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SEDIMENT TRANSPORT IN MONEY CREEK

J. B. Stall,¹ A.M. ASCE, N. L. Rupani,² and P. K. Kandaswamy,³ A.M. ASCE
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SYNOPSIS

Lake Bloomington, an impounding reservoir, has been subjected to detailed surveys in 1948, 1952 and 1955 to determine the deposition of sediment. During each of these surveys samples of the sediment were obtained. Particle size distribution analyses of 30 of these sediment samples were utilized to determine the tons of sediment deposited in the lake during each of these sedimentation periods. Postulating that sediment particles which had a diameter greater than 50 microns had been moved into the lake as bed material load, the total tons of this sized material was calculated based on the sediment samples.

An hydraulic study was made of the 2-1/2 mile reach of Money Creek immediately upstream from Lake Bloomington to determine its sediment-carrying capacity. A series of sediment samples were taken of the bed material of the Money Creek channel. Utilizing these data, curves of water discharge versus sediment discharge were computed utilizing three different methods: the Einstein procedure, the Schoklitsch formula, and the DuBoys formula.

At the lower end of this stream reach, immediately upstream from Lake Bloomington is a stream-gaging station for which flow records were available for each of the three sedimentation periods. Utilizing flow duration information from this stream gage, the total quantity of bed material moved through the Money Creek reach was calculated utilizing each of the three sediment transport relationships developed. The actual bed material-size sediment in Lake Bloomington is compared with the sediment transport as computed by the three methods.

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1. Associate Engr., Illinois State Water Survey Div., Urbana, Ill.
2. Formerly Asst. Engr., Illinois State Water Survey Div., Urbana, Ill.
3. Formerly Asst. Engr., Illinois State Water Survey Div., Urbana, Ill.

INTRODUCTION

Purpose

The purposes of this investigation are: (1) to attempt to compute the sediment moved as bed material load in Money Creek, Illinois, for a reach immediately upstream from a stream-gaging station and from Lake Bloomington, (2) to provide a comparison of three well-known bed-load formulas including the most recent one proposed by H. A. Einstein(1) and (3) to compare the results of these formulas with bed material measured by actual survey in Lake Bloomington.

Most sediment transport formulas are derived from laboratory flume studies. The importance of this study is believed to be the check of three formulas under natural conditions.

General

The city of Bloomington is located in the central part of McLean County, Illinois. A public water supply derived from wells was installed for the town in 1875. The wells were utilized until 1929 when Lake Bloomington was formed by the construction of an earth dam across Money Creek, a tributary to the Mackinaw River about 15 miles northeast of Bloomington. Since that time Lake Bloomington has been used for the public water supply. The lake has a drainage area of 61.0 square miles, a surface area of 487 acres, and had an original storage capacity of 2.17 billion gallons. Fig. 1 shows the location of Lake Bloomington and Money Creek watersheds.

A detailed survey was conducted on Lake Bloomington in August 1948 to determine the volume of sediment deposition. At that time a series of 19 cross sections of water depth and sediment thickness were taken on the lake as shown in Fig. 2. In August 1952 and in July 1955 soundings were repeated along these same cross sections to measure the further sediment deposition during the intervening periods. These cross sections were plotted and the total quantity of sediment (including the bed-load and wash-load) deposited in the lake was calculated by the method devised by the Soil Conservation Service.

(2)

Table 1 is a summary of the results of the three surveys. It will be seen that the capacity of the lake has been depleted every year at an average rate of about 0.46 percent of the original capacity. The 1955 survey showed a total loss in capacity of 791 acre feet or 258 million gallons.

During a period of low inflow and low lake level during 1954 less sediment was carried into the lake and a portion of the deposited sediment bed in the upper portion of the lake was exposed to air drying and consequently compacted. The specific weight of the sediment deposit in each segment of the lake was determined by a series of sediment samples obtained during each of the three surveys. A total of 30 samples were utilized to determine the tons of sediment deposited in the lake during each of the sedimentation periods.

Choice of Reach

Lake Bloomington and its watershed have been the subject of an hydrologic study by the State Water Survey since 1933.(3) The principal tributary to Lake Bloomington is Money Creek. Immediately upstream from the headwaters of the lake is located a stream-gaging station. This gage is sponsored

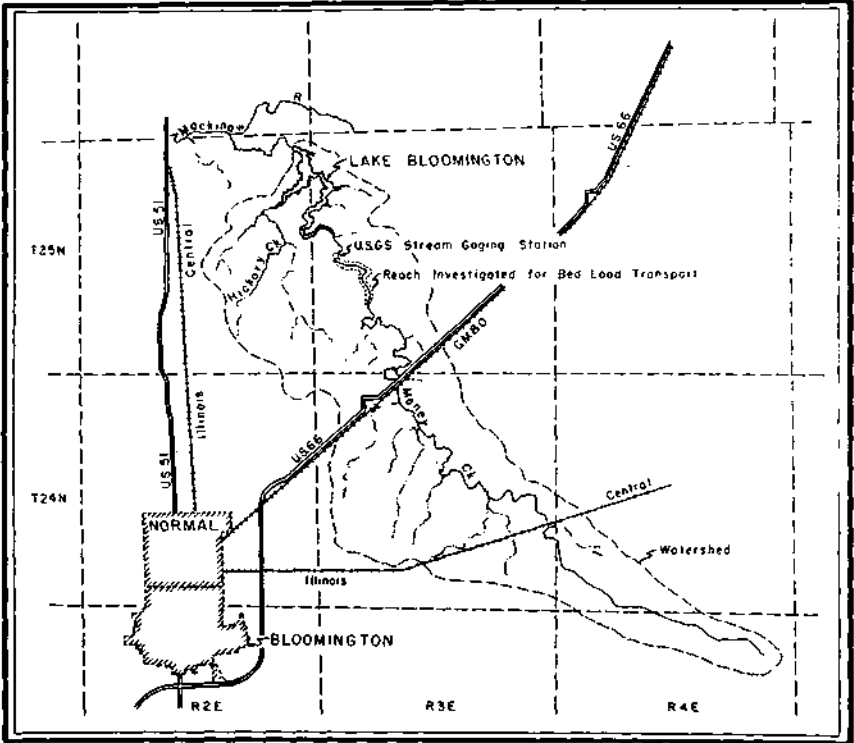


Figure 1 - Location of Money Creek and Lake Bloomington

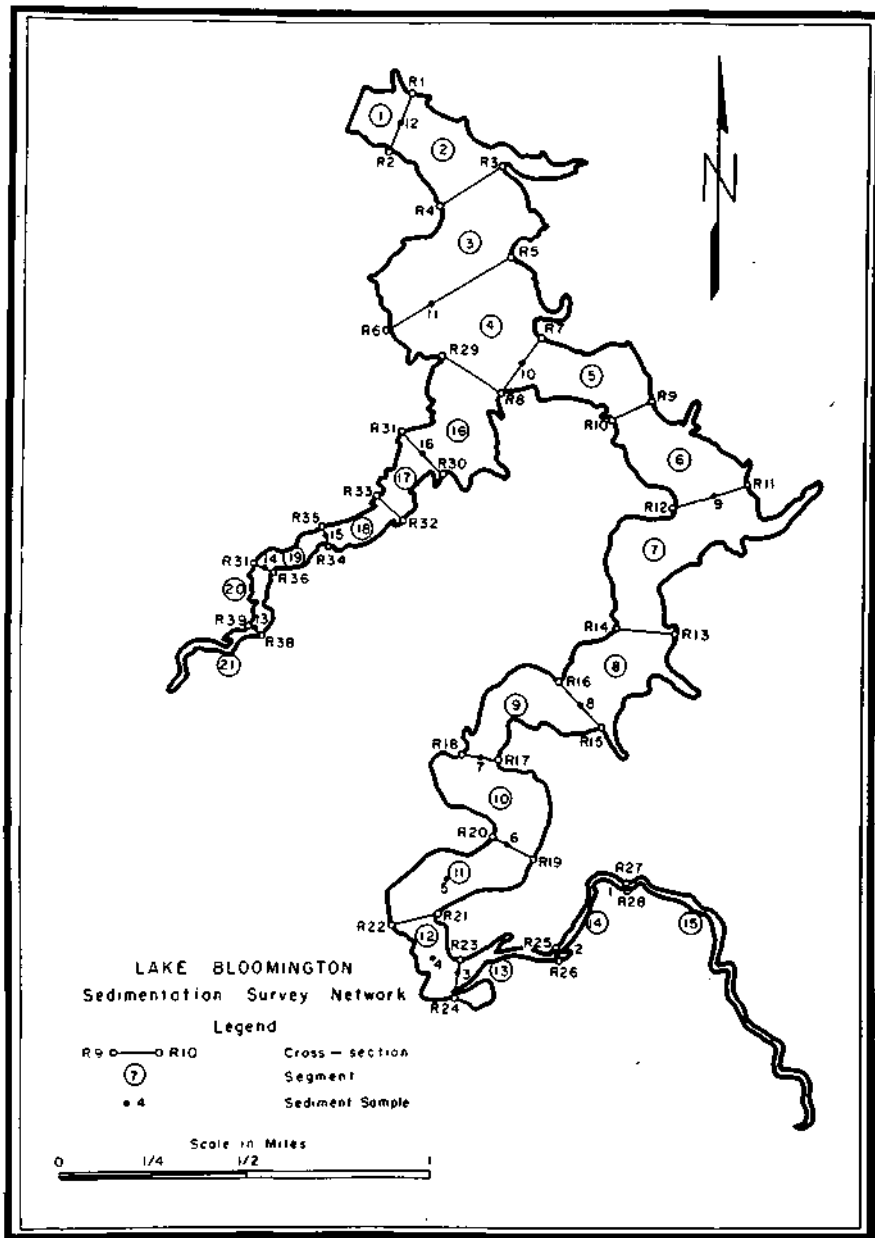


Figure 2 - Sedimentation Survey of Lake Bloomington

Table 1. Summary of Sedimentation Data
Lake Bloomington, Illinois

AGE

Dec. 1929 - Aug. 1943 - 18.7 yrs.
 Aug. 1948 - Aug. 1952 - 4.0 yrs.
 Aug. 1952 - July 1955 - 2.9 yrs.
 Dec. 1929 - July 1955 - 25.6 yrs.

WATERSHED

Total area - 61.0 sq. miles
 39,01+0 acres
 Land area - 60.2 sq. miles
 38,553 acres

RESERVOIR

Area at spillway crest - 487.2 acres

	<u>1929</u>	<u>1948</u>	<u>1952</u>	<u>1955</u>	<u>Units</u>
<u>Storage Capacity</u>	6654	6062	2905	5863	Acre-feet
	2168	1975	1924	1911	Mil. gal.
<u>Capacity per sq.mi.</u> of drainage area	109	99	97	96	Acre-feet

<u>SEDIMENTATION</u>	<u>1929-1948</u>	<u>1948-1952</u>	<u>1952-1955</u>	<u>1929-1955</u>	
Total	592	157	42	791	Acre-feet
<u>Average Annual Accumulation</u>					
From entire watershed ¹	31.7	39.3	14 ²	30.8	Acre-feet
Per sq.mile ¹	0.53	0.65	0.23 ²	0.51	Acre-feet
Per acre ¹	36.1	44.2	15.7 ²	34.7	Cubic-foot
Tons per acre ¹	0.74	0.91	0.34.	0.72	Tons

DEPLETION OF STORAGE

Loss of original capacity					
Total period	8.90	2.36	0.63 ²	11.89	Per cent
Per Year	0.48	0.59	0.21 ²	0.46	Per cent

¹ Land area only.

² Volume compacted due to drying.

by the State Water Survey and is operated by the United States Geological Survey. It records the drainage from 51.9 square miles of the total lake watershed. Records are available at this station from June 1933 to date. Because of the availability of these discharge records, the stream reach immediately upstream from this gage was given consideration for the present study.

In selecting a river reach for sediment transport calculations, it must be remembered that such a function can be applied only to a river reach of uniform flow. This means that the length of the channel must be sufficient to permit adequate determination of the over-all slope. Also the channel itself must be sufficiently uniform in shape, sediment composition, slope and outside effects such as bank vegetation, that it is possible to treat the reach as a uniform channel characterized by an over-all slope and by an average representative cross section. Practically, it is difficult to realize such an ideally uniform channel. After a field inspection of Money Creek, however, it was decided that this reach was sufficiently uniform to be utilized for bed-load calculations.

Hydraulic and Hydrologic Determinations

Hydraulic Properties of Channel

A field investigation was made of the 2-1/2 mile reach immediately upstream from the gaging station to determine hydraulic properties. A series of 13 cross sections was taken of the stream at approximately 1000-foot intervals. The slope of the water surface was determined to be 0.000905 by utilization of these 13 stations. The slope measurement was taken during a time when the average discharge was 160 cubic feet per second. Flow duration studies showed discharge equalled or exceeded this amount for six percent of the period of record.

To determine the stage-area relationship for the reach, each of the 13 cross sections was plotted in actual position in elevation. The average cross section for the reach was then determined by sliding all cross sections down the channel along the slope 0.000905 into the plane of the section at the lower end of the reach. This downstream cross section was at the stream-gaging station, giving a means of comparison between the mean cross section for the reach and the actual cross section of the gaging station. The stage-area curve for the reach determined in this manner is shown in Fig. 3.

In a similar manner the average stage-wetted perimeter curve was obtained and is shown in Fig. 4. In a wide channel like Money Creek the width of the channel was considered as the wetted perimeter for a known elevation. Consequently, corresponding values of the two curves made it possible to compute the hydraulic radius for the same elevation. These are shown in Fig. 5. The curve of stage versus hydraulic radius with bank friction was computed by means described by Einstein.⁽¹⁾ The stage-discharge relationship with and without bank friction is shown in Fig. 6 as compared to the actual stage-discharge relationship as measured at the gaging station.

In order to determine the width of the stream bed along which transportation takes place, the average widths of the stream at various stages of all of the 13 cross sections were plotted and are shown in Fig. 7. As reported by Chang⁽⁴⁾ and by Einstein⁽⁵⁾ the movement of bed material is reasonably expected to take place only along the bed portion of the channel. From Fig. 7 a

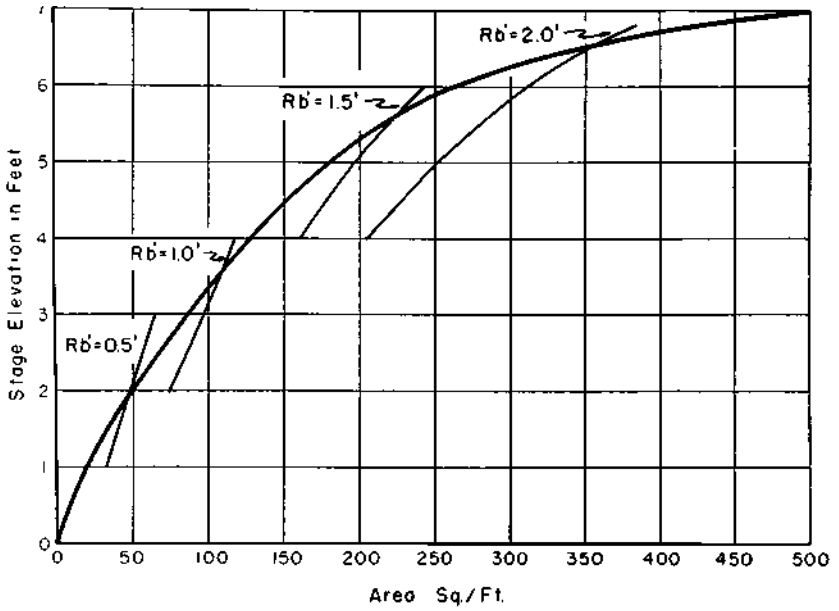


Figure 3 - Stage-Area Relationship for Money Creek

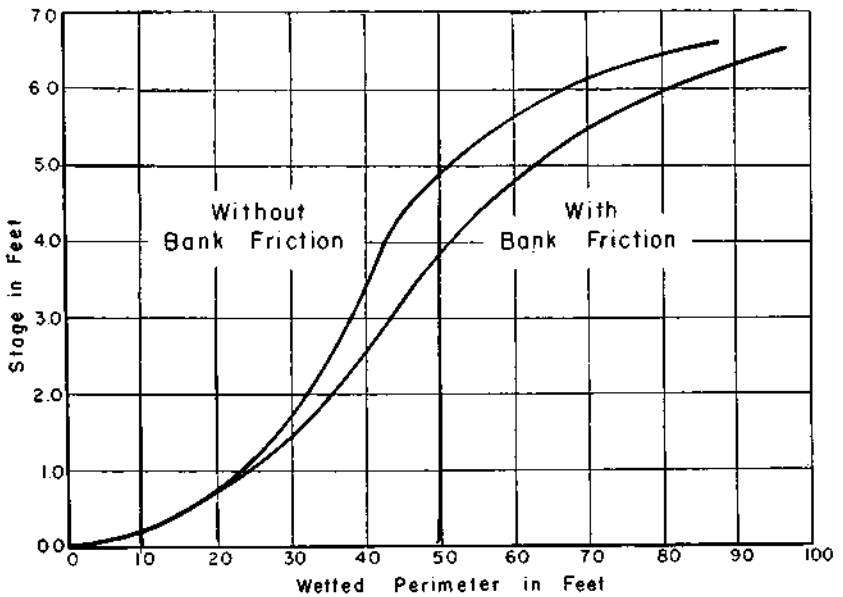


Figure 4 - Stage-Wetted Perimeter Relationship for Money Creek

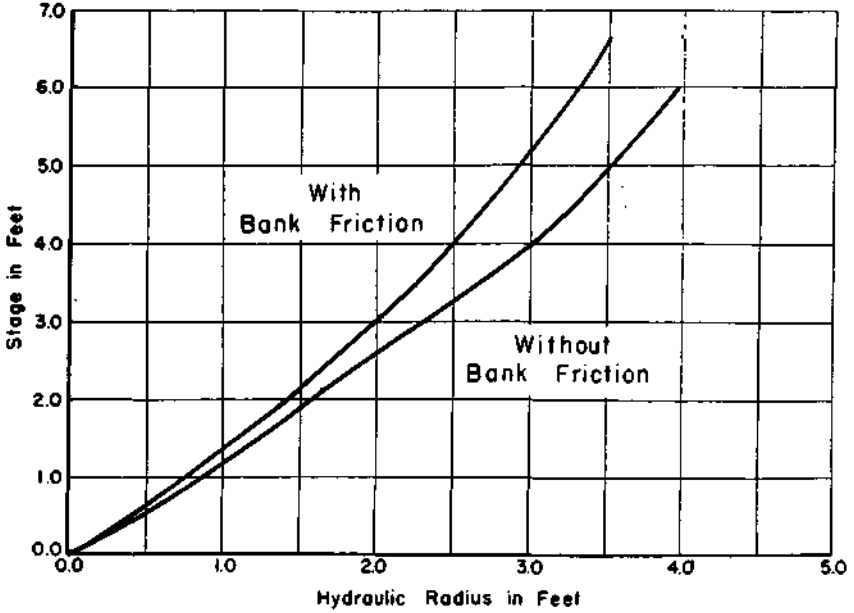


Figure 5 - Stage-Hydraulic Radius Relationship for Money Creek

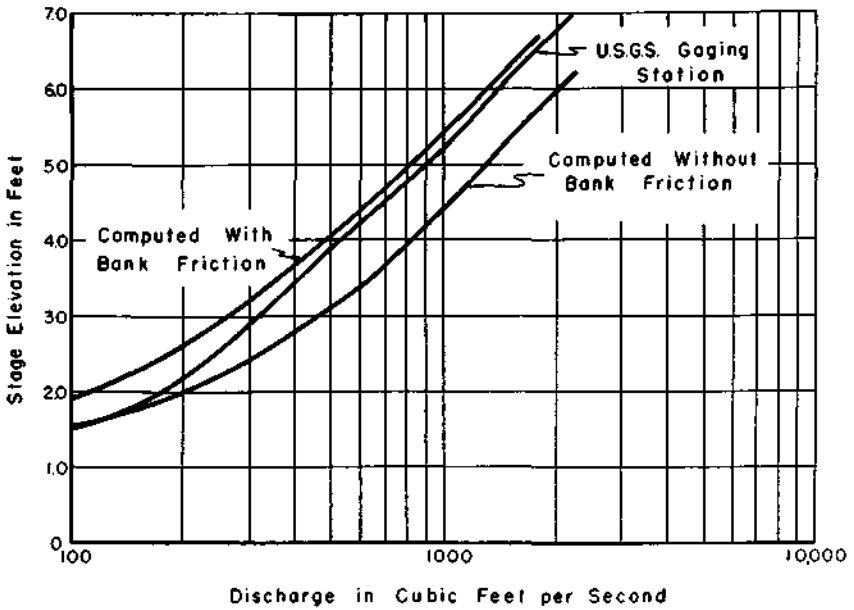


Figure 6 - Stage Discharge Relationship for Money Creek

width of 30 feet was arbitrarily chosen to represent the width of bed for the reach of Money Creek under consideration.

Computations of hydraulic properties need to be made only up to a stage corresponding to the highest flood that had occurred in this creek. This stage of seven feet was the maximum utilized in determining all hydraulic properties.

Bed-Sediment Samples

Since the bed-load formulas used in this study relate the grain-size composition of the bed material with the flow of the channel it is necessary to obtain representative samples of the bed material. A total of 18 samples of sediment was taken along the active channel and the flood plain of Money Creek. Samples were obtained by means of an auger or a pipe sampler and were taken to a depth of about 1.5 feet, the estimated depth of scour or active bed movement. The flood plain samples indicated 90 percent by weight to be finer than 50 microns, and it was concluded that these finer particles were deposited during the recession of flood flows. Consequently the flood plain samples were abandoned and five samples were chosen to represent the bed material in the active channel. The size distribution of this bed material based on these samples is shown in Table 2.

The data from Table 2 are plotted in Fig. 8, from which can be noted the characteristic grain sizes. The size which enters the Einstein equation of transport is $D_{35} = 0.195 \text{ mm} = 0.000639 \text{ feet}$ (grain size of which 35 percent is finer). The size characteristic for friction $D_{65} = 1.22 \text{ mm} = 0.0040 \text{ feet}$.

The sediment transport was calculated for grain sizes between 9.4 millimeters and 0.050 millimeters which represents 67 percent of the bed material. It is important to recognize however that 12 percent of the bed material is coarser than 9.4 millimeters. These gravel-size particles do not move for normal flows although an extremely small rate of transport may occur at high flood stages. Twenty-one percent of the bed material is finer than 0.050 millimeters. As much as 15 percent of this size may be expected to be a part of wash-load particles lodged behind the coarser grains. This can generally be neglected. No adjustments were made since a large percent of these finer materials were found in the bed. Though included in the bed material size distribution, no bed-load function exists for these finer particles. Calculations were made for individual sieve fractions using as representative the average grain sizes varying from 7.3 to 0.073 millimeters as shown in Table 2.

Calculation of Bed Material Deposited in the Lake

One of the most critical phases of the present investigation was the determination of the total quantity of sediment deposited in Lake Bloomington which was of the size moved as bed-material-load through the tributary creek, Money Creek. The quantities of sediment measured in the lake survey contained principally fine material which was undoubtedly moved into the lake as wash load. The 16 sediment samples taken during the 1948 and 1955 lake surveys were considered to be sufficient in number to indicate the sediment nature in the various segments of the reservoir. These samples were subjected to size distribution analyses. The locations of these sediment samples are shown in Fig. 2.

All particles of sizes 0.050 millimeters (50 microns) and more were considered as bed-load and particles of sizes smaller than 0.050 millimeters, in

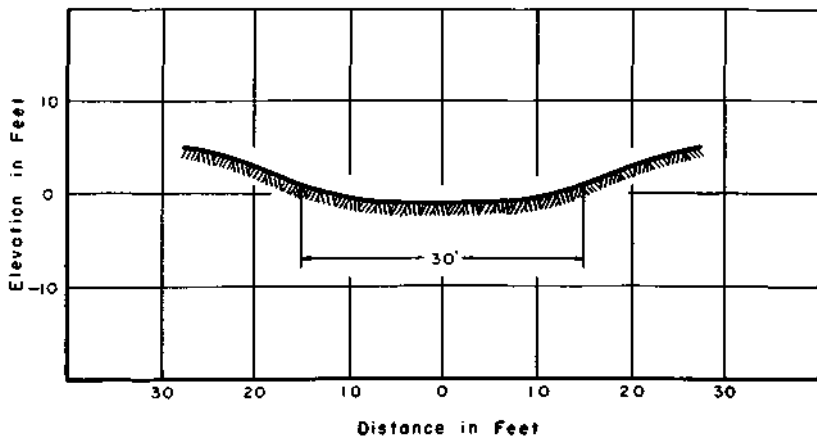


Figure 7 - Money Creek Average Cross Section

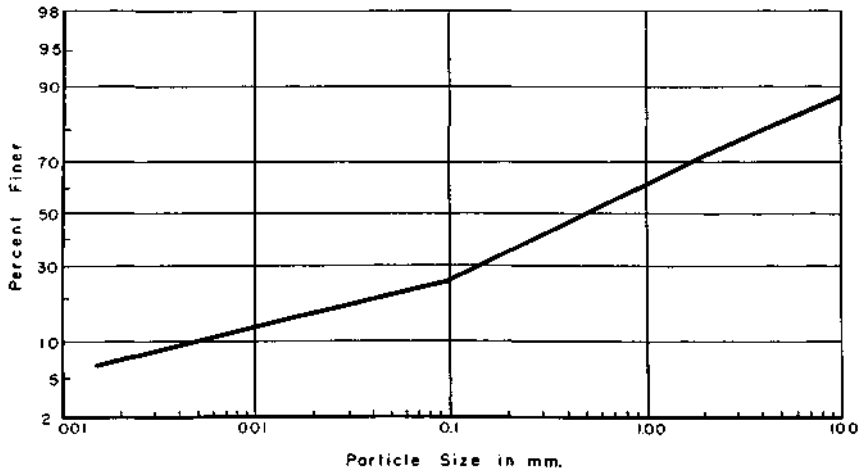


Figure 8 - Average Size Distribution of Bed Material Samples

Table 2. Average Size Distribution of Money Creek Bed Material Based on Five Samples

<u>Grain Size</u> <u>mm</u>	<u>Per Cent</u> <u>by Weight</u>	<u>Mean Diameter</u>	
		<u>mm</u>	<u>Feet</u>
Less than 0.050	21.0	-----	-----
0.050 to 0.097	4.0	0.073	0.000239
0.097 to 0.19	9.5	0.144	0.000472
0.19 to 0.37	10.7	0.28	0.000918
0.37 to 0.72	11.3	0.545	0.00179
0.72 to 1.1j.	10.7	1.06	0.00348
1.4 to 2.7	8.8	2.05	0.00672
2.7 to 5.2	7.0	3.95	0.01295
5.2 to 9.4	5.0	7.3	0.02393
Greater than 9.4	12.0	---	-----

Table 3. Sand Content of Lake Sediment Samples

<u>Sample</u> <u>No.</u>	<u>Per Cent Sand</u> <u>By weight</u> <u>(Diameter > 50 microns)</u>
1	97
2	53
3	22
4	33
5	17
6	11
7a	0.5
7b	0.5
8	0.5
9a	2.0
9b	7
10	2.5
11	1.0
12a	3.0
12b	2.5
13	18
14	2.5
15a	1.0
15b	0.5
16	1.0
"a"	Samples from upper portion of sediment deposit
"b"	Samples from lower portion of sediment deposit

the range of silt and clay, were considered wash load. Table 3 presents the results of these analyses, showing for each sample, the total percent by weight of material having a diameter greater than 50 microns. In only two of these samples was there material having a diameter in excess of 9.4 millimeters and in each of these cases the percentage was extremely small.

The sediment samples at the 16 locations in the lake were utilized to compute the total quantity of sediment in tons in each segment of the lake. The locations of the sediment samples and the lake segments are shown in Fig. 2. Table 4 shows the results of these calculations.

Reference to Figs. 1 and 2 shows that Money Creek is tributary to that portion of the lake containing segments 5 through 15 and Hickory Creek contributes the flow to segments 16 through 21. The two arms of the lake come together to form that portion of the lake containing segments 1, 2, 3 and 4. The tonnage of sediment as calculated in Table 4 for these four segments (1 through 4) was calculated by proportioning the total tonnage coming from Hickory Creek and from Money Creek. The drainage area of the Money Creek above the lake is 51.9 square miles and the drainage area on Hickory Creek above the lake is 10 square miles. The proportion of these two drainage areas was used to calculate the tonnage of bed material sediment deposited in these lower four segments of the reservoir. It will be noted from Table 4 that the total quantity of bed material, which can be assumed to have moved down Money Creek and into Lake Bloomington during the period 1929-1955, amounts to 60,527 tons.

Flow Duration Data

Computation of the bed load by any of the three formulas developed a relationship between sediment discharge in tons per day and water discharge in cubic feet per second. To determine the total quantity of material moved through this reach of the stream as bed load, it was necessary, therefore, to construct flow duration curves. It was desired that such data be available for each of the three periods during which sediment deposition was measured in Lake Bloomington. Water discharge data from the stream-gaging station on Money Creek were complete in this respect except for the period 1929 to 1933 and for the year 1941. To complete the flow record, the discharge at Money Creek was synthesized for these missing years.

Stream-gaging records were available for the neighboring Mackinaw River for the missing periods of record as well as for the complete period of record for Money Creek. The two drainage basins were assumed to be homogeneous in regard to their general flow characteristics. Flow duration curves for the two rivers were drawn based on the same period of records, namely 1934 through 1940 and 1942 through 1954. These were used to synthesize the missing records at Money Creek based upon the actual flow measurements on the Mackinaw River using a method described by Mitchell. (6) The actual flow records of Money Creek which is in the Mackinaw River basin have been published by the United States Geological Survey in their Water Supply Papers. (7)

To determine the bed-load quantity, the flow duration data for each of the three different periods were compiled as described above and shown in Table 5. Only flows above 110 cubic feet per second have been considered. This assumes that the quantity of bed material moved by lesser flow is negligible. Table 5 shows the duration of the high flows which have occurred at the Money

Table 4. Total Tonnage of Sand and Larger Material Deposited in Lake Bloomington

Segment	Sediment Tons	Samples	Per Cent Sand (> 50 microns)	Total Sand Tons	
				Hickory Cr. Arm of Lake	Money Cr. Arm of Lake
1	10,881	12a,12b	2.75	54	245
2	32,735	12a,12b	2.75	163	737
3	46,828	11	1.0	85	383
4	42,238	10,11	1.75	134	605
5	28,117	10	2.5	--	703
6	33,945	9a,9b	4.5	--	1528
7	49,294	9a,9b	4.5	--	2218
8	41,323	8	0.5	--	207
9	56,245	7a,7b,8	0.5	--	281
10	71,130	6	11	--	8594
11	99,630	5	17	--	16,937
12	41,840	3,4	27.5	--	11,506
13	27,527	2,3	37.5	--	10,323
14	5,061	1,2	75	--	3796
15	2,541	1	97	--	2464
16	45,285	16	1.0	453	---
17	16,407	16	1.0	164	---
18	11,960	15a,15b	2.75	329	---
19	15,182	14,15a,15b	2.17	329	---
20	16,477	13,14	10.3	1697	---
21	10,027	13	18	1805	---

50,527 Tons

Table 5. High Flow Duration During Lake Sedimentation Periods
Money Creek Gaging Station

Mean Discharge cfs	Duration in Days			Total
	1st Period	2nd Period	3rd Period	
	Dec. 1929 to Aug. 1948	Sept. 1948 to Aug. 1952	Sept. 1952 to July 1955	
110	376	124	59	559
165	199	58	20	277
245	103	34	11	148
355	58	18	6	82
520	31	10	3	44
760	11	2	--	13
1100	3	1	--	4

Creek gaging station during the three periods for which sedimentation was measured in Lake Bloomington. Discharges considered are mean values of incremental discharge ranging from 110 cubic feet per second to 1100 cubic feet per second.

Sediment Transport Calculations

The Einstein Procedure

H. A. Einstein developed and published in 1950 a complex procedure for computing the quantity of bed material transported by a stream.⁽¹⁾ This bed load function was applicable to an alluvial channel in an equilibrium state, which moves the material through which it flows.

The approach was based on the probability of movement of a particle of a particular diameter in the "bed layer." Movement in this layer was considered to occur by rolling and sliding on the bed or by making a series of short hops and was termed "surface creep." The thickness of the bed layer was postulated to be twice the grain diameter. The movement of particles was considered to be governed by statistical laws of probability and was so related to the flow. The average distance traveled by a particle between periods of deposition was assumed to be 100 diameters.

The concentration of particles having a particular diameter at the top of the bed layer is assumed to be equal to the concentration of suspended particles of the same diameter at this same boundary. This concentration is then related to the concentration of similar particles at any elevation in the vertical. By integration of the function the total sediment load of this diameter, per unit width, was determined at a representative vertical in the stream cross section at a given discharge. Load was calculated for a number of grain-size categories based on the samples of bed material. In calculating the total load of a mixture of particles, corrections were introduced for the "hiding factor" or interference of the larger grains with the smaller. A later publication by Einstein⁽⁸⁾ improves this correction.

The principal relationships utilized by Einstein in the bed load function are as follows:

$$q_s = \int_0^d c_y \bar{u}_y dy \quad (1)$$

$$c_y = c_a \left(\frac{d-y}{y} \frac{a}{d-a} \right)^z \quad (2)$$

$$z = \frac{v_s}{ku_*} \quad (3)$$

Where,

q_s = Rate of transportation of suspended load

d = Water depth

\bar{u}_y = Velocity at distance y above bed

c_y = Sediment concentration, weight per unit volume, of the fluid-sediment mixture at distance y above the bed

c_a = Sediment concentration at distance a above bed

v_s = Settling velocity of a sediment grain in still water

k = Universal constant of von Karman

u_* = Shear velocity at the bed

Einstein⁽¹⁾ considered the velocity distribution in open-channel flow over a sediment bed as being best described by the logarithmic formula based on von Karman's similarity theorem with the constants as proposed by Keulegan.⁽⁹⁾ He gave the vertical velocity distribution including the transition between the rough and smooth boundaries as:

$$\frac{\bar{u}_y}{u_*} = 5.75 \log_{10} \left(30.2 \frac{yx}{k_s} \right)$$

(4)

$$= 5.75 \log_{10} \left(30.2 \frac{y}{\Delta} \right)$$

wherein x is given as a function of k_s /

\bar{u}_y = the average point velocity at the distance y from the bed

$$u_* = \sqrt{T_0 / s_f} = \sqrt{S_e \cdot R \cdot g} \quad (\text{the shear velocity})(5)$$

S_f = the density of the water

S_e = the slope of the energy grade line

R = the hydraulic radius

g = the acceleration due to gravity

y = the distance from the bed

k_s = the roughness of the bed

x = a correction parameter

$$\Delta = k_s / x \quad \text{the apparent roughness of the surface} \quad (6)$$

$$\delta = \frac{11.6}{u_*} \quad \text{the thickness of the laminar sublayer of a smooth wall} \quad (7)$$

ν = the kinematic viscosity of the water

The value of k_s for uniform sediment equals the grain diameter as determined by sieving. The representative grain diameter of a sediment mixture is given by that sieve size of which 65 percent of the mixture by weight is finer.

The total rate of sediment transportation is the sum of the suspended and bed-load transport rate and is given by Einstein in Equation (63) of Reference (1) as,

$$i_T q_T = i_B q_B (P I_1 + I_2 + 1) \quad (8)$$

where:

i_T = Fraction of total load in a given size range

q_T = Total transport rate, weight per unit time and width

i_B = Fraction of bed load in a given size range

q_B = Bed load transport rate

I_1 = Integral value (Evaluation tables furnished by author)

I_2 = Integral value (Evaluation tables furnished by author)

P = Parameter of total transport

and,

$$P = \frac{1}{0.434} \log_{10} \left(\frac{30.2 x}{k_s / d} \right) \quad (9)$$

In the evaluation of sediment movement through the reach of Money Creek considered in this paper, the hydraulic character of the channel was computed in accord with the methods reported by Einstein. Detailed computations are not presented here but the resultant effects of bank friction are shown in Figs. 4 to 6 of this report. The computations of sediment movement are based on these hydraulic computations including the bank friction.

The relationship of sediment discharge to water discharge for Money Creek as determined by the Einstein procedure is tabulated in Table 6 and is shown graphically in Fig. 11. Table 6 shows the utilization of this relation and the flow duration data to determine the total bed material movement into Lake Bloomington during the sedimentation periods under consideration. Total transport calculated by this means amounts to 196,477 tons.

Schoklitsch Bed-Load Formula

The sediment movement in Money Creek was calculated by utilization of the Schoklitsch formula for uniform sand.

$$G = \frac{86.7}{\sqrt{d}} S^{1.5} B (q - q_0) \quad (10)$$

$$q_0 = \frac{0.00532 d}{S^{1.33}} \quad (11)$$

and the bed load for a mixture

$$G_t = a G_a + b G_b + c G_c \dots + m G_m \quad (12)$$

where

G = Bed load in tons per day

G_t = Total bed load for a mixture of particles

G_a = Quantity of bed load of a particular diameter

a = Percent weight of a particular diameter in a mixture

m = Number of size-gradation divisions in a mixture

d = Diameter of particle, inches

S = Hydraulic slope

B = Bed width, feet

q = Discharge, cfs

q_0 = Critical discharge at which movement of particle of diameter d , begins

The Schoklitsch⁽¹⁰⁾ formula serves to compute that portion of the total load of solids in the river which is transported (not in suspension) along the river bed by the tractive force of the stream. The Schoklitsch formula is based mainly on the classic flume experiments of G. K. Gilbert besides additional experimental data collected by Schoklitsch. It was developed for uniform grain material but there can be no valid objection to its being applied to mixtures as well. It has been verified and found to agree closely with the measurements in the River Danube and the Terek River.

In the application of this formula to a natural stream it was stated by Schoklitsch that the reach studied be relatively straight and the depths of water as uniform as possible in order that the width of the stream change as little as possible with stage. Table 7 shows the relation of area and width of Money Creek at the various discharges considered.

Table 8 summarizes the movement of bed-load material in Money Creek as calculated by the Schoklitsch formula. The relationship of sediment discharge to water discharge is plotted in Fig. 11. Table 8 shows the product of sediment discharge rating and flow duration information converted into total quantity of bed-load movement in tons for the various periods. Total transport by this method amounts to 79,065 tons.

DuBoys Formula

The sediment transport in Money Creek has been calculated by the DuBoys formula.

Table 6. Money Creek Sediment Discharge by
Einstein Bed Load Function
For particles 0.05 mm to 9.4 mm

Water Dis- charge cfs	Sediment Discharge Tons/Day	1st Period		2nd Period		3rd Period		
		Dura- tion Days	seal- merit Tons	Dura- tion Days	Sedi- ment Tons	Dura- tion Days	Sedi- ment Tons	Sedi- ment Tons
110	94	376	35,344	124	11,656	59	5,546	2,546
165	145	199	28,855	58	8,410	20	2,900	40,165
245	232	103	23,896	34	7,888	11	2,552	34,336
355	335	58	19,430	18	6,030	6	2,010	27,470
520	550	31	17,050	10	5,500	3	1,650	24,200
760	920	11	10,120	2	1,840	--	-----	11,960
1100	1450	3	<u>4,350</u>	1	<u>1,450</u>	--	<u>-----</u>	<u>5,800</u>
Total			139,045		42,774		14,658	196,477

Table 7. Money Creek Area-Width Relationship

Discharge cfs	Stage Feet	Area Sq.Ft.	Width Feet
110	1.98	49.5	25.0
165	2.38	63.0	26.5
245	2.91	83.0	28.5
355	3.49	106.0	30.4
520	4.13	133.5	32.3
760	4.84	171.0	35.3
1100	5.63	226.5	40.2

and the bed load for a mixture

$$G = (aq_a + bq_b + cq_c + \dots + nq_n) \gamma S_s \quad (14)$$

and

$$\mathcal{T} = \gamma y s \quad (15)$$

where

q = Transport rate of a particular diameter particle in volume per second per foot of width

C_s = Sediment parameter

\mathcal{T} = Intensity of bed shear

y = Unit weight of water

y = Depth of flow

s = Hydraulic slope

\mathcal{T}_c = Value of \mathcal{T} for which q_s is zero

S_s = Specific gravity of sediment particle

a = Percent weight of a particle diameter in a mixture

n = Number of size-gradation divisions in a mixture

G = Bed load total for mixture, pounds per second per foot of width

The DuBoys formula was one of the earliest published to determine the fluid transport of sediment. A great number of other formulas have been developed subsequently and have a similar nature. Johnson⁽¹¹⁾ tested a number of these and concluded that all formulas fitted equally well, thus indicating that the choice could be made on the basis of convenience. In order to utilize this formula, it was necessary to evaluate the parameters C_s and \mathcal{T}_c . The values summarized by Straub and published in Engineering Hydraulics⁽¹²⁾ were utilized. It was necessary, however, to extrapolate these relationships as shown in Fig. 9 for the relation of C_s to particle diameter, and in Fig. 10 for the relation of \mathcal{T}_c to particle diameter.

In Table 9 is summarized the results of the calculation of bed-load movement in Money Creek by the DuBoys formula. The relation of sediment discharge to water discharge has been plotted in Fig. 11. Table 9 shows the computation of the total sediment movement throughout this reach based on the flow duration of Money Creek for the three sedimentation periods. It will be noted that the total quantity of sediment moved calculated by this means amounts to 529,944 tons.

Discussion of Results

General

In Fig. 11 is shown the sediment discharge versus water discharge for Money Creek as determined by the three different methods. Table 10 shows the comparison of the quantity of sediment measured in Lake Bloomington

Table 8. Money Creek Sediment Discharge by Schoklitsch Bed Load Formula

For particles 0.05 mm to 9.4 mm

Water Discharge cfs	Sediment Discharge Tons/Day	1st Period		2nd Period		3rd Period		Total Sediment Tons
		Dura- tion Days	Sedi- ment Tons	Dura- tion Days	Sedi- ment Tons	Dura- tion Days	Sedi- ment Tons	
110	33.6	376	12,622	124	4,163	59	1981	18,766
165	59.8	199	11,894	58	3,467	20	1195	16,556
245	98.0	103	10,090	34	3,331	11	1078	14,499
355	152	58	8,797	18	2,730	6	910	12,437
520	232	31	7,192	10	2,320	3	696	10,208
760	349	11	3,842	2	698	--	----	4,540
1100 515	3		<u>1,544</u>	1	<u>515</u>	--	----	<u>2,059</u>
Total			55,981		17,224		5890	79,065

Table 9. Money Creek Sediment Discharge by DuBoys Bed Load Formula

For particles 0.05 mm to 9.4 mm

Water Discharge cfs	Sediment Discharge Tons/Day	1st Period		2nd Period		3rd Period		Total Sediment Tons
		Dura- tion Days	Sedi- ment Tons	Dura- tion Days	Sedi- ment Tons	Dura- tion Days	Sedi- ment Tons	
110	282	376	105,855	124	34,910	59	16,610	157,375
165	408	199	81,180	58	23,661	20	8,159	113,000
245	657	103	67,647	34	22,330	11	7,224	97,201
355	921	58	53,402	18	16,573	6	5,524	75,499
520	1295	31	40,130	10	12,945	3	3,884	56,959
760	1679	11	18,465	2	3,357	--	-----	21,822
1100	2022	3	<u>6,066</u>	1	<u>2,022</u>	--	-----	<u>8,088</u>
Total			372,745		115,798		41,401	529,944

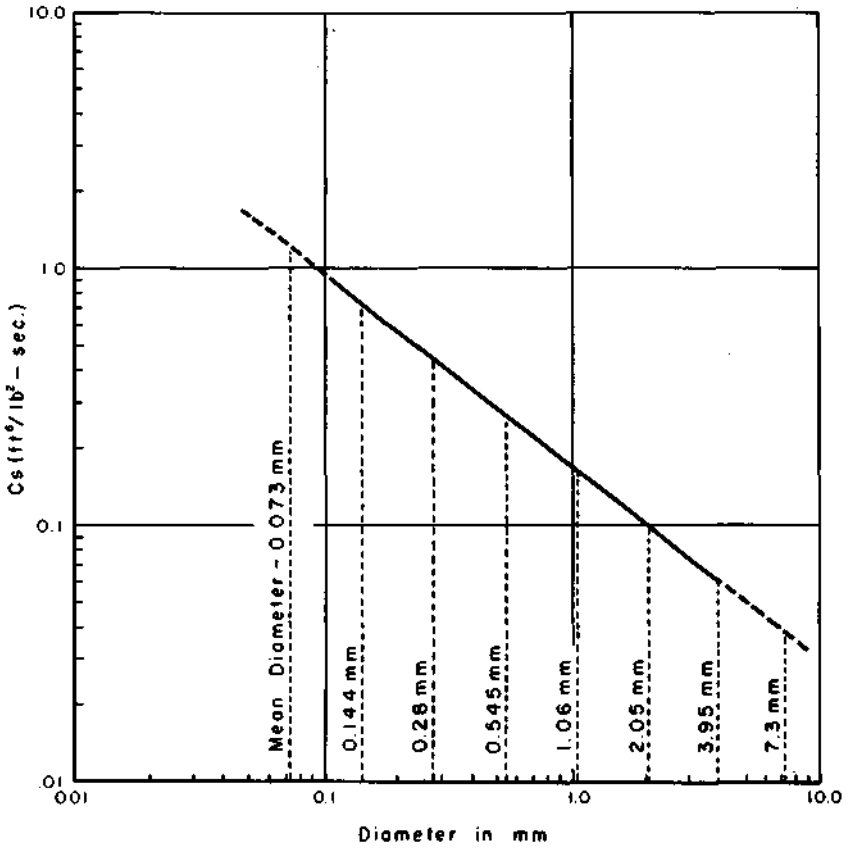


Figure 9 - Relationship of Sediment Parameter to Particle Diameter

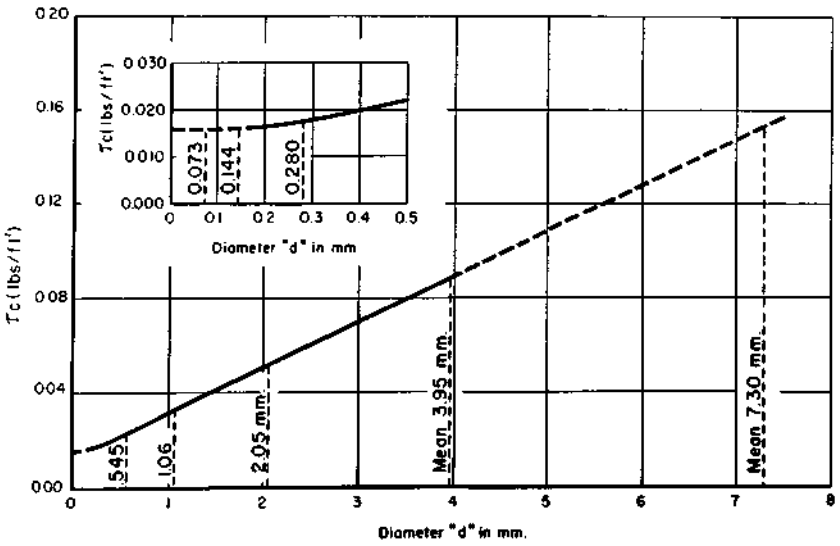


Figure 10 - Relationship of Critical Shear to Particle Diameter

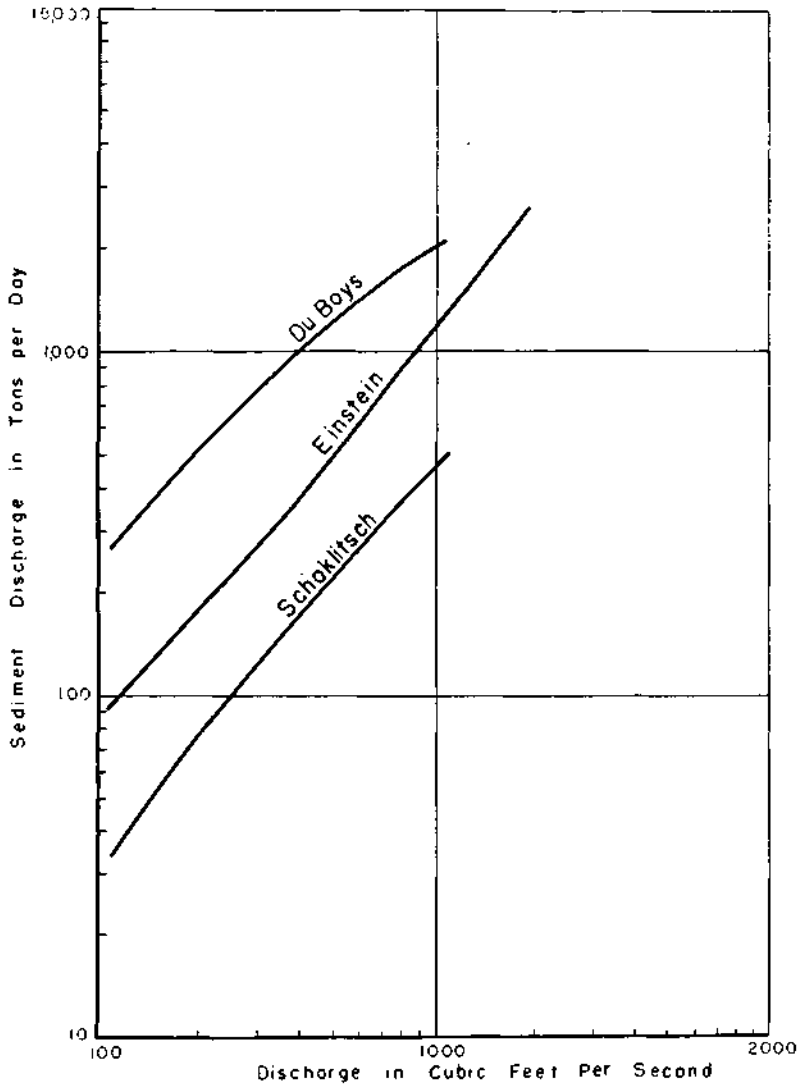


Figure 11 - Sediment Discharge Relation to Water Discharge

with the amount moved through the Money Creek reach, as computed by each of the three methods.

As shown in Table 10, the Schoklitsch formula gives results most nearly in agreement with the actual sediment measured in Lake Bloomington. For the total period of record, this formula gives results in tons only 31 percent greater than the measured quantity. In comparison the Einstein procedure gives results 225 percent too great, and the DuBoys formula, 776 percent too great.

The usefulness and limitations of the three methods utilized in this paper to compute sediment movement have been discussed in detail by Chien.⁽¹³⁾ Recognizing the limitations, these approaches merit continued study, trial, and improvement. These approaches were utilized on the Niobrara River near Cody, Nebraska⁽¹⁴⁾ in 1955. Results showed severe limitations to the Schoklitsch and DuBoys approaches and excellent results from the Einstein approach.

One recognized source of error in the use of a bed-load formula is the use of the water surface slope instead of the slope of the energy gradient. An accurate determination of the slope of the energy gradient requires the measurement of the velocity distribution at each end of the experimental reach. This observation is often eliminated and the resulting error involved in the slope determination is fairly large for the usual experimental conditions. It is of interest to note that Gilbert⁽¹⁵⁾ was undecided as to the proper value of slope to use and stated "I do not find it easy to decide which slope should be regarded as the true correlative of capacity for traction but as all of our laboratory data include the debris slope while the determinations of water slope are relatively infrequent the discussion of the results has adhered almost exclusively to the former. If the water slope is the true correlative then the use of the debris slope involves a systematic error."

Professor O'Brien and Lt. B. D. Rindlaub⁽¹⁶⁾ support Gilbert's selection in the statement "It is to be noted that the slope at the bottom is more nearly equal to the slope of the energy gradient than is the slope at the water surface and partly for this reason the data of G. K. Gilbert show less scattering than the data of more recent experimenters who have criticized Gilbert for not measuring the slope of the water surface in all of his experiments."

The Einstein Method

Sediment movement and river behavior are inherently complex since natural phenomena involve a great many variables. In applying the unified method presented by Einstein,⁽¹⁾ questions may arise such as: (1) Is it possible to obtain a truly representative bed material sample for size distribution characteristic curves? (2) What is the average (representative) diameter of the entire sediment mixture? (3) Is it possible to select an ideally uniform channel in nature? (4) Can the formulas for the hydraulics of the open channel be applied to such a complex problem as that of sediment transport phenomena? (5) If only one point of the size distribution curve is to be used for roughness height k_s , (D_{65}), how much confidence can the engineer have in its practical use? (6) Is the evaluation of bar resistance accurate and sufficient? (7) What is the lower limit in integrating suspended load? Many other questions may also arise. However, the engineer is warned against being discouraged by the absence of a better solution. In answer to the above questions it should be pointed out that the available information on the subject

Table 10. Bed Load Material Deposited in Lake Bloomington Compared to Computed Bed Material Movement in Money Creek.

<u>Sediment</u>	<u>Tons</u>			<u>Total</u>
	<u>1st Period</u>	<u>2nd Period</u>	<u>3rd Period</u>	
<u>Measured</u>				
Deposited in				
Money Creek Arm				
of Lake Bloomington	48,176	10,725	1,626	60,527
<u>Computed</u>				
(Per cent error shown below each value in tons)				
Einstein	139,045	42,774.	14,658	196,477
	189	299	802	225
Schoklitsch	55,981	17,224	5,890	79,065
	16	61	262	31
DuBoys	372,745	115,798	41,401	529,944
	674	980	2,446	776

of hydraulics of open channel flow has been applied as closely as possible. Other information is rationalized through practical experience and field measurements.

Schoklitsch and DuBoys Formulas

Although these two formulas are essentially the same in structure, their application to Money Creek gives results at great variance. It will be seen that the results of the Schoklitsch formula are more nearly in agreement with the survey data. One reason for the high values in the case of DuBoys may be the limitations to the evaluations of the parameters C_s and τ_c . Since the sediment parameter C_s expresses the relative susceptibility of a given sediment to movement and since the shear terms τ and τ_c involve the complex system of forces exerted by the flow upon the bed, the evaluation of these parameters by means of suitable experimental methods determines the reliability of the results. The adaptation of the values, utilized as shown in Figs. 9 and 10, to conditions in Money Creek is therefore somewhat questionable.

The Manning formula permits DuBoys relationship to be written in the following alternative form.

$$\begin{aligned}
 q &= C_s \tau (\tau - \tau_c) & (16) \\
 &= C_s \frac{r^2 s^{1.4}}{(1.49/n)^{1.2}} (q^{1.2} - q_0^{1.2})
 \end{aligned}$$

The exponent of the slope is 1.4 in this relationship as against 1.5 in the Schoklitsch formula while the exponent of q_0 is 1.2 as compared with 1.0. Although this shows general agreement the disparity between the results of the two formulas is well accounted for. In addition to this, it is a recognized fact⁽¹²⁾ that the DuBoys formula was based on an incorrect assumption as to the sliding motion of the sediment particles in movement.

CONCLUSION

By utilizing the measured quantity of sediment of bed-material size in Lake Bloomington as a check on the computed sediment which has moved through Money Creek it is concluded that the Schoklitsch formula gives the most reliable results, being only 31 percent in error for the over-all period of the study. The Einstein procedure gave results 225 percent high and the classic DuBoys formula gave results 776 percent high.

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