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LEAPING WEIRS AND OVERFLOW WEIRS FOR SEWERS

BY

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THESIS

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June 2, 1917

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY HAROLD EATON BABBITT

ENTITLED LEAPING WEIRS AND OVERFLOW WEIRS FOR SEWERS

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN MUNICIPAL AND SANITARY ENGINEERING.

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Recommendation concurred in:*

Committee on Final Examination*

*Required for doctor's degree but not for master's.

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FIGURE I.
LEAPING WEIR APPARATUS IN ACTION

---:PREFACE:---

The aim of this work has been to record the results of a series of tests on two types of sewer regulators without moving parts, the leaping weir and the overflow weir, and to present methods for their design. Some information has been collected concerning other tests on these two devices, a few examples of existing installations have been shown, and a brief historical resumè has been made.

The thesis has been divided into two parts: Part I treating the leaping weir and Part II the overflow weir.

Sewer regulators are used for relieving surcharged sewers of excess sewage. They may be used in combined sewers to divert the storm water, in overcharged separate sewers to divert the excess sewage into relief sewers, and in other cases to divert the sewage from one channel to another.

The types of regulators may be divided into two classes, those without moving parts and those with moving parts. Most of the moving part regulators depend on a float, which upon rising opens the gate to a relief outlet. Some of the devices are quite simple, others very complicated. Under certain conditions the moving part regulators will give satisfaction. Most of the moving part regulators are manufactured under the control of the patentee, and information as to their adaptability and capacity can be obtained from the manufacturer. After installation the devices usually require calibration.

The two regulators without moving parts are the leaping weir and the overflow weir. These are not patented and can be 'manufactured' easily in the field. The advantages and disadvantages of the weirs are discussed under each division of the subject.

The experimental work was done in the hydraulic laboratory of the College of Engineering at the University of Illinois. The study of the results and the deduction of the formulas was done after the completion of the laboratory work.

Literature on leaping weirs and overflow weirs is scarce. The most valuable information on the subject is to be found in American Sewerage Practice, Volume I, by Metcalf and Eddy. The following books have something to say of more or less value on these subjects:

"Sanitary Engineering" by E.C.S. Moore, and the second edition by Moore and Silcock.

"Sanitary Engineering" by Vernon Harcourt. p. 313

"Sewers and Drains for Populous Districts" by J.W. Adams. p. 133

"The Sewerage of Sea Coast Towns" by H.C. Adams. p.53

"The Cleaning and Sewerage of Cities" by R. Baumeister. pp. 5, 47, and 122.

"Sanitary Engineering" by Wood. Second Ed. p. 155

"Construction of Sewers" by Ogden. Chapter XI.

"The Main Drainage of Towns" by F.N. Taylor pp. 135-138

"Sewerage" by A.P. Folwell. p 170

"Sanitary-Engineering" by Baldwin Latham Second Edition p. 460

Other Publications

"The Elimination of Storm Water from Sewerage Systems" by D.E. Lloyd-Davis in Minutes of Proceedings of the Institution of Civil Engineers, Vol CLXIV p. 41

"The Milwaukee Sewerage System" by G.H. Benzenberg in Transactions of the American Society of Civil Engineers Vol. XXX p. 367 and 711

"The Walworth Run Sewer, Cleveland, Ohio" by W.C. Parmley in Transactions of the American Society of Civil Engineers Volume LV p. 341.

PART I

LEAPING WEIRS

CHAPTER I

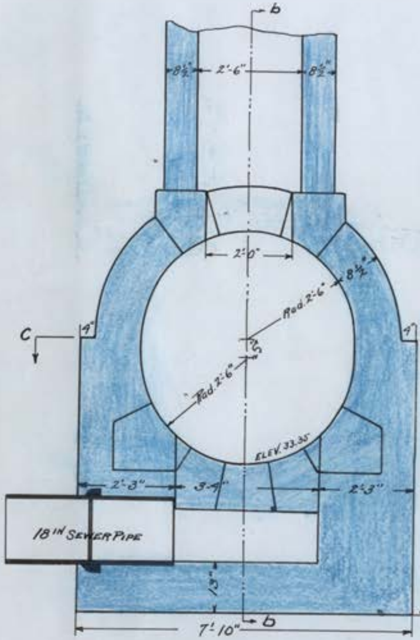
INTRODUCTION

Sect. 1. Definition:- A leaping weir is a device for controlling the amount of flow in a sewer. It operates automatically without moving parts, in such a manner that a relatively low flow in the sewer falls into a channel beneath the weir, whereas the higher velocities of larger quantities cause the stream to leap the gap opening into the channel below, and pass out through another channel. Figures 2, 5, and 6 show the details of typical leaping weirs.

Sect. 2. Purpose:- Under certain conditions in a combined sewerage system it is undesirable to conduct the entire storm flow to the point of dischalge of the dry weather flow. The outfall sewer can be relieved of the storm flow by inserting a leaping weir at its upper end and conducting the storm flow to some nearby outlet.

When a sewage treatment plant is placed at the outfall of a combined sewerage system it is undesirable to attempt to treat all of the storm water which will be delivered through the outfall. In some cases the amount of storm water will so dilute the ordinary dry weather flow as to permit the discharge of the untreated mixture into the body of water ordinarily receiving the treated dry weather flow, without causing a nuisance. A leaping weir is a device, without moving parts, which will successfully accomplish this purpose, so that the treatment plant can be entirely at rest during times of storm discharge from the sewerage system.

One of the original purposes of leaping or "separating" weir as mentioned in some of the books listed in the bibliography, was to collect the clear low water flow of a small stream as potable water, and to allow the muddy storm waters to pass by without



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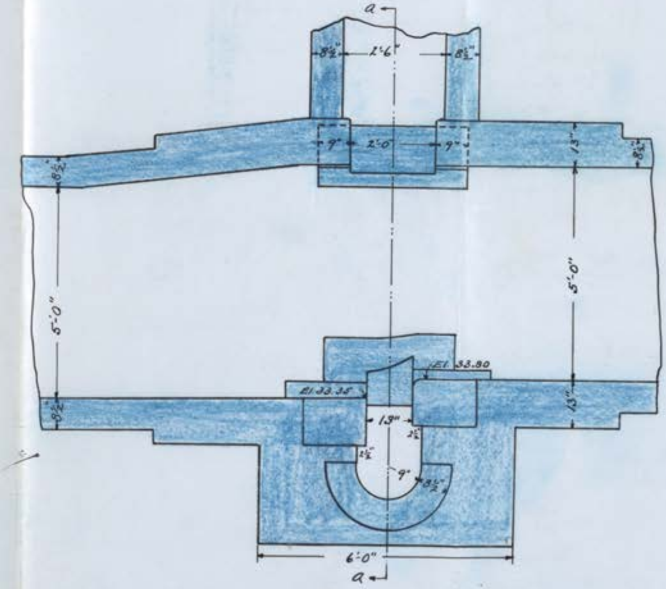

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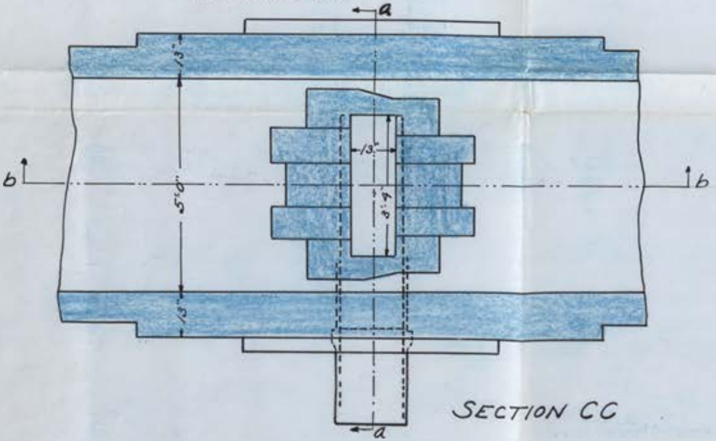


SECTION CC

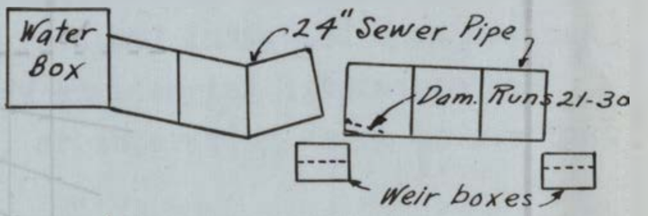
FIGURE 2
LEAPING WEIR
NORTH AVE. INTERCEPTER
MILWAUKEE, WIS.

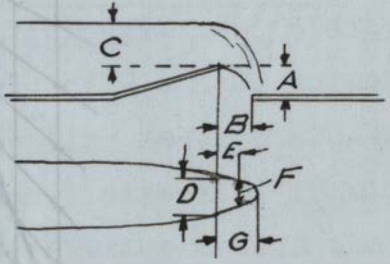
SYRACUSE INTERCEPTING SEWER BOARD
DIAGRAMS AND TABLES
LEAPING WEIR EXPERIMENTS

Run	Water Thru Meter		Intercepted Water		Disch. at Outlet		Slope
	Total	C.F.S.	Head Weir	C.F.S.	Head Weir	C.F.S.	
1	30	.5	.15	.2	.2	.3	.01
2	10	.15	.13	.15	0	0	"
3	10	.26	.15	.2	.07	.07	"
4	130	.36	.16	.21	.16	.21	"
5	117	.70	.18 Vel. opp. from .43' to .54	.22	.29	.48	"
6	69	.124	.11	.12	0	0	"
7	88	.512	.15	.2	.2	.29	" N.G
8	49	.438	.14	.18	.1	.12	"
9	52	.407	.15	.19	.15	.19	"
10	68	.708	.15	.19	.29	.5	"
11	50	.862	.18	.25	.35	.62	" NG
12	75	.4	.13	.16	.06	.06	"
13	40	.268	.14	.18	.07	.06	"
14	50	.443	.16	.21	.17	.22	" NG
15	60	.731	.17	.22	.25	.40	"
16	50	.758	.17	.22	.30	.52	"
17							" NG
18			.20	.28	.36	.67	"
19			.20	.28	.50	1.09	"
20			.22	.32	.52	1.13	"
21	Dam. 2' high in disch. pipe		.21	.31	.36	.67	"
22	"		.21	.31	.48	1.05	"
23	"		.19	.27	.05	.05	"
24	"		.20	.29	.10	.11	"
25	"		.21	.31	.20	.29	"
26	"		.21	.31	.45	.93	"
27	"		.15	.20	.00	.00	"
28	40	.33	.22	.33	.00	.00	"
29	40	.55	.30	.52	.01	.01	"
30			.30	.52	.05	.05	"
31			.31	.55	.15	.20	"
32			.31	.55	.20	.28	"
33			.31	.55	.36	.69	"

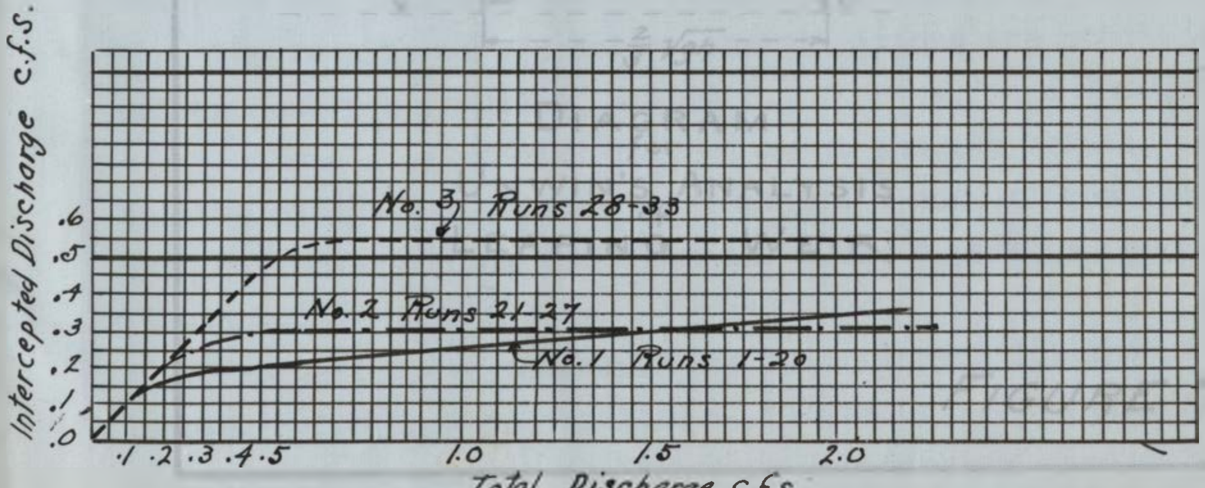
Run	A	B	C	D	E	F	G
2	.4	.4	.17	1.08	.21	.75	.4
3	"	"	.20	1.21	.25	1.17	"
4	"	"	.24	1.27	.25	1.00	"
5	"	"					
6	"	"	.19	1.04	.25	.8	.4
7	"	"	.29	1.35	"	1.2	"
8	"	"	.25	1.25	"	1.05	"
9	"	"	.27	1.3	"	1.12	"
10	"	"	.34	1.5	"	1.3	"
11	"	"	.36	1.50	"	1.4	"
12	"	"	.20	1.17	"	.93	"
13	"	"	.21	1.2	"	.96	"
14	"	"	.27	1.3	"	1.10	"
15	"	"	.31	1.4	"	1.25	"
16	"	"	.34	1.5	"	1.4	"
17	"	"					
18	"	"	.39	1.55	.25	1.4	.4
19	"	"	.44	1.7	"	1.5	"
20	"	"	.44	1.7	"	1.55	"
29	.3	.6	.27	1.4	"	1.25	"
30	"	"	.30	1.45	"	1.2	"

Dam. 2' hig



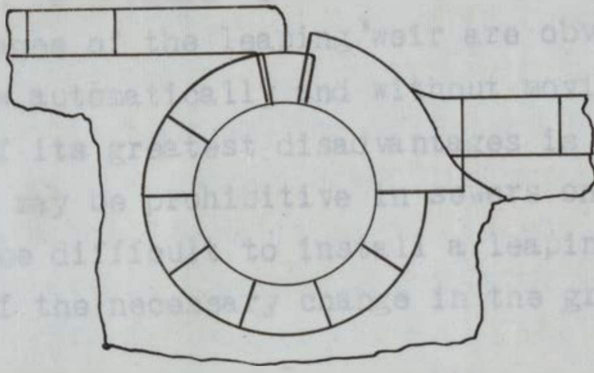


SKETCH



CURVES OF DISCHARGE FOR LEAPING WEIRS

FIGURE 3



BINNIE SEPARATING WEIR
BRADFORD, ENGLAND
WATERWORKS

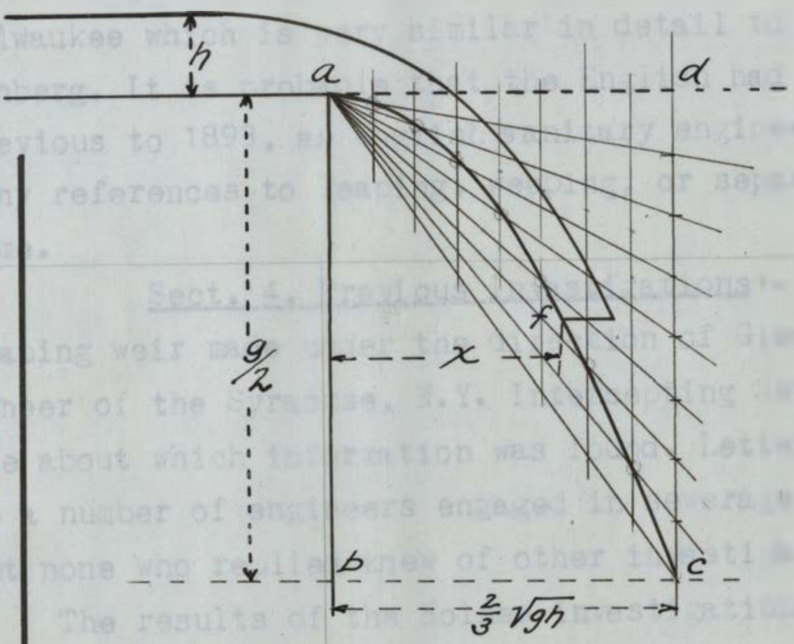


DIAGRAM
for
UNWIN'S ANALYSIS
of
LEAPING WEIR

FIGURE 4

interception. Such a device installed by Sir A.R. Binnie at Bradford England is shown in Figure 4.

The advantages of the leaping weir are obvious, but the fact that it operates automatically and without moving parts should be emphasized. One of its greatest disadvantages is the amount of head consumed, which may be prohibitive in sewers on flat grades. It would probably be difficult to install a leaping weir in an existing sewer because of the necessary change in the grade of the storm water outlet.

Sect. 3. Historical Resumé:-Leaping weirs were probably installed in the United States for the first time in Milwaukee, Wis. These weirs are mentioned by Benzenberg in Transactions of the American Society of Civil Engineers, Volume XXX, pp. 367 and 711 (Nov. 1893) Figure 2 shows the details of one of the exisitng weirs at Milwaukee which is very similar in detail to those described by Benzenberg. It is probable that the English had installed such devices previous to 1893, as English sanitary engineering literature has many references to leaping, weeping, or separating weirs before that time.

Sect. 4. Previous Investigations:- An investigation of a leaping weir made under the direction of Glenn D. Holmes, Chief Engineer of the Syracuse, N.Y. Intercepting Sewer Board was the only one about which information was found. Letters of inquiry were sent to a number of engineers engaged in sewerage design and construction but none who replied knew of other investigations.

The results of the Holmes investigations are published in "American Sewerage Practice" Volume I, by Metcalf and Eddy. The investigation was of a special form of weir, and its results are not extensive enough for general application. A copy of the results of the original observations and a sketch of the apparatus used is given in Figure 3.

The best known method in use by engineers for the design of leaping weirs is that credited to Professor W.C. Unwin. As quoted from the first edition of "Sanitary Engineering" by E.C.S. Moore, it is as follows:

".....This (the action of the weir) is effected by the velocity imparted to the water discharged over a weir causing it to follow a parabolic path; the distance the water is projected depends on the depth of water on the weir and the consequent amount of velocity. In Figure 4 let h be the head of water discharging over a weir, then, according to Professor Unwin, it is sufficiently accurate for practical purposes to assume the mean velocity of the water passing over the weir equals $2/3\sqrt{2gh}$. Then if x equal the width of the orifice ef, and y equal the difference of level ae of the two edges, and if a particle passes from a to f in t seconds, then

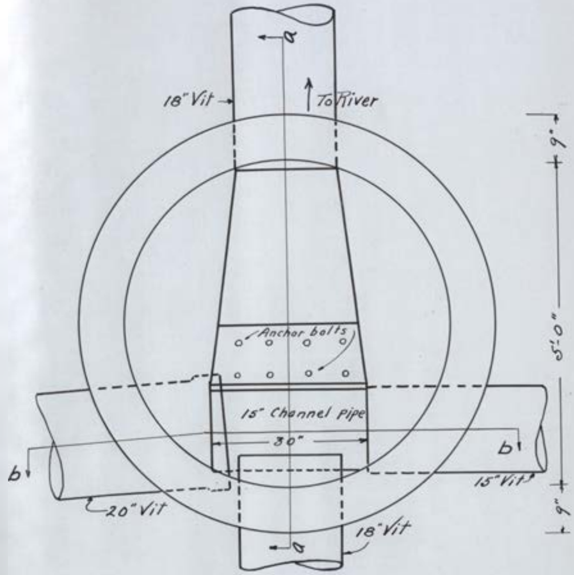
$$y = 1/3\ gt^2$$

$$x = 2/3\sqrt{2gh}\ t$$

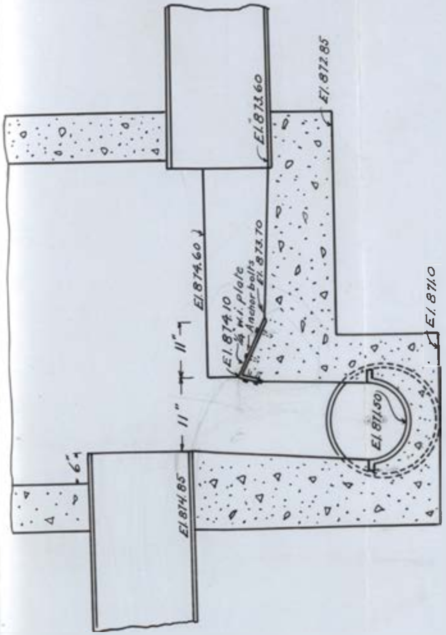
$$\text{Therefore } y = \frac{9}{16}\frac{x^2}{h}$$

This gives the width for any difference of level which the jet will just pass over with a head h. If in addition there is a velocity of approach h must include the head necessary to give that velocity. which is, $\frac{v^2}{2g}$ if v is the velocity of approach in feet per second. In order to describe the path of the jet, set off ab vertically to 1/2 g on any scale; and bc horizontally equal to $2/3\sqrt{gh}$; divide ad and dc into an equal number of equal parts, join a with the divisions on dc, and verticals through the divisions on ab, the intersections of these lines will give the parabolic path of the underside of the jet."

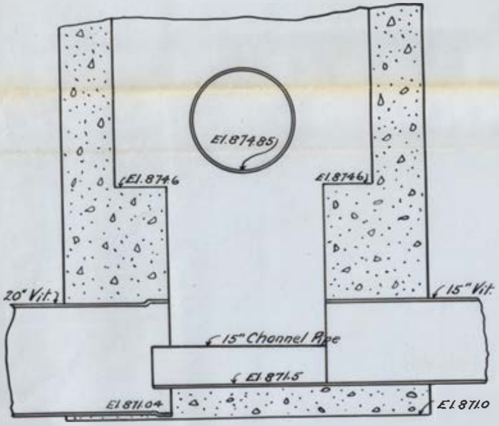
The results of the tests in the laboratory of the University of Illinois which are reported herein, led to the conclusion that this method is not correct. It is based on fallacies in the assumptions that first, the path of the center of gravity of the falling stream is a parabola with a vertical axis and with its apex at the point of leap, second that all of the particles in the stream describe the same path, and that the slope of the approaching stream can be neglected. The coordinates of the points of the falling stream in different tests have been plotted on logarithmic paper in Figures 11, 12 and 13 for the outside of the jet, in Figures 18, 19 and 20



Plan



Sec a-a



Sec b-b

Leaping Weir at Blackhoof St

FIGURE 5

CITY OF WAPAKONETA OHIO

INTERCEPTING SEWER

LEAPING WEIRS

SCALE ¾ IN = 1 FT

Traced from design by A.E. Kimberley, Consulting Eng'r
Columbus, Ohio

for the inside of the jet, and in Figures 24, 25, and 26 for the 'middle' curve or unbroken surface on the underside of the jet. The slope of these lines shows that none of the particles on these surfaces follows the path of a common parabola in its fall. The different slopes of the plotted lines on the figures mentioned shows the effect of both the velocity of approach and the slope of the approaching stream.

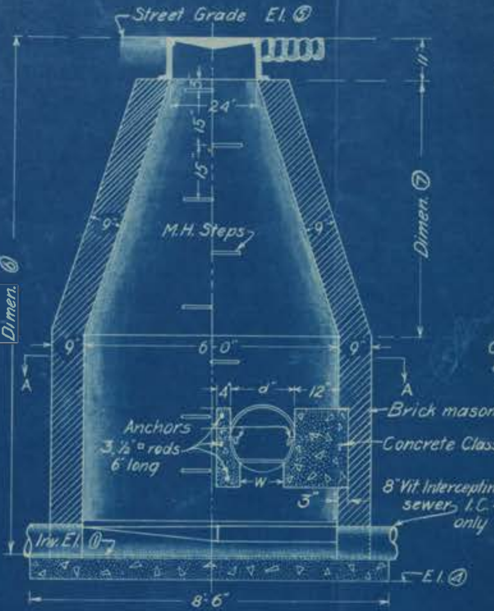
The design of a weir on this basis, particularly if the fall is greater than twelve to eighteen inches is likely to lead to markedly erroneous conclusions. Table I has been prepared to show the discrepancy between this method and the actual observations.

TABLE I

COMPARISON OF UNWIN'S LEAPING WEIR FORMULA
AND
DIRECT OBSERVATIONS

Run Number	Depth of flow in pipe, in feet.	Velocity of Approach Feet per Second	Velocity Head. Feet.	h Feet	VALUES OF X					
					Y = 1.0'			Y = 3.0'		
					UNWIN	Observed		UNWIN	Observed	
						Inside	Middle		Inside	Middle
25	0.12	1.66	.043	0.16	0.53	0.34	0.42	0.92	0.91	1.03
28	0.14	2.04	.065	0.21	0.61	0.25	0.49	1.06	0.85	1.13
214	0.20	3.30	.170	0.37	0.81	0.41	0.60	1.40	1.05	1.41
7	0.98	4.65	.338	1.32	1.53	1.07	----	2.64	1.92	----
19	0.94	4.05	.256	1.20	1.46	1.01	----	2.52	1.80	----
29	0.85	6.29	.617	1.47	1.61	1.24	----	2.78	2.00	----
195	1.20	5.55	.482	1.68	1.73	1.45	1.59	2.99	2.16	2.49
9	0.61	3.73	.218	0.83	1.21	0.79	----	2.10	1.59	----
206	0.63	4.40	.303	0.93	1.28	0.82	1.02	2.21	1.49	1.86
217	0.55	6.35	0.63	1.18	1.45	0.95	1.18	2.51	1.15	2.15

Sect. 5. Existing Installations:- The fact that existing installations designed by the Unwin method are reported as giving satisfactory results is probably due to the uncertainty concerning the amount of dilution necessary before the mixture of rain water and sewage may be discharged untreated without causing a nuisance. The British Royal Commission recommended a dilution of five to one, that is, the storm flow was to be six times the dry weather flow



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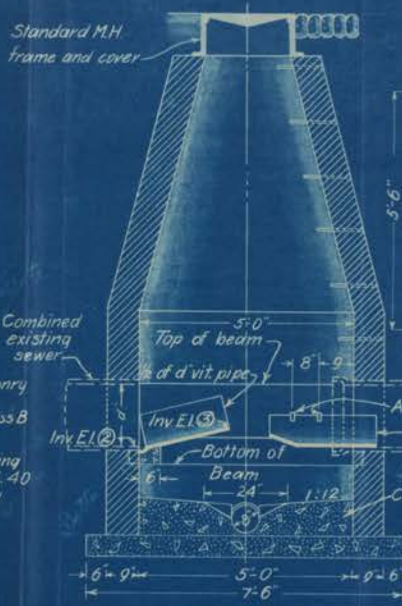
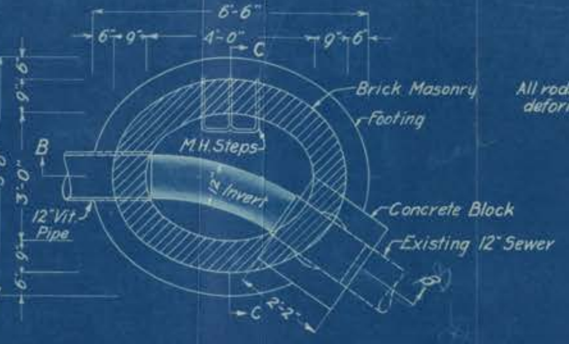
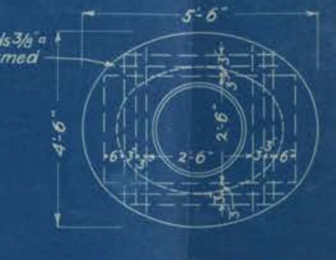


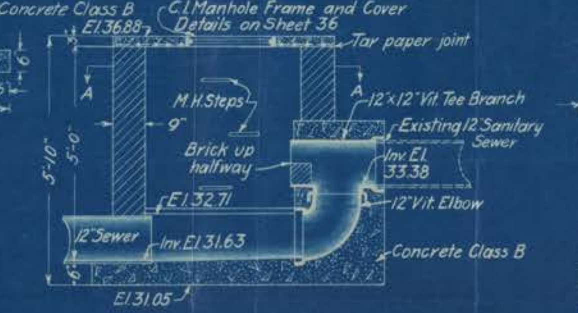
SECTION C-C



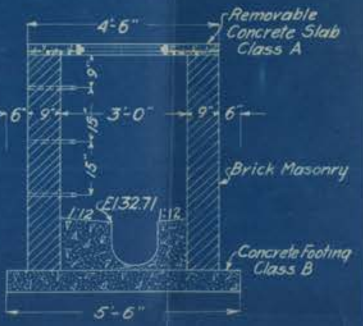
SECTON A-A



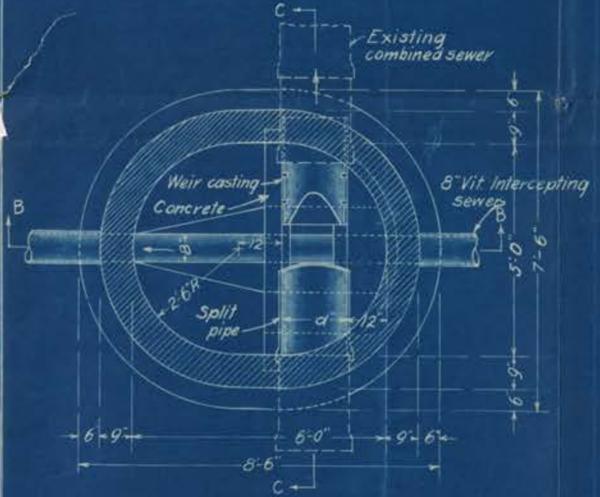
REINFORCEMENT PLAN



SECTION B-B



SECTION C-C



SECTIONAL PLAN A-A

SCALE IN FEET

DETAILS OF MANHOLE 61

TABLE OF DIMENSIONS FOR TYPE "C" I.C.

I.C. No.	W	d	Elevations					Dimensions	
			1	2	3	4	5	6	7
40	9"	12"	29.16	31.14	31.47	28.60	36.36	7'-2⅜"	4'-0"
41	9"	15"	33.25	34.10	34.52	32.69	44.0	10'-9"	6'-0"

DETAILS OF TYPE "C" I.C.

Figure 6

SYRACUSE INTERCEPTING SEWER BOARD

ADVANCE PRINT
CONTRACT 25

INTERCEPTING CHAMBERS

Glenn D. Holmes
Chief Engr

before it could be passed untreated into a stream.

There are many leaping weirs in existence. Figure 5 shows one of the weirs at Wapkoneta, Ohio, designed by A.E. Kimberley, by the 'Unwin' method, and reported as giving satisfaction. Figure 2 shows one of the weirs at Milwaukee. This is probably the earliest of the leaping weirs installed in the United States. Figure 3 shows, in diagrammatic form, the experimental weir used at Syracuse, N.Y. and Figure 6 shows one of the weirs installed on the basis of these tests. Figure 4 shows one of the early water supply weirs at Bradford, England.

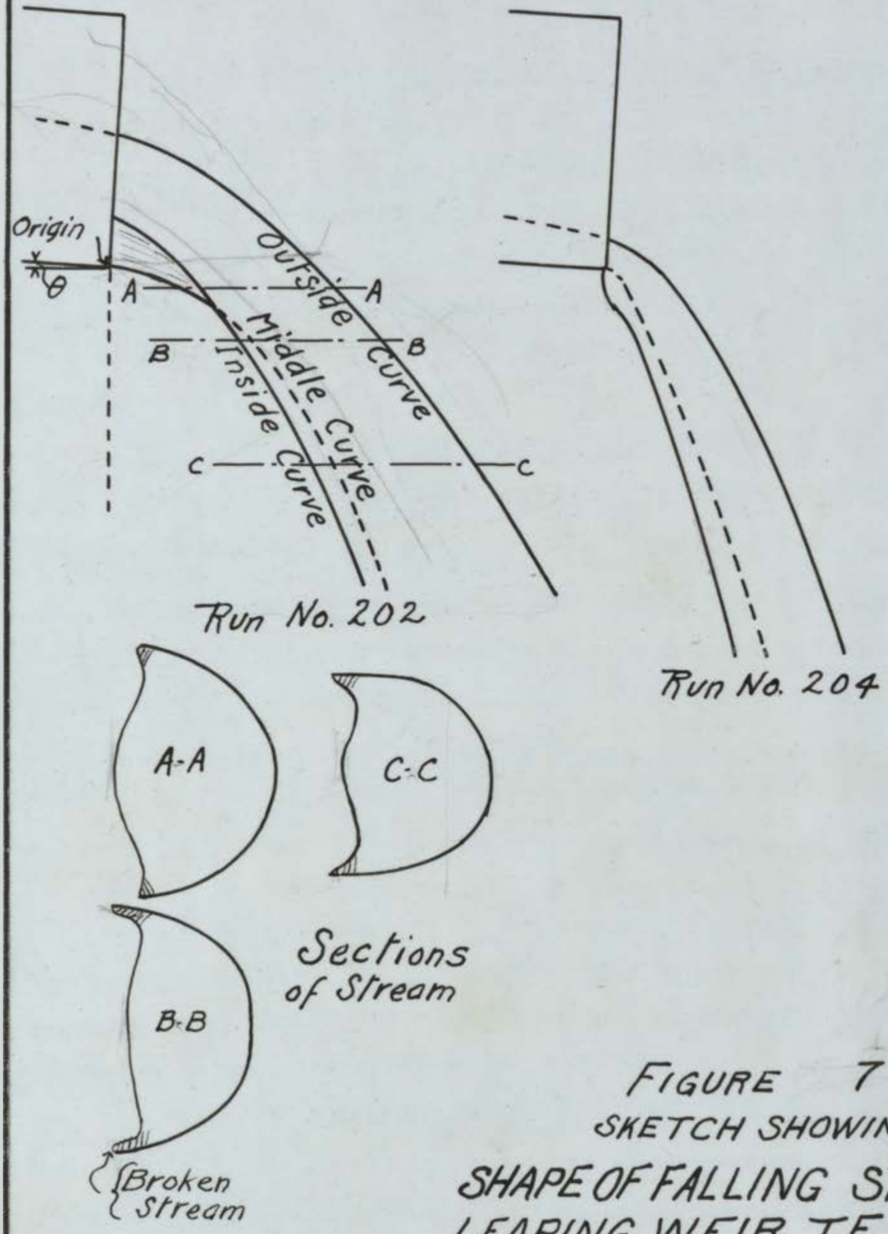


FIGURE 7
SKETCH SHOWING
SHAPE OF FALLING STREAM
LEAPING WEIR TESTS

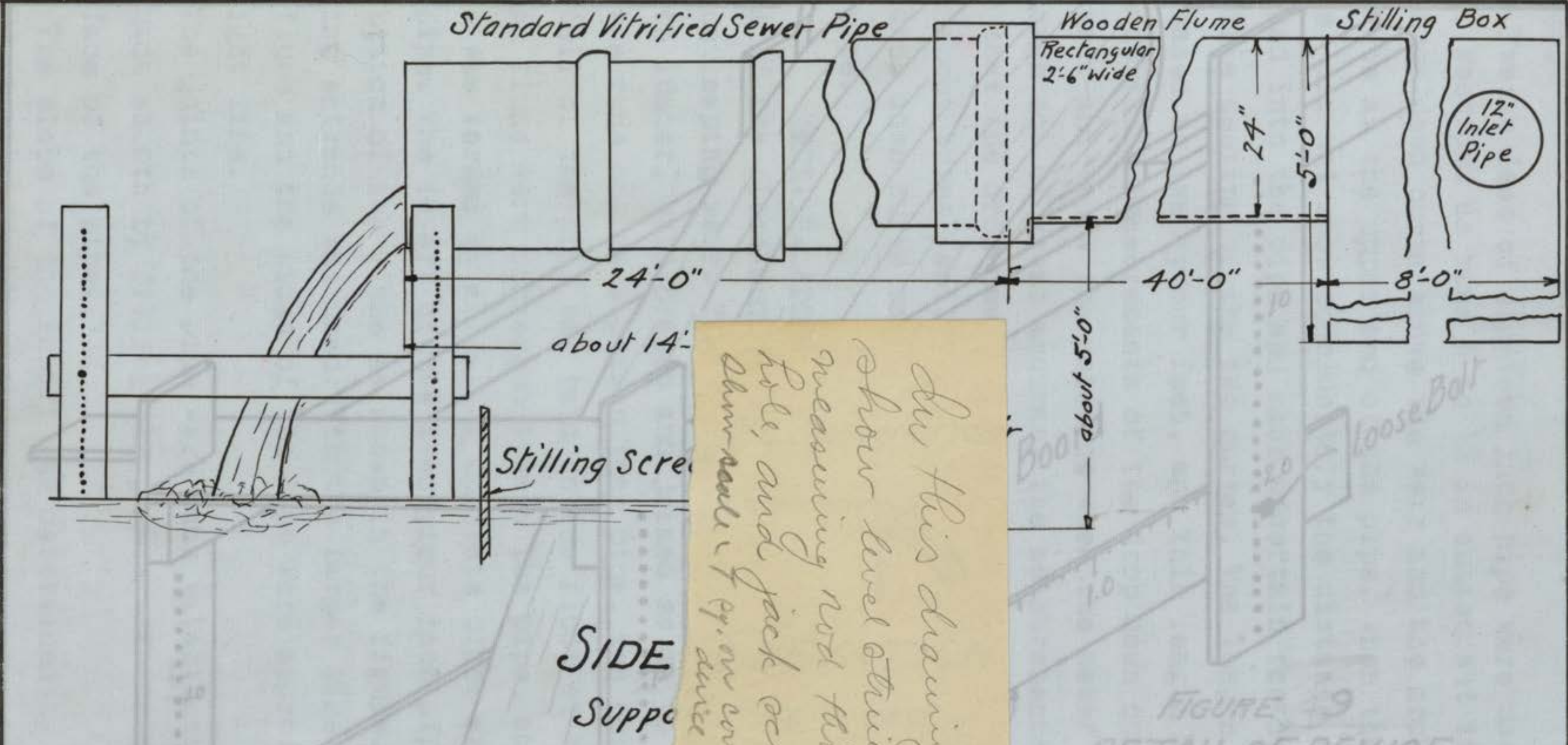
CHAPTER II

----:INVESTIGATIONS OF LEAPING WEIRS AT UNIVERSITY OF ILLINOIS:----

Sect. 6. Period Covered by Experimental Work:- The experimental work on the leaping weir was begun in May 1916. The first preliminary run was made on July 13, 1916, the interim being occupied with setting up the apparatus. The first run, the results of which are included in the following work was made on July 20. From then until November 18, the laboratory work was pushed vigorously. The latter part of the period was devoted to observations on overflow weirs as well as leaping weirs.

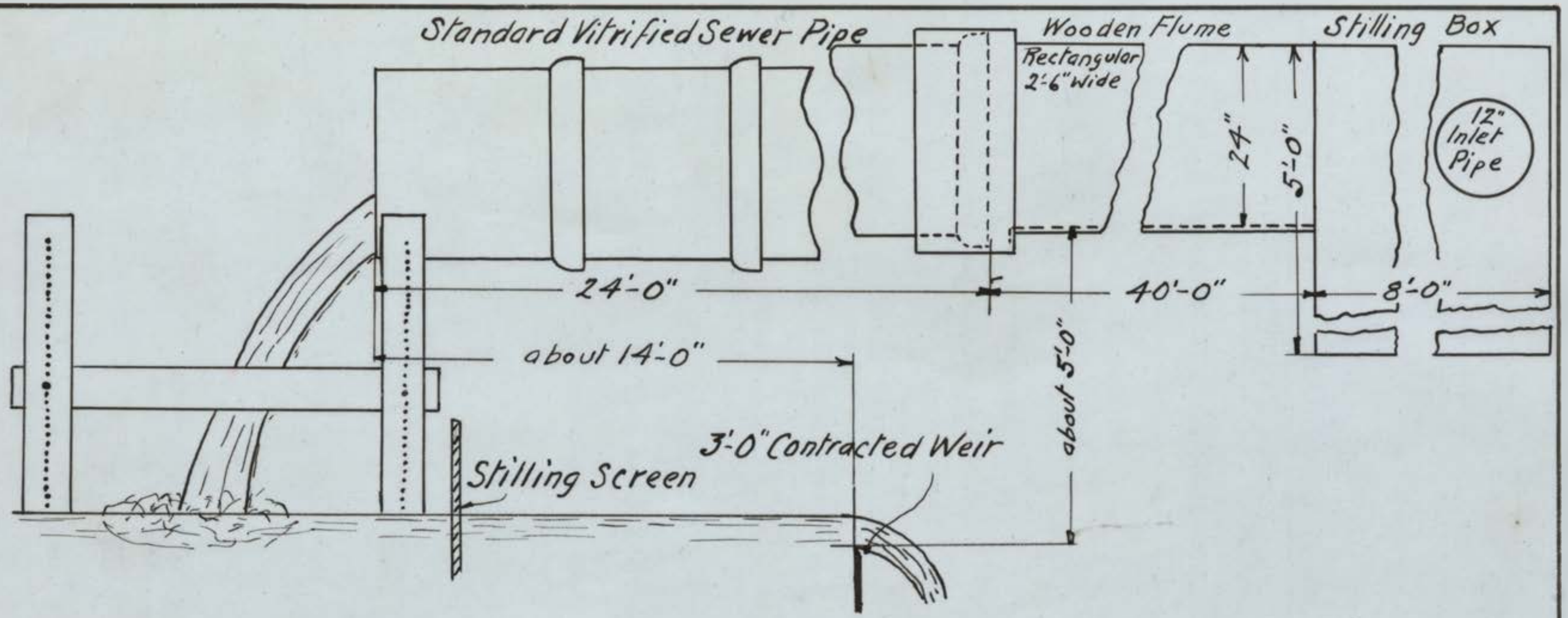
Sect. 7. Tests and Observations:- The type of leaping weir tested was that formed by the spigot end of a standard vitrified sewer pipe. The original plan was to measure the coordinates of certain points on the upper and lower surfaces of the stream leaping from such a pipe, and to express these coordinates in terms of the known conditions of the run. In this manner it was hoped to obtain a general expression which would make possible the location of the 'lower lip' of the weir for any required conditions of separation. It was found however, that the inside surface of the stream was extremely rough and broken, (see Figures 1 and 7) that is, the falling stream did not remain solid. As a result three curves were measured: the inside curve which represents the innermost line of drops of any consequence; the middle curve which represents the inner edge of the unbroken stream; and the outside curve which represents the smoot unbroken surface of the upper portion of the stream. In the first few inches of fall the inside and middle curves are coincident.

It was possible to observe the points on the outside curve with the greatest accuracy because of the smoothness of the surface. The coordinates of the other two curves could be approximated only to the nearest tenth to two tenths of a foot, because of the broken and rough condition of the stream. Even on the outside curve surging



In this drawing (Fig 8) show level string above pipe, measuring rod thrust thru a hole, and jack screw supports Show scale & fig. on coordinate measuring device & mark adjustable boards.

FIGURE 8
SKETCH SHOWING
APPARATUS
for
LEAPING WEIR
TESTS



SIDE ELEVATION

Supports Removed

FIGURE 8
SKETCH SHOWING
APPARATUS
for
LEAPING WEIR
TESTS

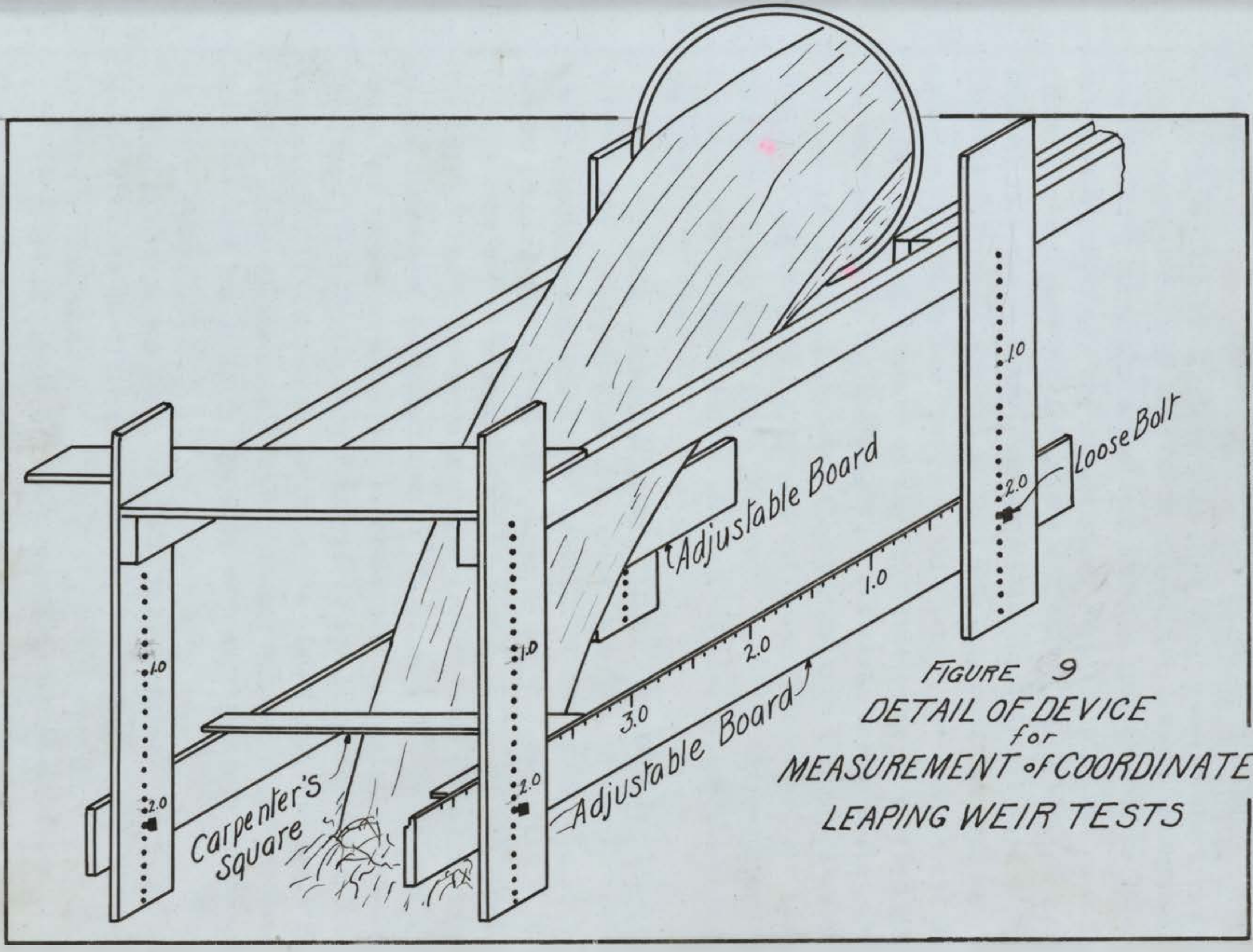


FIGURE 9
DETAIL OF DEVICE
for
MEASUREMENT of COORDINATES
LEAPING WEIR TESTS

waves and changes in the stream caused variations of 0.05 to 0.10 of a foot.

Twelve feet of eighteen inch pipe were used for the first ten runs, recorded as runs 6 to 15 in subsequent tables. Measurements of the drop down curve above the weir and the drop down curve below the entrance at the upper end of the pipe, when the pipe was on a flat grade, did not show conclusively the distance which the two curves reached into the pipe and made uncertain the determination of a possible meeting of the two curves. The length of the pipe was then increased to twentyfour feet, and this length was used in all subsequent runs. Measurements of the drop down curve were taken above the weir and below the entrance when the sewer was on a flat grade. Although not of great accuracy the measurements were sufficient to show that the drop down curve did not extend above the weir more than about three feet, which was well below the lowest point due to the drop down curve caused by the loss of head at the entrance of the pipe.

Sect. 8. Apparatus Used.- Figure 8 shows some of the details of the flume and pipe line which were used to convey the water to the leaping weir. The flume was built of one inch, matched, smooth lumber, well braced and joined so as to permit little leakage. For the runs on the eighteen inch pipe, the bottom of the outside of the bell of the pipe was laid on the floor of the flume, the sides of the flume were converged toward the pipe, and a rather abrupt bell mouth was formed by a flaring concrete block around the entrance to the pipe. The invert of the twentyfour inch pipe was laid flush with the bottom of the flume as shown in the figure. No bell mouth or converging entrance was used with the larger size of pipe. The sides of the flume and the sides of the pipe were approximately in the same straight line.

The joints of the pipe were made watertight and the inner surface was made smooth by filling the joints with cement, flush with the inner face of the pipe.

The slope of the invert was determined by measurements taken from a level string suspended above the pipe, by means of a measuring stick passed through holes in the pipe. These holes were some distance apart, and to make certain that the grade of the invert was smooth a small stream was run through the pipe and discrepancies in the grade were noted by the changes in the width and appearance of the stream. The slope of the invert was changed by means of screw jacks placed permanently as supports under the pipe. A relatively large change of the jack screw would make but little change in the elevation of the pipe above it, and because of the manner of support the pipes were maintained in good alignment both horizontally and vertically.

The coordinates of the leaping stream were measured by means of the apparatus shown on a large scale in Figure 9. The origin of coordinates was taken as the lip of the weir. The abscissas were measured out from a plumb line hung from the 'origin', the 'zero' being located on the scale of the movable board shown. Figure 1 is a photograph of the apparatus in action showing a portion of the measuring device in place.

The quantity of water passing over the leaping weir was measured on a standard weir placed in the laboratory weir channel. Different sizes of weirs were used in accordance with the amount of water being used. The weirs varied from one to three feet in length, with the maximum head on any weir not exceeding about fifteen inches. Both suppressed and contracted weirs were used. Because of the large rates of flow and the difficulty of maintaining air under the suppressed weir with high rates of flow, the three foot contracted weir was used more than any other.

The water was obtained from a sump in the basement of the laboratory and was raised by either one or both of two pumps. One of these pumps was a steam driven direct acting duplex pump and the other a centrifugal pump belt driven from a simple Corliss engine. Both pumps were driven to the limit of their capacity during various runs. The highest quantity obtained at any time with both pumps going was about six thousand gallons per minute during run number 490. The discharge from these two pumps passed through a twelve inch pipe to a stilling box (See Fig8) at the upper end of the flume. Adequate covering and baffles wereplaced on and in the box to prevent splashing and to force the water to issue into the flume in a fairly quiescent condition. A valve was placed on the twelve inch pipe immediately above the stilling box and variations in the rate of flow in the pipe were obtained partly by manipulation of this valve and partly by altering the speed of the two pumps. The rate of flow during a run was maintained constant by means of a connection between the discharge from the pumps and the standpipe in the laboratory in which a constant level was maintained by a Fisher automatic governor connected to the steam pipe on the steam pump. A measurement of the rate of flow through the apparatus was made after every coordinate observation in order to make sure that the conditions had not changed during the run. After the water had passed over the standard measuring weir it was led to the sump and used over again.

The depth of water in the sewer for different rates of flow was observed. The degree of accuracy of the depth measurements was low because of the roughness of the surface, the presence of standing waves and the occassional occurrence of the phenomenon of the hydraulic jump. Constant vigilance was necessary to guard against the presence of the latter condition during a run. The jump was easily broken up by placing anobstruction in the pipe until the water had backed up considerably and then suddenly removing the obstruction. Except for very low rates of flow the depth measurements were accurate only to the nearest 0.05 foot.

Sect. 9. Making A Run:- The order of procedure in making a run was as follows:

Adjust the coordinate measuring device and measure the zero of coordinates. Then remove the plumb bob from interference with further observations.

Take measurements from the level line down to the invert of the pipe to determine the slope, and adjust by means of jacks if not correct. Run a thin stream of water through the pipe to make sure that the alignment is good and adjust if unsatisfactory.

Start the steam pump pipe and fill the standpipe until the governor shut off the pump. Prime the centrifugal pump. Open the valve into the stilling box and start the centrifugal pump. Adjust the two pumps to about the proper speed with the valve above the stilling box at such an opening as to give the desired rate of flow over the leaping weir. The pumps were allowed to run for from two to five minutes until it was certain that there were no great fluctuations in the conditions.

Read the hook gage on the standard weir.

Read the coordinates to some point on the curve under observation and continue to alternate this and the preceding step until four or five points have been read on each curve.

Read the depth of the w ater at the lip of the weir.

Read the depth of water in the pipe.

It is believed from the results calculated from these observations that the greatest error in this process was in readings of the slope of the invert of the pipe. A relatively small variation in the slope will make a marked change in the coordinates of a point in the falling stream.

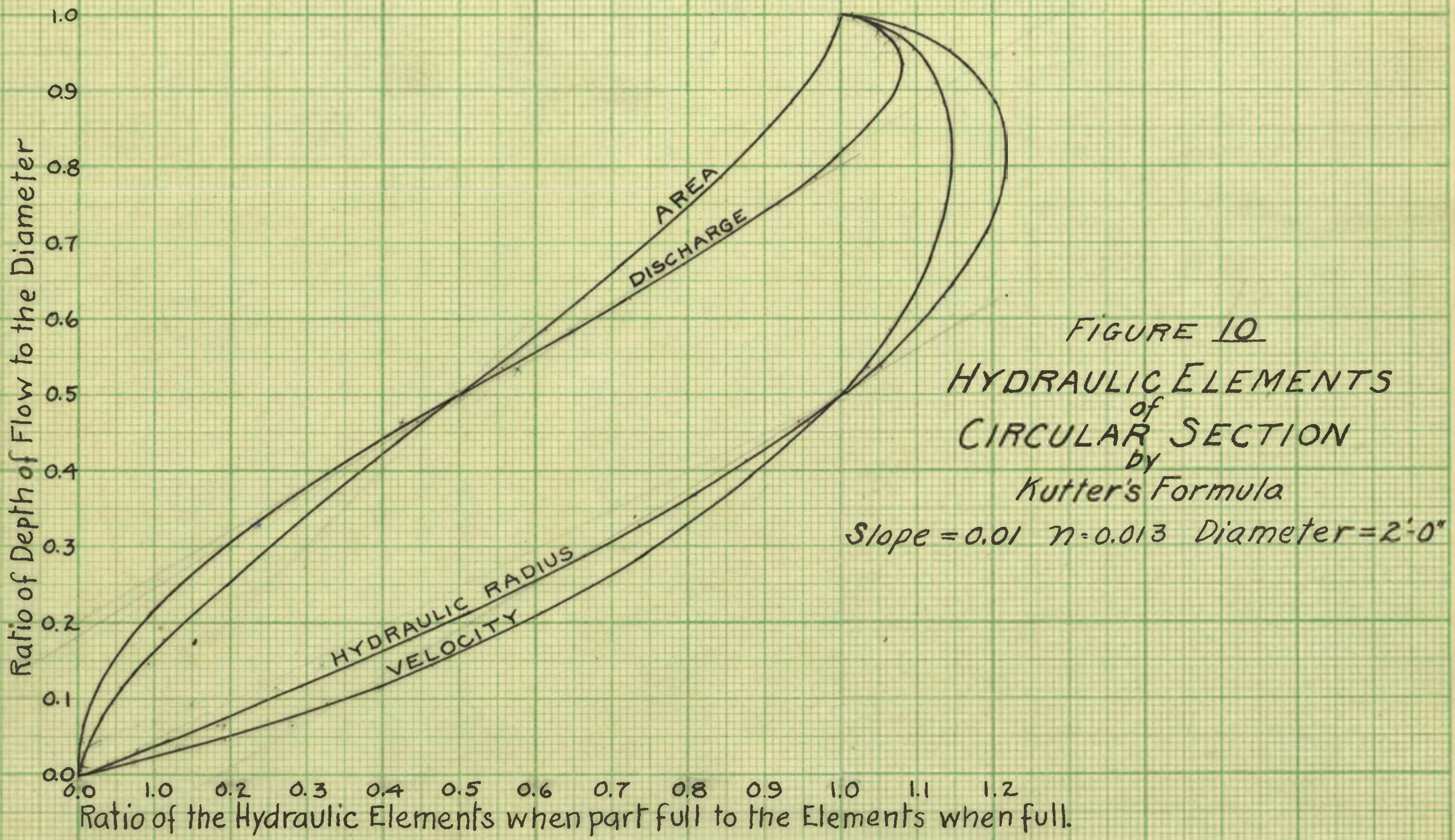


FIGURE 10

HYDRAULIC ELEMENTS
of
CIRCULAR SECTION
by
Kutter's Formula

Slope = 0.01 n = 0.013 Diameter = 2'-0"

CHAPTER III

----:RESULTS COMPUTED FROM OBSERVATIONS:----

Sect. 10. Direct Observations:- The slope of the invert and the depth of water on the lip of the weir and in the sewer are recorded in TableII. The coordinates of the points on the falling stream are recorded in Table III. These tables are made up from the direct observations recorded in the note book, by making the proper correction for the observed 'zero'.

Sect. 11. Rate Of Flow:- The rate of flow through the sewer was computed by applying the appropriate Francis weir formula to the observations made. The computation of the rate of flow for run number 29, is given as an example, as follows:

The hook gage reading on the three foot contracted weir was 1.295 feet. The zero reading on the gage was 0.767 feet. The difference between these two values was substituted for h in the expression $Q = 3.33(l - 0.2h)h^{3/2}$ The value of Q in this case is 6.38 cubic feet per second.

Sect. 12. Value of n In Kutter's Formula:- Before proceeding with velocity computations it was necessary to compute the exact depth of flow, the hydraulic radius, and the area of the cross section of the stream, for each run. The depths of flow as observed were too inaccurate for direct use, due to the rough condition of the surface of the flowing stream. The procedure followed was to compute the average value of n from the rate of flow and the observed depth of water in the pipe for all runs. Having obtained an average value of n by this means, the depth of flow for the individual runs was computed, and the computed depths were used in subsequent calculations, not the observed depths.

The ratio of the observed depth of flow to the diameter of the pipe was first computed. The other hydraulic elements were then read from Figure 10. The velocity of flow was then found by dividing the rate of flow by the area of the cross section of the stream. The velocity, hydraulic radius and slope were then substituted in Kutter's formula and n solved for directly. The average of all of these computations is shown for each slope in Table IV. The final average of all results, and the figure that was used throughout all the subsequent computations was that n = 0.013

TABLE IV

------:VALUE OF n IN KUTTER'S FORMULA DETERMINED:----

from

------- :OBSERVATIONS OF DEPTH OF FLOW:------

in

------:LEAPING WEIR EXPERIMENTS:------

Diameter of pipe in inches	18			24			
Slope	.007	.010	.018	.005	.006	.010	.014
Average value of n	.012	.013	.012	.011	.014	.016	.015

Sect. 13. Calculated Depth of Flow:- The depth of flow calculations are probably more accurate than the observations of the depth of flow, because the calculations represent the average of all of the observations. The computations were made as follows: A table was made up showing the rate of flow in eighteen and twenty-four inch pipes when flowing full on different slopes, as read from Swan and Horton's Hydraulic Diagrams, and checked by computations using Kutter's formula.

For any particular run the ratio between the flow when part full and the flow when full was next computed, and the ratio of the depth of flow to the diameter of the pipe was read from Figure 10. The actual depth of flow was then the product of this ratio and the diameter of the pipe. In a few cases the depth calculated in this manner was checked by a determination of the hydraulic radius and a direct substitution in Kutter's formula. In no case was a material discrepancy found between the results read from Figure 10 and the results as computed from Kutter's formula.

The results of the depth of flow computations are given in Table I.

Sect. 14. Velocity Heads.- The velocity head for each run was computed in order to determine the possible relation between the coordinates of the stream and the velocity head. The fact that the lines in Figures 11, 12, 13, 18, 19, 20, 24, 25, and 26 are not parallel indicates conclusively, as was shown on page 6, that the velocity head cannot act as is suggested in Unwin's analysis.

TABLE II

MISCELLANEOUS DATA FOR LEAPING WEIRS

RUN NUMBER	Slope of Invert	Discharge Cu.Ft. per Second	Depth of Flow in Feet		Velocity Feet per Second
			Observed	Calculated	
		18 inch pipe			
6	.005	1.356	---	0.44	3.20
7	.005	5.54	---	0.98	4.65
8	.005	4.02	---	0.79	4.37
9	.005	2.55	---	0.61	3.73
10	.005	.479	---	0.26	2.30
11	.023	6.33	---	0.66	8.40
12	.023	5.51	---	0.61	8.05
13	.023	4.33	---	0.54	7.91
14	.023	2.40	---	0.40	6.46
15	.023	0.66	---	0.22	4.16
16	.004	2.09	.76	0.59	3.27
17	.004	0.78	---	0.35	2.47
18	.004	.085	.25	0.14	1.12
19	.004	4.66	---	0.94	4.05
20	.004	3.76	.97	0.82	3.92
21	.007	6.21	.83	0.95	5.34
22	.007	3.98	.70	0.71	4.78
23	.007	1.93	.44	0.48	3.92
24	.007	0.80	.35	0.31	2.96
25	.007	.083	.12	0.12	1.66
26	.010	0.66	.25	0.27	3.28
27	.010	1.72	.44	0.41	4.33
28	.010	0.13	.14	0.14	2.04
29	.010	6.38	.81	0.85	6.29
30	.010	4.07	.65	0.56	5.49
31	.018	4.02	.54	0.55	6.77
32	.018	6.29	.75	0.70	7.71
33	.018	1.68	.33	0.35	5.30
34	.018	0.71	.21	0.24	4.55
35	.018	.076	.08	.13 0.25	2.45

RUN NUMBER	Slope of Invert	Discharge Cu.Ft. per Second.	Depth of Flow in Feet		Velocity Feet per Second
			Observ'd	Calculat'd	
		24 inch pipe			
195	.005	10.80	---	1.20	5.55
196	.005	6.48	0.33	0.92	4.91
197	.005	7.60	0.92	0.90	4.79
198	.005	4.64	0.69	0.74	4.21
199	.005	3.06	0.56	0.59	4.05
200	.005	1.62	0.42	0.43	3.19
201	.006	10.79	1 13	1.13	5.90
202	.006	9.14	0.99	1.03	5.55
203	.006	6.84	0.92	0.87	5.18
204	.006	0.585	0.25	0.27	2.40
205	.006	2.22	0.50	0.48	3.80
206	.006	3.75	0.63	0.63	4.40
207	.006	5.64	0.76	0.78	4.88
208	.009	0.514	0.23	0.25	2.64
209	.009	2.14	0.46	0.42	4.28
210	.009	3.86	---	0.57	5.17
211	.009	6.34	0.82	0.74	5.97
212	.009	8.51	1.06	0.87	6.40
213	.009	11.58	1.21	1.05	7.10
214	.014	0.364	0.15	0.20	3.30
215	.014	1.22	0.29	0.30	4.10
216	.014	3.17	0.51	0.46	5.70
217	.014	4.44	0.63	0.55	6.35
218	.014	8.10	0.94	0.75	7.53
219	.014	11.48	1.07	0.91	8.20

TABLE III

COORDINATES OF POINTS OBSERVED ON LEAPING WEIR

Distances in feet

RUN NUMBER	Coordinates from Lip of weir			
	Y	X		
	All Curves	Inside	Middle	Outside
6	0.71	0.53	----	1.24
	0.91	0.65	----	1.37
	1.21	0.77	----	1.54
	1.71	0.92	----	1.79
	2.21	1.10	----	2.01
	2.71	1.30	----	2.23
	3.21	1.46	----	2.44
7	2.71	1.87	----	3.18
	2.21	1.65	----	2.91
	1.71	1.40	----	2.63
	1.21	1.15	----	2.33
	0.71	0.89	----	1.93
8	0.71	0.89	----	1.64
	1.21	1.20	----	2.02
	1.71	1.43	----	2.24
	2.21	1.61	----	2.61
	2.71	1.77	----	2.86
9	3.21	1.69	----	2.74
	2.21	1.26	----	2.29
	1.71	1.10	----	2.05
	1.21	0.90	----	1.78
	0.71	0.62	----	1.44
10	0.71	0.36	----	0.88
	1.21	0.56	----	1.15
	1.71	0.75	----	1.37
	2.21	0.81	----	1.54
	3/21	1.14	----	1.85
11	0.18	0.35	----	1.80
	0.48	0.78	----	2.12
	0.98	1.40	----	2.60
	1.48	1.86	----	3.01
	1.98	2.12	----	3.37
	2.48	2.38	----	3.63
12	2.48	2.03	----	3.46
	1.98	1.77	----	3.10
	1.48	1.51	----	2.74
	0.98	1.25	----	2.35
	0.48	0.73	----	1.78
	0.18	0.33	----	1.57

RUN NUMBER	Coordinates from Lip of Weir			
	Y	X		
	All Curves	Inside	Middle	Outside
13	0.18	0.31	----	1.38
	0.48	0.73	----	1.71
	0.98	1.29	----	2.15
	1.48	1.41	----	2.55
	1.98	1.57	----	2.87
	2.48	1.88	----	3.18
14	2.48	1.5	----	2.77
	1.98	1.3	----	2.50
	1.48	1.16	----	2.21
	0.98	0.9	----	1.85
	0.48	0.63	----	1.38
	0.18	0.30	----	1.22
15	0.18	0.26	----	0.78
	0.48	0.4	0.63	1.07
	0.98	0.65	1.0	1.48
	1.48	0.86	1.26	1.79
	1.98	1.07	1.47	2.06
	2.98	1.51	----	2.49
16	0.49	0.59	0.74	1.17
	0.99	0.84	0.99	1.50
	1.49	0.95	1.30	1.81
	1.99	1.26	1.46	2.18
	3.49	1.33	1.43	2.70
17	0.49	0.29	0.49	0.83
	0.99	0.54	0.59	1.11
	1.49	0.70	0.80	1.36
	2.49	1.02	1.17	1.76
	3.49	1.33	1.43	2.07
18	3.49	0.93	----	1.26
	2.49	0.72	----	1.07
	1.49	0.45	----	0.82
	0.99	0.24	----	0.66
	0.49	0.04	----	0.46
19	0.49	0.69	----	1.54
	0.99	0.94	----	1.96
	1.49	1.25	----	2.30
	2.49	1.62	----	2.92
	3.49	1.93	----	3.38

TABLE III
(continued)

RUN NUMBER	Coordinates from Lip of Weir			
	Y	X		
	All Curves	Inside	Middle	Outside
20	3.49	1.88	----	3.15
	2.49	1.52	----	2.67
	1.49	1.05	----	2.11
	0.99	0.89	----	1.79
	0.49	0.64	----	1.41
21	0.53	0.8	----	1.93
	0.73	1.0	----	2.09
	1.23	1.3	----	2.51
	1.73	1.4	----	2.84
	2.73	1.8	----	3.46
22	2.73	1.42	----	3.10
	1.73	1.13	----	2.56
	1.23	0.94	----	2.24
	0.73	0.7	----	1.86
	0.53	0.65	----	1.76
23	0.53	0.5	----	1.28
	0.73	0.6	----	1.42
	1.23	0.84	----	1.77
	2.23	1.2	----	2.30
	3.23	1.4	----	2.78
24	3.23	1.22	1.42	2.27
	2.23	0.9	1.13	1.88
	1.23	0.54	0.7	1.43
	0.73	0.34	0.6	1.13
	0.53	0.3	0.55	1.00
25	0.53	0.05	----	0.58
	0.73	0.2	----	0.67
	1.23	0.4	0.5	0.88
	2.23	0.7	0.83	1.18
	3.73	1.1	1.22	1.54
26	0.57	0.33	0.33	1.09
	0.77	0.33	0.43	1.22
	1.07	0.38	0.58	1.42
	1.57	0.6	0.8	1.68
	2.57	0.85	1.15	2.12
	3.57	1.1	1.45	2.50
27	3.57	1.55	1.9	3.08
	2.57	1.2	1.55	2.61
	1.57	0.9	1.13	2.07
	1.07	0.7	0.9	1.78
	0.77	0.53	0.73	1.55
	0.57	0.5	0.63	1.46
28	0.57	0.13	0.13	0.69
	0.77	0.13	0.3	0.80
	1.07	0.3	0.5	0.95
	1.57	0.4	0.7	1.13
	2.57	0.7	1.0	1.45
	3.57	1.1	1.25	1.73
29	2.57	1.9	2.2	3.51
	1.57	1.4	1.73	2.86
	1.07	1.2	1.33	2.51
	0.77	1.1	1.13	2.24
	0.57	0.9	0.9	2.06
30	0.57	0.8	0.8	1.69
	0.77	0.93	1.03	1.85
	1.07	1.1	1.23	2.10
	1.57	1.2	1.5	2.42
	2.57	1.7	1.94	3.07
31	2.36	1.4	1.9	3.18
	1.36	1.02	1.4	2.50
	0.86	0.76	1.11	2.13
	0.56	0.61	0.86	1.84
32	0.56	0.66	0.91	2.17
	0.86	0.96	1.21	2.47
	1.36	1.22	1.47	2.95
	2.36	1.69	2.04	3.71
33	2.86	1.2	1.75	2.97
	1.86	0.9	1.33	2.41
	1.36	0.72	1.12	2.09
	0.86	0.51	0.81	1.73
	0.56	0.41	0.56	1.46
34	0.56	0.36	0.36	1.17
	0.86	0.41	0.56	1.44
	1.36	0.62	0.82	1.76
	2.36	0.94	1.24	2.30
	3.36	1.2	1.7	2.77
35	3.36	1.1	1.3	1.53
	2.36	0.84	1.02	1.35
	1.36	0.54	0.65	1.03
	0.86	0.44	0.44	0.83
	0.56	0.26	0.26	0.66

1.22
1.08
.97
.92
.80

TABLE III
(continued)

RUN NUMBER	Coordinates from Lip of Weir			
	Y	X		
	All Curves	Inside	Middle	Outside
195	0.63	1.2	1.3	2.32
	0.83	1.3	1.95	2.50
	1.13	1.5	1.6	2.74
	2.13	1.9	2.15	3.41
196	2.13	1.7	1.9	2.96
	1.13	1.25	1.4	2.35
	0.83	1.1	1.2	2.08
	0.63	1.0	1.15	1.91
197	0.63	0.95	1.15	1.87
	0.83	1.1	1.25	2.04
	1.13	1.25	1.45	2.18
	1.63	1.5	1.75	2.56
	2.63	2.0	2.15	3.18
198	2.63	1.55	1.70	2.84
	1.63	1.1	1.25	2.20
	1.13	1.0	1.15	2.05
	0.83	0.9	1.0	1.85
	0.63	0.8	0.95	1.70
199	0.63	0.65	0.8	1.48
	0.83	0.8	0.9	1.62
	1.13	0.9	1.05	1.83
	1.63	1.15	1.3	2.12
	2.63	1.45	1.6	2.59
200	2.63	1.1	1.3	2.18
	1.63	0.8	1.0	1.72
	1.13	0.6	0.73	1.47
	0.83	0.5	0.60	1.30
	0.63	0.5	0.5	1.18
201	2.07	1.9	2.03	3.40
	1.57	1.7	1.8	3.07
	1.07	1.45	1.6	2.82
	0.77	1.3	1.4	2.58
	0.57	1.2	1.2	2.40
202	0.57	1.1	1.15	2.13
	0.77	1.15	1.30	2.30
	1.07	1.3	1.45	2.56
	1.57	1.6	1.75	2.89
	2.57	2.0	2.2	3.53
203	2.57	1.85	2.05	3.27
	1.57	1.5	1.65	2.73
	1.07	1.15	1.35	2.39
	0.77	1.05	1.20	2.17
	0.57	0.9	1.05	2.02

RUN NUMBER	Coordinates from Lip of Weir			
	Y	X		
	All Curves	Inside	Middle	Outside
204	3.07	1.0	1.25	1.85
	2/07	0.7	0.9	1.52
	1.07	0.35	0.55	1.12
	0.77	0.3	0.4	0.96
	0.57	----	----	0.84
205	0.57	0.45	0.6	1.30
	0.77	0.5	0.7	1.44
	1.07	0.65	0.85	1.64
	2.07	1.0	1.2	2.18
	3.07	1.3	1.55	2.52
206	3.07	1.55	1.9	2.94
	2.07	1.2	1.5	2.48
	1.07	0.85	1.05	1.91
	0.77	0.65	0.85	1.68
	0.57	0.55	0.65	1.54
207	0.57	0.85	1.0	1.74
	0.77	0.95	1.1	1.90
	1.0	1.05	1.25	2.11
	1.5	1.25	1.5	2.45
	2.5	1.7	1.9	3.03
208	0.67	0.3	0.4	0.94
	0.97	0.35	0.6	1.14
	1.97	0.6	1.0	1.61
	2.97	1.0	1.35	1.97
209	2.97	1.4	1.7	2.79
	1.97	1.05	1.3	2.29
	0.97	0.65	0.9	1.69
	0.67	0.55	0.7	1.46
210	0.67	0.7	0.9	1.59
	0.97	0.85	1.1	1.90
	1.97	1.25	1.6	2.51
	2.97	1.6	2.0	3.01
211	2.47	1.6	2.0	3.28
	1.47	1.2	1.5	2.66
	0.91	1.0	1.25	2.36
	0.67	0.85	1.05	2.14
212	0.67	1.0	1.3	2.27
	0.97	1.2	1.45	2.50
	1.47	1.5	1.75	2.91
	1.97	1.7	2.0	3.23

TABLE III
(continued)

RUN NUMBER	Coordinates from Lip of Weir			
	Y	X		
	All Curves	Inside	Middle	Outside
213	1.97	2.0	2.25	---
	1.47	1.85	1.95	3.53
	0.97	1.6	1.7	3.18
	0.67	1.4	1.4	2.92
214	2.85	0.95	1.35	2.00
	1.85	0.71	0.96	1.61
	1.35	0.51	0.76	1.41
	0.85	0.37	0.52	1.13
215	0.85	0.50	0.7	1.51
	1.35	0.76	0.96	1.81
	1.85	0.96	1.21	2.12
	2.85	1.22	1.62	2.59
216	2.85	1.55	1.95	3,22
	1.85	1.16	1.46	2.66
	1.35	0.96	1.21	2.35
	0.85	0.72	0.92	1.98
217	0.85	0.87	1.12	2.09
	1.35	1.11	1.36	2.48
	1.85	1.26	1.61	2.80
	2.35	1.55	1.90	3.10
218	2.35	1.95	2.25	----
	1.85	1.66	1.96	3.38
	1.35	1.41	1.66	2.96
	0.85	1.17	1.37	2.67
219	0.85	1.42	1.52	2.95
	1.35	1.76	1/96	3.37
	1.85	1.96	2.16	----

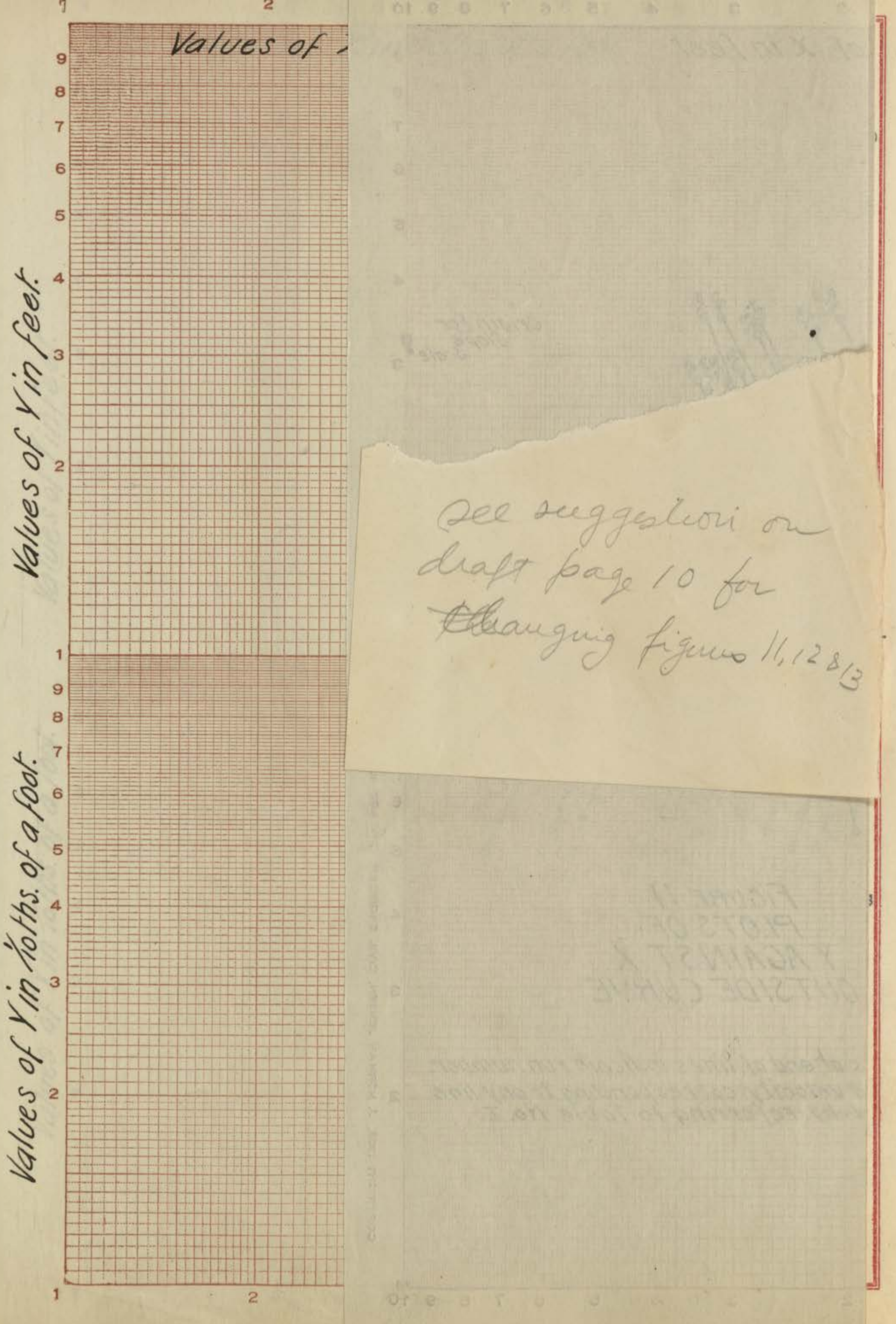
CHAPTER IV

------:DEDUCTION OF FORMULAS:------

Sect. 15. Rational Considerations:- The conditions in a flowing stream are complex in so far as the velocity of individual particles is concerned, as scarcely any two particles may have the same velocity. The average velocity of all the particles may not represent the velocity of any particle. As a flowing stream approaches a jump or free leap, as in this series of tests, the velocity of the stream increases, resulting in a lowering of the depth of flow, so that the actual depth of flow and the velocity at the point of leaping are different from the depth of flow and the velocity in the main channel. Although the individual particles may tend to fall in a parabolic path determined by the horizontal and vertical compenents of their velocity and the action of gravity, the effect of the other particles with different velocities will be to disturb this parabolic path. In consequence it is probable that the upper and lower edges of the falling stream will not remain parallel, and that no particle in the stream will follow a parabolic path unless it be due to such a combination of the various velocities of its neighbors as to render the resultant path of the group that of a parabola.

Assuming, however, that the velocities of all the particles of a flowing stream are the same, that the stream has a constant depth up to the point of leaping, and that the slope of the invert of the conduit is zero, the horizontal distance travelled in the time t will be vt, in which v is the initial horizontal velocity. The product vt is equal to the horizontal coordinate of any point on the falling stream as measured from the point of leaping as the origin. The distance which the same particle will drop in the time t will be $1/2\ gt^2$, which is the vertical coordinate of any point on the falling stream measured from the same origin. Using x and y to represent the values of the coordinates, and eliminating t it is found that $x = 1/4\ vy^{1/2}$ which is the equation of a common parabola, with a vertical axis and its apex in the origin.

The preceding assumption as to a constant depth of flow on a



See suggestion on draft page 10 for ~~Changing~~ figures 11, 12 & 13

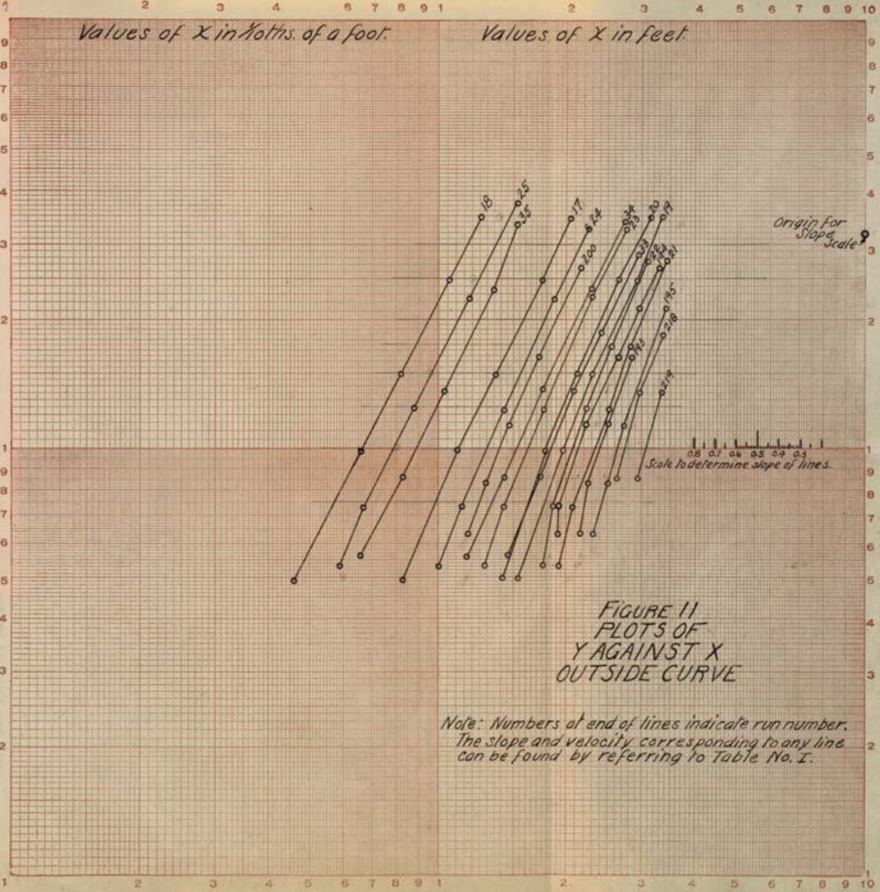


FIGURE 11
PLOTS OF
Y AGAINST X
OUTSIDE CURVE

Note: Numbers at end of lines indicate run number. The slope and velocity corresponding to any line can be found by referring to Table No. I.

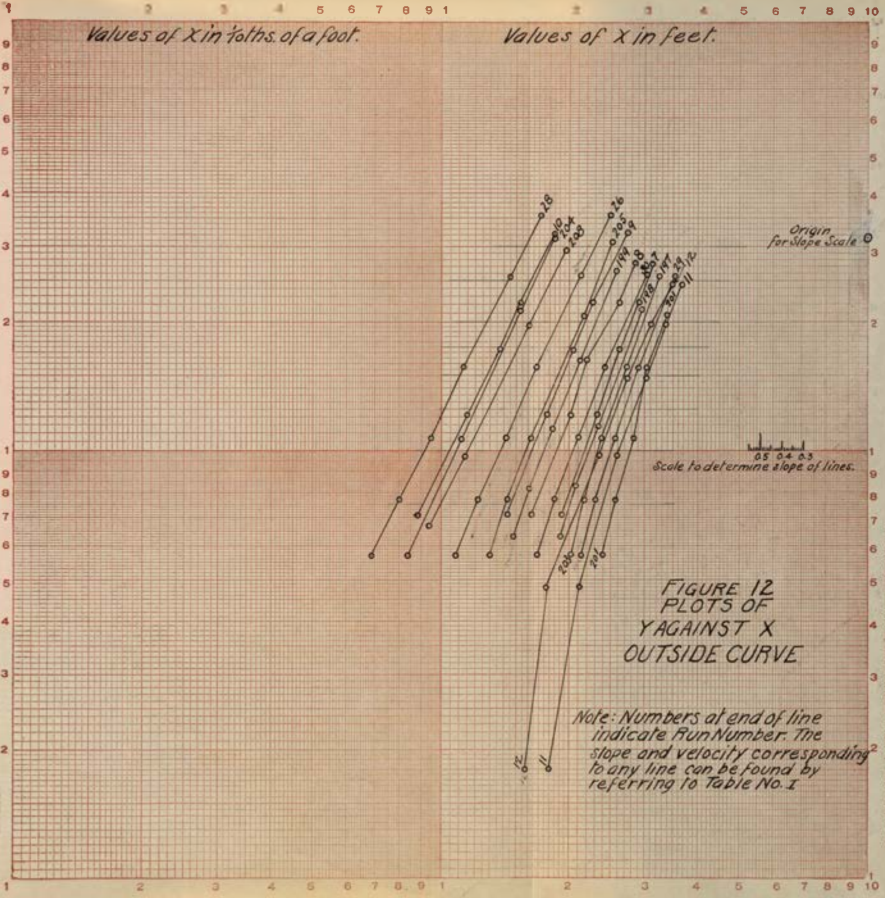


FIGURE 12
PLOTS OF
Y AGAINST X
OUTSIDE CURVE

Note: Numbers at end of line indicate Run Number. The slope and velocity corresponding to any line can be found by referring to Table No. I

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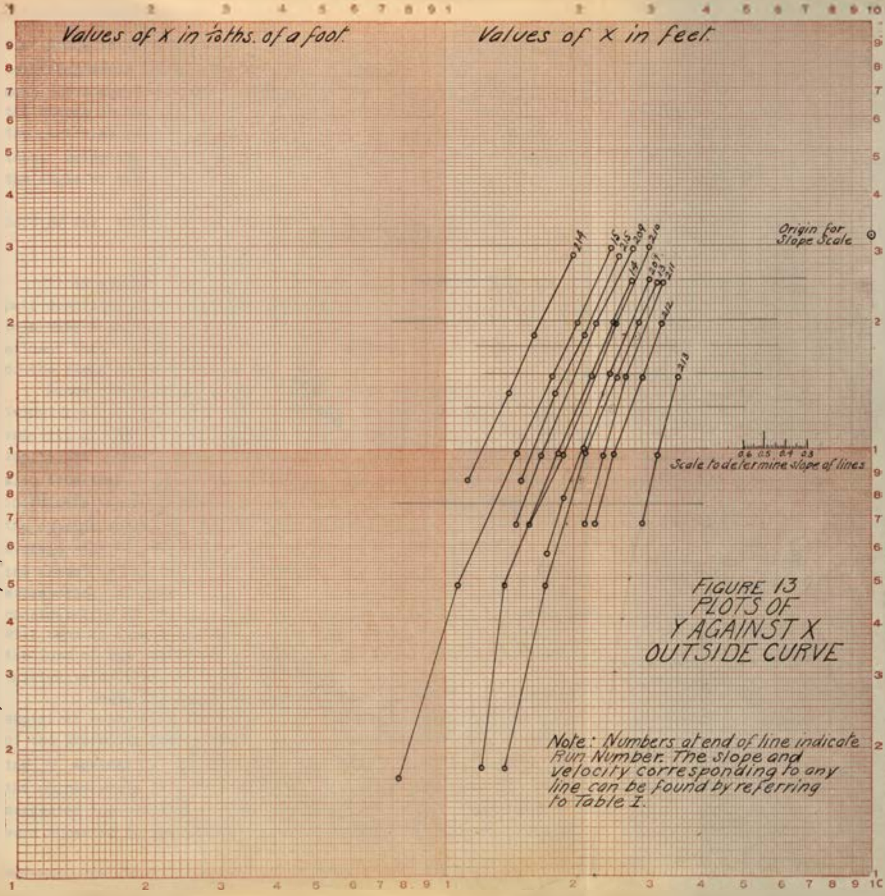

FIGURE 13
PLOTS OF
Y AGAINST X
OUTSIDE CURVE

Note: Numbers at end of line indicate Run Number. The slope and velocity corresponding to any line can be found by referring to Table I.

flat grade is an assumption of an impossible condition. The ordinary condition would be somewhat as shown in Figure 7 with a slight drop down curve above the lip. Disregarding the change in velocity due to the change in the section of the stream, and assuming that all of the particles have the same velocity equal to the average velocity, then, using the same notation, with the addition of ϕ to represent the angle of the slope with the horizontal, see Figure 7,

$$x = t\ v\cos\phi \quad \text{and} \quad y = t\ v\sin\phi + 1/2\ g\ t^2$$

Solving for t as before and equating it will be found that

$$x = V\cos\phi\left(\frac{-V\sin\phi \pm \sqrt{V^2\sin^2\phi + 2gY}}{g}\right)$$

It is evident from the form of this last expression that the path described by the falling particles is not that of a common parabola with a vertical axis and the apex at the origin. It is also evident that the characterof the path traversed is not independent of the slope of the pipe.

Since the average velocity of the stream is greater than the velocity at the bottom of the stream and is less than the average velocity at the point of leaping, it is evident that the last expression is not correct for the coordinates of the inside curve. It is also true that if the depth of waterin the pipe be added to the y coordinate the expression would be erroneous for the coordinates of the outside curve for similar reasons. If the relation between the average velocity and the top and bottom velocities were known, and the actual velocities of the particles were substituted for v in the expression, the result would still be incorrect as is evident from consideration of the fact that the mutual attraction of the particles will tend to impart the path of one upon the path of the other and the form of the inside and outside curves is an averaging of these initial velocities and later tendencies.

Sect. 16. Empirical Expressions:- Professor W.C. Unwin's empirical expression, referred to in section 4, based on a portion of the preceding theory would lead to the conclusion that by plotting y against x for any particular curve, on logarithmic paper the points would fall on a straight line whose slope was two. This suggested the plotting of such points for the three curves which were observed in the tests. Since the observations for the outside curve were the most accurate the points on the outside curve were plotted first.

Sect. 17. Outside Curves. General:- The coordinates y and x recorded in Table III, have been plotted for the majority of runs in Figures 11, 12 and 13. The points for any one run lie closely on a straight line, except for the higher velocities and slopes, where there is a slight tendency for the line to curve faster to the right as the value of y increases. It is probable that there is a tendency for these lines to approach a slope of two. This tendency is so slight however, that within the limits of error of the observations and the desired accuracy of the results, it can be ignored. Although the points lie on straight lines the slope of these lines is different for each run, which would indicate that the exponent of y is not a constant, but is dependent on either s or v or both. That is for v and s constant it is evident that

$$x = ky^m$$

It is possible that by plotting x against v with y constant that it may be discovered that $x = kv^n$. In Figure 14, x as read from Figures 11, 12 and 13 has been plotted against v for various constant values of y and lines drawn joining points on the same slope. Although these points do not lie as closely on a straight line as the plots of the x and y coordinates, the evidence is that for any particular value of y and s

$$x = kv^n$$

and therefore for any particular value of s

$$x = ky^m v^n$$

in which k is a function of s, m a function of s and v, and n a function of y and s.

Sect. 18. Outside Curve. Value of m. Exponent of y:- It will first be attempted to find the value of m, the exponent of y. For this purpose the slope of each xy line has been read from Figures 11, 12 and 13 and recorded in Table V. The values of v were vm then plotted against m for each line, but as the slope of the xy lines could not be read with any great degree of accuracy the natural tendency of the lines through the vm points was assumed and the slopes reread to see how closely they would conform to the assumed tendency. Three different relations for the vm lines were assumed and the lines replotted to coincide with these relations as closely as possible

The relation assumed first was that the points lay on a straight line for any one value of s and that the lines determined for different values of s were parallel. This would mean that the relation between v and m is in the form $v = pm+q$ in which p is a constant and represents the slope of the vm lines, and q is a variable dependent on the value of s. The value of p was scaled from the vm lines and found to be (-)24.6. The next step was to find the relation between q and s. For this purpose s was plotted against q. The points fell on a straight line, showing that the relation between q and s is in the form $s = p'q+k$. The values of p' and k as read from the sq line were 0.0059 and (-)0.0786 respectively. Substituting and transposing it was found that

$$m = 6.92s + 0.0407v + 0.542$$

The vm lines conforming to this equation were drawn for values of s and the values of v and m were read from Figures 11, 12 and 13 so as to conform as closely as possible to the above relation and these values were also plotted. The discrepancy between the calculated and observed lines made it evident that for the steeper invert slopes the slope of the vm lines (value of p above) was greater than that given by the preceding expression. For this reason the relation assumed first was considered as not correct, and the graphical analysis has not been included herein.

The next relation which was assumed was that the locus of the values of v when plotted against m was a conic whose equation was in the form $m = 0.5 - kv^p$ which was used in the form

$$\log k + p\log v = \log(0.5 - m)$$

The lines formed on logarithmic paper with values of $(0.5-m)$ as absiccas and values of v as ordinates for different but constant values of s were equally spaced for equal variations in s. It follows from the logarithmic equation above that when v equals unity $k = (0.5-m)$. It is therefore true that

$$\sin\phi = \frac{Cs}{\log 0.0068 - \log k}$$

in which Cs represents the distance which any $(0.5-m)v$ line is from the line representing the value of s equal to zero and ϕ is the complementary angle of the slope of these lines. By scaling from the plot it was found that $C = 12.5$ Then substituting and transposing

$$m = \frac{1}{2} - \frac{v^2}{10^{(2.17+28s)}}$$

The vm lines conforming to this relation were treated as for the relation previously assumed. In studying the results it seemed that as the velocities increased the values of m decreased too rapidly. It was also evident that the conic did not cross the X-X axis at the point $v = 0$, $m = 0.5$, and in view of the results obtained from subsequent studies of the middle and inside curves a third assumption was made with regard to the relation between v and m.

Figure 15 is a graphical representation of the relation finally assumed and adopted. This relation is such that the values of v when plotted against m for different values of s, fall on a series of straight lines converging at the point $v = 0$, $m = 0.57$. Under this assumption the relation between v and m can be expressed in the form $v = qm + p$ in which both q and p are functions of s. The values of q for different values of s were scaled from Figure 15 and plotted in Figure 16. The points fell upon a straight line giving the relation between q and s in the form $(-)q = rs + k$. By scaling and computation it was found that the relation between q and s was in the form $(-)q = 1180s + 20.1$ The relation between p and s was found similarly and plotted in Figure 16, and the relation found that $p = 0.57(1180s + 20.1)$. Substituting these values in the original form of the equation between v and m

$$m = 0.57 - \frac{v}{118s + 20.1}$$

The vm lines conforming to this relation have been plotted in Figure 15 and the lines drawn through the corresponding values of v and m as read from Tables II and V, for different slopes have also been plotted in this figure. The full lines represent the values as read from the tables; the dash lines represent the relation as expressed by the above equation.

The values of m for all three assumed relations were studied and the results were compared with the observations. The final expression for m gave the most accurate results and was more in conformity with the results obtained by similar studies of the inside and middle curves. It was therefore selected as the expression for

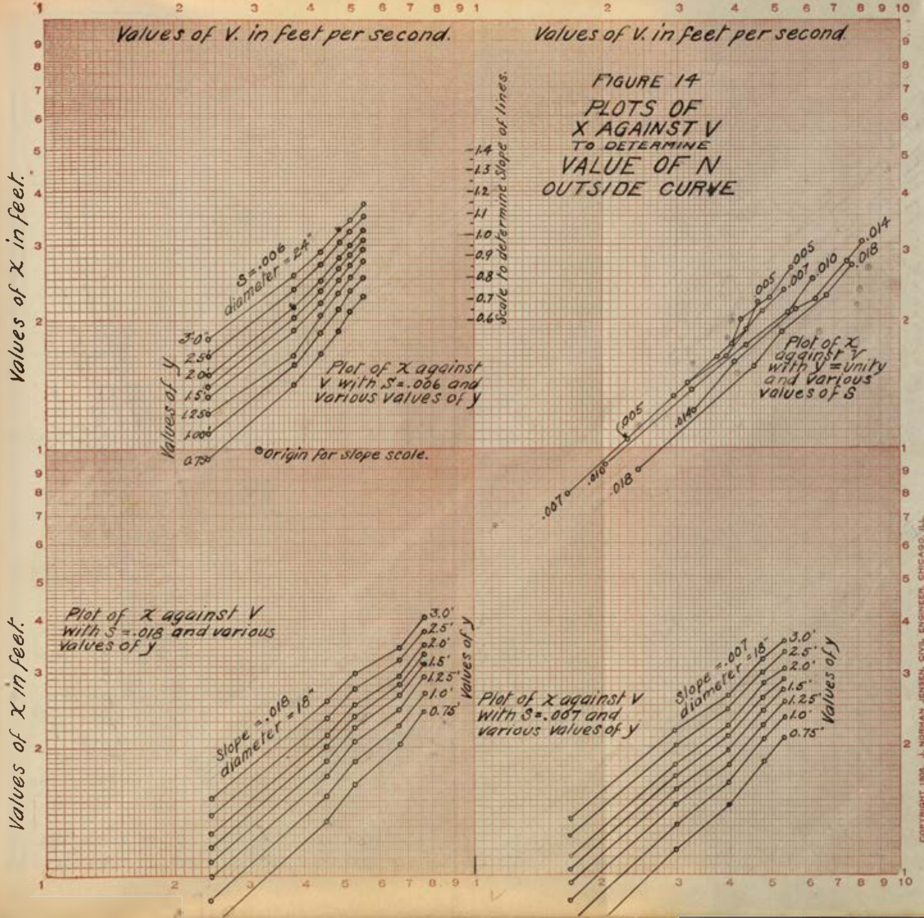

FIGURE 14

PLOTS OF
X AGAINST V
TO DETERMINE
VALUE OF N
OUTSIDE CURVE

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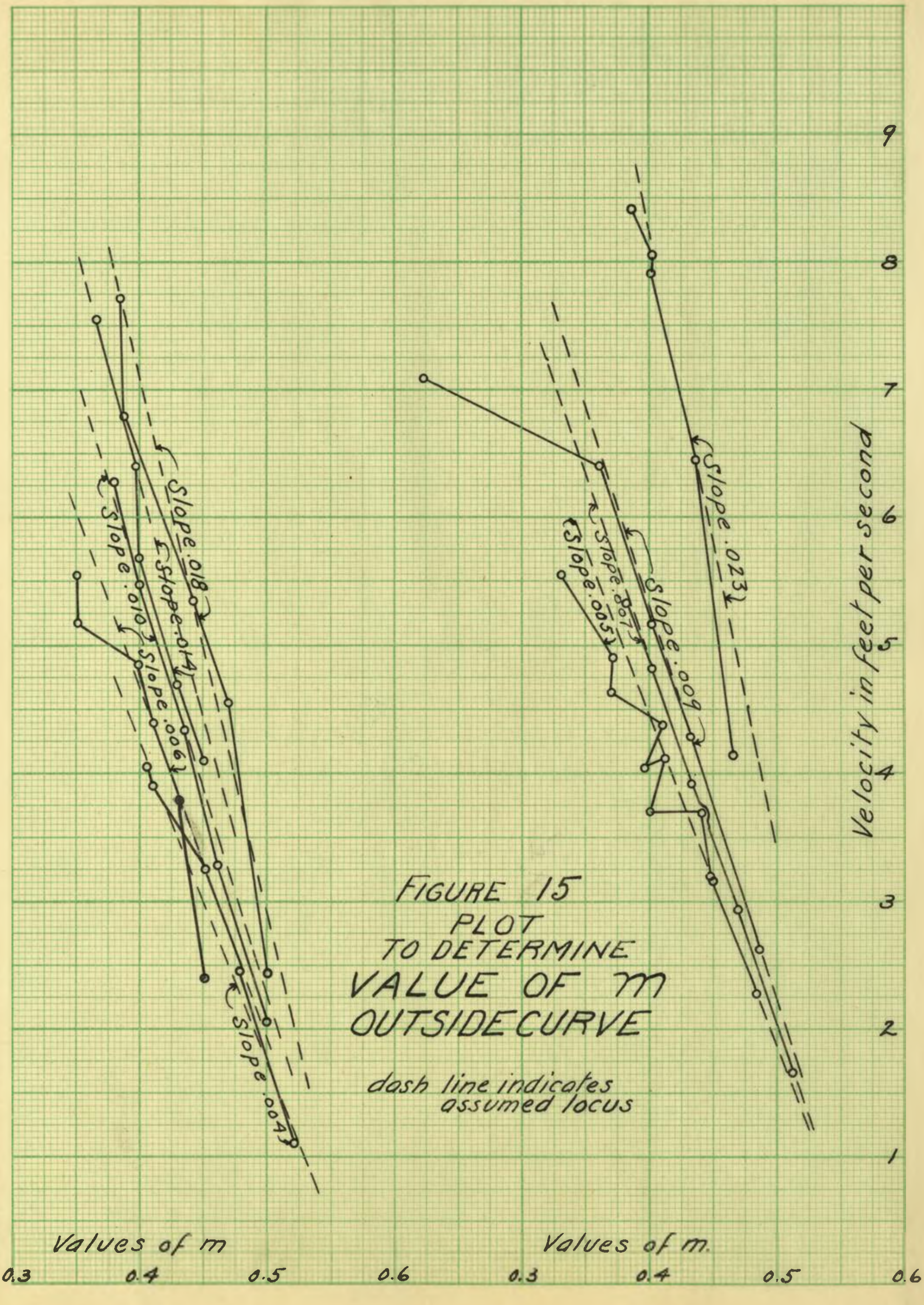

FIGURE 15
PLOT
TO DETERMINE
VALUE OF m
OUTSIDE CURVE

dash line indicates
assumed locus

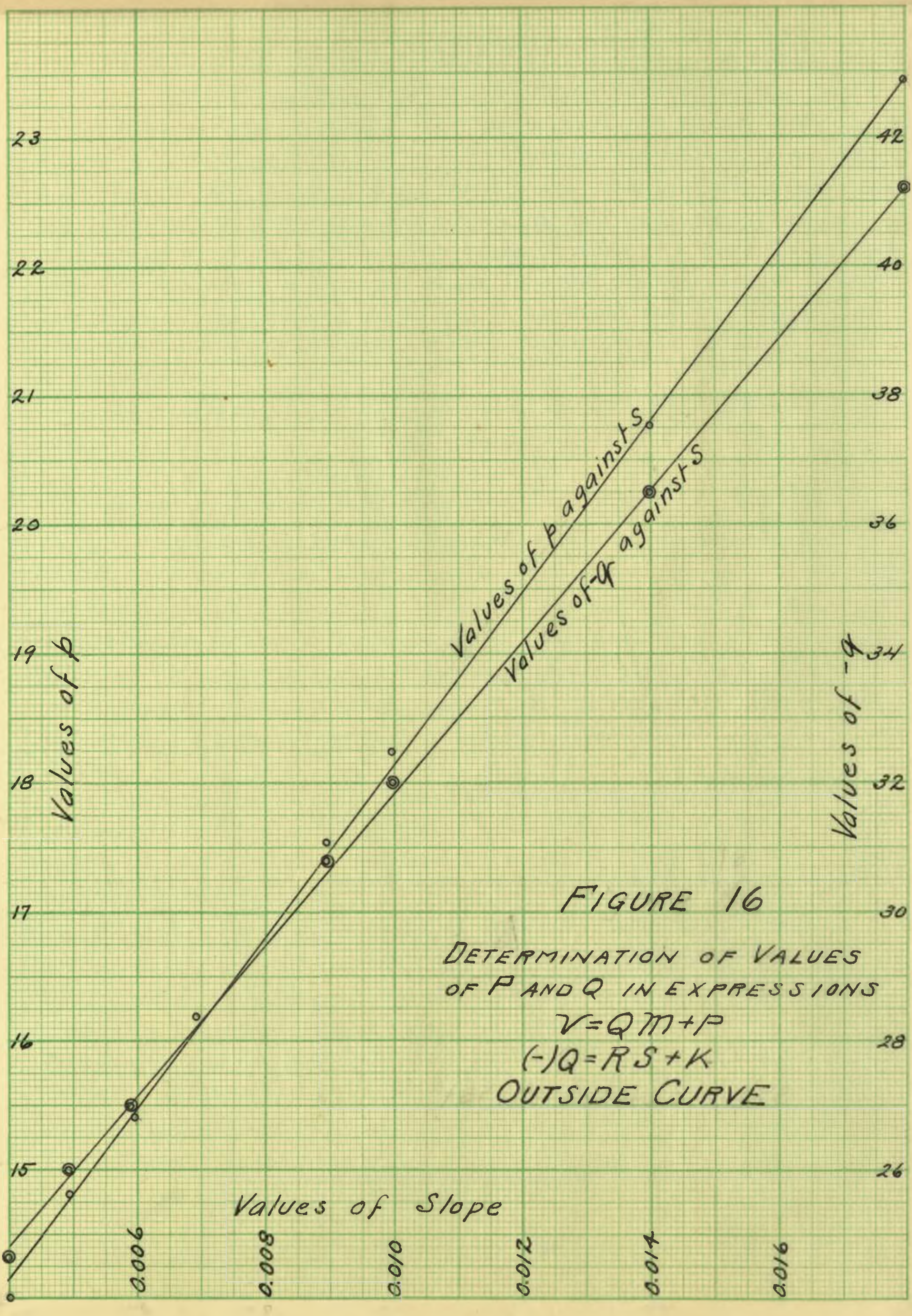

FIGURE 16

DETERMINATION OF VALUES OF P AND Q IN EXPRESSIONS

$V = QM + P$

$(-)Q = RS + K$

OUTSIDE CURVE

the exponent of y for the outside curve.

Sect. 19. Outside Curve. Value of n. The Exponent of v:-

If in the equation $x = ky^{m}v^{n}$ the value of y is unity, then $x = kv^{n}$. Then by holding y equal to unity and plotting v against x on logarithmic paper, the exponent of v can be determined. This has been done in Figure 14 and the value of n read as 1.0 regardless of the slope of the invert. The slope of the xv lines is not the same for different values of y as is shown in the figure, but since the value of x for any value of y is equal to the value of x when y is unity multiplied by y^{m} the preceding method of finding m is correct.

The equation of the outside curve has now been developed to the form

$$x = ky^{\left\{0.57 - \frac{v}{118s + 20.1}\right\}} v$$

It now remains to find some relation between k and x.

Sect. 20. Outside Curve. Value of k. the Coefficient:-

When y is unity the value of k is x/v. Values of k for different but corresponding values of x and v were computed for all runs and recorded in Table VI. A study of this table will show that the value of k is practically constant for any particular value of s. The values of k were plotted against s in Figure 17 and a line was drawn through the average value of k for each value of s recorded.

The equation of this line was determined by trial. It was concluded that the portion of the line within the limits of $s = .023$ and 0.010 was straight, and for values of s less than 0.010 the equation became that of a curve in the form $ks^{p} = 1$. Values of this relation were determined by trial in such a manner that the value of k for values of s greater than 0.010 was changed by less than 0.001, which is a greater change than the accuracy of the computations will permit. In other words the error introduced into the expression for the straight line portion by the expression for the curved portion is so small as to be beyond the limits of the accuracy of the computations. The value of k was finally expressed in terms of s as

$$k = \left(\frac{1}{10}\right)^{11}\left(\frac{1}{s}\right)^{4} - 7.5s + 0.543 + \phi$$

The line determined by this equation was plotted in Figure 17 and

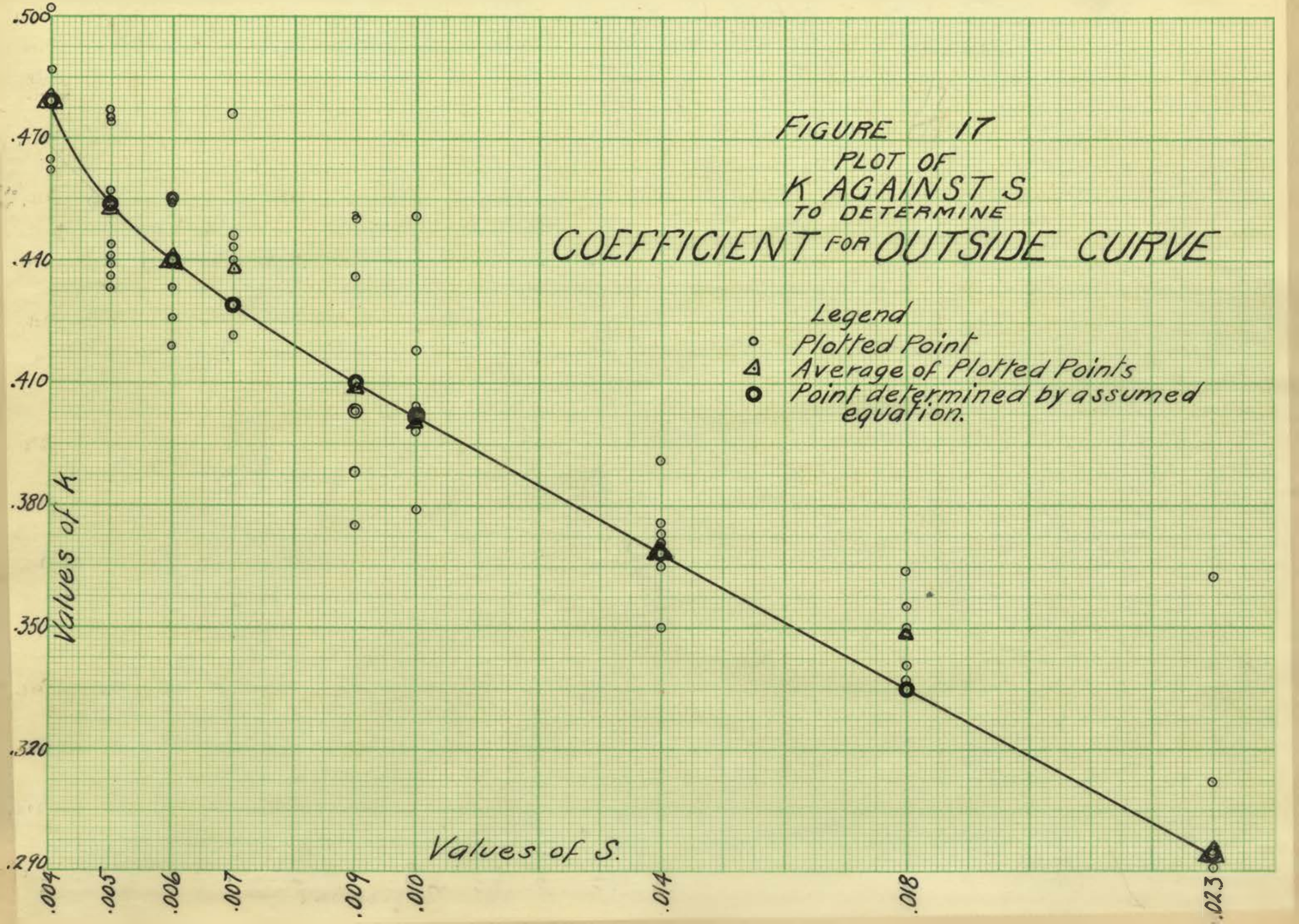
FIGURE 17
PLOT OF
K AGAINST S
TO DETERMINE
COEFFICIENT FOR OUTSIDE CURVE

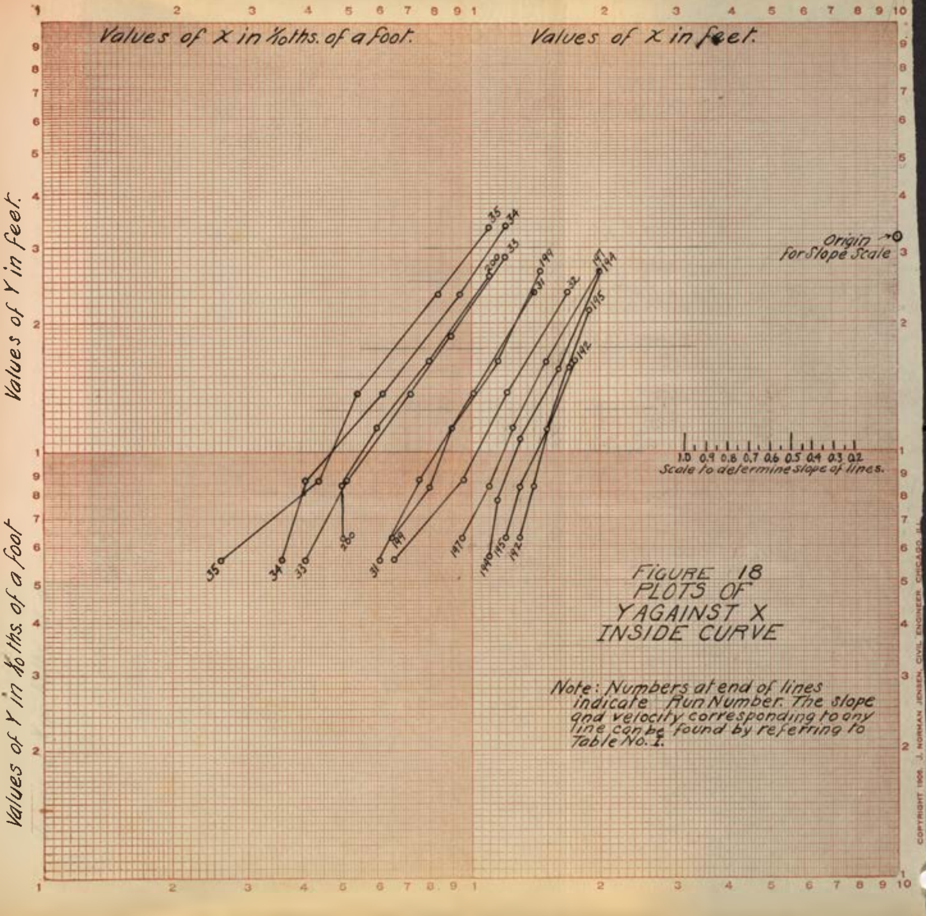

FIGURE 18
PLOTS OF
Y AGAINST X
INSIDE CURVE

Note: Numbers at end of lines indicate Run Number. The slope and velocity corresponding to any line can be found by referring to Table No. I.

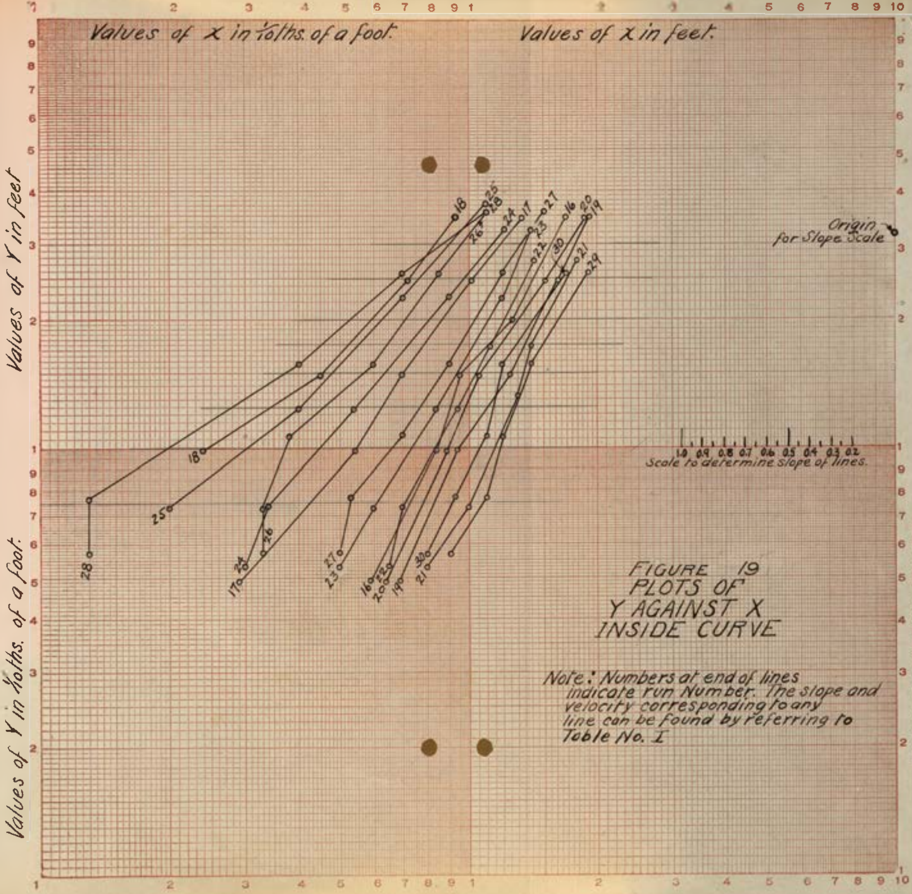

FIGURE 19
PLOTS OF
Y AGAINST X
INSIDE CURVE

Note: Numbers at end of lines indicate run Number. The slope and velocity corresponding to any line can be found by referring to Table No. I

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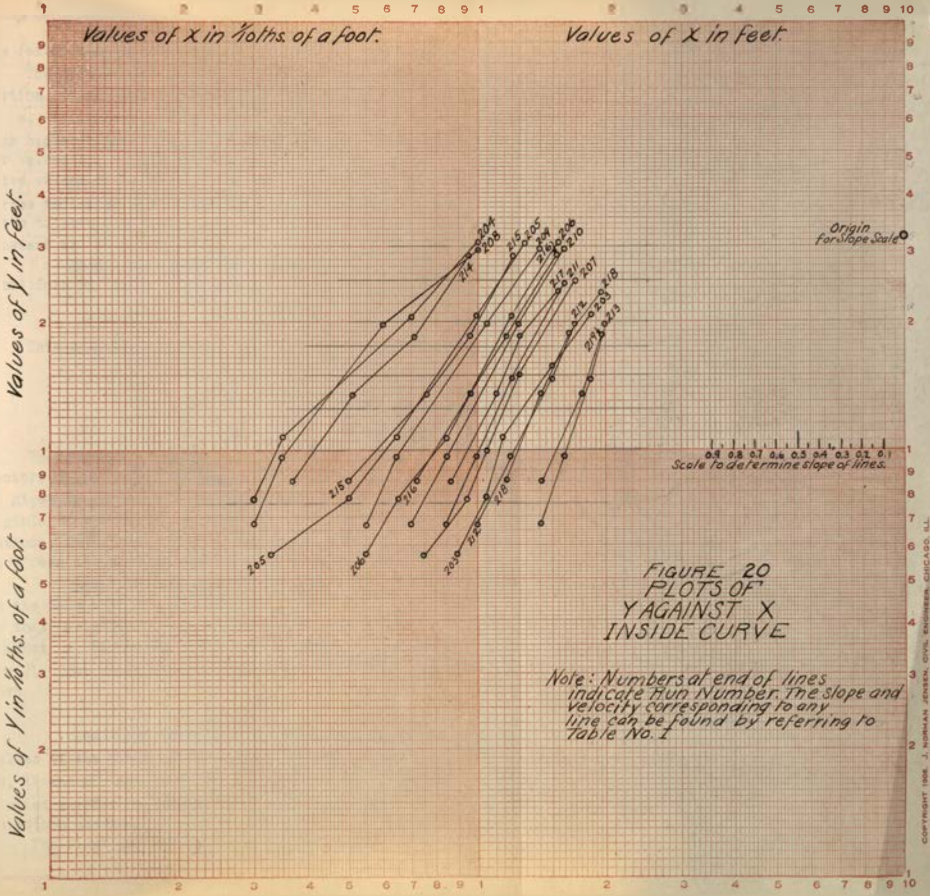

FIGURE 20
PLOTS OF
Y AGAINST X
INSIDE CURVE

Note: Numbers at end of lines indicate Run Number. The slope and velocity corresponding to any line can be found by referring to Table No. I

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the result coincides satisfactorily with the observations.

The term Ø is included in the above expression to care for that portion of the curve extending beyond the diagram for larger values of s. It is probable that the expression for k is not true for much higher slopes because the value of k would become negative for values of s greater than 0.072, which would, in turn, give negative values of x which is unreasonable. Subsequent studies of other curves indicates that the curve between k and s becomes asymptotic to some horizontal line, i.e. the value of k approaches a constant as s approaches infinity.

Sect. 21. Outside Curve. Final Form Of the Equation:- The final form of the equation for the outside curve is

$$x = \left\{\left(\frac{1}{10}\right)^{11}\left(\frac{1}{s}\right)^{4} - 7.5s + 0.543\right\} y^{\left\{0.57 - \frac{v}{118s+201}\right\}}$$

This equation is good only within the following limits:

Limits of s between 0.004 and 0.023
Limits of v between 1.0 and 8.5 feet per second
Limits of diameter between 18 and 24 inches.
Limits of y between 0.75 and 4.0 feet

There is no certainty as to the value of results computed from factors beyond these limits. A more extended discussion of this point is given in section 33. The value of the term $\left(\frac{1}{10}\right)^{11}\left(\frac{1}{s}\right)^{4}$ is negligible for values of s equal to or greater than 0.010. Short cut methods, tables and diagrams for the solution of this expression have been prepared and are discussed in section 35.

Sect. 22. Inside Curve. General:- The procedure followed in the deduction of the equation of the inside curve was similar to that followed for the outside curve. Values of y were plotted against x in Figures 18, 19 and 20, and of x against v in Figure 21, from which it is evident that the middle curve is also in the form

$$x = ky^{m}v^{n}$$

Sect. 24. Inside Curve. Value of m. Exponent of y:- The values of the slope of the xy lines read from the plots in Figures 18, 19 and 20 are recorded in Table V. The value of v was then plotted against corresponding values of m and but two different relations assumed, instead of three as for the outside curve.

The first assumed relation was that the points of the vm line

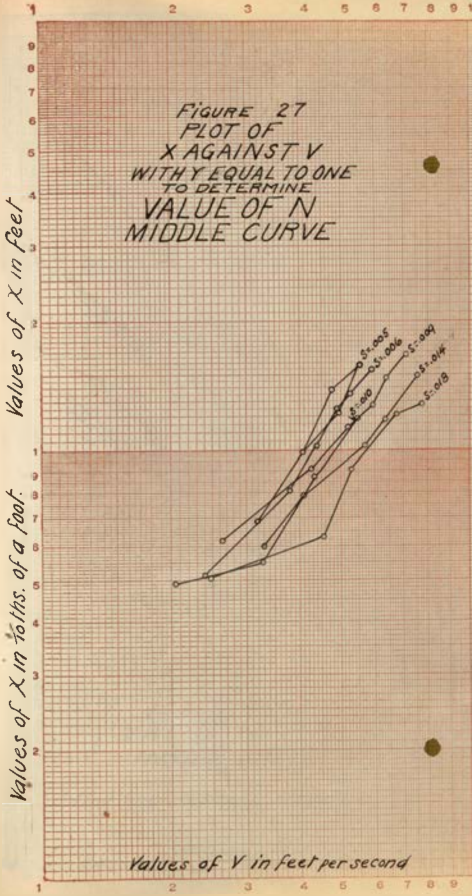

FIGURE 27
PLOT OF
X AGAINST V
WITH Y EQUAL TO ONE
TO DETERMINE
VALUE OF N
MIDDLE CURVE

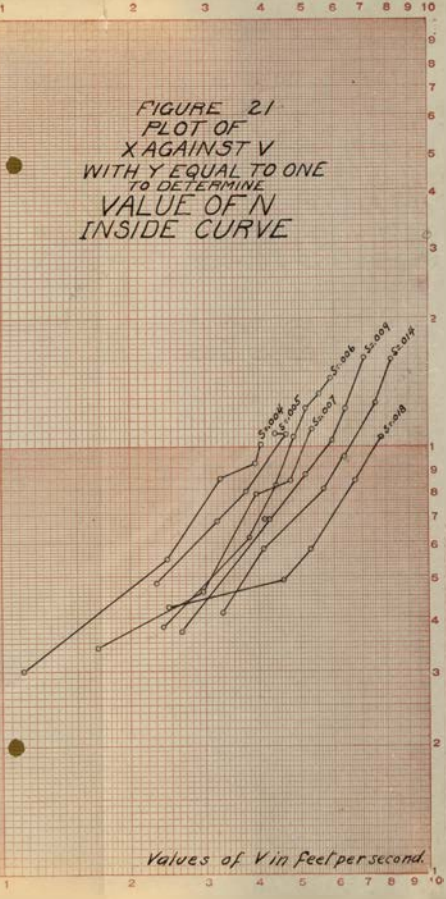

FIGURE 21
PLOT OF
X AGAINST V
WITH Y EQUAL TO ONE
TO DETERMINE
VALUE OF N
INSIDE CURVE

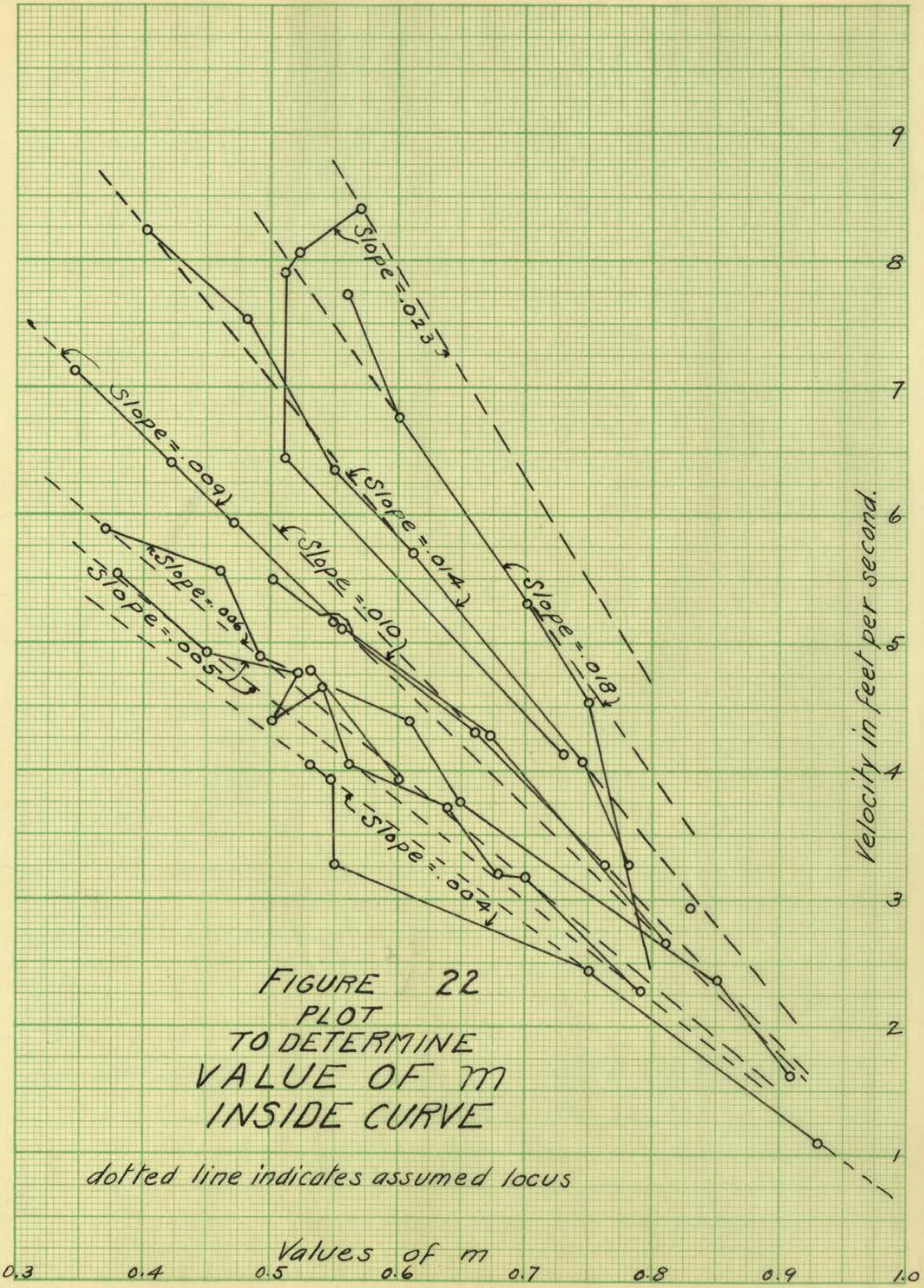

FIGURE 22
PLOT
TO DETERMINE
VALUE OF m
INSIDE CURVE

dotted line indicates assumed locus

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lay on a straight line for any one value of s and that the lines determined by different values of s were parallel and equally spaced. The equation for this relation was worked up, the lines plotted to agree with the equation and the values of m reread from the xy lines were also plotted against v. It was evident, as in the attemp for the outside curve, that this assumed relation did not give a sufficiently steep slope to the vm lines for the larger values of s. The relation was not used as a result.

The second relation assumed was similar to the final relation assumed for the outside curve, that is, that the values of v when plotted against m would fall on a series of straight lines determined by different values of s, and that these lines would converge at a point where $v = 0$ and $m = 1.095$. The relation between m, v, and s, was determined in the same manner as for the outside curve and the result reached that

$$m = 1.085 - \frac{v}{470s + 5.4}$$

The dash lines in Figure 22 have been plotted in accordance with this equation. The full lines are determined by the values of v and m as read from Tables II and V. The agreement between the assumed (dash) lines and the observed (full) lines is graphically shown. Although there is a greater discrepancy between the results for the inside than for the outside curves, the accuracy of the observations was also of a lower degree. The above expression has been accepted as final for the relation between m, v, and s for the inside curve.

Sect. 25. Inside Curve. Value of n. The Exponent of v:- The values of v have been plotted against x in Figure 21 with y as unity. As in the case of the outside curve, when y is unity $x = kv^n$ for the inside curve. Then by reading the slope of the vx line when y is unity, from Figure 21 the value of n was determined as 1.4

The equation for the inside curve has now been reduced to the form

$$x = k\,y^{\left\{1.085 - \frac{v}{470s+5.4}\right\}}\,v^{1.4}$$

It now remains to find some relation between k and x.

Sect. 26. Inside Curve. Value of k. The Coefficient:- Table

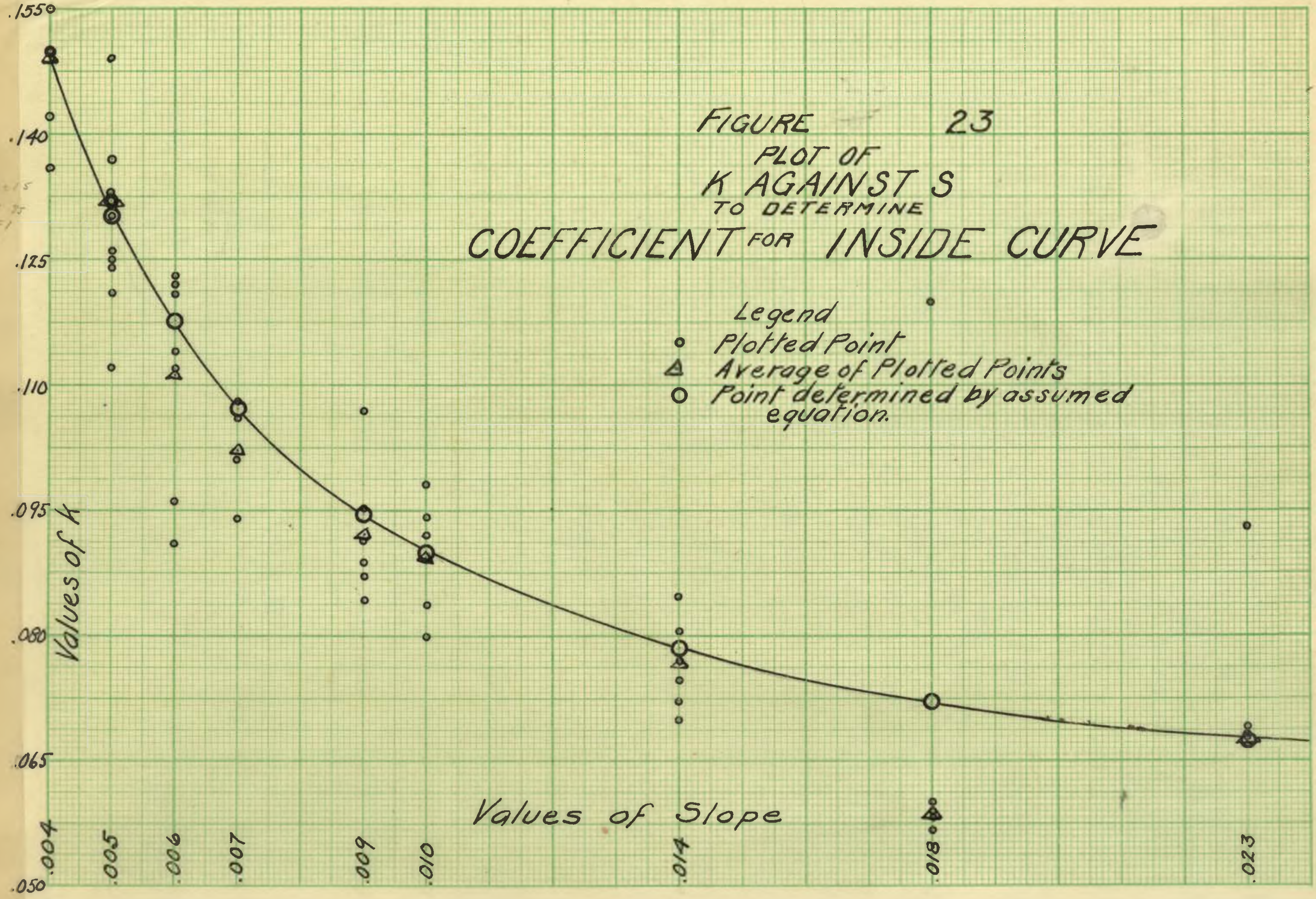


FIGURE 23

PLOT OF

K AGAINST S

TO DETERMINE

COEFFICIENT FOR INSIDE CURVE

V contains the values of k computed from the relation that $k = \frac{x}{v^{1.4}}$ when y is unity. It is evident from this table that the value of k is practically constant for any particular value of s, and that it is independent of values of v, or x. The values of k have been plotted against s in Figure 23. The equation of the line drawn through average values of k was determined by trial to be

$$k = \left\{ \frac{37}{10^5 S} + .039 \right\}$$

It is to be noted that this is in a somewhat different form from the expression for k for the outside curve. It was determined on the assumption that the curve followed the law $k = C/S^P + C_1$ which is the equation of a curve asymptotic to the Y-Y axis and to a line parallel to the X-X axis through the value of the ordinate C_1. This equation was rewritten in the form $\log K + P\log S = \log\left(\frac{C}{1-C_1}\right)$. Since C and C_1 are constant the coordinates of any point substituted in this expression should equal the result obtained by substituting the coordinates of any other point in the expression. The substitution of the two extreme points, with s = 0.004 and 0.023 was made and the results equated. C_1 was assumed to be 0.5 and the value of p determined as 1.5. The equation was then rewritten as $k = \frac{C}{S^{1.5}} + 0.5$ and the value of C determined by substituting the values of k and s for any point. The value of C was thus determined as 0.000033 and the resultant expression as $k = \frac{0.000033}{S^{1.5}} + 0.5$ This curve was plotted and found to be unsatisfactory. The values of p and C_1 were readjusted by trial and the different curves plotted until a satisfactory result was obtained. The trials were made with the knowledge that by increasing the exponent p of the denominator the curvature was increased; by adding a constant to the denominator the curve was shifted from left to right or right to left according as the sign of the constant was plus or minus; and that by increasing the value of C_1 the curve was raised, or by decreasing it the curve was lowered.

The line determined by the final value of k in terms of s is shown in Figure 23. This line seems to be more reasonable than the one for the outside curve because the value of k approaches a constant (in this case 0.039) as s approaches infinity. Error in extending the equation for steeper slopes than those observed should not be so great as in the former case.

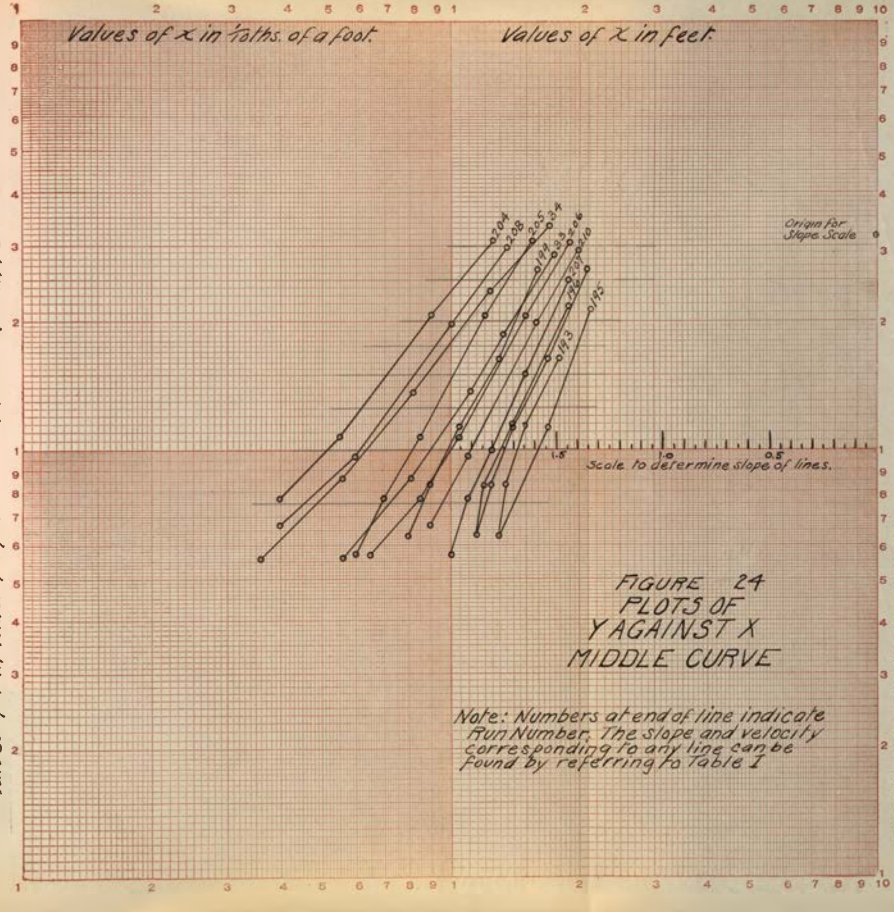


FIGURE 24
PLOTS OF
Y AGAINST X
MIDDLE CURVE

Note: Numbers at end of line indicate Run Number. The slope and velocity corresponding to any line can be found by referring to Table I

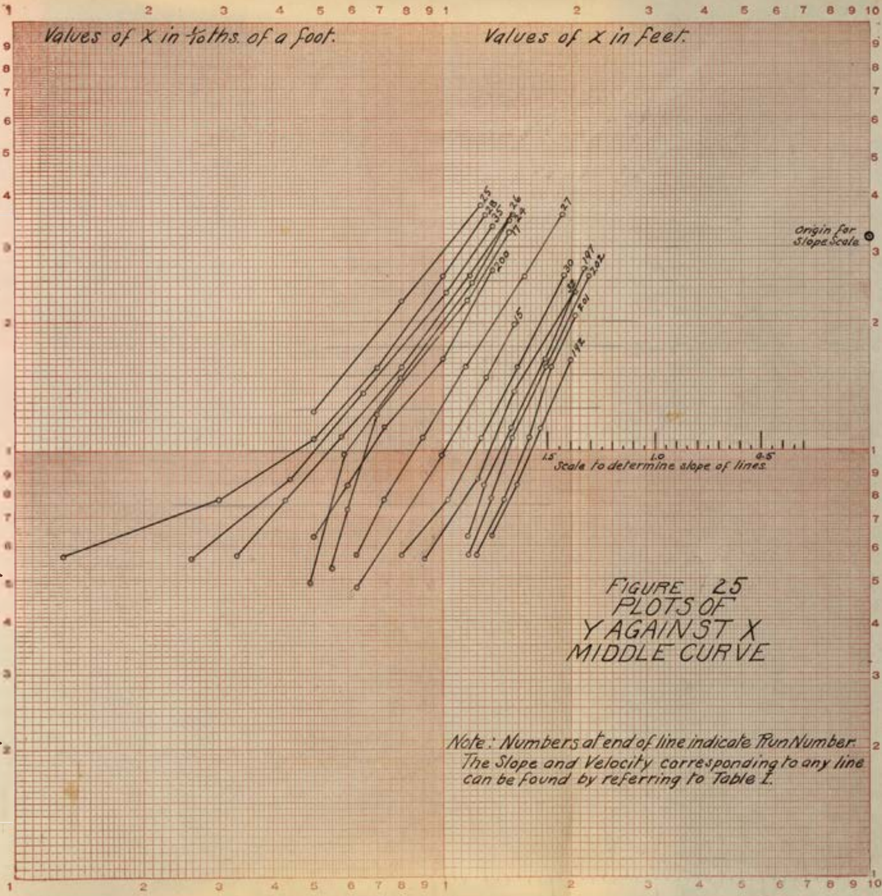


FIGURE 25
PLOTS OF
Y AGAINST X
MIDDLE CURVE

Note: Numbers at end of line indicate Run Number.
The Slope and Velocity corresponding to any line can be found by referring to Table I.

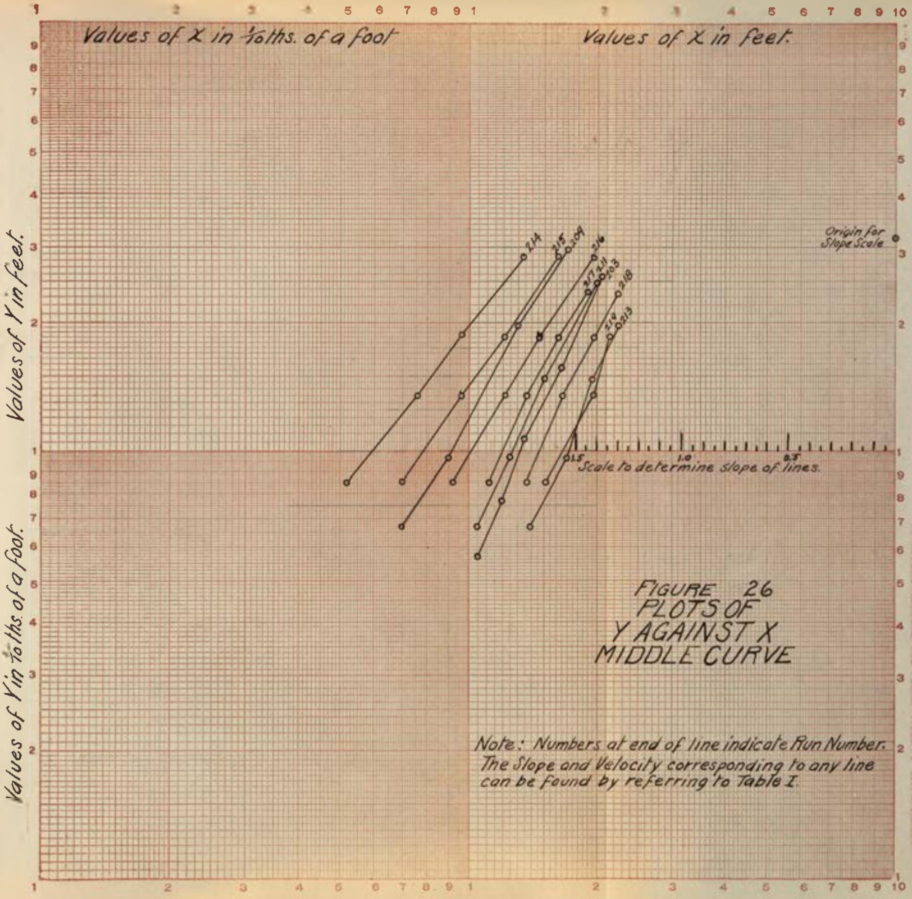


FIGURE 26
PLOTS OF
Y AGAINST X
MIDDLE CURVE

Note: Numbers at end of line indicate Run Number. The Slope and Velocity corresponding to any line can be found by referring to Table I.

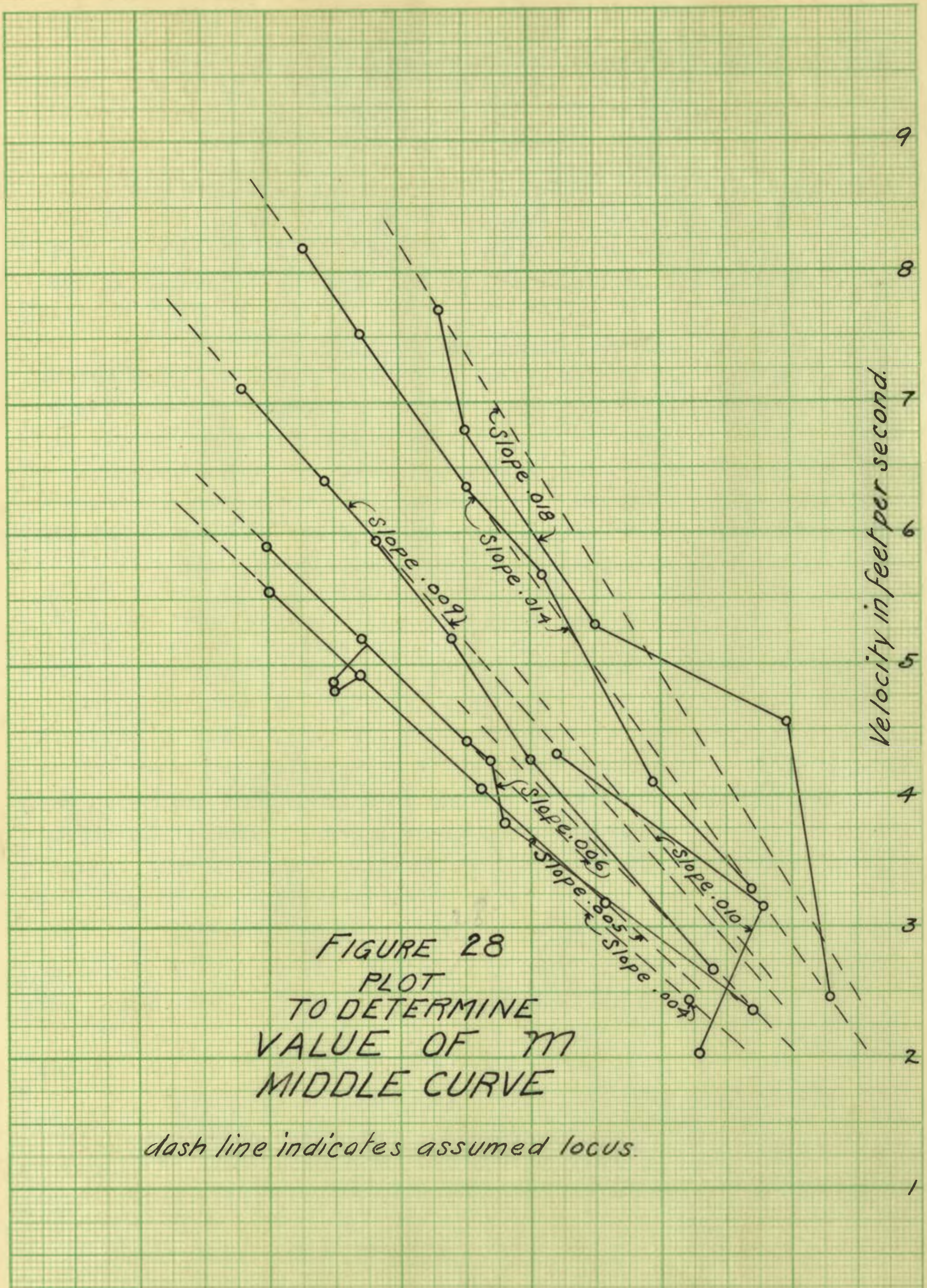


FIGURE 28
PLOT
TO DETERMINE
VALUE OF m
MIDDLE CURVE

dash line indicates assumed locus.

Sect. 27. Inside Curve. Final Form of the Equation:- The final form of the equation of the inside curve is

$$x = \left\{ \frac{0.00037}{s} + .039 \right\} y^{\left\{ 1.085 - \frac{v}{470s + 5.4} \right\}} v^{1.4}$$

A discussion of the accuracy of results to be obtained by extending this equation beyond the limits of the observations is to be found in section 33. Short cut methods, tables and diagrams have been prepared for the solution of the equation. They are discussed in section 35

Sect. 28. Middle Curve. General:- With the experience gained from the determination of the equations for the inside and outside curves the procedure for the determination of the equation of the middle curve was simplified.

The values of y were plotted against x in Figures 24, 25 and 26 and the values of x against v in Figure 27. The relation was found to be in the typical form $x = ky^m v^n$.

Sect. 29. Middle Curve. Value of m. Exponent Of y:- The values of v were plotted against m and the general tendency of the vm lines was observed. It was assumed that the vm points fell upon a straight line for any particular value of s, and that the lines determined by different values of s converged at a point whose coordinates were m equal to unity and v equal to zero. The equation of these lines was determined, as in the preceding examples, to be $m = 1 - \frac{v}{550s + 6.5}$ The lines determined by this expression have been plotted as dash lines in Figure 28 and the actual values of v and m have also been plotted as full lines in this figure. The agreement between the observed and calculated lines is sufficiently accurate for use.

Sect. 30. Middle Curve. Value Of n. Exponent of v:- The value of x was plotted against v with y as unity, in Figure 27 The slope of this line gives the exponent of v as 1.3

The equation is now in the form $x = ky^{\left\{ 1 - \frac{v}{550s + 6.5} \right\}} v^{1.3}$ and it remains to determine the value of k.

Sect. 31. Middle Curve. Value of k. The Coefficient- Table V contains values of k computed from the relation that $k = \frac{x}{v^{1.3}}$ when y equals one. These values are plotted against s in Figure 29, and the relation between k and s determined as before to be

$$k = \left\{ \frac{625}{(10)^{14} s^4} - 5s + .204 \right\}$$

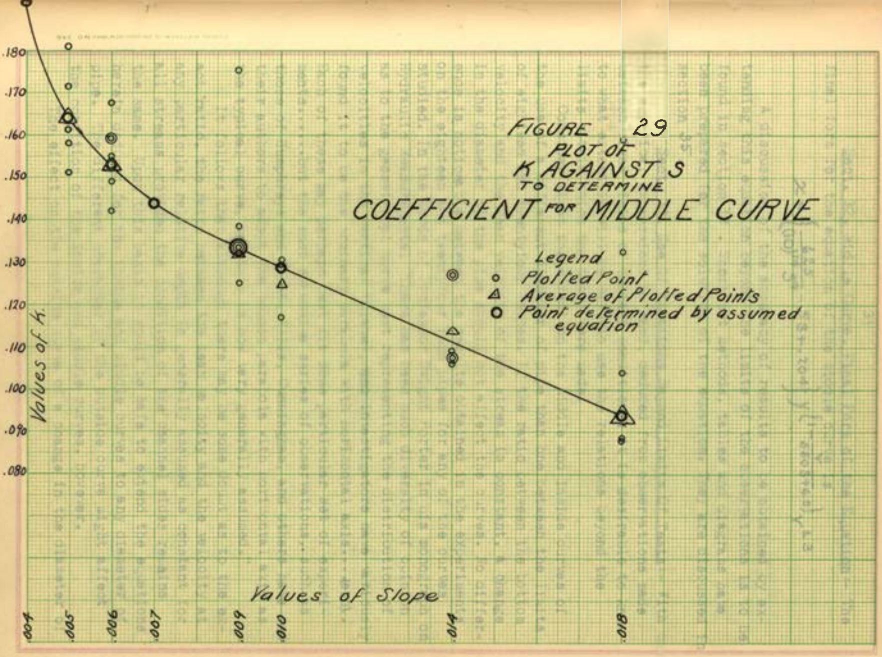


FIGURE 29

PLOT OF
K AGAINST S
TO DETERMINE
COEFFICIENT FOR MIDDLE CURVE

Sect. 32. Middle Curve. Final Form of the Equation:- The final form for the equation of the middle curve is

$$x = \left\{\frac{625}{(10)^{14} S^4} - 5S + .204\right\} y^{\left\{1 - \frac{v}{550S + 6.5}\right\}} v^{1.3}$$

A discussion of the accuracy of results to be obtained by extending this equation beyond the limits of the observations is to found in section 33 . Short cut methods, tables and diagrams have been prepared for the solution of the equation. They are discussed section 35.

Sect. 33. Use Of Equations Beyond Limits Of Tests:- With the empirical expressions in hand, deduced from observations made between fixed limits of certain conditions, it is desirable to kno to what extent it will be safe to use the relations beyond the limits within which the tests were made.

Consider first the effect on the middle and inside curves of the use of a different diameter of pipe than one between the limit of eighteen and twentyfour inches. If the ratio between the bottom velocity and the mean velocity of the stream is constant, a change in the diameter of the pipe should not affect the curves. No diffe ence is noticeable between the results obtained in the experiments on the eighteen and twentyfour inch pipes for any of the curves studied. In the words of Professor Dwight Porter in his monograph Hydraulic Measurements[1] "There has been much diversity of opinion as to the geometrical curve best representing the distribution of velocities in the vertical (plane), and investigators have various found it to be the ordinary parabola with horizontal axis...(etc). Each of these may perhaps best fit some particular set of experiments......but the most extensive series of observations, such as those of Humphreys and Abbot, Ellis, Cunningham, and others have l their authors to adopt the common parabola with horizontal axis as the typical curve, and this is now very generally assumed."

It follows that although there may be some doubt as to the ex act ratio, the ratio between the mean velocity and the velocity at any particular relative depth is generally assumed as constant for all streams provided the condition of the channel sides remains the same. Upon this basis it would be safe to extend the equation herein deduced for the inside and middle curves to any diameter of pipe. The different character of the outside curve might affect the condition of the inside and middle curves, however.

The effect on the outside curve of a change in the diameter c

the pipe may be quite different since the coordinates are to be
measured from the lip of the weir and not from the surface of the
stream. If the hydraulic elements of a circular section be shown
graphically, as in Figure 10, it is evident from the shape of the
draulic radius curve that if the diameter of the pipe be increase
to maintain the same hydraulic radius when the pipe is flowing pa
full, the depth of flow must be less. The greater the slope of th
invert the greater the velocity of flow, and the greater the hydra
lic radius the smaller the correction in the depth of flow for an
crease in the diameter. This means that the equation for the outs
curve if extended to large diameters of pipe would give too large
result for x for any one value of y, since the larger the diame
the lower the 'start' of the leaping point, that is, the stream in
the larger pipe has the greater handicap in the broad jump. A clos
approximation to the true curve could probably be reached by appl
ing the equation deduced herein with the value of y corrected b
the difference in the depth of flow between that in a twentyfour
inch pipe and in the larger pipe used when the velocity and slope
in each pipe are the same. That the difference in the result is
likely to be small is indicated by the insignificance of the diffe
ence in the results obtained in the eighteen and twentyfour inch
pipe tested. This correction for depth of flow would not be absolu
ly accurate because of the effect of the change of section of the
stream caused by the drop down curve. The steeper the slope the mo
accurate the correction because the drop down is less on steep tha
on flat slopes. The drop down curve becomes tangent to the surface
of the water in the pipe for any slope. Since the total drop down
a function of the slope as well as the depth of flow, a correction
for the depth of flow only is not all that is necessary, but will
aid in approaching the truth.

The equation of the drop down has not been computed. The gene
form for a circular channel would be very complicated, and would
of little benefit in this work. For any particular case it is comparatively easily determined by a 'cut and try' method.

Considering now the effect of applying the equations to flat
slopes than were tested, it is evident from Figures 17, 23 and 29
that the result might be a serious error because of the rapidity
which the values of k are changing. The values of v^n and y^m do
change very rapidly with changes in s so that they would have
little effect in reducing the rapidity of change in the value of
for the small slopes.

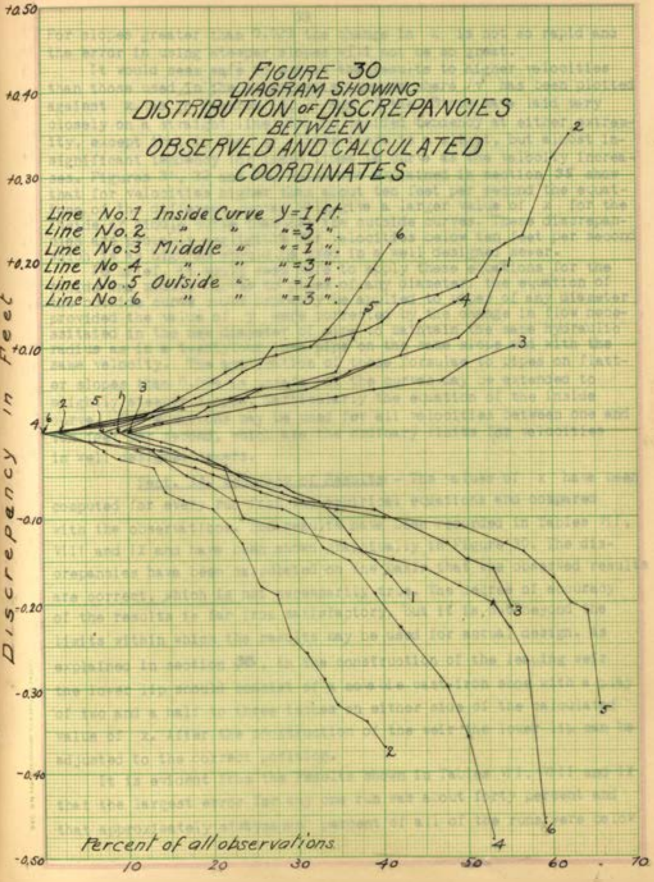


FIGURE 30
DIAGRAM SHOWING
DISTRIBUTION OF DISCREPANCIES
BETWEEN
OBSERVED AND CALCULATED
COORDINATES

For slopes greater than 0.023 the change in κ is not so rapid and the error in using steeper slopes will not be so great.

It would seem safe to extend the results to higher velocities than those used in the experiments because where v has been plotted against x, as in Figures 14, 21 and 27 the points have laid very closely on a straight line with no decided tendency at either extremity, except that in the xy lines there is a slight, but almost insignificant tendency for these lines to curve as the velocity increases. Figures 31, 32 and 34, which are explained in section 35 show that for velocities higher than about ten feet per second the equations of the different curves will give a larger value of x for the inside or middle curve than for the outside curves. These discrepancies are probably negligible for velocities below ten feet per second which is as high as should be used in a well designed sewer.

In brief, it would seem safe to apply these equations for the inside and middle curves to pipes of any diameter; the equation of the outside curve could probably be applied to pipes of any diameter provided the value of y were corrected by the change in flow necessitated in the new diameter of pipe to maintain the same hydraulic radius as in a twentyfour inch pipe on the same slope and with the same velocity. The application of these formulae to pipes on flatter slopes than 0.004 is unsafe but the curves may be extended to slightly steeper slopes, particularly the equation of the inside curve. The equations may be used for all velocities between one and ten feet per second, which are the ordinary limits for velocities in well designed sewers.

<u>Sect. 34. Accuracy of Results</u>. - The values of x have been computed for every run from the empirical equations and compared with the observations. The results have been recorded in Tables VII, VIII and IX and have been shown graphically in Figure 30. The discrepancies have been calculated on the basis that the observed results are correct, which is not necessarily true. The degree of accuracy of the results is far from satisfactory, but it is not beyond the limits within which the results may be used for actual design. As explained in section 38, in the construction of the leaping weir the lower lip should consist of a movable cast iron shoe with a play of two and a half to three inches on either side of the calculated value of x. After the construction of the weir the lower lip can be adjusted to the correct position.

It is evident from the results shown in Tables VII, VIII and IX that the largest error for any one run was about forty percent and that approximately sixtyseven percent of all of the runs were below

TABLE V

VALUES OF m, THE EXPONENT OF y, READ FROM THE LOGARITHMIC PLOTS OF y AGAINST x, FIGURES 11, 12, 13, 18, 19, 20, 24, 25 AND 26, FOR THE INSIDE, MIDDLE, AND OUTSIDE CURVES.

RUN NUMBER	Value of m		
	Inside	Middle	Outside
6	0.68	----	.445
7	0.54	----	0.37
8	0.50	----	0.41
9	0.64	----	0.40
10	0.79	----	0.48'
11	0.57	----	0.38
12	0.52	----	0.40
13	0.51	----	0.40
14	0.52	----	.435
15	0.73	66 ----	.465
16	0.55	----	0.45
17	0.75	.80,.60,.53 ----	0.48
18	0.93	----	0.52
19	0.53	----	0.41
20	0.54	----	0.41
21	----	----	0.38
22	0.53	----	0.40
23	0.60	----	0.43
24	0.83	80-86-72 ----	0.47
25	0.91	0-80	0.51
26	0.76	0.78	0.46
27	0.66	0.62	.435
28	1.20	0.73	0.50
29	----	----	0.38
30	0.50	----	0.40
31	0.60	0.55	.385
32	0.56	0.53	.385
33	0.70	0.65	0.44
34	0.75	0.80	0.47
35	0.80	0.83	0.50
195	0.38	0.40	0.33
196	0.45	0.47	0.37
197	0.52	0.45	----
198	----	?	0.41
199	0.56	0.56	.395
200	0.70	.655	0.45
201	0.37	0.40	----
202	0.46	0.43	0.35
203	0.55	0.47	0.35
204	0.85	0.77	0.45
205	0.65	0.58	0.43
206	0.61	0.55	0.41
207	0.49	0.45	0.40
208	0.81?	0.74	0.49
209	0.67	0.60	0.43
210	0.55	0.54	0.40
211	0.47	0.48	----
212	0.42	0.44	0.36
213	.345	0.38	0.24
214	0.78	0.77	0.47
215	.745	0.69	0.45
216	0.61	0.51	0.40
217	0055	0.55	.395
218	0.48	0.47	.365?
219	0.40	0.43	----

TABLE VI

VALUES OF K IN THE EMPIRICAL EXPRESSIONS FOR THE INSIDE, MIDDLE, AND OUTSIDE CURVES, AS COMPUTED FROM THE OBSERVED VALUES OF x AND y.

RUN NUMBER	Slope	Value of k		
		Inside	Middle	Outside
6	.005	.132	-----	.444
7	.005	.124	-----	.474
8	.005	.130	-----	.436
9	.005	.125	-----	.441
10	.005	.149	-----	.457
11	.023	.0697	-----	.312
12	.023	.0680	-----	.295
13	.023	.0658	-----	.276
14	.023	.0686	-----	.291
15	.023	.0929	.157	.362
16	.004	.162	.218	.487
17	.004	.155	.185	.462
18	.004	.256	-----	.598
19	.004	.142	-----	.502
20	.004	.136	-----	.465
21	.007	.1055	-----	.443
22	.007	.0943	-----	.440
23	.007	.1080	-----	.422
24	.007	.1009	-----	.446
25	.007	.1675	.218	.476
26	.010	.0795	.177	.418
27	.010	.0836	.129	.404
28	.010	.0920	.194	.451
29	.010	.0944	-----	.398
30	.010	.0978	.130	.379
31	.018	.0580	.132	.337
32	.018	.0603	.0892	.350
33	.018	.0565	.1042	.355
34	.018	.0589	.0878	.341
35	.018	.1198	.1590	.364
195	.005	.132	.171	.477
196	.005	.133	.158	.455
197	.005	.137	.181	----
198	.005	.126	----	.475
199	.005	.121	.161	.433
200	.005	.112	.151	.439
201	.006	.121	.155	----
202	.006	.122	.143	.455
203	.006	.123	.159	.454
204	.006	.112	.167	.455
205	.006	.096	.142	.419
206	.006	.091	.149	.426
207	.006	.114	.159	.433
208	.009	.0952	.175	.436
209	.009	.0888	.138	.403
210	.009	.0870	.133	.375
211	.009	.0844	.125	.388
212	.009	.0915	.133	.403
213	.009	.1040	.132	.451
214	.014	.0771	.127	.373
215	.014	.0805	.127	.391
216	.014	.0699	.106	.371
217	.014	.0718	.107	.350
218	.014	.0746	.109	.365
219	.014	.0845	.107	.376

TABLE VII
COORDINATES OF INSIDE CURVE
CALCULATED AND OBSERVED

RUN NUMBER	Value of X, in feet					
	Observed		Calculated		Discrepancy	
	Y=1	Y=3	Y=1	Y=3	Y=1	Y=3
6	0.67	1.40	0.66	1.39	-.01	-.01
7	1.07	1.92	1.11	1.90	.04	-.02
8	1.07	1.87	1.03	1.84	-.04	-.03
9	0.79	1.59	0.83	1.62	.04	.03
10	0.48	1.12	0.42	1.01	-.06	-.11
11	1.36	2.74	1.32	2.48	-.04	-.26
12	1.26	2.21	1.22	2.32	-.04	.11
13	1.19	2.08	1.19	2.30	0.00	.22
14	0.93	1.68	0.91	1.94	.02	.16
15	0.68	1.51	0.49	1.22	-.19	-.29
16	0.85	1.57	0.79	1.59	-.06	.02
17	0.55	1.20	0.53	1.21	-.02	.01
18	0.30	0.82	0.18	0.50	-.12	-.32
19	1.01	1.80	1.06	1.91	.05	.11
20	0.92	1.70	1.00	1.82	.08	.12
21	1.10	1.86	1.11	1.88	.01	.02
22	0.84	1.50	0.95	1.71	.11	.21
23	0.78	1.43	0.73	1.47	0.00	.04
24	0.46	1.15	0.48	1.06	.02	-.09
25	0.34	0.91	0.20	0.54	-.14	-.37
26	0.42	0.98	0.47	1.09	.05	.11
27	0.65	1.35	0.70	1.45	.05	.10
28	0.25	0.85	1.25	0.67	0.00	-.18
29	1.24	2.00	1.18	1.96	-.06	-.04
30	1.06	1.85	0.98	1.76	-.08	-.09
31	0.84	1.64	1.03	1.99	.19	.35
32	1.05	1.93	1.25	2.25	.20	.32
33	0.58	1.25	0.72	1.57	.14	.32
34	0.49	1.13	0.59	1.36	.10	.23
35	0.42	1.02	0.25	0.68	-.17	-.34
195	1.45	2.16	1.45	2.19	0.00	.03
196	1.23	1.97	1.21	2.00	-.02	.03
197	1.23	2.12	1.15	1.93	-.08	-.19
198	0.94	1.68	0.98	1.76	.04	.08
199	0.86	1.59	0.92	1.71	.06	.12
200	0.57	1.21	0.65	1.36	.08	.15
201	1.45	2.19	1.40	2.10	-.05	.01
202	1.33	2.18	1.28	2.20	-.05	.02
203	1.23	2.27	1.16	1.93	-.07	-.34
204	0.38	0.97	0.40	0.97	.02	0.00
205	0.62	1.27	0.76	1.51	.14	-.24
2;6	0.82	1.49	0.93	1.66	.11	-.17
207	1.05	1.80	1.06	1.73	.01	-.07
208	0.37	0.97	0.37	0.90	0.00	.07
209	0.68	1.40	0.72	1.45	.04	.05
210	0.87	1.58	0.93	1.69	.06	.11
211	1.03	1.75	1.13	1.88	.10	.13
212	1.23	2.10	1.26	2.02	.03	-.08
213	1.61	2.28	1.46	2.15	-.15	-.13
214	0.41	1.05	0.42	1.01	.01	-.04
215	0.58	1.31	0.57	1.30	-.01	-.01
216	0.80	1.59	0.90	1.75	.10	.16
217	0.95	1.75	1.05	1.92	.10	.17
218	1.26	2.02	1.33	2.20	.07	.18
219	1.60	2.28	1.50	2.34	-.10	.06

Note. Underlined figures indicate that the percent of error is greater than five.

TABLE VI II

COORDINATES OF MIDDLE CURVE

RUN NUMBER	Value of X, in Feet					
	Observed		Calculated		Discrepancy	
	y=1	y=3	y=1	y=3	y=1	y=3
6						
7						
8						
9						
10						
11						
12						
13						
14						
15#	1.00	1.98	0.44	1.04	-.56	-.94
16#	1.02	1.85	0.89	1.77	-.13	-.08
17#	0.60	1.35	0.62	1.36	.02	.01
18						
19						
20						
21						
22						
23						
24						
25#	0.42	1.03	0.29	0.73	-.13	-.30
26#	0.55	1.29	0.60	1.35	.05	.06
27#	0.87	1.72	0.87	1.75	0.00	.03
28#	0.49	1.13	0.33	0.83	-.16	-.30
29						
30	1.19	2.10	1.19	2.14	0.00	.04
31	1.21	2.18	1.12	2.13	-.09	-.05
32	1.26	2.30	1.32	2.38	.06	.08
33	0.91	1.81	0.82	1.72	-.09	-.09
34	0.63	1.54	0.67	1.48	.04	-.06
35#	0.51	1.24	0.30	0.76	-.21	-.48

RUN NUMBER	Value of X, in feet					
	Observed		Calculated		Discrepancy	
	y=1	y=3	y=1	y=3	y=1	y=3
195	1.59	2.49	1.55	2.40	-.04	-.09
196	1.22	2.22	1.32	2.19	.10	-.03
197	1.39	2.30	1.26	2.17	-.13	-.13
198						
199	0.09	1.83	1.02	1.87	.03	.04
200	0.68	1.43	0.74	1.52	.06	.09
201	1.55	2.38	1.54	2.39	-.01	.01
202	1.32	2.39	1.42	2.29	.10	-.10
203	1.35	2.20	1.29	2.18	-.06	-.02
204	0.52	1.22	0.52	1.20	0.00	-.02
205	0.81	1.54	0.87	1.69	.06	.15
206	1.02	1.86	1.05	1.92	.03	.06
217	1.25	2.08	1.19	2.07	-.06	-.01
208	0.62	1.46	0.47	1.10	-.15	-.36
209	0.91	1.72	0.89	1.77	-.02	.05
210	1.13	2.01	1.13	2.07	0.00	.06
211	1.28	2.20	1.26	2.33	.08	.13
212	1.49	2.43	1.50	2.44	.01	.01
213	1.69	2.73	1.62	2.50	-.07	-.23
214	0.60	1.41	0.52	1.22	-.08	-.19
215	0.79	1.68	0.70	1.53	-.09	-.15
216	1.02	2.01	1.06	2.04	.04	.03
217	1.18	2.15	1.22	2.23	.04	.08
218	1.50	2.53	1.52	2.52	.02	-.01
219	1.65	2.66	1.70	2.68	.05	.02

#note; The results for these runs are unreliable. The error is probably due to inexperience in observation

Underlined figures indicate that the percent of error is greater than five

TABLE IX

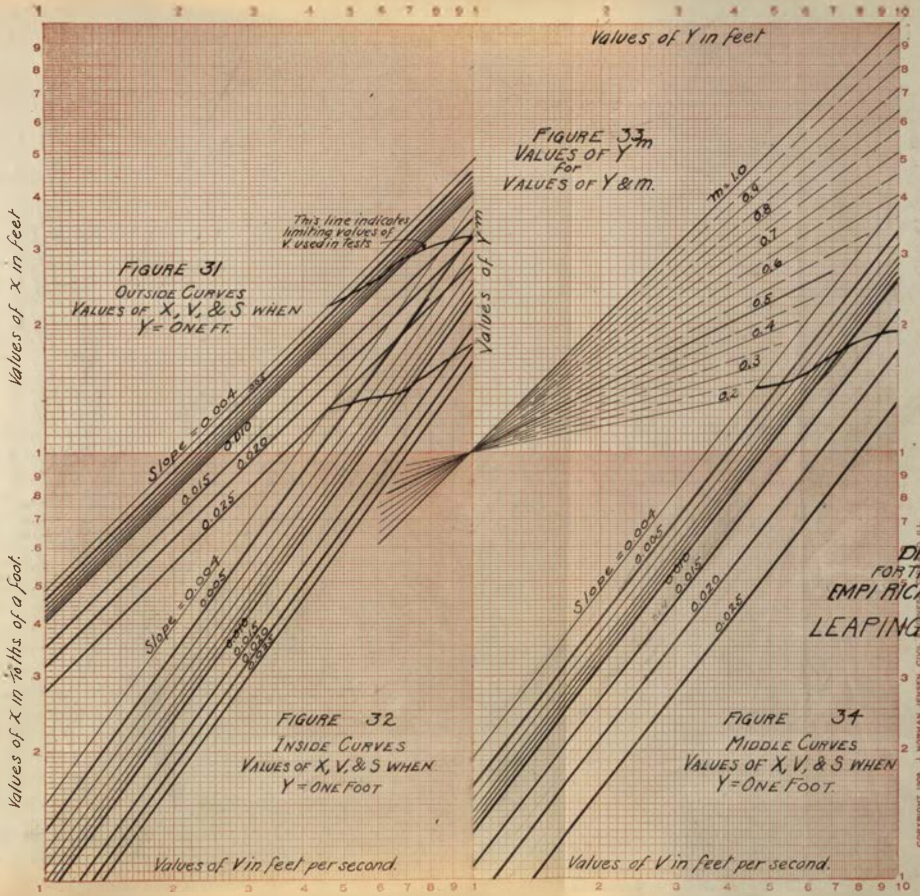
COORDINATES OF OUTSIDE CURVE

RUN NUMBER	Value of X, in Feet				Discrepancy	
	Observed		Calculated			
	y=1	y=3	y=1	y=3	y=1	y=3
6	1.42	2.35	1.45	2.36	.03	.01
7	2.20	3.27	2.10	3.24	-.10	-.03
8	1.90	2.96	1.96	3.04	.06	.08
9	1.64	2.63	1.69	2.70	.05	.07
10	1.05	1.79	1.04	1.78	-.01	-.01
11	2.62	3.99	2.41	3.72	-.21	-.27
12	2.37	3.69	2.30	3.56	-.07	-.13
13	2.18	3.42	2.25	3.48	.07	.06
14	1.88	3.01	1.85	3.07	-.03	.06
15#	1.50	2.48	1.18	2.02	-.32	-.46
16	1.59	2.55	1.55	2.51	-.04	-.04
17	1.14	1.94	1.16	1.95	.02	.01
18	0.67	1.17	0.54	0.97	-.13	-.20
19	2.03	3.18	1.92	3.00	-.11	-.18
20	1.82	2.88	1.86	2.92	.04	.04
21	2.36	3.57	2.25	3.44	-.11	-.13
22	2.10	3.23	2.01	3.12	-.09	-.11
23	1.65	2.66	1.65	2.64	0.00	-.02
24	1.32	2.20	1.25	2.07	-.07	-.13
25	0.79	1.37	0.70	1.23	-.09	-.14
26	1.37	2.13	1.30	2.16	-.07	.03
27	1.75	2.82	1.73	2.78	-.02	-.04
28	0.92	1.58	0.82	1.43	-.10	-.15
29	2.50	3.76	2.50	3.78	0.00	.02
30	2.08	3.18	2.19	3.40	.11	.22
31	2.28	3.46	2.15	3.35	-.13	-.11
32	2.70	4.05	2.50	3.85	-.20	-.20
33	1.88	3.00	1.71	2.77	-.17	-.13
34	1.55	2.60	1.46	2.42	-.09	-.18
35	0.89	1.54	0.89	1.56	0.00	.02

RUN NUMBER	Value of X, in Feet				Discrepancy	
	Observed		Calculated			
	y=1	y=3	y=1	y=3	y=1	y=3
195	2.65	3.88	2.51	3.72	-.14	-.16
196	2.23	3.40	2.21	3.38	-.02	-.02
197						
198	2.00	2.99	1.90	2.98	-.10	-.01
199	1.75	2.72	1.82	2.86	.07	.14
200	1.40	2.30	1.42	2.31	.02	.01
201						
202	2.52	3.75	2.41	3.59	-.11	-.16
203	2.35	3.45	2.24	3.43	-.11	-.02
204	1.09	1.81	1.05	1.80	-.04	-.01
205	1.59	2.55	1.65	2.64	.06	.09
206	1.87	2.90	1.91	3.00	.04	.10
207	2.11	3.29	2.14	3.30	.03	.01
208	1.15	1.99	1.07	1.84	-.08	-.15
209	1.72	2.79	1.72	2.76	0.00	-.03
210	1.94	3.03	2.08	3.22	.14	.19
211	2.32	3.50	2.39	3.60	.07	.10
212	2.58	3.75	2.60	3.87	.02	.12
213	3.20	4.21	2.89	4.19	-.11	-.02
214	1.23	2.00	1.27	2.17	.04	.17
215	1.60	2.66	1.47	2.43	-.13	-.23
216	2.11	3.29	2.03	3.19	-.08	-.10
217	2.22	3.41	2.26	3.50	.04	.09
218	2.75	----	2.70	4.02	-.05	----
219	3.08	----	2.91	4.25	-.17	----

#note; The results from this observation have disagreed with the other observations throughout all of the runs.

Underlined figures indicate that the percent of error is greater than five.



DIAGRAMS
FOR THE SOLUTION OF
EMPIRICAL EXPRESSIONS
FOR
LEAPING WEIR CURVES

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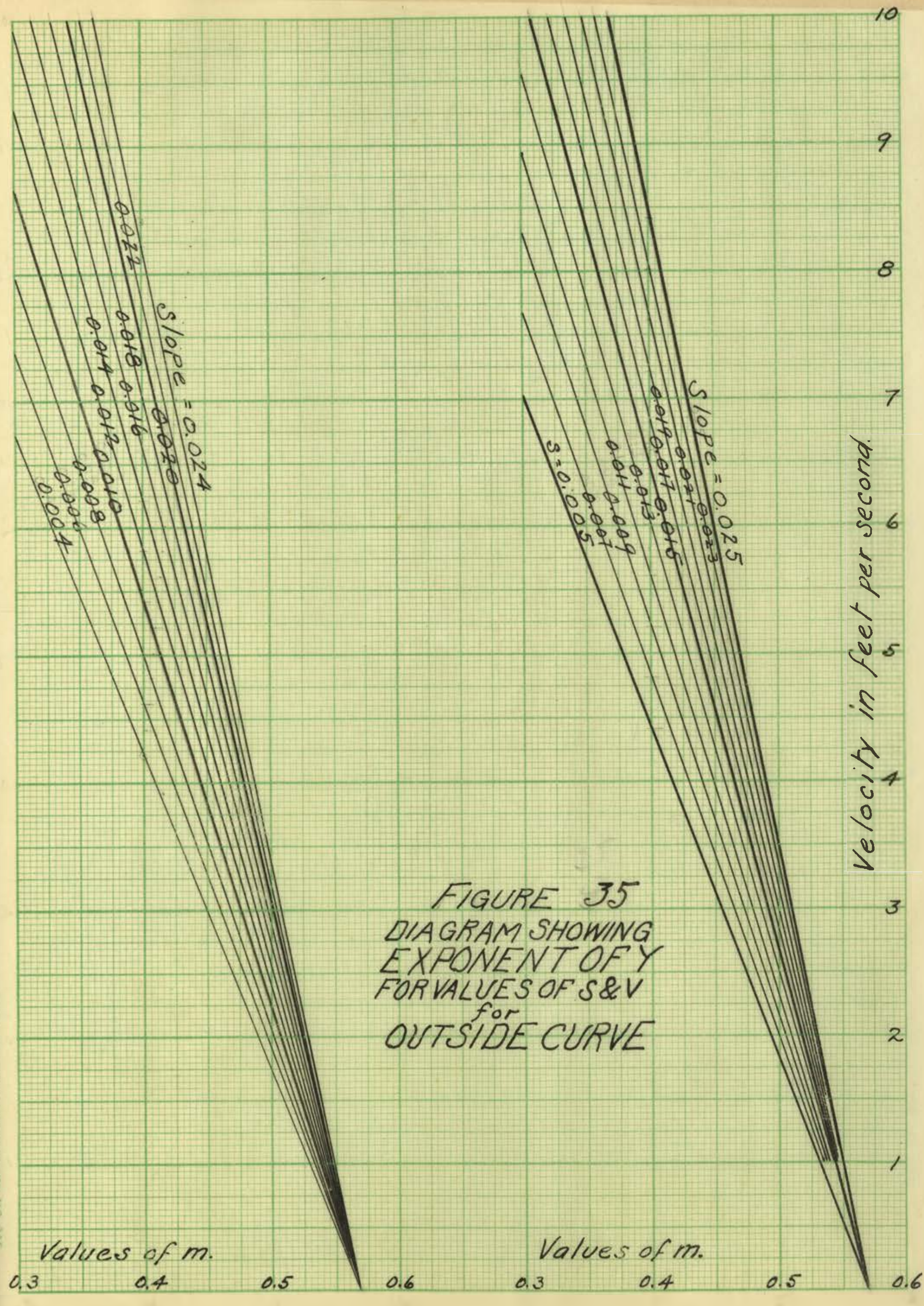


FIGURE 35
DIAGRAM SHOWING
EXPONENT OF Y
FOR VALUES OF S&V
for
OUTSIDE CURVE

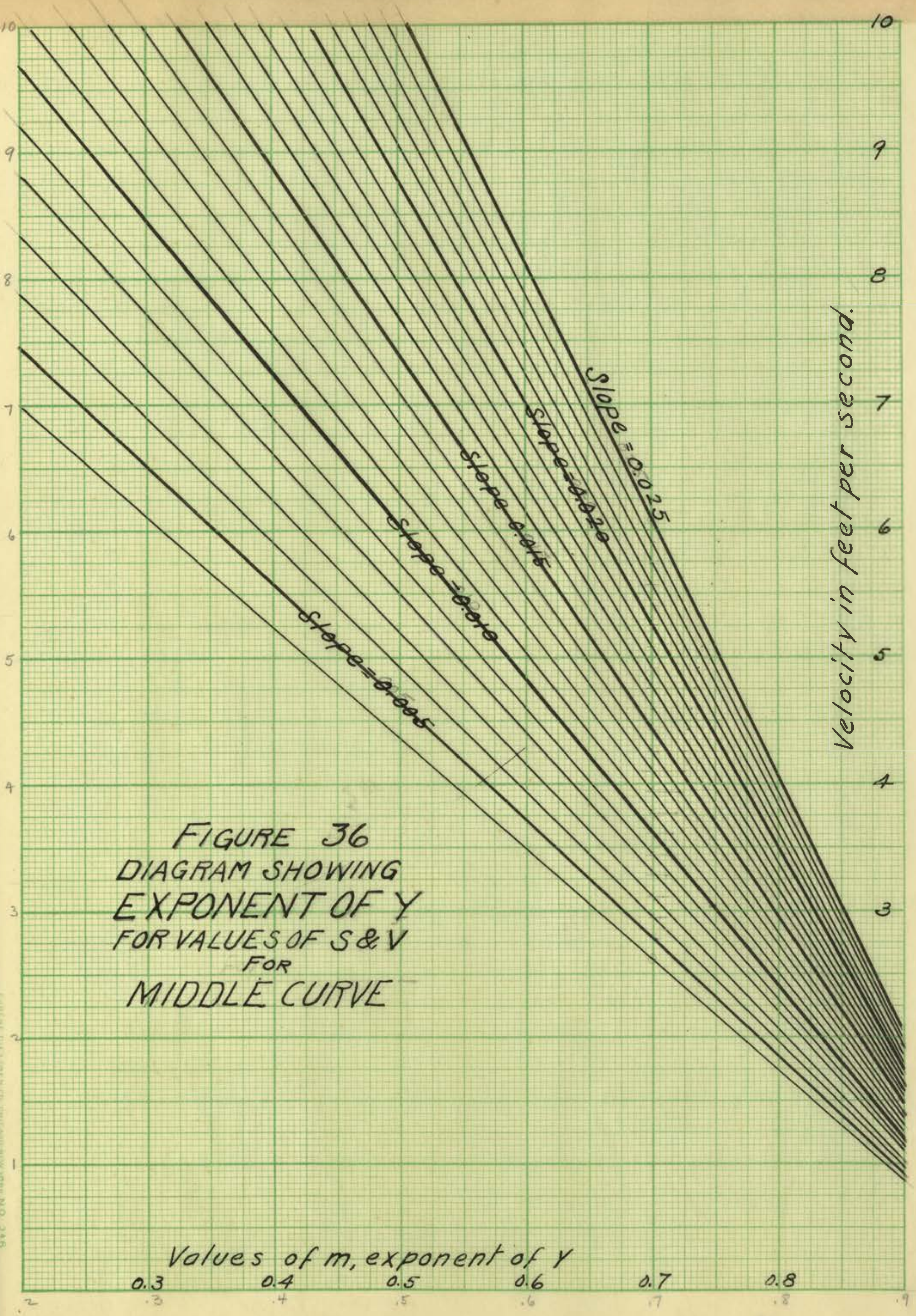


FIGURE 36
DIAGRAM SHOWING
EXPONENT OF Y
FOR VALUES OF S & V
FOR
MIDDLE CURVE

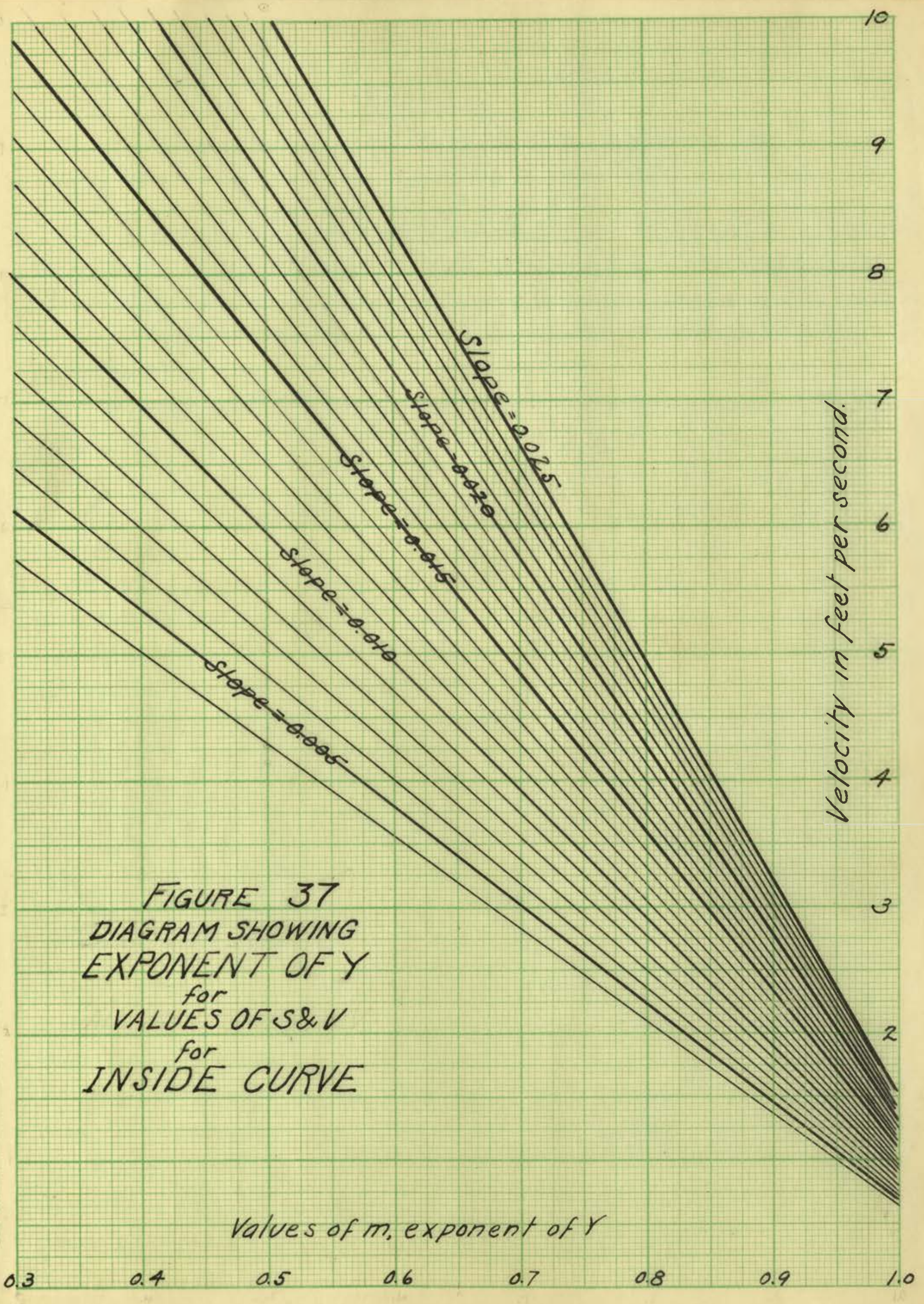


FIGURE 37
DIAGRAM SHOWING
EXPONENT OF Y
for
VALUES OF S&V
for
INSIDE CURVE

CHAPTER V

-----:DESIGN OF LEAPING WEIRS:-----

Sect. 35. Formulas and Diagrams:- In the design of a leaping weir by the method proposed in this thesis, it will be necessary to solve one or all of the equations for the inside, middle and outside curves. To facilitate this solution Figures 31, 32, 33 and 34 have been prepared.

In order that the coordinates of a point may be determined, the slope of the invert and the velocity of the approaching stream must be known. Figures 31, 32 and 34 will give the value of x when y is unity, on the respective curve desired. For any other value of y this value of x (when y is unity) must be multiplied by the new value of y raised to the mth. power. Figures 35, 36, and 37 give the values of m for different values of slope and velocity on the three different curves and Figure 33 gives the value of y^m for all ordinary values of y and m.

For example, let it be required to find the abscissa of a point whose ordinate is 2.5 feet on the outside curve of a stream with a velocity of approach of 4 feet per second, leaping from a sewer on a grade of 0.010 From Figure 31 the value of x when y is unity is 1.60 From Figure 35 the value of m when s is 0.010 and v is 4.0 is 0.445 From Figure 33 $2.5^{0.445}$ is 1.5 The required value of x is then (1.50)(1.60) or 2.40 feet.

Sect. 36. Investigation of Existing Weir:- The use of the diagrams in the investigation of an existing weir is more simple than their use in the design of a new weir.

Figure 5, showing the existing weir at Blackhoof St., Wapkoneta, Ohio, will be taken as an example for investigation. The diameter of the pipe is eighteen inches, and its slope, though not shown, will be assumed as 0.010 The coefficient of roughness in Kutter's formula will be taken as 0.015 for vitrified sewer pipe.

005

The coordinates of the lower lip of the weir are $x = 0.92$ and $y = 0.75$

The information ordinarily desired for a weir is the rate of flow over the weir which is necessary to start a discharge from the overflow and the rate of flow over the weir at which the dry weather intercepter will cease to discharge. The first rate of flow will also represent the full capacity of the intercepter.

The first condition requires that the coordinates of a point on the outside curve shall be (0.92; 0.75) This can be solved most easily by a method of trial. It is known that the value of x, when y is one, multiplied by 0.75^m is equal to 0.92

- all need to .005 as safe. Should have been used to .010

1st. Assume $v = 3$ ft. per second
then x for $y = 1.0$ (Fig 31) is 1.35 and m (Fig. 35) is 0.455
then y^m (Fig.33) is 0.88 and x is (1.35)(0.88) which gives a value too large

2nd. Assume $v = 2.5$ feet per second
then x for $y = 1.0$ is 1.13 and m is 0.474
then y^m is 0.88 and x is $(1.13)(0.88) = 0.99$, which again gives a value too large.

3rd. Assume $v = 2.3$ feet per second
then x for $y = 1$ is 1.04 and m is 0.476
then y^m is 0.88 and x is $(1.04)(0.88) = 0.92$

The velocity in the sewer as the overflow begins to discharge is therefore, 2.3 ft. per second. From Kutter's formula it is found that the velocity of flow when the sewer is full is 4.90 feet per second, and the rate of discharge is 8.6 cubic feet per second. From Figure 10, when the ratio of the velocity part full to the velocity when full is 0.47, the depth of flow is about 0.21 ft. and the rate of discharge is about 0.344 cubic feet per second. That is to say, the capacity of the dry weather intercepter should be 0.344 cubic feet per second, and the overflow will begin to discharge when the rate of flow in the main sewer exceeds this rate.

The second condition requires that the coordinates of a point on the middle[#] curve shall be (0.92; 0.75) A method of trial is to be followed;

1st. Assume $v = 4.3$ feet per second.
then x_m for $y = 1$ (fig.34) $= 1.10$ & M(Fig.36) $= .535$
then y^m (Fig.33) is 0.86 and x is (1.10)(0.86)
which equals 0.95. Too large a value.

2nd. Assume $v = 4.2$ feet per second
then x_m for $y = 1$ is 1.07 and m is 0.520
then y^m is 0.86 and x is $(1.07)(0.86) = 0.92$

When the dry weather intercepter ceases to discharge the velocity in the contributing sewer is 4.2 feet per second, which is 88% of the full velocity. From Figure 10 the rate of discharge is 31% of the full capacity of the sewer or 2.66 cubic feet per second, and the depth of flow is 0.58 feet. That is to say when the contributing sewer is discharging at about one third of its capacity the dry weather intercepter ceases to act.

Sect. 37. The Design Of A Leaping Weir:- Local conditions and other considerations must determine the diameter of the contributing or inlet sewer, the slope of this sewer, its coefficient of roughness, the full capacity of the dry weather intercepter, and the rate of discharge from the overflow when the intercepter is to cease discharging.

For the purpose of illustration all of these factors will be assumed as for the Blackhoof Street weir just studied. In this case the fixed conditions are: Diameter, 18 inches; slope 0.010; coefficient of roughness 0.015; capacity of the dry weather intercepter, 0.344 cubic feet per second; and the amount being discharged from the overflow when the intercepter ceases to discharge is 2.66 cubic feet per second.

An 18 inch sewer on a grade of 0.010 has a capacity, when full,

---

[#]Footnote: The inside curve is of little practical value as it represents little more than the spray due to the breaking up of the stream.

has a capacity of 8.6 cubic feet per second and a velocity of 4.9 feet per second. When discharging at the rate of 0.344 cubic feet per second, or at 4% of its full capacity, the velocity will be 47% of the full velocity or 2.3 feet per second. When discharging at the rate of 2.66 cubic feet per second, or at 31% of its full capacity, the velocity will be 88% of the full velocity or 4.2 feet per second. These ratios were read from Figure 10.

The abscissa of the middle curve, when $y = 1.0$ and $v = 4.2$ feet per second is 1.07 (Fig. 34) and the abscissa of the outside curve when $y = 1.0$ and $v = 2.3$ feet per second is 1.04 (Fig. 31) The value of m for the middle curve is 0.520 (Fig. 36) and for the outside curve is 0.476 (Fig. 35), therefore:

$$1.04\,y^{.476} = 1.07\,y^{.52} \quad \text{and} \quad y^{.044} = 0.973$$

which reduces to $y = 0.536$

It is not possible to read the abscissas of the curves from Figures 31, 32, and 35 to the nearest 0.01 foot, and a discrepancy of this much will materially affect the value of y when solved by the above process, for example, assume that

$1.04\,y^{.476} = 1.06\,y^{.52}$. Solving $y = 0.661$ or assume
$1.05\,y^{.476} = 1.06\,y^{.52}$. Solving $y = 0.832$

Since such a small discrepancy in reading the abscissas, when the value of y is one will make such a large difference in the final value of y, a method of trial in which the values of y are assumed until the abscissas of the points on the two curves become equal will give more accurate results. Following this procedure: it is evident from the values of the abscissas when $y = 1$ that the desired value of y is less than 1. It will first be assumed that $y = 0.5$. Then $(1.04)(0.5)^{.476} = (1.07)(0.5)^{.52}$ and solving $0.748 = 0.746$ Now assumed that $y = 0.75$, then $(1.04)(0.75)^{.476} = (1.07)(0.75)^{.52}$ and solving $0.907 = 0.910$

It becomes evident that the value of y can be selected within a relatively large range, providing the corresponding value of x is selected. Any value of y between 0.5 and 1.0 would probably be suitable, but one foot would probably give the most reliable results since the diagrams may not be reliable for values of y less than 0.75 of a foot.

The design of leaping weirs by this method will probably give greater accuracy than a design by the method credited to Unwin and described in section 4. The discrepancy between Unwin's method and the observations are shown in Table I. The discrepancy between the results of the preceding method and the observations are shown in Tables VII, VIII and IX, and in Figure 30

Sect. 38. Structural Features Of A Leaping Weir:- The lower lip of the leaping weir will be subjected to rough usage due to the impact of falling objects. To resist this wear the lip should be made of a heavy cast iron plate as shown in Figure 5. It is desirable to have this plate adjustable within 2 1/2 to 3 inches on either side of the computed value of x in order that proper adjustment may be made after installation.

In order to apply the preceding method of design the upper lip should be smooth and circular. No unusual protection against erosion need be given to the upper lip, unless the character of the sewage is unusually gritty, or the material of the weir is soft. In Milwaukee the upper and lower lips in brick and concrete sewers have been made of a hard granite: See. Figure 2.

It is sometimes desirable to place a grit chamber above the weir to protect the dry weather intercepter from the materials which would be dropped into it. Since the intercepter is usually smaller than the inlet sewer, there is a possibility that it may become clogged, particularly if the velocity is not maintained as high as in the influent sewer.

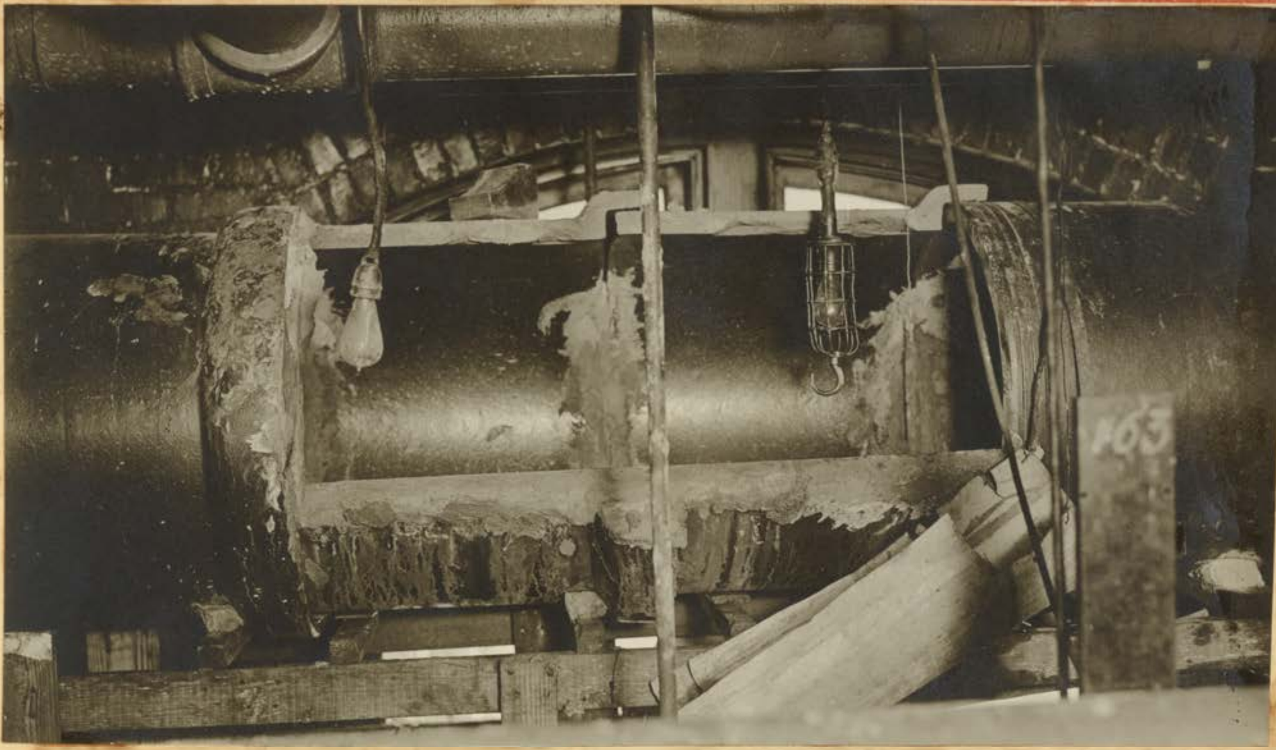


FIGURE 38
OVERFLOW WEIR READY FOR OPERATION

FIGURE 39
OVERFLOW WEIR IN OPERATION

PART II

OVERFLOW WEIRS

CHAPTER VI:

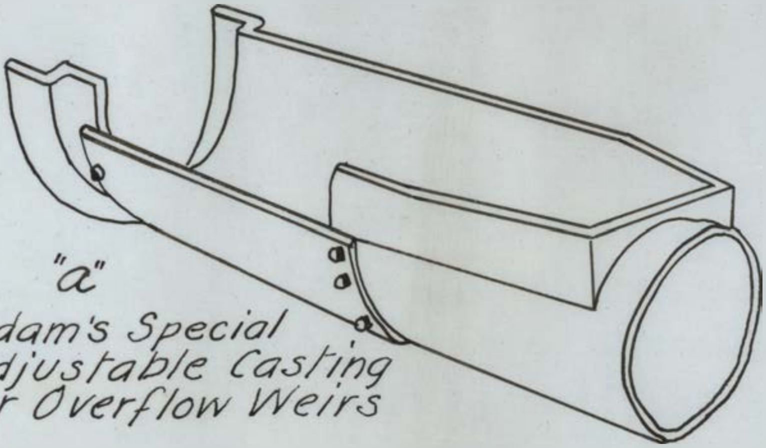
INTRODUCTION

Sect. 39. Definition:- The term overflow weir refers to an opening in the side of a conduit over which a portion of the contents of the conduit will spill when the depth of flow becomes sufficiently great. Photographs of the overflow weir used in the series of tests to be described are shown in Figures 38 and 39.

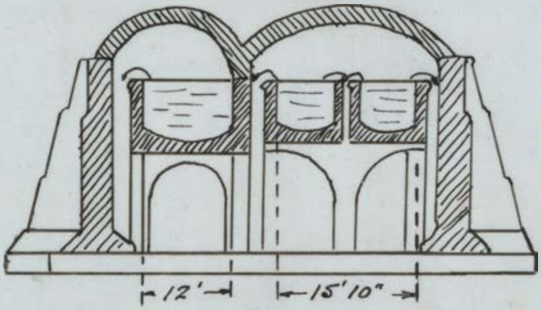
Sect. 40. Purpose:- An overflow weir, as used in sewers, is a device for controlling the amount of water to be carried in the sewer below the weir. Its purpose is to relieve the sewer of a portion of its contents in order to prevent overcharging, and consequent blocking up of the sewer.

Among the advantages of an overflow weir are: it does not consume any head for its operation; none of the gritty material in the contributing sewer is discharged into the relief sewer; it is easily constructed in an existing sewer; and when placed properly in a combined sewer only dilute sanitary sewage is removed, the undilute sanitary flow passing the weir because of inadequate depth to overflow. Its greatest disadvantage is that if the main sewer terminates in a treatment plant, during times of storm the plant must treat a large amount of dilute sewage.

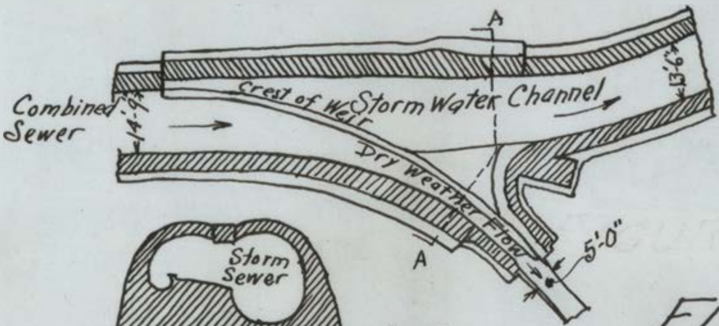
Sect. 41. Historical Resumé:- So far as could be determined by a search of existing literature no tests of overflow weirs in sewers have been made the results of which have been published. W.C. Parmley analyzes the hydraulics of an overflow weir in his art-



"a"
Adam's Special
Adjustable Casting
for Overflow Weirs



"b"
Overflow Weir
Northern Outfall Sewer
London



"C"
Overflow Weir at
Cleveland

FIGURE 40

icle on the "Walworth Run Sewer" in the Transactions of the American Society of Civil Engineers, Volume XL, page 341, and quoted in Chapt. VIII of this work. A bibliography of the references to overflow weirs which were found in the search has been given in the Preface.

The use of overflow and leaping weirs has apparently been more extensive in Europe than in the United States, if the number of times they are mentioned in the engineering literature of the two continents is to be taken as a criterion. No definite date as to the installation of the first overflow weir was found, but apparently the principle was applied as early as the first installation of extensive sewerage systems in the early part of the nineteenth century.

Sect. 42. Existing Installations:- There are many overflow weirs in existence, but the majority have some modification of the simple form used in these tests. Figure 40 "C", taken from Metcalf and Eddy's "American Sewerage Practice" Volume I shows the overflow weir in the Walworth Run sewer at Cleveland,Ohio. It is to be noted that this weir is built on a curve which would probably cause the discharge to be different from the same weir if built on a straight line. Figure 40"A" shows an adjustable casting for overflow weirs manufactured by the Adams Hydraulic Company of York, England. The edge of this weir is an element of the outside of the pipe, instead of the inside as is shown in Figure 38. Figure 40 "B" shows the over flow weir on the outfall of the London Main Drainage.

A somewhat complicated arrangement is shown in Figure 41. It is a weir designed by W.S. Shields for use at Lombard, Illinois.

The hydraulic elements of such installations as are shown in these figures cannot be determined with great accuracy since the factors entering into the problem are so many and so difficult to determine. In spite of the lack of definite information on which to base the design of these weirs they are usually said to give entire satisfaction. This is probably because little care is given to the

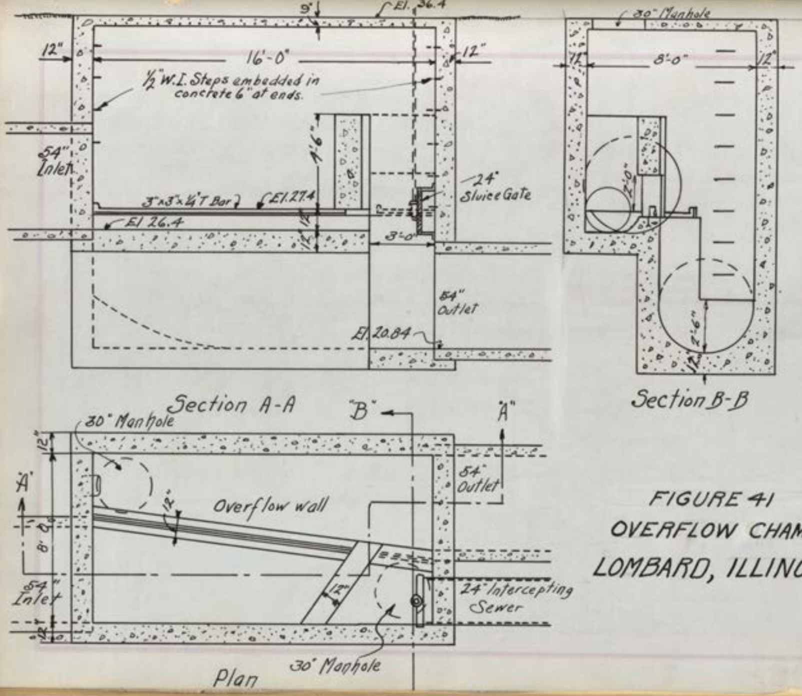
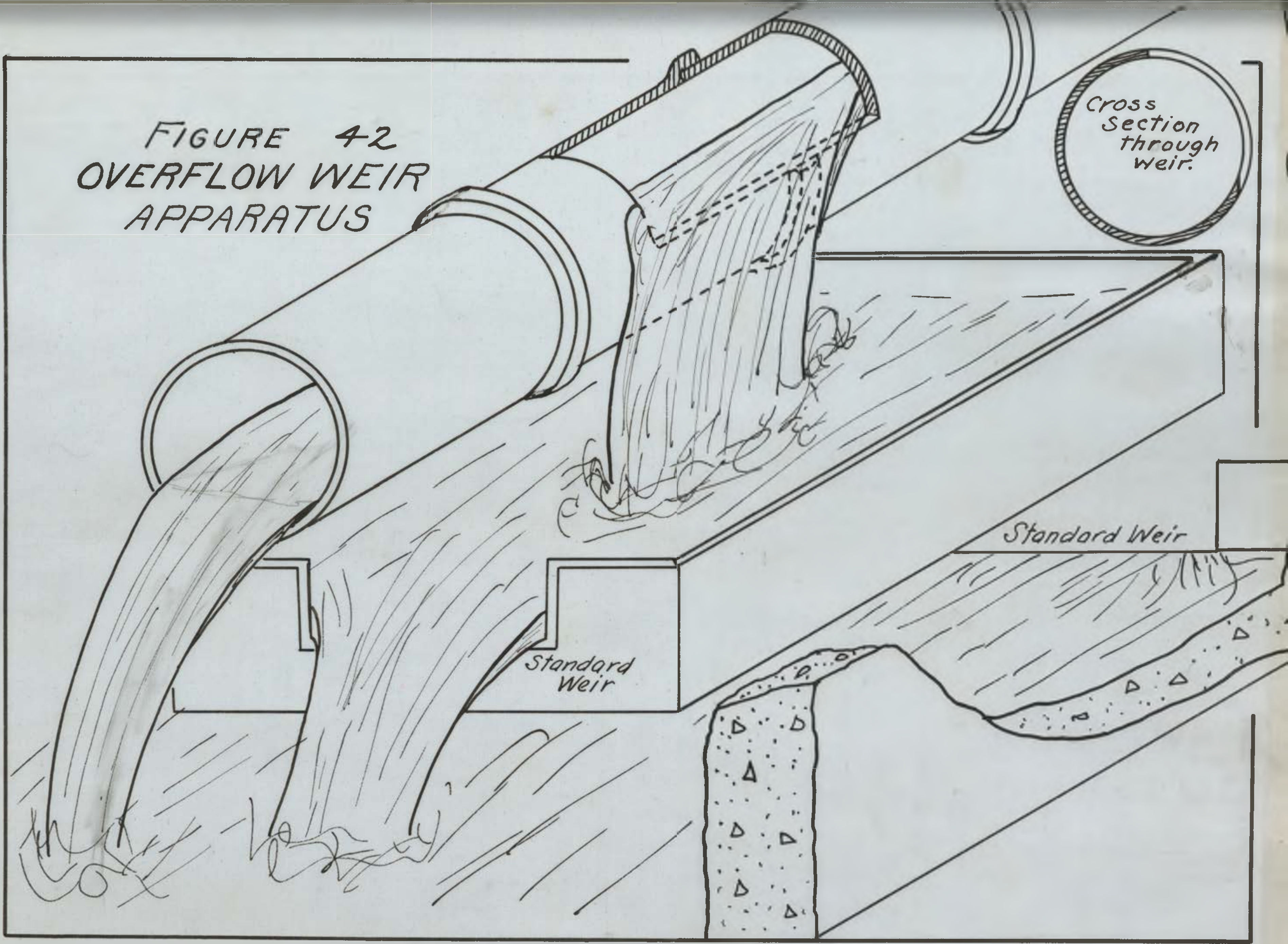


FIGURE 41
OVERFLOW CHAMBER
LOMBARD, ILLINOIS.

exact amount of sewage to be intercepted, and the purpose of relieving the main sewer is accomplished. As to whether this purpose might not have been accomplished at a smaller expense is an open question.

FIGURE 42
OVERFLOW WEIR
APPARATUS



-----:CHAPTER VII:-----

--:INVESTIGATIONS OF OVERFLOW WEIRS AT THE UNIVERSITY OF ILLINOIS:--

Sect. 43. Period Covered By Experimental Work:- The experimental work on the overflow weirs was run in conjunction with the tests on the leaping weirs. The apparatus for the leaping weirs was commenced in May 1916. The overflow weir was cut in the eighteen inch pipe on July 31, 1916. The first run which is recorded in Table X was made on August 5th. although some preliminary runs were made earlier in the month. The observations on the overflow weirs in the eighteen inch pipe were completed on August 18th. The observations on the overflow weir in the twentyfour inch pipe were commenced on September 23rd. and were completed on November 18th.

Sect. 44. Description Of Apparatus:- Figure 42 shows in diagrammatic form the equipment and arrangement used in the observations on the overflow weirs. The same sewer pipe, flume, cradle, measuring weirs, pumps, etc. were used in these tests as for the leaping weirs described in Chapter II. The only additional apparatus necessary for the observations on the overflow weirs was the weir box and appurtenances suspended beneath the overflow weir.

The overflow weir shown in Figure 38 consisted of a slot cut in the side of the sewer pipe. The upper portion of this slot was on the center line of the top of the pipe. The two sides were at right angles to the axis of the pipe, and were pointed up with cement so as to be sharp and square, and to follow the curve of the pipe, making a smooth surface on the inside of the pipe. The distance from the lower edge of the weir to the lower end of the sewer was never less than ten feet, and the distance from the upper end of the weir to the upper end of the sewer was fixed at eleven feet for all tests. These distances were sufficient to remove the weir from the turbulence occasioned by the water entering and leaving the pipe.

The lower edge of the slot, or the true weir, consisted of an iron bar ground to a sharp edge and set in cement mortar so that the upper edge formed an element of the inside of the pipe cylinder. The inside face was smoothed off with cement mortar and the outside face was angled so that the water fell freely from the weir, with air beneath the falling stream. A cross section of the pipe at the weir is shown in Figure 42.

Sect. 45. Making A Run:- After measuring the length of the weir and its height above the invert the order of procedure in making a run was as follows:

First; Take measurements from the level line suspended above the sewer down to the invert to determine the slope, and adjust the slope by means of the jacks supporting the cradle. The adjustment was assisted by running a small stream of water down the pipe, which indicated, by its uneveness, spots out of alignment.

Second; Start the steam pump and fill the stand pipe until the governor shut off the pump. Prime the centrifugal pump from the stand pipe. Open the valve into the stilling box and start the centrifugal pump. By means of the valve above the stilling box and the throttle on the engine and pump, adjust the two pumps to the proper speed to deliver the desired rate of flow. The pumps were then allowed to run for from two to five minutes until no fluctuations in conditions were apparent.

Third; Set the hook gage on the standard three foot weir to about the correct position. The entire discharge over and past the overflow weir were combined and discharged over this standard three foot weir.

Fourth; Set and read the hook gage in the weir box, measuring the discharge over the overflow weir, and immediately readjust and read the gage on the three foot weir.

Fifth; Observe the distance from the top of the pipe to the surface of the water at the following points; (a) 12 inches above the overflow weir, (b) at the upper end of the weir, (c) at the middle of the weir, (d) at the lower end of the weir, and (e) 12 inches below the weir. All of these points were not recorded for every run.

Sect. 46. Difficulties Observed:- When the eighteen inch pipe was put in place the outside of the bell rested on the bottom of the flume. In this position it was not possible to much more than half fill the pipe without overflowing the flume. Because of this the measurements on the weir placed half way up the eighteen inch pipe had to be abandoned. The twentyfour inch pipe was placed with its invert in line with the bottom of the flume. In this position it was possible to fill the pipe about three fourths full, when on a low slope. The pumps were working to the limit of their capacity. After having made several runs on the steep slopes it was noticed that at the upper end of the weir there was a sufficiently sudden change in the slope of the pipe to vitiate the results of runs numbers 220 to 298 inclusive.

The capacity of the weir box was taxed to the limit for rates of discharge over the overflow weir of more than three second feet. It was difficult to obtain good readings on the gage because of the turbulence of the large rates of flow.

Sect. 47. Summary of Direct Observations:- The diameter of the pipe, the slope of the invert, the height of the weir above the invert, the length of the weir, and the depth of water in the pipe and on the overflow weir were observed directly, and with the exception of the last two are recorded in Table X.

Sect. 48. Computed Results:- The rates of flow, n in Kutter's formula, and the depth of flow in the pipe were calculated in the same manner as for the leaping weirs described in Chapter III

The rates of flow in the pipe above the overflow weir, and the rate of discharge over the overflow weir are recorded, for each run, in Table X. The value of n in Kutter's formula was taken as 0.013 since the conditions were the same as for the leaping weir.

TABLE X

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of weir Feet	Rate of Disch.'g Second Feet		RUN NUMBER	Length of weir Feet.	Rate of Disch'g Second Feet	
		Q	q			Q	q
Diameter of pipe 18 inches							
Height of weir above invert 0.565 feet							
Slope .018							
37	1.307	6.61	0.136	44	2.146	6.54	0.222
38	1.307	5.81	0.0768	45	2.146	6.70	0.278
39	1.307	5.00	0.334	46	3/583	6.70	0.585
40	1.307	4.76	0.0155	47	3.583	6.32	0.446
41	2.146	4.80	0.0496	48	3.583	5.87	0.339
42	2.146	5.27	0.0825	49	3.583	5.49	0.243
43	2.176	5.86	0.156	50	3.583	5.16	0.164
Slope 0.010							
51	1.302	7.09	0.259	62	2.135	5.86	0.281
52	1.302	5.78	0.150	63	2.135	5.20	0.175
53	1.302	6.21	0.0956	64	2.135	4.75	0.0866
54	1.302	5.80	0.143	65	3.583	4.91	0.173
55	1.302	5.35	0.0956	66	3.583	5.11	0.253
56	1.302	4.43	0.0163	67	3.583	5.28	0.298
57	1.302	4.64	0.0448	68	3.583	5.65	0.390
58	2.135	7.10	0.475	69	3.583	6.05	0.496
59	2.135	7.10	0.500	70	3.583	6.53	0.625
60	2.135	6.53	0.379	71	3/583	7.15	0.827
61	2.135	6.06	0.312				
Slope .007							
72	1.307	6.70	0.280	82	2.146	6.16	0.386
73	1.307	6.21	0.199	83	2.146	6.59	0.466
74	1.307	5.66	0.168	84	2.146	6.87	0.516
75	1.307	5.31	0.132	85	3.583	6.93	0.836
76	1.307	4.96	0.0854	86	3.583	6.56	0.728
77	1.307	4.66	0.0496	87	3.583	6.18	0.615
78	2.146	4.45	0.0709	88	3.583	5.36	0.482
79	2.146	3.96	0.0294	89	3.583	4/99	0.271
80	2.146	5.05	0.182	90	3.583	3.97	0.0595
81	2.146	5.69	0.294	91	3.583	4.79	0.221
Slope 0.004							
92	1.323	6.65	0.379	102	2.156	4.61	0.219
93	1.323	6.04	0.303	103	2.156	5.36	0.1153
94	1.323	5.46	0.234	104	2.156	3.90	0.102
95	1.323	4/56	0.145	105	3.583	3.90	0.1348
96	1.323	3.98	0.0925	106	3.583	4.29	0.230
97	1.323	3.36	0.0346	107	3.583	4.85	0.358
98	2.156	6.50	0.577	108	3.583	5.59	0.586
99	2.156	5.83	0.490	109	3.583	6.10	0.780

Q = Rate above weir. q = Rate over weir.

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet		RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet	
		Q	q			Q	q
Diameter of pipe 18 inches							
Height of weir above invert, 0.565 feet							
Slope 0.004							
100	2.156	5.35	0.379	110	3.583	6.50	0.900
101	2.156	4.94	0.281	111	3.583	6.73	0.981
Height of weir above invert, 0.464 feet							
Slope 0.004							
112	1.292	6.67	0.674	122	2.135	6.90	1.131
113	1.292	6.13	0.584	123	3.583	6.93	1.647
114	1.292	5.41	0.478	124	3.583	6.62	1.548
115	1.292	3.87	0.270	125	3/583	6.62	1.56
116	1.292	2.96	0.1076	126	3.583	5.84	1.236
117	1.292	2.12	0.026	127	3.583	4.55	0.780
118	2.135	2.70	0.0971	128	3.583	3.79	0.603
119	2.135	3.60	0.291	129	3.583	3.25	0.387
120	2.135	5.34	0.665	130	3.583	2.45	0.1513
121	2.135	6.16	0.995				
Slope 0.007							
131	1.297	6.76	0.637	142	2.135	5.11	0.633
132	1.297	6.26	0.584	143	2.135	4.23	0.433
133	1.297	5.66	0.472	144	2.135	3.42	0.277
134	1.297	4.90	0.362	145	3.583	7.21	1.709
135	1.297	4.07	0.229	146	3.583	6.35	1.389
136	1.297	3.62	0.1752	147	3.583	5.85	1.238
137	1.297	2.70	0.0873	148	3.583	5.19	0.979
138	2.135	7.10	1.160	149	3.583	4.32	0.646
139	2.135	6.82	1.050	150	3.583	3.31	0.287
140	2.135	6.27	0.910	151	3.583	3/10	0.242
141	2.135	5.72	0.801				
Slope 0.010							
152	1.328	7.10	0.594	162	2.167	5.07	0.602
153	1.328	6.59	0.506	163	2.167	4.66	0.420
154	1.328	5.81	0.421	164	2.167	3.81	0.271
155	1.328	3.34	0.333?	165	2.167	3.00	0.118
156	1.328	3.10	0.272?	166	3.583	7.39	1.595
157	1.328	3.99	0.164	167	3.583	6.58	1.30
158	1.328	3.15	0.0758	168	3.583	6.00	1.058
159	2.167	7.40	1.125	169	3.583	5.30	0.900
160	2.167	6.74	0.918	170	3.583	4.61	0.638
161	2.167	5.95	0.750	171	3/583	3.79	0.402

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of Weir Feet	Rate Discharge Second Feet		RUN NUMBER	Length Weir Feet	Rate Discharge Second Feet	
		Q	q			Q	q
--------Diameter of pipe 18 inches--------- ---Height of weir above invert, 0.464 feet--							
----Slope 0.014----							
172	1.328	7.10	0.480	182	2.177	4.60	0.300
173	1.328	6.44	0.404	183	2.177	3.81	0.163
174	1.328	6.11	0.354	184	2.177	3.09	0.0539
175	1.328	5.41	0.281	185	3.583	3.04	0.0828
176	1.328	3.76	0.090	186	3.583	4.01	0.381
177	1.328	4.44	0.166	187	3.583	4.61	0.551
178	1.328	5.51	0.288	188	3.583	5.68	0.788
179	2.177	7.10	0.809	189	3.583	6.18	1.059
180	2.177	6.08	0.616	190	3.583	6.79	1.218
181	2.177	5.32	0.466	191	3.583	7.41	1.50
--------Diameter of pipe 24 inches--------- --Height of weir above invert 1.0 foot # ---							
----Slope 0.015----							
220	1.16	11.80	0.111	231	2.27	11.72	0.1630
221	1.16	11.49	0.0866	232	3.78	11.88	0.222
222	1.16	10.76	0.0526	233	3.78	12.70	0.348
223	1.16	10.65	0.046	234	3.78	12.46	0.308
224	1.16	10.00	0.0155	235	3.78	11.65	0.206
225	2.27	12.52	0.269	236	3.78	12.10	0.251
226	2.27	11.49	0.138	237	3.78	11.94	0.251
227	2.27	10.33	0.336	238	3/78	11.32	0.219
228	2.27	10.52	0.0483	239	3.78	11.22	0.1320
229	2.27	10.79	0.0796	240	3.78	9.38	0.0545
230	2.27	11.05	0.1170				
----Slope 0.010----							
241	1.18	11.35	0.156	258	2.28	10.70	0.1538
242	1.18	11.59	0.181	259	2.20	11.31	0.2115
243	1.18	11.78	0.191	260	2.28	10.88	0.1675
244	1.18	11.60	0.169	261	2.28	10.61	0.1420
245	1.18	11.38	0.164	262	2.28	9.65	0.0713
246	1.18	11.10	0.143	263	3.79	12.33	0.442
247	1.18	11.00	0.1411	264	3.79	12.20	0.425
248	1.18	10.96	0.1411	265	3.79	12.05	0.414
249	1.18	10.83	0.132	266	3.79	11.90	0.398
250	1.18	10.10	0.0925	267	3.79	11.78	0.373
251	2.28	12.18	0.292	268	3.79	11.74	0.366

\# Note:The results obtained from runs 220 to 298 were not used in reaching the conclusions, because of a probable error in measuring the slope for these runs

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet		RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet	
		Q	q			Q	q
Diameter of pipe 24 inches							
Height of weir above invert, 1.0 foot							
Slope 0.010							
252	2.28	12.10	0.301	269	3.79	11.62	0.332
253	2.28	10.96	0.1730	270	3.79	11.50	0.320
254	2.28	11.75	0.2465	271	3.79	11.11	0.244
255	2.28	12.11	0.284	272	3.79	10.52	0.132
256	2.28	11.98	0.2760	273	3.79	11.38	0.230
257	2.28	11.49	0.2195	274	3.79	11.22	0.223
Slope 0.007							
275	1.18	11.80	0.1582	287	2.27	9.34	0.0448
276	1.18	11.76	0.140	288	2.27	7.07	0.0074
277	1.18	10.38	0.0769	289	3.78	12.33	0.406
278	1.18	10.15	0.066	290	3.78	11.95	0.370
279	1.18	9.79	0.0447	291	3.78	11.05	0.219
280	1.18	9.50	0.357	292	3.78	11.81	0.3415
281	1.18	8.80	0.0061	293	3.78	10.90	0.2115
282	2.27	12.12	0.465	294	3.78	10.92	0.2005
283	2.27	10.74	0.1380	295	3.78	9.64	0.0799
284	2.27	11.80	0.228	296	3.78	10.05	0.1098
285	2.27	10.29	0.0911	297	3.78	9.41	0.066
286	2.27	9.58	0.0508	298	3.78	8.94	0.0368
Slope 0.004							
299	1.17	11.88	0.1690	312	2.27	9.96	0.1747
300	1.17	11.22	0.1420	313	2.27	9.88	0.1603
301	1.17	11.95	0.1783	314	2.27	10.71	0.1911
302	1.17	11/32	0.1450	315	2.27	11.62	0.2705
303	1.17	10.96	0.1290	316	2.27	12.50	0.370
304	1.17	8.73	0.0701	317	3.78	7.84	0.0634
305	1.17	7.13	0.0545	318	3.78	8.24	0.0702
306	1.17	6.76	0.0078	319	3.78	9.75	0.206
307	1.17	9.35	0.1502	320	3.78	10.36	0.256
308	2.27	6.41	0.0038	321	3.78	10.60	0.288
309	2.27	7.56	0.0673	322	3.78	10.92	0.3285
310	2.27	8.46	0.0725	323	3.78	11.73	0.469
311	2.27	9.35	0.1820	324	3.78	12.32	0.685
Height of weir above invert, 0.833 feet							
Slope 0.004							
325	1.18	5.15	— —	338	2.29	7.07	0.234
326	1.18	6.65	0.169	339	2.29	8.23	0.376
327	1.18	5.38	0.0181	340	2.29	8.82	0.407
328	1.18	6.20	0.1748	341	2.29	9.90	0.536
329	1.18	7.08	0.188	342	2.29	10.50	0.657
330	1.18	7.55	0.212	343	2.29	11.38	0.785

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet		RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet	
		Q	q			Q	q
Diameter of pipe 24 inches							
Height of weir above invert, 0.833 feet							
Slope 0.004							
331	1.18	8.30	0.298	344	3.79	6.30	0.223
332	1.18	9.13	0.348	345	3.79	7.19	0.335
333	1.18	9.85	0.409	346	3.79	7.78	0.510
334	1.18	10.57	0.470	347	3.79	9.21	0.690
335	1.18	11.48	0.465	348	3.79	10.45	0.910
336	2.29	6.56	0.257	349	3.79	11/78	1.482
337	2.29	6.92	0.210				
Slope 0.007							
350	1.18	6.56	0.0220	360	2.27	9.16	0.462
351	1.18	7.44	0.0834	361	2.27	10.25	0.576
352	1.18	8.34	0.191	362	2.27	11.06	0.721
353	1.18	9.02	0.234	363	2.27	11.92	0.907
354	1.18	9.61	0.288	364	3.79	6.90	0.211
355	1.18	9.60	0.281	365	3.79	7.87	0.368
356	1.18	11.35	0.474	366	3.79	8.80	0.586
357	1.18	11.80	0.599	367	3.79	9.98	0.895
358	2.27	6.41	0.0856	3368	3.79	10.63	1.045
359	2.27	8.44	0.383	369	3.79	11.80	1.669
Slope 0.010							
370	1.17	6.28	0.0397	382	2.27	10.40	0.496
371	1.17	7.79	0.0735	383	2.27	10.88	0.604
372	1.17	9.07	0.182	384	2.27	11.35	0.695
373	1.17	9.81	0.270	385	2.27	12.10	0.893
374	1.17	10.52	0.298	386	3.79	8.05	0.328
375	1.17	10.95	0.381	387	3.79	9.15	0.538
376	1.17	11.10	0.402	388	3.79	9.70	0.626
377	1.17	12.18	0.525	389	3.79	10.56	0.794
378	2.27	7.39	0.1115	390	3.79	11.22	0.964
379	2.27	7.81	0.1366	391	3.79	11.72	1.071
380	2.27	8.63	0.220	392	3.79	12.21	1.194
381	2.27	9.79	0.412				
Slope 0.015							
393	1.18	6.82	0.0133	406	2.29	11.12	0.656
394	1.18	8.38	0.1033	407	2.29	11.63	0.770
395	1.18	9.38	0.217	408	2.29	13.18	1.011
396	1/18	9.99	0.285	409	3.29	6.70	0.0339
397	1.18	10.68	0.348	410	3.29	7.33	0.0906
398	1.18	11.11	0.400	411	3.29	7.86	0.188
399	1.18	11.72	0.441	412	3.29	9.35	0.424
400	1.18	12.42	0.496	413	3.29	10.25	0.672
401	2.29	7.87	0.0857	414	3.29	10.61	0.760
402	2.29	9.06	0.262	415	3.29	10.86	0.860

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of Weir. Feet.	Rate of Discharge Cubic Feet per Second		RUN NUMBER	Length of Weir Feet	Rate of Discharge Cubic Feet per Second	
		Q	q			Q	q
Diameter of pipe 24 inches							
Height of weir above invert, 0.833 feet							
Slope 0.015							
403	2.29	9.87	0.419	416	3.29	10.78	0.835
404	2.29	10.38	0.525	417	3.29	12.20	1.182
405	2.29	10.70	0.566	418	3.29	12.41	1.253
Height of weir above invert 0.667 feet							
Slope 0.015							
419	1.18	5.61	0.117	435	2.28	10.68	0.984
420	1.18	5.95	0.1225	436	2.28	10.80	0.990
421	1.18	5.95	0.1252	437	2.28	11.15	1.130
422	1.18	7.30	0.1822	438	2.28	11.45	1.210
423	1.18	8.55	0.310	439	2.28	12.27	1.390
424	1.18	9.57	0.475	440	3.79	5.21	0.1228
425	1.18	9.59	0.450	441	3.79	6.99	0.491
426	1.18	10.50	0.586	442	3.79	8.15	0.740
427	1.18	11.18	0.676	443	3.79	8.76	0.906
428	1.18	11.80	0.725	444	3.79	9.44	1.058
429	1.18	12.44	0.805	445	3.79	10.38	1.308
430	2.28	5.88	0.0556	446	3.79	10.90	1.502
431	2.28	6.47	0.262	447	3.79	11.29	1.620
432	2.28	7.26	0.318	448	3.79	11.72	1.720
433	2.28	8.15	0.424	449	3.79	12.43	1.910
434	2.28	9.79	0.725				
Slope 0.010							
450	1.18	5.90	0.157	465	2.28	11.11	1.160
451	1.18	7.86	0.307	466	2.28	11.39	1.230
452	1.18	9.05	0.400	467	2.28	11.65	1.342
453	1.18	9.65	0.475	468	2.28	12.00	1.402
454	1.18	10.10	0.537	469	3.79	6.73	0.458
455	1.18	11.19	0.706	470	3.79	7.51	0.715
456	1.18	10.91	0.686	471	3.79	8.71	0.999
457	1.18	11.62	0.794	472	3.79	9.07	1.224
458	1.18	12.11	0.875	473	3.79	10.30	1.418
459	2.28	6.19	0.283	474	3.79	10.81	1.588
460	2.28	7.78	0.529	475	3.79	11.21	1.730
461	2.28	8.35	0.627	476	3.79	11.50	1.810
462	2.28	8.98	;.687	477	3.79	11.81	1.876
463	2.28	9.96	0.861	478	3.79	11.90	1.918
464	2.28	10.36	0.935	479	3.79	12.12	1.944
Slope 0.007							
480	1.18	5.68	0.0834	496	2.28	9.16	0.759
481	1.18	7.44	0.296	497	2.28	9.60	0.780
482	1.18	8.49	0.441	498	2.28	9.56	0.822
483	1.18	9.30	0.479	499	2.28	10.41	0.935
484	1.18	11.05	0.482	500	2.28	11.10	1.028

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

Run Number	Length of Weir. Feet.	Rate Discharge Second Feet Q	Rate Discharge Second Feet q	Run Number	Length of Weir. Feet.	Rate Discharge Second Feet Q	Rate Discharge Second Feet q
Diameter of pipe 24 inches							
Height of weir above invert, 0.667 feet							
Slope 0.007							
485	1.18	9.58	0.463	501	2/28	11.38	1.125
486	1.18	10.07	0.524	502	2.28	12.04	1.290
487	1.18	10.52	0.556	503	3.79	6.96	0.466
488	1.18	10.78	0.588	504	3.79	7.78	0.706
489	1.18	11.10	0.647	505	3.79	8.76	0.896
490	1.18	13.21	0.653	506	3.79	9.40	1.085
491	1.18	11.40	0.692	507	3.79	10.13	1.350
492	1.18	11.88	0.765	508	3.79	11.10	1.662
493	2.28	6.46	0.296	509	3.79	11.60	1.640
494	2.28	7.55	0.467	510	3.79	12.25	1.970
495	2.28	8.51	0.596				
Slope 0.004							
511	1.18	5.72	0.0120	526	2.29	8.04	0.548
512	1.18	6.82	0.214	527	2.29	8.91	0.741
513	1.18	7.69	0.291	528	2.29	9.52	0.893
514	1.18	8.69	0.484	529	2.29	10.10	1.08
515	1.18	9.16	0.541	530	2.29	11.42	1.33
516	1.18	10.05	0.636	531	2/29	11.88	1.44
517	1.18	10.51	0.706	532	3.79	6.79	0.537
518	1.18	10.85	0.724	533	3.79	7.96	0.840
519	1.18	10.85	0.721	534	3.79	8.97	1.12
520	1.18	11.21	0.744	535	3.79	9.98	1.38
521	1.18	11.42	0.765	536	3.79	10.59	1.59
522	1.18	11.63	0.794	537	3.79	10.75	1.64
523	2.29	5.16	0.216	538	3.79	11.32	1.89
524	2.24	5.99	0.264	539	3.79	-----	2.07
525	2.29	7.11	0.387				
Height of weir above invert, 0.500 feet							
Slope 0.004							
540	1.17	2.88	0.106	556	2.28	7.98	1.04
541	1.17	4/58	0.288	557	2.28	8.83	1.43
542	1.17	5.31	0.367	558	2.28	9.52	1.65
543	1/17	6.65	0.567	559	2.28	10.85	1.85
544	1.17	7.59	0.676	560	2.28	10.92	1.91
545	1.17	8.57	0.991	561	2.28	11.78	2.11
546	1.17	9.30	1.09	562	3.79	3.57	0.471
547	1.17	9.95	1.25	563	3.79	5.14	0.942
548	1.17	10.45	1.34	564	3.79	5.71	1.14
549	1.17	11.04	1.44	565	3.79	6.85	1.29
550	1.17	11.59	1.48	566	3.79	7.82	1.67
551	1.17	11.90	1.55	567	3.79	8.20	1.75
552	2.28	4.46	0.436	568	3.79	9.43	2.40

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet		RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet	
		Q	q			Q	q
Diameter of pipe 24 inches							
Height of weir above invert, 0.500 feet							
Slope 0.004							
553	2.28	5.35	0.665	569	3.79	10.08	2.64
554	2.28	6.30	0.871	570	3.79	10.60	2.88
555	2.28	7.10	0.991	571	3.79	11.00	3.04
Slope 0.007							
572	1.17	2.78	0.074	594	2.30	6.61	0.876
573	1.17	4.69	0.253	595	2.30	6.67	0.945
574	1.17	5.69	0.432	596	2.30	6.53	0.924
575	1.17	6.59	0.623	597	2.30	6.59	0.924
576	1.17	7.56	0.736	598	2.30	7.56	1.09
577	1.17	8.56	0.925	599	2.30	7.53	1.07
578	1.17	8.74	0.991	600	2.30	8.40	1.19
579	1.17	9.33	1.10	601	2.30	8.44	1.20
580	1.17	10.50	1.16	602	2.30	8.99	1.32
581	1.17	11.00	1.11	603	2.30	9.46	1.47
582	1.17	10.83	1.10	604	2.30	9.76	1.59
583	1.17	10.79	1.16	605	2.30	10.19	1.56
584	1.17	10.80	1.08	606	2.30	10.08	1.54
585	1.17	10.80	1.07	607	2.30	11.35	1 90
586	1.17	11.15	1.15	608	2.30	11.80	2.29
587	1.17	11.10	1.15	609	3.80	5.00	0.800
588	1.17	11.62	1.25	610	3.80	6.36	1.23
589	1.17	11.71	1.25	611	3.80	8.06	1.71
590	2.30	4/29	0.378	612	3.80	9.26	2.35
591	2.30	4.33	0.374	613	3.80	9.68	2.52
592	2.30	5.16	0.595	614	3.80	10.59	2.81
593	2.30	5.11	0.574				
Slope 0.010							
615	1.17	5.44	0.385	630	2.27	10.58	1.73
616	1.17	6.42	0.546	631	2.27	10.94	1.88
617	1.17	7.71	0.681	632	2.27	11.71	2.02
618	1.17	8.81	0.754	633	2.27	12.17	2.18
619	1.17	9.56	0.908	634	3.79	4.26	0.501
620	1.17	10.10	1.01	635	3.79	5.88	0.896
621	1.17	10.50	1.11	636	3.79	6.67	1.19
622	1.17	11.48	1.28	637	3.79	7.33	1.41
623	1.17	12.02	1.40	638	3.79	7.90	1.59
624	2.27	5.69	0.586	639	3.79	9.02	2.06
625	2.27	6.42	0.839	640	3.79	9.79	2.43
626	2.27	7.16	1.05	641	3.79	10.22	2.61
627	2.27	8.57	1.30	642	3.79	10.48	2.66
628	2.27	9.35	1.42	643	3.79	11.10	2.92

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet		RUN NUMBER	Length of Weir. Feet	Rate Discharge Second Feet	
		Q	q			Q	q
Diameter of pipe 24 inches							
Height of weir above invert, 0.500 feet							
Slope 0.010							
629	2.27	10.04	1.56	644	3.79	11.81	3.13
Slope 0.015							
645	1.16	5.46	0.354	660	2.28	9.44	1.24
646	1.16	6.33	0.404	661	2.28	9.79	1.35
647	1.16	7.10	0.441	662	2.28	10.23	1.42
648	1.16	7.73	0.521	663	2.28	10.70	1.55
649	1.16	8.76	0.666	664	2.28	11.03	1.65
650	1.16	9.72	0.751	665	2.28	11.62	1.92
651	1.16	9.94	0.896	666	3.80	5.98	0.860
652	1.16	10.38	1.03	667	3.80	7.13	1.18
653	1.16	11.05	1.11	668	3.80	7.92	1.42
654	1.16	11.48	1.18	669	3.80	8.62	1.66
655	1.16	11.88	1.27	670	3.80	9.26	1.89
				671	3.80	9.52	1.98
							.11
							.24
							.38

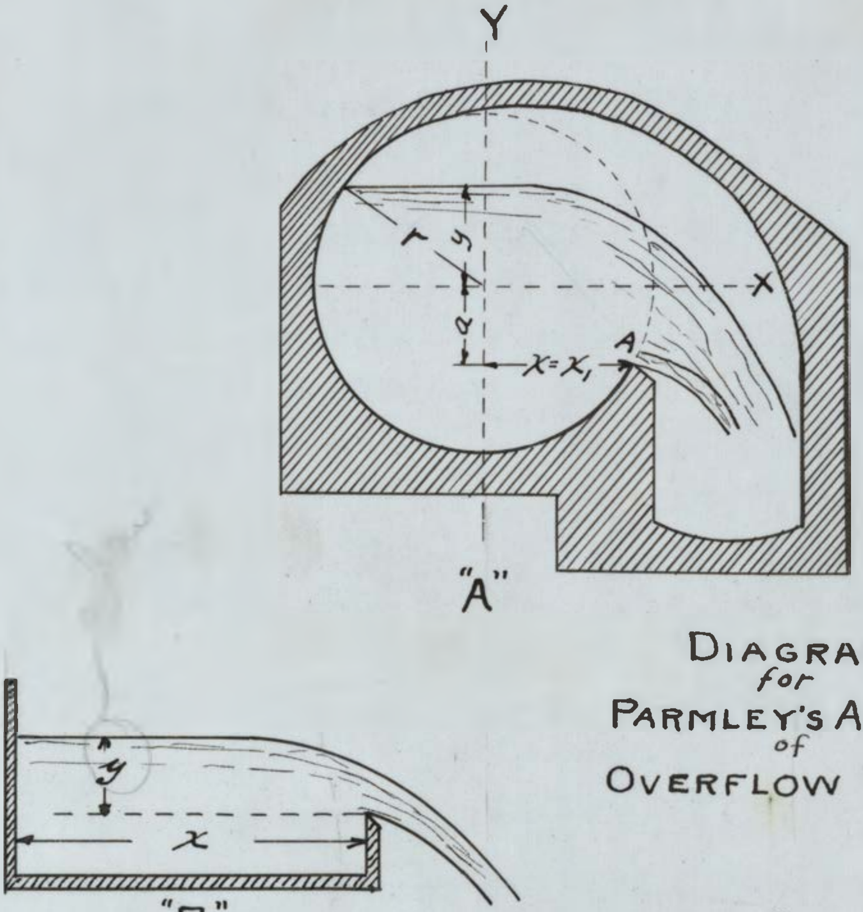
In redrawing 43B
change y to read h

TABLE X
(continued)

OBSERVATIONS ON OVERFLOW WEIRS

RUN NUMBER	Length of Weir Feet	Rate Discharge Second Feet		RUN NUMBER	Length of Weir Feet	Rate Discharge Second Feet	
		Q	q			Q	q
Diameter of pipe 24 inches							
Height of weir above invert, 0.500 feet							
Slope 0.010							
629	2.27	10.04	1.56	644	3.79	11.81	3.13
Slope 0.015							
645	1.16	5.46	0.354	660	2.28	9.44	1.24
646	1.16	6.33	0.404	661	2.28	9.79	1.35
647	1.16	7.10	0.441	662	2.28	10.23	1.42
648	1.16	7.73	0.521	663	2.28	10.70	1.55
649	1.16	8.76	0.666	664	2.28	11.03	1.65
650	1.16	9.72	0.751	665	2.28	11.62	1.92
651	1.16	9.94	0.896	666	3.80	5.98	0.860
652	1.16	10.38	1.03	667	3.80	7.13	1.18
653	1.16	11.05	1.11	668	3.80	7.92	1.42
654	1.16	11.48	1.18	669	3.80	8.62	1.66
655	1.16	11.88	1.27	670	3.80	9.26	1.89
656	2.28	6.39	0.691	671	3.80	9.52	1.98
657	2.28	7.05	0.805	672	3.80	9.90	2.11
658	2.28	8.09	0.966	673	3.80	10.18	2.24
659	2.28	8.79	1.11	674	3.80	10.70	2.38

Q = Rate above weir.
q = Rate over weir.



DIAGRAMS
for
PARMLEY'S ANALYSIS
of
OVERFLOW WEIRS

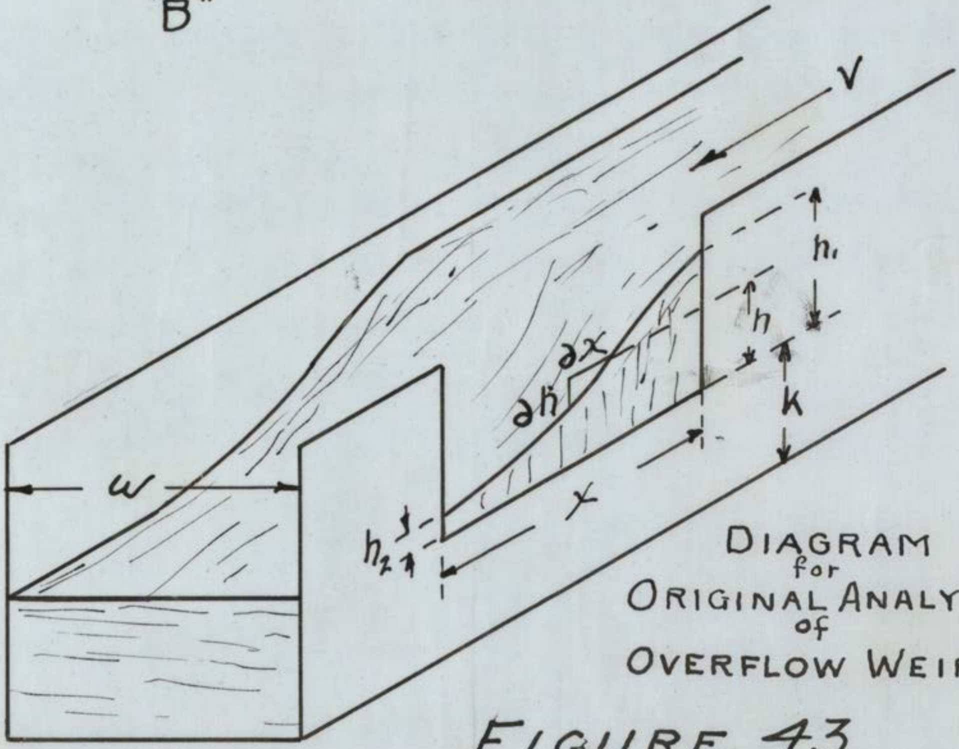


DIAGRAM
for
ORIGINAL ANALYSIS
of
OVERFLOW WEIRS

FIGURE 43

-----:CHAPTER VIII:-----

-----:LAW OF FLOW OVER OVERFLOW WEIRS:-----

Sect. 49. Rational Considerations:- W.C. Parmley in Transactions of the American Society of Civil Engineers, Volume LV, page 362 analyzes the rate of discharge over an overflow weir as follows:

"Let Figure 43A represent the cross section of the overflow chamber at the upper end of the weir, at the point where the water emerges from the sewer.

Let X and Y represent the axes of coordinates, with the origin in the axis of the sewer. Consider this section to represent a unit length of sewer.

Let A be the crest of the weir, and let $a + y$ be the depth of water over the weir.

Let the radius of the sewer equal r.

The coordinates of the weir are therefore, $x = x_1$ and $y = -a$

How long will it require for the water flowing over the weir to reduce the head of water on the weir from $a + y$ to any given lesser head?

Let dQ equal the volume of water discharged for the reduction of head dy, and let dt equal the time required for the discharge of the quantity dQ. We then have the equations

$$dQ = 2xdy = 2\sqrt{r^2 - y^2}\ dy$$

For the head $a + y$ the rate of discharge q = approximately

$$3.33\ (a + y)^{3/2}$$

Then $dQ = qdt = 3.33(a + y)^{3/2}\ dt$

Therefore $dt = \dfrac{0.6\sqrt{r^2 - y^2}}{(a+y)^{3/2}}\ dy$

Integrating between the limits y_1 and y_2 for any two heads upon the weir, gives the time required to reduced the head from y_1

to y_2.

It has not been possible however to integrate this equation and therefore it has been necessary to make use of it in the approximate form:

$$t = \sum \left[\frac{0.6\sqrt{r^2 - y^2}\,\Delta y}{(a+y)^{3/2}} \right]_{y_2}^{y_1}$$

Obtaining the Δt, for successive differences in head, Δy, between the limits of y_1 and y_2, and taking the sum of all these Δt's will give the approximate time t required.

This being a tedious process, an approximation can be made by reducing the circular sewer to a rectangular one of the same average width. In this case let Figure 43B represent the cross section of the rectangular sewer, with the weir at A, and with an initial depth of water y over the weir. Let the width of the channel W equal the average width of the circular sewer shown in Figure 43A to the left of the weir A. In this case the water overhanging the weir on the right is assumed to fall away by the force of gravity without interfering with the weir discharge over and back of the weir. In this case then we have

q = rate of discharge for head y, $= 3.33y^{3/2}$ and
Q = the total quantity discharged.

For an infinitesimal reduction in head, dy, we have

$$dQ = Wdy = qdt = 3/33y\,dt$$

therefore $dt = \dfrac{W}{3.33}\,y^{-3/2}\,dy$

Integrating between the limiting heads

$$t = \left(-\frac{W}{1.67\sqrt{y}}\right)_{y_2}^{y_1} = \frac{W}{1.67}\left(\frac{1}{y_2^{1/2}} - \frac{1}{y_1^{1/2}}\right)$$

If $y_2 = 0$, $t = \infty$, which shows......that theoretically it would require a weir of infinite length to reduce the water to a zer head.

The last formula is simple and easily applied, and does not give results varying greatly from those obtained from the differential equation for the circular sewer.

If the velocity in the sewer were constant while flowing the length of the weir, and if all the filaments in the entire cross section had the same velocity, the foregoing equations would give the time required to reduce the level of the water from one stage to another, and this time, multiplied by the velocity of flow in the sewer behind the weir would give the length of the weir required. These ideal conditions, however, are not obtained in practice. The velocity in the sewer is gradually retarded as the head becomes less, and, consequently, the sill must be lengthened somewhat in order to perform the same amount of work."

Parmley's method is not only difficult, but it is uncertain as to the value of the results, because of the assumptions on which it is based. An example has been solved by the method suggested by Parmley, at the end of Chapter IX. The uncertainty as to the correctness of the assumptions made in the Parmley analysis, and the variation from the experimental observations, together with the absence of experimental results by Parmley, tend to cast doubt on the value of the formula suggested by him.

The following analysis is based on somewhat different assumptions, which are also open to criticism. This analysis led to somewhat different conclusions which were helpful in making certain empirical assumptions as to the factors affecting the flow over the weir.

For the sake of simplicity the analysis will first be made for the discharge from an overflow weir in a rectangular flume, as shown in Figure 43C.

For any particular differential length of the weir it will be assumed that the discharge is

$$dq = 3.33\, h^{3/2}\, dx$$

It now remains to find h in terms of x.

From the figure it is evident that the time, dt, for the head to drop a distance dh is equal to the time, dt, for a particle to travel the distance dx. From the preceding analysis quoted from

$$dt = \frac{W\,h^{-3/2}}{3.33}\,dh$$

Now let V' represent the velocity in feet per second at the section in question, then

$$dx = V'dt = V'\frac{W\,h^{3/2}}{3.33}\,dh$$

therefore $dq = V'Wdh$

Since V' is a variable it remains to express V' in terms of h. With a constant slope V' varies only with the hydraulic radius. The hydraulic radius of the rectangular section is approximately

$$\frac{W(h+k)}{W+2(h+k)}$$

Then from the Chezy formula

$$V' = C_1\sqrt{\frac{W(h+k)}{W+2(h+k)}\,S}$$

and $$V = C_1\sqrt{\frac{W(h_1+k)}{W+2(h_1+k)}\,S}$$

from which $$V' = V\sqrt{\frac{(h+k)[W+2(h_1+k)]}{(h_1+k)[W+2(h+k)]}}$$

and $$Q = WV\int_{h_2}^{h_1}\sqrt{\frac{(h+k)[W+2(h_1+k)]}{(h_1+k)[W+2(h+k)]}}\,dh$$

This integration would result in an expression of no practical use. The difficulty lies in the expression for the hydraulic radius. If it were possible to express V' in more simple terms, an expression of greater value might be obtained.

The relation between the depth of flow and the hydraulic radius of a circular section is shown in Figure 10. An approximation to the form of the equation of the hydraulic radius curve, particularly the portion below the maximum point at 0.8 depth, when referred to the invert of the pipe as the origin of coordinates with horizontal and vertical axes, can be made by assuming it to be in the form of a parabola. By a series of trials the following equation was selected as representative of this curve: $y^2 - 1.6\,y + 0.52\,x = 0$

Now, up to the point in the preceding analysis for a rectangular channel where $dq = V'W\,dh$, all the steps are equally applicable to a circular section. If we substitute D, the diameter of the circle, for W, then

$$dq = V'D\,dh$$

In the preceding expression for x and y, $y = \frac{h+k}{D}$ and $x = \frac{4r}{D}$ where r represents the hydraulic radius at any depth. Substituting these values and solving

$$r = \left[1.6\left(\frac{h+k}{D}\right) - \left(\frac{h+k}{D}\right)^2\right]\frac{D}{2.08}$$

As before $$Q = DV\int_{h_2}^{h_1}\sqrt{\frac{1.6\left(\frac{h+k}{D}\right)-\left(\frac{h+k}{D}\right)^2}{1.6\left(\frac{h_1+k}{D}\right)-\left(\frac{h_1+k}{D}\right)^2}}\,dh$$

$$= \left[\frac{DV}{\sqrt{(h_1+k)(1.6-h_1-k)}}\right]\left[\frac{2}{3}(h+k)^{3/2}\right]_{h_2}^{h_1}\left[\frac{2}{3}\sqrt{(1.6-k-h)^3}\right]_{h_2}^{h_1}$$

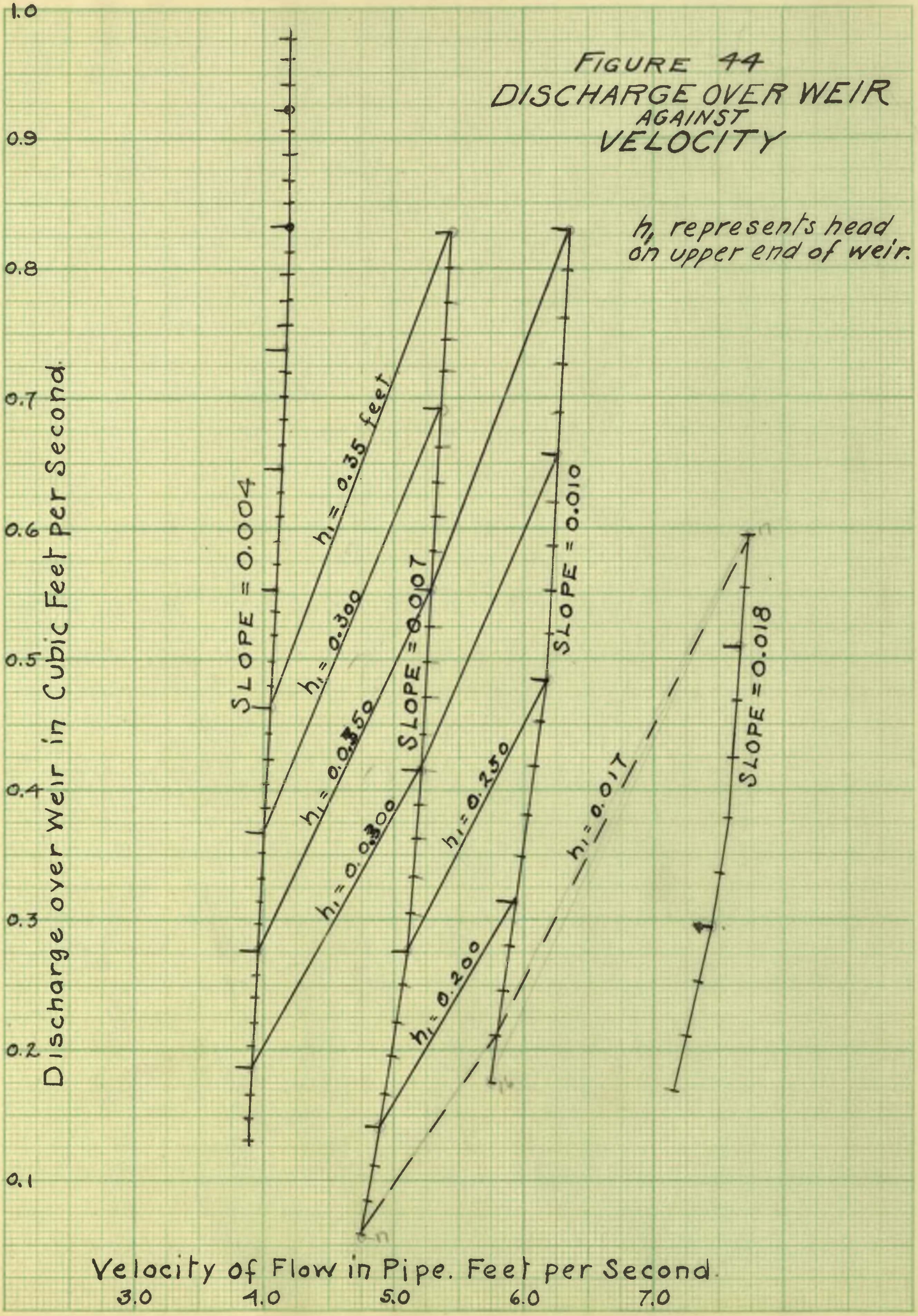
It becomes evident that any "rational" formula for Q which includes all of the factors affecting it, will be too cumbersome to serve a useful purpose. An empirical formula based on the preceding analysis might be more simple and quite as accurate.

Sect. 50. Empirical Expression:- From the preceding discussion it would seem that Q is dependent upon D, V, (h_1-h_2), and h_1. An important factor which does not appear here is the length of the weir. This, however, is dependent upon Q and (h_1-h_2).

An attempt was made to find the relation between Q, and the factors enumerated by following the procedure outlined below:

First, to compute the value of V, h_1 and h_2 for all of the runs. Second, To hold D, l (the length of the weir), and k constant and to plot Q against V. It became evident that each value of S (the slope) determined a different QV line. Points on the different QV lines which were determined by the same values of h_1 were joined. One of these plots is shown in Figure 44.

Third, Determine the proper scale for the values of h_1 on the QV lines and draw lines connecting equal values of h_1.



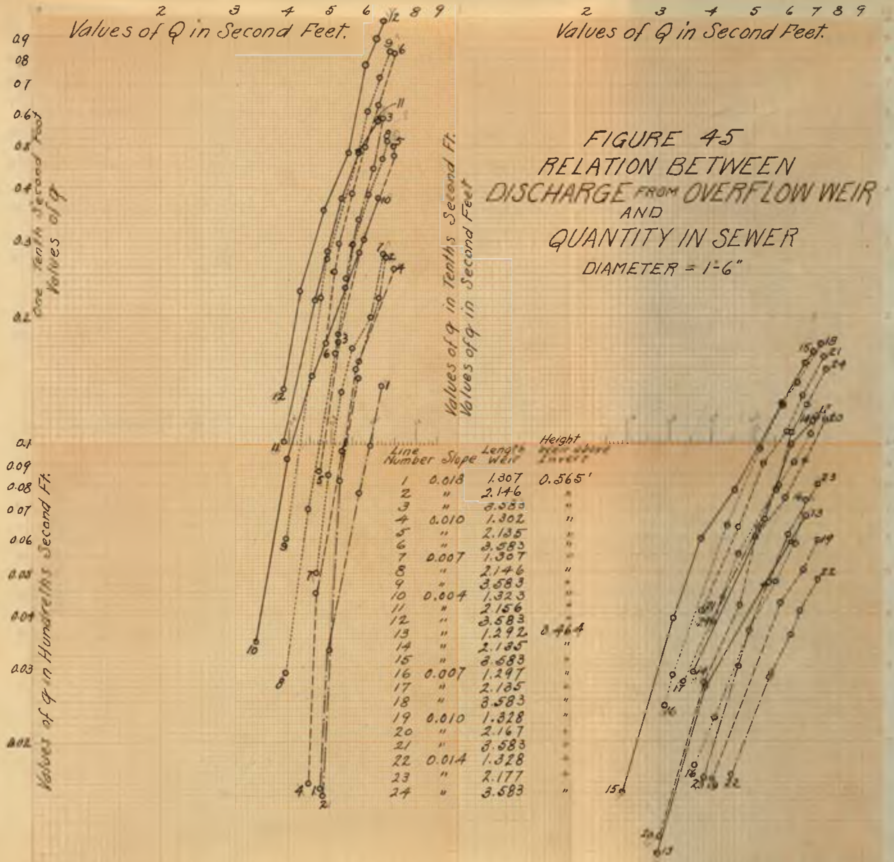
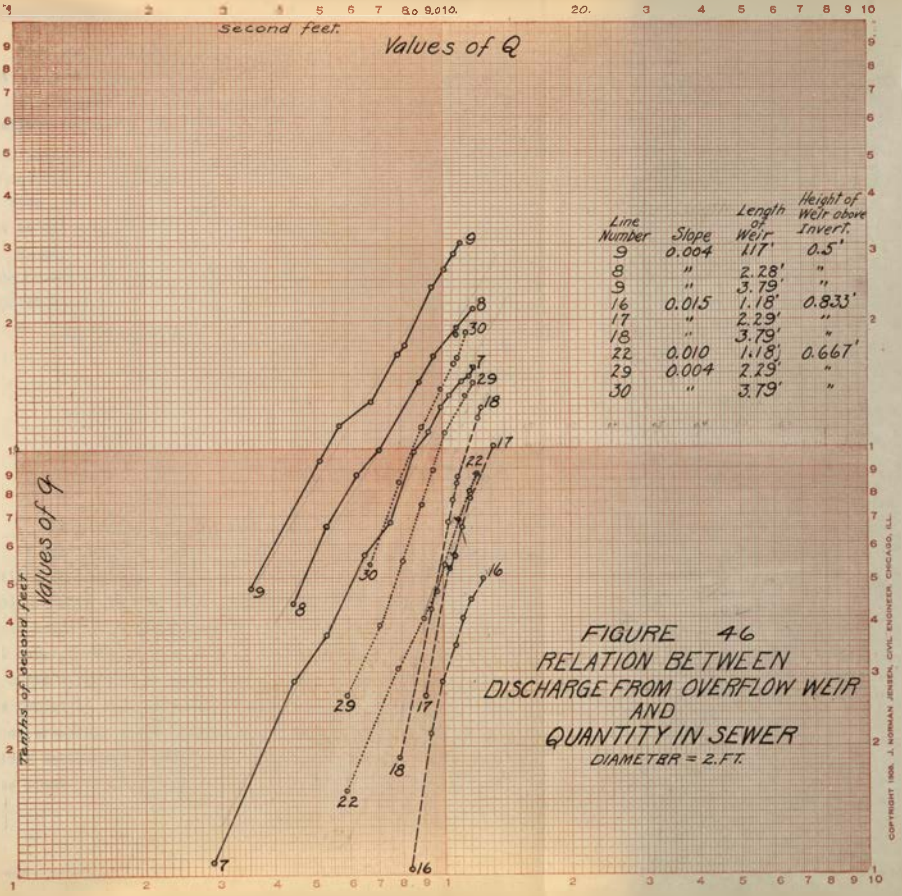


FIGURE 45
RELATION BETWEEN DISCHARGE FROM OVERFLOW WEIR AND QUANTITY IN SEWER
DIAMETER = 1'-6"

Line Number	Slope	Length Weir	Height weir above invert
1	0.018	1.307	0.565'
2	"	2.146	"
3	"	3.583	"
4	0.010	1.302	"
5	"	2.135	"
6	"	3.583	"
7	0.007	1.307	"
8	"	2.146	"
9	"	3.583	"
10	0.004	1.323	"
11	"	2.156	"
12	"	3.583	"
13	"	1.292	0.464
14	"	2.135	"
15	"	3.583	"
16	0.007	1.297	"
17	"	2.135	"
18	"	3.583	"
19	0.010	1.328	"
20	"	2.167	"
21	"	3.583	"
22	0.014	1.328	"
23	"	2.177	"
24	"	3.583	"



Line Number	Slope	Length of Weir	Height of Weir above Invert.
9	0.004	1.17'	0.5'
8	"	2.28'	"
9	"	3.79'	"
16	0.015	1.18'	0.833'
17	"	2.29'	"
18	"	3.79'	"
22	0.010	1.18'	0.667'
29	0.004	2.29'	"
30	"	3.79'	"

FIGURE 46
RELATION BETWEEN
DISCHARGE FROM OVERFLOW WEIR
AND
QUANTITY IN SEWER
DIAMETER = 2.FT.

From the appearance of the lines in Figure 44 it is evident that $Q = mV + n$ (very closely) in which m and n are functions of h_1.

It now remained to determine these functions. The resulting equations became so complicated, and so many inconsistencies appeared as to make advisable the abandonment of this line of reasoning for the solution of the problem. A basic objection to the expression in the form just studied is the appearance in it of h_1 and V, which must be calculated by a rather roundabout method.

Because of this latter objection an expression was sought which would give the expression for Q in terms of conditions which could be observed directly, or were easily computed. These variables are:

Q = The rate of flow in cubic feet per second in the sewer above the overflow weir

q = The rate of discharge in cubic feet per second over the overflow weir.

S = The slope of the invert

l = The length of the weir in feet

k = The height of the weir above the invert, in feet

d = The diameter of the pipe, in feet.

k' = The ratio of k to d, = k/d

It was thought possible, because of the nature of the results obtained in the tests, that there was some direct relation between Q and q. Since the preceding rational considerations, assisted by simple empirical assumptions, led to no valuable result, the values of Q were plotted against q to a natural scale. The appearance of the curve suggested an exponential relation between them, and they were replotted on logarithmic paper. Figure 45 is a plot of all of the observations made on eighteen inch pipe, and Figure 46 a few typical observations made on twentyfour inch pipe. The appearance of these lines is sufficient to lead to the conclusion that (very closely)

$$Q = k_1 q^m$$

For very small values of q the relation does not approximate a straight line when drawn on logarithmic paper, but since the values of q less than one tenth of a cubic foot per second are of but little practical value the value of the preceding expression is not impaired.

The terms, k_1 and m are probably functions of the other variables which were held constant in order to determine the relation between Q and q. These variables are S, l, k, and d. It is evident from the appearance of Figures 45 and 46 that the value of m is dependent upon d and k' only. The values of m were plotted against k' to a natural scale and found to lie very closely on a straight line for the value of d equal to eighteen inches. Since only three points were available for the location of this line for the twentyfour inch pipe and two points for the eighteen inch pipe, the result is not of great certainty. The equations of these lines were developed. Too few points were available for a more certain determination of the relations between m, d and k'. The values of these variables as observed in the experiments are therefore presented in Table XI.

TABLE XI

VALUES OF EXPONENT m IN RELATION $Q = k_1 q^m$ FOR OVERFLOW WEIRS

d	2'-0"				1'-6"	
k'	0.500	0.416	0.333	0.250	0.377	0.309
m	0.170	0.305	0.44	0.58	0.24	0.45
1/m	5.9	3.28	2.28	1.72	4.2	2.22

Further attempts were made to determine a general relation between d, k' and m were made, but they proved fruitless.

Sect. 51. Conclusions:- Sufficient observations have been made to determine conclusively that

$$Q = k_1 q^m$$

It was not possible, with the number of observations made, to determine satisfactorily a general relation between k_1, q, Q, and m.

Tables XI and XII have therefore been included, showing direct observations of these factors under various conditions. For the solution of any particular probelm the values can be selected from the tables and the equation solved. It is to be noted that the use of the formula beyond the limits of the experimental observations would not be possible because of the limitations of the tables.

Sect. 52. Accuracy of Results:- The expression $Q = k_1 q^m$ has been solved for each run and the result compared with the observed value of q. The percent which the difference between the computed quantity and the observed quantity was of the observed quantity has been plotted for all runs in which the rate of discharge over the overflow weir was equal to or greater than one cubic foot per second. For rates of discharge of less than one cubic foot per second the actual, and not the percent, difference was plotted. This change was made because the actual discrepancies were so high compared to the observed discharges, for the small rates, that the percentage discrepancy had little significance. It is to be understood that these discrepancies do not represent an actual error in either the observation or the computation, but that one or the other is probably in error to some extent.

Figure 47 shows that the largest percentage of error for any run (with a discharge greater than one cubic foot per second) was about 29, and that eighty percent of all of the runs in which large rates of discharge were observed have a discrepancy of less than 10 percent. For the smaller rates of discharge the greatest error is about 0.24 of a cubic foot per second, and 89 percent of the smaller discharges have a discrepancy of less than one tenth of a cubic foot per second.

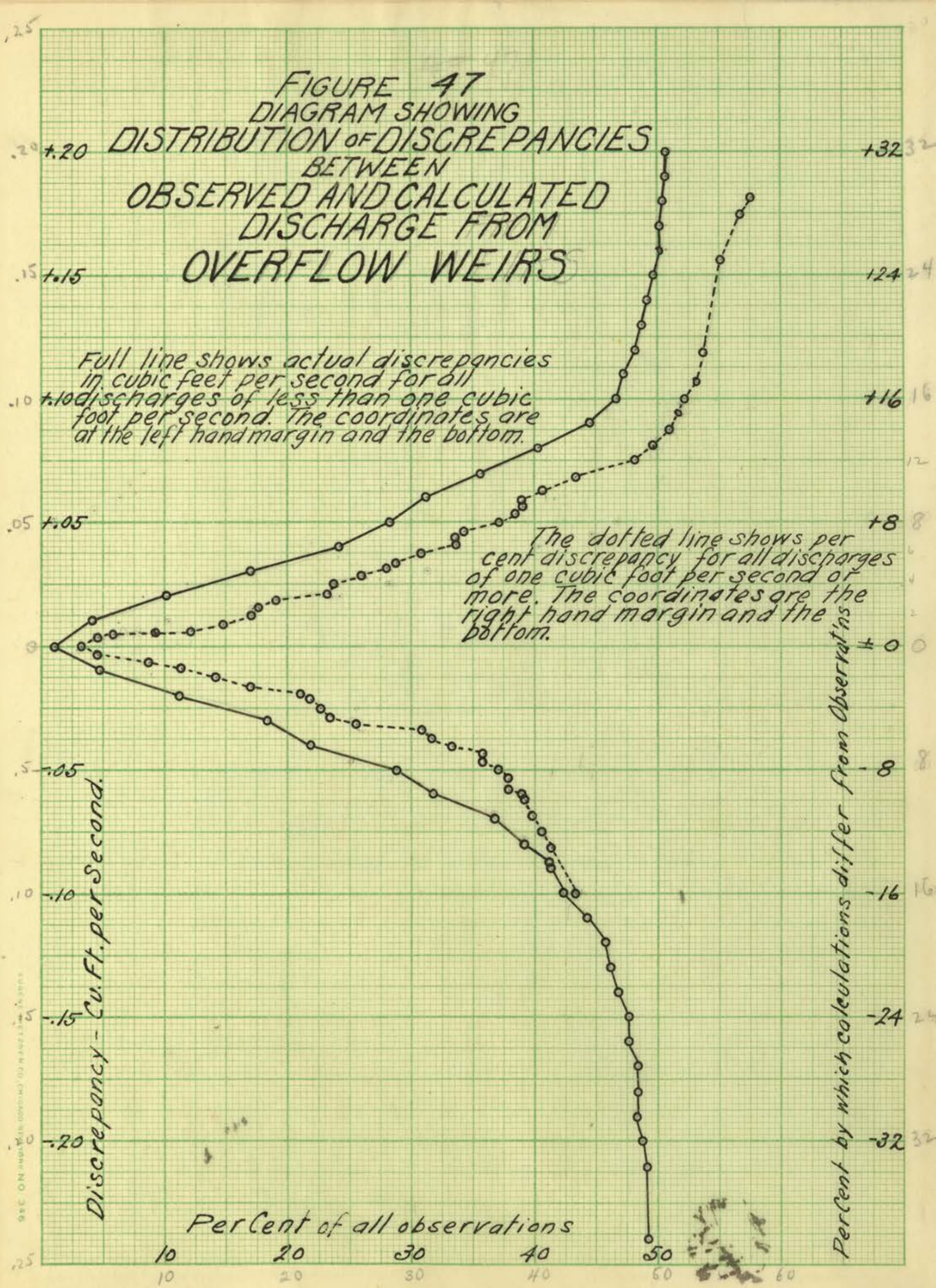


FIGURE 47
DIAGRAM SHOWING DISTRIBUTION OF DISCREPANCIES BETWEEN OBSERVED AND CALCULATED DISCHARGE FROM OVERFLOW WEIRS

Full line shows actual discrepancies in cubic feet per second for all discharges of less than one cubic foot per second. The coordinates are at the left hand margin and the bottom.

The dotted line shows per cent discrepancy for all discharges of one cubic foot per second or more. The coordinates are the right hand margin and the bottom.

TABLE XII

VALUES OF THE COEFFICIENT k, IN THE RELATION

$$Q = kQ^m$$

OVERFLOW WEIRS

Dia. ft.	k' ft.	l ft.	Values of slope ratio				
			.004	.007	.010	.015	.018
2.0	.500	1.17	16.0	----	----	----	----
2.0	.500	2.29	14.7	----	----	----	----
2.0	.500	3.79	13.3	----	----	----	----
2.0	.416	1.17	13.1	13.4	13.7	14.2	----
2.0	.416	2.29	11.7	11.9	12.2	12.7	----
2.0	.416	3.79	10.5	10.7	11.0	11.6	----
2.0	.333	1.17	12.6	12.8	13.1	13.5	----
2.0	.338	2.29	10.0	10.3	10.6	11.0	----
2.0	.333	3.79	8.5	8.8	9.0	9.5	----
2.0	.250	1.17	9.3	9.6	9.8	10.3	----
2.0	.250	2.29	7.0	7.3	7.5	8.0	----
2.0	.250	3.79	5.5	5.8	6.0	6.5	----
1.5	.377	1.31	8.6	9.0	9.5	----	10.6
1.5	.377	2.14	7.6	8.1	8.5	----	9.6
1.5	.377	3.58	6.6	7.0	7.4	----	8.6
1.5	.309	1.31	7.6	8.1	8.5	9.1#	----
1.5	.309	2.14	6.1	6.6	7.0	7.7#	----
1/5	.309	3.58	4.9	5.3	5.8	6.4#	----

k' represents the distance that the edge of the overflow weir is above the bottom of the invert

l represents the length of the weir

The slope for these runs was 0.014

----:CHAPTER IX:-----

-----:DESIGN OF OVERFLOW WEIRS:-----

Sect. 53. Methods:- The purpose of an overflow weir as described in Chapter VI, ~~was that~~ "~~the purpose of an overflow weir~~ "is to relieve a sewer of a certain proportion of its contents which threaten to overcharge it". It is usually desirable not to allow the overflow to begin until there has been a considerable dilution of the sanitary sewage, in order to render the portion removed from the sewer less offensive. This can be accomplished by placing the edge of the weir high in the sewer, but the higher the weir the longer it must be in order to discharge the same quantity.

So far as could be determined by inquiry among engineers, the only 'method' in use for the design of overflows was a rule of thumb guessing, except for the analysis quoted from Parmley. The value and accuracy of this formula or method have been discussed in section 49 on page 63, and the results to be obtained in the following example will serve to emphasize the conclusion.

In order to illustrate the method for the design of a weir in accordance with the results of the observations of this series of tests an example will be worked out in the following section.

Sect. 54. Example of Design:- The conditions to be assumed are: a twentyfour inch combined sewer on a grade of .01 with a coefficient of roughness of 0.015. At the time of sudden summer thunder showers the sewer does not carry away water fast enough and overflows at the manholes. It becomes desirable to construct an overflow weir which will relieve the sewer of one half of its contents, without spilling any of the dry weather flow which is assumed to be one fifth of the full capacity of the sewer.

By Kutter's formula it is found that the full capacity of the sewer is 19 cubic feet per second, which is equal to the value Q in the formula $Q = k_1 q^m$. Since one half of the capacity of the sewer is to be spilled $q = 9.5$ cubic feet per second. By consulting Figure 10 it is found that when the sewer is carrying one fifth of its full capacity it is flowing at a depth which is three tenths of its diameter, in this case six tenths of a foot. The value of k' is therefore 0.3. For the extra factor of safety a value of 0.333 will be assumed for k'. Then from Table XI m equals 0.44. Substituting these respective values of Q, q and m in the expression $Q = k_1 q^m$ it is found that $k_1 = 7.3$ By interpolation in Table XII the length of the weir is found to be about 2.5 feet.

By a very simple process it has been found that a weir thirty inches long placed 7.2 inches above the bottom of a twenty four inch sewer pipe on a grade of 0.01 will discharge one half of the full capacity of the sewer when this quantity is being delivered through the sewer, above the weir.

A solution of this problem will be made by the Parmley method and the results compared. Substituting in the formula given on page 62, $t = -\left(\frac{d}{1.67\sqrt{h}}\right)_{h_2}^{h_1} = \frac{d}{1.67}\left(\frac{1}{\sqrt{h_2}} - \frac{1}{\sqrt{h_1}}\right)$ The value of d in this case is 2, h_1 is about 0.8 x 2 or 1.6 and h_2 is 0.5 x 2 or 1.0 Therefore $t = \frac{2}{1.67}\left(\frac{1}{1} - \frac{1}{1.27}\right) = 0.24$ The velocity of flow when h_1 is 1.6 is about 7 feet per second therefore the length of the weir should be 1.68' = 7 x 0.24

Parmley states in his method that after having solved for the length of the weir by his method; "the sill must be lengthened to perform the same work". The difference between the value of 1.68 and 2.5 represents the necessary increase. The fact that the correct value was found at once by the formula $Q = k_1 q^m$ emphasizes the value of that formula.