TRANSFER FUNCTIONS FOR FAST ELECTROMAGNETIC AND CIRCUIT MODELING

BY

BASSEL ALESH

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Bachelor of Science in Electrical and Computer Engineering
in the College of Engineering of the
University of Illinois at Urbana-Champaign, 2017

Urbana, Illinois

Advisers:

Professor Andreas Cangellaris
Professor Jose Schutt-Aine

ABSTRACT

As the electronics industry continues to grow at a rapid pace, the need for faster and yet more accurate computer-aided simulation tools is at an all time high. Engineers need to perform accurate simulations in an expedient enough fashion to guide design decisions during the design stage of an electrical component or a system.

This thesis presents a set of such models used as part of a modeling and simulation framework for electromagnetic interference aware electronic system integration. By leveraging basic electromagnetic principles and boundary conditions, transfer functions were created to model different types of transmission lines and other circuit components in a way that allowed them to be integrated as building blocks of a larger system.

Keywords: Electromagnetics, Circuits, Simulations, Solvers, Modeling, Antennas, Transmission Lines, Shielded Cables

ACKNOWLEDGMENTS

First and foremost, I would like to thank my senior thesis advisor, Professor Jose Schutt-Aine, for his help and support throughout my year-and-a-half working under his supervision. His expertise and prowess in the field of electromagnetics is exceptional, and his ability to make a student feel welcome and part of the team is truly one of a kind. Stopping by his office for questions about research, classes, graduate school, or just life in general was a privelege that I appreciate dearly.

Next, I would like to thank Professor Andreas Cangellaris, who always challenged me and taught me to ask the right questions. Working closely with Professor Cangellaris on the bulk of my thesis proved to be a phenomenal learning experience that I will never forget, as he was an expert at explaining any concept pertaining to electromagnetics. Both Professor Schutt-Aine and Professor Cangellaris were incredible mentors, and helped me get the most out of my undergraduate research experience at Illinois.

I would also like to thank the members of the Synclesis group, Xu Chen, Robert Kummerer, Xiao Ma, and Gene Shiue, for all their support and assistance during my time as a member of their group.

This material is based upon work supported by the U.S. Army Small Business Innovation Research (SBIR) Program Office and and the U.S. Army Research Office under Contract No. W911NF-16-C-0125 and by Raytheon Company.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Motivation

Electromagnetic solvers are a necessary tool in modern electronic component design. The increasing demand for electronic devices over the past few decades called for a faster design process that enables a quicker release of products to market. Competition in the industry pushes engineers to design and prototype at faster paces, and without fast yet precise solvers at hand, this could become quite a challenge. Outside the consumer world, the demand for quick electromagnetic simulations is still high. This stems from the fact that designing for higher frequencies is becoming more desirable as the frequency spectrum becomes increasingly crowded and certain applications demand higher data rates.

Accurate simulations over such frequencies are challenging. This leads to a tradeoff between accuracy and speed when it comes to modeling different electrical components using current computer-aided tools. My group, Synclesis, led by Professor Jose Schutt-Aine and Professor Andreas Cangellaris, aims to use stochastical analysis to alleviate this tradeoff.

1.2 Purpose

This thesis goes about creating different blocks in the form of transfer functions for different circuit components. Models for a few basic antennas and filters were also implemented, but the bulk of this thesis focuses on the behavior of transmission lines in the proximity of an electric field. The calculations and simulations were all performed using Python, but could be implemented in C/C++ to obtain even faster solutions.

1.3 Outline

This thesis is organized into three chapters. Each chapter will include a brief description of the theory and background information surrounding the transfer function being discussed, with some sample code, examples, and plots to demonstrate functionality.

- Chapter 2 introduces basic antenna types such as the short-dipole and the half-wave dipole.
- Chapter 3 discusses the coupling behaviors of a two-wire and a single-wire-over-ground-plane transmission line configurations under a given set of input parameters.
- Chapter 4 includes the derivation of expressions describing the cable shield coupling behavior of a coaxial cable, that can eventually be extended to braided shields with multiple apertures.
- Chapter 5 concludes the thesis and discusses potential transfer functions to be implemented, and other future possibilities brought forth by this systematic approach of electromagnetic and circuit simulations.

CHAPTER 2

ANTENNAS

2.1 Introduction

Antennas can be represented as transfer functions when accounting for the incoming waves as an input parameter. The equations in this chapter are mentioned and discussed in [1]. The polarization, amplitude, and propagation direction of a wave are sufficient parameters that allow the calculation of the voltage at the load on the antennas end. Equation 2.1 is the main equation that determines the power delivered to a receiving antenna, whether from another antenna or from a nearby field.

$$P_{received} = S_{incident} A(\theta, \phi) \tag{2.1}$$

The $S_{incident}$ term represents the incident waves energy flux, or rate of energy transfer per unit area, in watts per square meter. It can be calculated in two ways, depending on the information available:

$$S_{incident} = \frac{P_{emitted}}{4r^2} = \frac{|E_{incident}|^2}{2} \tag{2.2}$$

The $A(\theta, \phi)$ term represents the effective area of the receiving antenna, and how much of that incoming flux its accepting. This term depends on the orientation of the antenna, both elevation and azimuth, relative to the incoming waves propagation vector. This is not determined by the waves polarization. It can be represented using the incoming wavelength and the antennas type-specific gain function, $G(\theta, \phi)$.

$$A(\theta, \phi) = \frac{G(\theta, \phi)\lambda^2}{4\pi} \tag{2.3}$$

2.2 Co-polarization

Whether or not the incoming wave actually stimulates the antenna and generates a current through it depends on its polarization, or how the electric fields are oriented. If oriented in the same direction as the antenna, then the antenna will observe a current through it. If the field is perpendicular to the antenna, then cross-polarization will occur, and no current will be observed across the antenna. This condition might not apply for all types of antennas, however.

2.3 Gain Functions of Different Antennas

Each type of antenna has a different gain function that depends on the elevation and azimuth of the incoming propagation vector. Different types of antennas and their gain functions are seen below.

Isotropic

$$G(\theta, \phi) = 1 \tag{2.4}$$

Short dipole

$$G(\theta, \phi) = 1.5sin^2(\theta) \tag{2.5}$$

Half-wave dipole

$$G(\theta, \phi) = \frac{1.64cos^2(\frac{\pi}{2}cos\theta)}{sin^2(\theta)} \tag{2.6}$$

Co-secant squared

$$G(\theta, \phi) = csc^2\theta \tag{2.7}$$

2.4 Voltage Output at the Load

After choosing the proper gain function from Equations 2.4 to 2.7, calculating the voltage at the load becomes quite simple. For the first case, the load is assumed to be matched to the antennas characteristic resistance. In that case, the voltage at the load is calculated next.

$$V = \sqrt{2P_{received}R} \tag{2.8}$$

If not matched to the antennas characteristic impedance, a mismatch factor has to be calculated to determine how much power is actually delivered to the load.

$$P_{delivered} = \frac{4R_{antenna}R_{load}}{(R_{antenna} + R_{load})^2 + (X_{antenna} + X_{load})^2} \times P_{received} \tag{2.9}$$

Using Equation 2.9 for the value of $P_{received}$ in Equation 2.8, the voltage at a mismatched load can be calculated. This concludes the process of evaluating a transfer function for antennas, with an incoming wave as the input, and a load voltage at the output.

2.5 Python Code for a Short Dipole

The input parameters needed for the first part of program are: field amplitude, field azimuth, field elevation, frequency, propagation elevation, propagation azimuth, antenna azimuth, antenna elevation, and the intrinsic impedance of the material. These will be used to determined the power received by the antenna in the presence of an incoming wave. The second part of the program requires the following input parameters: antenna resistance, antenna reactance, load resistance, load reactance. These will be used to determine output voltage would be at a specified load. The code can be seen below.

```
import numpy
import math

#Short Dipole Antenna Transfer Function

fieldAmplitude, fieldAzimuth, fieldElevation, frequency,
    propagationElevation, propagationAzimuth, antennaAzimuth,
    antennaElevation, eta = input("Seperated by spaces only,
    enter the following: field amplitude, field azimuth, field
    elevation, frequency, propagation elevation, propagation
    azimuth, antenna azimuth, antenna elevation, and the
    intrinsic impedance of the material. \n\n").split()
antennaAzimuth = float(antennaAzimuth)
antennaElevation = float(antennaElevation)
fieldAzimuth= float(fieldAzimuth)
fieldElevation = float(fieldElevation)
propagationElevation = float(propagationElevation)
propagationAzimuth = float(propagationAzimuth)
frequency = float(frequency)
fieldAmplitude = float(fieldAmplitude)
eta = float(eta)

antennaAzimuthRads = math.radians(antennaAzimuth)
antennaElevationRads = math.radians(antennaElevation)
fieldAzimuthRads = math.radians(fieldAzimuth)
fieldElevationRads = math.radians(fieldElevation)

gainTheta = antennaElevation - propagationElevation
gainRads = math.radians(gainTheta)
#This equation changes for the type of antenna used
gain = 1.5*pow(math.sin(gainRads),2)

incidentS = pow(fieldAmplitude,2)/(2*eta)
antennaX = math.sin(antennaElevationRads)*math.cos(
    antennaAzimuthRads)
```

```
antennaZ = math.cos(antennaElevationRads)
fieldX = math.sin(fieldElevationRads)*math.cos(fieldAzimuthRads
    )
fieldY = math.sin(fieldElevationRads)*math.sin(fieldAzimuthRads
    )
fieldZ = math.cos(fieldElevationRads)

wavelength = ((3*pow(10,8))/frequency)
effectiveArea = gain*pow(wavelength,2)/(4*math.pi)
if numpy.dot([antennaX,antennaY,antennaZ],[fieldX,fieldY,fieldZ
    ]) < 5e-2:
    copolarization = 0
else:
    copolarization = 1
powerReceived = copolarization*effectiveArea*incidentS

antennaResistance, antennaReactance, loadResistance,
    loadReactance = input("Seperated by spaces only, enter the
    following: antenna resistance, antenna reactance, load
    resistance, load reactance. \n\n").split()
antennaResistance = float(antennaResistance)
antennaReactance = float(antennaReactance)
loadResistance = float(loadResistance)
loadReactance = float(loadReactance)
MF = 4*antennaResistance*loadResistance/(pow((antennaResistance
    +loadResistance),2)+pow((antennaReactance+loadReactance),2)
    )
powerDelivered=MF*powerReceived
voltage = math.sqrt(2*powerDelivered*loadResistance)
```

2.6 Short Dipole Example

Consider a short dipole antenna oriented in the $\hat{z}$-direction. An incoming wave traveling in the $-\hat{x} - \hat{z}$ direction is present. The amplitude of the electric field is 120π volts per meter. The wave is propagating in free space with a frequency of $300\sqrt{2}$ MHz. Its electric field is polarized in the $\hat{z} - \hat{x}$-direction. The antenna has a characteristic impedance of 50 ohms with a 50 ohm load attached to it. The input parameters for the code are as follows:

```
field amplitude = 377
field azimuth = 180
field elevation = 45
frequency = 424 MHz
propagation elevation = 45
propagation azimuth = 0
antenna azimuth = 0
antenna elevation = 0
intrinsic impedance = 377
antenna resistance = 50
antenna reactance = 0
load resistance = 50
load reactance = 0
```

The results obtained from the Python script are then:

```
power received = 5.625 watts
power delivered = 5.625 watts (due to the matched load)
voltage at the load = 23.717 volts
```

Also tested with the code is the case where the field is polarized in the $\hat{x}$-direction. Since the antenna would then be cross-polarized, no output voltage would be seen at the load because no power would be received at the antenna. The result is checked by printing out the polarization value from the Python code:

```
copolarization = 0.0
```

CHAPTER 3

TRANSMISSION LINES

3.1 Introduction and ABCD Matrices

In this chapter, two types of transmission lines will be investigated: the two-wire and single-wire-above-ground transmission lines. The electromagnetic theory discussed in this chapter is crucial for the derivation of the shielded cable response in Chapter 4. By the end of this chapter, the voltage at the load and at the source for each of these configurations will be calculated in the presence of a nearby field. Essentially, a description of how electromagnetic coupling happens will be brought forth.

Different types of transmission lines can be modeled by a ABCD matrices. An ABCD matrix can be used to characterize the voltage and current at one of the two inputs of a two-port system, and the voltage and current at the other. Once obtained, the ABCD matrix can determine how a given circuit will behave once placed in a bigger system. Each term in the ABCD matrix represents a certain quantity, seen below.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{3.1}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} \tag{3.2}$$

Given the characteristic impedance Z_o, length (l), and propagation constant γ of a transmission line, its ABCD matrix can be determined. A transmission line can be modeled by Equation 3.3.

Parameter	Value
Line separation (D)	0.20 m
Wire radius (a)	0.0015 m
Line length (l)	30 m
Frequency	1 MHz

Table 3.1: Example line parameters

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} cosh(\gamma l) & jZ_o sinh(\gamma l) \\ j\frac{1}{Z_o} sinh(\gamma l) & cosh(\gamma l) \end{bmatrix} \tag{3.3}$$

The characteristic impedance and propagation constant can be evaluated if the values of L, R, C, and G (per unit length) are given. Using the equations below, the characteristic impedance and propagation constant can be calculated. Note that if the line is lossless, the propagation constant will only consist of the imaginary part that is the phase constant, β.

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{3.4}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{3.5}$$

If provided with the quantities above, then the procedure for obtaining the ABCD matrix for a transmission line is complete. Usually, the properties of a transmission line are given rather than the L, R, C, and G parameters themselves. These can be easily calculated depending on the type of transmission line used. For example, Equations 3.7 and 3.8 show the inductance and capacitance per unit length, respectively, of a two-wire transmission line.

The voltage gain across a transmission line can be calculated when the ABCD matrix is available. The open-circuit voltage gain for a transmission line is:

$$\frac{V_2}{V_1} = \frac{1}{A} \tag{3.6}$$

The input voltage source can come from an ideal source, an antenna, etc. For example, assume a two-wire lossless line system has the following parameters:

The inductance and capacitance per unit length can be expressed by:

$$L = \frac{\mu_0}{\pi} cosh^{-1}(\frac{D}{2a}) \tag{3.7}$$

$$C = \frac{\pi\epsilon_0}{cosh^{-1}(\frac{D}{2a})} \tag{3.8}$$

Plugging those into the equations for characteristic impedance (3.4) and propagati on constant (3.5) above to get:

$$Z_o = \sqrt{\frac{L}{C}} \approx 586 \tag{3.9}$$

$$\gamma = \frac{j\omega}{c} = \frac{j2\pi \times 10^6}{3 \times 10^8} \approx j0.021 \tag{3.10}$$

The A-parameter can then be calculated to be:

$$A = cosh(\gamma l) = cosh(j0.021 \times 30) = 0.808 \tag{3.11}$$

and the voltage gain is:

$$\frac{V_2}{V_1} = \frac{1}{A} = 1.238 \tag{3.12}$$

3.2 Electromagnetics Background

For the remainder of this chapter, the transmission lines described will be on the $x - z$ plane, with the length of the wire extending in the positive-$\hat{x}$ direction. The scenarios will involve an incident plane wave and describe the behavior of the voltage and current along the line due to coupling. As a result of reflections on the line, there will be both incident and reflected fields in the vicinity of the line.

Recalling the following from Maxwell's equations:

$$\nabla \times E = -j\omega\mu_0 H \tag{3.13}$$

and the following from Stoke's theorem:

$$\int_C E \cdot dl = -j\omega\mu_0 \iint_S H \cdot ds \tag{3.14}$$

The equation for the voltage can be derived to be:

$$V(x) = -\int_0^d E_z(x,z)dz \tag{3.15}$$

Notice that Equation (3.15) only considers the z-component of the electric field to determine voltage. Based on our setup, the wires are above each other in the z-direction. Since voltage can be expressed as the electric field multiplied by the distance that the field covers, or the amount of field in a given direction, this solution is not too foreign. The magnetic field, however, also induces a voltage on the line. Next, because the incident and scattered magnetic fields on the line compose a differential voltage on the line, and the current induced on the line is only due to the scattered magnetic fields, a source voltage arises to compensate for the missing incident magnetic field factor. More specifically, because of the incident magnetic flux field, a distributed voltage source forms on the line. This source voltage, $V'_{S1}(x)$, becomes crucial for the derivations in this chapter. It equates to:

$$V'_{S1}(x) = -j\omega\mu_0 \int_0^d H_y^{inc}(x,z)dz \tag{3.16}$$

The magnetic field component that induces such currents or source voltages along the line is the y-component, as seen in Equation 3.16's source voltage expression, because it is the component that is perpendicular to the $x-z$ plane and causes the flux between the wires that eventually allows for current to be induced. Finally, the first expression for the distributed voltages and currents on the line reduces to:

$$\frac{dV(x)}{dx} + j\omega L' I(x) = V'_{S1}(x) \tag{3.17}$$

The next expression will follow the same procedure but depend on a distributed current source along the line instead. Text [2] goes over the steps in detail, but the final result for the second expression simplifies to:

$$\frac{dI(x)}{dx} + j\omega C' V(x) = I'_{S1}(x) \tag{3.18}$$

The current source per unit length term, $I'S1(x)$, is given as:

$$I'_{S1}(x) = -j\omega C' \int_0^d E_z^{inc}(x,z)dz \tag{3.19}$$

In Equations 3.17 and 3.19, the per-unit-length inductance and capacitance values simplify when the height of the two wire system is much larger than the wire radii (d >> a) to be:

$$L' = \frac{\mu}{\pi} ln(\frac{D}{2a}) \tag{3.20}$$

$$C' = \frac{\pi\epsilon}{ln(\frac{D}{2a})} \tag{3.21}$$

A better way to visualize the induced per-unit-length voltages and currents is seen in Figure 3.1. Notice that these are per-unit-length parameters and need to be multiplied by the necessary division size to give a proper voltage or current.

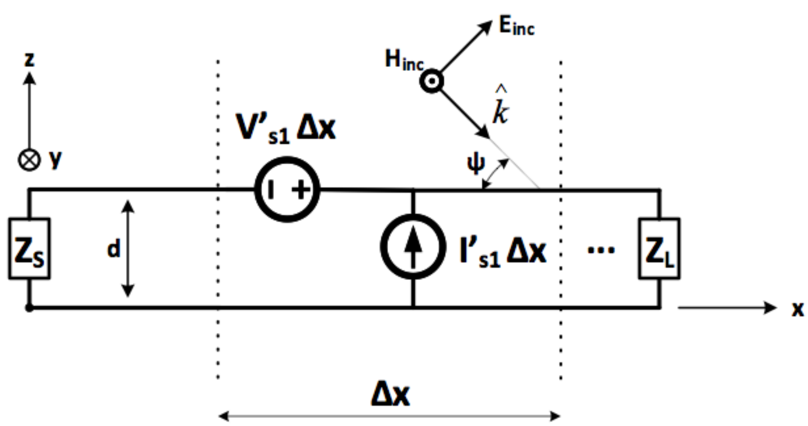

Figure 3.1: Distributed sources along the transmission line

The text [2] defines source vectors to represent the superposed current and voltage sources as a result of the incident plane wave. Source vectors are essentially impulse functions along the line distributed in constant intervals.

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\int_0^L e^{\gamma x_s}[V'_{S1}(x_s) + Z_c I'_{S1}(x_s)]dx_s \\ -\frac{1}{2}\int_0^L e^{\gamma(L-x_s)}[V'_{S1}(x_s) - Z_c I'_{S1}(x_s)]dx_s \end{bmatrix} \tag{3.22}$$

Using the source vectors with Equations 3.16 and 3.19 for the per-unit-length sources, the source vectors turn out to be:

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}(\frac{E_0(cos\alpha sin\psi cos\phi + sin\alpha sin\phi)jkdsin\psi}{\gamma - jkcos\psi cos\phi} - E_0 dcos\psi\alpha)(1 - e^{(\gamma - jkcos\psi cos\phi)L} \\ -\frac{1}{2}e^{\gamma L}(\frac{E_0(cos\alpha sin\psi cos\phi + sin\alpha sin\phi)jkdsin\psi}{\gamma + jkcos\psi cos\phi} + E_0 dcos\psi\alpha)(1 - e^{-(\gamma + jkcos\psi cos\phi)L} \end{bmatrix} \tag{3.23}$$

Finally, the voltages and currents at the ends of the line can be computed using the simple matrix below, with ρ corresponding to the reflection coefficient at each side of the line (with ρ_1 being at the source and ρ_2 being at the load), γ as the propagation constant, Z_c as the characteristic impedance of the line, and L as the length of the line.

$$\begin{bmatrix} I(0) \\ I(L) \end{bmatrix} = \frac{1}{Z_c} \begin{bmatrix} 1-\rho_1 & 0 \\ 0 & 1-\rho_2 \end{bmatrix} \begin{bmatrix} -\rho_1 & e^{\gamma L} \\ e^{\gamma L} & -\rho_2 \end{bmatrix}^{-1} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \tag{3.24}$$

$$\begin{bmatrix} V(0) \\ V(L) \end{bmatrix} = \begin{bmatrix} 1+\rho_1 & 0 \\ 0 & 1+\rho_2 \end{bmatrix} \begin{bmatrix} -\rho_1 & e^{\gamma L} \\ e^{\gamma L} & -\rho_2 \end{bmatrix}^{-1} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \tag{3.25}$$

Also worth noting is the reflection coefficient at a load at any end of the line. The Z_c term represents the characteristic impedance of the line and the Z_L term represents the load impedance. If the load is matched to the characteristic impedance, the reflection coefficient equates to zero and no waves are reflected back. If it is an open or short, the wave is fully reflected back with $\rho = 1$ (for an open) or $\rho = -1$ (for a short).

In the equations above, α is the polarization of the incoming wave; $\alpha = 0$ corresponds to a vertically-polarized wave and $\alpha = 90$ corresponds to a horizontally polarized wave. ϕ and ψ correspond to the wave's propagation direction. ψ corresponds to what is typically referred to as θ, or the elevation angle. In fact, in Chapter 4, θ will be used to express the elevation angle, d is the separation between the two wires and l is the distance along the line.

This source vector can be used to calculate the voltage at the load using the following relationship, where L is the length of the line (the location of the load at the output):

$$V(l = L) = \frac{(1+\rho_2)(-S_1 e^{\gamma L} - S_2\rho_1)}{\rho_1\rho_2 - e^{2\gamma L}} \tag{3.26}$$

Note that this is only the coupling voltage, and that the voltage from the source attached to the line needs to be added to this result. Here ρ corresponds to the load reflection coefficient on each end. The same relationship can be derived for the current at the load:

$$I(l = L) = \frac{(\rho_2 - 1)(S_1 e^{\gamma L} + S_2 \rho_1)}{\rho_1 \rho_2 - e^{2\gamma L}} \tag{3.27}$$

3.3 Coupling Example

From the reference text [2], an example of a two-wire transmission line's response to an incident wave is provided. In the example, the normalized current at the load (l = L) of the line is plotted against $\frac{kd}{2}$ to show periodicity along frequency. Here, $\alpha = \phi = 0°$ (vertical polarization) and $\psi = 60°$.

The length of the line is 30 meters, the line separation is 0.2 meters, and the wire radius is 0.0015 meters, as with the example in Table 3.1. The loads are given as $Z_1 = Z_2 = \frac{Z_o}{2} \approx 293\Omega$, in relation to the characteristic impedance of the line from Equation 3.9 (Z_1 is the impedance at the source and Z_2 is the impedance at the load). The magnitude of the current is normalized with the magnitude of the incident electric field and plotted in Figure 3.2. The voltage can be plotted on the same plot to show the relationship between the two on the line. Both the current and voltage can be seen to take a somewhat sinusoidal pattern, which will be the case of any induced voltage or current by an electromagnetic field. The peak value of this normalized current turns out to be around 0.4 $\frac{mA}{V/m}$, which is quite high! Chapter 4 goes over shielded cables and how their coupling behavior is much less significant than that of an exposed two-wire transmission line.

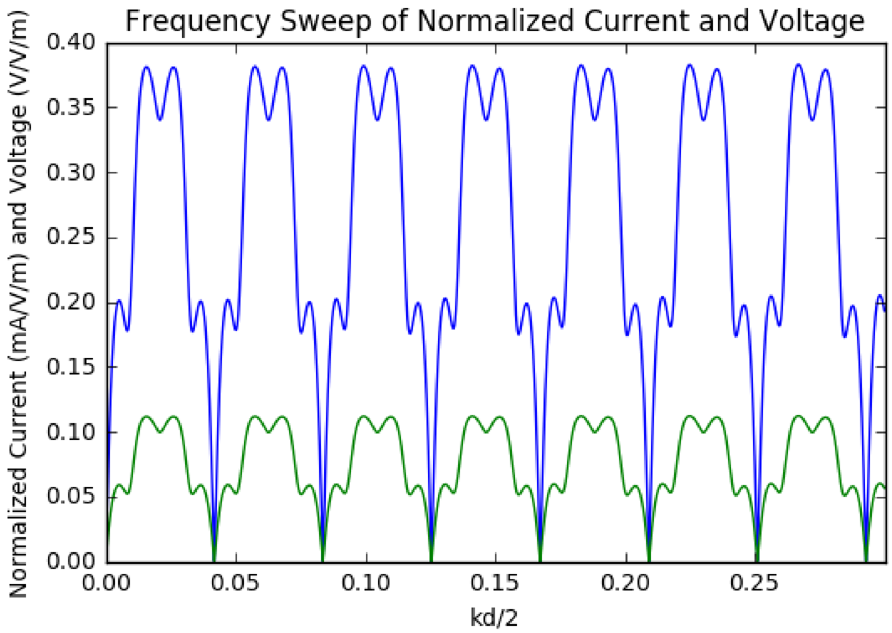

Figure 3.2: Normalized current magnitude (blue) and normalized voltage magnitude (green) frequency sweep

More tests can be performed on this line: for example, if it was horizontally polarized, with $\alpha = 90°$, we would expect there to be no response to the nearby field on the line. This is exactly what the results showed. Another test would be to make the wave propagate straight down onto the line by making $\psi = 90°$. This would yield the result in Figure 3.3. The results are reassuring: with $\psi = 90°$, the electric field is aligned parallel to the line (in this example). The periodicity of the current comes out cleaner, although its magnitude seems to hit a lower value than in Figure 3.2.

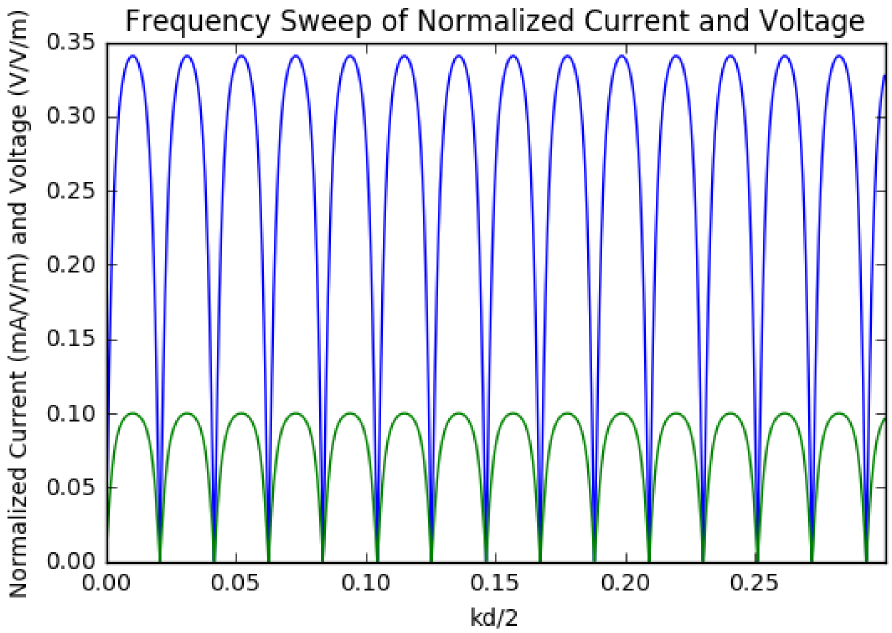

Figure 3.3: Normalized current magnitude (blue) and normalized voltage magnitude (green) frequency sweep with $\psi = 90°$

3.4 Running the Program

The full program can be run through the command line when given the proper input parameters. The code takes the voltage at the load due to coupling at a specified frequency and uses the ABCD matrix result from Equation 3.12 to calculate the additional voltage due to a voltage source at the load (with the appropriate gain). The two voltages (one from coupling, the other from the source adjusted with the necessary gain) are added together to give the total voltage at the load. The parameters can be entered in the following order:

1. Wire radius
2. Line separation
3. Line length
4. Sheet resistance
5. Voltage at the beginning of the line

6. Load impedance at the beginning of the line
7. Load impedance at the end of the line
8. ψ
9. ϕ
10. α
11. Specified frequency of the incident wave
12. Field magnitude

```
python tl.py 0.0015 0.2 30 0 10 293 293 0 60 0 1000000 1
```

Where the seventh entry, 10, corresponds to a source voltage of 10 V and 293 corresponds to the load impedances at both ends, in ohms. Essentially, this program tests a two-wire transmission line's voltage at the load. The final result is printed below.

```
Voltage Due to Coupling:
0.0401604605305
Load Voltage:
(12.4047489677+0j)
```

The fact that the program takes input parameters as such makes it much easier to experiment with different input parameters and properly test out the transmission line. Notice that the coupling voltage is quite low: at 1 MHz, this voltage is about 0.0406 volts. This value might seem low, but it is a result of a field of magnitude 1 V/meter. Although an additional 0.0406 volts on a wire could be very concerning to begin with, an increase in the field magnitude could cause an even greater coupling voltage at the output. This example is not exactly realistic, as the line is assumed to be lossless, which is not the case in a real-world example. Nevertheless, it does give a good representation of why two-wire transmission lines are not a good idea in sensitive systems. Chapter 4 does go over shielded cables, however, and the results there are more comforting. First, however, this chapter will conclude with a discussion of a single-wire configuration placed over a ground plane, and what exactly changes in our source vector equation from 3.22 to give the desired results.

3.5 Single Wire over a Ground Plane

Before moving to Chapter 4, it is worth noting what happens to the transmission line formed when a single wire is placed above a ground plane. The ground plane, in this scenario, is a perfect electric conductor (PEC). Figure 3.4 shows what this setup looks like. Without diving into the math again, the changes that apply here are easily spotted. First, the distance between the wire and the ground is h, or how high the line is off the ground. Next, since this is a PEC, it is worth noting the following behavior. The presence of the PEC results in a reflected field such that the superposition of the incident and reflected fields results in zero tangential electric fields on the PEC plane. Therefore, Equation 3.23 becomes:

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} -(\frac{E_0(cos\alpha sin\psi cos\phi + sin\alpha sin\phi)jkhsin\psi}{\gamma - jkcos\psi cos\phi} - E_0 hcos\psi\alpha)(1 - e^{(\gamma - jkcos\psi cos\phi)L} \\ -e^{\gamma L}(\frac{E_0(cos\alpha sin\psi cos\phi + sin\alpha sin\phi)jkhsin\psi}{\gamma + jkcos\psi cos\phi} + E_0 hcos\psi\alpha)(1 - e^{-(\gamma + jkcos\psi cos\phi)L} \end{bmatrix} \tag{3.28}$$

Equations 3.26 and 3.27 can be used to get the voltage and current at the loads. Essentially what this configuration does is thinking of the ground as just another return path (just as the second wire in the first configuration), but now the magnitude of the sources doubled. It is important to keep in mind that to obtain L' and C' for the line, the distance, D, needs to be replaced by two times the height, h, in Equations 3.20 and 3.21.

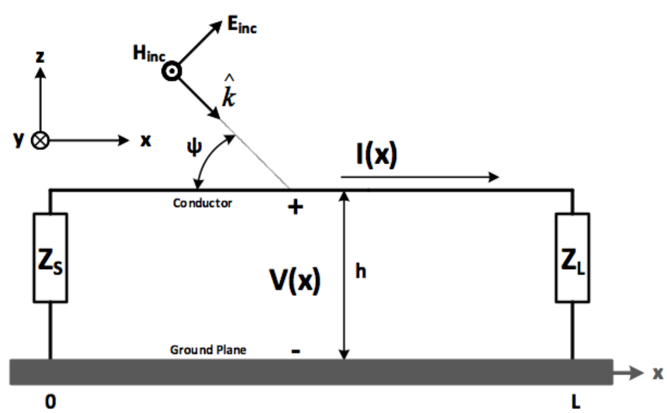

Figure 3.4: Single line over perfectly conducting ground plane

CHAPTER 4

SHIELDED CABLES

The final part of this thesis deals with finding a solution for the voltage induced on the inner wire of a shielded cable due to an incident electromagnetic field. In the previous chapter, it was observed that a nearby incident field can induce a reasonably high voltage on exposed transmission lines. For most systems, this configuration is not favorable because of these high potential levels of interference. For that reason, different types of shielded cable shields exist today, ranging from standard coaxial cables to braided cables and more.[1] This chapter will mainly examine the coaxial cable. Using the equations from the previous chapter, the response for the voltage on the inner conductor will be derived. The proposed solution utilizes the finite difference method to solve the differential equations that describe line behavior under the presence of an incident field.

4.1 Setting up the Problem

Figure 4.1 describes the coaxial cable that this chapter will investigate. The cable has two conductors: outer and inner. The incident field causes induced voltage and current sources along the outer conductor, referred to as $V_{ss}(x)$ and $I_{ss}(x)$, respectively. These sources are per-unit-length, and exist along the entire line. As a result of these sources, however, a voltage difference $V_s(x)$ across the outer conductor appears, and a current $I_s(x)$ flows through it too. A perfect shield does not allow any of the incident field across the outer conductor and into the inner conductor, due to the boundary conditions on both sides of the outer conductor. However, the voltage and current formed on the line, $V_s(x)$ and $I_s(x)$, do induce per-unit-length sources on the inner conductor through the finite-conductivity shield. By thinking of

[1] Types of cable shielding, www.alphawire.com [3]

the dielectric as some sort of resistor, it can be thought to cause a voltage drop across the impedance of the outer conductor and thus induce a current source, $I_{si}(x)$, on the inner conductor. The same can be thought of the outer current $I_s(x)$ causing an induced voltage $V_{si}(x)$ on the inner conductor. From [2], impedance that allows for such a phenomenon is called the transfer impedance, Z_t and is defined by different properties of the line and frequency and relates $V_s(x)$ and $I_{si}(x)$. Similarly, the admittance is referred to as the transfer admittance, Y_t, and relates $I_s(x)$ and $V_{si}(x)$. Figure 4.2 shows the shielded cable scenario that will be considered. It is important to note that ψ was replaced with θ and is defined differently according to the diagram. For the main derivation below, the exterior loads, Z_1^e and Z_2^e, will be treated as infinite (i.e. open terminations on the exterior). In the final section of this chapter, a different approach will be taken to solve for the configuration where the terminations are shorted (Z_1^e and Z_2^e equal to zero).

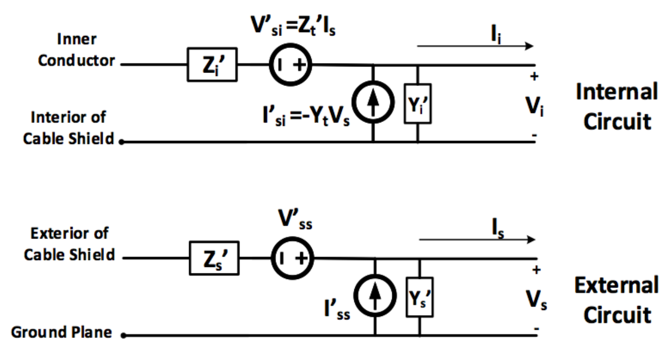

Figure 4.1: Outer (bottom) and inner (top) conductor circuit diagrams

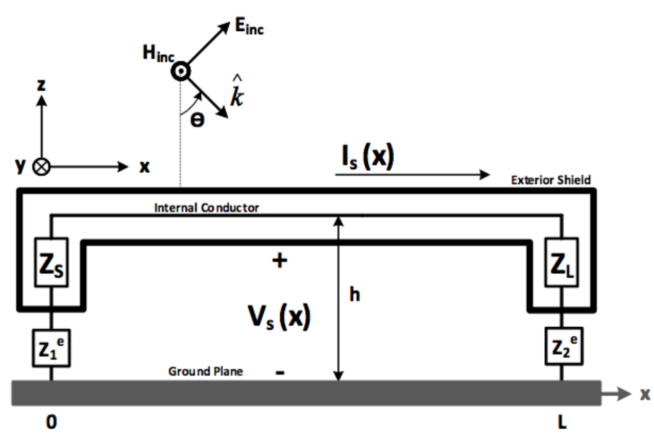

Figure 4.2: Shielded cable scenario with an incident wave

This gives the following relationships between the outer conductor and inner conductor:

$$V'_{si} = Z_t I_s \tag{4.1}$$

$$I'_{si} = -Y_t V_s \tag{4.2}$$

The transfer impedance is defined by the following equation:

$$Z_t = R'_0 * \frac{(1+j)\frac{\Delta}{\delta}}{sinh((1+j)\frac{\Delta}{\delta})} \quad \frac{\Omega}{m} \tag{4.3}$$

From [2], R'_0 is the DC per-unit-length resistance (units of $\frac{\Omega}{m}$) and is defined as:

$$R'_0 = \frac{1}{\pi\sigma(b-a)(b+a)} \approx \frac{1}{2\pi\sigma a \Delta_{wall}} \quad \frac{\Omega}{m} \tag{4.4}$$

where σ is the electric conductivity of the outer conductor. Δ_{wall} is the wall thickness, which is given by $b-a$ (b is the outer conductor radius and a is the inner wire radius). δ is the skin depth of the outer conductor and is a function of frequency. It is defined as:

$$\delta = \sqrt{\frac{1}{\pi \cdot f \cdot \sigma \cdot \mu}} \tag{4.5}$$

Here, μ is the permeability of the outer conductor. At higher frequencies, the DC per-unit-length resistance can be simplified and cause the entire transfer impedance to be approximated as the following at frequencies in the MHz range (which happen to be our typical frequencies of interest):

$$Z_t = \frac{2\sqrt{2}e^{-(1+1j)\frac{\Delta_{wall}}{\delta}} e^{j\frac{\pi}{4}}}{2\pi a \delta \sigma} \quad \frac{\Omega}{m} \tag{4.6}$$

4.2 Solving for $V_s(x)$ and $I_s(x)$

Now that the necessary equations have been set up, the solution can proceed. From Equations 3.16-3.19, the following relationship between the voltages and currents and induced voltages and currents can be realized:

$$\frac{dV_s(x)}{dx} + j\omega L' I_s(x) = V'_{ss}(x) \tag{4.7}$$

$$\frac{dI_s(x)}{dx} + j\omega C' V_s(x) = I'_{ss}(x) \tag{4.8}$$

Here, the induced per-unit-length voltage and current sources are the same as before, since it grows along the $+\hat{x}$-direction and is stacked along the $+\hat{z}$-direction. They turn out to be:

$$V'_{ss}(x) = -j\omega\mu_0 \int_0^d H_y^{inc}(x,z)dz \tag{4.9}$$

$$I'_{ss}(x) = -j\omega C' \int_0^d E_z^{inc}(x,z)dz \tag{4.10}$$

Here, C' is given by $\frac{2\pi\epsilon}{ln(2h/a)}$ and L' is given by $\frac{\mu}{2\pi}ln(\frac{2h}{a})$ from [2], since the outer conductor is treated like a single wire over a ground plane. The reference system for this problem is the same one from Chapter 3, but with a θ instead of ψ (although they have the same definition). Using the defined reference system, the equations above become:

$$V'_{ss}(x) = -j\omega\mu_0 \frac{|E|}{\eta} cos\alpha e^{-jksin\theta cos\phi x} \frac{2sin(kcos\alpha cos\theta)h}{kcos\alpha cos\theta} \tag{4.11}$$

$$I'_{ss}(x) = -j\omega C' |E| sin\theta cos\alpha e^{-jksin\theta cos\phi x} \frac{2sin(kcos\alpha cos\theta)h}{kcos\alpha cos\theta} \tag{4.12}$$

The dependence on x is reassuring, as the incident wave is a plane wave, and some periodicity is expected along the line (in the $+\hat{x}$-direction, that is). For the remainder of this solution, the line will be approximated as a concatenation of segments of length Δ such that:

$$\Delta = \frac{\lambda}{15} \tag{4.13}$$

where λ is the wavelength of the frequency of interest. This resolution ensures accurate approximation of the wave phenomena on the line. The indexing will start at $n = 1$, and end at $n = N$, where N is the total number of divisions on the line. When plugging in x-values into Equations 4.9 and 4.10, the following relationship is used:

$$x = (n-1) \cdot \Delta \tag{4.14}$$

Next, it is worth noting some boundary conditions from Equation 4.7. Since the ends of the outer conductor are left as open circuits, $I_s(x)$ will be zero at those points. This gives the following relationship between the discretized $V_s(x)$ and $V'_{ss}(x)$ for $n = 1$ and $n = N$, the start point and end point of the line, respectively:

$$\frac{dV_s(x)}{dx} = V'_{ss}(x) \tag{4.15}$$

$$\frac{V_{s,2} - V_{s,1}}{\Delta} = V'_{ss}(x)|_{x=\frac{\Delta}{2}} = g_{\frac{1}{2}} \tag{4.16}$$

$$\frac{V_{s,N} - V_{s,N-1}}{\Delta} = V'_{ss}(x)|_{x=N-\frac{\Delta}{2}} = g_{N-\frac{1}{2}} \tag{4.17}$$

The g coefficients will be discussed after the following derivation. For all points but the end points discussed above, Equations 4.7 and 4.8 can be rearranged to give the relationship seen below.

$$\frac{d^2V_s(x)}{dx^2} + j\omega L'\frac{dI_s}{dx} = \frac{dV'_{ss}(x)}{dx}$$

$$\frac{d^2V_s(x)}{dx^2} + j\omega L'(-j\omega C'V_s(x) + I'_{ss}(x)) = \frac{dV'_{ss}(x)}{dx}$$

$$\frac{d^2V_s(x)}{dx^2} + j\omega L'(-j\omega C'V_s(x) + I'_{ss}(x)) = \frac{dV'_{ss}(x)}{dx}$$

$$\frac{d^2V_s(x)}{dx^2} + \omega^2 L'C'V_s(x) = \frac{dV'_{ss}(x)}{dx} - j\omega L'I'_{ss}(x) \tag{4.18}$$

The right side of the equation will be referred to as $f(x)$, and its discretized form will be referred to as f_n, where n is the current point on the line that it is representing (equivalent to $f(n\Delta)$, essentially). Instead of f_n, the end points will be represented with $g_{\frac{1}{2}}$ and $g_{N-\frac{1}{2}}$ as seen above.

The second derivative in the finite difference method is described as:

$$\frac{d^2V(x)}{dx^2} = \frac{V_{n+1} - 2V_n + V_{n-1}}{\Delta^2} \tag{4.19}$$

Applying this definition to Equation 4.18, the resulting relationship for $V_s(x)$ simplifies to:

$$V_{s,n+1} - 2V_{s,n} + V_{s,n-1} + (k\Delta)^2 V_{s,n} = f_n\Delta^2 \quad [V] \tag{4.20}$$

where k is equal to $\omega\sqrt{L'C'}$. Combining Equations 4.16, 4.17, and 4.20 can result in the tridiagonal matrix in Equation 4.21. By solving for the column of unknowns on the right side of the tridiagonal matrix, the discretized $V_s(x)$ values along the outer conductor can be obtained. In the solution presented in this thesis, the tridiagonal matrix will be broken down to a banded matrix, with a leading zero in the first row and a trailing zero in the third row, and will then be solved using the `solve_banded` function in Python.

$$\begin{bmatrix} -1 & 1 & 0 & \dots & \dots & 0 \\ 1 & (k\Delta)^2 - 2 & 1 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \dots & \dots & 1 & (k\Delta)^2 - 2 & 1 \\ 0 & \dots & \dots & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_{N-1} \\ V_N \end{bmatrix} = \begin{bmatrix} g_{\frac{1}{2}}\Delta \\ f_2\Delta^2 \\ . \\ . \\ f_{N-1}\Delta^2 \\ g_{N-\frac{1}{2}}\Delta \end{bmatrix} \tag{4.21}$$

The last step is to solve for $I_s(x)$. Keeping in mind that $I_s = 0$ when $n = 1$ or $n = N$, Equation 4.7 can be manipulated to give the following:

$$I_{s,n} = \frac{1}{j\omega L'}(V'_{ss,n} - \frac{V_{s,n+1} - V_{s,n-1}}{2\Delta}) \quad [A] \tag{4.22}$$

For its discretized form, $V'_{ss,n} = V'_{ss}(n\Delta)$ and V_s has already been solved for from the tridiagonal matrix. Hence, the remaining I_s values for $n \in [2, N-1]$ are obtained and the solution can proceed.

4.3 Solving for $V_i(x)$ and $I_i(x)$

Using Equations 4.1, 4.2 and 4.6, the resulting per-unit-length current and voltage sources on the inner conductor can be evaluated. In the case of a solid tubular shield for the coaxial cable, Y_t happens to be negligible, according to [2]. This is because the solid sheath happens to be a very good shield for the electric field. When apertures are introduced, however, this term is no longer negligible and can be incorporated into the derivations that follow.

Since the source voltages and currents for the inner conductor are now present, the same tridiagonal matrix from Equation 4.21 can be built but with a different f_k column vector and slightly different coefficients. The inner conductor will be connected to two loads, one at the beginning, Z_S, and one at the end, Z_L. As a result, $I_i \neq 0$ at the end points. Since current has to be defined to flow in accordance with a certain reference system, the two equations for $I_{i,1}$ and $I_{i,N}$ will be defined as:

$$I_{i,1} = \frac{-V_{i,1}}{Z_S} \quad [A] \tag{4.23}$$

$$I_{i,N} = \frac{V_{i,N}}{Z_L} \quad [A] \tag{4.24}$$

The current at $n = 1$ is negative is for consistency purposes relating to how the current should be defined to flow in one direction. As with Equations 4.16 and 4.17, the following relationship can be obtained:

$$\frac{V_{i,2} - V_{i,1}}{\Delta} + j\omega L' I_{i,1} = V'_{si}(x)|_{x=\frac{\Delta}{2}} = g_{\frac{1}{2}}$$

$$\frac{V_{i,2} - V_{i,1}}{\Delta} + j\omega L'(\frac{-V_{i,1}}{Z_S}) = V'_{si}(x)|_{x=\frac{\Delta}{2}} = g_{\frac{1}{2}}$$

$$V'_{si}(x)|_{x=\frac{\Delta}{2}} = g_{\frac{1}{2}} = \frac{V'_{si,1} + V'_{si,2}}{2}$$

$$V_{i,2} - (1 + \frac{j\omega L'\Delta}{Z_S})V_{i,1} = g_{\frac{1}{2}}\Delta \quad [V] \tag{4.25}$$

The same analysis can be performed to give the resulting relationship for the end point of the inner conductor:

$$V'_{si}(x)|_{x=N-\frac{\Delta}{2}} = g_{N-\frac{1}{2}} = \frac{V'_{si,N} + V'_{si,N-1}}{2}$$

$$V_{i,N-1} + (1 + \frac{j\omega L'\Delta}{Z_L})V_{i,N} = g_{N-\frac{1}{2}}\Delta \quad [V] \tag{4.26}$$

As for the remaining values in the f_k column, the right-hand side of Equation 4.18 can be discretized to give the relationship seen in Equation 4.27 below. One relationship that was used before in Equation 4.22 but was not elaborated upon was that of finding the first derivative at a certain point using discretized values. The definition of the first derivative implies a "forward derivative". This means that $\frac{dV}{dx} = \frac{V_{n+1}-V_n}{\Delta}$ in our equations above. Generally, it is a good idea to have the average of both the backward and forward derivatives, especially when all "previous" and "next" points are available for the calculation. Since the ends of the inner conductor are handled separately under their own boundary condition, this averaging poses no problem in how the f_k column vector is defined. Equation 4.27 allows for the definition of f_k for $n \in [2, N-1]$ that will be used in the remainder of this solution.

$$f_n = \frac{V'_{si,n+1} - V'_{si,n-1}}{2\Delta} - j\omega L' I'_{si,n} \quad [\frac{V}{m^2}] \tag{4.27}$$

For the inner cable, the per-unit-length inductance and capacitance expressions change to those of a coaxial cable from [2]: $L' = \frac{\mu_0}{2\pi}\, ln\frac{b}{a}$ and $C' = 2\pi\epsilon_0\, ln\frac{b}{a}$. Since the relationship in Equation 4.20 still holds, the tridiagonal matrix for the inner conductor can finally be constructed.

$$\begin{bmatrix} -1-\frac{j\omega L'\Delta}{Z_S} & 1 & 0 & \dots & \dots & 0 \\ 1 & (k\Delta)^2-2 & 1 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \dots & \dots & 1 & (k\Delta)^2-2 & 1 \\ 0 & \dots & \dots & 0 & -1 & 1+\frac{j\omega L'\Delta}{Z_L} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_{N-1} \\ V_N \end{bmatrix} = \begin{bmatrix} g_{\frac{1}{2}}\Delta \\ f_2\Delta^2 \\ . \\ . \\ f_{N-1}\Delta^2 \\ g_{N-\frac{1}{2}}\Delta \end{bmatrix} \tag{4.28}$$

By solving for the V_n column matrix, the voltage on the interior conductor,

V_i, can be obtained. As with Equation 4.22, the current I_i can be obtained using Equation 4.29.

$$I_{i,n} = \frac{1}{j\omega L'}(V'_{si,n} - \frac{V_{i,n+1} - V_{i,n-1}}{2\Delta}) \quad [A] \tag{4.29}$$

This concludes the derivation for the voltage and current on the inner conductor! Section 4.4 walks through an example of a shielded coaxial line and the voltage and current it builds up on the inner conductor as a result of an incident field.

4.4 Shielded Cable Example

The following section will utilize the procedure in the previous section to solve for the voltage and current on the inner conductor of a 30-meter shielded coaxial line terminated with 50 Ω loads on both ends on the interior with the ends of the shield not grounded.

The frequency of interest is 200 MHz. The incident wave has an electric field magnitude of $1\,\frac{V}{m}$, and $\alpha = \phi = 0°$ and $\theta = 90°$. Based on a semi-rigid UT-141 cable from Micro-Coax in [4] and [5], the outer conductor has a radius (b) of 3.581 mm and the inner conductor has a radius (a) of 0.9195 mm. These will be used to calculated Δ in Equation 4.30 below for the transfer impedance (this should not be confused with the Δ used to refer to the size of a division along the wire).

One thing to note is that Z_t heavily depends on this wall-thickness Δ, which is justified given that a thicker wall would mean less voltage coupled from the outer conductor to the inner conductor. For this example, Δ will be manually entered as 0.1 mm instead of following the UT-141 cable specifications. The reason for this is that with the actual parameters of the wall-thickness, the resulting voltage and current on the inner conductor would be extremely low and might give the impression that the simulation is, as a whole, unnecessary (do engineers care about an extra $\sim 10^{-200}$ volts showing up on their transmission line?), therefore a thinner wall is imagined and used in the subsequent simulations. The resulting transfer impedance for this problem is calculated below.

$$Z_t = \frac{2\sqrt{2}e^{-(1+1j)\frac{\Delta}{\delta}}e^{j\frac{\pi}{4}}}{2\pi a\delta\sigma} = -3.16 \cdot 10^{-10} - j5.97 \cdot 10^{-10} \quad \frac{\Omega}{m} \tag{4.30}$$

The conductivity of the copper sheath is $5.96 \cdot 10^{7} \frac{S}{m}$. The inductance per-unit-length and capacitance per-unit-length are calculated for both the outer shield and the inner cable:

Outer Shield

$$L' = \frac{\mu_0}{2\pi} ln(\frac{2h}{a}) = 3.44 \cdot 10^{-7} \quad [\frac{H}{m}] \tag{4.31}$$

$$C' = \frac{2\pi\epsilon_0}{ln(2h/a)} = 3.23 \cdot 10^{-11} \quad [\frac{F}{m}] \tag{4.32}$$

Inner Conductor

$$L' = \frac{\mu_0}{2\pi}\, ln\frac{b}{a} = 2.72 \cdot 10^{-7} \quad [\frac{H}{m}] \tag{4.33}$$

$$C' = 2\pi\epsilon_0\, ln\frac{b}{a} = 4.09 \cdot 10^{-11} \quad [\frac{F}{m}] \tag{4.34}$$

Now, the induced voltage and current sources on the exterior conductor, $V'_{ss}(x)$ and $I'_{ss}(x)$, can be plotted using Equations 4.11 and 4.12.

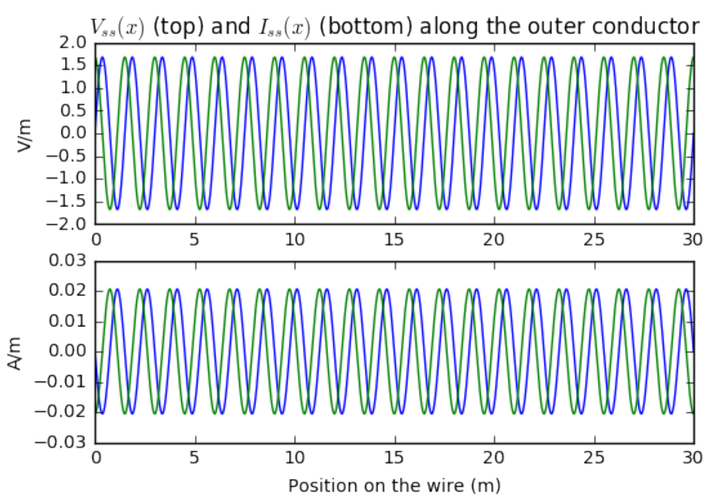

Figure 4.3: Real (blue) and imaginary (green) plots of $V'_{ss}(x)$ (top) and $I'_{ss}(x)$ (bottom) along the outer conductor

Next, by solving the first tridiagonal matrix for the outer conductor, $V_s(x)$ and $I_s(x)$ can be retrieved. The sinusoidal behavior is expected, as the incident wave is a plane wave and propagates in the $\hat{x}$-direction along the wire. The magnitude of this voltage and current is also plotted for a better idea of the periodicity of these quantities along the line. These plots are seen in Figure 4.4.

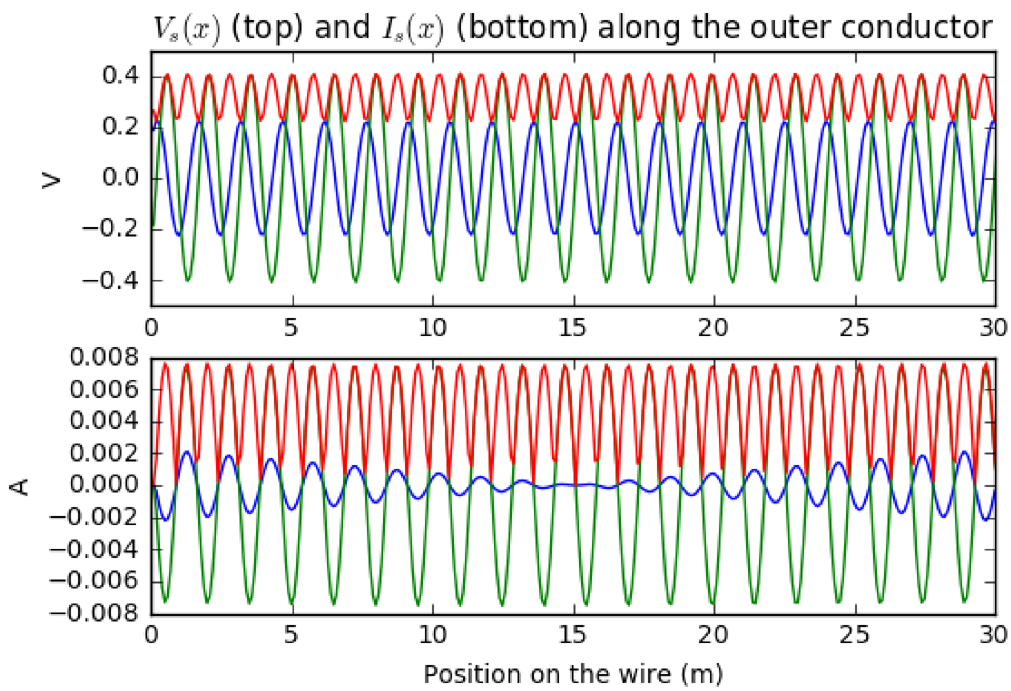

Figure 4.4: Real (blue), imaginary (green), and magnitude (red) plots of $V_s(x)$ (top) and $I_s(x)$ (bottom) along the outer conductor

Using the calculated transfer impedance from Equation 4.30, the plot for $V'_{si}(x)$ can be obtained. Since it was previously concluded that Y_t is negligible for this type of cable, there will be $I'_{si}(x)$ sources along the inner conductor. Figure 4.5 shows this voltage per-unit-length.

A quick observation that can be made is that the plot is still sinusoidal, therefore the solution is still consistent with the coupling behavior on the outer conductor. Another observation is that the magnitude of this induced voltage is now really small. This is justified by the low transfer impedance value and also by the very reason a shielded coaxial cable is used: the voltage coupled to the inner conductor is trivial!

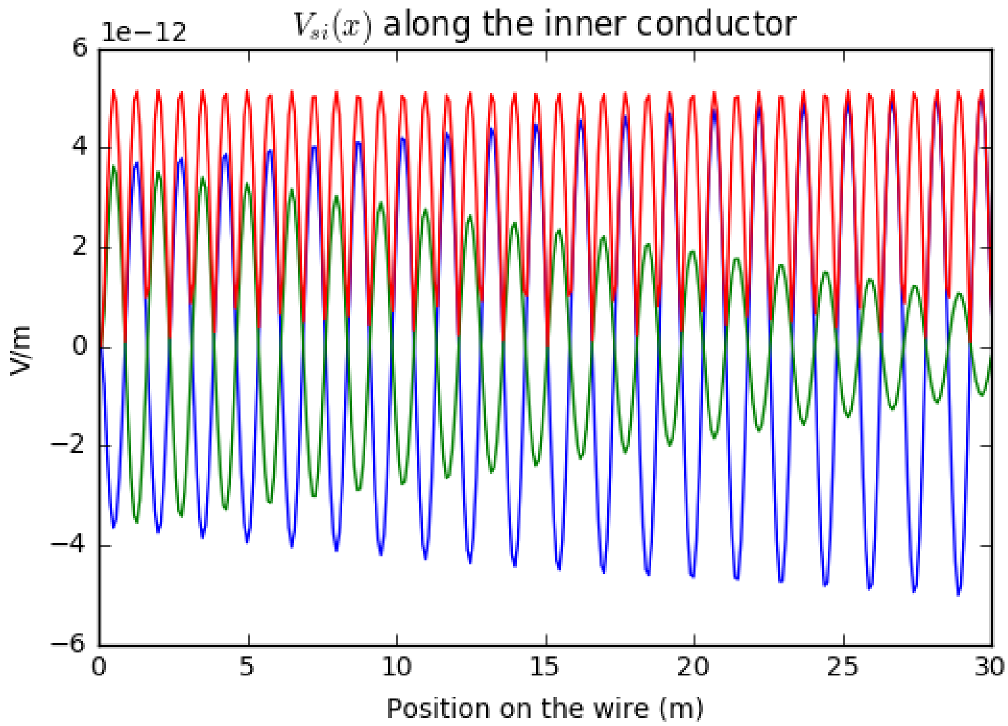

Figure 4.5: Real (blue), imaginary (green), and magnitude (red) plots of $V'_{si}(x)$ along the inner conductor

Finally, by solving the final tridiagonal matrix and Equation 4.29, $V_i(x)$ and $I_i(x)$ along the inner conductor can be obtained. The plots are seen in Figure 4.6 with $V_i(x)$ on the top and $I_i(x)$ on the bottom. A few interesting trends can be spotted in these two plots. First of all, the magnitude of $V_i(x)$ and $I_i(x)$ appear to increase and decrease along the line. This is due to the superposition of the per-unit-length sources that were plotted on Figure 4.4. The slight dip in magnitude around the 15 meter mark is due to the choice of frequency and length of the wire that cause nulls at different points along the line. For the outer conductor, this relationship is simple: at the midpoint of the wire, $I_s(x)$ will be equal to zero amperes. On the inner conductor, however, this trend also depends on the choice of wavelength (specified by the operating frequency). For this example, the wavelength happens to be 1.5 meters, and $V_i(x)$ and $I_i(x)$ nicely experience that slight dip in magnitude around the midpoint of 15 meters. Playing around with different operating frequencies and wire lengths provides a better understanding of this relationship.

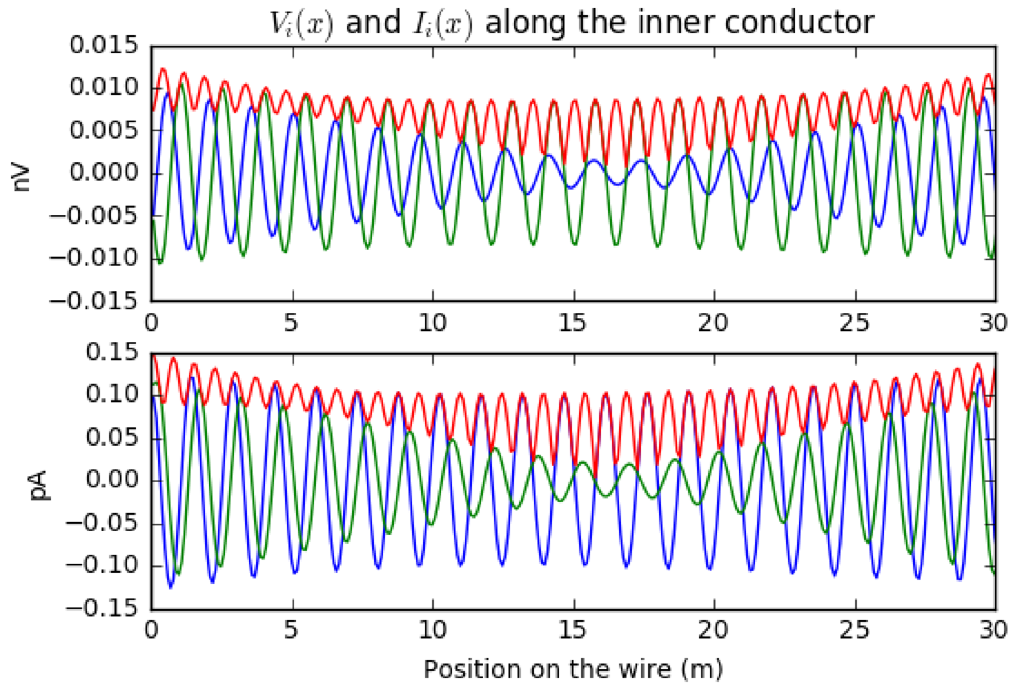

Figure 4.6: Real (blue), imaginary (green), and magnitude (red) plots of $V_i(x)$ (top) and $I_i(x)$ (bottom) along the inner conductor

At an order of $\sim 10^{-11}$, the magnitudes of $V_i(x)$ and $I_i(x)$ are quite small. If apertures are introduced to the cable, the equation for Z_t would change. Also, Y_t would no longer be negligible and would have to be accounted for. Then, the voltages and currents on the inner conductor become more of a concern.

As a sanity check, the code written up for the shielded cable problem can be tested for other incident waves. For example, if the change is made from the previous example so that $\theta = 0°$, and no E_z component exists, and it might be easy to assume that no coupling occurs on the exterior cable to begin with. This is not the case, however, because H_y is still non-zero and a non-zero $V_{ss}(x)$ forms on the exterior cable. Solving the tridiagonal matrix gives non-zero magnitudes of $V_s(x)$ and $I_s(x)$ on the exterior and thus there would be some induced voltage and current per-unit-length sources on the inner conductor.

Another test case could be for when the same parameters as the main example hold, except now $\alpha = 90°$. This is very similar to the sanity check performed in Chapter 3 on the two-wire transmission line. As expected, there will be no voltages or currents induced on both the outer and inner conductors, since now both E_z and H_y are zero. Because the electric field is now polarized horizontally, meaning perpendicular to the field of incidence, the incident wave will induce no sources on the cable.

4.5 Shielded Cable Simulation with the Shield Grounded at the End Points

If the shielded cable were to have its exteriors not open, and instead connected to the ground, the solution above would need to change. By changing the Z_1^e and Z_2^e load values from Figure 4.2 from being open to shorted to zero. An easy way to approach this is to solve for the currents on the exterior, $I_s(x)$, using Equation 4.8 instead of 4.7. The tridiagonal matrix would be adjusted using the following procedure:

$$\frac{d^2 I_s(x)}{dx^2} + j\omega C' \frac{dV_s}{dx} = \frac{dI'_{ss}(x)}{dx}$$

$$\frac{d^2 I_s(x)}{dx^2} + j\omega C'(-j\omega L' I_s(x) + V'_{ss}(x)) = \frac{dI'_{ss}(x)}{dx}$$

$$\frac{d^2 I_s(x)}{dx^2} + \omega^2 L'C' I_s(x) = \frac{dI'_{ss}(x)}{dx} - j\omega C' V'_{ss}(x) \tag{4.35}$$

Again, the right-hand side of Equation 4.33 above will be referred to as the f_k column. When broken up into discrete components, Equation 4.33 reduces to a similar form seen in Equation 4.20.

$$\frac{d^2 I(x)}{dx^2} = \frac{I_{n+1} - 2I_n + I_{n-1}}{\Delta^2} \tag{4.36}$$

$$I_{s,n+1} - 2I_{s,n} + I_{s,n-1} + (k\Delta)^2 I_{s,n} = f_n \Delta^2 \quad [A] \tag{4.37}$$

The endpoints on the line defined by the quantities $g_{\frac{1}{2}}$ and $g_{N-\frac{1}{2}}$, seen below.

$$\frac{I_{s,2} - I_{s,1}}{\Delta} = I'_{ss}(x)|_{x=\frac{\Delta}{2}} = g_{\frac{1}{2}} \tag{4.38}$$

$$\frac{I_{s,N} - I_{s,N-1}}{\Delta} = I'_{ss}(x)|_{x=N-\frac{\Delta}{2}} = g_{N-\frac{1}{2}} \tag{4.39}$$

where k, again, is equal to $\omega\sqrt{L'C'}$. The tridiagonal matrix is then assembled and solved for $I_s(x)$ below.

$$\begin{bmatrix} -1 & 1 & 0 & \dots & \dots & 0 \\ 1 & (k\Delta)^2-2 & 1 & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \dots & \dots & 1 & (k\Delta)^2-2 & 1 \\ 0 & \dots & \dots & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_{N-1} \\ I_N \end{bmatrix} = \begin{bmatrix} g_{\frac{1}{2}}\Delta \\ f_2\Delta^2 \\ . \\ . \\ f_{N-1}\Delta^2 \\ g_{N-\frac{1}{2}}\Delta \end{bmatrix} \tag{4.40}$$

The next step would be to plug that into Equation 4.8 and finally solving for $V_s(x)$. Thus, the voltages and currents on the exterior circuit are solved for, and the procedure can continue to solve for $V_i(x)$ and $I_i(x)$ as done before. As a reference, the example from Section 4.4 is revisited with the same input parameters but now with a grounded shield, as opposed to an open. Following the aforementioned steps, the final solution for $I_i(x)$ and $V_i(x)$ is plotted in Figure 4.7.

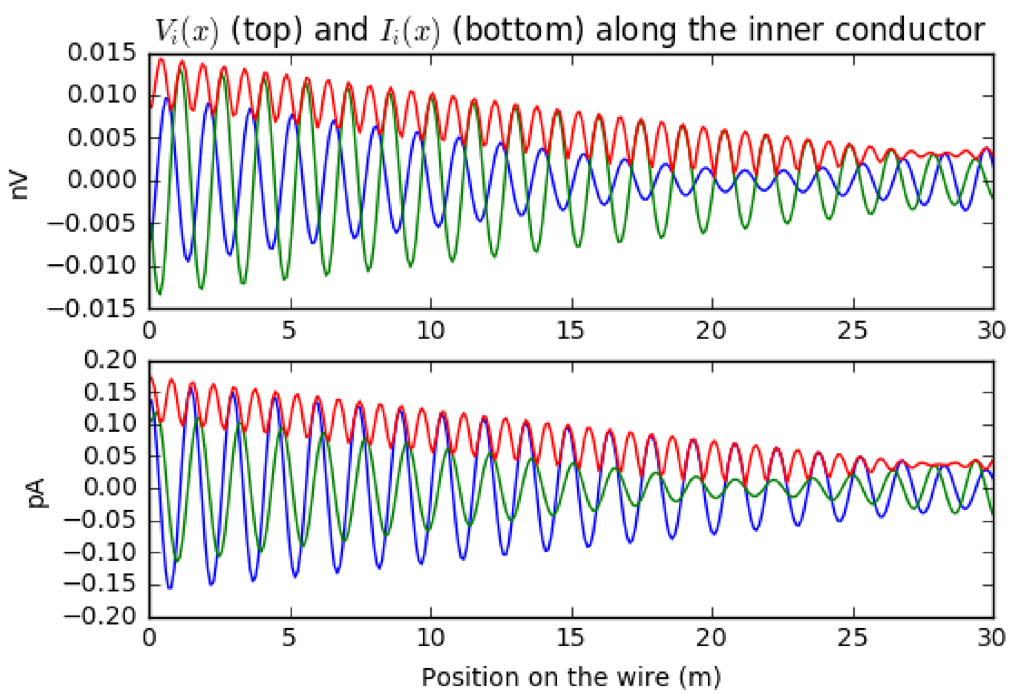

Figure 4.7: Real (blue), imaginary (green), and magnitude (red) plots of $V_i(x)$ (top) and $I_i(x)$ (bottom) along the inner conductor with a grounded outer shield

The voltage and current current seem to decrease along the line, but that is due to the chosen line length and incident field wavelength of 1.5 meters. With the shield that was not grounded in Section 4.4, a wavelength of 1.5 meters was a resonant wavelength since it is an integer multiple of the line length, 30 meters. The open ends of the shield force V_s to be a maximum at each end of the line, and will consequently force a similar standing wave behavior on V_i. As a result, V_i is seen to be the smallest around the center of the line in Figure 4.6. With the shield grounded in Figure 4.7, this resonant wavelength does not support a standing wave (due to the connection to ground), but instead simply solves for the voltages along the line that would meet the given boundary conditions when superposed. Another experiment with a 40 meter line yields the plot in Figure 4.8. Clearly, the V_i and I_i do not explicitly decrease along the line in this case, meaning that the trend observed in Figure 4.7 is not inclusive of all configurations.

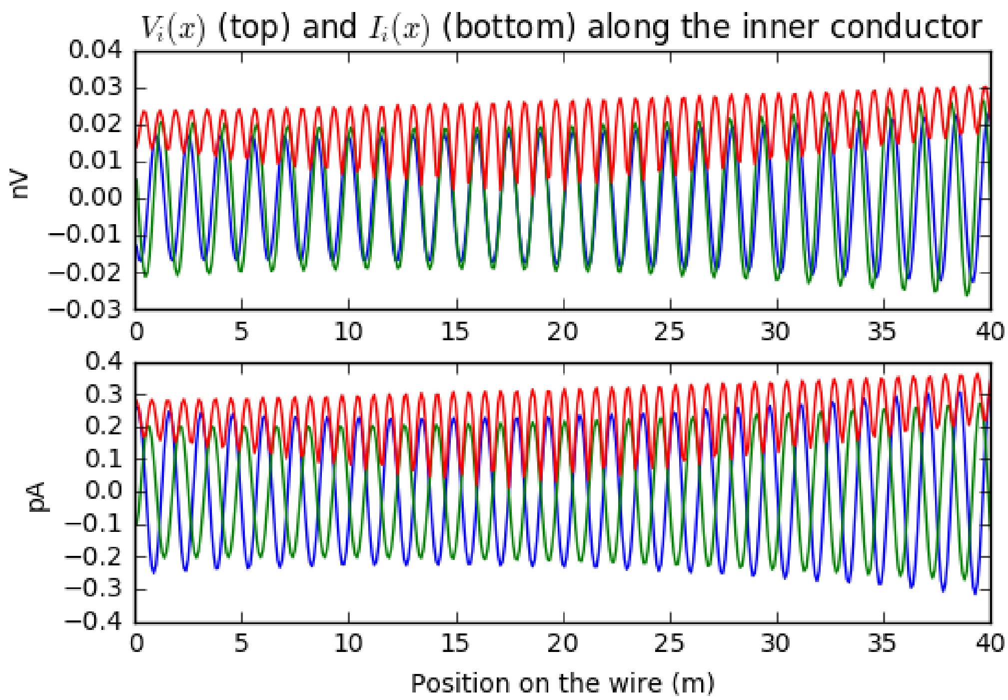

Figure 4.8: Real (blue), imaginary (green), and magnitude (red) plots of $V_i(x)$ (top) and $I_i(x)$ (bottom) along a 40 meter line

4.6 Additional Notes on the Shielded Cable Simulation

The derivations above are implemented using the code in Appendix A.2, and would be used in similar command-line argument form to plot the V_i and I_i as a final result.

Now that a couple of a sanity checks were completed, it is worth mentioning why such induced voltages and currents from electromagnetic fields are so important. With the example above, the electric field applied had a magnitude of 1 $\frac{V}{m}$. If a stronger field were to be applied, the voltage on the inner conductor would be larger. If a strong enough incident wave were to be applied to a sensitive device, it could severely damage it by shorting it or interrupting an important communication protocol. If the cable provided had apertures or was imperfect, damaging results could be induced without requiring a necessarily strong electric field. This sheds some light on the importance of Electromagnetic Compliance (EMC) and Electromagnetic Interference (EMI) when it comes to designing electronic devices.

CHAPTER 5

CONCLUSION AND FUTURE WORK

This thesis investigated different antenna and transmission line simulations and concluded with the derivation of a solution for analyzing the voltages and currents on shielded cables. As touched on in Section 4.5, electromagnetic simulations are very important as they assess and test not only the functionality of an electronic device, but also ensure that no interference would cause any harm to its circuitry. By stepping through the solutions in Python, quick simulations can be performed on an electronic component of interest without the need for computer-aided tests and analyses.

Although the thesis concluded with the solution for a shielded coaxial cable, some interesting next steps would be to test it for braided shields that have a non-zero transfer admittance. Another topic worth investigating is the effect of connectors on these simulations. For example, if an antenna was hooked up to a shielded cable, the connector at that interface could cause some noticeable differences in the coupling behavior at high frequencies.

The work presented in this thesis could be expanded into a library of different circuit components that could be cascaded together to mimic the behavior of a real-world system. If done precisely, this could expedite the simulations that need to be performed when designing any given circuit or system.

APPENDIX A

CODE APPENDIX

A.1 Transmission Line Code

```
### INPUT PARAMETERS ###
### in the order below ###
# 1. wireRadius (radius of each wire)
# 2. lineSeparation (separation between the two wires)
# 3. length
# 4. sheetResistance (0 in a lossless line)
# 5. sourceVoltage (coming from the antenna or other source)
# 6. load1
# 7. load2
# 8. phi
# 9. psi
# 10. alpha
# 11. frequency
# 12. fieldMagnitude

from cmath import sinh, cosh, pi, sqrt, cos, sin, polar, acosh
import decimal
from numpy import exp, linspace, array, arange
from scipy import real, imag
from scipy.integrate import quad
import matplotlib.pyplot as plt
from pylab import *

#Import Inputs from Terminal by running python filename.py
   followed by the input parameters seperated by spaces and
```

```
    then pressing Enter.
# line properties
wireRadius = float(sys.argv[1])
lineSeparation = float(sys.argv[2])
length = float(sys.argv[3])
sheetResistance = float(sys.argv[4])

# load impedances and source voltage
sourceVoltage = float(sys.argv[5])
load1 = float(sys.argv[6])
load2 = float(sys.argv[7])

# incident wave properties
phi = (float(sys.argv[8])*pi)/180
psi = (float(sys.argv[9])*pi)/180
alpha = (float(sys.argv[10])*pi)/180
selected_frequency = float(sys.argv[11])
fieldMagnitude = float(sys.argv[12])

# environment
c = 3*10**8
permeability = 1.2566370614*10**(-6)
permittivity = 8.85418782*10**(-12)
imaginaryPermittivity = 0

# get inductance, capacitance, resistance, and conductance per
    unit length
def getInductancePerUnitLength(wireRadius, lineSeparation):
    L = (permeability/pi)*acosh(lineSeparation/(2*wireRadius))
    return L

def getCapacitancePerUnitLength(wireRadius, lineSeparation):
    C = pi*permittivity/acosh(lineSeparation/(2*wireRadius))
    return C

def getResistancePerUnitLength(sheetResistance, wireRadius):
```

```
    R = sheetResistance/(pi*wireRadius)
    return R

def getConductancePerUnitLength(wireRadius, lineSeparation):
    G = pi*imaginaryPermittivity/acosh(lineSeparation/(2*
        wireRadius))
    return G


# call the functions above to obtain L, C, R, and G per unit
    length
L = getInductancePerUnitLength(wireRadius, lineSeparation)
C = getCapacitancePerUnitLength(wireRadius, lineSeparation)
R = getResistancePerUnitLength(sheetResistance, wireRadius)
G = getConductancePerUnitLength(wireRadius, lineSeparation)


fmax=0.3*c/(lineSeparation*pi)
f=linspace(0,fmax,100000)
k=2*pi*f/c
xrange = k*lineSeparation/2
omega = 2*pi*f

# calculate the propagation constant and characteristic
    impedance
def getPropagationConstant(L, R, C, G, omega):
    w = omega
    propagationConstant = sqrt((R+1j*w*L)*(G+1j*w*C))
    return propagationConstant

def getCharacteristicImpedance(L, R, C, G, omega):
    w = omega
    characteristicImpedance = sqrt((R+1j*w*L)/(G+1j*w*C))
    return characteristicImpedance

# call the functions above
```

```
propagationConstant = getPropagationConstant(L, R, C, G, omega)
characteristicImpedance = getCharacteristicImpedance(L, R, C, G
    , omega)

# define main coupling function with designated input
    parameters
def coupling_current(lineLength, fieldMagnitude, phi, psi,
    alpha, load1, load2, lineSeparation, L, R, C, G, omega):
    k=omega/c
    length = lineLength
    fieldMag = fieldMagnitude
    d = lineSeparation
    Zc = getCharacteristicImpedance(L, R, C, G, omega)
    p1 = (load1-Zc)/(load2+Zc)
    p2 = (load2-Zc)/(load2+Zc)
    prop = getPropagationConstant(L, R, C, G, omega)

    #S1 vector
    S1atop=fieldMag*(cos(alpha)*sin(psi)*cos(phi)+sin(alpha)*
        sin(phi))*1j*k*d*sin(psi)
    S1abottom=prop-1j*k*cos(psi)*cos(phi)
    S1b=fieldMag*d*cos(psi)*cos(alpha)
    S1ab=(S1atop/S1abottom)-S1b
    S1c=1-exp((prop-1j*k*cos(psi)*cos(phi))*length)
    S1=-0.5*S1ab*S1c

    #S2 vector
    S2atop=fieldMag*(cos(alpha)*sin(psi)*cos(phi)+sin(alpha)*
        sin(phi))*1j*k*d*sin(psi)
    S2abottom=prop+1j*k*cos(psi)*cos(phi)
    S2b=fieldMag*d*cos(psi)*cos(alpha)
    S2ab=(S2atop/S2abottom)+S2b
    S2c=1-exp(-(prop+1j*k*cos(psi)*cos(phi))*length)
    S2=-0.5*S2ab*S2c*exp(prop*length)

    # determinant of inverse vector
```

```
    det=((p2*p1)-exp(2*length*prop))*Zc

    # denominator of final expression
    K=1/(det)

    # calculating the final coupling load current
    iLoad=(p2-1)*(S1*exp(length*prop)+S2*p1)*K

    # returning the magnitude
    return abs(iLoad)

def coupling_voltage(lineLength, fieldMagnitude, phi, psi,
        alpha, load1, load2, lineSeparation, L, R, C, G, omega):
    k = omega/c
    length = lineLength
    fieldMag = fieldMagnitude
    d = lineSeparation
    Zc = getCharacteristicImpedance(L, R, C, G, omega)
    p1 = (load1-Zc)/(load2+Zc)
    p2 = (load2-Zc)/(load2+Zc)
    prop = getPropagationConstant(L, R, C, G, omega)

    #S1 vector
    S1atop=fieldMag*(cos(alpha)*sin(psi)*cos(phi)+sin(alpha)*
        sin(phi))*1j*k*d*sin(psi)
    S1abottom=prop-1j*k*cos(psi)*cos(phi)
    S1b=fieldMag*d*cos(psi)*cos(alpha)
    S1ab=(S1atop/S1abottom)-S1b
    S1c=1-exp((prop-1j*k*cos(psi)*cos(phi))*length)
    S1=-0.5*S1ab*S1c

    #S2 vector
    S2atop=fieldMag*(cos(alpha)*sin(psi)*cos(phi)+sin(alpha)*
        sin(phi))*1j*k*d*sin(psi)
    S2abottom=prop+1j*k*cos(psi)*cos(phi)
    S2b=fieldMag*d*cos(psi)*cos(alpha)
```

```
    S2ab=(S2atop/S2abottom)+S2b
    S2c=1-exp(-(prop+1j*k*cos(psi)*cos(phi))*length)
    S2=-0.5*S2ab*S2c*exp(prop*length)

    # determinant of inverse vector
    det=((p2*p1)-exp(2*length*prop))

    # denominator of final expression
    K=1/(det)

    # calculating the final coupling load current
    vLoad=(p2+1)*(-S1*exp(prop*length)-S2*p1)*K

    # returning the magnitude
    return abs(vLoad)


# frequency sweep to view periodicity - only works in a
   lossless case with a constant characteristic impedance and
   propagation constant


# call the coupling function
voltage_result = coupling_voltage(length,fieldMagnitude,phi,psi
    ,alpha,load1, load2,lineSeparation, L, R, C, G, omega)
current_result = coupling_current(length,fieldMagnitude,phi,psi
    ,alpha,load1, load2,lineSeparation, L, R, C, G, omega)

# plot the voltage due to coupling versus kd/2
plot(xrange,current_result*1000/fieldMagnitude)
plot(xrange,voltage_result*1000/fieldMagnitude)
# set up the plot
xlim([0,0.3])
ylim([0,1.0])
plt.title('Frequency Sweep of Normalized Voltage Due to
   Coupling')
```

```
plt.xlabel('kd/2')
plt.ylabel('Normalized Voltage')
show()
```

A.2 Shielded Cable Code

```
import numpy as np
from numpy import exp, linspace, array, arange
from cmath import sinh, cosh, pi, sqrt, cos, sin, polar, acosh
from scipy import real, imag
from scipy.integrate import quad
from scipy.linalg import solve_banded
import matplotlib.pyplot as plt
from pylab import *
import decimal

# input frequency
frequency = float(sys.argv[1])*10**(6)

c = 3*10**8
w = frequency*2*pi
wavelength = c/frequency

# field parameters from the user
alpha = float(sys.argv[2])*pi/180
phi = float(sys.argv[3])*pi/180
theta = float(sys.argv[4])*pi/180
Emag = float(sys.argv[5])

# predefined field parameters
eta = 120*pi
k = cos(phi)*2*pi/wavelength

# line parameters
```

```
length = float(sys.argv[6])
delta = wavelength/15
x = linspace(0,length, 10000)
h = float(sys.argv[7])

#transfer impedance properties
conductivity = float(sys.argv[8])
b = float(sys.argv[9])*10**(-3)
a = float(sys.argv[10])*10**(-3)
permeability = 1.2566370614*10**(-6)
permittivity = 8.85418782*10**(-12)

# load impedances on the interior
Zl = float(sys.argv[11])
Zs = float(sys.argv[12])


# line properties
# deltaBA = b - a ### uncomment this line and remove the one
   below for the correct results
deltaBA = 0.0001
skinDepth = sqrt(1/(pi*frequency*conductivity*permeability))
Zt = 2*exp(-(1+1j)*deltaBA/skinDepth)*exp(1j*pi/4)*sqrt(2)/(2*
   pi*a*skinDepth*conductivity)

# per unit length parameters for outer shield
Cs= 2*pi*permittivity/(log(2*h/b))
Ls = (permeability/(2*pi))*log(2*h/b)

# per unit length parameters for inner cable
Li = (permeability/(2*pi))*log(b/a)
Ci = 2*pi*permittivity/log(b/a)

# line properties
deltaBA = b - a
skinDepth = sqrt(1/(pi*frequency*conductivity*permeability))
```

```
# setting up the E-field
Ez = Emag*sin(theta)*cos(alpha)

# setting up the H-field --- TODO: add cos(phi) factor to
   account for no voltage scenario
Hy = -1*cos(alpha)*Emag/eta

# Is(x) and Vs(x) for the exterior cable
Iss = -1j*w*Cs*Ez*exp(-1j*k*sin(theta)*x)*2*sin(k*cos(theta)*h)
   /(k*cos(theta))
Vss = -1j*w*permeability*Hy*exp(-1j*k*sin(theta)*x)*2*sin(k*cos
   (theta)*h)/(k*cos(theta))

# setting up the first tridiagonal matrix endpoints
g1 = -1j*w*permeability*Hy*exp(-1j*k*sin(theta)*(delta/2))*2*
   sin(k*cos(theta)*h)/(k*cos(theta))
endPoint = length-(delta/2)
gN = -1j*w*permeability*Hy*exp(-1j*k*sin(theta)*(endPoint))*2*
   sin(k*cos(theta)*h)/(k*cos(theta))

# delta
divisions = length/delta

# setting up the fk column
N = int(divisions + 1)
n = linspace(1, N, N)
def sourceCurrent(x1):
    i = -1j*w*Cs*Ez*exp(-1j*k*sin(theta)*x1)*2*sin(k*cos(theta)
       *h)/(k*cos(theta))
    return i

def sourceVoltage(x1):
    v = -1j*w*permeability*Hy*exp(-1j*k*sin(theta)*x1)*2*sin(k*
       cos(theta)*h)/(k*cos(theta))
    return v
```

```
def functionExterior(x1):
    flipper = -1j*k*sin(theta)*sourceVoltage(x1) -1j*w*Ls*
        sourceCurrent(x1)
    return flipper

fkExterior=[]
fkExterior.append(g1*delta)
for i in range(N-2):
    a = functionExterior((i+1)*delta)
    fkExterior.append(a*delta*delta)
fkExterior.append(gN*delta)

# solving for Vs by setting up the banded matrix
top = np.array([0, 1])
for i in range(N-2):
    top = np.append(top, 1)

# referred to as k in the derivations
b = w*w*Ls*Cs*delta*delta - 2
middle = np.array([-1])
for i in range(N-2):
    middle = np.append(middle, b)
middle = np.append(middle, 1)

bottom = np.array([1])
for i in range(N-3):
    bottom = np.append(bottom, 1)
bottom = np.append(bottom, -1)
bottom = np.append(bottom, 0)

# building the full banded matrix for Vs
full = np.array([top, middle, bottom])

# final Vs solution
Vs = solve_banded((1,1), full, fkExterior)
```

```
# solving for Is
Is = np.array([0])
for i in range(N-2):
    newCurrWL = -(0.5*(Vs[i+2]-Vs[i])/delta)+sourceVoltage((i
        +1)*delta)
    newCurr = newCurrWL/(1j*w*Ls)
    Is = np.append(Is, newCurr)
Is = np.append(Is, 0)

# Vsi and Isi calculation
Vsi = Zt*Is
Yt = 0 # for the shielded cable with no apertures, transfer
    admittance is negligible
Isi = -Yt*Is

# setting up the second fk column
fkInteriorFirst = (Vsi[1]+Vsi[0])/2 # Vs @ x = delta/2
fkInteriorEnd = (Vsi[N-1]+Vsi[N-2])/2 # Vs @ x = length - delta
    /2

fkInterior = []
fkInterior.append(fkInteriorFirst*delta)
for i in range(N-2):
    a = 0.5*(Vsi[i+2]-Vsi[i])/delta - 1j*w*Li*Isi[i+1]
    fkInterior.append(a*delta*delta)
fkInterior.append(fkInteriorEnd*delta)


# solving for Vi
topInterior= np.array([0, 1])
for i in range(N-2):
    topInterior = np.append(topInterior, 1)

# referred to as k in the derivations
bInterior = w*w*Li*Ci*delta*delta - 2
```

```
middleInterior = np.array([-1-(1j*w*Li*delta)/Zs])
for i in range(N-2):
    middleInterior = np.append(middleInterior, bInterior)
middleInterior = np.append(middleInterior, 1+(1j*w*Li*delta)/Zl
    )

bottomInterior = np.array([1])
for i in range(N-3):
    bottomInterior = np.append(bottomInterior, 1)
bottomInterior = np.append(bottomInterior, -1)
bottomInterior = np.append(bottomInterior, 0)

# building the full banded matrix for Vi
fullInterior = np.array([topInterior, middleInterior,
    bottomInterior])

# final Vi solution
Vi = solve_banded((1,1), fullInterior, fkInterior)

# solving for Ii at its first endpoint
Ii = np.array([-Vi[0]/Zs])

# solving for remaining Ii values except last
for i in range(N-2):
    currentInteriorWL = -(0.5*(Vi[i+2]-Vi[i])/delta)+Vsi[i+1]
    currentInterior = currentInteriorWL/(1j*w*Li)
    Ii = np.append(Ii, currentInterior)

# solving for second Ii endpoint, and thus the solution for Ii
    is complete
Ii = np.append(Ii, Vi[N-1]/Zl)

# plotting Vi
plt.subplot(2, 1, 1)
plt.plot(n*delta, real(Vi)*10**9)
plt.plot(n*delta, imag(Vi)*10**9)
```

```
plt.plot(n*delta, abs(Vi)*10**9)
plt.title('$V_{i}(x)$ (top) and $I_{i}(x)$ (bottom) along the
    inner conductor')
plt.ylabel('nV')

# plotting Ii
plt.subplot(2, 1, 2)
plt.plot(n*delta, real(Ii)*10**12)
plt.plot(n*delta, imag(Ii)*10**12)
plt.plot(n*delta, abs(Ii)*10**12)

plt.xlabel('Position on the wire (m)')
plt.ylabel('pA')
plt.show()
```

REFERENCES

[1] E. Kudeki, "ECE 350 Lecture Notes," Aug 2016.

[2] F. M. Tesche, M. V. Ianoz, and T. Karlsson, *EMC Analysis Methods and Computational Models.* John Wiley and Sons, 1997.

[3] AlphaWire. (2015, Dec) Types of cable shielding. http://www.alphawire.com/en/Company/Blog/2015/December/Types%20of%20Cable%20Shielding. [Online]. Available: http://www.alphawire.com/en/Company/Blog/2015/December/Types%20of%20Cable%20Shielding

[4] "Standard copper 50-ohm semi-rigid cable," Micro-Coax, UT-141-100-TP, rev. D.

[5] J. P. Rohrbaugh, "Shield transfer impedance model of a multi-branched braid shielded cable harness," *2013 IEEE International Symposium on Electromagnetic Compatibility*, Aug 2013.