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ON DECODING EUCLIDEAN GEOMETRY CODES

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Abstract

In this report, an improved decoding algorithm for Euclidean Geometry codes is presented. It will be shown that this class of codes can be orthogonalized in less than or equal to 3 steps. That is, it requires no more than 3 steps of majority logic in decoding these codes. This result greatly reduces the decoding complexity without reducing the error-correcting capabilities of the codes.

The proposed decoding algorithm is a general one. In fact, it is applicable for all codes that are constructed from finite geometries. The application to Projective Geometry codes will be presented in a separate report.

1. Introduction

Majority-logic decoding[5] has been of interest to coding theorists and engineers for two important reasons. First, majority-logic decoding can be very simply implemented. That is, it is attractive from a practical point of view. Next, majority-logic decoding inherently does not perform bounded-distance decoding. Thus it can automatically correct more error patterns than those guaranteed by the decoding algorithm itself without additional cost.

The finite geometry codes, namely, the Euclidean Geometry[10] and Projective Geometry[3,8,9] codes, form an important subclass of the cyclic codes that are majority-logic decodable. Although these codes seem to be somewhat less powerful than the well-known BCH codes,[1,4] these two types of codes are comtetitive in many situations. The reason for this is that the finite geometry codes can be simply implemented.

The decoding complexity of the finite geometry codes grows exponentially with L,[11] the number of levels (or steps) of majority logic required. It is desirable, therefore, to decode these codes in as few steps as possible. Unfortunately, the existing algorithms for this class of codes often require that L be large.

In this report, an improved decoding algorithm for Euclidean Geometry (EG) codes is presented. It will be shown that EG codes can be orthogonalized in no more than 3 steps. That is, EG codes can be majority-logic decoded in less than or equal to 3 steps. The results greatly reduce the decoding complexity of EG codes without reducing the error-correcting capability of the codes. Thus they should make EG codes very attractive for practical use on error-control systems.

The concept behind the improved decoding algorithm for EG codes is applicable for the decoding of Projective Geometry (PG) codes. The results on the decoding of PG codes are presented in a separate report.

In Section 2 of this report, some of the properties of EG codes and the existing decoding algorithms for the codes are briefly reviewed. An improved decoding algorithm is presented in Section 3. Using this improved decoding algorithm, it will be shown in Section 4 that EG codes can be majority-logic decoded in less than or equal to 3 steps. Finally, a conclusion is made in Section 5.

In the following sections the reader will be assumed to be familiar with the concept of orthogonality[5,6] in majority-logic decoding and the structure of Euclidean geometries.[2] Where possible the notation and conventions employed in Reference 6 will be used.

2. Euclidean Geometry Codes

In this section, we will first briefly review some of the properties of EG codes. Then we will discuss the Reed decoding algorithm[7] and the Weldon's modified decoding algorithms[6,11] for these codes.

For a prime p, an m-dimensional Euclidean geometry over $GF(p^s)$ is denoted by $EG(m,p^s)$. Each of the p^{ms} points of this geometry can be uniquely associated with a field element of the finite field $GF(p^{ms})$. Thus, a point of the geometry can be represented as some power of α, where α is a primitive element of $GF(p^{ms})$. The point at the origin corresponds to the element 0 in $GF(p^{ms})$. A point is also called a 0-flat. A line or 1-flat through the point α^{e_o} consists of the p^s points α^j such that

$$\alpha^j = \alpha^{e_o} + \beta^i \alpha^{e_1}$$

where α^{e_1} is a point different from α^{e_o}, β is a primitive element of $GF(p^s)$, and β^i take on all possible elements of $GF(p^s)$. In general the set of p^{rs} points linearly dependent on (r+1) points not in an (r-1)-flat forms an r-flat.

It is convenient to consider each of the non-zero elements of $GF(p^{ms})$ as a location number of a cyclic code of length $p^{ms}-1$ over GF(p). An r-flat can then be associated with a polynomial that has coefficient 1 in positions corresponding to the p^{rs} points of the flat and zero elsewhere. This polynomial will be represented as a polynomial in the algebra of polynomials modulo $x^{p^{ms}-1}-1$ over GF(p).

Now, an r-th order EG code of length $p^{ms}-1$ with symbols from GF(p) has the property that its null space contains all (r+1)-flats of $EG(m,p^s)$ which do not pass through the origin.[6,11]

From the geometric structure of the EG codes, it has been shown that the Reed algorithm[7] can be used for decoding these codes.[6,10] The central idea is that the (parity) check sums* corresponding to all (r+1)-flats intersecting on a given r-flat are orthogonal on the check sum corresponding to the given r-flat. The number of (r+1)-flats (excluding the one that passes through the origin) that intersect on a given r-flat is equal to[6,11]

$$J = \frac{p^{(m-r)s}-1}{p^s-1} \tag{1}$$

Since the check sums corresponding to all (r+1)-flats are known to the decoder, the check sum corresponding to all r-flats can be correctly determined by a majority voting provided that $[\frac{J}{2}]$** or fewer errors occurred.

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*A check sum corresponding to an r-flat is the inner product of the received vector (or noise vector) and the vector corresponding to the r-flat.

**[x] is equal to the largest integer less than or equal to x.

Note that the number in Eq.(1) increases as r decreases. It is possible to find at least as many as J check sums corresponding to some r-flats that are orthogonal on the check sum corresponding to a given (r-1)-flat. Thus, the check sums corresponding to all (r-1)-flats can also be correctly determined by majority-logic decision provided that $[\frac{J}{2}]$ or fewer errors occurred. This decoding process can be repeated until the error digits corresponding to all 0-flats are determined. It requires (r+1) levels or steps of majority logic to finish the decoding.

The complexity of the decoding of EG codes grows exponentially with the number of decoding steps employed. It is important, therefore, to try to cut down the number of decoding steps. In this regard Weldon[6,11] has proposed two modified decoding algorithms. Both of the algorithms can correct as many guaranteed errors as the original algorithm.

The first of the Weldon's modified algorithms applies only for the case that ms is a composite number. The decoding proceeds in the same way as the original algorithm until the c-flats are determined, where g.c.d.(c,m) = f $\neq$ 1. Then all $(\frac{c}{f} -1)$ flats of $EG(\frac{m}{f},p^{sf})$ instead of (c-1)-flats of $EG(m,p^s)$ are determined. This trick may be used again and again to reduce the number of decoding steps. The application of this improved algorithm is rather limited, however, because the codes that can be applied deteriorate rapidly as their length increases.[6,11] In addition, it still requires a large number of decoding steps if r is large.

The second modified algorithm by Weldon requires only two steps in decoding. In the first step, all r-flats are determined in exactly the same way as the original algorithm. In the second step, the 0-flats are determined from these r-flats using the idea of non-orthogonal check sums due to Rudolph.[8]

Though this algorithm reduces the number of decoding steps to two, the decoder may not cost less than the decoder using the original algorithm. The reason is that a single majority gate with a very large number of inputs has to be used.

3. An Improved Decoding Algorithm

In this section we propose an improved decoding algorithm which is also a modification of the Reed algorithm. This decoding algorithm can also apply to Projective Geometry codes.

In the first step of the improved decoding algorithm, r-flats are determined from the sets of (r+1)-flats in exactly the same way as in the original Reed algorithm. Let k be the smallest number such that a set of at least J r-flats that are orthogonal on a given k-flat can be constructed. Obviously, k is less than or equal to (r-1). Then, in the second step of decoding, each of the k-flats is determined from the set of r-flats that are orthogonal on the given k-flat. This process can be repeated until all noise digits or 0-flats are determined.

For an example, consider the (31,16) code over GF(2). This code contains all the 3-flats of EG(5,2) in its null space. The first of the modified decoding algorithm by Weldon does not apply to this code. The Reed algorithm for this code requires 3 decoding steps. The number J is equal to 6. Using the improved decoding algorithm, it is possible to decode this code in 2 steps. First, 2-flats are determined from the sets of 3-flats that are orthogonal on the given 2-flats. Next, error-digits corresponding to all 0-flats are determined from the sets of 2 flats that are orthogonal on the given 0-flats. It can be shown by actual construction that at least 9 2-flats that are orthogonal on a given 0-flat can be formed. Thus, the improved algorithm can correct the same number of guaranteed errors in 2 steps.

In general the problem of finding the smallest k such that a set of at least J r-flats orthogonal on a given k-flat can be constructed has not been solved. Thus the number of decoding steps that can be reduced by the improved algorithm can not be expressed explicitly. However, utilizing the concept behind the new algorithm, it can be shown in the next section that EG codes are orthogonalizable in less than or equal to 3 steps.

4. Further Applications and Important Results

The basic idea of the improved decoding algorithm introduced in the last section can be utilized to prove that EG codes can be majority-logic decoded in no more than 3 steps. In order to prove this, we need a series of lemmas.

Lemma 1. In $EG(m,p^s)$, if it is possible to construct I r-flats that are orthogonal on any given k-flat which passes through the origin, then it is possible to construct at least (I-1) r-flats that are orthogonal on a given k-flat.

Proof: A k-flat that passes through a point α^{e_o} consists of the points α^j of the form

$$\alpha^j = \alpha^{e_o} + \beta^{i_1}\alpha^{e_1} + \beta^{i_2}\alpha^{e_2} + \ldots + \beta^{i_k}\alpha^{e_k}$$

where $\alpha^{e_o}, \alpha^{e_1}, \ldots, \alpha^{e_k}$ are elements of $GF(p^{ms})$ and are linearly independent, $\beta^{i_1}, \beta^{i_2}, \ldots, \beta^{i_k}$ take on all possible combinations of values in $GF(p^s)$. This k-flat can be considered the coset of the k-flat that passes through the origin with $\alpha^{e_1}, \alpha^{e_2}, \ldots, \alpha^{e_k}$ as basis elements and with α^{e_o} as the coset leader. Thus, a k-flat that passes through the origin can be transformed into a k-flat that passes through a given point α^{e_o}. If α^{e_o} is on the original k-flat, then the transformed k-flat is the same as the original k-flat. Therefore, a set of J r-flats that are orthogonal on a given k-flat which passes through the origin can be transformed into a set of (J-1) or J r-flats that are orthogonal on a k-flat which passes through a particular point, depending on whether the particular point is on one of the J original r-flats or not.

Q.E.D.

Lemma 2. It is possible to construct $I = (\frac{p^{\ell ms}-1}{p^{ms}-1} -1)$ m-flats that are orthogonal on a given point in $EG(\ell m, p^s)$.

Proof: Let α be a primitive element of $GF(p^{\ell ms})$, and

$$\beta = \alpha^{I+1}$$

then $0, 1, \beta, \beta^2, \ldots, \beta^{p^{ms}-2}$ are elements of $GF(p^{ms})$. Furthermore, they form an m-flat of $EG(\ell m, p^s)$ which passes through the origin. Now any two of the following I+1 m-flats

$$F_i = \{0, \alpha^i, \alpha^i\beta, \ldots, \alpha^i\beta^{p^{ms}-2}\} \qquad (2)$$

$$i = 0, 1, \ldots, I$$

contain no other common element besides the origin. If $\alpha^{i_1}\beta^{j_1} = \alpha^{i_2}\beta^{j_2}$, and $i_1 \geq i_2$, then $\alpha^{i_1-i_2} = \beta^{j_2-j_1}$. This is impossible because $0 \leq i_1-i_2 \leq I$. Thus the I+1 m-flats in Eq.(2) are orthogonal on the point of origin. By Lemma 1, a set of $I = \frac{p^{\ell ms}-1}{p^{ms}-1} -1$ m-flats that are orthogonal on a given point can be constructed from the set of m-flats in Eq.(2).

Q.E.D.

From Lemma 2 it can be shown that an m-th order EG code associated with $EG(\ell m, p^s)$ can be orthogonalized in two steps. At the first step of decoding, m-flats are determined from the sets of J (m+1)-flats that are orthogonal on the given m-flats, where $J = \frac{p^{(\ell-1)ms}-1}{p^s-1} - 1$. Since the value of J is not greater than that of I in Lemma 2, at the second step of decoding, all 0-flats can be determined from the sets of at least J m-flats that are orthogonal on the given 0-flats. Thus we have the next lemma.*

Lemma 3 An m-th order EG code associated with $EG(\ell m, p^s)$ is 2-step orthogonalizable.

Lemma 4 If $r > m$, then it is possible to construct $I = \frac{(p^{\ell ms}-1)}{p^{(r-m)s}(p^{ms}-1)} - 1$ r-flats that are orthogonal on a given (r-m)-flat in $EG(\ell m, p^s)$.

Proof: Let F be the given (r-m)-flat. Then F consists of $p^{(r-m)s}$ points. If F and an m-flat F_i in Eq.(2) contain no common point besides 0, then F and this particular F_i form an r-flat. If there is another m-flat in Eq.(2) that does not have any point (except the 0 point) in common with the r-flat thus formed, then the second r-flat can be formed from this m-flat and F. Since the m-flats in Eq.(2) are orthogonal on the origin, the two r-flats thus formed are orthogonal on F. This construction process can be repeated to construct more r-flats orthogonal on F. The number of such r-flats that can be formed is lower bounded by I.

Q.E.D.

Now consider an r-th order EG code with the associated geometry $EG(2m, p^s)$, where $r > m$. The number J is equal to

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*This result has been obtained by Weldon[11] from a different approach when he presented his first modified decoding algorithm.

$$J = \frac{p^{(2m-r)s}-1}{p^s-1} - 1$$

This code can be orthogonalized in 3 steps. At the first step, r-flats are determined from (r+1)-flats in the conventional way. At the second step, (r-m)-flats are determined from r-flats. From Lemma 4, it is possible to construct $I_1 = \frac{p^{2ms}-1}{p^{(r-m)s}(p^{ms}-1)} - 1$ r-flats that are orthogonal on a given (r-m)-flat. Notice that I_1 is not less than J. At the last step, all 0-flats are determined from (r-m)-flats. From Lemma 2, it is possible to construct $I_2 = \frac{p^{2ms}-1}{p^m-1} - 1 = p^{ms}$ m-flats that are orthogonal on a given 0-flat. Since $r-m < m$, at least I_2 (r-m)-flats that are orthogonal on a given 0-flat can be constructed. Notice again that I_2 is not less than J. Therefore, we have the following Lemma.

Lemma 5 If $r > m$, an r-th order EG code with the associated geometry $EG(2m,p^s)$ can be 3-step orthogonalizable.

Lemma 6 If it is possible to construct I r-flats that are orthogonal on a given k-flat in $EG(m,p^s)$, then it is also possible to construct I (r+1)-flats that are orthogonal on a given (k+1)-flat in $EG(m+1,p^s)$, and vice versa.

Proof: Let it be assumed first that all flats pass through the origin. It is well known that $GF(p^{ms})$ associated with $EG(m,p^s)$ can be considered an m-dimensional vector space over $GF(p^s)$. A k-flat of $EG(m,p^s)$ can be considered a k-dimensional subspace of $GF(p^{ms})$. Let $\{\alpha_1, \alpha_2, \ldots, \alpha_k\}$ be the basis of the k-flat, and $A = \{\alpha_{k+1}, \alpha_{k+1}, \ldots, \alpha_m\}$ be the set of other basis elements of $GF(p^{ms})$. Then, without loss of generality, an r-flat that contains the k-flat can be constructed by taking a linear combination of the k-flat and some other (r-k) basis elements formed from the set A.

On the other hand, let $\{\eta_1, \eta_2, \ldots, \eta_{k+1}\}$ be the basis of a (k+1)-flat of $EG(m+1,p^s)$, and $B = \{\eta_{k+2},\ldots,\eta_{m+1}\}$ be the set of other basis elements of $GF(p^{(m+1,s)})$ that are not on the (k+1)-flat. Then an (r+1)-flat that contains the (k+1)-flat can be constructed by a linear combination of the (k+1)-flat and some other (r-k) basis elements formed from the set B.

Notice that the number of elements in A is equal to that of elements in B. An element in A can be uniquely associated with an element in B. Thus an r-flat that contains a given k-flat in $EG(m,p^s)$ can be uniquely associated with an (r+1)-flat that contains a given (k+1)-flat in $EG(m+1,p^s)$. Therefore, the lemma is true for all flats that pass through the origin. By the same argument as in the proof of Lemma 1, the lemma is true in general.

Q.E.D.

A direct consequence of Lemma 6 is Lemma 7.

Lemma 7 If $r > \frac{m}{2}$, then an r-th order EG code associated with $EG(m,p^s)$ can be orthogonalized in 3 steps.

Proof: By Lemma 5, the lemma is true for even value of m. Thus we only have to consider the case that m is odd. Let $m = 2\bar{m} + 1$. Then $\bar{m} < r < 2\bar{m}$. The number J is equal to

$$J = \frac{p^{(2\bar{m}-r+1)s}-1}{p^s-1} - 1$$

By Lemma 6 and Lemma 4, it is possible to construct

$$I_1 = \frac{p^{2\bar{m}s}-1}{p^{(r-\bar{m}-1)s}(p^{\bar{m}s}-1)}$$

r-flats that are orthogonal on a given $(r-\overline{m})$-flat in $EG(2\overline{m}+1,p^s)$. Also, by Lemma 6 and Lemma 2, it is possible to construct at least

$$I_2 = \frac{p^{2\overline{m}s}-1}{p^{\overline{m}s}-1}$$

$(r-\overline{m})$-flats that are orthogonal on a given 0-flat in $EG(2\overline{m}+1,p^s)$. Since both I_1 and I_2 are not less than J, the lemma is proved.

Q.E.D.

<u>Lemma 8</u> If $r \leq \frac{m}{2}$, an r-th order EG code associated with $EG(m,p^s)$ can be orthogonalized in 1 or 2 steps.

<u>Proof</u>: If r is equal to zero, then an r-th order EG code is one step orthogonalizable. If m is divisible by r, the code is orthogonalizable in 2 steps by Lemma 2. If m is not divisible by r, let k be the smallest number such that m+k is divisible by r, that is, $m+k = \ell \cdot r$ for some integer ℓ. By Lemma 4, it is possible to construct $I = \frac{p^{\ell rs}-1}{p^{ks}(p^{rs}-1)} -1$ (r+k)-flats that are orthogonal on a given k-flat in $EG(\ell r,p^s)$. In addition, by Lemma 6, it is possible to construct I r-flats that are orthogonal on a given 0-flat in $EG(m,p^s)$. Since the number I is not less than the number $J = \frac{p^{(m-r)}-1}{p^s-1} -1 = \frac{p^{(\ell r-k-r)s}-1}{p^s-1} -1$ of the code, the lemma is proved.

Q.E.D.

Combining Lemma 7 and Lemma 8, we have our theorem on decoding EG codes.

<u>Theorem</u> An r-th order EG code can be orthogonalized in less than or equal to 3 steps.

In summary, the decoding of an r-th order EG code can be described as follows. At the first step of decoding, r-flats are determined from (r+1)-flats. If $r = 0$, then this is the end of decoding. At the second step of decoding, all 0-flats are determined from the sets of r-flats if $r \leq m/2$, otherwise $(r-[\frac{m}{2}])$-flats are determined from the sets of r-flats. In the case $r \leq m/2$, no more steps have to be taken. If $r > m/2$, all 0-flats are determined from the sets of $(r-[\frac{m}{2}])$-flats in the third or last step of decoding. This decoding procedure is depicted as follows:

a. $r = 0$ $\qquad 1 \longrightarrow 0$ $\qquad$ (1 step)

b. $0 < r \leq \frac{m}{2}$ $\qquad (r+1) \longrightarrow r \longrightarrow 0$ $\qquad$ (2 steps)

c. $r > \frac{m}{2}$ $\qquad (r+1) \longrightarrow r \longrightarrow r-[\frac{m}{2}] \longrightarrow 0$ $\qquad$ (3 steps)

5. Conclusion

In this report we have proposed a general improved majority logic decoding algorithm for codes that are constructed from finite geometries. The application to Euclidean Geometry codes has been further discussed. In particular, we have shown that EG codes can be orthogonalized in less than or equal to 3 steps. That is, these codes can be majority-logic decoded in no more than 3 steps.

The proposed improvement is a real one. The decoding complexity of EG codes can be reduced enormously in most cases by the improved decoding algorithm. This should make EG codes very attractive for practical use on error-control systems.

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13. ABSTRACT

In this report, an improved decoding algorithm for Euclidean Geometry codes is presented. It will be shown that this class of codes can be orthogonalized in less than or equal to 3 steps. That is, it requires no more than 3 steps of majority logic in decoding these codes. This results greatly reduces the decoding complexity without reducing the error-correcting capabilities of the codes.

The proposed decoding algorithm is a general one. In fact, it is applicable for all codes that are constructed from finite geometries. The application to Projective Geometry codes will be presented in a separate report.

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