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Abstract

The current trend in the metal-cutting industry is to find ways to completely eliminate or drastically reduce cutting fluid use in most machining operations. Recent advances in tool and machine technology have made it possible to perform some machining without cutting fluid use or with minimum quantity lubrication (MQL). Drilling takes a key position in the realization of dry or MQL machining. Economical mass machining of common metals (e.g., tool and construction-grade steels) requires knowledge of the work piece characteristics as well as the optimal machining conditions. In this study we investigated the effects of using MQL in drilling 1020 and 4140 steels using HSS tools with different coatings and geometries. The treatments selected for MQL in this study are commonly used by industry under flood cooling for these materials. A full factorial experiment was conducted, and the regression models for both surface finish and hole size were generated. The regression models were then used in a Pareto optimization study, and the trade-off between surface finish and hole size deviation from the nominal size was reported. The results showed a definite increase in tool life and better or very acceptable surface quality and size of holes drilled when using MQL compared with flood cooling.

Executive Summary

The objective of this study was to investigate the machinability of 1020 steel and 4140 steel using minimum quantity lubrication (MQL) and flood cooling. The study used four different tools. An experiment plan was developed and data for tool life, surface finish, and hole deviation were collected for 1020 steel and 4041 steel under MQL. A second experiment under flood cooling was conducted for those tools that showed longer life. The machining community could easily use the results of this study as a guide for their machining operations when cutting 1020 and 4140 steels using HSS drill bits under the specified machining parameters and conditions. The regression models generated in this study were used in a Pareto optimization investigation. The trade-off between the surface finish and deviation of the hole size from the nominal size was investigated. A summary of the experiment results is reported below.

Summary of the Study Results for Drilling 1020 Steel: MQL Case

Part 1 of this research was conducted to find the machinability of 1020 steel using four different HSS drill bits with MQL and flood applications by varying the cutting speed and feed rate. Table 1 shows the maximum life, surface finish, and hole size for the four drills used in this study for 1020 steel under MQL cooling. Note that if the first, second, and third best surface and hole size were similar, then they all were reported. Otherwise, only the best case was reported.

Table 2 shows the feed and speed for the best maximum life, surface finish, and hole size reported in Table 1. Note that the same best hole size was achieved using three sets of feed and speed for Tool 305. All three sets are reported.

Table 1: Best Maximum Life, Surface Finish, and Hole Size Using MQL.

	Drill 205	**Drill 305**	**Drill 651**	**Drill 657**
Best Maximum Life	1320	1260	900	900
2nd Best Maximum Life	960	N.S.T.R.*	660	840
3rd Best Maximum Life	N.S.T.R.*	N.S.T.R.*	570	N.S.T.R.*
Best Average Surface Finish (micro inches)	287.85	234.5	238.27	175.0
2nd Best Average Surface Finish (micro)	308.64	N.S.T.R.*	238.76	N.S.T.R.*
Best Average Hole Size (in)	0.5050	0.5050	0.5030	0.5030
2nd Best Average Hole Size (in)	0.5065	N.S.T.R.*	N.S.T.R.*	N.S.T.R.*

*Not significant

Table 2: Feed and Speed for Best Maximum Life, Surface Finish, and Hole Size Under MQL.

	Drill 205		Drill 305		Drill 651		Drill 657	
	Speed (SFM)	**Feed (IPR)**	**Speed (SFM)**	**Feed (IPR)**	**Speed (SFM)**	**Feed (IPR)**	**Speed (SFM)**	**Feed (IPR)**
Best Maximum Life	100	0.008	100	0.008	80	0.006	80	0.006
2nd Best Maximum Life	80	0.008	N.S.T.R.*	N.S.T.R.*	100	0.006	80	0.008
3rd Best Maximum Life	N.S.T.R.*	N.S.T.R.*	N.S.T.R.*	N.S.T.R.*	80	0.010	N.S.T.R.*	N.S.T.R.*
Best Average Surface Finish	100	0.006	100	0.010	120	0.006	100	0.008
2nd Best Average	120	0.008	N.S.T.R.*	N.S.T.R.*	100	0.010	N.S.T.R.*	N.S.T.R.*
Best Average Hole Size	100	0.008	100	0.008	80	0.010	80	0.010
	N.A.!	N.A. !	120	0.006	N.A. !	N.A. !	N.A. !	N.A. !
	N.A. !	N.A. !	120	0.008	N.A. !	N.A. !	N.A. !	N.A. !
2nd Best Average Hole Size	120	0.008	N.S.T.R.*	N.S.T.R.*	N.S.T.R.*	N.S.T.R.*	N.S.T.R.*	N.S.T.R.*

!Not Applicable
*Not significant

Summary of the Study Results for Drilling 1020 Steel: Flood Cooling

The drill bits that achieved a tool life of greater than 900 holes, Drill 205 and Drill 305, were also tested with flood cooling under maximum tool life conditions under MQL. Table 3 shows the best maximum life, surface finish, and hole size under flood cooling. Table 4 shows the feed and speed for best maximum life, surface finish, and hole size using flood cooling.

Summary of Study Results for Drilling 4140 Steel: MQL

Part 2 of this study was conducted to find the effects of drilling a 1 inch deep hole into a block of 4140 steel using four different (titanium, cobalt, and regular) 0.5 inch high-speed steel drill bits. Two feed rates (0.006 and 0.008 IPR) and two speeds (60 and 80 SFM) for a total of 16 combinations of treatments were performed on a CNC Bridgeport milling machine under a mist coolant for MQL. Table 5 shows the best maximum life, surface finish, and hole size. Table 6 shows the feed and speed for best maximum life, surface finish, and hole size reported in Table 5.

Table 3: Best Maximum Life, Surface Finish, and Hole Size Using Flood Cooling.

	Drill 205	Drill 305
Best Maximum Life	10	10
Best Average Surface Finish (micro inches)	169	N.S.T.R.*
Best Average Hole Size (in)	0.5120	N.S.T.R.

*

*Not significant

Table 4: Feed and Speed for Best Maximum Life, Surface Finish, and Hole Size Under Flood Cooling.

	Drill 205		Drill 305	
	Speed (SFM)	Feed (IPR)	Speed (SFM)	Feed (IPR)
Best Maximum Life	100	0.008	100	0.008
Best Average Surface Finish	100	0.008	N.S.T.R.*	N.S.T.R.*
Best Average Hole Size	100	0.008	N.S.T.R.*	N.S.T.R.*

*Not significant

Table 5: Best Maximum Life, Surface Finish, and Hole Size Using MQL.

	Drill 205	Drill 305	Drill 651	Drill 657
Best Maximum Life	< 10	30	230	270
Best Average Surface Finish (micro inches)	30.5	32.0		30.0

Table 6: Feed and Speed for Best Maximum Life, Surface Finish, and Hole Size Under MQL.

	Drill 205		Drill 305		Drill 651		Drill 657	
	Speed (SFM)	Feed (IPR)	Speed (SFM)	Feed (IPR)	Speed (SFM)	Feed (IPR)	Speed (SFM)	Feed (IPR)
Best Maximum Life	All Treatments	All Treatments	80	0.006	80	0.006	80	0.008
Best Average Surface Finish	60	0.006	60	0.006	120	0.006	60	0.006

Summary of Study Results for Drilling 4140 Steel: Flood Cooling

The HSS tools that provided a tool life greater than 230 holes, Drill 651 and Drill 657, were also tested with flood cooling under the conditions that provided maximum tool life under MQL. Table 7 shows the best maximum life, surface finish, and hole size using flood cooling. Table 8 shows the feed and speed for the best maximum life, surface finish, and hole size reported in Table 7.

Table 7: Best Maximum Life, Surface Finish, and Hole Size Using Flood Cooling.

	Drill 651	Drill 657
Best Maximum Life	240	10
Best Average Surface Finish (micro inches)	73	65
Best Average Hole Size (in)	0.5000	0.5160

Table 8: Feed and Speed for Best Maximum Life, Surface Finish, and Hole Size Under Flood Cooling.

	Drill 651		Drill 657	
	Speed (SFM)	Feed (IPR)	Speed (SFM)	Feed (IPR)
Best Maximum Life	80	0.006	80	0.008
Best Average Surface Finish (micro inches)	80	0.006	80	0.008
Best Average Hole size (in)	80	0.006	80	0.008

Introduction

The current trend in the metal-cutting industry is to find ways to completely eliminate or drastically reduce cutting fluid use in most machining operations. In fact, an increasing number of countries view the use of coolants in machining ferrous and nonferrous components as undesirable for economical, health, and environmental reasons. In a German study, Heins (1997) reported that coolant and coolant management costs are between 7.5% and 17% of the total manufacturing cost compared with only 4% for cutting tools. Sreejith and Ngoi (2000) stated that lubrication represents 16 to 20% of the product cost. Quaile (2000) reported that the coolant cost is approximately 15% of the life-cycle operational cost of a machining process.

Chalmers (1999) reported that more than 100 million gallons of metalworking fluids are used in the U.S. each year, and that 1.2 million employees are exposed to them and to their potential health hazards. The savings in cutting fluid and other related costs would be very significant if micro-lubrication (minimum quantity lubrication or MQL) is adopted, particularly in common machining operations (e.g., milling and drilling) that are currently conducted with flood application.

Minimum quantity lubrication administers traditional metal removal fluids (oils and water miscible) at very low levels (.02 gallons/min or lower). These are once-through systems; there is no need to collect the applied fluid. MQL systems are considerably more cost-effective than flood application systems. McCabe (2002) reported that according to automakers, the annual operating cost of a flood application-based machining system is estimated to be between $350,000 and $1,000,000. The cost for an MQL system is between $100,000 and $300,000. In the same study, he reported that the component cost was reduced by 45% when minimum quantity lubrication was used compared with flood cooling in drilling aluminum.

Horkos (2006) compared the cost of flood coolant with the MQL performed by a Japanese cutting tool manufacturer (Figure 1). Figure 1 depicts a sharp cost reduction using MQL compared with flood cooling.

The challenge in using MQL for machining is to provide substitutes for the four critical functions of flood cooling. Although it is generally thought that MQL systems can supply excellent lubrication, the results on acceptable cooling are not conclusive. Recent advances in tool and machine technology have made it possible to perform some machining without cutting fluid use or with MQL. Drilling takes a key position in the realization of dry machining. The main problem in dry drilling of steels is the reliable removal of chips from the drilled hole. Another problem is the tendency of the drill to jam in the hole if its diameter expands too much as a result of a high tool temperature (Klocke et al. 1995).

The development of various coating technologies that would improve wear resistance for various tools has found the integration of hard coatings with cutting tool substrate materials to be the most successful innovation in this regard (Quinto 1996; Sahoo et al. 2002). McCabe et al. (2001) reported that coating drills with a variety of standard products raised the hole-producing capability of twist drills from 25 to approximately 225 holes when cutting aluminum. The tool geometry and cutting conditions were further optimized, which raised its drilling capacity to

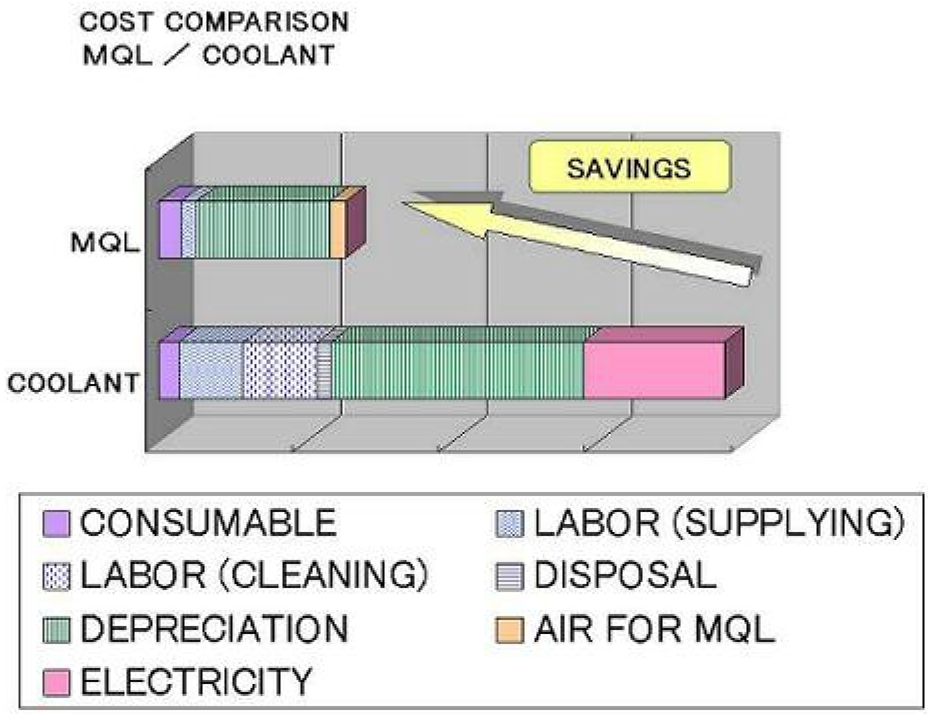


Figure 1: Cost comparison of coolant and MQL.

5000 holes. Nouari et al. (2003) reported that with large cutting speeds and low feed, good surface quality and dimensional accuracy can be obtained with optimum drill geometry when machining aluminum. They also reported that tool life was increased significantly when optimized drill geometry was coated with a diamond film in the same experiment.

Klocke and Eisenblatter (1996) reported that dry drilling was not possible because of the strong tendency of the aluminum to adhere to the tool. It was found that even a minimum quantity of cutting fluid that is fed towards the contact zone suffices to achieve a drilling operation that meets the stipulated quality characteristics. Braga et al. (2003) conducted a study in which the objective was to test the MQL technique to drill aluminum silicon alloy with a solid carbide drill. They showed that drilling aluminum can be successfully achieved with MQL.

One concern of MQL is that the metalworking fluids mist themselves, and consequently, are potential health hazards. The standard advisory committee convened by the United States Occupational Health and Safety Administration (OSHA) in 1997 found that exposure to metalworking fluids may result in cancer, asthma, hypersensitivity pneumonitis, other respiratory disorders, dermatitis, and other health conditions.

The optimal selection of machining parameters such as speed and feed rates is a critical issue when determining the use of machining parts. In the real world, there are multiple objectives that often compete with each other. They should be optimized simultaneously, and the trade-off among them should be studied. Aman and Hari (2005) discussed various techniques to optimize machining processes.

More realistic decision-making becomes possible when there are several alternatives to select. A trade-off is frequently used in decision making. A trade-off is defined as a reduction in one criterion to gain a unit improvement in another. Therefore, to choose the best compromise among different solutions, the decision-maker must bring his or her preferences to the design process. Formally, the best trade-off mathematically is defined as the Pareto optimization. A point is a Pareto optimal point if all the objectives cannot be improved at the same time. Das developed a new method called Normal Boundary Intersection (NBI), which generates equally spaced Pareto points on the Pareto front. Kim and de Weck (2004) enhanced the bi-objective adaptive weighted sum method that generates an even spread of Pareto points on non-convex regions of multi-objective problems. He also showed that his method is more effective for visualization of the Pareto front mesh. Kim and Kim (2004) proposed a new method for interactive Multi-Objective Programming (MOP) to increase the effectiveness of both the NBI method and Interactive Weighted Tchebycheff Procedure (IWTP). Galperin (1997) studied and compared the Pareto analysis with the balance space approach and demonstrated the differences and interrelationship between them.

Costs associated with procurement, filtration, separation, disposal, and record-keeping for the US Environmental Protection Agency (USEPA) for coolant are increasing. Already, the costs to dispose of coolant are higher than the initial cost of the coolant, and prices are still rising. Even stricter regulations are under consideration for coolant usage, disposal, and worker protection. As a result, coolant in wet machining operations is a crucial economic issue. An alternative, machining with MQL, is gaining acceptance as a cost-saving and environmentally friendly option in place of some wet machining processes.

Additional research of MQL is needed in all metalworking processes that use flood coolants as a cooling option. At this point, there have been no studies conducted to determine the cutting effects of high-speed steel drill bits when drilling holes into 1020 and 4140 steel. The objective of this research is to study the machinability of 1020 and 4140 steel using four different high-speed steel drill bits with MQL and the trade-off between surface finish and the deviation from nominal hole size.

Research Objectives

This project aimed to study the effects of feed, speed, and coating when drilling a 1 inch deep hole into a block of 1020 and 4140 steels using four different half-inch steel drill bits. The drill bits were made of high-speed steel (two Titanium-coated, one high-speed steel-cobalt combination, and one regular high-speed steel). The drilling was performed on a CNC Bridgeport milling machine under a mist coolant. The objectives of this research were to:

1. evaluate the effects of speed and feed rate on the surface finish in drilling 4140 and 1020 steels;
2. evaluate the effects of speed and feed rate on hole size in drilling 4140 and 1020 steels;
3. evaluate the interaction effect of speed and feed rate on the surface finish in drilling 4140 and 1020 steels;
4. evaluate the interaction effect of speed and feed rate on hole size in drilling 4140 and 1020 steels;
5. determine the correlation between the surface finish and the number of holes drilled for each tool and each treatment when drilling 4140 and 1020 steels;
6. make recommendations for feasible solutions based on the study results; and
7. investigate the effects of the levels of the optimal machining conditions for MQL under flood cooling.

Methods, Procedures, and Results

The following sections focus on the methods, procedures, and results used to conduct this study. Also, the experimental procedures, drilling tools and equipment, and the equipment used for data collection are discussed.

Design of Experiment

This study was conducted using a randomized factorial experimental design, as shown in Tables 9 and 10 for 4041 and 1020 steel, respectively. The two independent variables were cutting speed and feed rates. The depth of the hole was 1 inch throughout for all drilling operations. The two dependent variables were surface finish and hole size (inner diameter, ID). The speed and feed are reported in square feet per minute (SFM) and inches per revolution (IPR).

Cutting Tools

The tools used were high-speed steel (HSS) and cobalt drill bits manufactured by Guhring, Inc. with the following specifications/dimensions (Table 11).

Drilling Equipment

A computer numeric-controlled Bridgeport vertical milling machine, Discovery Torq- Cut 22, was used to perform the drilling operations for this study, as shown in Figure 2.

Table 9: Factorial Experiment Layout for 4041 Steel.

Drill #	**Speed=60SFM**	**Speed=80SFM**
Feed= 0.006 IPR	Treatment 1	Treatment 2
Feed=0.008IPR	Treatment 3	Treatment 4

Table 10: Factorial Experiment Layout for 1020 Steel.

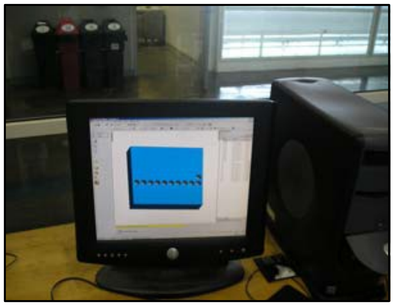
Drill #	**Speed=80SFM**	**Speed=100SFM**	**Speed=120SFM**
Feed= 0.006IPR	Treatment 1	Treatment 2	Treatment 3
Feed=0.008IPR	Treatment 4	Treatment 5	Treatment 6
Feed=0.01IPR	Treatment 7	Treatment 8	Treatment 9

Table 11: Specifications and Dimensions of Guhring, Inc. Drill Bits.

Tool Specification	Diameter (in)	Coating	Cutting Angle (deg)
Drill 205	0.500	No coating	118
Drill 306	0.500	Cobalt	118
Drill 651	0.500	Titanium	118
Drill 657	0.500	Titanium	130

Figure 2: Bridgeport vertical milling machine, Discovery Torq-Cut 22.

Figure 3(a) shows a 3-D model of a block generated on a feature cam. Figure 3(c)-(g) shows the actual drilling process using MQL. The work piece material was 4140 and 1020 steels billets, flame cut to a workable size of 7×6×2 inches, as shown after being drilled, in Figure 4.



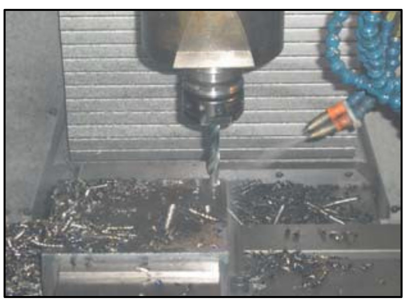
(a)

(b)

(c)

(d)

(e)



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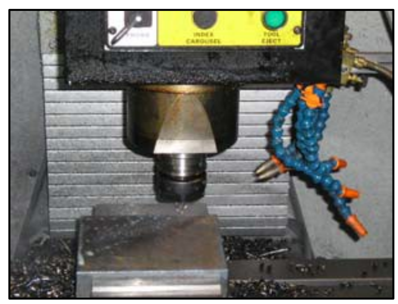

(g)

Figure 3: Machining by using MQL.

Figure 4: Drilled work pieces.

Drilling Procedure

1. Pick a work piece at random from the batch.
2. Turn on the three-axis CNC milling machine.
3. Open safety door.
4. Place billet into manual vice clamp and center.
5. Using a standard hand file, two engraved markings are made on the upper left corner to indicate the initial hole of the sequence.
6. Zero-out the center and use a half-inch drill bit for all three axes.
7. Place misters about 6 inches from the spot drill and aim directly onto the spot drill bit point.
8. Initiate the drilling program for desired drilling parameters.
9. Pause the machine after the first 10 spot-drilled points and automatic tool change.
10. Adjust mister to be about 6 inches away from the drill bit at about a 45-degree angle and aim directly on lower one-quarter portion of the drill bit cutting end.
11. Once the initial 10-hole sequence has been drilled 1 inch deep, the machine stops and brings forth the table/vice/billet for removal.
12. The vice is then loosened; billet is removed, and then placed on its left side.
13. Earmuffs are worn for noise protection, and the holes are cleared of any debris using an air nozzle.
14. The billet is then placed on a nearby table with the holes facing upward for hole size

measurement.

Data Collection

Each treatment was repeated until the tool failed. The tool was declared failed if:

- Three consecutive inner diameter readings were greater or equal to 0.51 inches.

or

- The hole diameter became smaller than the very first hole drilled.

The criterion has been determined to be very feasible by the die and mold industry tool makers.

The data collected were the surface finish and hole diameter. All data were collected and saved on a spreadsheet.

Inner Diameter Measuring Procedure

1. Using a standard digital caliper, the inside diameter of the first and every 10th hole were measured and recorded on a spreadsheet.
2. If the inner diameter of the hole was greater than 0.51 inches, the previous two holes were then measured. If three consecutive readings that were greater or equal to 0.51 inches were recorded, the tool was declared failed.
3. If the previous two holes did not depict the same failure result of greater than or equal to 0.51 inches, the drilling process was repeated for another sequence of 30 holes.

Measuring Surface Finish

1. The surface finish of all the holes drilled was measured at the end of every day.
2. A Mitutoyo surface finish profilometer, model no. 211, was used to measure the surface finish.
3. The work piece was set on a clamping vice for surface finish measurement (Figure 5).
4. The cut-off length for the measurement was set at 0.1 inches.
5. The stylus was inserted and a startup button was pushed to take the [Ra] reading. Two readings of surface finish were recorded for every 10th hole of each row of drilled holes.

Data Analyses

The analysis of variance was conducted for each tool and for both surface finish and hole size for all treatments. The purpose was to investigate the significant effects of each response variable. The following steps were performed in the analysis:

1. Check the F-value to find out if the model is significant.
2. Perform significance tests for the main and interaction effects for independent variables.
3. Check the R-square and Adj. R-square values. Perform any transformation of model if needed.
4. Reduce the model to find out the significant effects.

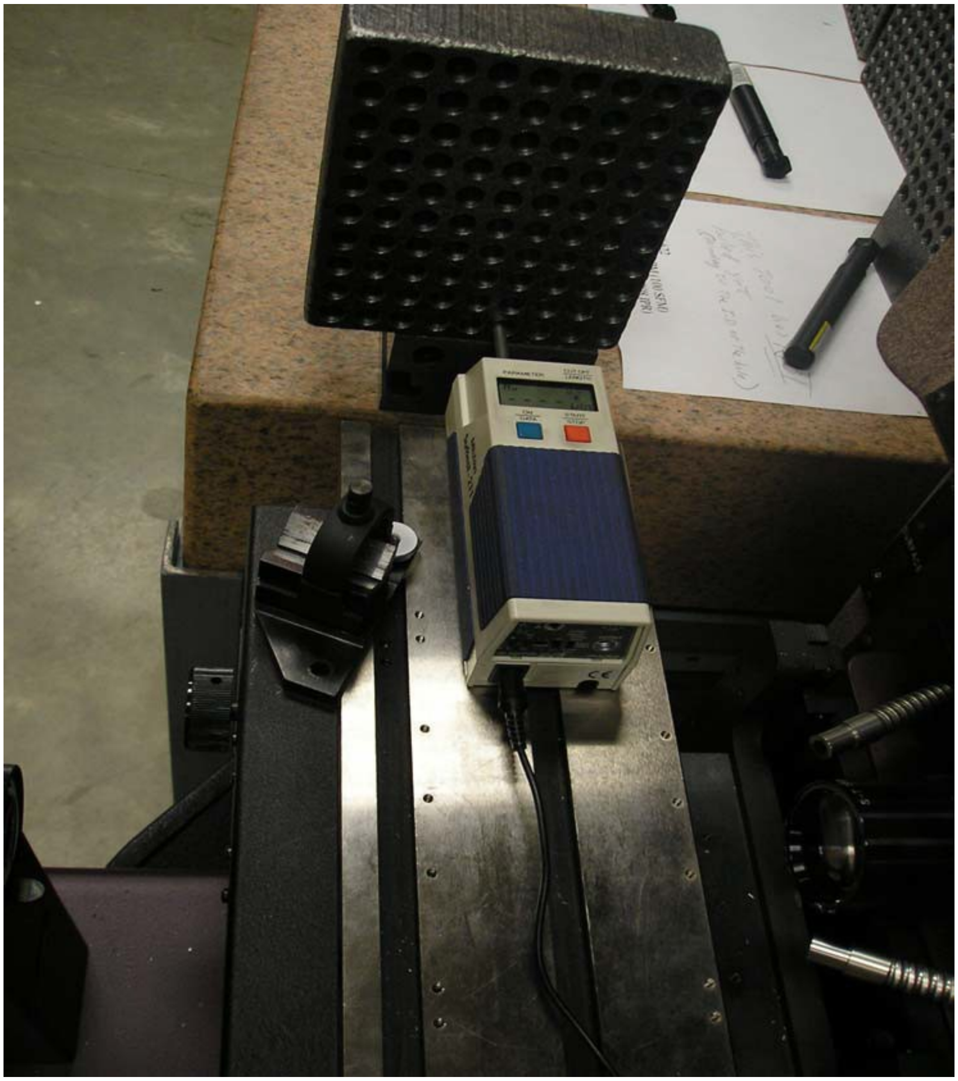

Figure 5: Profilometer.

Assumptions

1. Individual measurement differences and errors were normally distributed within each group.
2. Size of the variance in the distribution of individual differences and random errors was identical in each group.
3. Individual differences and measurement errors were independent from group to group.

To check the first assumption, the residual was plotted vs. the predicted value for all treatments and for both hole size and surface finish (see Appendix A). The plots confirm that the data were normally distributed. To check the second assumption, the residuals were plotted vs. the predicted values (see Appendix A). No pattern was observed. Therefore, the data have a constant variance.

The sources of the outliers in the hypothesis were many. These included excessive vibrations, material homogeneity, and potential errors in fixturing and instrument readings. A measure of influence is the Cook's distance, which was a scaled measure of the difference between the fitted values with and without the k^{th} observation in the model. That is:

$$D_k = \frac{1}{p+1} s^2 \sum_{i=1}^{n} (y_i(k) - y_i)^2$$

D_k = Cook's distance
p = number of regressor variable in the model
s = standard deviation
$y_i(k)$ = fitted value for i^{th} observation when k^{th} observation is omitted
y_i = i^{th} observation

A large value indicates that the k^{th} observation was influential. Based on this statistic, some of the outlying data in the analysis of surface finish and inner diameter have been removed. The analysis of variance and the regression models were conducted after the omission of the outliers from data based on the Cook's distance method.

An analysis of variance was performed and the results were reported in Appendix B. The F- statistics test was performed to ensure that the model was significant at a 5% confidence level. The analysis of variance was conducted, and the important factors and interactions at the 5% confidence level were identified. The following were the prediction models for surface finish and inner diameter deviation using four different HSS drill bits. The regression model was of the form:

$$S_f(S,F) = A_0 + A_1 S + A_2 F + A_3 S^2 + A_4 F^2 + A_5 S F \quad (1)$$

$$H_s(S,F) = B_0 + B_1 S + B_2 F + B_3 S^2 + B_4 F^2 + B_5 S F \quad (2)$$

Where S and F are speed and feed, respectively. S_f and H_s are the surface finish as measured by R_a and the hole diameter, respectively. The coefficients A's and B's are reported in Table 12 and Table 13, respectively.

The R-squared and Adjusted R-squared values of the regression models for 1020 steel are reported in Tables 14 and 15, respectively. The R-squared and Adjusted R-squared values for 1020 steel is above 0.9, therefore all the regression models are good predictors for 1020 steel. Either R-squared or Adjusted R-squared values are less than 0.9, except for the hole size for Drill 657, for 4140 steel. Therefore, only the regression model for hole size for Drill 657 is a good predictor. The authors decided to conduct the Pareto optimization study for 1020 steel regression models only.

Table 12: Coefficients of the Regression Models for 1020 Steel.

Tool	Surface Finish					Hole Size				
	A1	A2	A3	A4	A5	B1	B2	B3	B4	B5
Drill 205	-5.89327	192805	0.05207	-5967869	-913.47936	0.00053769	-4.50992	-0.00000249	268.06685	0
Drill 305	20.55421	-152962	-0.14251	0	869.49781	0.00053366	-5.25426	0	465.53860	-0.01950
Drill 651	1.41061	97789	-0.04029	-10052562	579.54642	0.00002852	0	0	-8.62163	0.01429
Drill 657	0	90841	0	-5935234	0	0.00021732	1.65497	-0.00000148	-214.71018	0.00999

Table 13: Coefficients of the Regression Models for 4140 Steel.

Tool	Surface Finish				Hole Size			
	A0	A1	A2	A5	B0	B1	B2	B5
Drill 205	N.E.D.G.*	N.E.D.G.*	N.E.D.G.*	N.E.D.G.*.	N.E.D.G.*	N.E.D.G.*	N.E.D.G.*	N.E.D.G.*
Drill 305	97.88	53.63	6.62	-6.62	0.51	-0.875E-003	-2.542E-004	5.542E-004
Drill 651	97.05	40.54	-1.16	-3.94	0.50	-2.792E-004	-2.542E-004	5.542E-004
Drill 657	116.99	41.87	23.37	20.01	0.51	5.597E-003	-3.937E-003	4.987E-003

*** Not Enough Data Generated due to early tool failure**

Table 14: The R-squared and Adj R-squared Values for the Regression Models for 1020 Steel.

Tool	Surface Finish		Inner Diameter Deviation	
	R-squared	Adj R-squared	R-squared	Adj R-squared
Drill 205	0.9783	0.9780	0.9276	0.9245
Drill 305	0.9708	0.9701	0.8630	0.8541
Drill 651	0.9456	0.9449	0.9474	0.9455
Drill 657	0.9286	0.9278	0.9069	0.9038

Table 15: R-squared and Adj R-squared Values for the Regression Models for 4140 Steel.

Tool	Surface Finish		Hole Size	
	R-squared	Adj R-squared	R-squared	Adj R-squared
Drill 205	N.E.D.G.*	N.E.D.G.	N.E.D.G.	N.E.D.G.
Drill 305	0.9128	0.6512	0.8512	0.6279
Drill 651	0.8546	0.8348	0.8258	0.7997
Drill 657	0.9008	0.8926	0.9406	0.9366

*** Not enough data generated due to early tool failure**

Multi-Objective Optimization (MOP)

The general mathematical formulation of a Multi-Objective Optimization is:

$$\text{Minimize} \quad F(x) = \left[f_1(x) \quad f_2(x) \quad . \quad . \quad f_n(x) \right]^T, \qquad n \geq 2$$

$$\mathrm{x} \in C,$$

where

$$C = \{x : h(x) = 0, g(x) \leq 0, x_l \leq x \geq x_u\}$$
$$h(x) \textit{ is equality constraint and}$$
$$g(x) \textit{ is non equality constraint}$$

"*C*" denotes the feasible set defined by equality and inequality constraints and explicit variable bounds. The space in which the objective vector forms is called the *objective space,* and the image of the feasible set under *F* is called the *attained set*. The goal here was to minimize both the *Surface Roughness* as measured by the R_a value of the surface finish, S_f and *Hole Deviation*, as measured by the deviation of the hole diameter from its nominal value, H_s, while satisfying the bounds on *Speed* and *Feed*. That is:

$$\text{Minimize } F(x) = \begin{bmatrix} S_f(S,F) \\ H_s(S,F) \end{bmatrix}$$

Subject to

$$80 \ SFM < S < 120 \ SFM$$
$$0.006 \ IPR < F < 0.01 \ IPR$$

There are no equality constraints, $h(x)$, and inequality constraints, $g(x)$, constraints for the above MOP. Because surface roughness, hole deviation, feed, and speed have a different order of magnitude, they were scaled. That is:

$$S = 100s; F = \frac{f}{100}; s_f = \frac{S_f}{100}; \ h_s = 100H_s \, (3)$$

Substitute the set of Equation (3) into Equation (1) and (2) to get:

$$s_f(s, f) = A_1 s + 10^{-4} A_2 f + 10^{-6} A_3 f + 10^{-6} A_4 s^2 + A_5 sf \, (4)$$
$$h_s(s, f) = 10^4 B_1 s + B_2 f + 10^{-2} B_3 f + 10^{-2} B_4 s + 10^2 B_5 sf \ (5)$$

Where *sf*, h_S, *s*, *and f* are the scaled surface finish, hole deviation, speed, and feed, respectively. Equation (4) and (5) are used to generate a series of surface plots. They are reported in Appendix D.

The following optimization problem is defined using the scaled objectives and variables. That is:

$$\text{Minimize} \quad \begin{bmatrix} s_f(s,f) \\ h_s(s,f) \end{bmatrix}$$

Subject to:

$$0.80 < s < 1.20$$
$$0.6 < f < 1.0$$

The Normal Boundary Intersection (NBI) method (see Appendix E for details) is used to generate the Pareto front and its image in the design space for Drill 205, 305, 651, and 657. The Pareto front and its image in the design space for Drill 205 are shown in Figures 6 and 7, respectively.

The Pareto front and its image in the design space for Drill 305 are shown in Figure 8, Figure 1, and Figure 9, respectively.

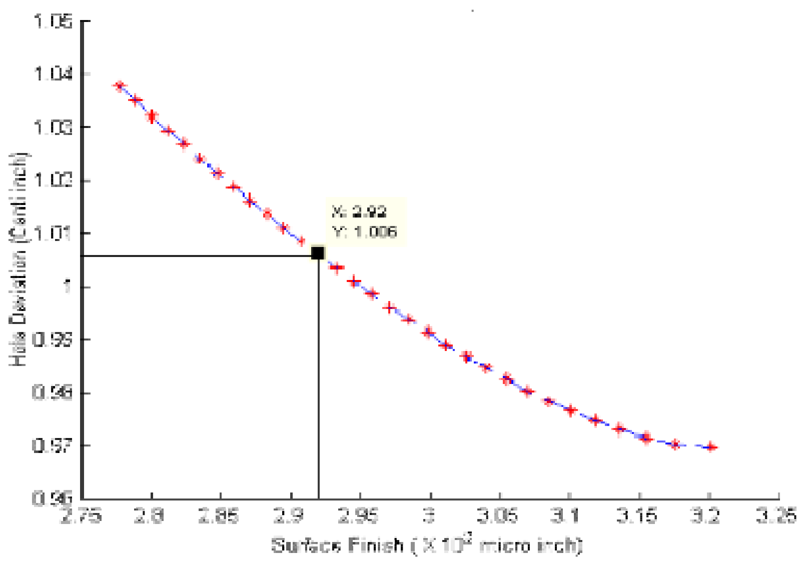

Figure 6: Pareto front for Drill 205.

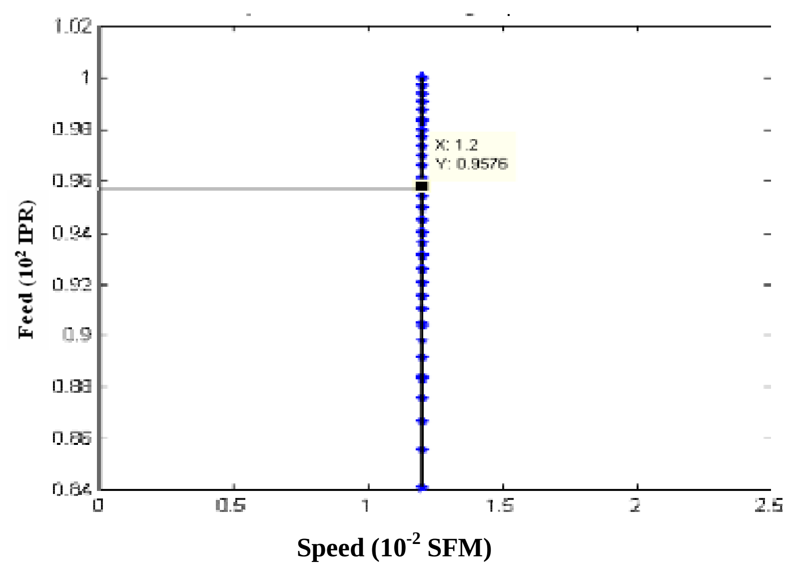

Figure 7: Image of the Pareto front in the design space.

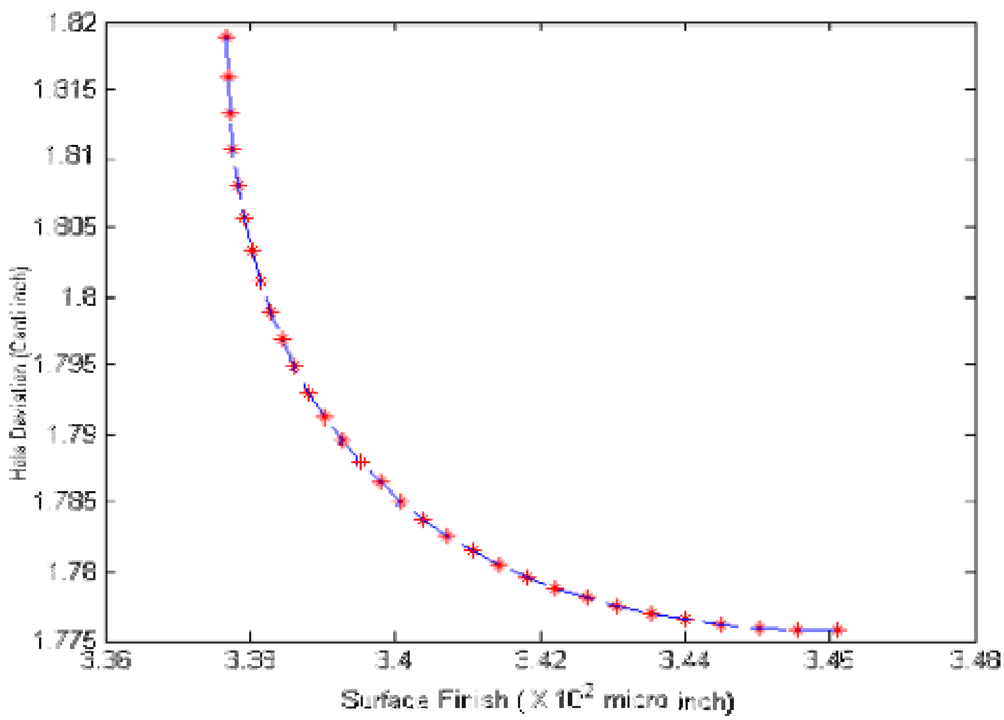

Figure 8: Pareto front for Drill 305.

For Drill 651 and Drill 657, the surface finish and hole size deviation were not competing, and a single point that minimizes both criteria was found. The optimal feed and speed and the corresponding surface finish and hole size deviation for Drill 651 and 657 were reported in Table 16.

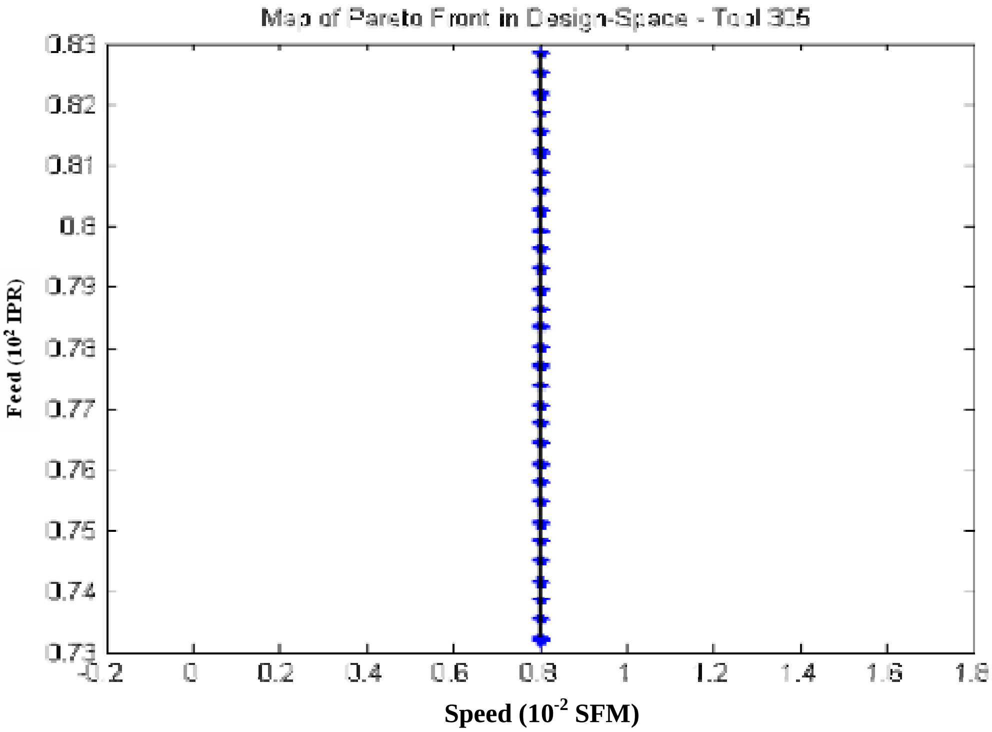

Figure 9: Image of Pareto front in design space for Drill 305.

Table 16: Optimal, Feed, Speed, Surface Finish, and Hole Size Deviations for Drills 651 and 657.

Tool	Speed (SFM)	Feed (IPR)	Surface Finish	le Size deviation
Drill 651	120	0.006	240.00	4.00
Drill 657	80	0.010	314.89	4.30

The surface finish and hole size as a function of tool life was plotted and reported in Appendix C. A summary of tool life and the trend for surface finish and hole size were reported in Table 17 through Table 29.

Table 17: Life, Surface Finish, and Hole Size Trends for Speed=80 SFM and Feed= 0.006 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	60	No trend observed	No trend observed
Drill 305	60	Increase in hole size	No trend observed
Drill 651	900	Increase in hole size	Increased up to 110th
Drill 657	900	Increase in hole size	Increase in surface finish

Table 18: Life, Surface Finish, and Hole Size Trends for Speed=80 SFM and Feed= 0.008 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	960	Increase in hole size	No trend observed
Drill 305	60	Increase in hole size	No trend observed
Drill 651	420	Increase in hole size till 60th	Some increase
Drill 657	840	Increase in hole size	Increase in surface finish

Table 19: Life, Surface Finish, and Hole Size Trends for Speed=80 SFM and Feed= 0.010 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	30	Increase in hole size	Increased till 50th hole
Drill 305	90	Increase in hole size	Improved till 80th hole
Drill 651	570	No trend observed	Increase till 190th hole
Drill 657	480	No trend observed	Increase in surface finish

Table 20: Life, Surface Finish, and Hole Size Trends for Speed=100 SFM and Feed= 0.006 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	80	Increase in hole size	No trend observed
Drill 305	90	Increase in hole size	Increase in surface finish
Drill 651	660	No trend observed	Increase in surface finish
Drill 657	420	Increase in hole size	Increase in surface finish

Table 21: Life, Surface Finish, and Hole Size Trends for Speed=100 SFM and Feed= 0.008 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	1320	No trend observed	No trend observed
Drill 305	1260	No trend observed	Increase in surface finish
Drill 651	390	Decrease in hole size	Increase in surface finish
Drill 657	120	Increase close to failure	No trend observed

Table 22: Life, Surface Finish, and Hole Size Trends for Speed=100 SFM and Feed= 0.010 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	450	Decrease in hole size	No trend observed
Drill 305	30	Increase in hole size	Increase followed by a decrease
Drill 651	210	Increase in hole size	Increase in surface finish
Drill 657	420	No trend observed	Increase in surface finish

Table 23: Life, Surface Finish, and Hole Size Trends for Speed=120 SFM and Feed= 0.006 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	90	Increase in hole size	No trend observed
Drill 305	210	Increase till 60th hole, then a?	No trend observed
Drill 651	330	Increase till 120th hole then a?	No trend observed
Drill 657	330	No change till 120th hole then an increase in hole size	Increase in surface finish

Table 24: Life, Surface Finish, and Hole Size Trends for Speed=120 SFM and Feed= 0.008 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	240	Increase till 90th hole then a decrease	No trend observed
Drill 305	330	No trend observed	No trend observed
Drill 651	180	No trend observed	Increase in surface finish
Drill 657	420	No trend observed	Increase in surface finish

Table 25: Life, Surface Finish, and Hole Size Trends for Speed=120 SFM and Feed= 0.010 IPR for 1020 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	90	Increase in hole size	Increase in surface finish
Drill 305	80	Increase in hole size	Increase in surface finish
Drill 651	360	No trend observed	No trend observed
Drill 657	420	Decrease in hole size	Increase in surface finish

Table 26: Life, Surface Finish, and Hole Size Trends for Speed=60 SFM and Feed= 0.006 IPR for 4140 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	10	Increase in hole size	Premature to report (early failure)
Drill 305	20	Increase in hole size	Increase in surface finish
Drill 651	90	Improve in hole size	Increase in surface finish
Drill 657	60	Improve in hole size	Increase in surface finish

Table 27: Life, Surface Finish, and Hole Size Trends for Speed=60 SFM and Feed= 0.008 IPR for 4140 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	10	Increase in hole size	Premature to report (early failure)
Drill 305	10	Increase in hole size	Premature to report (early failure)
Drill 651	80	Improve in hole size	Increase in surface finish
Drill 657	150	Improve in hole size	Increase in surface finish

Table 28: Life, Surface Finish, and Hole Size Trends for Speed=80 SFM and Feed= 0.006 IPR for 4140 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	10	Increase in hole size	Premature to report (early failure)
Drill 305	20	Increase in hole size	Increase in surface finish
Drill 651	230	Improvement in hole size	Decrease in surface finish
Drill 657	10	Increase in hole size	Premature to report (early failure)

Table 29: Life, Surface Finish, and Hole Size Trends for Speed=80 SFM and Feed= 0.008 IPR for 4140 Steel.

Tool	Tool Life	Hole Size	Surface Finish
Drill 205	10	Improvement in hole size	Insufficient data
Drill 305	10	Increase in hole size	Insufficient data
Drill 651	50	Increase in hole size	Increase in surface finish
Drill 657	265	No trend observed	Slight increase

Conclusions and Recommendations

The study undertaken using MQL and flood application when drilling 1020 steel revealed that:

1. Drill 205 provided the best tool life and a better inner diameter hole under Micro lubrication. It provided the best surface finish under flood application.
2. Drill 305 provided the best tool life and a better inner diameter hole under Micro lubrication. It provided the best surface finish under flood application.
3. Drill 657 provided the best tool life and a better diameter hole under Micro lubrication. It provided the best surface finish under flood application.

The study undertaken using MQL and a flood application when drilling 4140 steel revealed that:

1. Drill 657 provided the best tool life and the best surface finish under Micro lubrication.
2. Drill 651 provided the best tool life and the best surface finish under Micro lubrication.
3. It would seem that the tool with the greatest cutting angle of 130° managed the greatest number of holes. This tool was Drill 657, whereas the other remaining tools only had a cutting angle of 118°. Another observation was that Drills 657 and 651 were titanium coated, whereas Drills 205 and 305 were not. This could be the reason why both Drills 657 and 651 drilled a significantly greater number of holes than Drills 205 and 305.
4. It was worth noting that the only major difference among all these drills seems to be the coatings. The titanium-coated drills have clearly out-performed the cobalt and HSCO cobalt drills under most treatments when using MQL and flood applications

Potential future studies include:

1. Varying the fluid application rate when mist cooling to determine the potential effect on the qualities considered in this study, namely surface finish, hole size, and tool life.
2. Study mist characteristics under both flood and MQL conditions for various levels of the cutting variables
3. Extend the method to other work piece and tool materials.

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Appendix A: Plots of Residuals vs. Normal Quantile

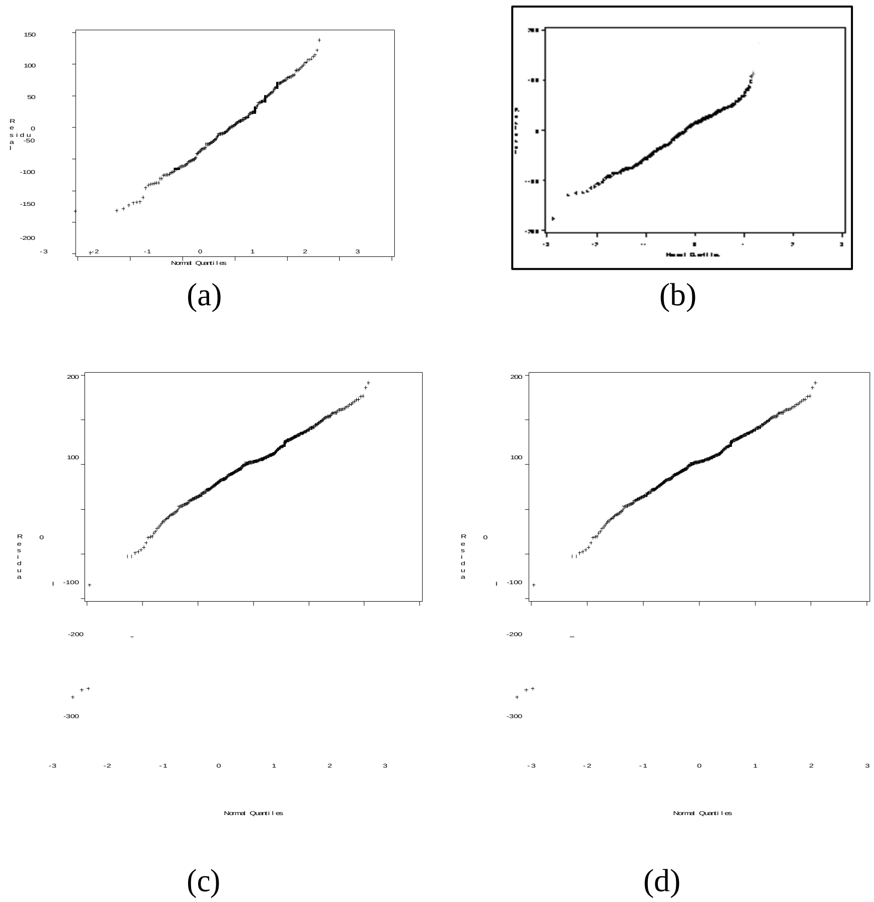
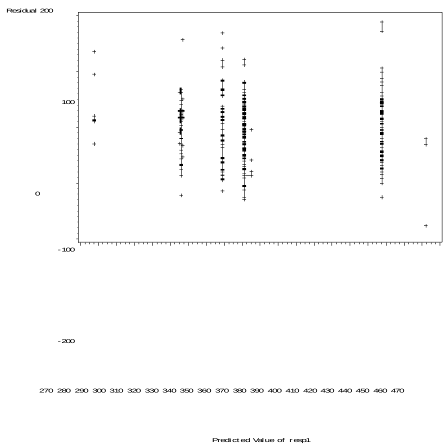
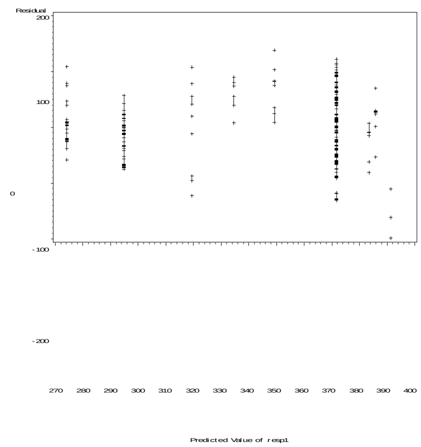
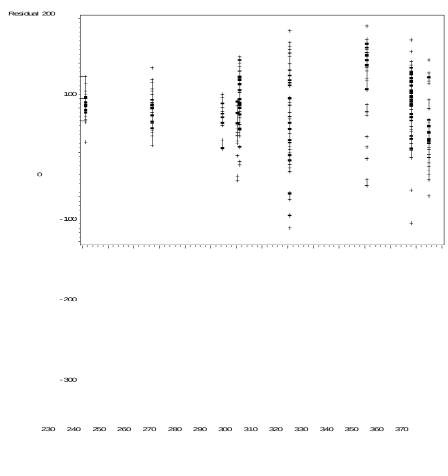


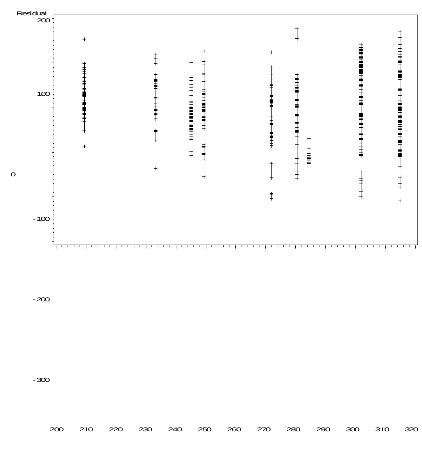
Figure A-1: Plot of residuals vs. normal quantile; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.





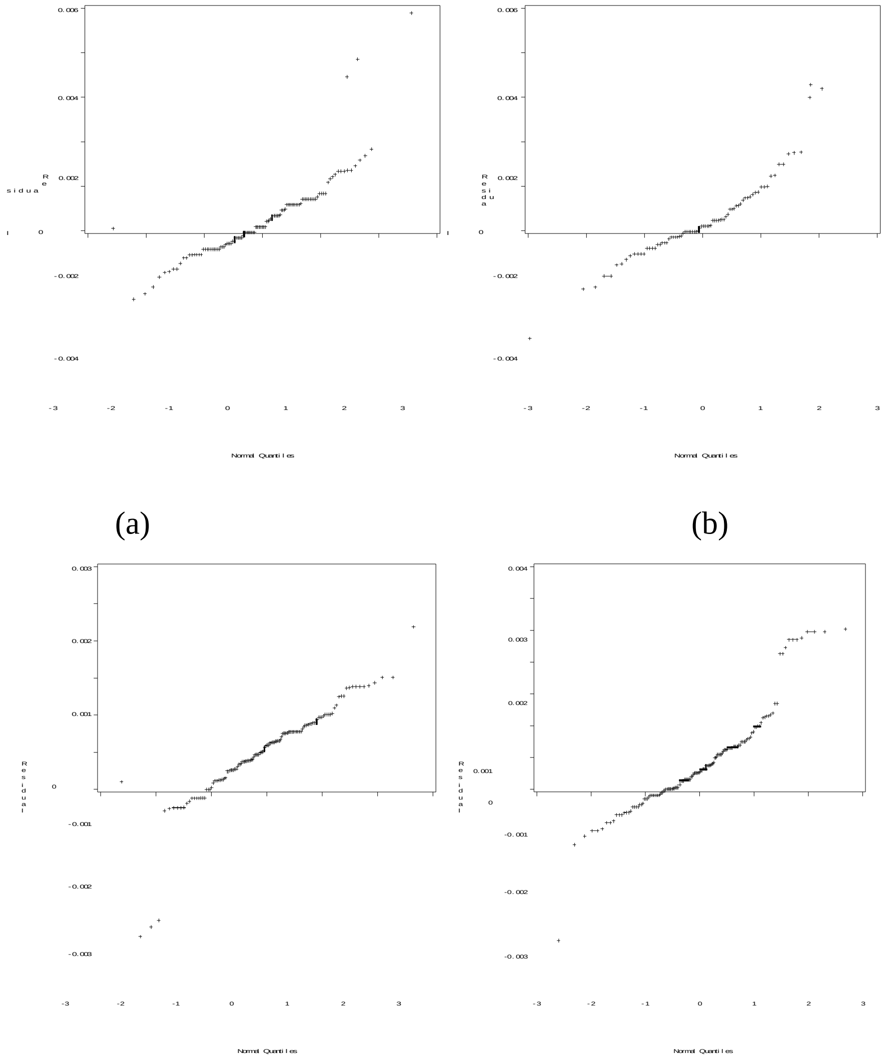
(a) (b)





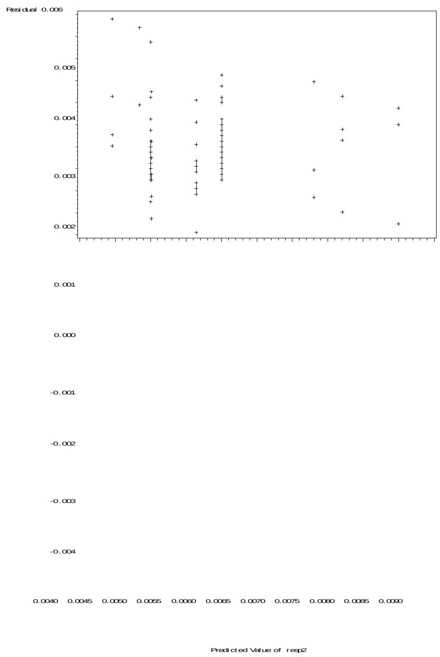
(c) (d)

Figure A-2: plot of residuals vs. predicted value; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

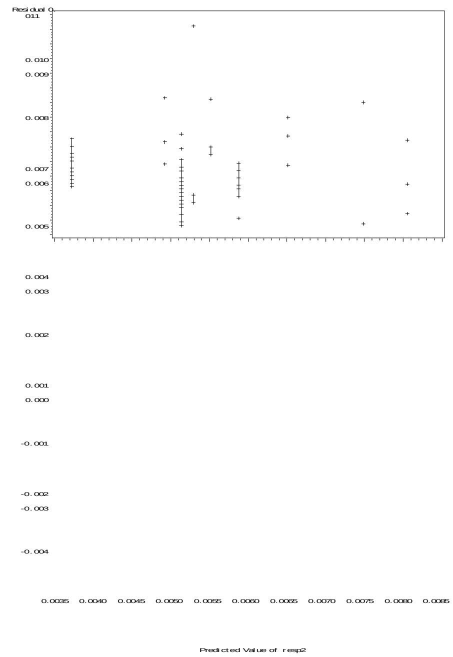


(a) (b) (c) (d)

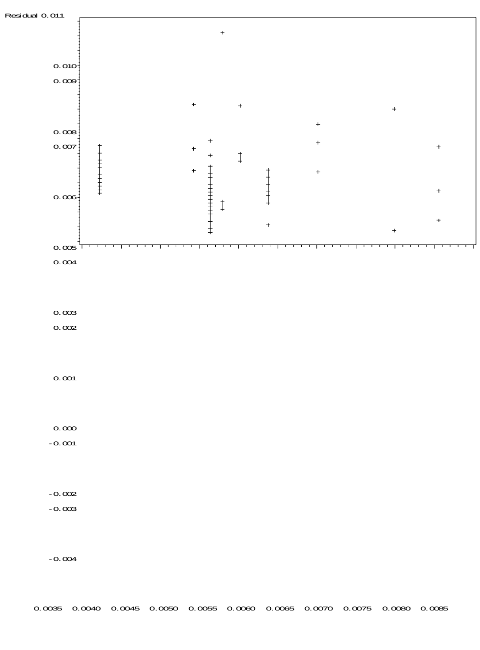
Figure A-3: Plot of residuals vs. normal quantile; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.



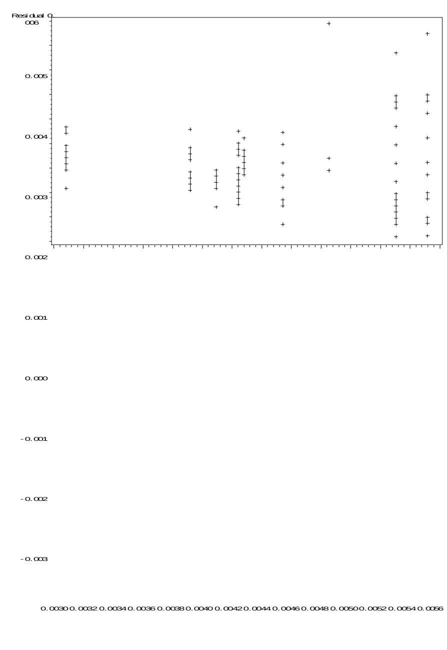
(a)



(b)

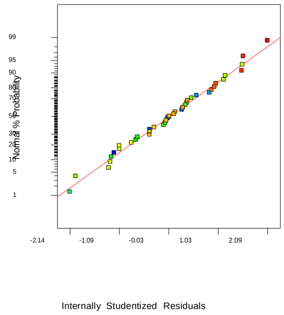


(c)

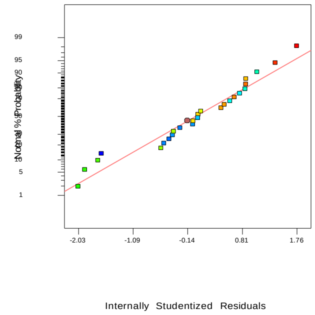


(d)

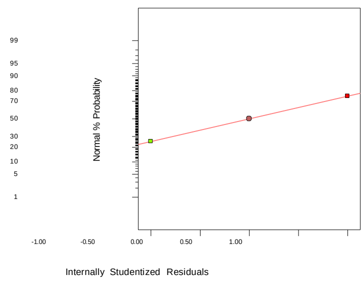
Figure A-4: plot of residuals vs. predicted value; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.



(a)

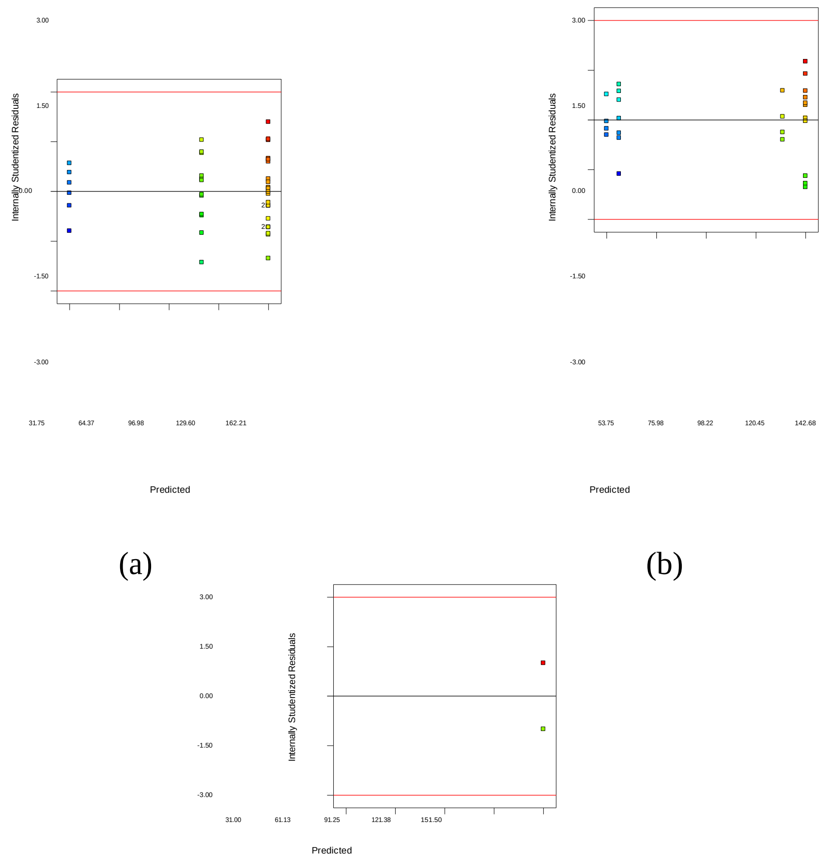


(b)



(c)

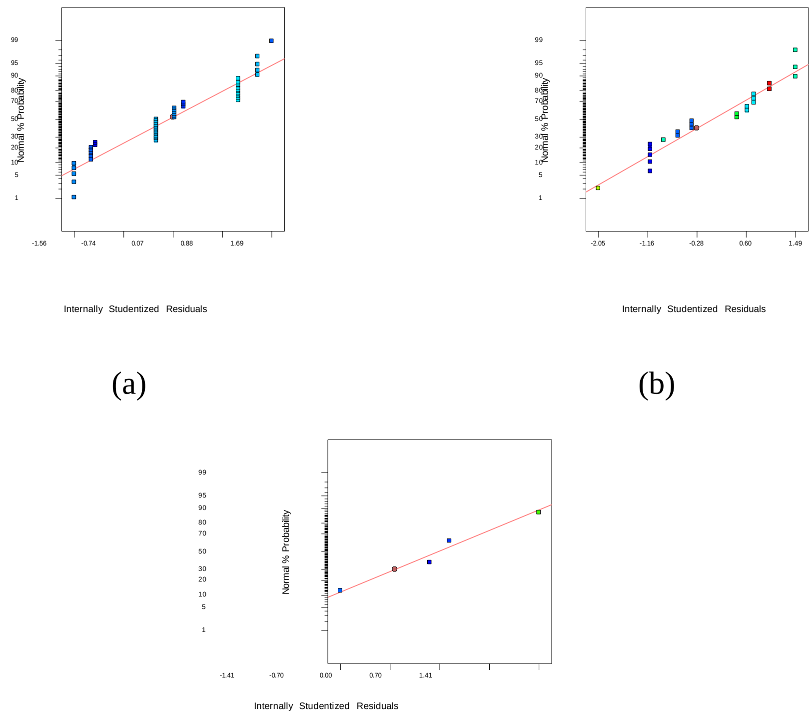
Figure A-5: Normal plot of residuals in data for surface finish for 4140 steel; (a) Drill 658; (b) Drill 651; (c) Drill 305.



(a) (b)

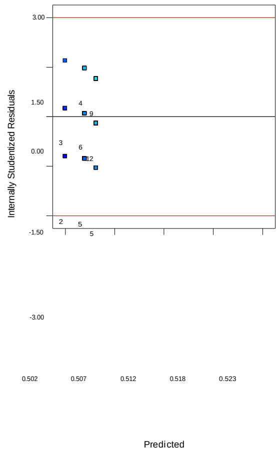
(c)

Figure A-6: Residual vs. predicted values; (a) Drill 657; (b) Drill 651; (c) Drill 305.

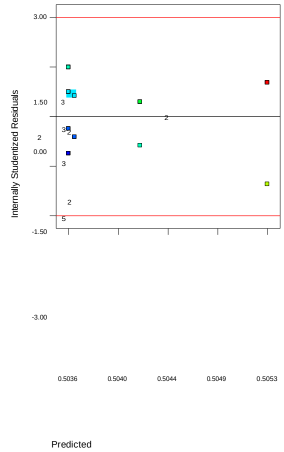


(c)

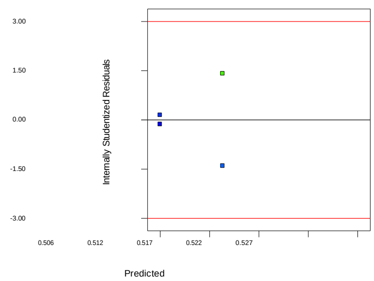
Figure A-7: Normal plots of residuals for hole size for steel 4140; (a) Drill 657; (b) Drill 651; (c) Drill 305.



(a)



(b)



(c)

Figure A-8: Residual vs. predicted values for hole size and steel 4140; (a) Drill 657; (b) Drill 651; (c) Drill 305.

Appendix B: Analysis of Variance Results for Surface Finish for 1020 Steel

Table B-1: Analysis of Variance for Surface Finish, Drill 205.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	43251930	8650386	2673.21	<.0001
Error	296	957843	3235.95732		
Uncorrected Total	301	44209773			

Root MSE	56.88548	R-Square	0.9783
Dependent Mean	376.55150	Adj R-Sq	0.9780
Coeff Var	15.10696		

Parameter Estimates

Variable	DF	Parameter Error	Standard f Value	Pr > |f|	Variable
speed	1	-5.89327	2.52072	5.4756	0.0201
feed	1	192805	27908	47.7481	<.0001
feedsq	1	-5967869	1938789	9.4864	0.0023
speedsq	1	0.05207	0.01646	9.9856	0.0017
speedfeed	1	-913.47936	210.04699	18.9225	<.0001

Response 1 = (-5.89327*Speed) + (192805*Feed) + (-5967869*Feed*Feed) +
(0.05207*Speed*Speed) + (-913.47936*Speed*Feed)

Table B-2: Analysis of Variance for Surface Finish, Drill 305.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	23080009	4616002	1597.30	<.0001
Error	185	534627	2889.87711		
Uncorrected Total	190	23614637			

Root MSE	53.75758	R-Square	0.9774
Dependent Mean	345.90263	Adj R-Sq	0.9767
Coeff Var	15.54125		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
speed	1	24.48854	4.41177	5.55	<.0001
feed	1	-203150	60406	-3.36	0.0009
feedsq	1	9090213	3574171	2.54	0.0118
speedsq	1	-0.15593	0.02546	-6.12	<.0001
speedfeed	1	657.34356	202.38869	3.25	0.0014

Response 1 = (24.48854*Speed) + (-203150*Feed) + (9090213*feedsq) +
(0 15593*speedsq) + (657 34356*speedfeed)

Table B-3: Analysis of Variance for Surface Finish, Drill 651.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	39537233	7907447	1355.64	<.0001
Error	390	2274873	5833.00664		
Uncorrected Total	395	41812106			

Root MSE	76.37412	R-Square	0.9456
Dependent Mean	313.50380	Adj R-Sq	0.9449
Coeff Var	24.36147		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
speed	1	1.41061	2.53685	31.36	0.5785
feed	1	97789	32229	9.18.9	0.0026
feedsq	1	-10052562	2036736	24.4036	<.0001
speedsq	1	-0.04029	0.01487	7.3441	0.0070
speedfeed	1	579.54642	131.12447	19.5364	<.0001

Response 1 = (1.41061*Speed) + (97789*Feed) + (-10052562*Feed*Feed) + (0.04029*Speed*Speed) + (579.54642*Speed*Feed)

Table B-4: Analysis of Variance for Surface Finish, Drill 657.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	33034275	6606855	1116.51	<.0001
Error	429	2538564	5917.39827		
Uncorrected Total	434	35572839			

Root MSE	76.92463	R-Square	0.9286
Dependent Mean	273.99885	Adj R-Sq	0.9278
Coeff Var	28.07480		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
speed	1	0.61765	2.09643	0.0841	0.7684
feed	1	90841	26049	12.1801	0.0005
feedsq	1	-5935234	1697212	12.25	0.0005
speedsq	1	-0.01119	0.01270	0.7744	0.3789
speedfeed	1	-15.32430	133.55110	0.0121	0.9087

Response 1 = (90841*Feed) + (-5935234*Feed*Feed)

Table B-5: Analysis of Variance for Hole Size Deviation, Drill 205.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	0.00399	0.00079889	294.84	<.0001
Error	115	0.00031160	0.00000271		
Uncorrected Total	120	0.00431			

Root MSE	0.00165	R-Square	0.9276
Dependent Mean	0.00571	Adj R-Sq	0.9245
Coeff Var	28.84692		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
speed	1	0.00053769	0.00010559	25.9081	0.7684
feed	1	-4.50992	1.19894	14.1376	0.0003
feedsq	1	268.06685	80.93303	10.9561	0.0012
speedsq	1	-0.00000249	6.942304E-7	12.8881	0.0005
speedfeed	1	-0.00493	0.00844	0.3364	0.5601

Response = (0.00053769*Speed) + (-4.50992*Feed) + (268.06685*Feed*Feed) + (0.00000249*Speed*Speed)

Table B-6: Analysis of Variance for Hole Size Deviation, Drill 305.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	0.00235	0.00047074	97.04	<.0001
Error	77	0.00037354	0.00000485		
Uncorrected Total	82	0.00273			

Root MSE	0.00220	R-Square	0.8630
Dependent Mean	0.00528	Adj R-Sq	0.8541
Coeff Var	41.71108		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
speed	1	0.00053366	0.00020828	6.5536	0.0124
feed	1	-5.25426	2.67230	3.8809	0.0529
feedsq	1	465.53860	156.07624	8.8804	0.0038
speedsq	1	-0.00000204	0.00000121	2.8224	0.0974
speedfeed	1	-0.01950	0.00974	4.00	0.0487

Response = (0.00053366*Speed) + (-5.25426*Feed) + (465.53860*Feed*Feed) + (-0.01950*Speed*Feed)

Table B-7: Analysis of Variance for Hole Size Deviation, Drill 651.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	0.00316	0.00063220	497.46	<.0001
Error	138	0.00017538	0.00000127		
Uncorrected Total	143	0.00334			

Root MSE	0.00113	R-Square	0.9474
Dependent Mean	0.00459	Adj R-Sq	0.9455
Coeff Var	24.53487		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
speed	1	0.00002852	0.00006125	0.2209	0.6422
feed	1	1.65497	0.77911	4.4944	0.0354
feedsq	1	-214.71018	49.38948	18.9225	<.0001
speedsq	1	-8.62163E-7	3.599665E-7	5.76	0.0180
speedfeed	1	0.01429	0.00323	19.5364	<.0001

Response = (0.00002852*Speed) + (1.65497*Feed) + (-214.71018*Feed*Feed) + (-8.62163*Speed*Speed) + (0.01429*Speed*Feed)

Table B-8: Analysis of Variance for Hole Size Deviation, Drill 657.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	0.00314	0.00062880	292.30	<.0001
Error	150	0.00032269	0.00000215		
Uncorrected Total	155	0.00347			

Root MSE	0.00147	R-Square	0.9069
Dependent Mean	0.00444	Adj R-Sq	0.9038
Coeff Var	33.00762		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
speed	1	0.00021732	0.00006628	10.758	0.0013
feed	1	-1.15854	0.82577	1.96	0.1627
feedsq	1	-12.25764	53.85238	0.529	0.8203
speedsq	1	-0.00000148	4.033727E-7	13.4689	0.0003
speedfeed	1	0.00999	0.00427	5.4756	0.0207

Response = (0.00021732*Speed) + (-0.00000148*Speed*Speed) + (0.00999*Speed*Feed)

Table B-9: Analysis of Variance for Surface Finish for 4140 Steel, Drill 657.

Analysis of Variance Results for Surface Finish and Hole Size for 4140 Steel

	Term	Effect	SumSqr	% Contribtn
Require	Intercept			
Model	A-Speed	83.7321	21612.1	51.0933
Model	B-Feed	46.7321	6732	15.9152
Model	AB	-40.0179	4936.51	11.6704
Error	Lack Of Fit		0	0
Error	Pure Error		9018.66	21.3211
	Lenth's ME	25.4026		
	Lenth's SME	31.3512		

ANOVA for selected factorial model
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	81932.33	3	27310.78	109.02	<0.0001	significant
A-Speed	*21612.11*	*1*	*21612.11*	*86.27*	*< 0.0001*	
B-Feed	*6732.00*	*1*	*6732.00*	*26.87*	*< 0.0001*	
AB	*4936.51*	*1*	*4936.51*	*19.71*	*< 0.0001*	
Pure Error	9018.66	36	250.52			
Cor Total	90950.99	39				

Std. Dev.	15.83	R-Squared	0.9008
Mean	129.36	Adj R-Squared	0.8926
C.V. %	12.24	Pred R-Squared	N/A
PRESS	N/A	Adeq Precision	26.066

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High	VIF
Intercept	116.99	1	4.51	107.85	126.13	
A-Speed	41.87	1	4.51	32.72	51.01	3.21
B-Feed	23.37	1	4.51	14.22	32.51	1.87
AB	-20.01	1	4.51	-29.15	-10.87	2.85

Final Equation in Terms of Coded Factors:
Surface Finish = (116.99) + (41.87 * Speed) + (23.37 * Feed) – (20.01 * Speed * Feed)

Table B-10: Analysis of Variance for Surface Finish, Drill 651.

	Term	Effect	SumSqr	%Contribution
Require	Intercept			
Model	A-Speed	81.0731	35830.7	82.0975
Model	B-Feed	-2.32305	29.4185	0.0674054
Model	AB	-7.85877	336.675	0.77141
Error	Lack Of Fit		0	0
Error	Pure Error		7447.31	17.0637
	Lenth's ME	14.9675		
	Lenth's SME	18.6267		

ANOVA for selected factorial model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	43778.93	3	14592.98	43.11	< 0.0001
A-Speed	*35830.70*	*1*	*35830.70*	*105.85*	*< 0.0001*
B-Feed	*29.42*	*1*	*29.42*	*0.087*	*0.7709*
AB	*336.68*	*1*	*336.68*	*0.99*	*0.3295*
Pure Error	7447.31	22	338.51		
Cor Total	51226.24	25			

Std. Dev.	18.40	R-Squared	0.8546
Mean	104.98	Adj R-Squared	0.8348
C.V. %	17.53	Pred R-Squared	0.8134
PRESS	9556.46	Adeq Precision	12.323

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High
Intercept	97.05	1	3.94	88.88	105.23
A-Speed	40.54	1	3.94	32.37	48.71
B-Feed	-1.16	1	3.94	-9.33	7.01
AB	-3.93	1	3.94	-12.10	4.24

Final Equation in Terms of Coded Factors:

Surface Finish = +97.05 + 40.54 * Speed -1.16 * Feed -3.94 * Speed * Feed

Table B-11: Analysis of Variance for Surface Finish, Drill 305.

	Term	Effect	SumSqr	% Contribution
Require	Intercept			
Model	A-Speed	107.25	13145.8	88.2322
Model	B-Feed	13.25	200.643	1.34668
Model	AB	-13.25	200.643	1.34668
Error	Lack Of Fit		0	0
Error	Pure Error		1352	9.07439
	Lenth's ME	116.911		
	Lenth's SME	206.028		

ANOVA for selected factorial model
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	14154.20	3	4718.07	3.49	0.3704	not significant
A-Speed	*13145.79*	*1*	*13145.79*	*9.72*	*0.1976*	
B-Feed	*200.64*	*1*	*200.64*	*0.15*	*0.7659*	
AB	*200.64*	*1*	*200.64*	*0.15*	*0.7659*	
Pure Error	352.00	1	1352.00			
Cor Total	15506.20	4				

Std. Dev.	36.77	R-Squared	0.9128
Mean	108.60	Adj R-Squared	0.6512
C.V. %	33.86	Pred R-Squared	N/A
PRESS	N/A	Adeq Precision	3.664

Final Equation in Terms of Coded Factors:

Surface Finish = (97.88) + (53.63 * Speed) + (6.62 * Feed) – (6.62 * Speed * Feed)

Table B-12: Analysis of Variance for Surface Finish, Drill 205.

	Term	Effect	SumSqr	% Contribtn
Require	Intercept			
Model	A-Speed	116.75	13630.6	95.8401
Model	B-Feed	23.75	564.063	3.96607
Model	AB	-5.25	27.5625	0.193799
	Lenth's ME	276.36		
	Lenth's SME	816.591		

Response 1 Surface Finish

ANOVA for selected factorial model

Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value
Model	14222.19	3	4740.73	
A-Speed	*13630.56*	*1*	*13630.56*	
B-Feed	*564.06*	*1*	*564.06*	
AB	*27.56*	*1*	*27.56*	
Pure Error	0.000	0		
Cor Total	14222.19	3		

Std. Dev.	
R-Squared	1.0000
Mean	103.38
Adj R-Squared	
C.V. %	
Pred R-Squared	N/A
PRESS	N/A
Adeq Precision	0.000

Factor	Coefficient Estimate	df	Standard Error	95% CI Low
Intercept	103.38	1		
A-Speed	58.37	1		
B-Feed	11.88	1		
AB	-2.63	1		

Final Equation in Terms of Coded Factors:

Surface Finish = (103.38) + (58.37 * Speed) + (11.88 * Feed) - (2.63 * Speed * Feed)

Table B-13: Analysis of Variance for Hole Size Deviation, Drill 657.

	Term	Effect	SumSqr	% Contribution
Require	Intercept			
Model	A-Speed	0.0111936	0.000394078	42.5107
Model	B-Feed	-0.00787308	0.000194954	21.0305
Model	AB	-0.00997308	0.000312825	33.7457
Error	Lack Of Fit		0	0
Error	Pure Error		2.51513E-005	2.71317
	Lenth's ME	0.00451253		
	Lenth's SME	0.00555607		

ANOVA for selected factorial model
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	3.983E-004	3	1.328E-004	232.28	< 0.0001	significant
A-Speed	*3.941E-004*	*1*	*3.941E-004*	*689.40*	*< 0.0001*	
B-Feed	*1.950E-004*	*1*	*1.950E-004*	*341.06*	*< 0.0001*	
AB	*3.128E-004*	*1*	*3.128E-004*	*547.26*	*< 0.0001*	
Pure Error	2.515E-005	44	5.716E-007			
Cor Total	4.235E-004	47				

Std. Dev.	7.561E-004	R-Squared	0.9406
Mean	0.50	Adj R-Squared	0.9366
C.V. %	0.15	Pred R-Squared	N/A
PRESS	N/A	Adeq Precision	96.982

Factor	Coefficient Estimate	df	Standard Error	95%CI Low	95%CI High	VIF
Intercept	0.51	1	2.132E-004	0.51.51		
A-Speed	5.597E-003	1	2.132E-004	5.167E-003	6.026E-003	3.76
B-Feed	-3.937E-003	1	2.132E-004	-4.366E-003	-3.507E-003	1.90
AB	-4.987E-003	1	2.132E-004	-5.416E-003	-4.557E-003	3.39

Final Equation in Terms of Coded Factors:

Hole Diameter = (0.51) + (5.597E-003 * Speed) – (3.937E-003 * Feed) - (4.987E-003 * Speed * Feed)

Table B-14: Analysis of Variance for Hole Size Deviation, Drill 651.

	Term	Effect	SumSqr	% Contribtn
Require	Intercept			
Model	A-Speed	-0.000558333	1.26197E-006	14.2797
Model	B-Feed	-0.000508333	1.04606E-006	11.8366
Model	AB	0.00110833	4.97281E-006	56.2694
Error	Lack Of Fit		0	0
Error	Pure Error		1.55667E-006	17.6143
	Lenth's ME	0.000444837		
	Lenth's SME	0.000554777		

ANOVA for selected factorial model
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	7.382E-006	3	2.461E-006	31.61	< 0.0001	significant
A-Speed	*1.262E-006*	*1*	*1.262E-006*	*16.21*	*0.0007*	
B-Feed	*1.046E-006*	*1*	*1.046E-006*	*13.44*	*0.0015*	
AB	*4.973E-006*	*1*	*4.973E-006*	*63.89*	*< 0.0001*	
Pure Error	1.557E-006	20	7.783E-008			
Cor Total	8.938E-006	23				

Std. Dev.	2.790E-004	R-Squared	0.8258
Mean	0.50	Adj R-Squared	0.7997
C.V. %	0.055	Pred R-Squared	0.7446
PRESS	2.283E-006	Adeq Precision	14.633

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High	VIF
Intercept	0.50	1	6.933E-005	0.50	0.50	
A-Speed	-2.792E-004	1	6.933E-005	-4.238E-004	-1.345E-004	1.22
B-Feed	-2.542E-004	1	6.933E-005	-3.988E-004	-1.095E-004	1.22
AB	5.542E-004	1	6.933E-005	4.095E-004	6.988E-004	1.11

Final Equation in Terms of Coded Factors:

Hole Diameter = (0.50) - (2.792E-004 * Speed) – (2.542E-004 * Feed) + (5.542E-004 * Speed * Feed)

Table B-15: Analysis of Variance for Hole Size Deviation, Drill 305.

	Term	Effect	SumSqr	% Contribtn
Require	Intercept			
Model	A-Speed	-0.01175	0.000184083	49.297
Model	B-Feed	0.00875	0.000102083	27.3376
Model	AB	-0.00525	3.675E-005	9.84155
Error	Lack Of Fit		0	0
Error	Pure Error		5.05E-005	13.5238
	Lenth's ME	0.0263671		
	Lenth's SME	0.0399881		

ANOVA for selected factorial model
Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	2.888E-004	3	9.628E-005	3.81	0.2147
A-Speed	*1.841E-004*	*1*	*1.841E-004*	*7.29*	*0.1142*
B-Feed	*1.021E-004*	*1*	*1.021E-004*	*4.04*	*0.1821*
AB	*3.675E-005*	*1*	*3.675E-005*	*1.46*	*0.3510*
Pure Error	5.050E-005	2	2.525E-005		
Cor Total	3.393E-004	5			

Std. Dev.	5.025E-003	R-Squared	0.8512
Mean	0.51	Adj R-Squared	0.6279
C.V. %	0.98	Pred R-Squared	N/A
PRESS	N/A	Adeq Precision	4.997

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High
Intercept	0.51	1	2.176E-003	0.50	0.52
A-Speed	-5.875E-003	1	2.176E-003	-0.015	3.487E-003
B-Feed	4.375E-003	1	2.176E-003	-4.987E-003	0.014
AB	-2.625E-003	1	2.176E-003	-0.012	6.737E-003

Final Equation in Terms of Coded Factors:

Hole Diameter = (0.51) - (.875E-003 * Speed) + (4.375E-003 * Feed) - (2.625E-003 * Speed * Feed)

Table B-16: Analysis of Variance for Hole Size Deviation, Drill 205.

	Term	Effect	SumSqr	% Contribtn
Require	Intercept			
Model	A-Speed	-0.003	9E-006	2.43902
Model	B-Feed	-0.018	0.000324	87.8049
Model	AB	-0.006	3.6E-005	9.7561
	Lenth's ME	0.114356		
	Lenth's SME	0.3379		

Response 2 Hole Diameter

ANOVA for selected factorial model Analysis of variance table [Partial sum of squares - Type III]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	3.690E-004	3	1.230E-004		
A-Speed	*9.000E-006*	*1*	*9.000E-006*		
B-Feed	*3.240E-004*	*1*	*3.240E-004*		
AB	*3.600E-005*	*1*	*3.600E-005*		
Pure Error	0.000	0			
Cor Total	3.690E-004	3			

Std. Dev.		R-Squared	1.0000
Mean	0.52	Adj R-Squared	
C.V. %		Pred R-Squared	N/A
PRESS	N/A	Adeq Precision	0.000

Factor	Coefficient Estimate	df	Standard Error	95% CI Low	95% CI High
Intercept	0.52	1			
A-Speed	-1.500E-003	1			
B-Feed	-9.000E-003	1			
AB-3.000E-003		1			

Final Equation in Terms of Coded Factors:

Hole Diameter = (0.52) - (1.500E-003 * Speed) - (9.000E-003 * Feed) - (3.000E-003 * Speed * Feed)

Appendix C: Plots of Surface Finish and Hole Size vs. Number of Holes Drilled

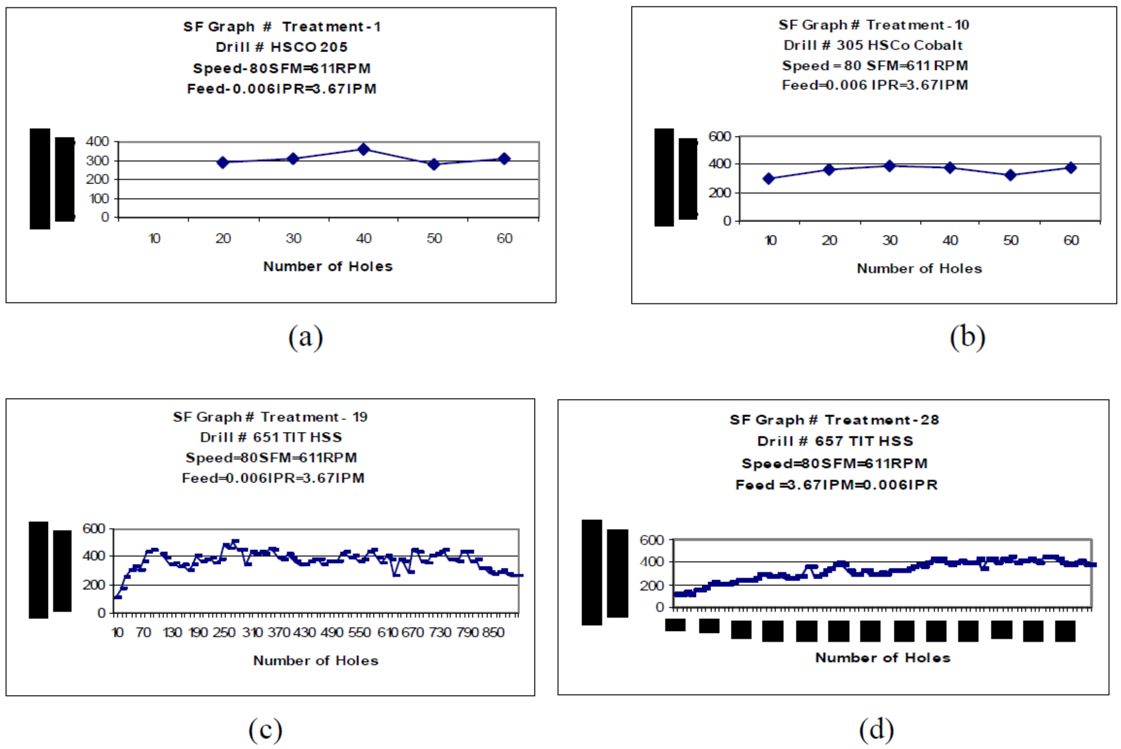


Figure C-1: Surface finish vs. number of holes drilled for speed of 80 SFM, feed of 0.006IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

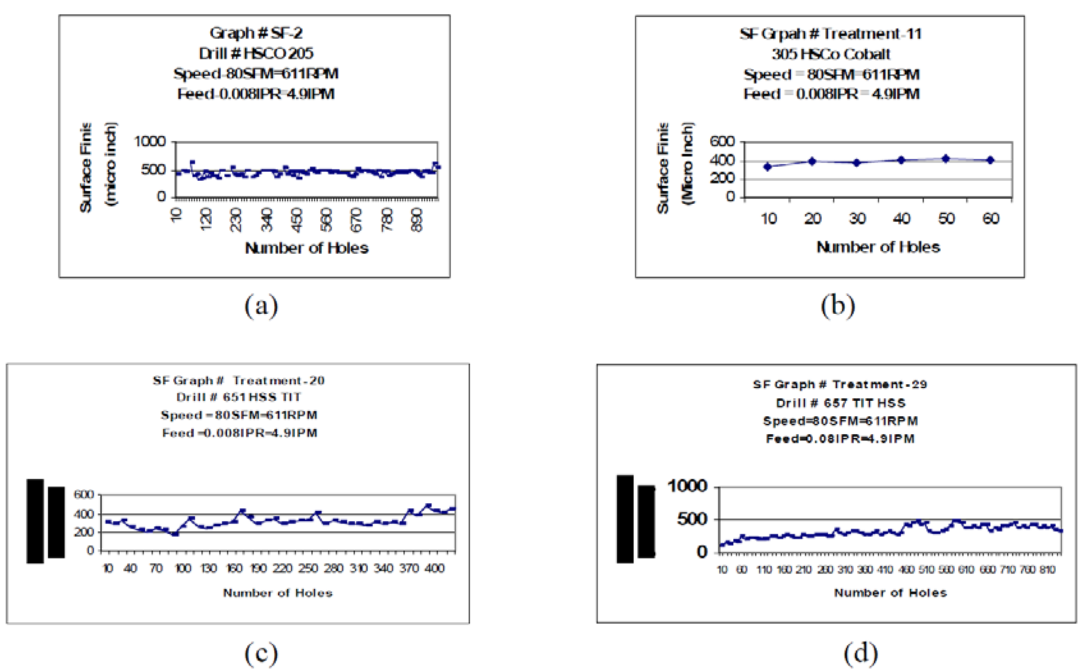


Figure C-2: Surface finish vs. number of holes drilled for a speed of 80 SFM, feed of 0.008IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

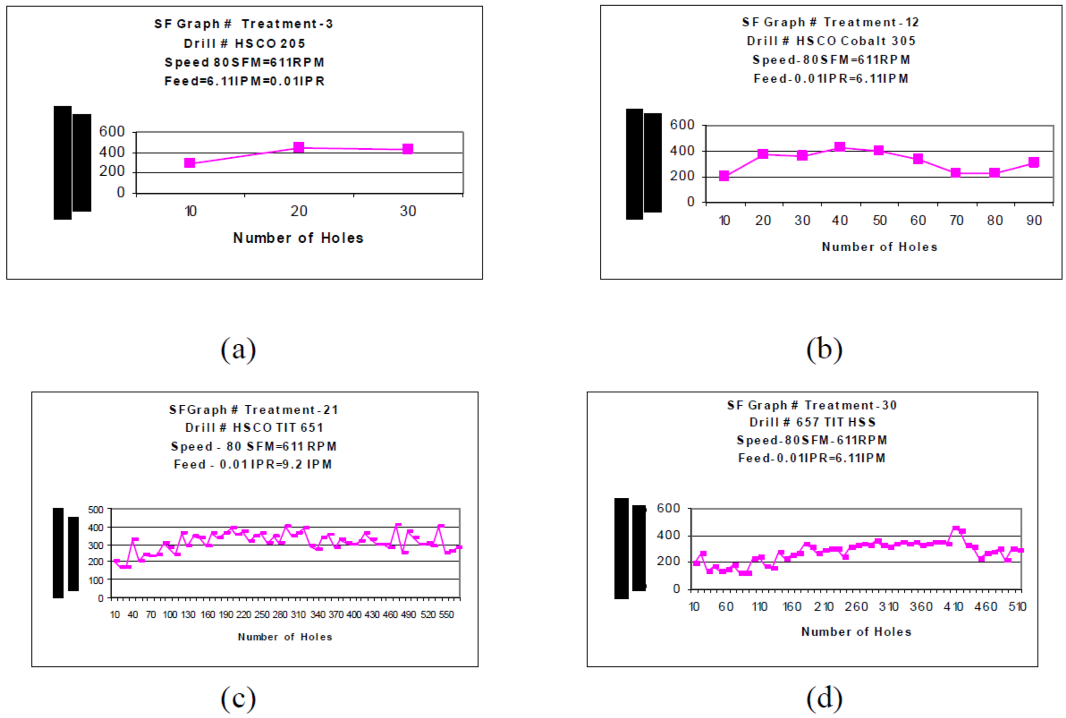


Figure C-3: Surface finish vs. number of holes drilled for a speed of 80 SFM, feed of 0.01IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

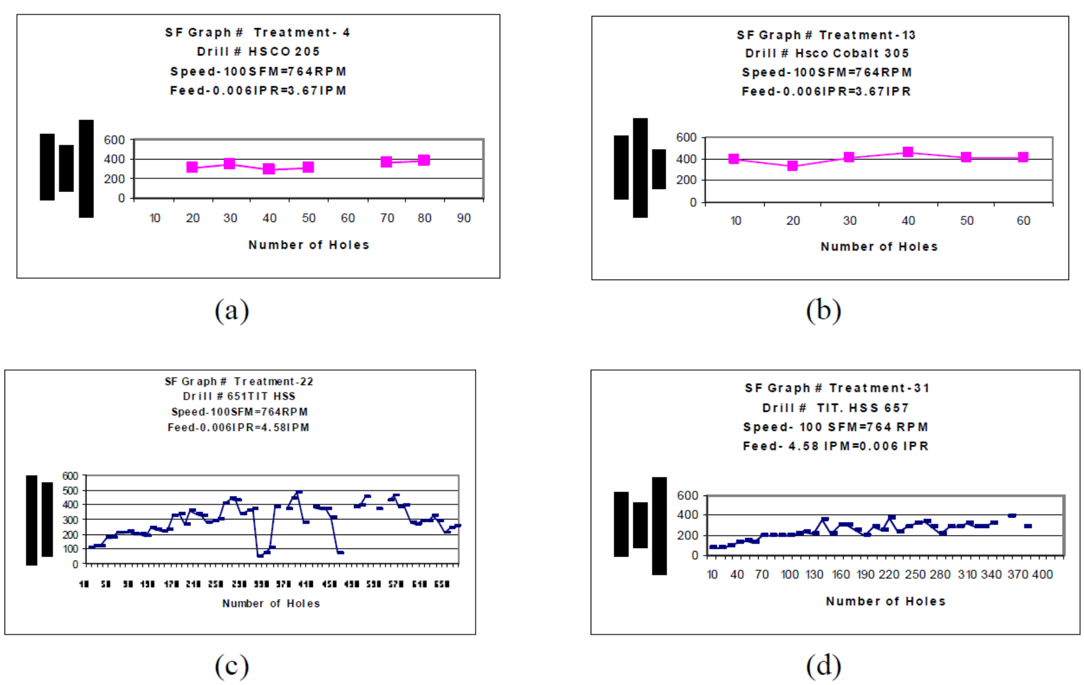


Figure C-4: Surface finish vs. number of holes drilled for a speed of 100 SFM, feed of 0.006IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

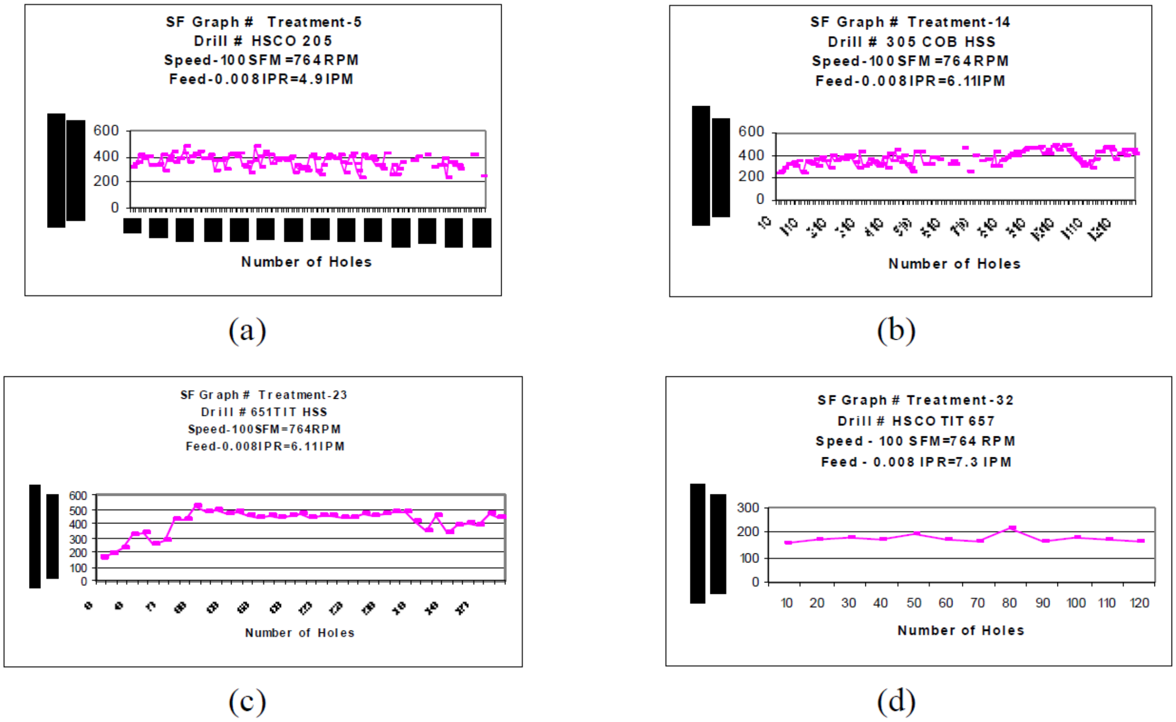


Figure C-5: Surface finish vs. number of holes drilled for a speed of 100 SFM, feed of 0.008IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

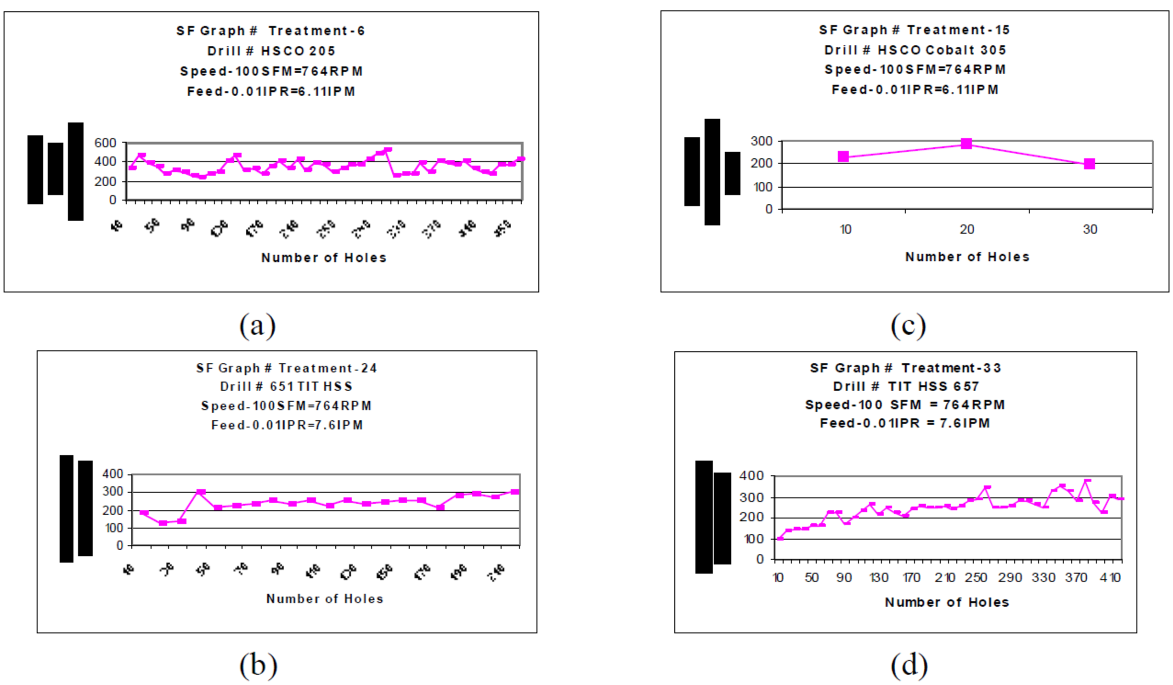


Figure C-6: Surface finish vs. number of holes drilled for a speed of 100 SFM, feed of 0.01IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

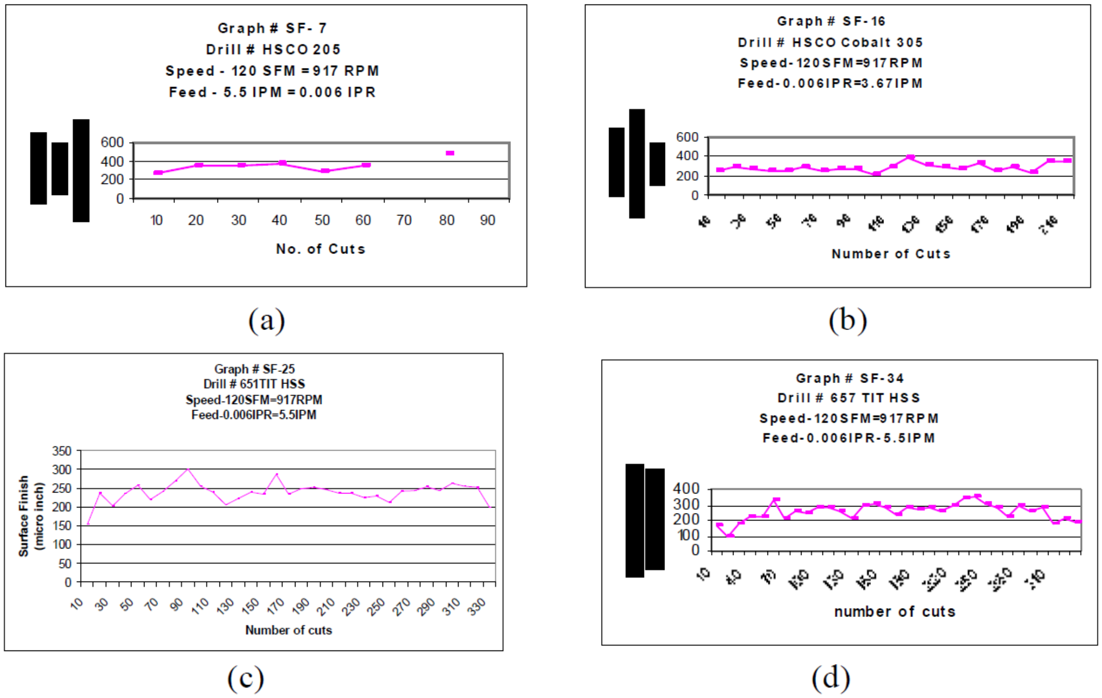


Figure C-7: Surface finish vs. number of holes drilled for a speed of 120 SFM, feed of 0.006IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

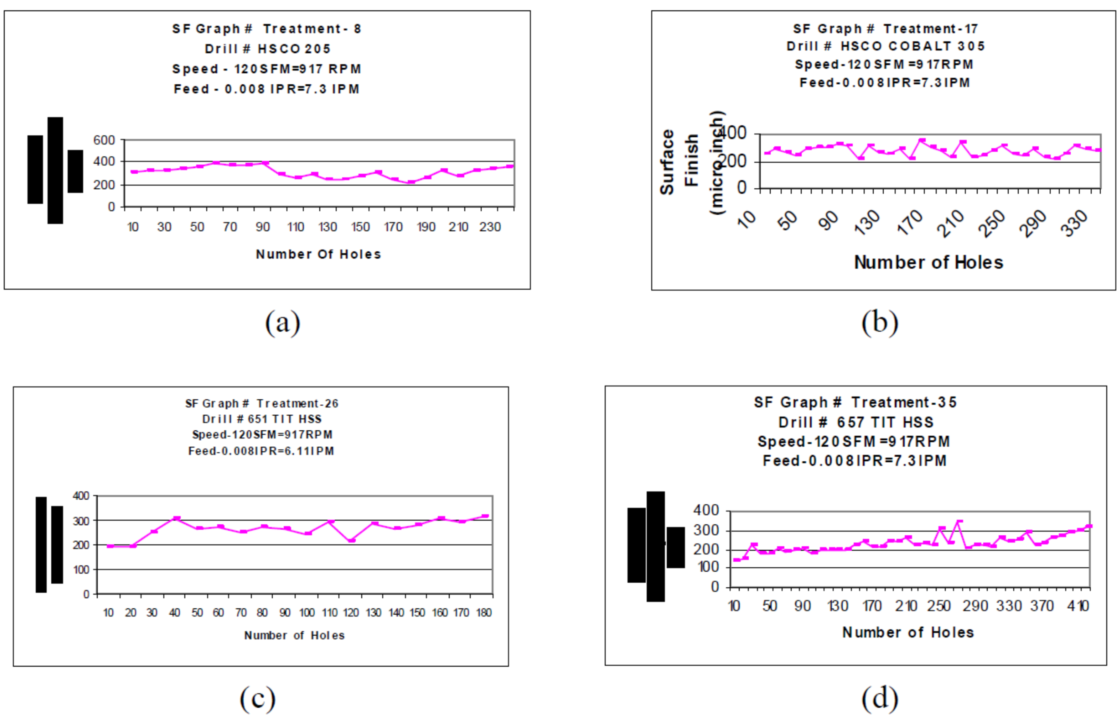
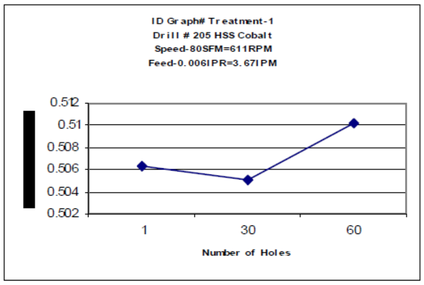
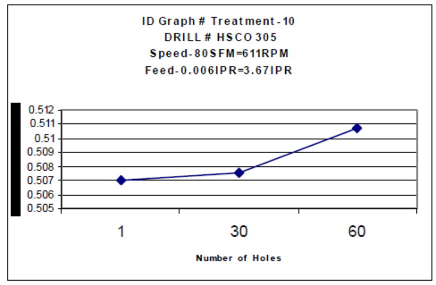


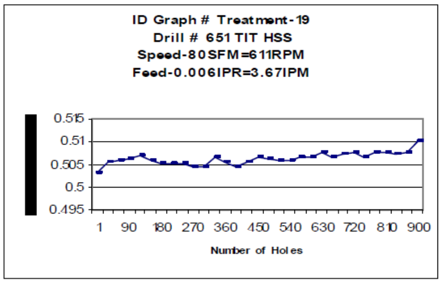
Figure C-8: Surface finish vs. number of holes drilled for a speed of 120 SFM, feed of 0.008IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.



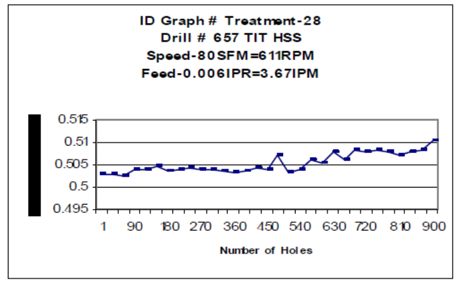
(a)



(b)

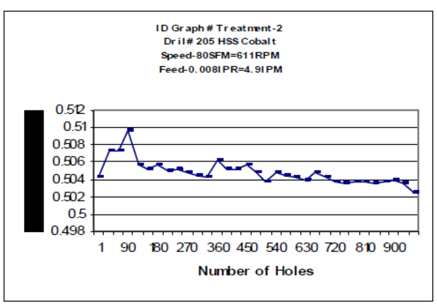


(c)

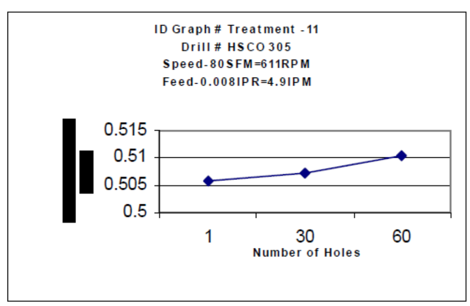


(d)

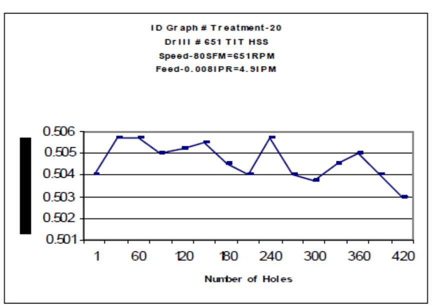
Figure C-9: Hole size vs. number of holes drilled for a speed of 80 SFM, feed of 0.006IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.



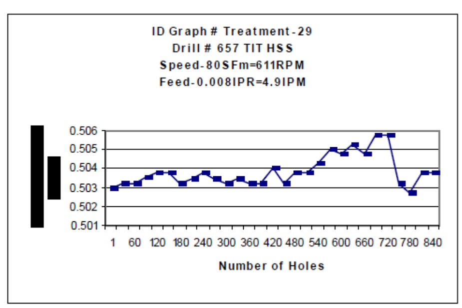
(a)



(b)



(c)



(d)

Figure C-10: Hole size vs. number of holes drilled for a speed of 80 SFM, feed of 0.008IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

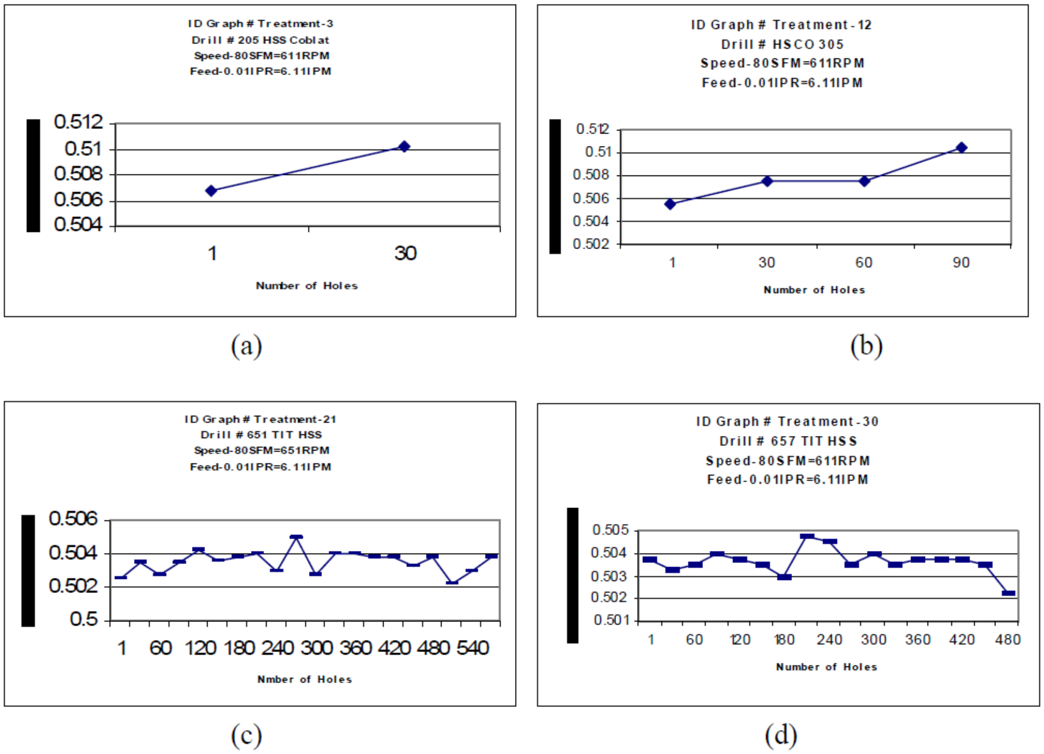


Figure C-11: Hole size vs. number of holes drilled for a speed of 80 SFM, feed of 0.01IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

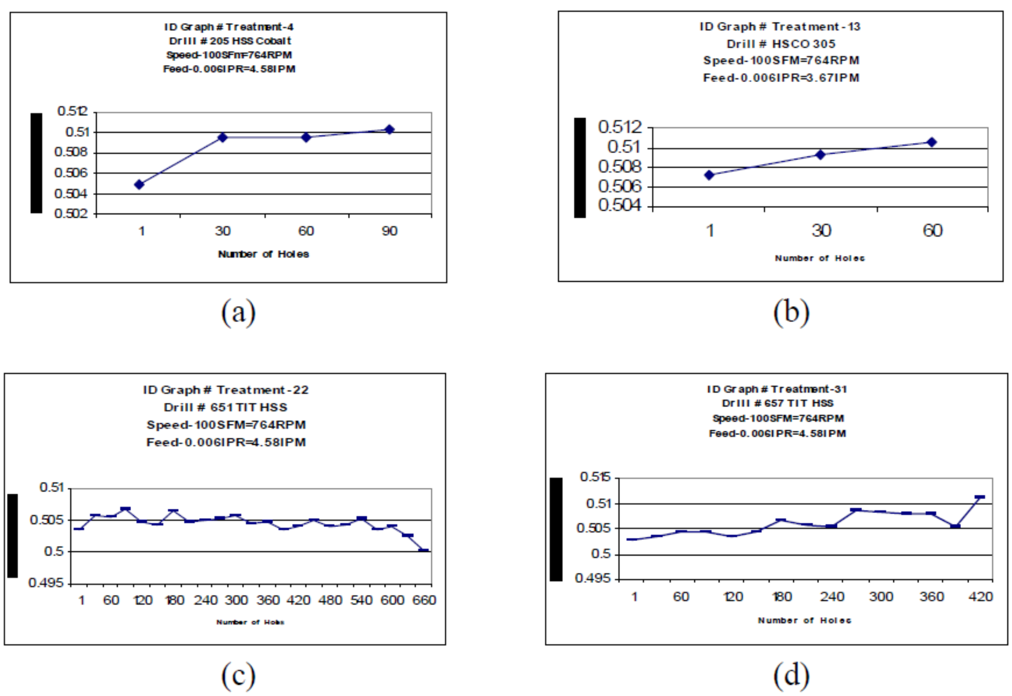
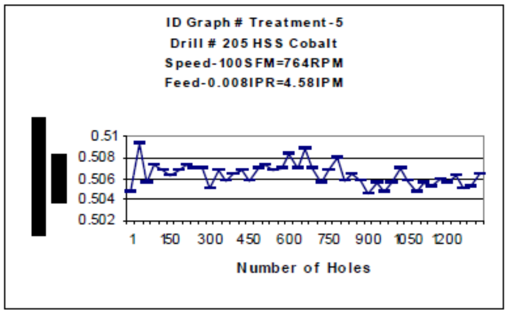
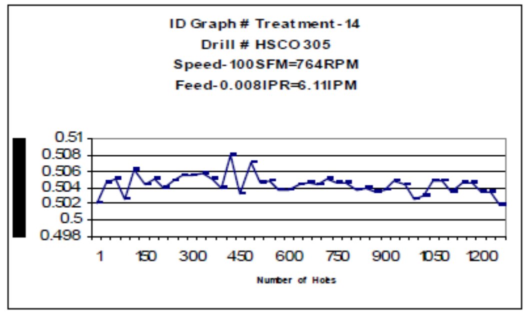


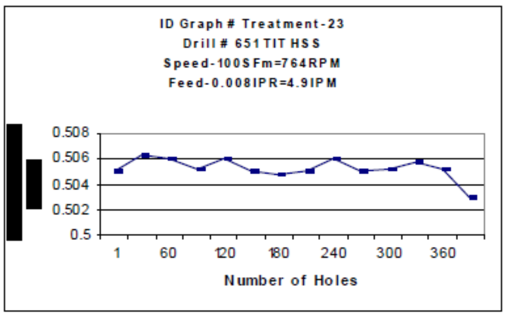
Figure C-12: Hole size vs. number of holes drilled for a speed of 100 SFM, feed of 0.006IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.



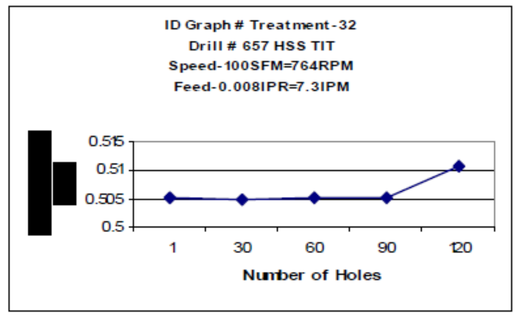
(a)



(b)



(c)



(d)

Figure C-13: Hole size vs. number of holes drilled for a speed of 100 SFM, feed of 0.008IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

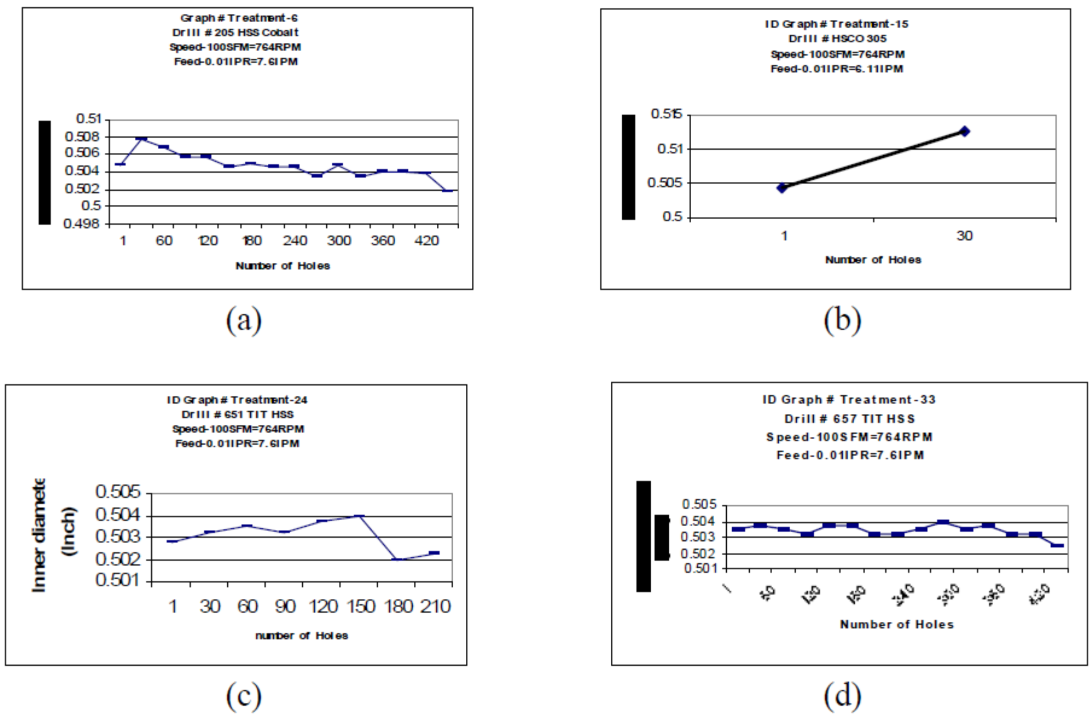


Figure C-14: Hole size vs. number of holes drilled for a speed of 100 SFM, feed of 0.01IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

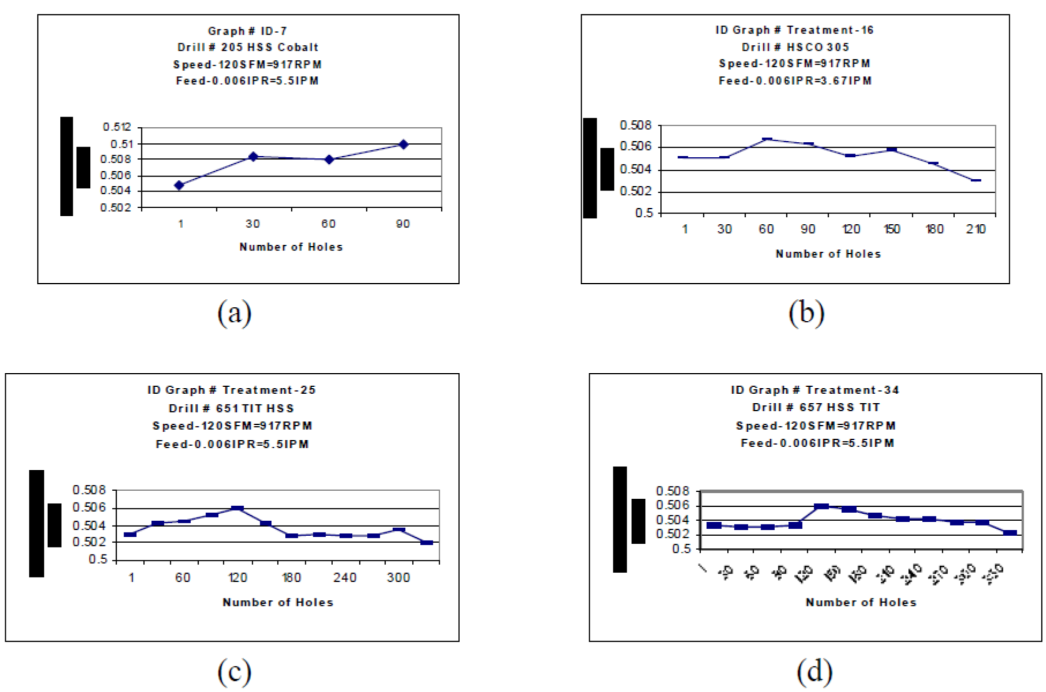


Figure C-15: Hole size vs. number of holes drilled for a speed of 120 SFM, feed of 0.006IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

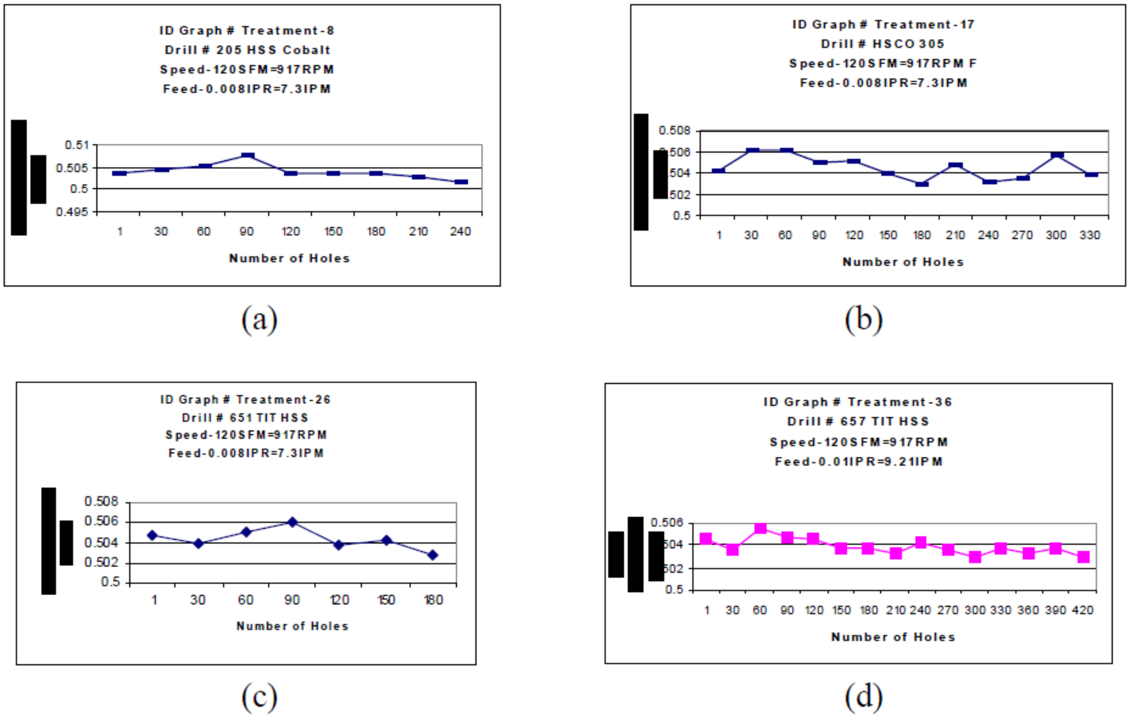


Figure C-16: Hole size vs. number of holes drilled for a speed of 120 SFM, feed of 0.008IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

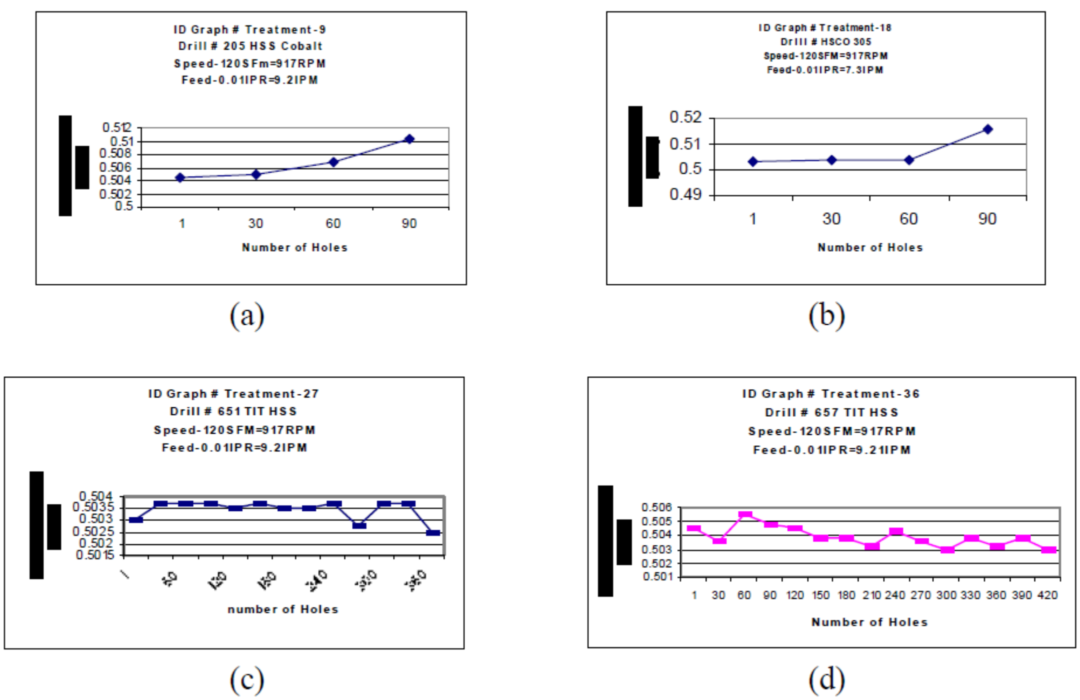


Figure C-17: Hole size vs. number of holes drilled for a speed of 120 SFM, feed of 0.01IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

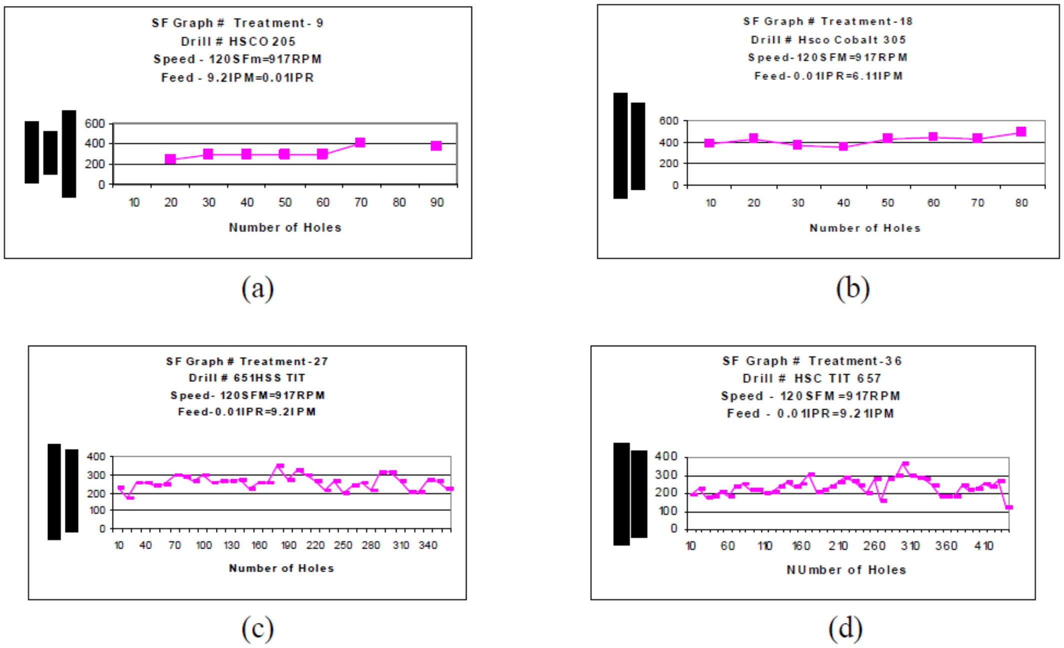


Figure C-18: Surface finish vs. number of holes drilled for a speed of 120 SFM, feed of 0.01IPR for 1020 steel; (a) Drill 205; (b) Drill 305; (c) Drill 651; (d) Drill 657.

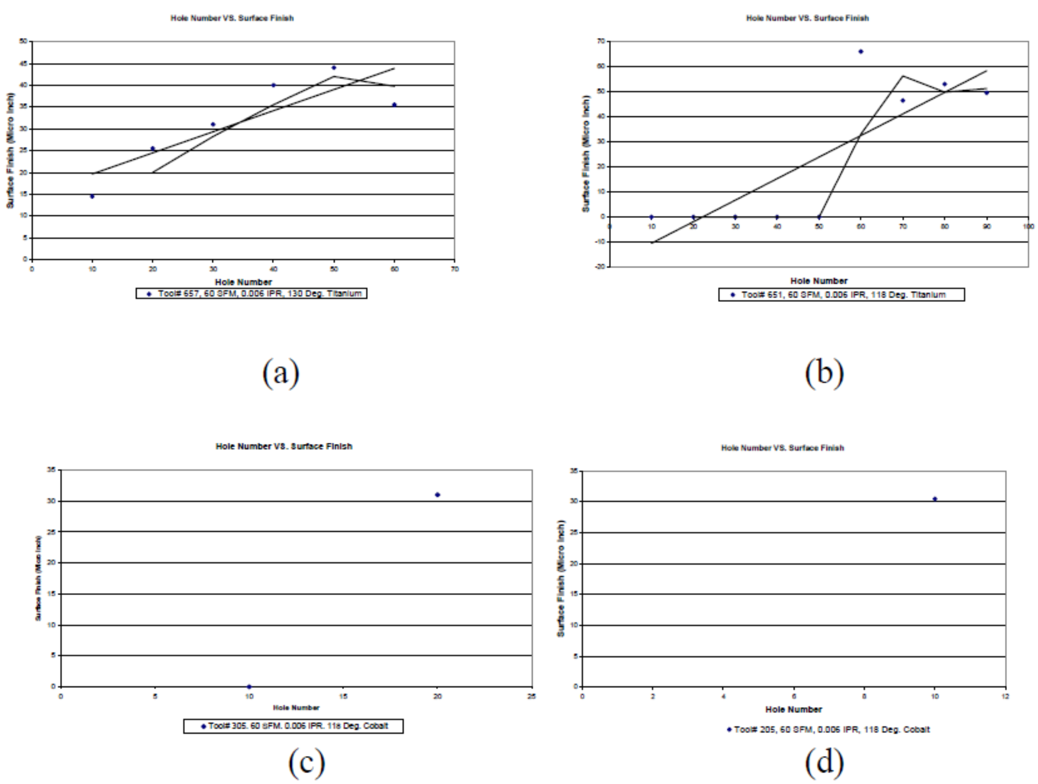


Figure C-19: Surface finish vs. number of holes drilled for speed of 60 SFM and feed of 0.006 IPR for 4140 steel; (a) Drill 657; (b) Drill 651; (c) Drill 305; (d) Drill 205.

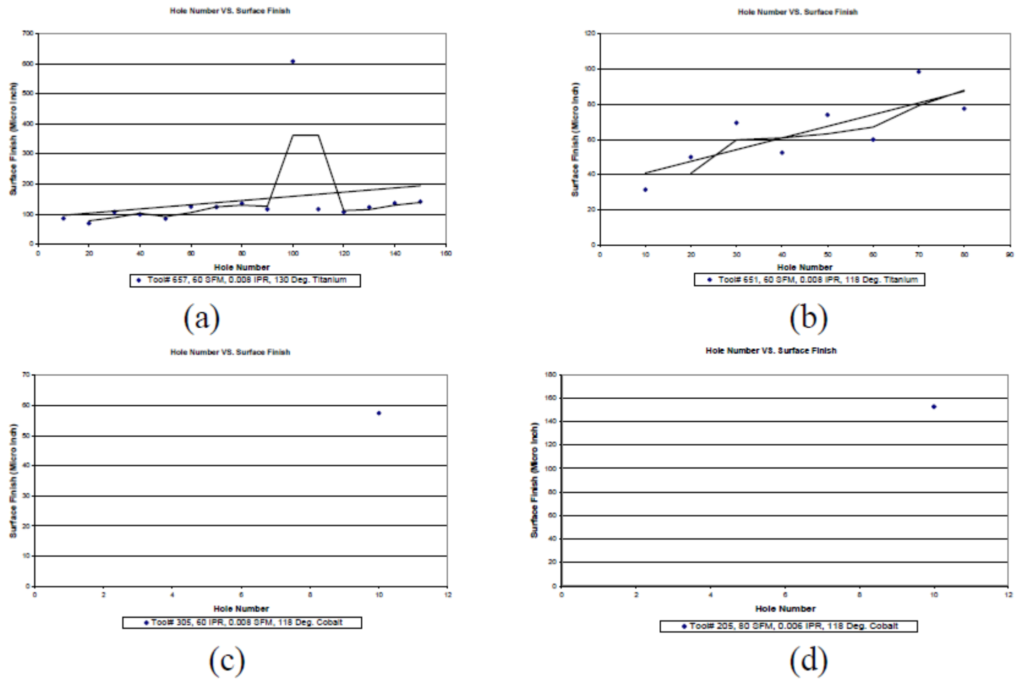


Figure C-20: Surface finish vs. number of holes drilled for speed of 60 SFM and feed of 0.008 IPR for 4140 steel; (a) Drill 657; (b) Drill 651; (c) Drill 305; (d) Drill 205.

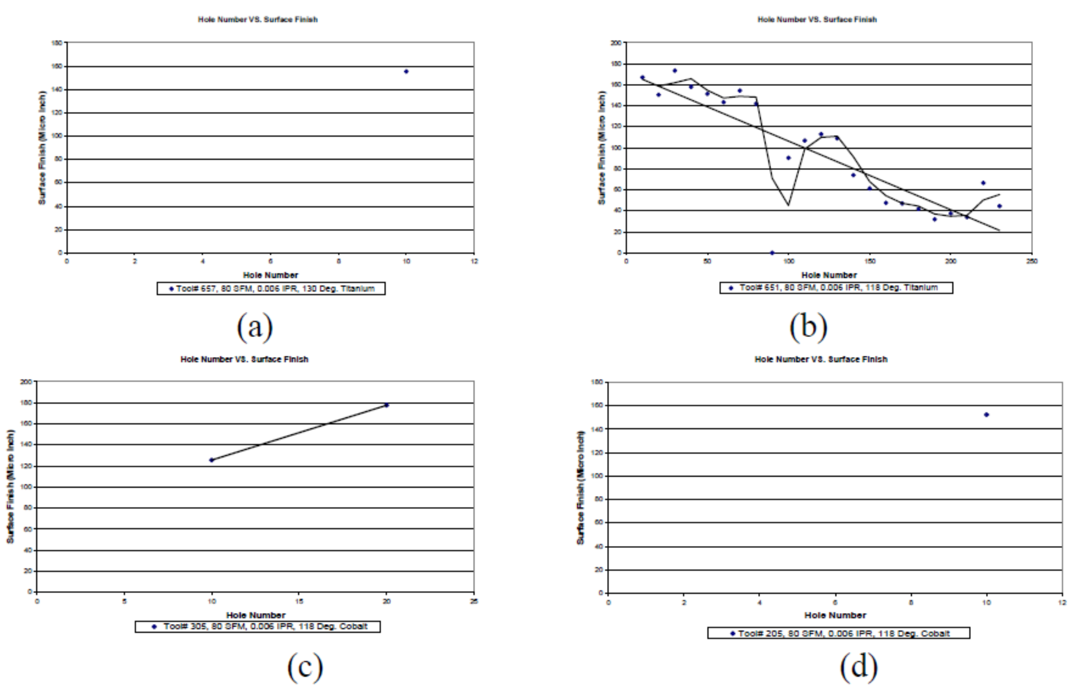


Figure C-21: Surface finish vs. number of holes drilled for speed of 80 SFM and feed of 0.006 IPR for 4140 steel; (a) Drill 657; (b) Drill 651; (c) Drill 305; (d) Drill 205.

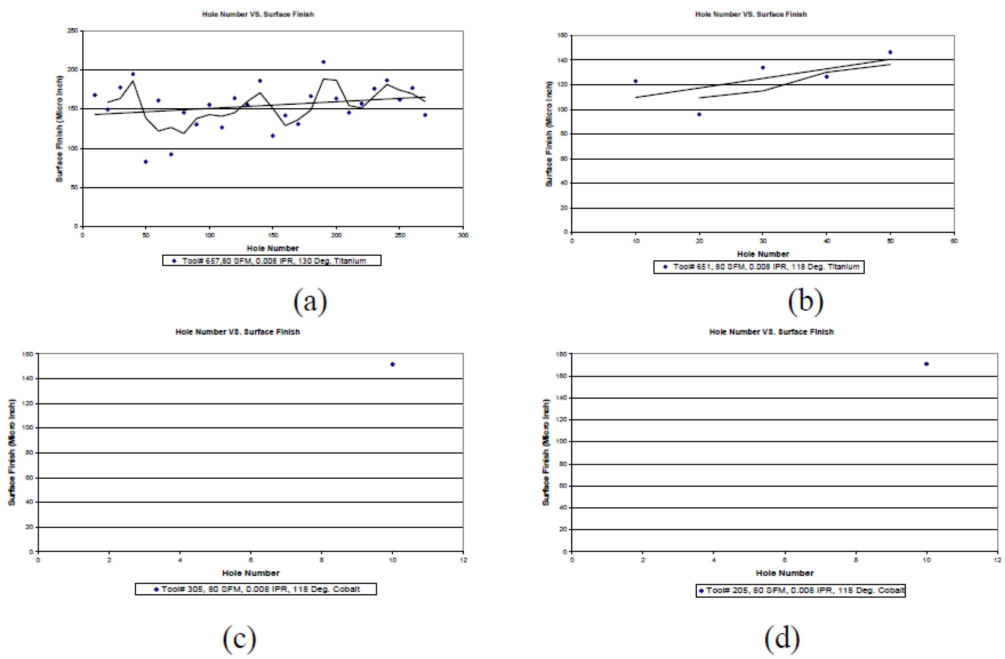


Figure C-22: Surface finish vs. number of holes drilled for speed of 80 SFM and feed of 0.008 IPR for 4140 steel; (a) Drill 657; (b) Drill 651; (c) Drill 305; (d) Drill 205.

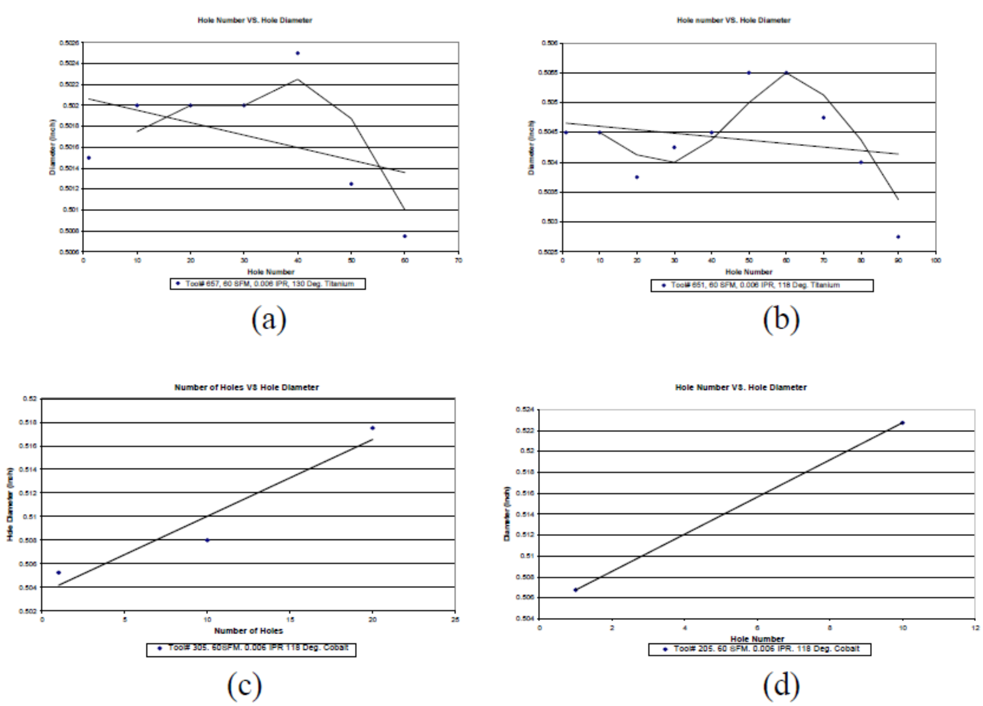


Figure C-23: Hole size vs. number of holes drilled for speed of 60 SFM and feed of 0.006 IPR for 4140 steel; (a) Drill 657; (b) Drill 651; (c) Drill 305; (d) Drill 205.

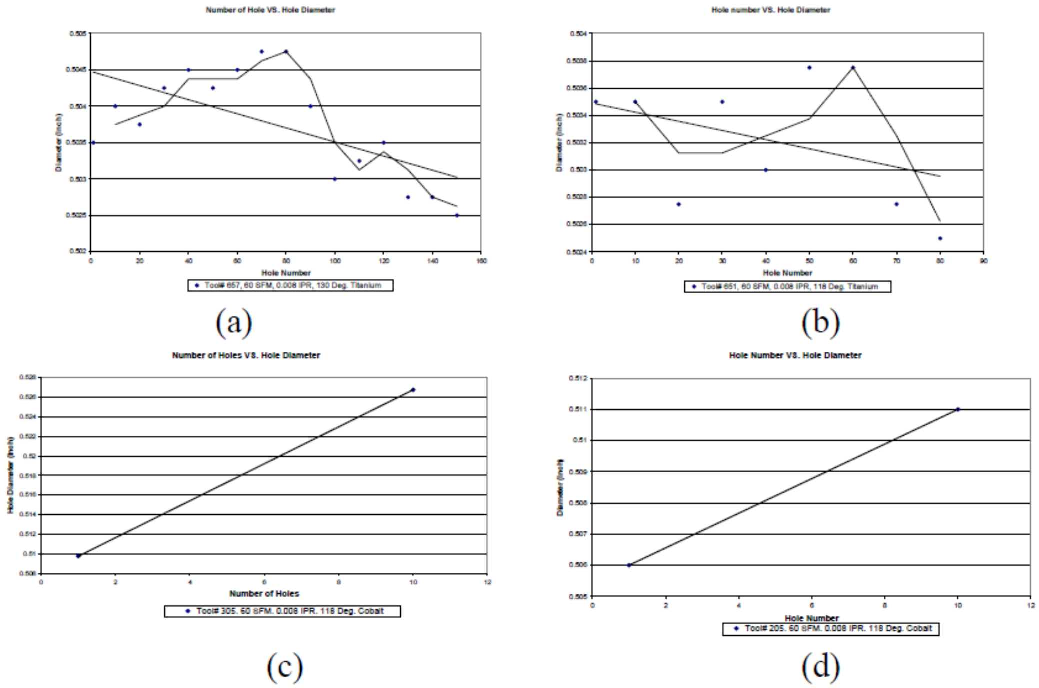


Figure C-24: Hole size vs. number of holes drilled for speed of 60 SFM and feed of 0.008 IPR for 4140 steel; (a) Drill 657; (b) Drill 651; (c) Drill 305; (d) Drill 205.

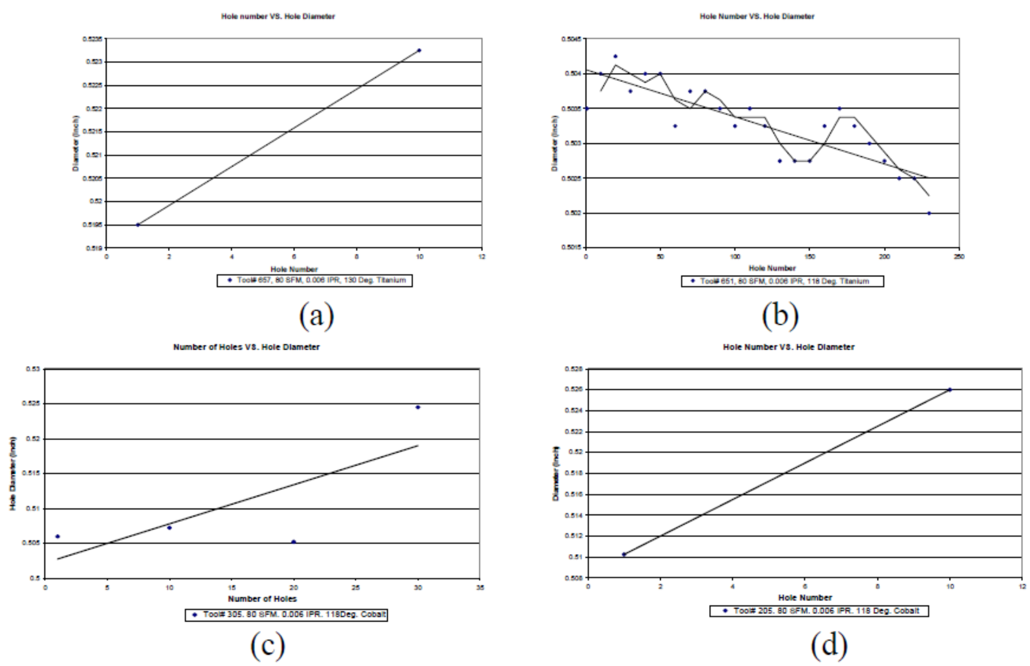


Figure C-25: Hole size vs. number of holes drilled for speed of 80 SFM and feed of 0.006 IPR for 4140 steel; (a) Drill 657; (b) Drill 651; (c) Drill 305; (d) Drill 205

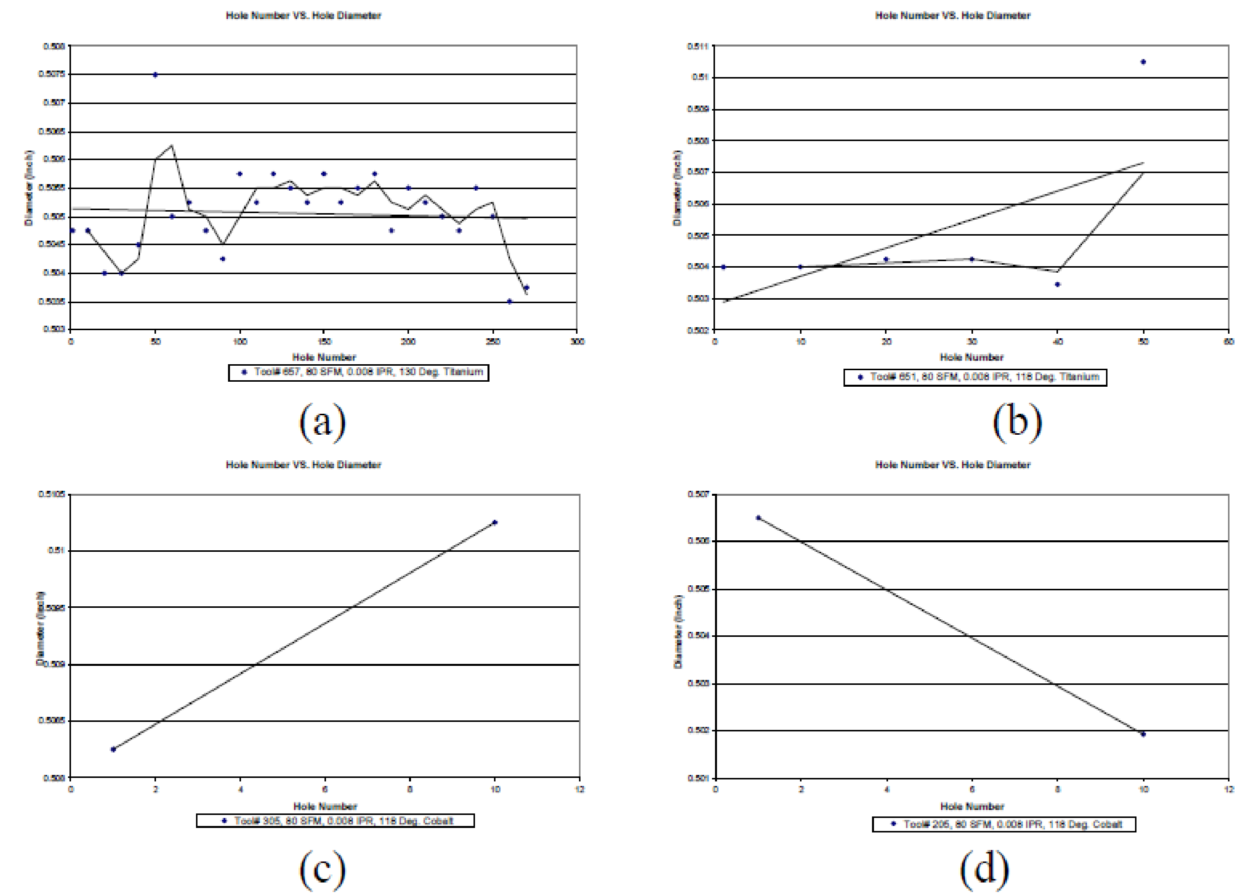
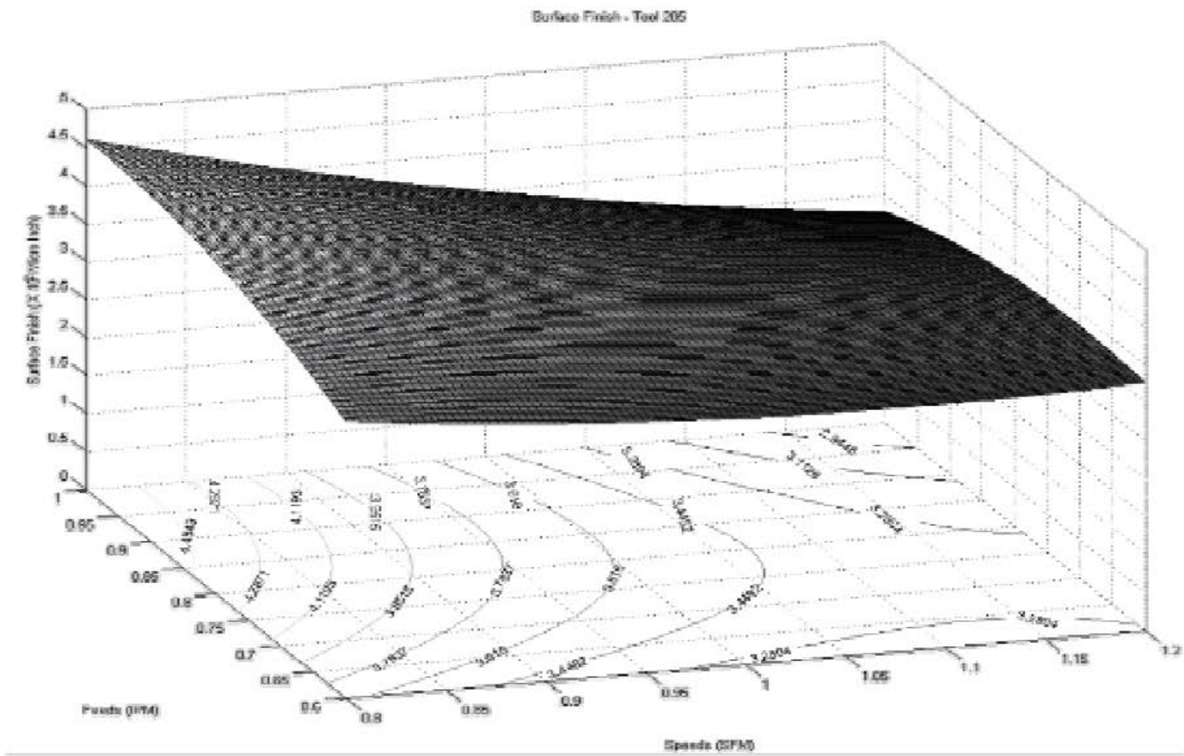
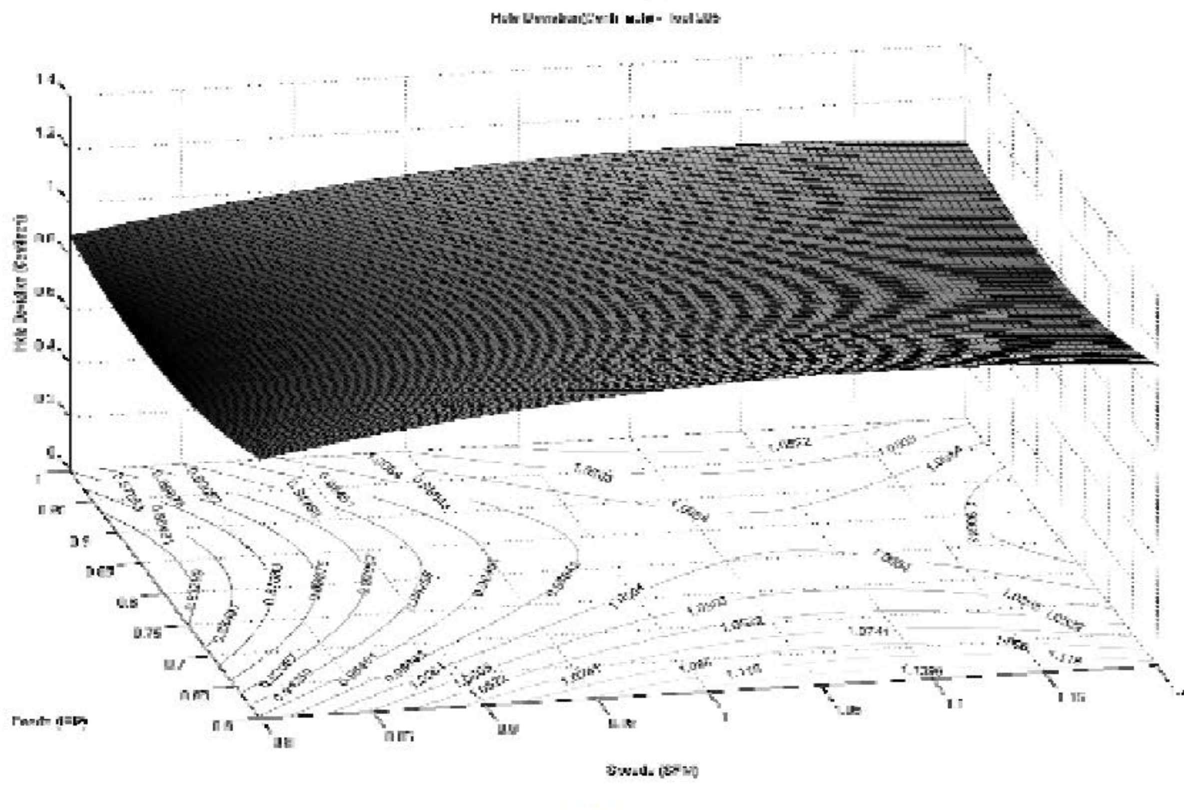


Figure C-26: Hole size vs. number of holes drilled for speed of 80 SFM and feed of 0.008 IPR for 4140 steel; (a) Drill 657; (b) Drill 651; (c) Drill 305; (d) Drill 205.

Appendix D: 3-D Plots of Surface Finish and Hole Size Deviation for 1020 Steel



(a)



(b)

Figure D-1: 3-D plot for Drill 205; (a) Surface finish; (b) Hole size deviation.

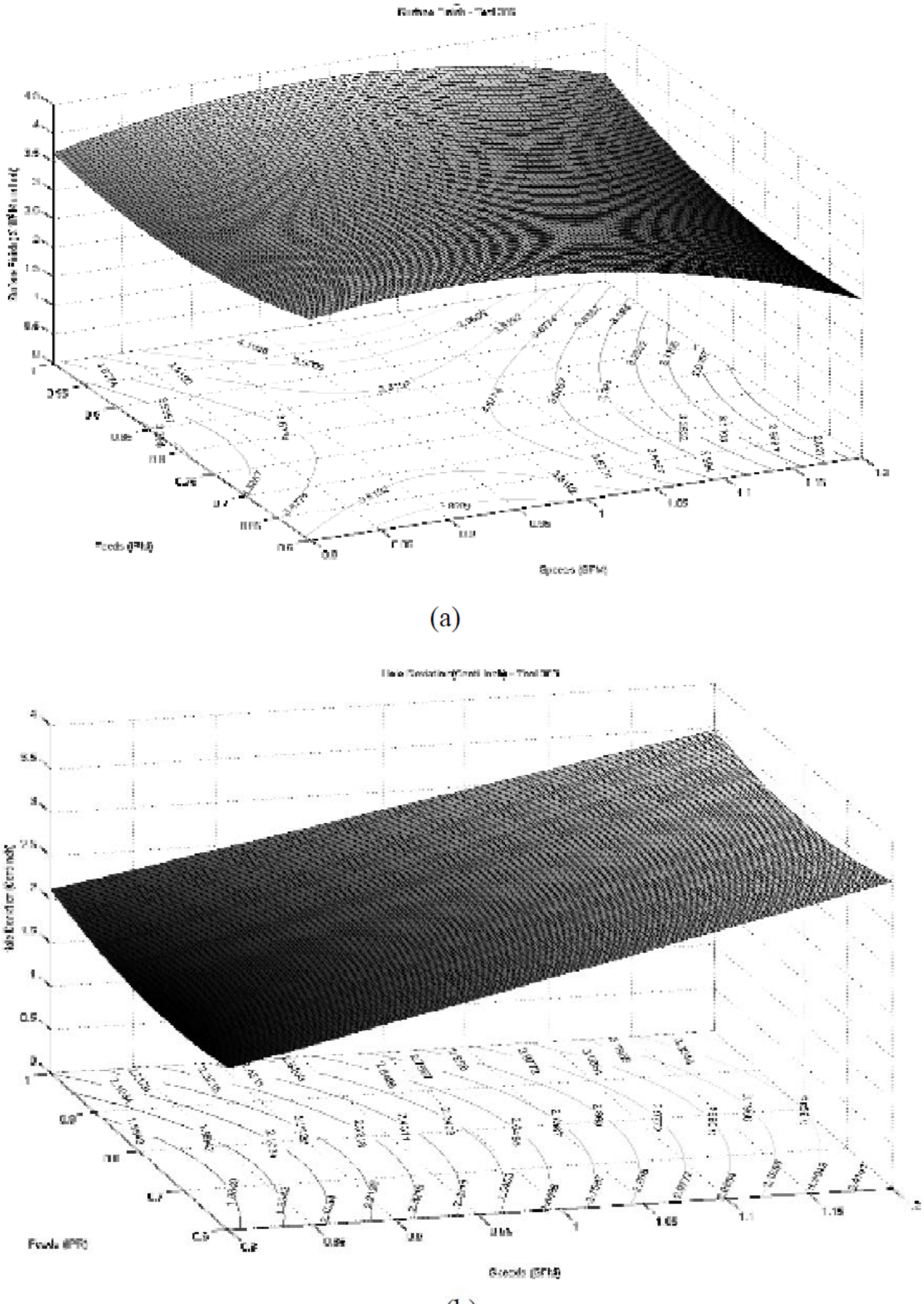
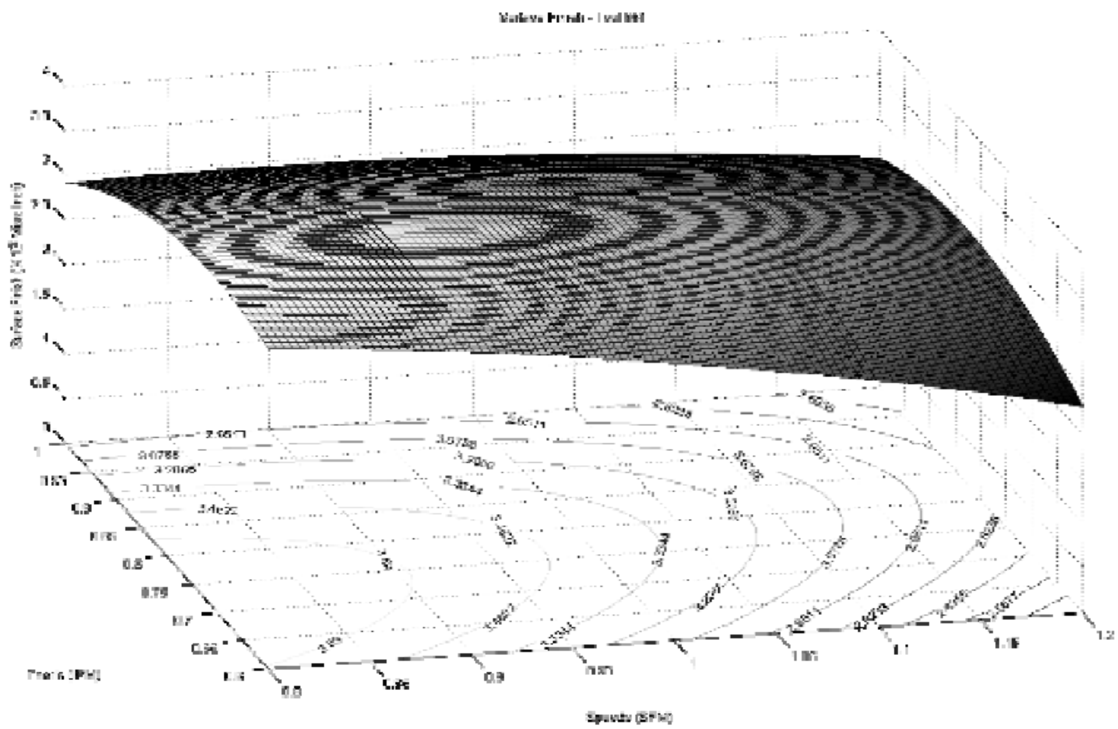
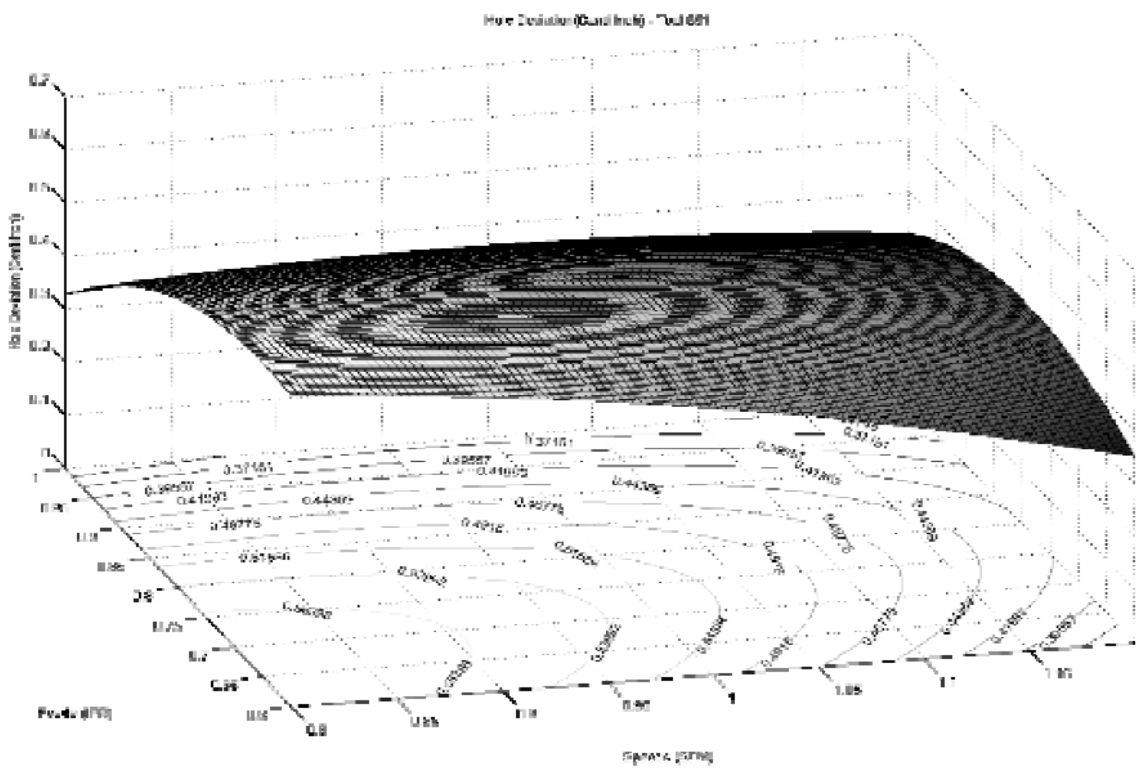


Figure D-2: 3-D plot for Drill 305; (a) Surface finish; (b) Hole size deviation.

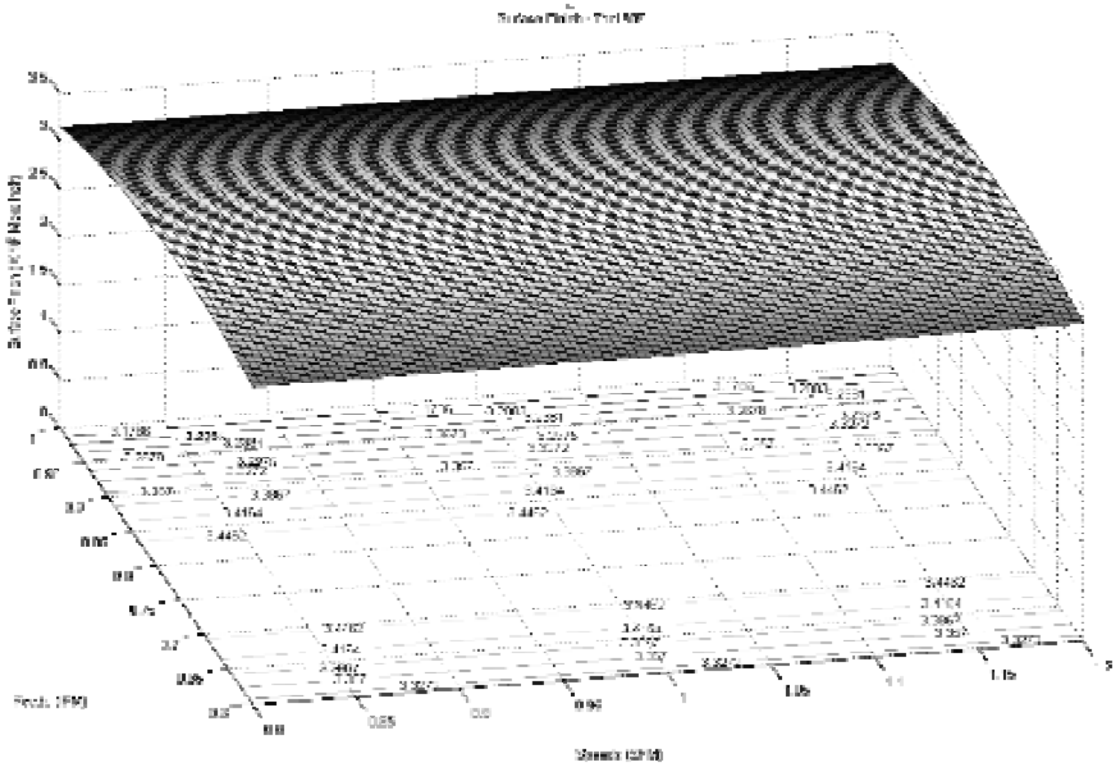


(a)

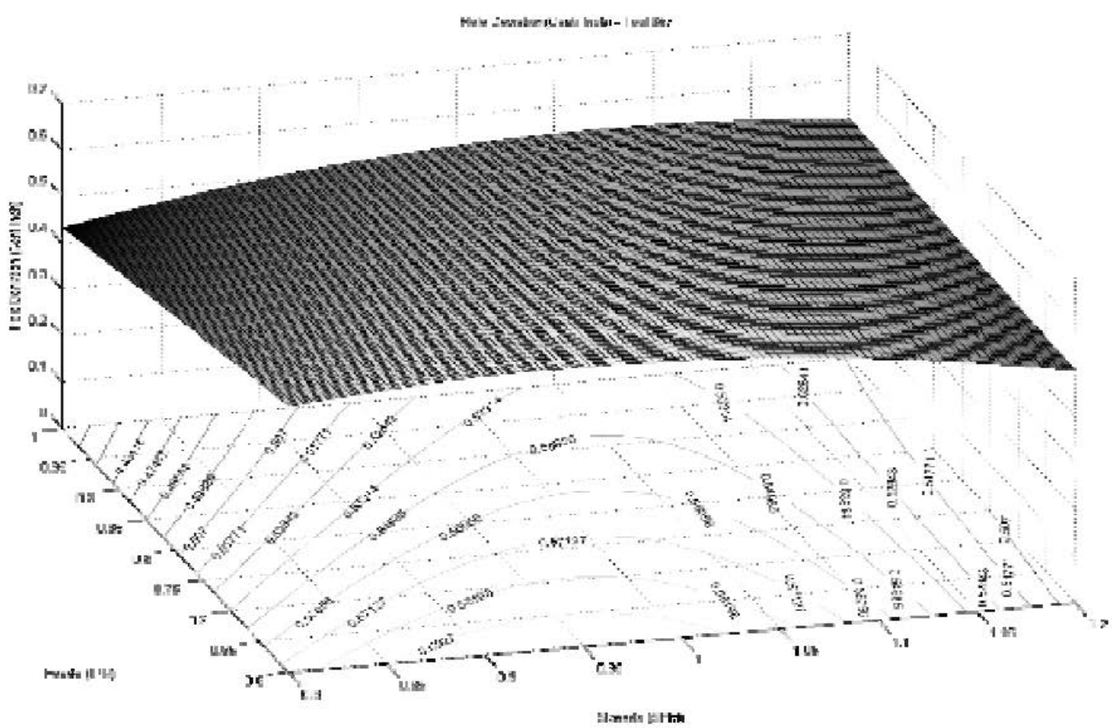


(b)

Figure D-3: 3-D plot for Drill 651; (a) Surface finish; (b) Hole size deviation.



(a)



(b)

Figure D-4: 3-D plot for Drill 657; (a) Surface finish; (b) Hole size deviation.

Appendix E: Normal Boundary Intersection (NBI)

NBI is a new method proposed by Das and Dennis to explore Pareto optimal points on the Pareto surface for competing objectives. This method generates evenly distributed Pareto points on the Pareto front.

Convex Hull of Individual Minima (CHIM)

$$\text{Let } f_1(x) = \textit{Surface Finish,}$$

$$f_2(x) = \textit{Hole Size Deviation}$$

The vector F^* containing the individual minima, f_i^* $i = 1{,}2$ of the two objectives is called the *shadow minimum*. That is:

$$F^* = \begin{bmatrix} f_1^* \\ f_2^* \end{bmatrix}$$

Let Φ be the *2* x *2* matrix whose i^{th} column is $F(x_i^*) - F^*$. That is:

$$\Phi = \begin{bmatrix} f(x_1^*) - f_1^* & f(x_2^*) - f_1^* \\ f(x_1^*) - f_2^* & f(x_2^*) - f_2^* \end{bmatrix},$$

Since $f(x_i^*) = f_i^*$ and x_i^* minimizes $f_i(x)$, we have:

$$\Phi(i,i) = 0 \quad \text{i=1,2}$$

Hence,

$$\Phi = \begin{bmatrix} 0 & f(x_2^*) - f_1^* \\ f(x_1^*) - f_2^* & 0 \end{bmatrix}$$

The *CHIM* is defined as the set of points that are convex combinations of the columns of Φ, i.e., $\{\Phi\beta : \beta_i \geq 0, \sum_i \beta_i = 1\}$. Figure E-1 shows the *Shadow Minimum*, *Pareto curve*, and *CHIM* of a bi-objective space. Note the *Shadow minimum* is shifted to the origin so that all the objectives are positive, i.e.,

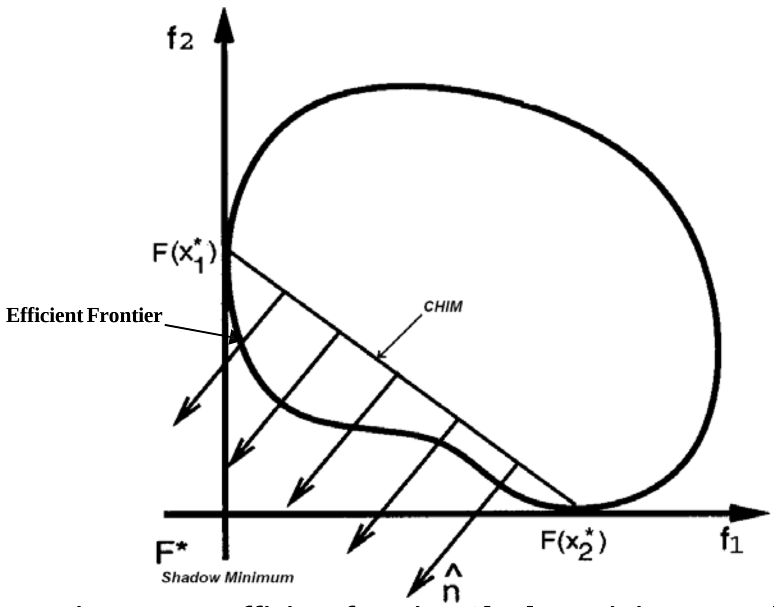


Figure E-1: Efficient frontier, Shadow minimum, and CHIM.

Normal Boundary Intersection Subproblem (NBI_β)

The NBI_β sub-problem is formulated to find feasible Pareto optimal points on the convex part of the Pareto curve farthest from the *CHIM*. An NBI_β sub-problem starts by subdividing the *CHIM* into an equal number of divisions. Then for each point on the CHIM, the furthest point from the CHIM along normal to CHIM is identified. These points are points on the Pareto front. This is shown graphically in Figure E-1.

If vector β represents barycentric coordinates, then $\Phi\beta$ represents a point on the *CHIM*. Let the unit normal emanating from the point $\Phi\beta$ on the *CHIM* towards the origin be $\hat{n}$. Then, $\Phi\beta + t\hat{n}, t \in \Re$ represents the set of points along the unit normal. Therefore, maximizing the distance *t* from the point, $\Phi\beta$, solves the problem of finding the Pareto point. This NBI_β subproblem can be mathematically formulated as:

$$\max_{x,t} \quad t$$

Subject to

$$\Phi\beta + t\,\hat{n} = F(x),$$
$$0.80 < s < 1.20,$$
$$0.6 < f < 1.0,$$

The constraint $\Phi\beta + t\hat{n} = F(x)$ makes sure that the point *x* is mapped by F to a point on the normal, while the bounds on *speed* and *feed* ensure the feasibility of *x* with respect to the original MOP. Thus, for various values of barycentric coordinates β, the solutions of the NBI_β subproblem yields corresponding Pareto points that are evenly spread on the boundary of the Pareto curve.