\pi$ "-theorem is a formalization of Rauleigh's method of dimensional analysis. It provides a method for computing non dimensional parameters for a given set of variables. In this study, the VaschyBuckingham theorem is used to identify the driving parameters of the airborne stage [R.C.7]. The following variables were identified as descriptors of the airborne stage, where L is a unit length, M is a unit mass and T is a unit time:

1. Morphological variables

- body length $l$, with $[l]=\mathrm{L}$
- mass $m$, with $[m]=\mathrm{M}$
- radius of the elytra curvature $R$, with $[R]=\mathrm{L}$


## 2. Kinematic variables

- velocity of the center of mass at take-off $V_{0}$ (resultant), with $\left[V_{0}\right]=\mathrm{LT}^{-1}$
- take-off angle $\alpha$ (in radians)
- acceleration of the center of mass at take-off $A c c_{0}$, with $\left[A c c_{0}\right]=\mathrm{LT}^{-2}$
- jump height $h$, with $[h]=\mathrm{L}$
- landing distance $d$, with $[d]=\mathrm{L}$
- energy available to perform the jump $E_{\text {input }}$, with $\left[E_{\text {input }}\right]=\mathrm{ML}^{2} \mathrm{~T}^{-2}$.


### 3.5.2 Non-dimensional Parameters

All driving parameters can be described using three dimensions: L, M and T. Three of the driving parameters were selected as repeated variables to apply the $\pi$-theorem: the mass $m$,
the length $l$ and the energy input $E_{\text {input }}$. The six remaining variables can be expressed in terms of $m, l$ and $E_{\text {input }}$ in the following way:

- Morphology:

$$
\begin{equation*}
\Pi_{R}=\frac{R}{l} \tag{16}
\end{equation*}
$$

- Kinematic:

$$
\begin{equation*}
\Pi_{h}=\frac{h}{l} \tag{17}
\end{equation*}
$$

- Dynamic:

$$
\begin{gather*}
\Pi_{E}=\frac{V_{0} \sqrt{m}}{\sqrt{E_{\text {input }}}} \Rightarrow \Pi_{E}^{2}=\frac{V_{0}^{2} m}{E_{\text {input }}} \propto \frac{E_{\text {kinetic }}}{E_{\text {input }}}  \tag{18}\\
\Pi_{\alpha}=\alpha  \tag{19}\\
\Pi_{W}=\frac{A c c_{0} m l}{E_{\text {input }}} \tag{20}
\end{gather*}
$$

$\Pi_{h}$ will be referred as the relative jump height and $\Pi_{R}$ as the relative radius of the elytra. $\Pi_{E}$ and $\Pi_{W}$ can be considered as efficiency parameters, as $\Pi_{E}$ is the ratio of the energy output divided by the energy input and $\Pi_{W}$ represents the work done to accelerate the center of mass at take-off (output) per energy input. Assuming the conservation of energy from the pre-jump stage to the airborne stage, we assume $E_{\text {input }}=E_{\text {airborne }}$, which we assimilate to $E_{\text {biom,spring }}$ and $E_{\text {muscles }}$ as a first approximation. As $\Pi_{E}$ and $\Pi_{W}$ are physically closely related (representing an energy output over an energy input) only $\Pi_{W}$ will be considered for the remainder of this study.

## 4 Results

### 4.1 Pre-jump Model Results

The force drop as the peghold/lip mechanism snapped into the loaded position was measured to be $W=$ $1.6 m N$ for a Melanotus spp. specimen. There are multiple force drops in the measurement (Fig. 24), but video timing was used to confirm that the force drop corresponded to the "snap" of the beetle hinge.

Implementing Eq. 14 with a friction coefficient of $\mu=0.142$ [33] and a measured friction wedge angle of $\alpha=25.2^{\circ}$ (see Fig.23) gives a beam deflection force of $P=2.44 m N$. Assuming a hollow cylindrical peg


Figure 23: Geometry and force vector used to calculate the stiffness of the peghold/lip mechanism. with outer diameter $\sim 1 \mathrm{~mm}$, wall thickness $\sim 200 \mu \mathrm{~m}$, length $\mathrm{L}=4 \mathrm{~mm}$, and Young's modulus of $E \sim 7 G P a$ (from Vincent et al. mechanical characterisation of insect cuticle [34]), the deflection of the beam $\delta$ is $0.25 \mu m$ (by Eq. 13).

This value is likely too low, further work is needed to refine the specific snap-fit model properties.

The force drop is used to estimate the stiffness contribution of this mechanism to the hinge stiffness $K$. Rough estimates of the slip distance across the mesosternal lip width of $s \sim 50 \mu m$ (see Fig.23) gives a launch torque $T_{l}$ of 80 nNm (i.e. $T_{l}=W . s$ ). This torque acting over an impulse angular displacement of $\theta \sim 0.1$ radians at a distance away from the center of rotation of $R \sim 0.5 \mathrm{~mm}$ gives a hinge stiffness of $K_{h}=\frac{T}{\theta}=8 * 10^{-7} \mathrm{Nmrad}^{-1}$ [R.C.6]. Further experiment will increase mesearuement fidelity and allow analysis of the effect of the geometry on the force drop. Melanotus sp., Parallelosthetus attenuatus, Alaus oculatus and Melanactes sp. specimens will be tested in the near future.


Figure 24: The force needed to actuate the hinge of a Melanotus spp. beetle showing the force drop during the snap-fit and subsequent breakage as the motion was pushed past biological limits.

### 4.2 Take-off Model Results

The equation of motion of the crank-slider mechanism (Fig.20) was derived and solved for the torque $T$. The hinge stiffness $K$ was then calculated assuming that the torque necessary to create the downwards motion of the hinge is linearly proportional to $\gamma$. In the take-off model, the hinge represents the only energy storage mechanism. Hence it releases all the energy available to jump (i.e. the energy stored in the biomechanical springs after the prejump stage). Analysis of the high-speed recording of the take-off stage (Fig.14) provided angular velocities and accelerations (Fig. 15), as well as the body's angle in the energy storage position. Fig. 25 shows the calculated results for the stiffness K resulting of the jumps of five different species.[R.C.6]


Figure 25: Coefficient of stiffness K calculated in terms of the mass (right) of 4 beetles species (left), namely Alaus oculatus ( $\mathrm{n}=1$ ), Ampedus linteus ( $\mathrm{n}=1$ ), Melanotus spp. ( $\mathrm{n}=4$ ), Melanactes sp. ( $\mathrm{n}=1$ ) and Parallelosthetus attenuatus $(\mathrm{n}=3)$. On the right: zoom in the results for Ampedus linteus, Melanotus spp. and Parallelosthetus attenuatus.

Further investigations will include improving the take-off model in order to take into account where (and how) the energy is stored in the body.

### 4.3 Airborne Model Results

### 4.3.1 Two-masses Model Simulation

The equation of motion of the two mass-rigid body model (Fig. 22) was derived and a MATLAB [31] simulation was developed to replicate the airborne motion of the beetle.

The jumps of Melanotus spp. ( $\mathrm{n}=2$ ), Parallelosthetus attenuatus ( $\mathrm{n}=2$ ) and Ampedus linteus $(\mathrm{n}=1)$ were simulated as part of this study. Only jumps that were perpendicular to the camera's view were used for further analysis. The beetles opened their wings during the descent stage in approximately half of the jumps, a behavior that is not taken into account by the simulation.


Figure 26: Left: The airborne trajectory reconstructed from high speed video recordings. Right: A MATLAB [31] simulation of the airborne trajectory calculated using the two-mass rigid body model for a jump of a Melanotus spp. individual.

The model was able to predict the maximum height $h$ and the landing distance $d$ (Fig.16) with less than $10 \%$ error. Further simulations of the remaining beetles filmed will enable a higher confidence in this value[R.C.6]. Given that the ballistic motion model was able to predict the jump kinematics of the click beetle even when they deployed their wings during the descent supports the hypothesis that click beetles open their wings to stabilize themselves, and not to cover distance.

### 4.3.2 Energetics Approach

$E_{\text {ballistic }}$ and $E_{\text {rotation }}$ were derived considering:

$$
\begin{align*}
& E_{\text {ballistic }}=\frac{1}{2} m v^{2}  \tag{21}\\
& E_{\text {rotation }}=\frac{1}{2} I \omega^{2} \tag{22}
\end{align*}
$$

where $v$ and $\omega$ are the average linear velocity of the center of mass at take-off and the angular velocity of the body at take-off respectively. $E_{\text {ballistic }}$ and $E_{\text {rotation }}$ are calculated considering the mean values for $v$ and $\omega$ of all the jumps analyzed for each beetle. Fig. 27 shows the dominance of $E_{\text {ballistic }}$ over $E_{\text {rotation }}$. Fig. 28 presents the calculated total energy $E_{\text {airborne }}$ $\left(E_{\text {airborne }}=E_{\text {ballistic }}+E_{\text {rotation }}\right)$ and Fig. 29 shows the correlation between the energy per kg mass and the mass.


Figure 27: Relationship between $E_{\text {ballistic }}$ (left) and $E_{\text {rotation }}$ (right) and the mass Alaus oculatus ( $\mathrm{n}=1$ ), Ampedus linteus $(\mathrm{n}=1)$, Melanotus spp. ( $\mathrm{n}=8$ ), Hemicrepidius sp. ( $\mathrm{n}=1$ ), and Parallelosthethus attenuatus $(\mathrm{n}=4)$. The difference in magnitude of $E_{\text {ballistic }}$ and $E_{\text {rotation }}$ for each specimen shows the dominance of $E_{\text {ballistic }}$ over $E_{\text {rotation }}$.


Figure 28: Relationship between $E_{\text {airborne }}$ and the mass of 5 beetles species, namely Alaus oculatus ( $\mathrm{n}=1$ ), Ampedus linteus ( $\mathrm{n}=1$ ), Hemicrepidius sp. ( $\mathrm{n}=1$ ), Melanotus spp. ( $\mathrm{n}=8$ ), and Parallelosthethus attenuatus $(\mathrm{n}=4)$.


Figure 29: $\quad E_{\text {airborne }} / m$ in relation to the mass of Alaus oculatus $(\mathrm{n}=1)$, Ampedus linteus ( $\mathrm{n}=1$ ), Hemicrepidius sp. ( $\mathrm{n}=1$ ), Melanotus spp. ( $\mathrm{n}=8$ ), and Parallelosthethus attenuatus ( $\mathrm{n}=4$ ).

Kaschek calculated $E_{\text {ballistic }}$ and $E_{\text {rotation }}$ similarly for beetles in the same length range as Melanotus spp., Parallelosthetus attenuatus and Hemicrepidius sp.. However, comparison with Kaschek's values for $E_{\text {ballistic }}$ and $E_{\text {rotation }}$ is not possible at this time as Kaschek measured the mass of live animals differently, after each jump [26]. The mass used in this study is the dry mass (Sec.2). However, the values calculated for $E_{\text {ballistic }}, E_{\text {rotation }}$ and the energy per kg mass fall in the same range as the ones obtained by Kaschek: Kaschek derives energies ranging from $0.515 \mathrm{~J} / \mathrm{kg}$ (Cidnopus aeruginosus) to $2.62 \mathrm{~J} / \mathrm{kg}$ (Athous haemorrhoidalis) [26], whereas we obtain energies ranging from $0.48 \mathrm{~J} / \mathrm{kg}$ to $1.4 \mathrm{~J} / \mathrm{kg}$.

### 4.3.3 Non-dimensional Analysis Approach

The relative radius of the elytra was calculated for 8 specimens (Fig. 30) [R.C.7]. Additional measurements will be taken on a larger number of specimens in order to identify potential patterns.


Figure 30: Relative elytra radius of Melanotus spp. ( $\mathrm{n}=6$ ) and Parallelosthetus attenuatus ( $\mathrm{n}=2$ ) specimens.

The average relative jump height $\Pi_{h}$ was calculated for each specimen (Fig. 31). Fig. 31 shows that the relative jump height decreases as the mass increases.[R.C.7].


Figure 31: Relative jump height $\Pi_{h}$ in relation to the mass for Alaus oculatus ( $\mathrm{n}=1$ ), Ampedus linteus $(\mathrm{n}=1)$, Hemicrepidius sp. $(\mathrm{n}=1)$, Melanotus spp. $(\mathrm{n}=8)$, and Parallelosthethus attenuatus $(\mathrm{n}=4)$.

Fig. 32 shows the relation of $\Pi_{W}$ and the mass of the animal. Preliminary results show that $\Pi_{W}$ is superior to 1 . While the energy is conserved, the time it takes to store the energy is band-width limited (muscles) whereas the energy is released kinematically (biomechanical
springs). This results in a much faster energy release (a couple milliseconds) than energy storage (a couple seconds), thus leading to power amplification. $\Pi_{W}$ increases as the beetle gets larger and heavier. [R.C.7]


Figure 32: $\Pi_{W}$ in relation to the mass of Alaus oculatus $(\mathrm{n}=1)$, Ampedus linteus $(\mathrm{n}=1)$, Hemicrepidius sp. ( $\mathrm{n}=1$ ), Melanotus spp. $(\mathrm{n}=8)$, and Parallelosthethus attenuatus $(\mathrm{n}=4)$.

For a given unit energy input, the work produced by the beetle's body to jump is presented Fig. 33 (on Fig.33, E=1).


Figure 33: $\quad \Pi_{W}$ given a fixed energy input $E_{\text {airborne }}=1$ in relation to the mass for Alaus oculatus ( $\mathrm{n}=1$ ), Ampedus linteus $(\mathrm{n}=1)$, Hemicrepidius sp. ( $\mathrm{n}=1$ ), Melanotus spp. ( $\mathrm{n}=8$ ), and Parallelosthethus attenuatus $(\mathrm{n}=4)$.

Fig. 33 should be understood from a roboticist standpoint: given a mass (or body length,
as $m \propto l$, see Fig. 5), and energy input (E, coming from the motor, battery, etc), Fig. 33 defines the work that needs to be produced by the body (or the structure) to accelerate the center of mass upwards, i.e. the acceleration that one must achieve with its design, in order for a jumping robot to take-off like a click beetle. For a given set of design parameters \{m,l,E\}, Fig. 33 provides a biological solution, as a starting point for design optimization. One could consider click beetles as a first set of prototypes, which jump with the desired acceleration for a given $\{\mathrm{m}, \mathrm{l}, \mathrm{E}\}$. [R.C.7]

## 5 Conclusions and Future Work

### 5.1 Summary of the Work and Original Contributions

This study presented a biomechanical analysis of jump of the click beetle for the design of a self-righting robot. The original contributions were as follows:

- [R.C.1] The three consecutive stages of the jump were identified and analyzed. In particular, the angular velocity and acceleration of the body during the take-off stage was extracted. The stages of the jump were identified and analyzed using high-speed video recordings, environmental scanning electron micrographs, and morphological measurements of nine click beetle species.
- [R.C.2] The correlation between the mass and the length of click beetles from various species indicated isometric scaling. Morphological measurements of nine click beetles species showed that the mass is proportional to the length cubed.
- [R.C.3] The hinge structure was detailed and assumed to be the source of the clicking sound characteristic to the jump of click beetles. Scanning electron micrographs showed the existence of a friction plate on the peghold and conformal edge on the mesosternal lip, as well as the existence of hairs along the mesosternal cavity to guide the motion of the peghold.
- [R.C.4] It was shown that the trajectory of the body during the airborne stage follows a ballistic motion. The parameters calculated during the airborne stage showed that the trajectory of the center of mass of the beetle followed a ballistic motion for $97 \%$ of the jumps. Preliminary results of the airborne stage energetic analysis confirm it by highlighting the dominance of the ballistic motion over the body rotation as an energy release mechanism.
- [R.C.5] The wing deployment behavior was observed during the airborne stage and increased the likelihood of the beetle landing on its feet.
- [R.C.6] An energetic analysis of the jump stages was developed, as well as kinematic and dynamic models for each stage of the jump. The hinge
stiffness, the equivalent body stiffness and a simulation of the airborne trajectory were derived. The kinematics of the jump were derived based on data extracted from high-speed video recordings of six click beetle species. A micro-mechanical experiment was used to provide preliminary data on the hinge stiffness. The equivalent stiffness of the body was calculated during take-off stage. Moreover, the airborne model was able to simulate the jump trajectory of Melanotus spp., Parallelosthetus attenuatus and Ampedus linteus specimens within $10 \%$.
- [R.C.7] A non-dimensional analysis enabled the identification and correlation of the dominant morphological and kinematic parameters during the airborne stage. The non-dimensional analysis of the airborne stage showed power amplification throughout the jump.


### 5.2 Future Work

Future work will include collecting morphological data from more individuals and from other species of large click beetles. More experiments are planned to confirm the energy exchange assumptions (such as CT-Scans and dissections) and to measure any body deflections during the pre-jump and take-off stages. In addition, the pre-jump, take-off and airborne models will be validated through physical prototypes. Various head+prothorax and mesothorax+metathorax+abdomen units masses will be tested in order to evaluate the influence of the mass ratio and the total mass on the jump. The non-dimensional analysis methodology will be used to study the jump of other small animals with fast energy release mechanisms, such as trap jaw ants.

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## Appendix A Specimens Collecting Methods and Conservation

Click beetles were collected in various locations in dead wood (Melanotus sp.) or using panel traps with alcohol lures (all species). The lures were 100 ml of a $90 \%$ ethanol solution. Ethanol (volatile organic compound) is produced by many stressed of decomposing plants and attracts beetles to places where they can lay their eggs or feed. The panel traps used are designed to look like trees. In the future, different lures will be used.

The click beetles were fed with a $10 \%$ sugar - $90 \%$ water solution once a week. Once the beetles died, they are placed in a $70 \%$ ethanol solution and conserved in a fridge to prevent the evaporation of the alcohol.

One CT Scan was conducted during Spring 2017 on a live Melanotus spp. The sample was constrained from moving in a tube. However, the resulting images were blurry. Further measurements should be conducted on dead animals to prevent the sample from moving.

## Appendix B Dry Mass Measurement

During Summer 2017, we observed that the mass of live beetles varied across days and drops once the beetle dies and "dries". In this study, the mass of each individual is defined as the dry mass of the specimen: to obtain a uniform measurement of the mass of the beetles studied, we manually dried the specimens by placing them in an oven (Chicago Surgical \& Electrical Co. 300) at $28^{\circ} \mathrm{C}$ for 2 to 3 weeks, until mass stabilization. The mass was measured every two days using a Fisher Scientific accuSeries ${ }^{\circledR}$. Fig. 34 shows the mass variation of a Melanotus spp. specimen across days. The mass of the live animal is approximatively decreased by half during the drying process.


Figure 34: Variation of the mass of a Melanotus spp. specimen alive and while drying in an oven at $28^{\circ} \mathrm{C}$.

## Appendix C Body Length Measurement

Length measurements were conducted on dead beetles (previously in ethanol). The wet body was flexible enough to be flexed around the hinge and enable length measurements of the abdomen $\left(l_{1}\right)$ (from the base to the apex) and of the length of the head and pronotum $\left(l_{2}\right)$ as shown Fig. 35.


Figure 35: The body length was measured by adding the lengths of the 2 body segments.

## Appendix D Elytra Curvature Measurement

The radius of the elytra ( R ) was measured by fitting a circle of radius R on the dorsal edge of the elytra (around the contact point with the ground when the beetle is on the dorsal side). The body segment considered goes from the middle of the 3rd sternite of the abdomen (Fig. 36.a.) to the base of the wings (base). Fig. 36 shows parts of the circle for Melanotus spp. (a and b) and Paralellosthetus attenuatus (c).


Figure 36: a. Abdomen segment definition. b. Part of the circle of radius R for a Melanotus spp. specimen. c. Part of the circle of radius R for a Parallelosthetus attenuatus specimen.

## Appendix E Peghold Shape Variations Across Species

Fig. 37 shows the variation in shapes of the peghold of 2 Melanotus spp. specimens. Fig. 13 (Sect. 2) presents the geometry of an Alaus oculatus animal.


Figure 37: ESEM micrographs of 2 Melanotus spp. specimens. a. ventral view of the peghold (specimen 1), b. dorsal view of the peghold, facing the mesosternal lip (specimen 1), c. side view of the peghold (specimen 1), d. ventral view of the peghold (specimen 2), e. dorsal view of the peghold, facing the mesosternal lip (specimen 2), f. side view of the peghold (specimen 2).

## Appendix F External Hair Variations Across Species

High resolution pictures show the different nature of the hair on the back of the elytra across beetle species. Fig. 38 shows the very smooth surface of Parallelosthetus attenuatus (Fig.38.a), compared to hairy Melanotus spp.(Fig.38.b). Fig. 38.c. shows the absence of hair around the point of contact of the elytra with the ground (Melanotus spp. specimen). We do not know if this absence of hair is due to the jump or to another phenomenon.


Figure 38: Pictures of the back of the elytra for a. Parallelosthetus attenuatus b. Melanotus spp. c. Melanotus spp.

## Appendix G Micro-mechanical Experiment

The micro-mechanical experiment set up to derive the hinge's stiffness is presented Fig. 39. A red nylon thread was attached to the prothorax of each specimen using polyurethane-based adhesive, and allowed to dry for a couple of days in the lab.


Figure 39: Micro-mechanical experiment set up to measure the force required to load/release the hinge (snap-fit mechanism). a,b: Dry beetles' mesothorax and abdomen are placed in foam and mounted on a linear stage. c: Starting from a neutral position, a force is applied on the prothorax by linearly translating the stage, causing the hinge to rotate and the hinge to load. The force is measured using a force transducer.

## Appendix H Take-off model calculations

Definition and assumptions The take-off model describes the motion of the beetle from the beginning of the energy release to the moment it leaves the ground. A crank-slider model is used to describe the motion. The angle $\gamma$ describes how the body goes from the arching position back to a flat position : the take-off model considers $\gamma$ going from $\gamma_{0}$ to $180^{\circ}$ where $\gamma_{0}$ ranges approximately between 165 and $170^{\circ}$.

We define :

- $l_{1}, m_{1}$ as the length and mass of the first link, respectively.
- $l_{2}, m_{2}$ as the length and mass of the second link, respectively.

And we assume that:

- the two links are homogeneous, the mass being equally distributed
- the joints at the points $\mathrm{A}, \mathrm{H}$ and B are perfect
- the motion is the $(\vec{x}, \vec{y})$ plane

The take-off model parameters are defined Fig. 40.


Figure 40: Take-off model parameters definition

Derivation of the equation of motion To obtain the equation of motion, we proceed by isolating 1 (system composed of Link 1) and 2. The 2nd Newton law gives:

$$
\begin{equation*}
\{\mathcal{D}(S / R)\}=\left\{\mathcal{F}_{\bar{S} \rightarrow S}\right\} \tag{23}
\end{equation*}
$$

Where $S=\{1\}+\{2\}$ and $\mathrm{R}(\vec{x}, \vec{y}, \vec{z})$ is any reference frame. Thus,

$$
\begin{equation*}
\{\mathcal{D}(1 / R)\}+\{\mathcal{D}(2 / R)\}=\left\{\mathcal{F}_{\overline{1} \rightarrow 1}\right\}+\left\{\mathcal{F}_{\overline{2} \rightarrow 2}\right\} \tag{24}
\end{equation*}
$$

## Isolating $\{1\}$

The exterior forces applied on $\{1\}$ are :

$$
\left\{\mathcal{F}_{0 \rightarrow 1}\right\}=\left\{\begin{array}{c}
\overrightarrow{\mathcal{M}_{0 / 1}^{A}}  \tag{25}\\
\overrightarrow{\mathcal{F}_{0 \rightarrow 1}}
\end{array}\right\}_{A}=\left\{\begin{array}{c}
0 \\
F_{01 x} \vec{x}+F_{01 y} \vec{y}
\end{array}\right\}_{A}
$$

With $T=T_{2 \rightarrow 1}$ (and T is negative) :

$$
\left\{\mathcal{F}_{2 \rightarrow 1}\right\}=\left\{\begin{array}{c}
\overrightarrow{\mathcal{M}_{2 / 1}^{H}}  \tag{26}\\
\overrightarrow{\mathcal{F}_{2 \rightarrow 1}}
\end{array}\right\}_{H}=\left\{\begin{array}{c}
T \vec{z} \\
F_{21 x} \vec{x}+F_{21 y} \vec{y}
\end{array}\right\}_{H}
$$

Writing the torsors at $G_{1}$ :

$$
\begin{gather*}
\overrightarrow{\mathcal{M}_{0 / 1}^{G_{1}}}=\overrightarrow{\mathcal{M}_{0 / 1}^{A}}+\overrightarrow{G_{1} A} \wedge\left(F_{01 x} \vec{x}+F_{01 y} \vec{y}\right)  \tag{27}\\
\left\{\mathcal{F}_{0 \rightarrow 1}\right\}=\left\{\begin{array}{c}
-\frac{l_{1}}{2} \cos (\alpha) F_{01 y}+\frac{l_{1}}{2} F_{01 x} \sin (\alpha) \\
F_{01 x} \vec{x}+F_{01 y} \vec{y}
\end{array}\right\}_{A} \tag{28}
\end{gather*}
$$

Similarly,

$$
\left\{\mathcal{F}_{2 \rightarrow 1}\right\}=\left\{\begin{array}{c}
\left(T+\frac{l_{1}}{2} \cos (\alpha) F_{21 y}-\frac{l_{1}}{2} \sin (\alpha) F_{21 x}\right) \vec{z}  \tag{29}\\
F_{21 x} \vec{x}+F_{21 y} \vec{y}
\end{array}\right\}_{A}
$$

Moreover,

$$
\begin{gather*}
\{\mathcal{D}(1 / R)\}_{G_{1}}=\left\{\begin{array}{c}
m_{1} \overrightarrow{\Gamma(1 / R)} \\
\overline{\delta_{G_{1}}(1 / R)}
\end{array}\right\}_{G_{1}}  \tag{30}\\
\{\mathcal{D}(1 / R)\}_{G_{1}}=\left\{\begin{array}{c}
m_{1} a_{G_{1} x} \vec{x}+m_{1} a_{G_{1} y} \vec{y} \\
I_{G_{1}} \ddot{\alpha} \vec{z}
\end{array}\right\}_{G_{1}} \tag{31}
\end{gather*}
$$

Thus,

$$
\left\{\begin{array}{c}
F_{01 x}+F_{21 x}=m_{1} a_{G_{1} x}  \tag{32}\\
F_{01 y}+F_{21 y}=m_{2} a_{G_{1} y} \\
I_{G_{1}} \ddot{\alpha}=T+\frac{l_{1}}{2} \cos (\alpha) F_{21 y}-\frac{l_{1}}{2} \sin (\alpha) F_{21 x}
\end{array}\right.
$$

## Isolation of $\{2\}$

The exterior forces applied on $\{2\}$ are :

$$
\left\{\mathcal{F}_{1 \rightarrow 2}\right\}=\left\{\begin{array}{c}
\overrightarrow{\mathcal{M}_{1 / 2}^{H}}  \tag{33}\\
\overrightarrow{\mathcal{F}_{1 \rightarrow 2}}
\end{array}\right\}_{H}=\left\{\begin{array}{c}
-T \vec{z} \\
F_{12 x} \vec{x}+F_{12 y} \vec{y}
\end{array}\right\}_{H}=\left\{\begin{array}{c}
\left(-T-\frac{l_{2}}{2} \sin (\beta) F_{12 x}-\frac{l_{2}}{2} F_{12 y} \cos (\beta)\right) \vec{z} \\
F_{12 x} \vec{x}+F_{12 y} \vec{y}
\end{array}\right\}_{G_{2}}
$$

And :

$$
\begin{gather*}
\left\{\mathcal{F}_{0 \rightarrow 2}\right\}=\left\{\begin{array}{c}
\overrightarrow{\mathcal{M}_{0 / 2}^{P}} \\
\left.\overrightarrow{\mathcal{F}_{0 \rightarrow 2}}\right\}_{P}=\left\{\begin{array}{c}
\overrightarrow{0} \\
F_{02 y} \vec{y}
\end{array}\right\}_{P}=\left\{\begin{array}{c}
\left(\frac{l_{2}}{2} \sin (\beta) F_{02 y} \cos (\beta) \vec{z}\right. \\
F_{02 y} \vec{y}
\end{array}\right\}_{G_{2}} \\
\{\mathcal{D}(2 / R)\}_{G_{2}}=\left\{\begin{array}{c}
m_{2} \overrightarrow{\Gamma(2 / R)} \\
\overline{\delta_{G_{2}}(2 / R)}
\end{array}\right\}_{G_{2}} \\
\{\mathcal{D}(2 / R)\}_{G_{2}}=\left\{\begin{array}{c}
m_{2} a_{G_{2} x} \vec{x}+m_{2} a_{G_{2} y} \vec{y} \\
I_{G_{2}} \ddot{\beta} \vec{z}
\end{array}\right\}_{G_{2}}
\end{array}\right. \tag{34}
\end{gather*}
$$

Thus,

$$
\left\{\begin{array}{c}
F_{02 y}=m_{2} a_{G_{2} x}  \tag{37}\\
F_{02 y}+F_{12 y}=m_{2} a_{G_{2} y} \\
I_{G_{2}} \ddot{\beta}=-T-\frac{l_{2}}{2} \cos (\beta) F_{12 y}-\frac{l_{2}}{2} \sin (\beta) F_{12 y}+\frac{l_{2}}{2} F_{02 y} \cos (\beta)
\end{array}\right.
$$

Combining 32 and 37, we obtain :

$$
\begin{gather*}
T=k \gamma=\frac{l_{1} l_{2} \cos (\alpha) \cos (\beta)\left[1+m_{1} a_{G_{1 y}}\right]}{l_{2} \cos (\beta)+l_{1} \cos (\alpha)\left[1+m_{1} a_{G_{1 y}}\right]}\left(\frac{2 I_{G_{1}} \ddot{\alpha}}{l_{1} \cos (\alpha)\left[1+m_{1} a_{G_{1} y}\right]}-\frac{m_{1} a_{G_{1} y}}{1+m_{1} a_{G_{1} y}}\right. \\
\left.-\frac{\tan (\alpha)}{m_{1} a_{G_{1} x}+m_{2} a_{G_{2} x}-\frac{I_{G_{2}} \ddot{\beta}}{l_{2} \cos (\beta)}+l_{2} \tan (\beta) m_{2} a_{G_{2} x}}-\frac{m_{2} a_{G_{2} x}}{\cos (\beta)}+m_{1} a_{G_{1} y}+m_{2} a_{G_{2} x}\right) \tag{38}
\end{gather*}
$$

Where : $I_{G_{1}}=\frac{m_{1} l_{1}^{2}}{12}$ and $I_{G_{2}}=\frac{m_{2} l_{2}^{2}}{12}$
Several geometric relations can be derived:

$$
\begin{gather*}
x_{G_{1}}=\frac{l_{1}}{2} \cos (\alpha) \\
x_{G_{2}}=l_{1} \cos (\alpha)+\frac{l_{2}}{2} \cos (\beta)  \tag{39}\\
y_{G_{2}}=\frac{l_{1}}{2} \sin (\alpha) \\
y_{G_{2}}=-l_{1} \sin (\alpha)+\frac{l_{2}}{2} \sin (\beta)
\end{gather*}
$$

From 39, we derive :

$$
\begin{gather*}
a_{G_{1} x}=-\frac{l_{1}}{2} \ddot{\alpha} \sin (\alpha)-\frac{l_{1}}{2} \dot{\alpha}^{2} \cos (\alpha) \\
a_{G_{2} x}=-l_{1} \ddot{\alpha} \sin (\alpha)-l_{2} \dot{\alpha}^{2} \cos (\alpha)-\frac{l_{2}}{2} \ddot{\beta} \cos (\beta)-\frac{l_{2}}{2} \dot{\beta}^{2} \sin (\beta) \\
a_{G_{1} y}=-\frac{l_{1}}{2} \ddot{\alpha} \cos (\alpha)+\frac{l_{1}}{2} \dot{\alpha}^{2} \sin (\alpha)  \tag{40}\\
a_{G_{2} y}=-l_{1} \ddot{\alpha} \cos (\alpha)-l_{2} \dot{\alpha}^{2} \sin (\alpha)+\frac{l_{2}}{2} \ddot{\beta} \cos (\beta)-\frac{l_{2}}{2} \dot{\beta}^{2} \sin (\beta)
\end{gather*}
$$

