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OF OPTIMAL REGULATORS FOR
HIGH-ORDER LINEAR PLANTS**

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AN APPROXIMATE DESIGN OF OPTIMAL REGULATORS
FOR HIGH-ORDER LINEAR PLANTS

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ABSTRACT

It is shown that the state feedback matrix of a linear system optimal with respect to a quadratic performance index can be expanded in a MacLaurin series in parameters which change the order of the system. The first two terms of this series are employed in an "optimally sensitive design" for a high-order plant. The result of the optimally sensitive design is superior to that achieved by a conventional low-order design, while the amount of computation is considerably lower than it is required for a high-order design. An example of a second order design for a fifth order plant is given.

INTRODUCTION

This paper proposes a method for designing approximately optimal regulators for high-order linear plants with quadratic performance indices. The method is motivated by the fact that the application of the existing design procedure [1,2] to high-order plants represents a computationally difficult and cumbersome task. It is well known that the number of scalar equations, which correspond to the matrix Riccati equation, increases with the square of the order of the plant equation. A conventional attempt to avoid this difficulty is to neglect some small time constants, moments of inertia and similar "parasitic" parameters which increase the equation order. In the sequel the approach based on such a "low-order" description of the plant is called the low-order design. The design based on a "high-order" description of the plant, in which these "parasitic" parameters are not neglected, is called the high-order design. At the present time a designer is left with the dilemma: either to apply the high-order design which is computationally involved, or to use a low-order design which is simpler, but which may result in an unsatisfactory system performance.

The method of optimally sensitive design proposed in this paper provides an analytical tool for resolving this dilemma. It results in better system performance than achieved by the low-order design and requires considerably less computation than the high-order design. In this method attention is focused upon the dependence of the optimal

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feedback gain matrix K on small parameters whose presence increases the order of the plant equation. It is proved in this paper that K can be expanded in a MacLaurin series with respect to these parameters. The first term of this expansion corresponds to the low-order design. The second and, if necessary, few more terms are used as an "optimally sensitive" correction of the low-order design. The effectiveness of the optimally sensitive design is illustrated by a second order design for a fifth order plant.

This paper makes use of the singular perturbation theory of ordinary differential equations [3-6]. It represents further extension and application of the method proposed in an earlier paper [7].

LOW-ORDER AND HIGH-ORDER DESIGNS

In order to motivate the statement of the problem given in the section "Optimally Sensitive Design" high-order and low-order designs are compared in this section.

Let

$$\frac{dx}{dt} = Fx + Gu \quad (1)$$

be the low-order description of a physical plant, where x is an n -dimensional state vector and u is an r -dimensional control vector. Equation (1) has been derived by neglecting the small parameters μ_1, \dots, μ_m , whose presence could increase the equation order. A more accurate description of the same physical plant is possible if the presence of these parameters is taken into account. Let the small parameters be linear functions of a scalar parameter λ , $\mu_i = \rho_i \lambda$, where ρ_i , $i=1, \dots, m$, are some known coefficients. Then the high-order description of the plant is

$$\frac{d\tilde{x}}{dt} = A_1 \tilde{x} + A_2 \tilde{z} + B_1 u, \quad (2a)$$

$$\lambda \frac{d\tilde{z}}{dt} = A_3 \tilde{x} + A_4 \tilde{z} + B_2 u, \quad (2b)$$

where \tilde{x} is an n -dimensional vector and \tilde{z} is an m -dimensional vector [7]. The matrices A_1, A_2, A_3, A_4, B_1 and B_2 are differentiable functions of λ , $A_1=A_1(\lambda), \dots, B_2=B_2(\lambda)$, and their values at $\lambda=0$ are denoted by C_1, C_2, C_3, C_4, D_1 and D_2 , respectively. Since the descriptions (1) and (2) must coincide with $\lambda=0$, F and G are defined as

$$F = C_1 - C_2 C_4^{-1} C_3, \quad G = D_1 - C_2 C_4^{-1} D_2. \quad (3)$$

It is apparent from (3) that a necessary condition for the compatibility of (1) and (2) is that C_4 is nonsingular. (This condition is a preliminary one. In the sequel a stronger condition will be imposed on C_4 .) At $\lambda=0$ the variable z is no longer a part of the state vector and can be expressed as a linear function of x and u ,

$$z = -C_4^{-1}(C_3x + D_2u). \quad (4)$$

The transition from (1) to (2) is due to a small perturbation of λ at $\lambda=0$. Without loss of generality this perturbation is assumed positive. It changes the order of the equation from n to $n+m$, ("singular perturbation"). The notation \tilde{x} indicates the perturbed n -dimensional part of the $(n+m)$ -dimensional state vector, while \tilde{z} is the additional m -dimensional part introduced by the same perturbation $\lambda > 0$.

The design objective is to find a control u such that the performance index (5) is minimized,

$$J = \frac{1}{2}y'(T) \pi y(T) + \frac{1}{2} \int_0^T (y'Qy + u'Ru) dt, \quad (5)$$

where y is an n -dimensional output vector, R is a positive definite, and π and Q are positive semidefinite matrices and prime denotes transposition.

The low-order design is considered first. In this case the low-order equation (1) is the side condition for the minimization of (5) and, without loss of generality, the output y is assumed to be equal to the state, $y=x$. Then the well known solution to the above optimal control problem is

$$u^* = -Sx, \quad (6)$$

where S is the low-order-optimal feedback gain matrix,

$$S = R^{-1}G'M. \quad (7)$$

The symmetric matrix M is the solution of the equation

$$\frac{dM}{dt} = -MF - F'M + MGR^{-1}G'M - Q \quad (8)$$

with the end condition

$$M(T) = \pi. \quad (9)$$

Hence, the low-order design consists of solving a system of $\frac{n(n+1)}{2}$ scalar differential equations (8). To check the applicability of the low-order design the behavior of the plant (2) with the low-order optimal feedback gain S must be analyzed. For the purpose of this paper it suffices to show by an example that the result of a low-order design can be not only far from the optimum, but also unstable.

Example 1. In a voltage regulator problem [8] the plant is defined by the block diagram in Fig. 1 where s is Laplace variable and

$$T_1 = 5, \quad T_2 = 2, \quad T_3 = 0.07, \quad T_4 = 0.04, \quad T_5 = 0.1 \quad (10a)$$

$$a_1 = 2.5, \quad a_2 = 3.2, \quad a_3 = 6, \quad a_4 = 3, \quad a_5 = 3. \quad (10b)$$

It is customary to attempt a low-order design by neglecting the time constants which are ten or more times smaller than the dominant ones. In this example T_3 , T_4 , and T_5 are twenty to fifty times smaller than T_2 and are neglected in the low-order model. The state equations for the low-order model are

$$\frac{dx}{dt} = \begin{bmatrix} -\frac{1}{T_1} & \frac{a_1}{T_1} \\ 0 & -\frac{1}{T_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{a}{T_2} \end{bmatrix} u, \quad (11)$$

where $a = a_2 a_3 a_4 a_5$. Let the performance index be

$$J = \frac{1}{2} \int_0^{\infty} (y^2 + u^2) dt, \quad (12)$$

where the output is $y=x_1$.

The feedback gain matrix $S = [s_1, s_2]$ obtained from (7) and (8) is

$$S = [.9578 \quad .0996]. \quad (13)$$

The applicability of this low-order design depends on whether the regulator S can be used in connection with the fifth-order plant or not. The answer is negative since the resulting feedback system Fig. 2 is unstable. Moreover, this feedback system is unstable even if the time constants T_3 , T_4 , and T_5 have two times smaller values than in (10a).

As the above example shows it is very likely that the result of a low-order design is not directly applicable to a higher order plant.

The minimization of (5) with side condition (2) is referred to as the high-order design. Let in this case $y = \tilde{x}$ and

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3/\lambda & A_4/\lambda \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2/\lambda \end{bmatrix}, \quad (14a)$$

$$Q_0 = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}, \quad \pi_0 = \begin{bmatrix} \pi & 0 \\ 0 & 0 \end{bmatrix}. \quad (14b)$$

Then the result of the high-order design is

$$\tilde{u}^* = -[K_1 \tilde{x} + K_2 \tilde{z}], \quad (15)$$

where $K = [K_1, K_2]$ is the high-order-optimal feedback gain matrix,

$$K = R^{-1} B' P. \quad (16)$$

The symmetric matrix P is the solution of the equation

$$\frac{dP}{dt} = -PA - A'P + PBR^{-1}B'P - Q_0, \quad (17)$$

with the end condition

$$P(T) = \pi_0. \quad (18)$$

In the high-order design $\frac{(n+m)(n+m+1)}{2}$ scalar equations (17) must be solved. No use is made of the result of the previous low-order design.

OPTIMALLY SENSITIVE DESIGN

A question which immediately arises from the comparison of the above two designs is whether it is possible to develop an approximation method which will represent a compromise between a simple but unsatisfactory low-order design and an optimal but computationally involved high-order design. In other words, a method is sought which will result in a performance superior to the performance achievable by the low-order design, but with an amount of computation considerably lower than it is needed for the high-order design.

The optimally Sensitive Design developed in the following sections consists of approximating the high-order-optimal gain matrix $K(\lambda)$ by its truncated MacLaurin series in λ ,

$$K(\lambda) \approx K(0) + \Delta K, \quad (19)$$

where

$$\Delta K = \left. \frac{\partial K}{\partial \lambda} \right|_{\lambda=0} \lambda.$$

There are two problems to be solved in the development of this method. Firstly, it must be shown under which conditions the optimally sensitive design is possible, that is when is the matrix $K(\lambda)$ continuous and differentiable with respect to λ at $\lambda=0$. Secondly, a procedure must be derived for computing $\frac{\partial K}{\partial \lambda}$ at $\lambda=0$. The practical applicability of the optimally sensitive design depends on the simplicity of this procedure.

CONTINUITY OF $K(\lambda)$ AT $\lambda = 0$

In order to make the analysis of the solution $K(\lambda)$ of (17) more convenient, (16) and (17) are rewritten as follows:

$$K_1 = R^{-1}(B_1'P_1 + B_2'H_2'), \quad K_2 = R^{-1}(\lambda B_1'H_2 + B_2'H_3) \quad (20)$$

$$\frac{dP_1}{dt} = -P_1A_1 - A_1'P_1 - H_2A_3 - A_3'H_2 + P_1E_1P_1 + P_1EH_2' + H_2E'P_1 + H_2E_2H_2' - Q, \quad (21a)$$

$$\lambda \frac{dH_2}{dt} = -P_1A_2 - H_2A_4 - \lambda A_1'H_2 - A_3'H_3 + \lambda P_1E_1H_2 + P_1EH_3 + \lambda H_2E'H_2 + H_2E_2H_3, \quad (21b)$$

$$\lambda \frac{dH_3}{dt} = -\lambda A_2' H_2 - \lambda H_2' A_2 - H_3 A_4 - A_4' H_3 + \lambda^2 H_2' E_1 H_2 + \lambda H_2' E H_3 + \lambda H_3 E' H_2 + H_3 E_2 H_3, \quad (21c)$$

where $E_1 = B_1 R^{-1} B_1'$, $E_2 = B_2 R^{-1} B_2'$, $E = B_1 R^{-1} B_2'$, and where P_1 , H_2 and H_3 are $n \times n$, $n \times m$ and $m \times m$ matrices defined by

$$P = \begin{bmatrix} P_1 & \lambda H_2 \\ \lambda H_2' & \lambda H_3 \end{bmatrix}.$$

The end condition (18) is partitioned into

$$P_1(T) = \pi, \quad H_2(T) = 0, \quad H_3(T) = 0. \quad (22)$$

In view of classical theorems on the continuous dependence of solutions of ordinary differential equations on parameters, $P_1 = P_1(\lambda)$, $H_2 = H_2(\lambda)$ and $H_3 = H_3(\lambda)$ are continuous functions of λ when $\lambda > 0$. For the optimally sensitive design, however, it is necessary that these functions be continuous at $\lambda=0$ as well, that is that they have the following properties:

Property 1: As $\lambda \rightarrow 0^+$ the limits

$$P_1(\lambda) \rightarrow L_1, \quad H_2(\lambda) \rightarrow L_2, \quad H_3(\lambda) \rightarrow L_3$$

exist and are unique.

Property 2. L_1 , L_2 , and L_3 satisfy equation

$$\frac{dL_1}{dt} = -L_1 C_1 - C_1' L_1 - L_2 C_3 - C_3' L_2 + L_1 N_1 L_1 + L_1 N L_2' + L_2 N' L_1 + L_2 N_2 L_2' - Q, \quad (23a)$$

$$0 = -L_1 C_2 - L_2 C_4 - C_3' L_3 + L_1 N L_3 + L_2 N_2 L_3, \quad (23b)$$

$$0 = -L_3 C_4 - C_4' L_3 + L_3 N_2 L_3, \quad (23c)$$

which is obtained by letting $\lambda=0$ in (21). N_1 , N_2 and N are the values of E_1 , E_2 and E at $\lambda=0$. The end condition for (23) is $L_1(T) = \pi$.

If $P_1(\lambda)$, $H_2(\lambda)$ and $H_3(\lambda)$ have the Properties 1 and 2 then the low-order design can be used as the first step of the optimally sensitive design. This is shown by substituting in (23a) the roots

$$L_2 = -L_1 C_2 C_4^{-1}, \quad L_3 = 0, \quad (24)$$

of the algebraic equations (23b) and (23c),

$$\begin{aligned} \frac{dL_1}{dt} = & -L_1 (C_1 - C_2 C_4^{-1} C_3) - (C_1 - C_2 C_4^{-1} C_3)' L_1 \\ & + L_1 (D_1 - C_2 C_4^{-1} D_2) R^{-1} (D_1 - C_2 C_4^{-1} D_2)' L_1 - Q, \end{aligned}$$

and noticing that, by virtue of (3),

$$\frac{dL_1}{dt} = -L_1 F - F' L_1 + L_1 G R^{-1} G' L_1 - Q.$$

This equation in L_1 is identical to the low-order Riccati equation (8) in M. Therefore $L_1 = M$ and it follows from (20) and (24) that, as $\lambda \rightarrow 0$,

$$K_1(\lambda) \rightarrow R^{-1} (D_1 - C_2 C_4^{-1} D_2)' M = S, \quad K_2(\lambda) \rightarrow 0. \quad (25)$$

Theorem 1. For the functions $P_1(\lambda)$, $H_2(\lambda)$ and $H_3(\lambda)$ to have the properties 1 and 2 it is sufficient that C_4 be negative definite.

This theorem is proved by applying a more general theorem due to Tikhonov [3], that is by showing that the roots (24) are the stable steady state solution of the following auxiliary equations

$$\frac{dL_3}{d\tau} = -L_3 C_4 - C_4' L_3 + L_3 N_2 L_3, \quad (26a)$$

$$\frac{dL_2}{d\tau} = -L_1 C_2 - L_2 C_4 - C_3' L_3 + L_1 N L_3 + L_2 N_2 L_3. \quad (26b)$$

In (26), L_1 and t are considered as fixed parameters and the only independent variable is the "fast mode time" τ . Let $L_3(\tau, \phi_3)$ and $L_2(\tau, \phi_2, \phi_3)$ be the solution of (26) for any end condition $L_3(\tau, \phi_3) = \phi_3$ and $L_2(\tau, \phi_2, \phi_3) = \phi_2$.

Proposition. If C_4 is negative definite, then, as $\tau \rightarrow -\infty$,

$$L_3(\tau, \phi_3) \rightarrow 0, \quad (I)$$

$$L_2(\tau, \phi_2, \phi_3) \rightarrow -L_1 C_2 C_4^{-1}. \quad (II)$$

To prove this proposition, define a Lyapunov function,

$$V(L_3) = \frac{1}{2} \text{trace}(L_3)^2,$$

whose derivative in view of (26a) is,

$$\frac{dV}{d\tau} = \text{trace}(-L_3 C_4 L_3 - L_3 C_4' L_3 + L_3^{3/2} N_2 L_3^{3/2}).$$

It is seen that if C_4 is negative definite $\frac{dV}{d\tau}$ must be positive definite which proves part (I). Part (II) is obtained by substituting $L_3 = 0$ in the linear equation (25b) which is stable when C_4 is negative definite.

DIFFERENTIABILITY OF $K(\lambda)$ AT $\lambda=0$

When $\lambda > 0$ the classical theorems on differentiability of solutions of ordinary differential equations with respect to parameters guarantee the existence and uniqueness of the derivatives $\tilde{W}_1 = \frac{\partial P_1}{\partial \lambda}$, $\tilde{W}_2 = \frac{\partial H_2}{\partial \lambda}$ and $\tilde{W}_3 = \frac{\partial H_3}{\partial \lambda}$.

These functions are the solutions of the sensitivity equations which can be obtained by differentiating (21) with respect to λ . The end conditions for such equations are zero, since the terminal conditions (22) do not depend on λ . Then (20) can be differentiated with respect to λ ,

$$\frac{\partial \tilde{K}_1}{\partial \lambda} = R^{-1} (\tilde{\beta}'_1 P_1 + B'_1 \tilde{W}_1 + \tilde{\beta}'_2 H'_2 + B'_2 \tilde{W}'_2) \quad (27a)$$

$$\frac{\partial \tilde{K}_2}{\partial \lambda} = R^{-1} (B'_1 H_2 + \lambda \beta'_1 H_2 + \lambda B'_1 \tilde{W}_2 + \beta'_2 H_3 + B'_2 \tilde{W}_3) \quad (27b)$$

where $\tilde{\beta}'_1 = \frac{\partial B_1}{\partial \lambda}$ and $\tilde{\beta}'_2 = \frac{\partial B_2}{\partial \lambda}$. The tilde stresses the fact that all the above quantities are evaluated at $\lambda > 0$. However, the optimally sensitive design requires the derivative $\frac{\partial K}{\partial \lambda}$ to be evaluated at $\lambda = 0$. Again the classical theorems are not applicable and the singular perturbation theory must be used.

Theorem 2. Functions \tilde{W}_1 , \tilde{W}_2 and \tilde{W}_3 have Properties 3 and 4 if the following three conditions are satisfied:

- (1) C_4 is negative definite;
- (2) A_1 , A_2 and B_1 have continuous first partial derivatives with respect to λ ;
- (3) A_3 , A_4 and B_2 have continuous second partial derivatives with respect to λ .

Property 3. As $\lambda \rightarrow 0^+$ the limits

$$\tilde{W}_1 \rightarrow W_1, \quad \tilde{W}_2 \rightarrow W_2, \quad \tilde{W}_3 \rightarrow W_3$$

exist and are unique.

Property 4. The functions W_1 , W_2 and W_3 are the solutions of the equations

$$\begin{aligned} \frac{dW_1}{dt} = & -W_1 C_1 - L_1 \alpha_1 - \alpha'_1 L_1 - C'_1 W_1 - W_2 C_3 - L_2 \alpha_3 - \alpha'_3 L'_2 - C'_3 W'_2 + W_1 N_1 L_1 + L_1 \Gamma_1 L_1 \\ & + L_1 N_1 W_1 + W_1 N L'_2 + L_1 \Gamma L'_2 + L_1 N W'_2 + W_2 N' L_1 + L_2 \Gamma' L_1 + L_2 N' W_1 \\ & + W_2 N_2 L'_2 + L_2 \Gamma_2 L'_2 + L_2 N_2 W'_2, \end{aligned} \quad (28a)$$

$$\begin{aligned} 0 = & -\frac{dL_2}{dt} - W_1 C_2 - L_1 \alpha_2 - W_2 C_4 - L_2 \alpha_4 - C'_1 L_2 - C'_3 W_3 + L_1 N_1 L_2 + L_1 N W_3 \\ & + L_2 N' L_2 + L_2 N_2 W_3, \end{aligned} \quad (28b)$$

$$0 = C'_2 L_2 + L'_2 C_2 + W_3 C_4 + C'_4 W_3, \quad (28c)$$

where

$$\alpha_i = \left. \frac{\partial A_i}{\partial \lambda} \right|_{\lambda=0}, i=1, \dots, 4; \quad \beta_j = \left. \frac{\partial B_j}{\partial \lambda} \right|_{\lambda=0}, j=1, 2$$

$$\Gamma_j = \left. \frac{\partial E_j}{\partial \lambda} \right|_{\lambda=0}, j=1, 2; \quad \Gamma = \left. \frac{\partial E}{\partial \lambda} \right|_{\lambda=0}.$$

Equation (28) is obtained by differentiating (21) with respect to λ and then setting $\lambda=0$. The end condition for (28) is given by

$$W_1(T) = \int_T^{-\infty} [-H_2(\tau)C_3 - C_3'H_2'(\tau) + \pi NH_2'(\tau) + H_2(\tau)N'\pi + H_2(\tau)N_2H_2'(\tau) - \pi CC_3 - C_3'C'\pi + \pi(NC' + CN')\pi - \pi CN_2C'\pi] d\tau \quad (29)$$

where $C = C_2C_4^{-1}$ and where $H_2(\tau)$ is the solution of

$$\frac{dH_2}{d\tau} = -\pi C_2 - H_2C_4 \quad (30)$$

with the end condition $H_2(T) = 0$.

Theorem 2 immediately follows from a more general theorem due to Vasileva [4].

In view of (27) and Property 3,

$$\frac{\partial \tilde{K}_1}{\partial \lambda} \rightarrow R^{-1}(\beta_1'L_1 + D_1'W_1 + \beta_2'L_2 + D_2'W_2) = \left. \frac{\partial K_1}{\partial \lambda} \right|_{\lambda=0}, \quad (31a)$$

$$\frac{\partial \tilde{K}_2}{\partial \lambda} \rightarrow R^{-1}(D_1'L_2 + D_2'W_3) = \left. \frac{\partial K_2}{\partial \lambda} \right|_{\lambda=0}, \quad (31b)$$

and the analytical basis for the optimally sensitive design is completed since W_1 , W_2 and W_3 can be obtained from (28).

It should be noted from (29) and (30) that if there is no terminal cost, $\pi=0$, the final condition $W_1(T) = 0$.

DESIGN FOR TIME-INVARIANT SYSTEMS

If the matrices A and B do not depend on time, if $T = \infty$ and if (2) is completely controllable and observable [1], then the feedback gain matrix $K(\lambda)$ is time-invariant and the high-order design consists of solving a system of $\frac{(n+m)(n+m+1)}{2}$ quadratic equations,

$$0 = -PA - A'P + PBR^{-1}B'P - Q_0. \quad (32)$$

In this case the optimally sensitive design is especially simple. The matrices W_1 , W_2 and W_3 are time-invariant and (28) becomes

$$0 = -W_1 C_1 - L_1 \alpha_1 - \alpha_1' L_1 - C_1' W_1 - W_2 C_3 - L_2 \alpha_3 - \alpha_3' L_2 - C_3' W_2 + W_1 N_1 L_1 + L_1 \Gamma_1 L_1 + L_1 N_1 W_1 \\ + W_1 N L_2' + L_1 \Gamma L_2' + L_1 N W_2' + W_2 N' L_1 + L_2 \Gamma' L_1 + L_2 N' W_1 + W_2 N_2 L_2' + L_2 \Gamma_2 L_2' + L_2 N_2 W_2', \quad (33a)$$

$$0 = -W_1 C_2 - L_1 \alpha_2 - W_2 C_4 - L_2 \alpha_4 - C_1' L_2 - C_3' W_3 + L_1 N_1 L_2 + L_1 N W_3 + L_2 N' L_2 + L_2 N_2 W_3, \quad (33b)$$

$$0 = C_2' L_2 + L_2' C_2 + W_3 C_4 + C_4' W_3. \quad (33c)$$

Hence, the optimally sensitive design consists of the following steps:

1. solving $\frac{m(m+1)}{2}$ linear scalar equations (33c),
2. inverting the $m \times m$ matrix C_4 ,
3. solving $\frac{n(n+1)}{2}$ linear scalar equations (33a).

The computational simplicity of the above procedure compared with solving (32) is threefold. Firstly, all equations (33) are linear. Secondly, (33) represents $m(n-1)$ less scalar equations than (32). Thirdly, steps 1 through 3 can be performed consecutively.

It should be noted that for the above time-invariant problem the sufficient condition of theorems 1 and 2 is satisfied even if C_4 is not negative definite, but is similar to a negative definite matrix.

Since at the present time there is no direct way to estimate how close is the result of an optimally sensitive design to the optimum, a typical design example is worked out which illustrates both the simplicity of the computational procedure and the applicability of the obtained result.

Example 2. Consider the plant in Fig. 1 and performance index (12) and let the output be $y = \tilde{x}_1$. The small parameters T_3 , T_4 and T_5 are expressed as

$$T_3 = 0.7\lambda, \quad T_4 = 0.4\lambda, \quad T_5 = \lambda.$$

Then, the plant state equation (2) is written as

$$\frac{d\tilde{x}}{dt} = \begin{bmatrix} -0.2 & -0.5 \\ 0 & -0.5 \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 & 0 & 0 \\ 1.6 & 0 & 0 \end{bmatrix} \tilde{z}$$

$$\lambda \frac{d\tilde{z}}{dt} = \begin{bmatrix} -10/7 & 60/7 & 0 \\ 0 & -2.5 & 7.5 \\ 0 & 0 & -1 \end{bmatrix} \tilde{z} + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \tilde{u}.$$

Since the eigenvalues of C_4 are all negative and distinct, it is similar to a negative definite matrix and hence Theorems 1 and 2 hold. From the low-order design of Example 1,

$$L_1 = M = \begin{bmatrix} 0.2064 & 0.0111 \\ 0.0111 & 0.0011 \end{bmatrix},$$

$$L_2 = -MC_2 C_4^{-1} = \begin{bmatrix} 0.0124 & 0.0425 & 0.3193 \\ 0.0013 & 0.0044 & 0.0332 \end{bmatrix}.$$

Let

$$W_1 = \begin{bmatrix} w_1 & w_2 \\ w_2 & w_3 \end{bmatrix}, \quad W_2 = \begin{bmatrix} w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} w_{10} & w_{11} & w_{12} \\ w_{11} & w_{13} & w_{14} \\ w_{12} & w_{14} & w_{15} \end{bmatrix}.$$

Then (33) is solved in three steps.

Step 1.

$$\begin{aligned} w_{10} &= 0.0014, \\ w_{11} &= 14/55/(60/7 w_{10} + 0.0071) = 0.0049, \\ w_{12} &= 7/17(7.5 w_{11} + 0.0531) = 0.0372, \\ w_{13} &= 24/7 w_{11} = 0.0170, \\ w_{14} &= 2/7(60/7 w_{12} + 7.5 w_{13}) = 0.1275, \\ w_{15} &= 7.5 w_{14} = 0.9567. \end{aligned}$$

Step 2.

$$\begin{aligned} w_4 &= 1.12 w_2 - 0.0766, \\ w_5 &= 3.4286 w_4 - 0.1499, \\ w_6 &= 7.5 w_5 - 2.7492, \\ w_7 &= 1.12 w_3 - 0.0039, \\ w_8 &= 3.4286 w_7 - 0.0076, \\ w_9 &= 7.5 w_8 - 0.1428. \end{aligned}$$

Step 3.

$$\begin{aligned} 0.4 w_1 + 5.7474 w_6 &= 0 \\ -0.5 w_1 + 0.7 w_2 + 0.2988 w_6 + 2.8737 w_9 &= 0, \\ -w_2 + w_3 + 0.5976 w_9 &= 0. \end{aligned}$$

Solving (33a) and (33b)

$$\begin{aligned} w_1 &= 1.9265, & w_2 &= 0.2005, & w_3 &= 0.0208, \\ w_4 &= 0.1479, & w_5 &= 0.3571, & w_6 &= -0.1341, \\ w_7 &= 0.0195, & w_8 &= 0.0591, & w_9 &= 0.3003. \end{aligned}$$

Finally, equation (31) gives

$$\left. \frac{\partial K_1}{\partial \lambda} \right|_{\lambda=0} = [3 w_6 \quad 3 w_9] = [-0.4023 \quad 0.9010],$$

$$\left. \frac{\partial K_2}{\partial \lambda} \right|_{\lambda=0} = [3 w_{12} \quad 3 w_{14} \quad 3 w_{15}] = [0.1116 \quad 0.3827 \quad 2.8702].$$

The above value of $\frac{\partial K}{\partial \lambda}$ is general in the sense that it may serve not only for $\lambda=0.1$ as required in this example, but also for a range of values of λ . In order to see how large this range is, the above result of the optimally sensitive design is compared with the result of the optimal high-order design. The table below gives the values of the performance index J obtained by the low-order design, the optimally sensitive design and the high-order design for the initial condition $x_1(0) = -10$, $x_2(0) = z_1(0) = z_2(0) = z_3(0) = 0$. (Similar results are obtained for other initial conditions.)

TABLE OF VALUES OF THE PERFORMANCE INDEX J FOR $0 < \lambda \leq 0.5$

λ	0.025	0.050	0.075	0.1	0.2	0.3	0.4	0.5
Low-order design	13.80	24.86	∞	∞	∞	∞	∞	∞
Optimally sensitive design	12.601	14.66	16.57	18.36	25.03	31.57	38.76	45.55
High-order design	12.599	14.64	16.49	18.19	23.90	28.50	32.42	35.86

It is seen that the range of λ in which the low-order design can be used is as narrow as $0 \leq \lambda \leq 0.025$, while the optimally sensitive design can be applied in about twelve times larger range, $0 \leq \lambda \leq 0.3$.

It should be noted that in this example the high-order design consists in solving a system of fifteen quadratic equations (32), while the linear equations for the optimally sensitive design are easily solvable by hand.

For values of T_3 , T_4 and T_5 given in (10a) $\lambda=0.1$ and in view of (19) the optimally sensitive gain matrix is

$$K(0) + 0.1 \frac{\partial K}{\partial \lambda} = [0.9176 \quad 0.1897 \quad 0.0111 \quad 0.0383 \quad 0.2870].$$

The block-diagram of the resulting regulator system is given in Fig. 3. It is of interest to compare the matrix $K(0) + 0.1 \frac{\partial K}{\partial \lambda}$ with the high-order optimal matrix $K(0.1)$,

$$K(0.1) = [0.9243 \quad 0.1711 \quad 0.0161 \quad 0.0392 \quad 0.2644].$$

The comparison of the responses (Fig. 4) of the optimal (solid line) and the optimally sensitive (dotted line) systems gives another indication of the validity of the optimally sensitive design.

CONCLUSIONS

It is proved that in linear optimal systems in which the matrix C_4 is negative definite (or similar to a negative definite matrix) the state feedback matrix $K(\lambda)$ can be expanded in a MacLaurin series with respect to

a small parameter λ which changes the order of the plant. This result made it possible to design an approximately optimal $(n+m)$ -th order system on an n -th order model of the plant. It is shown that an "optimally sensitive design," which is computationally simpler than a high-order design, results in system performances superior to that achieved by a conventional low-order design.

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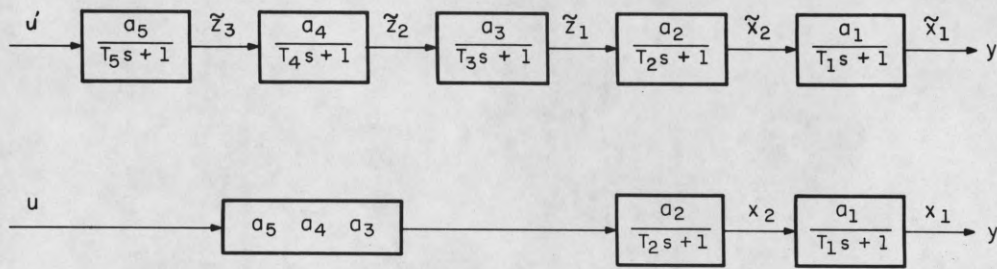
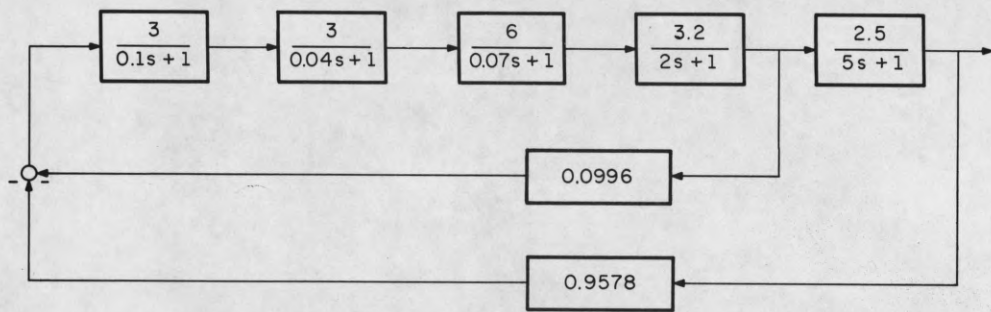
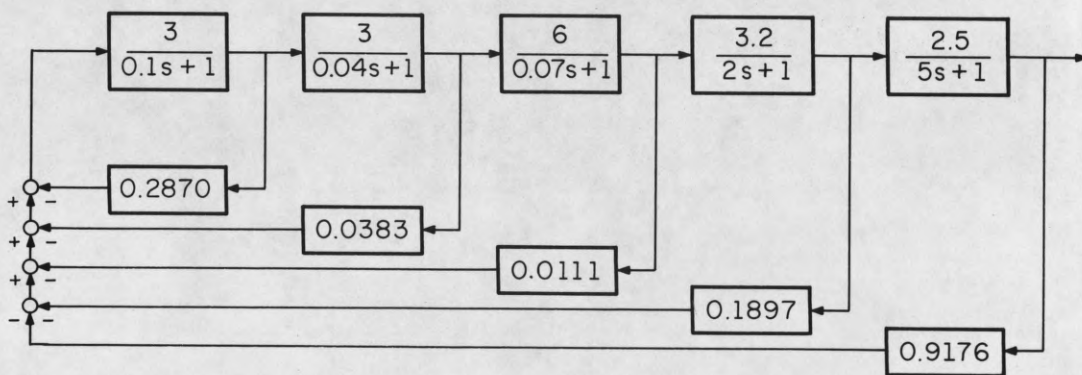


Fig. 1. High-order and low-order models of the plant.



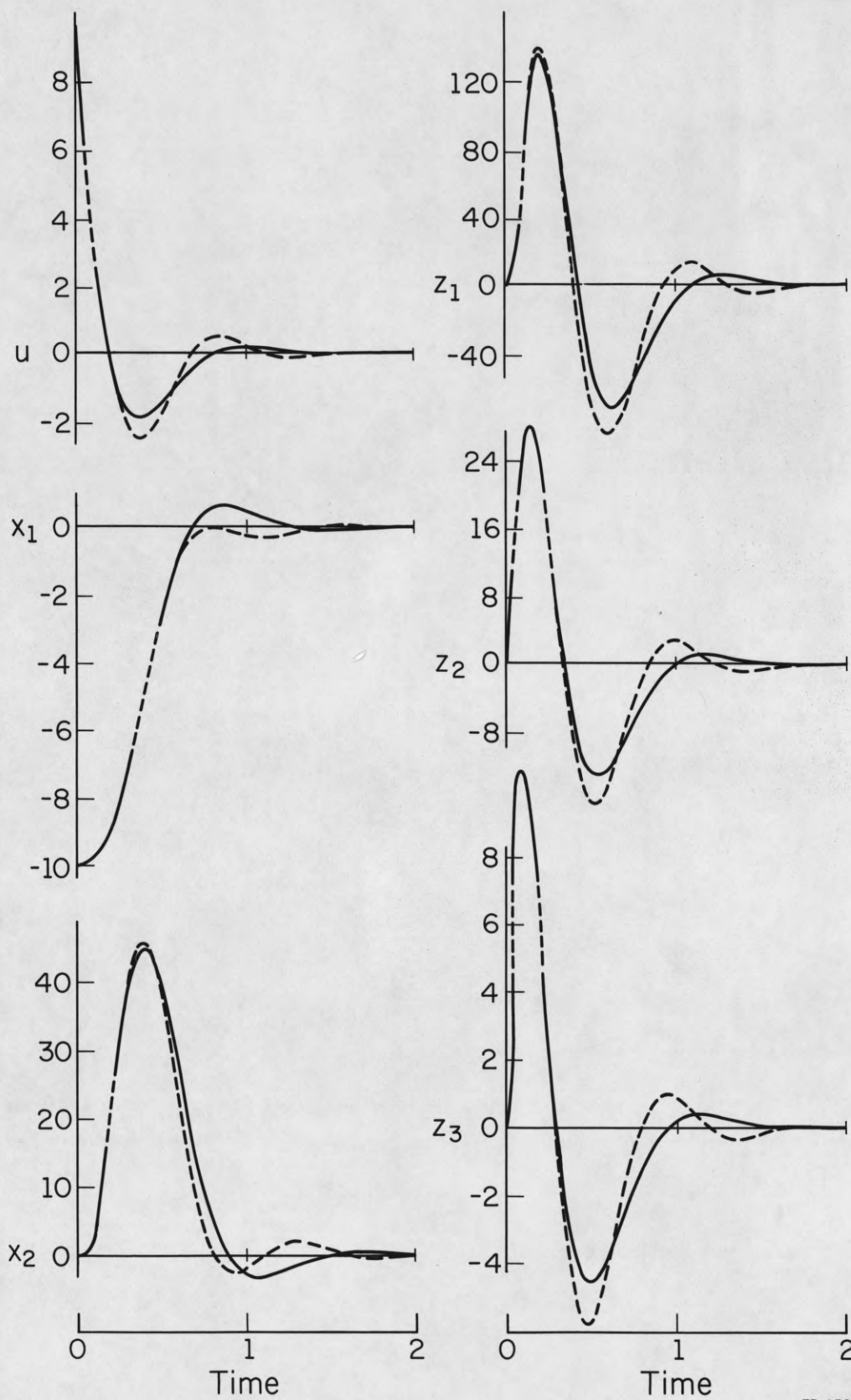
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Fig. 2. System with the low-order-optimal regulator.



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Fig. 3. System with the optimally sensitive regulator



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Fig. 4. Responses of the high-order optimal (solid line) and optimally sensitive (dotted line) systems.

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13. ABSTRACT It is show that the state feedback matrix of a linear system optimal with respect to a quadratic performance index can be expanded in a MacLaurin series in parameters which change the order of the system. The first two terms of this series are employed in an "optimally sensitive design" for a high-order plant. The result of the optimally sensitive design is superior to that achieved by a conventional low-order design, while the amount of computation is considerably lower than it is required for a high-order design. An example of a second order design for a fifth order plant is given.			

14.

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Change of System Order
 Closed Loop Optimal-Control
 Higher-Order System
 Linear System
 Lower-order System
 Optimally Sensitive Design
 Singular Perturbation
 Small Parameters
 State Feed-back Regulator