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LOWER AND UPPER-BOUNDS FOR THE GENERAL JUNCTION ROUTING PROBLEM

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VLSI layout design consists of two phases: *placement* and *routing*. In the placement phase circuit modules are positioned, and in the routing phase they are interconnected. After the placement phase the area unoccupied by the modules, called the *routing area*, is decomposed into sub-regions. Thereafter, the routing phase is divided into two stages: *global* routing and *local* routing. In the global routing stage, nets are assigned to various sub-regions, and in the local routing stage, the nets are assigned to the tracks and columns. In general, the sub-regions, in a placement of rectangular modules, can be "L"-, "S"-, "T"-, or "X"-shaped *junctions* (the simple rectangular channel is a special case of these general-shaped junctions).

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LOWER AND UPPER-BOUNDS FOR THE GENERAL JUNCTION ROUTING PROBLEM †

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Abstract

VLSI layout design consists of two phases: *placement* and *routing*. In the placement phase circuit modules are positioned, and in the routing phase they are interconnected. After the placement phase the area unoccupied by the modules, called the *routing area*, is decomposed into sub-regions. Thereafter, the routing phase is divided into two stages: *global* routing and *local* routing. In the global routing stage, nets are assigned to various sub-regions, and in the local routing stage, the nets are assigned to the tracks and columns. In general, the sub-regions, in a placement of rectangular modules, can be "L"-, "S"-, "T"-, or "X"-shaped *junctions* (the simple rectangular channel is a special case of these general-shaped junctions).

In this paper, we present new non-trivial lower and upper bounds for local routers for general shaped junctions. To the best of our knowledge, these are the first known theoretical results for these problems. For routing of two-terminal nets in arbitrary junctions, we provide optimal results by showing upper bounds which match the universal lower bounds. In the case of routing of three-terminal nets, our upper bounds match the existential lower bounds for the case of "L"-shaped junctions. For instance, we show $t_1 + t_2 = d_1 + d_2$ for routing two-terminal nets, and $t_1 + t_2 = \frac{3}{2}(d_1 + d_2)$ for routing three-terminal nets in some "L"-shaped junctions, where d_1 and d_2 are the densities of the two associated channels and t_1 and t_2 are their widths. All our lower bounds are valid for both the knock-knee and the Manhattan routing models, while our upper bounds are only valid for the knock-knee routing model.

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1. Introduction

A widely used strategy in VLSI layout design is the so called *building-block* layout strategy, which usually consists of two phases: the placement phase and the routing phase [5, 6, 13, 16, 25, 34]. In the placement phase, the circuit blocks of arbitrary sizes are positioned on a layout surface and the *routing area* between blocks is defined by specifying a set of disjoint rectangles (not necessarily unique) (see Figure 1). In the routing phase, the interconnections among the blocks are carried out in two phases of *global* routing and *local* routing. The global router first assigns nets to the various rectangles of the routing area. A net connecting terminals lying in more than one rectangle is divided into several sub-nets, each of which belongs entirely to one rectangle [32, 14, 23, 33]. The local routers determine the routing of sub-nets inside each rectangle [7, 11, 28, 1, 15, 21, 29, 35, 27, 19, 8, 12].

The terminals of a sub-net are of two types: *fixed* and *free*. The position of a fixed terminal is predetermined (for example, by the connections of nets on the circuit blocks), whereas the position of a free terminal can be moved along the side of the rectangle it belongs. The problem of routing nets in a

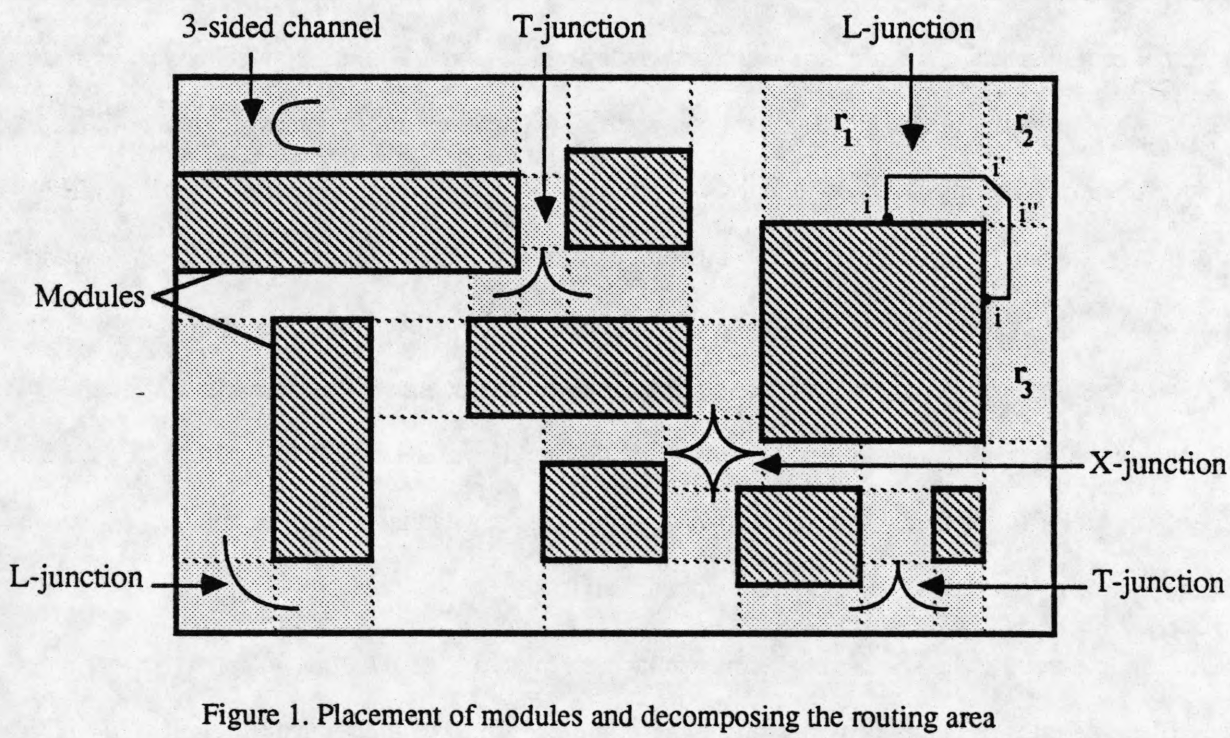

Figure 1. Placement of modules and decomposing the routing area

rectangle with fixed terminals on two opposite sides of the rectangle and free terminals on the other two sides, is called the *channel routing problem* (CRP). The problem of routing nets in a rectangle with fixed terminals on all four sides of the rectangle is called the *switchbox routing problem* (SRP). For the example shown in Figure (1), the global router first divides net i, passing through the rectangles r_1, r_2 and r_3, into three sub-nets i–i' in r_1, i'–i'' in r_2 and i''–i in r_3. The introduced terminals i' and i'' on the common boundaries of r_1–r_2 and r_2–r_3 are fixed or free depending on the routing sequence of these rectangles. If r_1 is routed before r_2, then terminal i' is free for r_1 and fixed for r_2. If, instead, r_2 is routed before r_1, then i' is free for r_2 and fixed for r_1. It is easy to see that when both r_1 and r_3 are routed before r_2, the problem of routing sub-nets in rectangle r_2 is a switchbox routing problem.

The traditional layout systems use channel routers and switchbox routers for routing rectangles. The best known results for these routers are listed in Table 1. It is also known that in general, because of cyclic precedence constraints in the placement, all the rectangles can not be treated as channel routing problems [5, 6, 13, 16, 34]. Hence, switchbox routing problems are unavoidable. It is known that the general switchbox routing problems are harder to route and require excessive routing area (compared to the channel routing problems), which makes them undesirable. To avoid routing switchboxes, one can form "L-", "T-" and "X"-shaped *junctions* by combining switchboxes and channels together [5, 12, 19, 24]. The problem of routing in these junctions is referred to as the *junction routing problem* (*JRP*). For instance, r_1, r_2 and r_3 in Figure (1) can be combined to form an "L"-junction. Intuitively, routing such junctions is easier than routing switchboxes since they are less restrictive. Thus far, the only algorithms known for routing in junctions are heuristic in nature. In this paper, we propose the first provably good algorithms to route in general shaped junctions of Figure (1) (i.e., "L-", "S-", "T-", and "X"-shaped). In the first part of this paper, we discuss lower bounds on the channel widths of a junction, and, in the second part, we discuss the corresponding upper bounds.

An important issue when considering the layout problem is the *routing model*. The routing model describes the rules of layout of wires during the routing phase. There are three different kinds of routing models: Manhattan, knock-knee, and restricted-overlap [30]. In the Manhattan model, two wires may

Results / Geometry	Lower bounds		Upper bounds		
	2-term.	Multi-term.	2-term.	3-term.	multi-term.
2-sided CRP	d	d	d [PS84]	$\frac{3d}{2}$ [PS84]	$\frac{3d}{2}+O(\sqrt{d \log d})$ [GK87]
3-sided CRP	d	$\frac{3d}{2}$ [SZ88]	d [PS84]	$\frac{3d}{2}$ [PS84]	$2d-1$ [PS84]
Switchbox	$2(d_h+d_v)$ [MZ88]	$2(d_h+d_v)$ [MZ88]	———	———	———

Table 1: Previous Results

share a grid point only by crossing at that point but the wires are not allowed to overlap [1,4]. In the knock-knee model wires may share a grid point either by crossing or by bending at that point; again, the wires are not allowed to overlap [22, 26, 18]. On the other hand, in the restricted-overlap model, two wires are allowed to overlap for $O(1)$ units (usually 1 or 2) [2, 3, 8, 10]. In this paper, we will only discuss the non-overlap models. Our lower bounds are valid for both the Manhattan and the knock-knee models, but our upper bounds are only valid for the knock-knee model.

Other important issues for the consideration of layout problems are the assumptions about the *placement model*. The placement model describes the rules for the placement of blocks and restrictions on the adjustments of the routing region permitted during the routing phase. All the previous results summarized in Table 1 are derived under the assumption that all the allowable operations do not change the density [1] of the routing problem. In this paper, we assume a placement model that permits one to move circuit blocks in the routing phase (in order to facilitate routing), under the restriction that the densities of all the channels associated with the junction do not change. These assumptions are required since all our bounds are expressed in terms of the densities of the problems, and they are quite realistic since changing the densities of previously routed regions could make their routings unusable.

In the Section 2, we first give the definitions for the general junctions. In Section 3, we present lower bounds for "L", "S", "T", and "X"-junctions. In Section 4, we first present upper bounds for routing

[1] Channel density is an important parameter used in measuring the complexity of a channel routing problem.

two-terminal nets in "L"- and "S"-junctions, and then we use these bounds to obtain upper bounds for other junctions. Later, we develop similar bounds for three-terminal nets. Finally, we state some open problems and suggest an alternative technique for decomposing the routing area based on these results.

2. Definitions and Notation

We call two rectangles *adjacent* if they share a non-zero length of their boundaries. We refer to a collection of adjacent rectangles as a *junction*. In the following, we define four types of junctions and the corresponding junction routing problems by combining routing problems of several adjacent rectangles together:

An *L–junction* $L(t_1, t_2, x_0, y_0)$ is an "L"-shaped region that is the union of the following four rectangular regions (assuming without loss of generality [2], that $t_1 \geq t_2$) (see Figure 2(a)):

$$L = \{(x, y) : x < 0, -t_1 \leq y \leq 0\}$$

$$T = \{(x, y) : 0 \leq x \leq t_2, 0 < y\}$$

$$J_1 = \{(x, y) : 0 \leq x \leq x_0, -t_1 \leq y \leq 0\}$$

$$J_2 = \{(x, y) : 0 \leq x \leq t_2, -y_0 \leq y \leq 0\}$$

where $0 \leq x_0 \leq t_2$ (distance between the origin and the vertical segment of the "dent") and $0 \leq y_0 \leq t_1$ (distance between the origin and the horizontal segment of the "dent") are the offset parameters predetermined by the positions of the surrounding circuit blocks. We refer to sets L and T as the *left* and *top* channels and to the set $J_1 \cup J_2$ as the *junction area*. Any shortest Manhattan path between (0, 0) and $(x_0, -y_0)$ is called the *bottleneck* of the junction area. A simple L-junction (one without a "dent") corresponds to $x_0 = t_2$ and $y_0 = t_1$.

An *S–junction* $S(t_1, t_2, x_0, y_0)$ is an "S"-shaped region that is the union of the following three rectangular regions (assuming without loss of generality, that $t_1 \geq t_2$) (see Figure 2(b)).

[2] This assumption is justified, by the symmetry of the rectangular regions and by an appropriate numbering of the regions.

$$L = \{(x,y) : x < 0, -t_1 \le y \le 0\}$$

$$R = \{(x,y) : x > x_0, -y_0 \le y \le t_2 - y_0\}$$

$$J = \{(x,y) : 0 \le x \le x_0, -t_1 \le y \le t_2 - y_0\}$$

where $0 \le x_0$ (distance between the origin and the right end of R), and $0 \le y_0 \le t_1$ (distance between the lower shore of R and the upper shore of L) are the offset parameters of the junction which are determined by the positions of the surrounding circuit blocks. We refer to sets L and R as the *left* and *right* channels and to set J as the *junction area*. Any shortest Manhattan path between $(0,0)$ and $(x_0, -y_0)$ is called the *bottleneck* of the junction area. There are two distinct configurations of S-junctions (as shown in Figure 2(b)): (1) when its left and right channels are a "containing" pair [3], and (2) when its they are a "non-containing" pair. By the symmetry of Left and Right channels under rotations and reflections, it is easy to check that these two configurations encompass every type of S-junction. The three-sided channel is a special case of an S-junction, where $t_2 = 0$.

A *T-junction* $T(t_1, t_2, t_3, x_0, y_0)$ is a "T"-shaped region that is the union of the following five rectangular regions (assuming without loss of generality, that $t_1 \ge t_3$)(see Figure 2(c)):

$$L = \{(x,y) : x < 0, -t_1 \le y \le 0\}$$

$$T = \{(x,y) : 0 \le x \le t_2, \max(0, t_3 - y_0) < y\}$$

$$R = \{(x,y) : t_2 < x, -y_0 \le y \le t_3 - y_0\}$$

$$J_1 = \{(x,y) : 0 \le x \le x_0, -t_1 \le y \le 0\}$$

$$J_2 = \{(x,y) : 0 \le x \le t_2, -y_0 \le y \le \max(0, t_3 - y_0)\}$$

where $0 \le x_0 \le t_2$ (distance between the origin and the right end of set R) and $0 \le y_0 \le t_1$ (distance between the origin and the lower shore of R) are the offset parameters predetermined by the positions of the surrounding circuit blocks. We refer to sets L, R, and T as the *left, right,* and *top* channels, respectively and to the set $J_1 \cup J_2$ as the *junction area*. Any shortest Manhattan path between $(0,0)$ and

[3] Two horizontal (or vertical) channels are said to be a "containing" pair, if extensions of the shore-lines of one channel lie entirely inside the other channel, else they are said to be a "non-containing" pair.

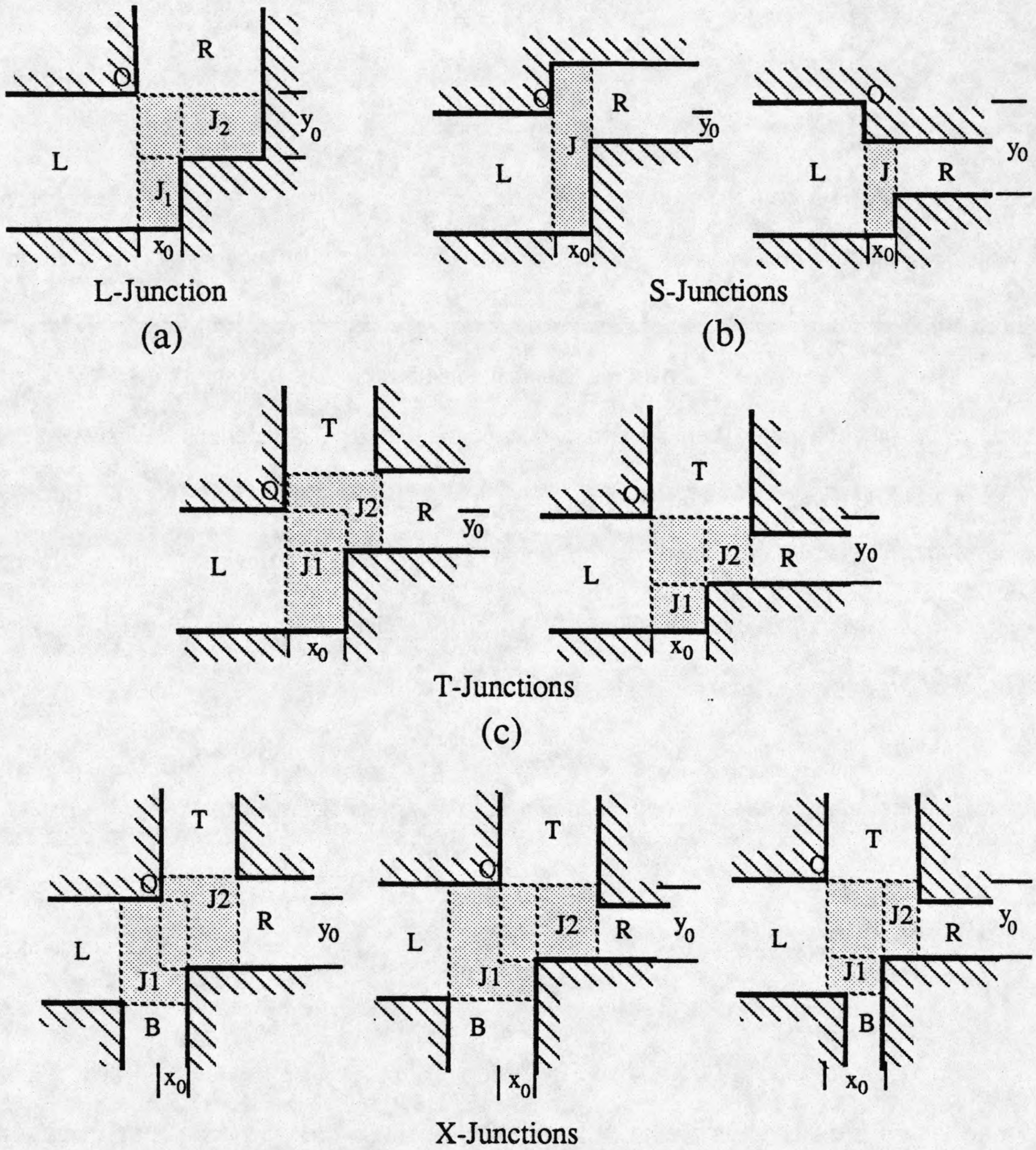

Figure 2. Different kinds of junctions

$(x_0, -y_0)$ is called the *bottleneck* of the junction area. There are two distinct configurations of T-junctions (as shown in the Figure 2(c)): (1) when its horizontal (left and right) channels are a "containing" pair, and (2) when its horizontal channels are a "non-containing" pair.

An $X-junction$ $X(t_1, t_2, t_3, t_4, x_0, y_0)$ is an "X"-shaped region that is the union of the following six rectangular regions (assuming without loss of generality that $t_1 \geq t_3$ and $t_2 \geq t_4$) (see Figure 2(d)):

$$L = \{(x, y) : x < \min(0, x_0 - t_4), -t_1 \leq y \leq 0\}$$

$$T = \{(x, y) : 0 \leq x \leq t_2, \max(0, t_3 - y_0) < y\}$$

$$R = \{(x, y) : t_2 < x, -y_0 \leq y \leq t_3 - y_0\}$$

$$B = \{(x, y) : x_0 - t_4 \leq x \leq x_0, y < -t_1\}$$

$$J_1 = \{(x, y) : \min(0, x_0 - t_4) \leq x \leq x_0, -t_1 \leq y \leq 0\}$$

$$J_2 = \{(x, y) : 0 \leq x \leq t_2, -y_0 \leq y \leq \max(0, t_3 - y_0)\}$$

where $0 \leq x_0 \leq t_2 + t_4$ (distance between the origin or the left shore of T and the right shore of B) and $0 \leq y_0 \leq t_1$ (distance between the origin and the lower shore of R) are the offset parameters determined by the positions of the surrounding circuit blocks. We refer to sets L, R, T, and B as the *left*, *right*, *top*, and *bottom* channels, respectively, and to the set $J_1 \cup J_2$ as the *junction area*. Any shortest Manhattan path between (0, 0) and $(x_0, -y_0)$ is called the *bottleneck* of the junction area. There are three distinct configurations of X-junctions, as shown in the Figure 2(d), (1) when both its horizontal and vertical channel pairs are "containing", (2) when both its horizontal and vertical channel pairs are "non-containing", and (3) when its horizontal channels are "containing" and the vertical channels are "non-containing" or vice-versa.

In general, the above defined junctions can be more appropriately viewed as *multi−way junctions*. For instance, S- and L- junctions can be thought of as 2–*way* junctions, T- and X-junctions can be thought of as 3– and 4–*way* junctions, respectively. However, since their shapes resemble "L"-, "S"-, "T"- and "X"-shapes in special cases, we prefer to use the previously introduced terminology. This is consistent with the conventional definitions of "L"-, "T"-, and "X"-channels used in the literature [19, 12, 24].

Next, we define some of the fundamental notions required for any routing problem. We introduce this notation for the case of a simple channel routing problem. A *net* N is a collection of terminals:

$N = (\{T_1, \cdots, T_t\}, \{B_1, \cdots, B_b\}, R_r, L_l)$, which are to be connected[4] [9]. In a rectangle of width w, the upper terminal T_j (here T_j is considered as a symbol) is located at grid point $(T_j, w+1)$ (here T_j is considered as an integer), and the lower terminal B_j is at $(B_j, 0)$. R_r and L_l, the right and left free terminals of N lying on the right and left sides of the rectangle, represent connections of N with its sub-nets lying in different rectangles. Both r and l can be either 0 or 1. N is called a *j–terminal* net if $t + b + r + l \leq j$. The distance between the upper and lower terminals, w, is referred to as the *width* of the channel. The *density* d of a channel routing problem is defined as the maximum, over all cross-sections C, of the *local density* $\delta(C)$ at cross-section C:

$$\delta(C) = |\ \{N \mid N \text{ is a net with } \min\{T_1, B_1, L_l\} \leq C \text{ and } \max\{T_u, B_l, R_r\} \geq C+1\}\ | .$$

We can similarly define the *junction–area density* D_j in the junction area, as the number of nets required to cross its *bottleneck*. A routing problem in a k-way junction is specified in terms of the densities of the associated channels and the junction area. For example, a routing problem in an L-junction is specified by the densities in the Left channel d_1, Top channel d_2, and its junction-area D_j. The *capacity* of a cross-sectional cut C, $\kappa(C)$, is defined as the length of C.

3. Lower Bounds for various Junction Routing Problems

For the channel routing problem, the density is a trivial lower bound on the channel width [11, 17]. Similarly, a trivial lower bound on the total channel width of a k-way junction is $\sum_{i=1}^{k} t_i \geq \sum_{i=1}^{k} d_i$, where t_i and d_i are the width and density of the i-th associated channel. In this section, we present a set of non-trivial lower bounds for routing in general types of junctions. In proving these lower bounds we use the following observation: the capacity of a chosen cross-sectional cut has to be at least as big as the number of nets required to cross it. In the same way that the density of a channel routing problem provides a trivial lower bound on the channel width, the junction-area density, as defined in the previous section,

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[4] This definition of nets is defined for a channel routing problem, where there is at most one terminal of a net on the left or the right side. In the case of nets in a switchbox routing problem, there may be more than one terminals of a net on the left or the right side of the rectangle.

provides a lower bound on the length of the bottleneck. This in turn implies a lower bound on the junction "size", which is captured in the sum of the widths of its associated channels. We refer to such a lower bound as a *universal* lower bound since it depends on the junction density of any given problem. In contrast, we also prove *existential* lower bounds on junction size by constructing specific worst-case routing problems. Any routing problem that violates the universal lower bound condition can not be routed; so we assume that the global router has checked for this condition at every bottleneck and guarantees of no such violation during the local routing phase. In constructing the existential worst-case routing problems, we ensure that the universal lower bound is not violated. Our technique consists of presenting a specific terminal arrangement and choosing proper sets of cross-sectional cuts. Then, by arguing that the capacity (length) of these cuts has to be more than their densities (number of nets required to cross it), we obtain our lower bounds. In the following, we first discuss the existential lower bound for L-junctions. Next, we obtain lower bounds for S-junction, T-junction, and X-junction using the same technique.

3.1. Lower Bounds for L-Junctions

As indicated previously, we denote the densities of the Left and the Top channels of an L-junction by d_1 and d_2, and their widths by t_1 and t_2, respectively. The offset parameters of the L-junction are represented by x_0 and y_0. We assume without loss of generality, that $t_1 \geq d_1$ and $t_2 \geq d_2$ which follows from the trivial lower bounds on channel widths.

Consider the routing problem with terminal arrangement as shown in Figure (3). The set of $d_1/2$ terminals, L_B, on the bottom shore of the Left channel is divided into sets s_1 and s_2 of cardinalities n_1 and n_2 (one of them could possibly be empty). Terminals of s_1 are connected to terminals on the rightmost vertical segment (above the dent) of the junction area, as shown in the Figure (3), and the n_2 terminals of s_2 are connected to terminals on the leftmost vertical segment (below the dent) of the junction area. Similarly, L_T, the set of $d_1/2$ terminals on the top shore of the Left channel is divided into two sets, s_3 and s_4, where the n_3 terminals of s_3 are connected to terminals on the rightmost (upper) vertical seg-

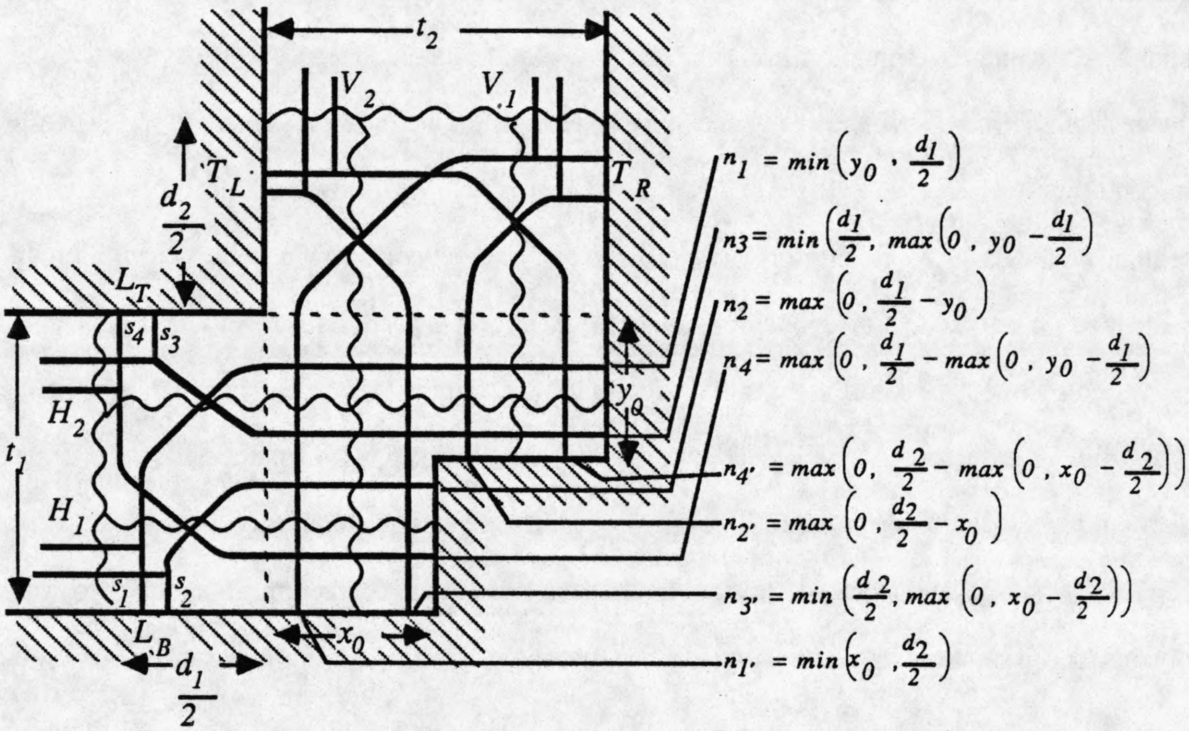

Figure 3. Lower Bound example for an L-junction

ment of the junction area and the n_4 terminals of s_4 are connected to terminals on the leftmost (lower) vertical segment of the junction area. All the d_1 terminals of L_B and L_T are also connected to other terminals in the Left channel, thereby giving a density of d_1 in the Left channel. We introduce similar sets T_R and T_L on the right and left shores of the Top channel, which are partitioned into subsets $s_{1'}$, $s_{2'}$, $s_{3'}$, and $s_{4'}$. These sets are connected to the bottom segments of the junction area as shown in Figure (3).

The partitions of sets L_B, L_T, T_R and T_L are chosen such that, the connection of nets does not violate the junction-density, D_j, in other words

$$|s_1| + |s_3| + |s_{1'}| + |s_{3'}| \le x_0 + y_0.$$

In order to find the cardinality of the various sets s_i, we discuss the sequence in which the terminals of the Left channel are connected. First of all, we connect as many of the $d_1/2$ terminals of L_B to the y_0 locations on the rightmost segment of the junction area, thus $|s_1| = n_1 = \min(y_0, d_1/2)$. Then, we connect any remaining terminals of L_B to the leftmost segment of the junction area, thus $|s_2| = n_2 = \max(0, d_1/2 - y_0)$. Next, we try to connect the terminals of L_T to the remaining terminals (if any) on the rightmost segment, and finally we connect all the remaining terminals of the top shore to the remaining terminals on the leftmost segment. Hence, $|s_3| = n_3 = \min(d_1/2, \max(0, y_0 - d_1/2))$ and $|s_4| = n_4 = \max(0, d_1/2 - \max(0, y_0 - d_1/2))$. The terminals of the Top channel are connected in a similar fashion; so

$$|s_{1'}| = n_{1'} = \min(x_0, \frac{d_2}{2}),$$

$$|s_{2'}| = n_{2'} = \max(0, \frac{d_2}{2} - x_0),$$

$$|s_{3'}| = n_{3'} = \min(\frac{d_2}{2}, \max(0, x_0 - \frac{d_2}{2}))$$

$$\text{and } |s_{4'}| = n_{4'} = \max(0, \frac{d_2}{2} - \max(0, x_0 - \frac{d_2}{2})).$$

It is easy to check that

$$n_1 + n_2 = \frac{d_1}{2} = n_3 + n_4, \qquad n_1 + n_3 = \min(y_0, d_1),$$

$$n_{1'} + n_{2'} = \frac{d_2}{2} = n_{3'} + n_{4'}, \qquad n_{1'} + n_{3'} = \min(x_0, d_2).$$

$$n_1 + n_3 + n_{1'} + n_{3'} \le x_0 + y_0$$

For any feasible routing of this problem, we require that the capacity of any cross-sectional cut is at least as big as the number of nets required to cross it. This gives us a way to prove lower bounds on $t_1 + t_2$. In the following we consider four sets of cuts H_1, H_2, V_1, and V_2 as shown in Figure (3). For example, H_1 consists of a vertical segment, between the shores of the Left channel (of length t_1) and a horizontal segment (of length $d_1/2 + x_0$). Although H_1 is a pair of segments, we still refer to H_1 as a cross-sectional cut.

For cut H_1, we have

$$\kappa(H_1) = t_1 + (\frac{d_1}{2} + x_0)$$

$$\delta(H_1) = d_1 + \frac{d_1}{2} + \min(x_0\,,\, d_2) + \max(0\,,\, \frac{d_1}{2} - \max(0\,,\, y_0 - \frac{d_1}{2})).$$

Combining these equations, we get

$$t_1 \geq d_1 + \min(0\,,\, d_2 - x_0) + \max(0\,,\, \frac{d_1}{2} - \max(0\,,\, y_0 - \frac{d_1}{2})). \tag{1}$$

For cut H_2, we have

$$\kappa(H_2) = t_1 + (\frac{d_1}{2} + t_2)$$

$$\delta(H_2) = d_1 + \frac{d_1}{2} + d_2 + \min(y_0\,,\, \frac{d_1}{2}).$$

Combining these equations, we get

$$t_1 + t_2 \geq d_1 + d_2 + \min(y_0\,,\, \frac{d_1}{2}). \tag{2}$$

For cut V_1, we have

$$\kappa(V_1) = t_2 + (\frac{d_2}{2} + y_0)$$

$$\delta(V_1) = d_2 + \frac{d_2}{2} + \min(y_0\,,\, d_1) + \max(0\,,\, \frac{d_2}{2} - \max(0\,,\, x_0 - \frac{d_2}{2})).$$

Combining these equations, we get

$$t_2 \geq d_2 + \min(0\,,\, d_1 - y_0) + \max(0\,,\, \frac{d_2}{2} - \max(0\,,\, x_0 - \frac{d_2}{2})). \tag{3}$$

For cut V_2, we have

$$\kappa(V_2) = t_2 + (\frac{d_2}{2} + t_1)$$

$$\delta(V_2) = d_2 + \frac{d_2}{2} + d_1 + \min(x_0\,,\, \frac{d_2}{2}).$$

Combining these equations, we get

$$t_1 + t_2 \geq d_1 + d_2 + \min(x_0\,,\, \frac{d_2}{2}). \tag{4}$$

By combining inequalities of Equations (1) and (3), we get

$$t_1 + t_2 \geq d_1 + d_2 + \min(0\,,\, d_1 - y_0) + \max(0\,,\, \frac{d_1}{2} - \max(0\,,\, y_0 - \frac{d_1}{2})) \qquad (5)$$

$$+ \min(0\,,\, d_2 - x_0) + \max(0\,,\, \frac{d_2}{2} - \max(0\,,\, x_0 - \frac{d_2}{2})).$$

By combining inequalities of Equations (2) and (4), we get

$$t_1 + t_2 \geq d_1 + d_2 + \max(\min(x_0\,,\, \frac{d_2}{2})\,,\, \min(y_0\,,\, \frac{d_1}{2})). \qquad (6)$$

Both Equations (5) and (6) give nontrivial lower bounds for all the values of x_0 and y_0, but, for particular values of the offset parameters, one of them will give a better bound than the other. This kind of analysis is summarized in the Figure (4), which shows the regions of dominance for the two equations. For example, if $x_0 \leq d_2/2$ and $y_0 \leq d_1/2$, then inequality of (5) gives

$$t_1 + t_2 \geq d_1 + d_2 + 0 + \frac{d_1}{2} + 0 + \frac{d_2}{2}$$

$$= \frac{3}{2}d_1 + \frac{3}{2}d_2. \qquad (7)$$

And inequality of (6) gives

$$t_1 + t_2 \geq d_1 + d_2 + \max(x_0\,,\, y_0). \qquad (8)$$

It is easy to check that for the above mentioned range of values for x_0 and y_0, lower bound of Equation

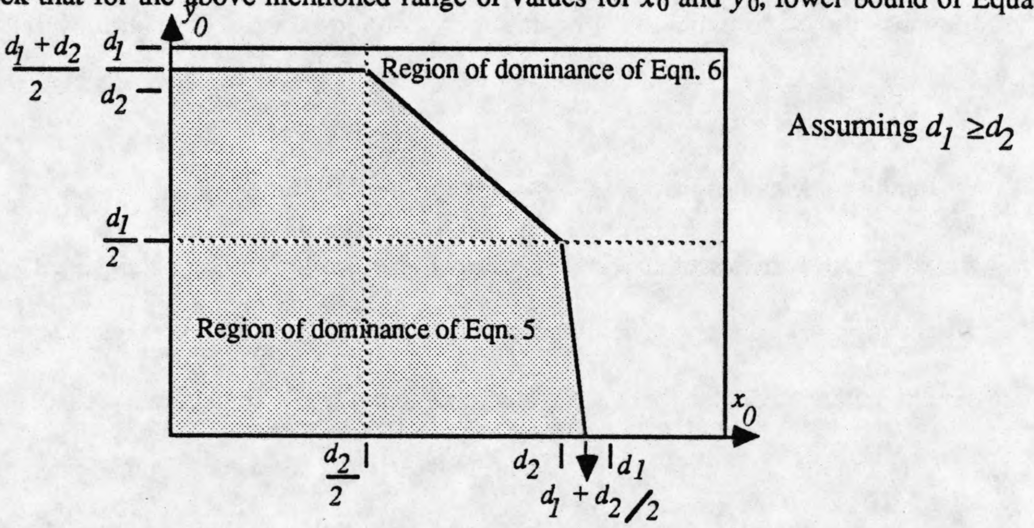

Figure 4. Tradeoff regions of Equations (5) and (6)

(7) is always better than the bound of Equation (8) [5]. For the case when $x_0 \geq d_2/2$ and $y_0 \geq d_1/2$, Equation (5) gives

$$t_1 + t_2 \geq d_1 + d_2 + \max(0\,, d_1 - y_0) + \min(0\,, d_1 - y_0) + \max(0\,, d_2 - x_0) + \min(0\,, d_2 - x_0)$$
$$\geq 2d_1 + 2d_2 - x_0 - y_0. \tag{9}$$

And, Equation (6) gives

$$t_1 + t_2 \geq d_1 + d_2 + \max(\frac{d_1}{2}\,, \frac{d_2}{2}). \tag{10}$$

It is easy to check that Equation (9) gives better bounds than Equation (10) if $(d_1/2 + d_2/2) \leq x_0 + y_0 \leq d_1 + d_2 - \max(d_1/2\,, d_2/2)$. For a simple L-junction, where $x_0 = t_2$ and $y_0 = t_1$ (without any "dent"), we have $t_1 + t_2 \geq d_1 + d_2 + \max(d_1/2\,, d_2/2)$.

We have obtained lower bounds for various range of values of the offset parameters x_0 and y_0. In order to obtain the strongest existential lower bound we choose the worst case settings of x_0 and y_0. Thus, we can write the following theorem.

Theorem 1: The existential lower bound for any general L-junction is $t_1 + t_2 \geq \frac{3}{2}(d_1 + d_2)$.

In the following all the existential lower bounds are stated for the worst case settings of the offset parameters of the junctions.

3.2. Lower Bounds for S-Junctions

We denote the densities of the Left and Right channels of an S-junction by d_1 and d_2 and their widths by t_1 and t_2, respectively. The junction area is described by one rectangle. The offset parameters of the junction are denoted by x_0 and y_0. We assume without loss of generality, that $t_1 \geq d_1$ and $t_2 \geq d_2$, which follows from the trivial lower bounds. Moreover, in the case of an S-junction, we also assume that the height of the junction area (i.e., $t_1 + t_2 - y_0$) is $\geq V_j$, where V_j is the maximum column density in the

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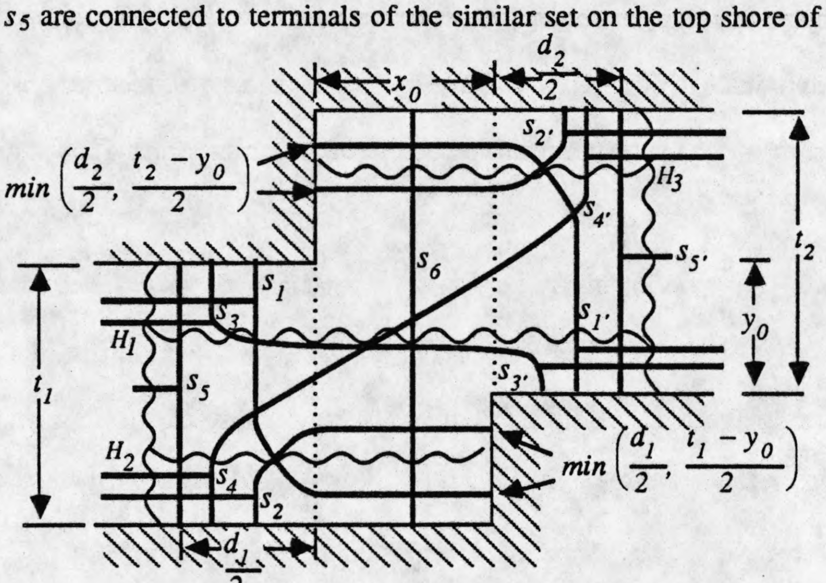
[5] Note: The lower bound of Equation (7) is the same as combining the lower bounds for two separate three-sided channels, for which one can show $t \geq \frac{3}{2}d$, for three and more terminal nets.

junction area, referred to as the *vertical density* of the S-junction. This is obvious, since the junction area is like a channel whose width has to be at least as big as its density. This sort of lower bound is also referred to as a universal lower bound.

Consider the routing problem with terminal arrangement as shown in Figure (5). The set of $d_1/2$ terminals, L_T, on the top shore of the Left channel is partitioned into three subsets s_1, s_3, and s_5, one or two of them could possibly be empty. The n_1 terminals of set s_1 are connected to terminals on the bottom half of the right segment of the junction area, as shown in Figure (5), and the n_3 terminals of set s_3 are connected to equal number of terminals of the bottom shore of the Right channel, and the rest n_5 terminals of s_5 are connected to a similar set of terminals of the bottom shore of the Left channel. Similarly, L_B, the set of $d_1/2$ terminals of the top shore of the Left channel is partitioned into three subsets s_2, s_4, and s_5. The n_2 terminals of s_2 are connected to terminals on the top half of the right segment of the junction area, n_4 terminals of s_4 are connected to terminals on the top shore of the Right channel, and finally the n_5 terminals of s_5 are connected to terminals of the similar set on the top shore of the Left channel.



$$n_{3'} = n_3 = |s_3| = |s_{3'}| = min\left(max\left(0, \frac{d_1}{2} - \frac{t_1 - y_0}{2}\right), max\left(0, \frac{d_2}{2} - \frac{t_2 - y_0}{2}\right)\right)$$

$$n_4 = |s_4| = max\left(0, max\left(0, \frac{d_1}{2} - \frac{t_1 - y_0}{2}\right) - max\left(0, \frac{d_2}{2} - \frac{t_2 - y_0}{2}\right)\right)$$

$$n_{4'} = |s_{4'}| = max\left(0, max\left(0, \frac{d_2}{2} - \frac{t_2 - y_0}{2}\right) - max\left(0, \frac{d_1}{2} - \frac{t_1 - y_0}{2}\right)\right)$$

Figure 5. Lower Bound example for an S-junction

All the d_1 terminals of L_T and L_B are also connected to other terminals in the Left channel, which results in a channel density of d_1 in the Left channel. The sets of $d_2/2$ terminals of the top and bottom shore of the Right channel, R_T and R_B, respectively, are partitioned into subsets $s_{1'}$, $s_{2'}$, $s_{3'}$, $s_{4'}$ and $s_{5'}$ and connected in a similar fashion, where $|s_3| = |s_{3'}|$ and $|s_4| = |s_{4'}|$. Moreover, x_0 terminals on the top shore (or set s_6) of the junction area are connected to x_0 terminals on its bottom shore.

It is easy to check that the arrangement of nets in our routing problem does not violate the junction density requirements, i.e.,

$$|s_3| + |s_4| + |s_6| \le x_0 + y_0.$$

The cardinality of the various sets s_i is decided by the sequence in which terminals are connected. First of all, we connect as many of the $d_1/2$ terminals of the top shore of the Left channel with the bottom $(t_1 - y_0)/2$ locations on the right segment of the junction area, thus $n_1 = \min(d_1/2\ , (t_1 - y_0)/2)$. Then, we connect an equal number of terminals from the bottom shore to the top half of the right segment. Similar connections are made in the Right channel, thus $n_{1'} = n_{2'} = \min(d_2/2\ , (t_2 - y_0)/2)$. In general, there will be some leftover terminals in both channels. We try to use as many of them as possible (i.e., until there are no more leftover in one of the channels) by connecting terminals of the top shore of one channel to terminals on the bottom shore of the other channel. Thus, $n_3 = n_4 = \min(\max(0\ , d_1/2 - (t_1 - y_0)/2)\ , \max(0\ , d_2/2 - (t_2 - y_0)/2)) = n_{3'} = n_{4'}$. And finally, the remaining $n_5 = \max(0\ , \max(0\ , d_1/2 - (t_1 - y_0)/2) - \max(0\ , d_2/2 - (t_2 - y_0)/2))$ terminals of the top shore of the Left channel are connected to equal number of terminals of its bottom shore. Similarly, $n_{5'} = \max(0\ , d_2/2 - (t_2 - y_0)/2) - \max(0\ , d_1/2 - (t_1 - y_0)/2))$ terminals of the top shore of the Right channel are connected with equal number of terminals on its bottom shore.

It is easy to check that

$$n_1 + n_3 + n_5 = \frac{d_1}{2} = n_2 + n_4 + n_5$$

$$n_{1'} + n_{3'} + n_{5'} = \frac{d_2}{2} = n_{2'} + n_{4'} + n_{5'}$$

$$n_{1'} + n_{4'} + n_{5'} = \frac{d_2}{2}.$$

For any feasible routing of the problem given in Figure (5), we require that the capacity of any set of cuts is at least as big as the number of nets required to cross it. In the following we consider three cuts H_1, H_2 and H_3 as indicated in the Figure (5).

For cut H_1, we have

$$\kappa(H_1) = t_1 + (\frac{d_1}{2} + x_0 + \frac{d_2}{2}) + t_2$$

$$\delta(H_1) = d_1 + \frac{d_1}{2} + x_0 + \frac{d_2}{2} + d_2.$$

Combining these equations, we get

$$t_1 + t_2 \geq d_1 + d_2. \tag{11}$$

For cut H_2, we have

$$\kappa(H_2) = t_1 + (\frac{d_1}{2} + x_0)$$

$$\delta(H_2) = d_1 + \frac{d_1}{2} + x_0 + \min(\frac{d_1}{2}, \frac{t_1 - y_0}{2}).$$

Combining these equations, we get

$$t_1 \geq d_1 + \min(\frac{d_1}{2}, \frac{t_1 - y_0}{2})$$

$$\text{or } t_1 \geq \min(\frac{3}{2}d_1, 2d_1 - y_0). \tag{12}$$

For cut H_3, we have

$$\kappa(H_3) = t_2 + (\frac{d_2}{2} + x_0)$$

$$\delta(H_3) = d_2 + \frac{d_2}{2} + x_0 + \min(\frac{d_2}{2}, \frac{t_2 - y_0}{2}).$$

Combining these equations, we get

$$t_2 \geq d_2 + \min(\frac{d_2}{2}, \frac{t_2 - y_0}{2})$$

$$\text{or } t_2 \geq \min(\frac{3}{2}d_2, 2d_2 - y_0). \tag{13}$$

Inequality of Equation of (11) gives only a trivial bound, but by combining Equations (12) and (13), we get the following non-trivial bound

$$t_1 + t_2 \geq \max(\min(\frac{3}{2}d_1 , 2d_1 - y_0) , \min(\frac{3}{2}d_2 , 2d_2 - y_0)). \qquad (14)$$

For example, if $y_0 \leq \min(\frac{d_1}{2} , \frac{d_2}{2})$ then

$$t_1 + t_2 \geq \frac{3}{2}(d_1 + d_2).$$

It is easy to check that the arrangement of Figure (5) can also be used for the second type of S-junction (i.e., "containing" type) shown in Figure 4(b). Thus we can write the following theorem.

Theorem 2: The existential lower bound for any general S-junction is $t_1 + t_2 \geq \frac{3}{2}(d_1 + d_2)$.

As noted previously, a three-sided channel is a special case of an S-junction. We use the terminal arrangement as shown in Figure (6) for proving identical lower bounds for the three-sided problem. This arrangement is similar to the one in Figure (5), except that it is restricted to one channel. The $d_1/2$ terminals of L_T and L_R are partitioned into subsets s_1, s_2, s_3, and s_4 where

$$|s_1| = |s_3| = \min(\frac{d_1}{2}, \frac{t_1 - y_0}{2}),$$

$$|s_2| = |s_4| = \max(0 , \frac{d_1 - t_1 + y_0}{2}).$$

In addition x_0 terminals on the top shore of the junction area are connected to x_0 terminals on its bottom

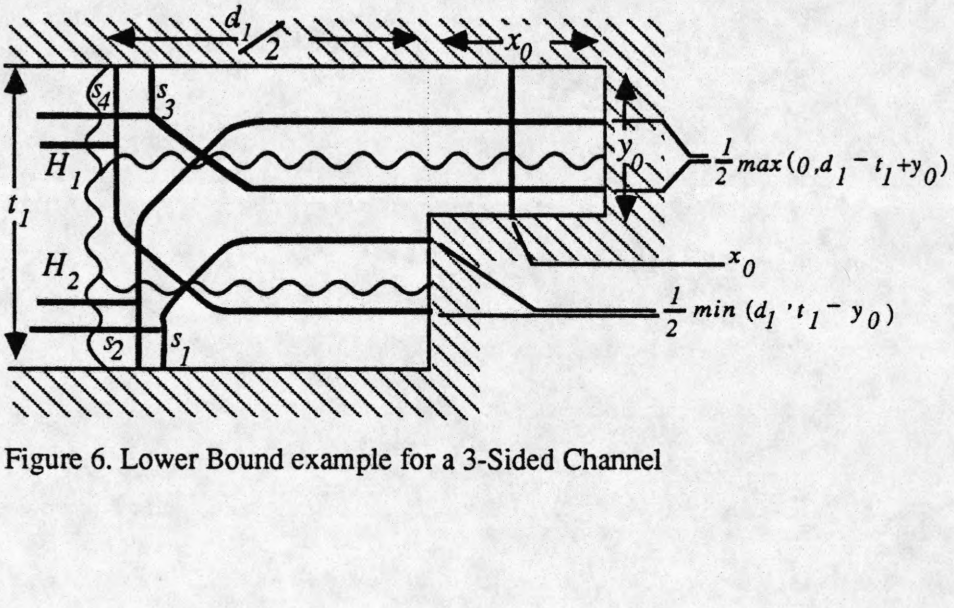


Figure 6. Lower Bound example for a 3-Sided Channel

shore. In the following we consider two cuts H_1, and H_2 as shown in Figure (6).

For cut H_1, we have

$$\kappa(H_1) = t_1 + (\frac{d_1}{2} + x_0)$$

$$\delta(H_1) = d_1 + \frac{d_1}{2} + x_0 + \max(0\,,\, \frac{d_1 - t_1 + y_0}{2}).$$

Combining these equations, we get

$$t_1 \geq d_1 + \max(0\,,\, \frac{d_1 - t_1 + y_0}{2}). \tag{15}$$

For cut H_2, we have

$$\kappa(H_2) = t_1 + \frac{d_1}{2}$$

$$\delta(H_2) = d_1 + \frac{d_1}{2} + \min(\frac{d_1}{2}\,,\, \frac{t_1 - y_0}{2}).$$

Combining these equations, we get

$$t_1 \geq d_1 + \min(\frac{d_1}{2}\,,\, \frac{t_1 - y_0}{2}). \tag{16}$$

For $y_0 \leq t_1 - d_1$, the lower bounds of Equations (15) and (16) are: $t_1 \geq d_1$ and $t_1 \geq \frac{3}{2}d_1$. This arrangement of terminals will be used in constructing the terminal arrangement for the T-junction. Thus, we can write the following theorem.

Theorem 3: The existential lower bound for any general three-sided channel is $t \geq \frac{3}{2}d$.

This lower bound matches the lower bound proved in [31] for a "simple" three-sided channel.

3.3. Lower Bounds for T-Junctions

We denote the densities of the Left, Right, and Top channels of a T-junction by d_1, d_2 and d_3, and their widths by t_1, t_2 and t_3, respectively. The junction area is described by the union of two rectangles. The shape parameters of the junction are denoted by x_0 and y_0. We assume without loss of generality, that $t_1 \geq d_1$, $t_2 \geq d_2$, and $t_3 \geq d_3$, which follows from the trivial lower bounds.

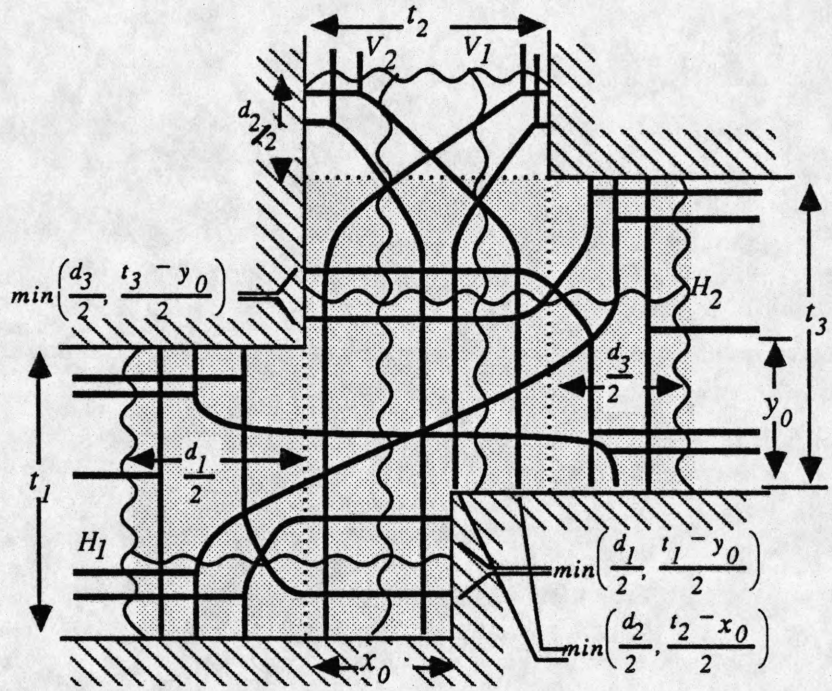


Figure 7. Lower Bound example for T-junction

Consider the routing problem with terminal arrangement as shown in Figure (7). The sets of $d_1/2$ terminals on the top and bottom shores of the Left channel, and the sets of $d_3/2$ terminals of the top and bottom shores of the Right channel are connected to each other in a fashion similar to the arrangement shown in Figure (5) for the S-junction. The sets of $d_2/2$ terminals on the left and right shores of the Top channel are connected to bottom segments of the junction area in a fashion similar to the arrangement shown in Figure (6), for the three-sided channel. We choose four different cuts H_1, H_2, V_1, and V_2, as shown in Figure (7).

For cut H_1, we have

$$\kappa(H_1) = t_1 + (\frac{d_1}{2} + x_0)$$

$$\delta(H_1) = d_1 + \frac{d_1}{2} + \max(0, d_2 - t_2 + x_0) + \min(\frac{d_1}{2}, \frac{t_1 - y_0}{2}).$$

Combining these equations, we get

$$t_1 \geq \frac{3}{2}d_1 + \max(0\,,\, d_2 - t_2 + x_0) + \min(0,\, \frac{t_1 - d_1 - y_0}{2}). \tag{17}$$

For cut H_2, we have

$$\kappa(H_2) = t_3 + (\frac{d_3}{2} + t_2)$$

$$\delta(H_2) = d_3 + \frac{d_3}{2} + d_2 + \min(\frac{d_3}{2}\,,\, \frac{t_1 - y_0}{2}).$$

Combining these equations, we get

$$t_2 + t_3 \geq d_2 + \frac{3}{2}d_3 + \min(0,\, \frac{t_1 - d_3 - y_0}{2}). \tag{18}$$

For cut V_1, we have

$$\kappa(V_1) = t_2 + (\frac{d_2}{2} + t_3)$$

$$\delta(V_1) = d_2 + \frac{d_2}{2} + \min(\frac{d_2}{2}\,,\, \frac{t_2 - x_0}{2}) + \min(d_3,\, t_3 - y_0)$$

$$+ \min(\max(0,\, d_1 - t_1 + y_0),\, \max(0,\, d_3 - t_3 + y_0)).$$

Combining these equations, we get

$$t_2 + t_3 \geq \frac{3}{2}d_2 + d_3 + \min(0,\, \frac{t_2 - d_2 - x_0}{2}) + \min(0,\, t_3 - d_3 - y_0) \tag{19}$$

$$+ \min(\max(0,\, d_1 - t_1 + y_0),\, \max(0,\, d_3 - t_3 + y_0)).$$

For cut V_2, we have

$$\kappa(V_2) = t_2 + (\frac{d_2}{2} + t_3 + t_1 - y_0)$$

$$\delta(V_2) = d_2 + \frac{d_2}{2} + \max(0,\, \frac{d_2 - t_2 + x_0}{2}) + \min(d_1\,,\, t_1 - y_0) + \min(d_3\,,\, t_3 - y_0)$$

$$+ \min(\max(0,\, d_1 - t_1 + y_0),\, \max(0,\, d_3 - t_3 + y_0)).$$

Combining these equations, we get

$$t_1 + t_2 + t_3 \geq d_1 + d_2 + d_3 + y_0 + \min(0, t_1 - d_1 - y_0) + \max(0\,, \frac{d_2 - t_2 + x_0}{2}) + \min(0, t_3 - d_3 - y_0)$$

$$+ \min(\max(0, d_1 - t_1 + y_0), \max(0, d_3 - t_3 + y_0)). \qquad (20)$$

If $y_0 \leq \min(t_1 - d_1, t_3 - d_3)$, and $x_0 \leq t_2 - d_2$, the lower bounds of Equations (17) thru (20) are as follows:

$$t_1 \geq \frac{3}{2} d_1$$

$$t_2 + t_3 \geq d_2 + \frac{3}{2} d_3$$

$$t_2 + t_3 \geq \frac{3}{2} d_2 + d_3$$

$$t_1 + t_2 + t_3 \geq d_1 + d_2 + d_3 + y_0.$$

These equations can be summarized as follows

$$t_1 + t_2 + t_3 \geq d_1 + d_2 + d_3 + \max(\frac{d_1 + d_2}{2}, \frac{d_1 + d_3}{2}, y_0).$$

The construction for the other form of T-junction (i.e., "containing" type) is identical and all the bounds would be similar. Thus, we can write the following theorem.

Theorem 4: The existential lower bound for any general T-junction is $t_1 + t_2 + t_3 \geq \frac{3}{2}(d_1 + d_2) + d_3$.

3.4. Lower Bounds for X-Junctions

We denote the densities of the four channels (in a clockwise sequence, starting at the Left channel) by d_1, d_2, d_3, and d_4, respectively, and their channel widths by t_1, t_2, t_3, and t_4 respectively. As before, the shape (offset) parameters of the X-junction are x_0 and y_0. In all the following we assume that $t_1 \geq d_1$, $t_2 \geq d_2$, $t_3 \geq d_3$, and $t_4 \geq d_4$ which follows from the trivial lower bounds on channel widths. Without loss of generality, we assume $d_1 \geq d_3$ and $d_2 \geq d_4$.

Consider the routing problem with terminal arrangement as shown in Figure (8). The sets of $d_1/2$ terminals on the top and bottom shores of the Left channel, and the sets of $d_3/2$ terminals of the top and bottom shores of the Right channel are connected to each other in a fashion similar to the arrangement

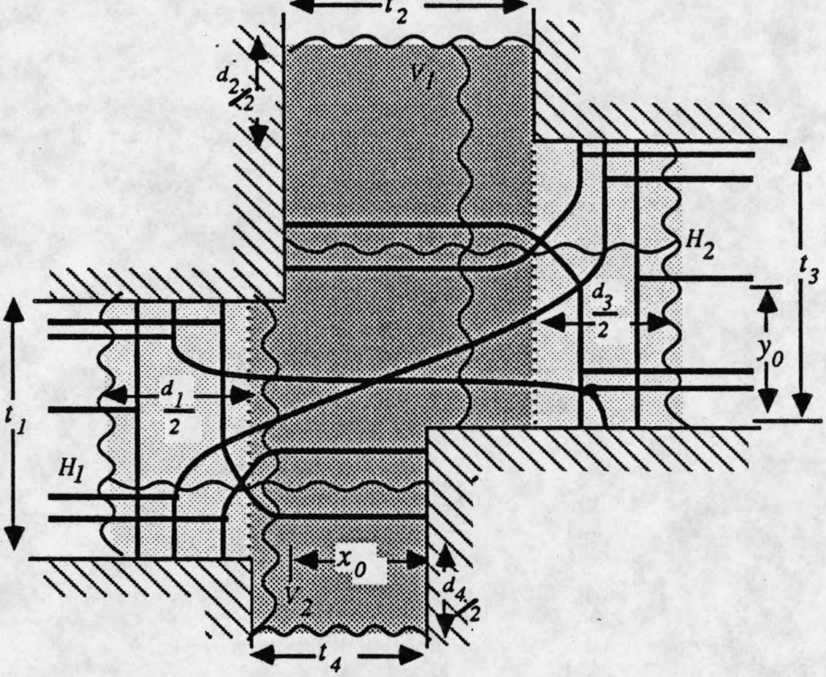


Figure 8. Lower Bound example for X-junction

shown in Figure (5). Also, the sets of $d_2/2$ terminals on the left and right shores of the Top channel, and the sets of $d_4/2$ terminals on the left and right shores of the Bottom channels are connected to each other in a similar fashion. We choose four different cross-sectional cuts indicated by H_1, H_2, V_1 and V_2. The capacities of these cuts are equal to their lengths which are represented as $\kappa(H_1)$, $\kappa(H_2)$, $\kappa(V_1)$ and $\kappa(V_2)$, and the densities of these cuts are the number of nets required to cross them which are represented as $\delta(H_1)$, $\delta(H_2)$, $\delta(V_1)$ and $\delta(V_2)$, respectively.

For cut H_1, we have

$$\kappa(H_1) = t_1 + (\frac{d_1}{2} + t_4)$$

$$\delta(H_1) = d_1 + \frac{d_1}{2} + \min(\frac{d_1}{2}, \frac{t_1 - y_0}{2}) + \min(d_4, t_4 - x_0)$$

$$+ \min(\max(0, d_2 - t_2 + x_0), \max(0, d_4 - t_4 + x_0)).$$

Combining these equations, we get

$$t_1 + t_4 \geq \frac{3}{2}d_1 + d_4 + \min(0, \frac{t_1 - d_1 - y_0}{2}) + \min(0, t_4 - d_4 - x_0) \quad (21)$$

$$+ \min(\max(0, d_2 - t_2 + x_0), \max(0, d_4 - t_4 + x_0)).$$

For cut H_2, we have

$$\kappa(H_2) = t_3 + (\frac{d_3}{2} + t_2)$$

$$\delta(H_2) = d_3 + \frac{d_3}{2} + \min(\frac{d_3}{2}, \frac{t_3 - y_0}{2}) + \min(d_2, t_2 - x_0)$$

$$+ \min(\max(0, d_2 - t_2 + x_0), \max(0, d_4 - t_4 + x_0)).$$

Combining these equations, we get

$$t_2 + t_3 \geq d_2 + \frac{3}{2}d_3 + \min(0, \frac{t_3 - d_3 - x_0}{2}) + \min(0, t_2 - d_2 - y_0) \quad (22)$$

$$+ \min(\max(0, d_2 - t_2 + x_0), \max(0, d_4 - t_4 + x_0)).$$

For cut V_1, we have

$$\kappa(V_1) = t_2 + (\frac{d_2}{2} + t_3)$$

$$\delta(V_1) = d_2 + \frac{d_2}{2} + \min(\frac{d_2}{2}, \frac{t_2 - x_0}{2}) + \min(d_3, t_3 - y_0)$$

$$+ \min(\max(0, d_1 - t_1 + y_0), \max(0, d_3 - t_3 + y_0)).$$

Combining these equations, we get

$$t_2 + t_3 \geq \frac{3}{2}d_2 + d_3 + \min(0, \frac{t_2 - d_2 - x_0}{2}) + \min(0, t_3 - d_3 - y_0) \quad (23)$$

$$+ \min(\max(0, d_1 - t_1 + y_0), \max(0, d_3 - t_3 + y_0)).$$

For cut V_2, we have

$$\kappa(V_2) = t_4 + (\frac{d_4}{2} + t_1)$$

$$\delta(V_2) = t_1 + \frac{d_4}{2} + t_4 + \min(\frac{d_4}{2}, \frac{t_4 - x_0}{2}) + \min(d_1, t_1 - y_0)$$

$$+ \min(\max(0, d_1 - t_1 + y_0), \max(0, d_3 - t_3 + y_0)).$$

Combining these equations, we get

$$t_1 + t_4 \geq d_1 + \frac{3}{2}d_4 + \min(0, \frac{t_4 - d_4 - x_0}{2}) + \min(0, t_1 - d_1 - y_0) \tag{24}$$

$$+ \min(\max(0, d_1 - t_1 + y_0), \max(0, d_3 - t_3 + y_0)).$$

Lower Bounds given by Equations (21) thru (24) hold for every choice of x_0 and y_0. However, one of these equations gives the best possible bound for given choice of values for the offset parameters. For example in the case, when $y_0 \leq \min(t_1 - d_1, t_3 - d_3)$ and $x_0 \leq \min(t_2 - d_2, t_4 - d_4)$, Equations (21) thru (24) become as follows:

$$t_1 + t_4 \geq \frac{3}{2}d_1 + d_4$$

$$t_2 + t_3 \geq d_2 + \frac{3}{2}d_3$$

$$t_2 + t_3 \geq \frac{3}{2}d_2 + d_3$$

$$t_1 + t_4 \geq d_1 + \frac{3}{2}d_4.$$

These equations can be summarized as follows:

$$t_1 + t_2 + t_3 + t_4 \geq d_1 + d_2 + d_3 + d_4 + \max(\frac{d_1 + d_2}{2}, \frac{d_1 + d_3}{2}, \frac{d_2 + d_4}{2}, \frac{d_3 + d_4}{2}).$$

It is easy to check that by using cuts H and V (Figure 8), one can show that

$$t_1 + t_2 + t_3 + t_4 \geq d_1 + d_2 + d_3 + d_4 + \max(x_0, y_0).$$

The construction for the other two forms of X-junction is identical and all the bounds would be similar.

Thus we can write the following theorem.

Theorem 5: The existential lower bound for an X-junction is $t_1 + t_2 + t_3 + t_4 \geq \frac{3}{2}(d_1 + d_2) + d_3 + d_4$.

4. Upper bounds for the General Junction Routing Problems

In the previous Sections, we proved lower bounds on the junction size in terms of the channel densities. In this section, we present a set of upper bounds on the junction size for the general junction routing problem. Upper bounds for a given junction routing problem are said to be *matching* the lower bounds if they equal the existential lower bounds for the worst case instance. On the other hand, upper bounds for a given routing problem are said to be *optimal* if they match the universal lower bounds, i.e., an optimal router uses minimum possible area (of the channels and junction region) for every instance. In this section we assume that for the junction routing problem (*JRP*), the widths of all its channels are no less than their corresponding densities, and the junction bottleneck is no less than the junction-area density. For any k-way *JRP*, we will prove $t_i \leq d_i$, $i = 1, \cdots, k$ for two-terminal-net case and $t_i \leq \frac{3}{2} d_i$, $i = 1, \cdots, k$ for the three-terminal net case. Our upper bounds for the case of two-terminal-net *JRP* s are optimal, and the upper bounds for the case of three-terminal nets are matching bounds for "L", and "S"-junction routing problems.

A general junction consists of a junction area (which is the union of up to two rectangles) and associted channels, as defined previously. One approach for routing a general junction is by decomposing it into subproblems of routing separately in the channels and the junction area. The sequence of routing the various subproblems is important. For example, if the channels are routed first then the junction area routing problem becomes a switchbox routing problem. As mentioned before, in general the problem of routing in a switchbox requires excessive area which makes them unfavorable. Our approach is to first route the junction area and then the associated channels. Thus, our junction router is composed of two parts: a *junction area router* and a *channel router*. We develop a junction area router for the general junction areas. After routing the junction area, routing in the associated channels is achieved by the existing three-sided channel router of [22]. We first discuss the case of 2-terminal nets. The case of multi-terminal nets is solved by decomposing multi-terminal nets into two-terminal nets.

In Section 4.1, we present the upper bound for routing a two-terminal-net simple L- and S-*JRP*, where the junction area is a single rectangle. In Section 4.2, we derive upper bounds for the case of *JRP* s with general junction area, which is the union of two rectangles. In Section 4.3, by decomposing three-terminal nets into two-terminal nets, we develop upper bounds for the three-terminal-net general *JRP* s. In Section 5, we summarize the results of this paper and indicate a technique for generalizing our upper bounds for the case of more than three-terminal nets.

4.1. Upper Bound for 2-Terminal Simple Junction Routing Problem

A simple junction routing problem is a *JRP* where the junction area is a single rectangle, whereas a general junction routing problem is a *JRP* where the junction area consists of union of two rectangles. In this section, we develop routers for the two junctions with simple junction areas, i.e., L- and S-junctions, which are then used in the next section to develop routers for junctions with general junction areas.

4.1.1. Simple L-junction Router

The simple L-junction router plays a fundamental role in the proof of the upper bounds for the general *JRP*. As is shown later, a general junction routing problem can be routed by using the simple L-junction router for appropriately chosen parts of the problem.

A routing problem in an L-junction consists of routing in the associated channels and the junction area. In our approach we first route the junction area and then the associated channels, as shown in Figure (9). In the following, we give an optimal junction area router.

While routing the junction-area (of the L-junction of Figure 9), we represent the crossings of nets from the Left and Top channels into the junction area by introducing terminals on its left and top sides. A terminal is introduced on the left boundary of the junction area, if and only if there is some net with one terminal to the left of this boundary and another terminal to the right. Terminals on the top boundary are introduced similarly. These terminals are free to move along the boundary they lie on, since the corresponding channels are routed only after we finish routing the junction area. In the rest of this paper,

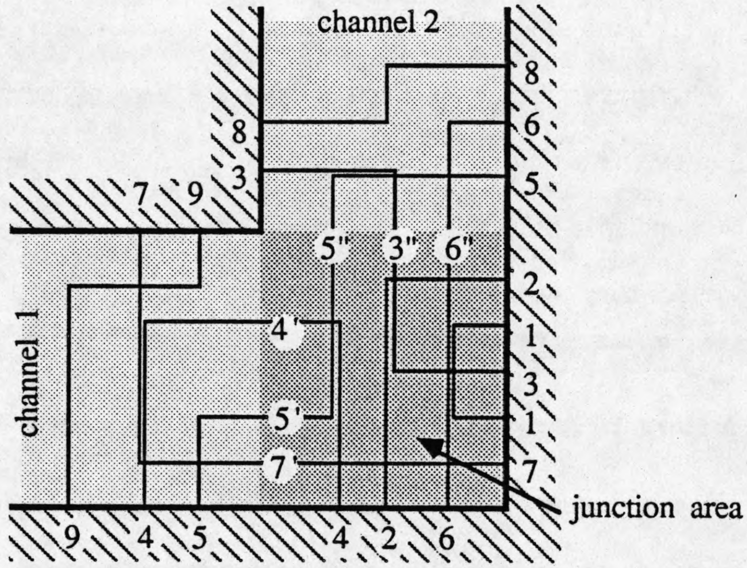


Figure 9. L-junction Routing Problem

we represent the required crossings from one side of the cut to the other side of the cut by *introduced terminals*. These introduced terminals are free or fixed depending on the sequence of routing.

A *simple–junction–area routing problem* (simple-*JaRP*, for short) is specified by a set of nets with terminals on sides *A*, *B*, *C*, and *D* of a rectangular junction area. The terminals on the adjacent sides *A* and *B* are *fixed*, and the terminals on sides *C* and *D* are *free*. Moreover, unlike the conventional switchbox routing problem, the simple *JaRP* permits one to use the channel areas to complete the connections of its nets. These "extended" connections outside the junction area are completed during the routing of the corresponding channels. Figure 10(a) specifies the *JaRP* of the example in Figure (9). Terminals 4′, 5′ and 7′ are free on side *D*, and terminals 3″, 5″ and 6″ are free on side *C*.

Since the position of terminals on sides *C* and *D* is free, different terminal sequences result in different number of nets crossing a (horizontal or vertical) cut, hence the previous definition of local density is not appropriate. We redefine the local density of a (horizontal or vertical) cut in the junction area as the minimum, over all terminal sequences on *C* and *D*, of the number of nets crossing the cut. The difficulty of routing the junction area arises from the fact that some cross sectional cuts are oversaturated, i.e., the

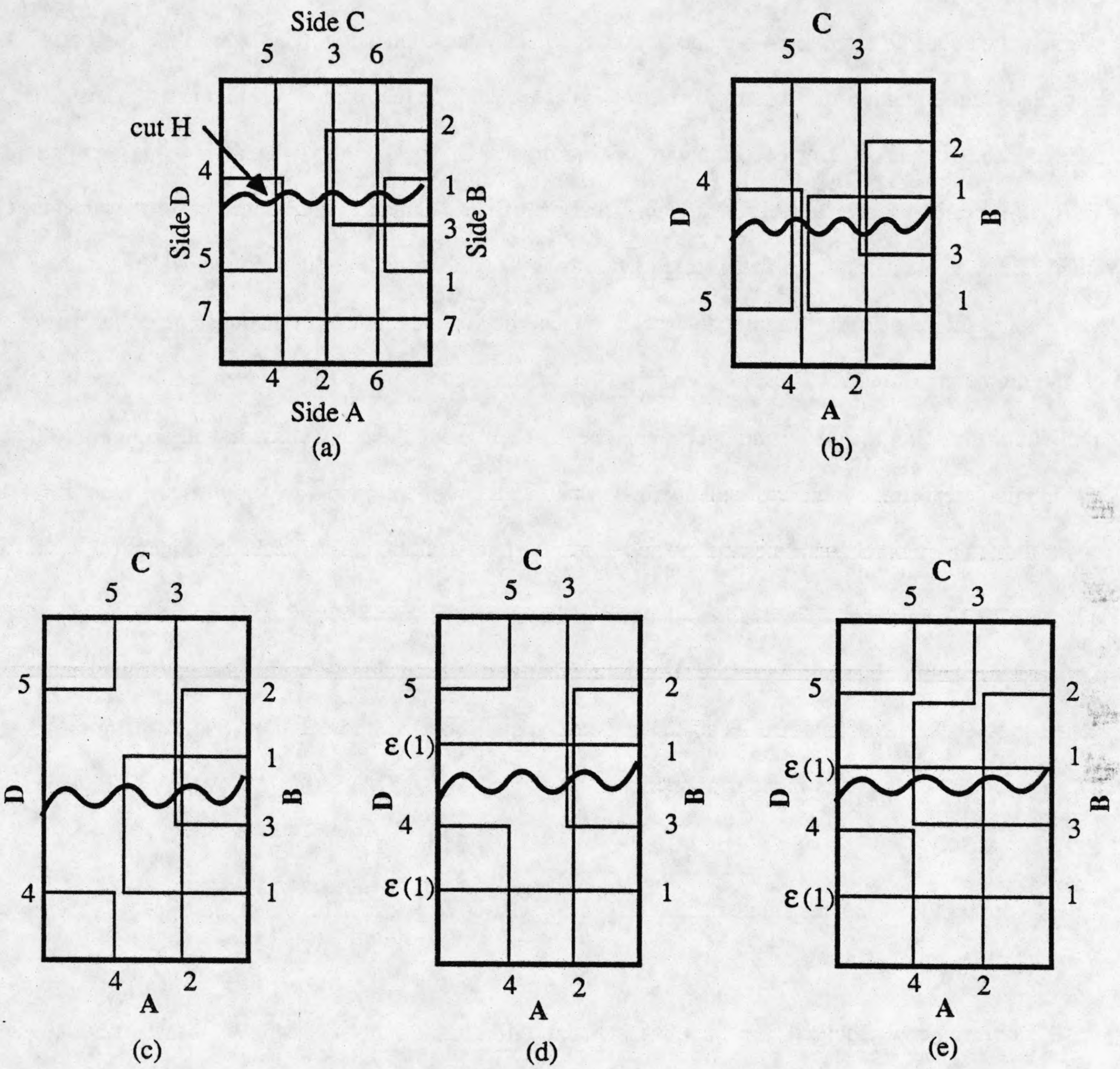


Figure 10. Simple-junction-area Routing

density of the cut exceeds its capacity. For example, the capacity of the indicated horizontal cut H, in Figure 10(a), is 3 and the number of required crossings is 4. The existing switchbox routing algorithm can not be directly used in this situation. Therefore, it is impossible to complete the routing entirely inside the junction area without increasing its area. We resolve this problem by dividing a net crossing an oversaturated cut into two parts which do not cross this cut, and connecting them later in the appropriate channels. Consequently, the density of the cut is decreased. For the example of Figure 10(c), net 1 is divided into two nets by using two empty grid points on side D, specified by two pairs 1-$\varepsilon(1)$ (see Figure 10 (d)). (Terminal label $\varepsilon(i)$ represents the *extension* of net i which is not connected in the junction area.) These two nets are then considered as two "new" two-terminal nets in the junction area routing. Since the positions of both the terminals labeled $\varepsilon(1)$ are free on side D, one of them can be placed above H and the other below H. Thus, the density of H is decreased by one. This process can be repeatedly applied for every cut till there is no oversaturated cut. Then, using the channel routing algorithm of [22], the junction area can be routed without any increase in area. In the following, we only discuss the case of oversaturated horizontal cuts, since the method to resolve oversaturation of vertical cuts is identical.

We call a terminal of a net belonging to side X (where $X \in \{A, B, C, D\}$) as its *x–terminal*. Denote a net with one terminal on side A and the other terminal on side B as an AB-net. Similarly, we define for AC-nets, BC-nets, BD-nets, CD-nets, AD-nets, AA-nets, and BB-nets. (Note: There are no CC-nets or DD-nets.) The number of such nets is denoted as N_{ab}, N_{ac}, N_{bd}, N_{bc}, N_{cd}, N_{ad}, N_{aa}, and N_{bb}, respectively.

Lemma 1: For an oversaturated horizontal cut H, i.e., $\delta(H) > \kappa(H)$, there is at least one AB, BB or BC net crossing it.

Proof: (by contradiction) We claim, if there are no AB, BB, BC nets crossing H, then H can not be oversaturated. The AA-nets do not contribute to $\delta(H)$ since both its terminals lie on one side of the horizontal cut H. Also, the BD-nets do not contribute to $\delta(H)$, since by proper choice of the location of their d-terminals one can avoid the crossings of any BD nets at H. We also ignore the AC-nets while counting oversaturation of H, because each such net adds exactly one to $\delta(H)$ and $\kappa(H)$ (since its a-terminal uses one terminal on side A). Hence for oversaturation, we only consider AD and CD-nets. By arranging the free d-terminals of AD and CD-nets, we ensure that the d-terminals of all the CD-nets lie above the d-

terminals of AD-nets. The d-terminals of CD-nets are arranged consecutively starting at the top track; the d-terminals of AD-nets are arranged consecutively starting at the bottom track (See Figure (11)). Note that such an arrangement ensures that only one kind of nets (either AD or CD) cross H, if any. We assume without loss of generality, that AD-nets cross H. Since, each such net uses one terminal of side A, the total number of AD-nets is less than the length of H. Hence H can not be oversaturated. This proves our claim. □

Lemma 2: For a horizontal cut H with oversaturation $y = \delta(H) - \kappa(H)$, the total number of AB, BB, and BC nets crossing H is at least y.

Proof: This follows trivially from the previous Lemma, because for every unit of oversaturation there is at least one AB, BB or BC net so for y units of oversaturation there are at least y such nets.□

In fact, we can claim a slightly stronger result.

Lemma 3: For a horizontal cut H with oversaturation $y = \delta(H) - \kappa(H)$, the total number of AB and BB nets crossing H is at least y.

Theorem 6: For a horizontal cut H with oversaturation $y = \delta(H) - \kappa(H) > 0$, there are at least $2y$ empty grid points on side D of the simple-junction area.

Proof: We denote the lengths of sides A and B by l_a and l_b. The capacity of a horizontal cut H is $\kappa(H) = l_a$.

Using Lemma 1, for the worst-case of oversaturation, all the AB, BC and the BB-nets of the problem could cross H. We assume without loss of generality, that the d-terminals of CD and AD-nets have been arranged such that, all the CD nets appear "above" the AD-nets, as described in Lemma 1. The distance between cut H and side C is at least $N_{ab} + N_{bb}$, and the distance between cut H and side A is at least $N_{bc} + N_{bb}$. If $N_{cd} \geq N_{ab} + N_{bb}$ then only CD-nets cross H, and if $N_{ad} \geq N_{bc} + N_{bb}$ then only AD-nets cross H, otherwise neither of them cross H. It is easy to check that the arrangement of terminals as shown in Figure (11) has the maximum possible density for the horizontal cut H. We prove our theorem statement by showing that there are at least $2y$ empty grid points on D, for this worst-case example. Let

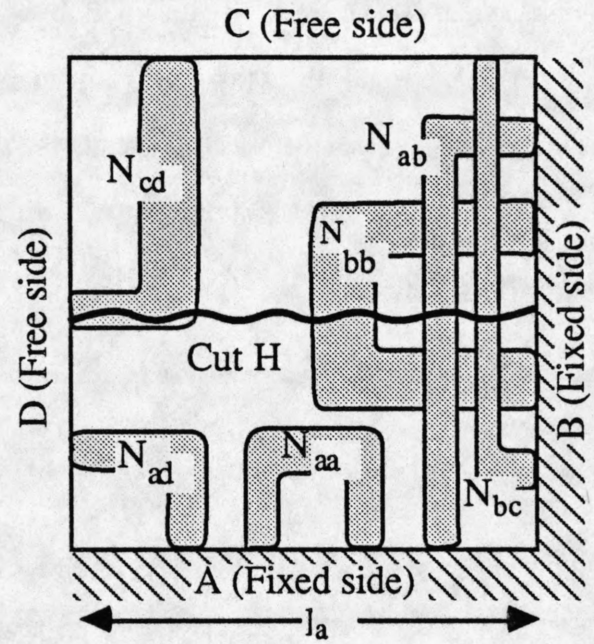


Figure 11. Worst case arrangement with maximum crossings across H

x_d be the number of empty grid points on the side D. We write the following equations by counting the number of terminals on each side of the boundary,

$$N_{ad} + N_{ab} + 2N_{aa} \le l_a \tag{25}$$

$$N_{ab} + N_{bc} + 2N_{bb} \le l_b \tag{26}$$

$$N_{cd} + N_{bc} \le l_c = l_a \tag{27}$$

$$N_{cd} + N_{ad} + x_d = l_d = l_b. \tag{28}$$

From Equation (28), we obtain

$$x_d = l_b - N_{cd} - N_{ad}. \tag{29}$$

The density of H, $\delta(H)$, can be written as follows (refer to Figure 11):

$$\delta(H) \le N_{ab} + N_{bb} + N_{bc} + \max\{(N_{cd} - N_{ab} - N_{bb}), (N_{ad} - N_{bc} - N_{bb}), 0\}. \tag{30}$$

The first three terms of Equation (30), represent the number of AB, BC, and BB-nets crossing H and the last term represents the three different cases where (1) only CD-nets cross H, (2) only AD-nets cross H. and (3) no AD or CD-nets cross H. In the following we consider these three cases separately.

Case 1: (CD-nets crossing H)

$$\text{i.e., } (N_{cd} - N_{ab} - N_{bb}) \geq (N_{ad} - N_{bb} - N_{bc})$$

$$\text{and } (N_{cd} - N_{ab} - N_{bb}) \geq 0.$$

$$\delta(H) \leq N_{ab} + N_{bb} + N_{bc} + N_{cd} - N_{ab} - N_{bb}.$$

$\delta(H) \leq N_{bc} + N_{cd} \leq l_a$. Hence, there is no oversaturation in this case.

Case 2: (AD-nets crossing H)

$$\text{i.e., } (N_{ad} - N_{bc} - N_{bb}) \geq (N_{cd} - N_{ab} - N_{bb})$$

$$\text{and } (N_{ad} - N_{bc} - N_{bb}) \geq 0.$$

$$\delta(H) \leq N_{ab} + N_{bb} + N_{bc} + N_{ad} - N_{bc} - N_{bb}.$$

$\delta(H) \leq N_{ab} + N_{ad} \leq l_a$. Hence, there is no oversaturation in this case.

Case 3: (no AD or CD-nets crossing H)

$$\text{i.e., } (N_{cd} - N_{ab} - N_{bb}) \leq 0$$

$$\text{and } (N_{ad} - N_{bb} - N_{bc}) \leq 0.$$

Then, $\delta(H) \leq N_{ab} + N_{bb} + N_{bc}.$ Hence, there may be oversaturation in this case. (31)

The amount of oversaturation is $y = \delta(H) - l_a$.

$$y \leq N_{ab} + N_{bb} + N_{bc} - l_a. \quad (32)$$

Using Equations (29) and (32), we obtain

$$x_d - 2y \geq l_b - N_{cd} - N_{ad} - 2(N_{ab} + N_{bb} + N_{bc} - l_a)$$

$$x_d - 2y \geq (l_b - N_{ab} - N_{bc} - 2N_{bb}) + (l_a - N_{ab} - N_{ad}) - (l_a - N_{bc} - N_{cd}).$$

From Equations (25) and (27), we get $l_a \geq \max\{(N_{ab} + N_{ad}),(N_{bc} + N_{cd})\}$, and from Equation (30), we get $l_b \geq N_{ab} + N_{bc} + 2N_{bb}$. Hence, $x_d - 2y \geq 0$ or $x_d \geq 2y$. This completes the proof of the theorem. □

Using Lemmas 1, 2 and 6, and Theorem 6, we can formulate the following algorithm for resolving oversaturation of all the horizontal cuts of a simple junction area.

Step 1:

Arrange the c and d-terminals of AC and BD-nets on the free sides C and D so these nets can be connected by straight wires. Then, remove all AC and BD-nets, and the corresponding tracks and columns.

Step 2:

Arrange the d-terminals of all AD and CD-nets, such that the d-terminals of CD-nets are above the d-terminals of AD-nets. Place the d-terminals of CD-nets consecutively starting at the top track. Place the d-terminals of AD-nets consecutively starting at the bottom track.

Step 3:

Calculate the density of each horizontal cut. If there is no oversaturated horizontal cut, then go to Step 4. Else, choose a cut H, such that $\kappa(H) < \delta(H)$. Let $y = \delta(H) - \kappa(H)$ be the amount of oversaturation of H. Find $2y$ empty grid points on side D (follows from Theorem 6) and choose y AB, BB or BC-nets which cross H. Replace each of the chosen AB-net by an AD – and a BD-net, each BB-nets by two BD-nets, each BC-nets by a BD- and a CD-net. Go to Step 1.

Step 4:

Since there are no oversaturated cuts in the junction area, route the junction area by using the three-sided router [6] of [22], where terminals of sides A ,B and D are considered fixed.

Step 5:

Finally, combine the routing of Step 4 with the straight wire connections of nets removed in Step 1. Stop.

The example of Figures 10 (a) thru (e) illustrate the above algorithm. By Step 1, c-terminal of net 6 and d-terminal of net 7 are aligned as shown in Figure 10(a). Then, they are routed by straight wires. After which we remove these nets and the corresponding track and column (Figure 10(b)). Using Step 2, d-terminals of nets 4 and 5 are rearranged (Figure 10(c)). The cut H is an oversaturated cut. Next, by Step 3 we resolve the oversaturation of H. Net 1 crossing H is replaced by two BD-nets connecting each of its b-terminals to one empty grid points on D (Figure 10(d)). In the remaining problem there are no

[6] The channel router of [22] can be easily modified to give an optimal router for the three-sided channel.

more oversaturated cuts and hence it can be routed without using any extra columns or tracks (Figure 10(e)).

After finishing the junction area routing terminals on sides C and D are fixed, so we need to route two three-sided channels (Left and Top), which can be achieved optimally using [22]. We can set $l_a = d_2$ and $l_b = d_1$. Then, the time complexity of the above algorithm can be related to the associated channel densities.

Theorem 7: The above junction area routing algorithm is guaranteed to terminate with no oversaturated cuts. The time complexity of this algorithm is $O(d_1 + d_2)$.

Proof: Each time when the algorithm reduces the oversaturation of a cut by one, it removes at least one net from the junction area. Since the total number of nets is $\leq d_1 + d_2$, the algorithm goes through Step 1 thru Step 3 at most $d_1 + d_2$ times. The initial arrangement of AD, CD, BC-nets on sides C and D requires $O(d_1 + d_2)$ time. The density of all the cuts can also be computed in the same time. After which removal of each oversaturating net takes $O(1)$ time. Finally, Step 4 requires $O(d_1)$ time to route the three-sided channel using [22]. Hence, the complexity of the whole algorithm is $O(d_1 + d_2)$. □

The channel router of [22] in the worst case requires $O(n)$ time, where n is the number of columns of the channel. Hence, the time complexity of our simple L-junction router for two-terminal nets is $O(d_1 + d_2) + n_1 + n_2$, where n_1 and n_2 are the numbers of the columns in the Left and Top channels. Then, we can write the following theorem.

Theorem 8: A two-terminal nets can be optimally routed in a simple L-junction with $t_1 = d_1$ and $t_2 = d_2$. The time complexity of the algorithm is $O(d_1 + d_2) + n_1 + n_2$.

4.1.2. S-junction Router

In this subsection, we discuss the problem of routing in an S-junction, which is another one of the fundamental problems. As discussed in the next section, this router can be used to design routers for general T- and X-junctions. A routing problem in an S-junction consists of routing in the associated channels and the junction area. Similar to the case of L-junction, we first route the junction area and then the associated channels (see Figure (12)). In the following we give an optimal router for the junction area of an S-junction.

As discussed before, we assume that the junction density D_j is no more than the bottleneck of the junction. Moreover, the density of any vertical cut in the junction area, called V_j, is no more than the height $h_j = t_1 + t_2 - y_0$ of the junction area rectangle. We discuss the problem of routing for both configurations of the S-junctions, namely the "containing" and the "non-containing" type (Figure 2(b)). For the "containing" type of S-junction of Figure (12) the routing of nets is carried out as follows:

Step 1:

Choose an arbitrary sequence of the introduced terminals on the right opening of the junction area. Route the junction rectangle J using the three-sided channel router of [22] . This determines a sequence of introduced terminals on the left opening of J.

Step 2:

Use the three-sided channel router for routing the Left and Right channels of the junction.

The initial choice of the sequence of terminals in Step 1 does not change the densities of the associated channels and the junction area. Unfortunately, this scheme can not be used in the case of the "non-containing" configuration of the S-junction. This is because, by an inappropriate choice of the sequence

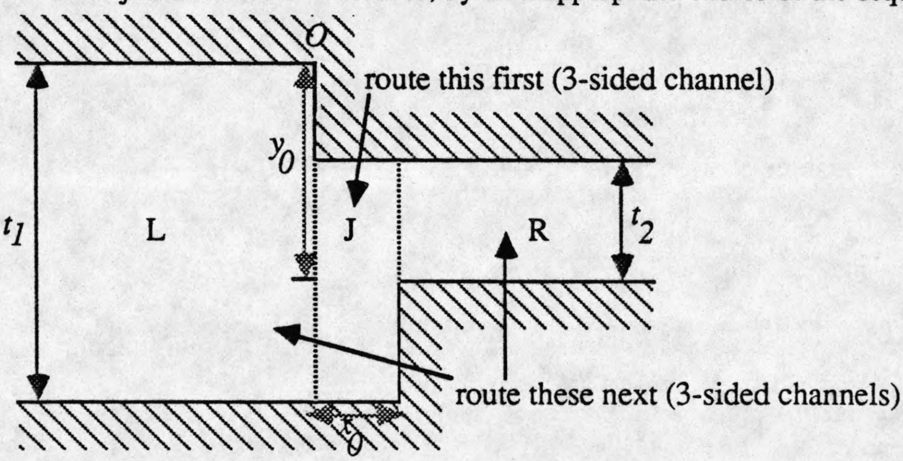


Figure 12. Routing a 'Containing' S-junction

of terminals on the right opening, we may not be able to route the junction area due to the fixed sequence on its left segment. Also, some of the nets connecting terminals of the junction area may need to be extended and connected in the Right channel. Therefore, the crossing sequence at the right (or the left) opening can not be arbitrarily fixed. In the following, we devise a different scheme to route in the "non-containing" type of S-junction.

Here again we first router the junction area J and then the associated channels. If there are no over-saturated horizontal or vertical cuts in the junction area, it can be routed by using a switchbox router. In our problem, there are no oversaturated vertical cuts, however, there might be some oversaturated horizontal cuts. Therefore, some of the nets of J have to be extended beyond its left and right openings and to be connected in the associated channels. The choice of extended nets and the proper sequence of terminals at the left and right openings is found as follows: (Figure 13)

(a) remove the oversaturation of all the horizontal cuts by expanding the switchbox horizontally (by adding extra columns adjacent to J in the channel area);

(b) route the expanded switchbox using the existing algorithms;

(c) retain the routing of the junction area and discard the routing in the extended area (including the routing in the added columns).

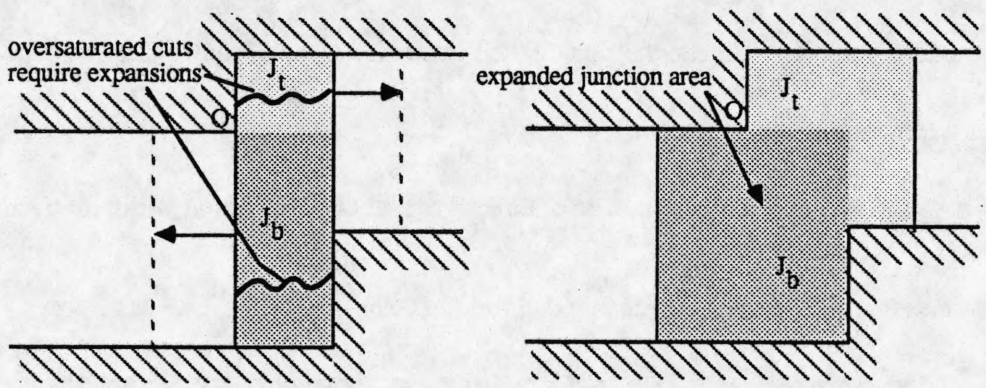


Figure 13. Routing a 'Non-Containing' S-junction

The "non-containing" type S-junction can be routed using the following algorithm:

Step 1:

Choose arbitrary sequences of the introduced terminals on the left and right openings of J. Divide J into two parts J_t and J_b, where J_t is the region above the origin, and J_b is the region below the origin. Calculate the horizontal densities V_t and V_b in regions J_t and J_b (i.e., maximum over all the horizontal cuts in that region), respectively.

Step 2:

If V_t and V_b are no bigger than x_0 (capacity of any horizontal cut), go to Step 3. Otherwise, if V_t is bigger than x_0, then introduce $x_t = V_t - x_0$ columns on the right side of J in the Right channel. Shift the terminals on the right opening of J to the right side of the extension part (keeping the same sequence). Similarly, if V_b is bigger than x_0 then extend J in the left direction.

Step 3:

Route the modified junction area using the existing switchbox router of [15].

Step 4:

Remove the routing in the extension regions.

Step 5:

Route the Left and the Right channels using the three-sided channel router [22].

Thus, we can state the following theorem.

Theorem 9: A two-terminal nets can be optimally routed in a S-junction with $t_1 = d_1$ and $t_2 = d_2$.

4.2. Upper Bounds for 2-Terminal General Junction Routing Problem

In this section, we route general junctions by using the above developed simple L-junction and S-junction routers. The strategy used here is the same as in the previous section; first route the junction area and then the associated channels. The most general kind of junction can be routed by a straight forward decomposition of the junction into sub-regions, which are simple L- and S-junctions, and three-sided channels. According to the definition, the general junction area is an union of two rectangles J_1 and J_2. If one can route this junction area without using any extra area, by connecting the nets which cause the problem of oversaturation in the channel area (as we did for simple L-*JRP*), then one can route the general junction optimally. The most general junction defined in this paper is the X-junction, and L-, and T-junction can be considered as special cases of the X-junction, so we illustrate our approach by discussing upper bounds only for X-junctions.

For a general X-junction shown in Figure (14), we call the overlap region of J_1 and J_2 as $\bar{J}$, where $\bar{J} = \{(x, y): 0 \le x \le x_0, -y_0 \le y \le 0\}$. The junction area $J_1 \cup J_2$ routing can be done by routing the two

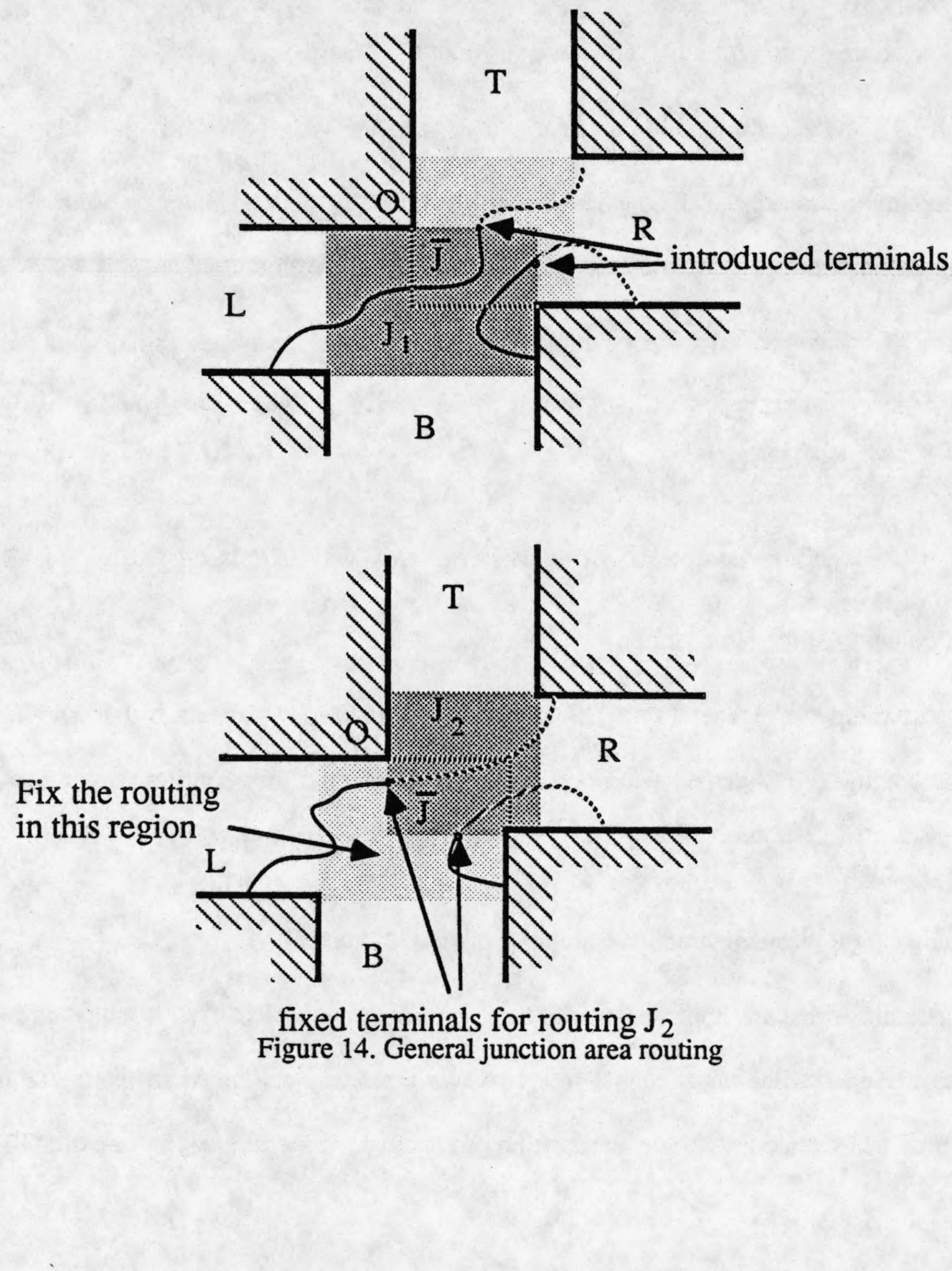


Figure 14. General junction area routing

junction area rectangles J_1 and J_2. To route J_1, we introduce terminals on the top and right sides of $\bar{J}$ representing the connections from region $L \cup J_1 \cup B$ to region $T \cup J_2 \cup R$. According to our assumptions about the universal lower bounds, the number of nets crossing these two sides is no more than the capacity of these two sides. We can always choose a sequence of these introduced terminals. By considering the introduced terminals as fixed terminals, the routing in J_1 is a simple-*JaRP*, and can be solved by using the previous algorithm. To route J_2, we erase the routing of nets inside region $\bar{J}$, but fix the crossing sequence at the left and bottom sides of $\bar{J}$ which is determined during the routing of J_1. The introduced terminals on the top and right sides of $\bar{J}$ are now erased. Finally, routing in J_2 is another simple-*JaRP* and can be completed optimally. This gives us the following theorem.

Theorem 10 : The general junction area for two-terminal nets can be routed optimally.

After finishing the routing of the general junction area, the associated channels are routed optimally using the channel router of [22]. We conclude this section with the following theorems.

Theorem 11 : The general k-way junction routing problem for two-terminal nets can be routed optimally, i.e., $t_i = d_i$, $i = 1, \cdots, k$, where t_i and d_i are respectively the width and density of the ith channel.

Theorem 12 : The time complexity of the router of an k-way *JRP* is $O(\sum_{i=1}^{k} d_i + \sum_{i=1}^{k} n_i)$, where n_i is the number of columns of the ith channel.

We can also route more general types of "T"- and "X"-junction, i.e., when $x_0 \geq t_2$ and $y_0 \geq t_1$. This is done by first routing the S-shaped region of the junction area and then routing the remaining parts as L-junctions or three-sided channels, as illustrated from the example in Figure (15).

4.3. Upper Bounds for 3-terminal Junction Routing Problems

In this section we discuss upper bounds of three-terminal nets JRPs. This is achieved by appropriately decomposing each three-terminal net into two two-terminal nets and then using the routers discussed in the previous sections. The decomposition of nets is done by duplicating one of their terminals.

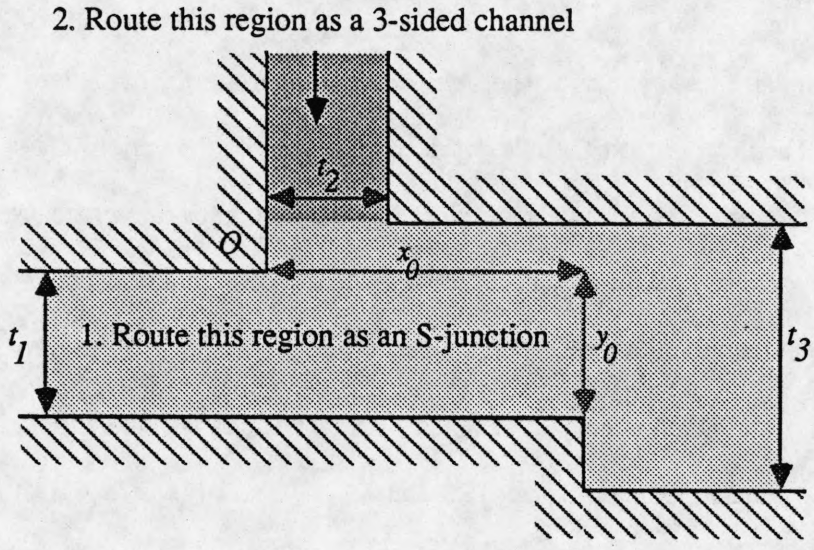


Figure 15. General junction area decomposition

The duplicated terminal is placed in a new track or column which is inserted besides the terminal that is duplicated, and both these terminals are interconnected. Thus, during the decomposition procedure the number of tracks and columns increases. As we will show next, by a proper choice of decomposition of nets this increase in the size of the junction area can be bounded. In fact, the required increase in the junction area is less than the required increases in the channels by the best known channel routers. In other words, the existing channel routers are the "bottleneck" in our routing algorithm. Our upper bounds match the existential lower bounds in the case of L-junction.

Once the decomposition procedure transforms all three-terminal nets into two-terminal nets, the transformed two-terminal nets junction area routing problem is carried out by the algorithms of the previous section. We only discuss the decomposition in the cases of simple L-*JaRP* and S-*JRP*, since the general junction area routing can be achieved by combining these routers, as discussed earlier. For the three-sided channel routing problem of three-terminal nets, we still use the channel router of [22] which uses $\frac{3d}{2}$ tracks in the worst case (which is a matched bound) for a routing problem with density d. It therefore suffices to show that the increase in the width and the height of the junction area are bound by half of their original values. This proves that $\sum_{i=1}^{k} t_i \le \frac{3}{2} \sum_{i=1}^{k} d_i$ for a three-terminal-net k-way *JRP*.

In the following we first classify nets according to their terminal positions on the sides of a simple junction area. Then, we present the rules of decomposition for the various classes of nets. By counting the number of the duplicated terminals introduced by the decomposition procedure on each side, an upper bound for the simple simple *JaRP* is obtained. Finally, by combining the upper bounds for channel routing and the junction area routing, we obtain the result for the general *JRP*.

The sides of the junction area are denoted by A, B, C and D, where the terminals of A and B are fixed and the terminals of C and D are free. Let X_1, X_2 and X_3 (where $X_i \in \{A, B, C, D\}$, i=1,2,3) be the positions of the three terminals of a net N. Then, we call net N an $X_1 X_2 X_3$-net. For example, an *AAB*-net has two of its terminals on side A and another terminal on side B. It is easy to check that there are a total of nineteen different types of nets in a simple junction area routing problem, according to their terminal positions. These nets are classified in four sets.

Set I: *AA*−, *AC*−, *AD*−, *BB*−, *BC*−, *BD*−, and *CD*− nets.

Set II: *AAA*−, and *BBB*− nets.

Set III: *AAB*−, *ABB*−, *AAC*−, *BBC*−, *AAD*−, and *BBD*− nets.

Set IV: *ABC*−, *ABD*−, *ACD*−, and *BCD*− nets.

We denote the number of these nets in a given routing problem by N_{aa}, N_{ac}, $\cdots$, N_{bcd}, respectively. As a rule, while decomposing a net, we introduce duplicated terminals only on the sides where the net originally has at least one of its terminals. When we say an $X_1 X_2 X_3$ net is decomposed with a duplicated terminal on side X_i, for i = 1, 2, or 3, we mean that the X_i-terminal of the net is duplicated. In other words, if for an $X_1 X_2 X_3$ net the duplicated terminal is on X_2, it is decomposed into an $X_1 X_2$-net and another $X_2 X_3$ net. In order to duplicate a terminal on a side, say on side A (or side B), we insert a column (resp. a row) and duplicate that terminal thus increasing the width (resp. height) of the simple junction area by one (Figure 16). There are three different ways to decompose an $X_1 X_2 X_3$ net depend-

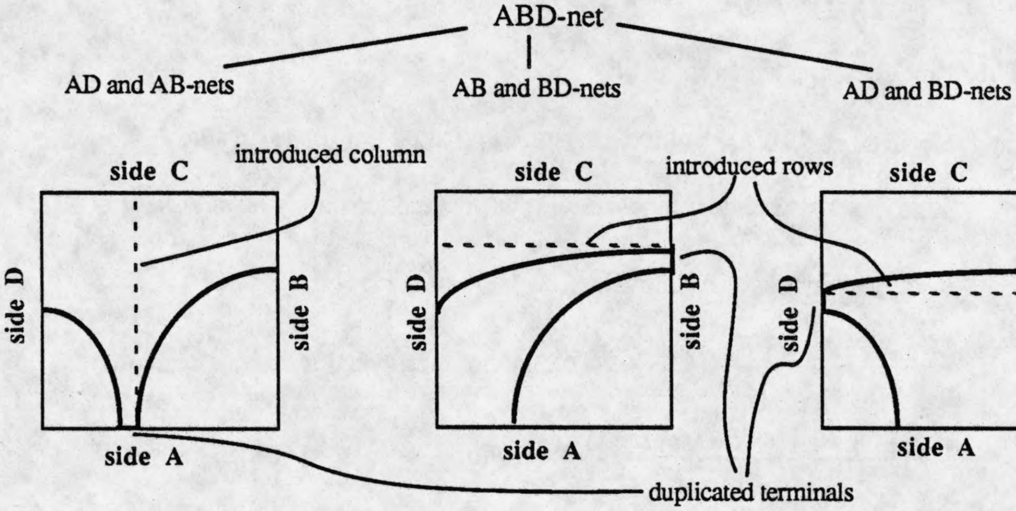


Figure 16. Decomposition of a Three-Terminal Net

ing on the choice of the side where the duplicated terminal lies on. For instance, an *ABD* net has the three different decompositions shown in Figure (16):

(1) *ABD*-net → *AD*– and *AB*–nets with duplicated terminal on side *A*;

(2) *ABD*-net → *AB*– and *BD*–nets with duplicated terminal on side *B*;

(3) *ABD*-net → *AD*– and *BD*–nets with duplicated terminal on side *D*.

Since we add one column (resp. one row) for every duplicated terminal, the total amount of increase in any side of the junction area is equal to the number of duplicated terminals on that side.

Our strategy is to distribute the duplicated terminals evenly around the sides of the junction area. When the context is clear, we will use $N_{x_1x_2x_3}$ to express both the number of $X_1X_2X_3$-nets (original definition) and the $X_1X_2X_3$-nets. For instance, when we say that αN_{aab} nets with duplicated terminals on side *A* we really mean that "αN_{aab} AAB-nets are decomposed with duplicated terminals on side *A*". We propose the following decomposition scheme:

The nets in **Set I** are not decomposed.

The nets in **Set II** are decomposed as follows:

N_{aaa} nets are decomposed with duplicated terminals on side A

N_{bbb} nets are decomposed with duplicated terminals on side B.

The nets in **Set III** are decomposed as follows:

$\frac{2(N_{aab}+N_{aac}+N_{aad})}{3}+\frac{N_{abb}}{3}$ nets with duplicated terminals on side A

$\frac{2(N_{abb}+N_{bbc}+N_{bbd})}{3}+\frac{N_{aab}}{3}$ nets with duplicated terminals on side B

$\frac{N_{aac}+N_{bbc}}{3}$ nets with duplicated terminals on side C

$\frac{N_{aad}+N_{bbd}}{3}$ nets with duplicated terminals on side D.

The nets in **Set IV** are decomposed as follows:

$\frac{N_{abc}+N_{abd}+N_{acd}}{3}$ nets with duplicated terminal on side A

$\frac{N_{abc}+N_{abd}+N_{bcd}}{3}$ nets with duplicated terminal on side B

$\frac{N_{abc}+N_{acd}+N_{bcd}}{3}$ nets with duplicated terminal on side C

$\frac{N_{abd}+N_{acd}+N_{bcd}}{3}$ nets with duplicated terminal on side D.

In this way all nets of three-terminals are divided into two-terminal nets. The total number of duplicated terminals on side A is

$$=N_{aaa}+\frac{2(N_{aab}+N_{aac}+N_{aad})}{3}+\frac{N_{abb}}{3}+\frac{N_{abc}+N_{abd}+N_{acd}}{3}.$$

By counting the number terminals on side A, we can write for the width of the junction area, l_a,

$$l_a \geq 3N_{aaa}+2(N_{aab}+N_{aac}+N_{aad})+N_{abb}+N_{abc}+N_{abd}+N_{acd}.$$

Hence, we obtain that the number of duplicated terminals on side A is less than $\frac{l_a}{3}$, which means the increase on side A is less than one third of its original value. The same argument can be applied to the increase of sides B, C, and D. The terminals on sides C are free, therefore its duplicated terminals can be aligned in columns containing the duplicated terminals of A. This implies that increases in one side can be shared by the opposite side. Similarly, the duplicated terminals of sides D and B can be aligned in rows. Hence, we have the following theorem.

Theorem 13: The three-terminal nets in a simple junction area can be routed by increasing the width and height of the junction area, by no more than one third of its original size.

Next, we discuss the case of general junctions. We again decompose the three-terminal nets into two-terminal nets by duplicating terminals of the net on the junction area boundary. We illustrate the technique for a junction area which is the union of two rectangles, as shown in Figure (17). We partition the boundary of $J_1 \cup J_2$ into twelve pieces, i.e., $A, A', \ldots, F, F'$. The nets are decomposed in a fashion similar to the case of a simple junction area: $\frac{i}{3} N_{x_1 x_2 x_3}$ nets of type $X_1 X_2 X_3$ are decomposed with the duplicated terminals on side X_j, where $X_j \in \{A, B, C, D, E, F, A', B', C', D', E', F'\}$ and $i \in \{0, 1, 2, 3\}$ is the number of terminals of such nets on side X_j. It is easy to check that the number of duplicated terminals on any side X_j (i.e., the increase in its length) is no more than its original size. However, since each of the sides with fixed terminals (namely, A, A', B, and B') face a side with free

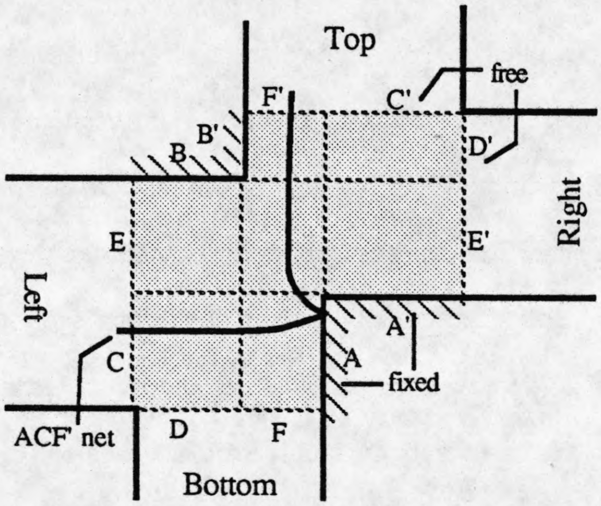


Figure 17. Decomposition of 3-terminal Nets in a General Junction Area

terminals, their increases can be shared. Hence, the total increase on the left opening $C \cup E$ (resp. top, right and bottom openings) is no more than one third its original size. Thus, we can write the following theorem.

Theorem 14: The three-terminal nets in a general junction area can be routed by increasing the width (distance between left and right opening) and height (distance between top and bottom opening) of the junction by no more than one third of their original sizes.

Finally, applying the three-sided channel router of [22], the associated channels can be routed in $\frac{3d_i}{2}$, where d_i is the density of the i-th associated channel. Hence, we obtain the following theorem.

Theorem 15: The three-terminal-net k-way junction routing problems can be routed with $\sum_{i=1}^{k} t_i \le \sum_{i=1}^{k} \frac{3d_i}{2}$, for $k = 2, 3, 4$.

5. Discussion and Open Problems

We have presented new techniques to prove lower and upper bounds for general junction routing problems. All the results of this paper are summarized in Table 2. These are the first known theoretical results for the problems of junction routing.

From Table 2, it is clear that for two-terminal nets the universal lower bounds match the upper bounds for all junctions. Hence, these results are optimal. For three-terminal nets the lower bounds for the case of L-junction also match the upper bounds. However, there is a gap for the case of other junctions, which is an open problem. The upper bound results for multi-terminal nets (more than 3-terminals) as shown in the Table can be obtained trivially, i.e., by duplicating every terminal of the junction area. Actually, these bounds can be improved by using the decomposition technique outlined for the case of three-terminal nets [20]. By our scheme for the multi-terminal router, one can expect better upper bounds if there is an improved multi-terminal router for the three-sided channel.

All the routers presented in this paper, except the S-junction router, give layouts which are three-layer wirable. In general, the S-junction router gives layouts which are four-layer wirable. Our upper

$\sum_i t_i$ / junctions	Lower bounds		Upper bounds		
	2-terminal	multi-terminal	2-terminal	3-terminal	multi-terminal
L-junction	$\geq d_1 + d_2$	$\geq \frac{3}{2}(d_1 + d_2)$	$\leq d_1 + d_2$	$\leq \frac{3}{2}(d_1 + d_2)$	$\leq 2(d_1 + d_2)$
S-junction	$\geq d_1 + d_2$	$\geq \frac{3}{2}(d_1 + d_2)$	$\leq d_1 + d_2$	$\leq 2(d_1 + d_2)$	$\leq 2(d_1 + d_2)$
T-junction	$\geq d_1 + d_2 + d_3$	$\geq \frac{3}{2}(d_1 + d_2) + d_3$	$\leq d_1 + d_2 + d_3$	$\leq \frac{3}{2}(d_1 + d_2 + d_3)$	$\leq 2(d_1 + d_2 + d_3)$
X-junction	$\geq d_1 + d_2 + d_3 + d_4$	$\geq \frac{3}{2}(d_1 + d_2) + d_3 + d_4$	$\leq d_1 + d_2 + d_3 + d_4$	$\leq \frac{3}{2}(d_1 + d_2 + d_3 + d_4)$	$\leq 2(d_1 + d_2 + d_3 + d_4)$

Table 2: Summary of Our Results

bounds are valid only for the knock-knee routing model, while it is an open problem to find upper bounds for the Manhattan model. In the case of the overlap model both the problems of finding lower and upper bounds are open.

This paper has given some indications that routing a junction requires less area than routing a switchbox. This implies that dividing the routing region into channels and junctions is likely to be better than dividing into channels and switchboxes. This is still an open problem. In fact, there are scores of other problems related to the decomposition of the routing region in junctions. For example, the problem of determining the "best" sequence to route the various junctions. Finally, the problem of routing in more than 4-way junctions is another interesting problem.

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