REPORT R-459 MARCH, 1970 UILU-ENG-70-204

COORDINATED SCIENCE LABORATORY

DYNAMIC MODEL OF A CONTINUOUS COLD ROLLING MILL

M. JAMSHIDI
P. KOKOTOVIĆ

UNIVERSITY OF ILLINOIS - URBANA, ILLINOIS

DYNAMIC MODEL OF A CONTINUOUS COLD ROLLING MILL

by

M. Jamshidi and P. Kokotovic

This work was supported in part by the Joint Services Electronics Program (U.S. Army, U.S. Navy, U.S. Air Force) under Contract DAAB-07-67-C-0199; also in part by the U. S. Air Force under Grant AFOSR 68-1579A; and also in part the Alcoa Foundation.

DYNAMIC MODEL OF A CONTINUOUS COLD ROLLING MILL

by

M. Jamshidi and P. Kokotović

Abstract

The dynamic model of a continuous cold reduction mill is developed. The model makes use of the force-torque equation expansion technique developed in an earlier report and presents the generalized (7N+8)th order state equation for an N-stand rolling mill. The equations of interstand tensions are found to be highly nonlinear with the arguments delayed due to the transition of strip between the stands. The model developed can be used in the design of a controller for the entire rolling process.

Acknowledgments

The authors are thankful to Mrs. Sandra Bowles, Mrs. Barbara Champagne, Mrs. Sherry Kallembach, Mrs. Rose Lane, and Mrs. Nicola Stillings for their superb typing of the report.

I. INTRODUCTION

In this report a nonlinear model of an N-stand rolling mill is developed. The model represents an improvement on previously proposed models [1,2].

Figure 1 shows an N-stand mill presented with coiler, decoiler, roll screw-down and drive motors.

As seen, the decoiler feeds the strip into the first stand where its thickness is reduced by a combination of interstand tension and roll separating force. The rolls of each stand are driven by a dc-motor which in turn is connected to its generator. The generators, the circuit breakers and emergency stopping circuits are not shown for simplicity of the diagram [3]. The drive motors can be controlled by both armature and field, while the screwdown motors are only armature controlled.

The model developed in this report will be used in the design of a controller for the rolling process. Each stand, coiler and decoiler are considered as dynamic subsystems coupled by an elastic strip. In general the coupled model is nonlinear and its order is high. Consequently, straight forward applications of optimal control theory would be virtually impossible and some approximation methods are needed. The development of this model takes into consideration properties of a class of near optimum design methods. In particular, it is adapted for an application of singular perturbation [4-6], decoupling methods [7], three time scale design method [8] and design of systems with time lags [9-12]. This model, although assumes the electrical features of

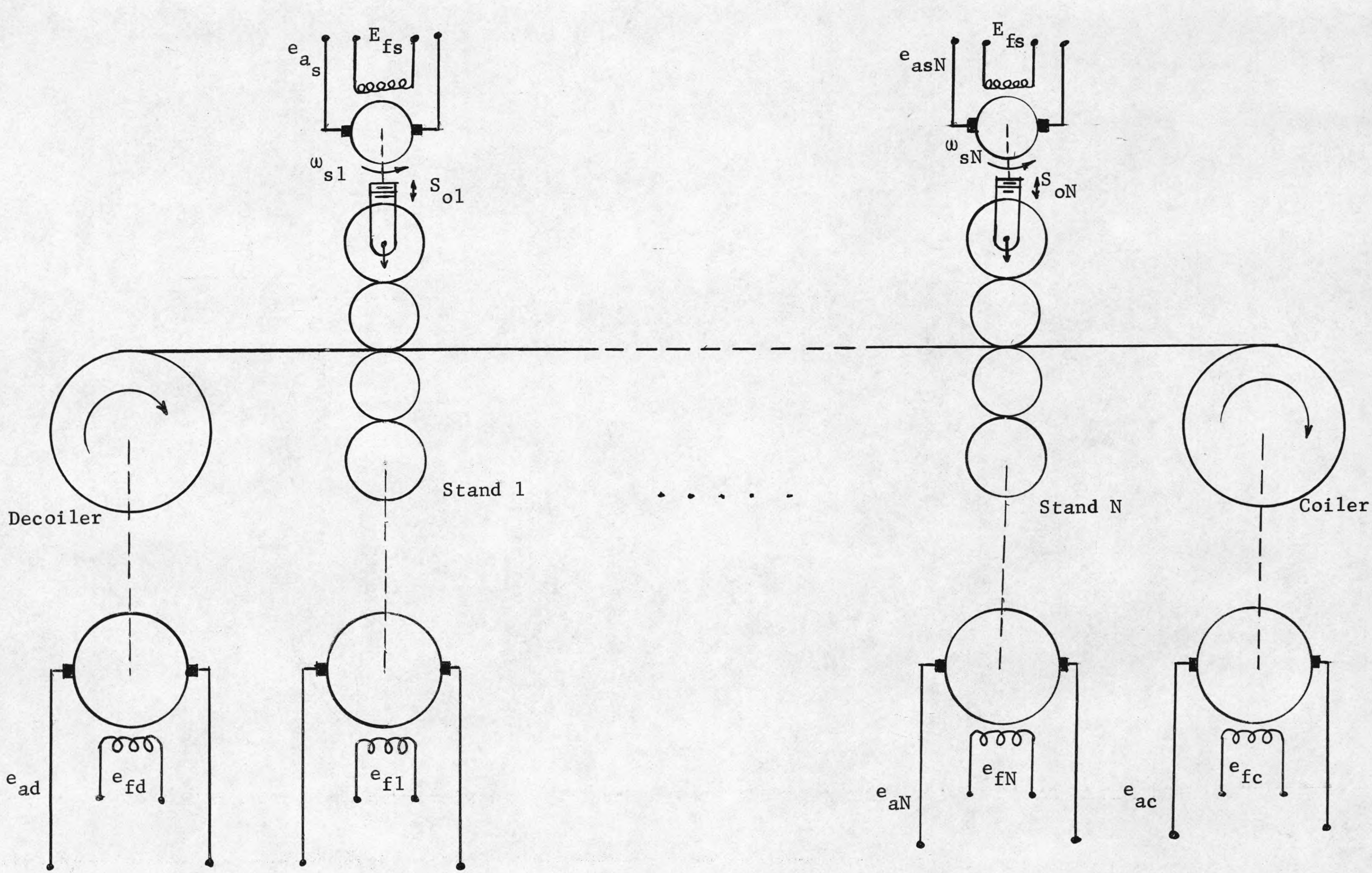

Figure 1. An N-stand continuous cold rolling mill.

Figure 1, remains general in its mechanical subsystems and formulation of state equations for the mill in interstand variables. The N-stand mill system of Figure 1 is divided into two major subsystems: mechanical and electrical.

2. MECHANICAL SUBSYSTEM

A brief review of rolling theory will explain the deformation process, interstand relations and other rolling equations relating mill process variables such as thickness, tension, torque, force, and coefficients of friction. Appendix 1 presents the notation and nomenclature for the entire report.

2.1 Deformation Process

In general, there is no "exact" rolling theory presented in the literature. The "exactness" of a rolling theory depends on assumptions involved. Thus far, the theory of Orowan [13] is most "exact". It permits variation of both yield stress and coefficient of friction. It is furthermore useful for both cold and hot strip rolling. Orowan's theory has been extended by Finne, et al [14]. However, due to its complexity, this theory is approximated by Bland and Ford [15].

The theory of Bland and Ford is used throughout our analysis. Using this theory several authors have developed graphical techniques and iterative methods for computations of force and torque to be applied to the rolls during the process [1,2,16-20]. In an earlier report we developed a computational procedure for determining the coefficients of a three-term Taylor series expansion of rolling force and torque [21].

The deformation of strip takes place at each stand's roll gap. A roll gap configuration is shown in Figure 2.

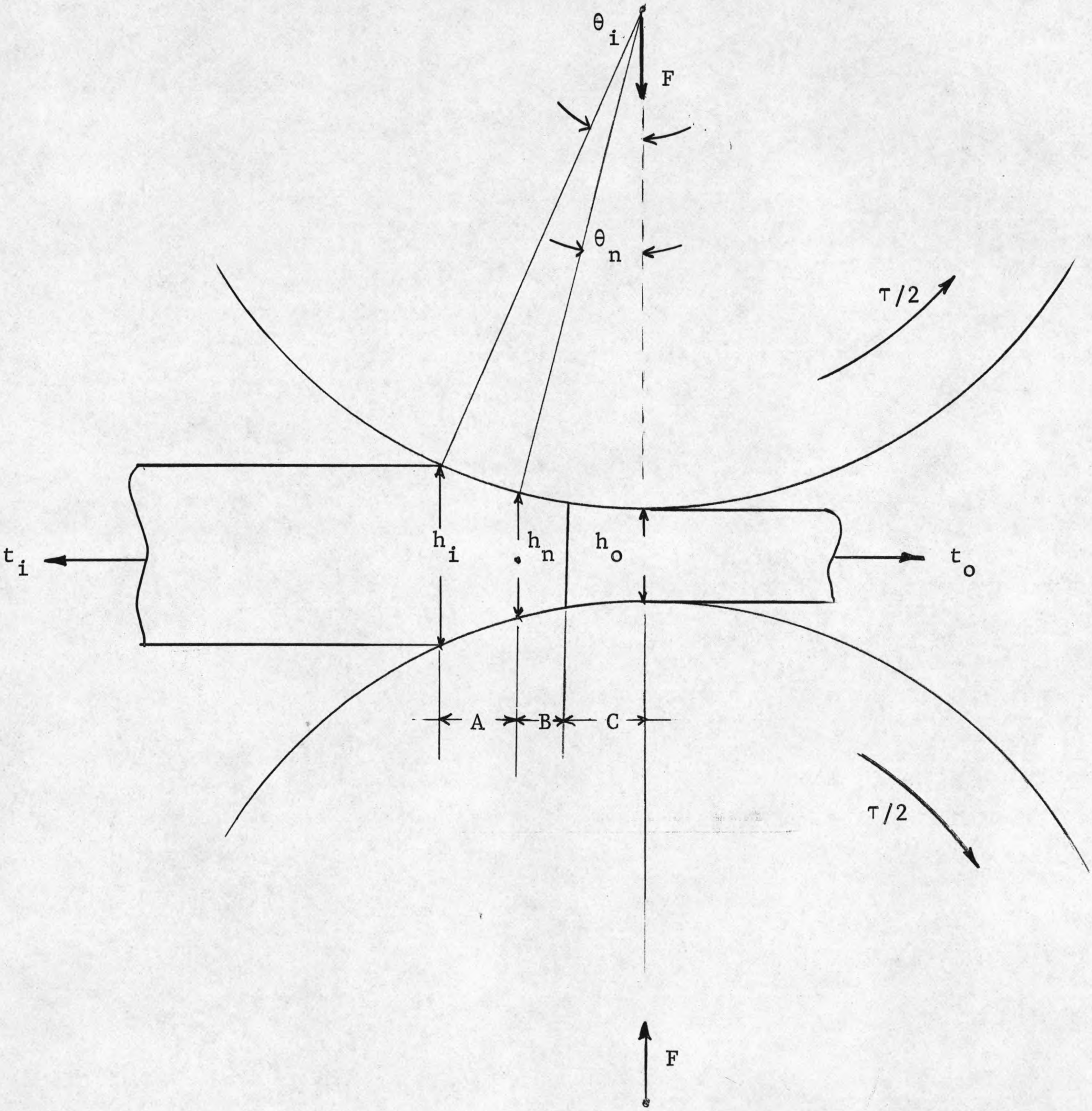

Figure 2. A roll gap configuration.

As seen the arc of contact is divided into three segments:

A is the elastic arc at the entry.

B is the plastic arc.

C is the elastic recovery arc at the exit.

In the two elastic regions the frictional force exerted by the rolls to the strip is commonly assumed to [13-15] be proportional to roll pressure with proportionality being the coefficient of friction. It is in these two regions where the rolling metal is said to be "slipping" [13-20]. In region B, however, the frictional force increases up to the magnitude of the yield stress in shear and a plastic shear occurs in the metal. The surface of the rolling metal is then said to be "sticking" to the rolls [13]. One of the basic assumptions made in [13] is that the deformation in the plastic region is a "homogeneous compression", i.e. the strip is divided into a number of vertical segments which remain plane and perpendicular to the direction of rolling. Three of these planes are shown in Figure 2. As seen from this figure, in the neighborhood of the entry plane these vertical segments (Planes) are "squeezed backward" while close to the exit plane they are "squeezed forward". In the middle of the gap there is a segment where the strip is being pulled out to its right and pushed back to its left. Such a segment is called the "neutral plane" [13,20].

The rolling theory of Bland and Ford calculates roll force and torque items of the "mill variables", h_i, h_o, t_i, t_o, and μ,

$$F = F(h_i, h_o, t_i, t_o, \mu) \tag{2.1}$$

$$t = t(h_i, h_o, t_i, t_o, \mu) \ . \tag{2.2}$$

As it will become clear later a third equation for the neutral point thickness, h_n (See Figure 2) is also required.

$$h_n = h_n(h_i, h_o, t_i, t_o, \mu) \tag{2.3}$$

Analytic expressions for (2.1, 2 and 3) developed by Bland and Ford are given in Appendix 2.

These expressions are complicated nonlinear functions of mill variables and a simplification of their calculation is needed. In [18] a graphical method was proposed. Some computer iterative methods have been suggested [1,2], where small (1%) incremental changes of different mill variables are made to perform linearization of the force-torque equations. A computationally simpler and in the same time more accurate approximation is given in [21] where 2.1, 2 and 3 are expanded using explicit differentiation. The truncated series are

$$\hat{F} = \langle k, q \rangle + \langle q, Kq \rangle \tag{2.4}$$

$$\hat{\tau} = \langle \ell, q \rangle + \langle q, Lq \rangle \tag{2.5}$$

$$\hat{h}_n = \langle m, q \rangle + \langle q, Mq \rangle \tag{2.6}$$

where q is a vector with the mill variables as its components.

$$q^T = [\hat{h}_i \ \hat{h}_o \ \hat{t}_i \ \hat{t}_o \ \hat{\mu}], \tag{2.7}$$

k is a 5X1 normalized vector, $k_i = \frac{q_i^*}{F^*} \frac{\partial F}{\partial q_i}$ i = 1,...5 and K is a 5X5 symmetric matrix whose elements are, $k_{ij} = \frac{1}{2} \frac{q_i^* q_j^*}{F^*} \frac{\partial^2 F}{\partial q_i \partial q_j}$. Similar definitions apply for ℓ, L, m, and M for (2.5) and (2.6). The computational scheme developed in [21] consists of two main subroutines which allows all the mill coefficients to be obtained in less than 20 seconds on a CDC-1604 computer.

A common assumption in all rolling theories is that the volume of the material per unit time passing through each vertical segment is constant, i.e.

$$v_i h_i = ... = v_n h_n = ... = v_o h_o \quad . \tag{2.8}$$

2.2. Stand Relations

The output tension of the jth stand is assumed to be equal to the input tension of the (j+1)th stand, i.e.

$$t_{i,j+1}(t) = t_{oj}(t) \quad . \tag{2.9}$$

The output tension is related to the strip velocity by the elasticity principle (Hook's law) [22-24],

$$\dot{\hat{t}}_{oj} = \frac{E}{T} (\hat{v}_{i,j+1} - \hat{v}_{o,j}) \tag{2.10}$$

where it is assumed that the delay time T (transit time between stands j and j+1) is constant [14,18]. The input thickens at the (j+1)th stand is the same as the output thickens at the jth stand delayed by T, i.e.

$$h_{i,j+1}(t) = h_{o,j}(t-T) \ . \tag{2.11}$$

As noted before when force is applied to the rolls an elastic deformation occurs to the "mill housing" and the rolls, i.e. the entire structure (stand and its four highs) is considered to be a stiff spring with a modulus of elasticity (spring constant) M' [1,2,25]. Thus,

$$F_j = M'(S_j - S_{o,j}) \tag{2.12}$$

where S_j is the screw-down setting (separation of rolls during rolling, positive upward) of the jth stand and $S_{o,j}$ is the screw-down setting when $F_j = 0$. If the elastic recovery of the metal is neglected as in the case in Bland and Ford's theory [1,2,15,25] then the output thickness $h_{o,j} = S_j$ and (2.12) in incremental form becomes [2],

$$\hat{h}_{o,j} = \left(\frac{1}{h_{o,j}}\right) S_{o,j} + \left(\frac{F}{Mh_{o,j}}\right) \hat{F}_j \ . \tag{2.13}$$

3. ELECTRICAL SUBSYSTEM

The electrical subsystem consists of the control rectifier motor groups for coiler, decoiler, stands, drives, and screw-down servomechanisms. In order to preserve generolity, our model is derived with the assumption that the armature voltages of the motors are control variables. In a solution of an optimum control problem the dynamics of the generators or rectifiers will have to be included, and the motor armature voltages will become state variables.

In this section the dynamic equations of motors are given. A block diagram of both mechanical and electrical subsystems shows the interconnection between process variables.

3.1 Coiler and Decoiler

The coiler and decoiler are driven by a dc motor which is controlled by both armature and field voltages.

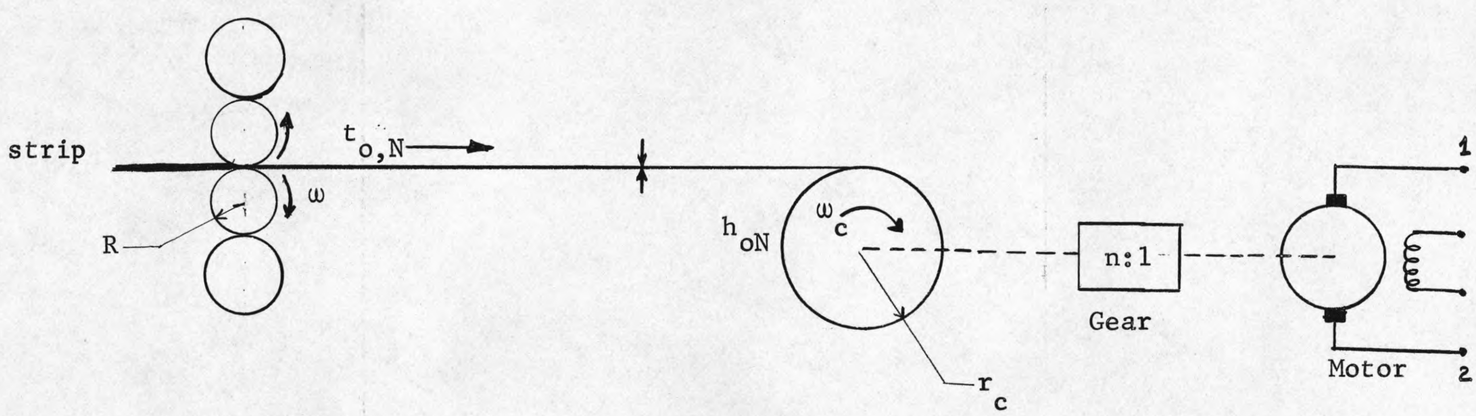

Figure 3. Coiler schematic.

The torque equation at the coiler's shaft, Figure 3, is [23,26].

$$\Sigma\tau = \tau_c = J_c\dot{\omega}_c + \dot{J}_c(\omega_c - \omega\ ) + B_c\omega_c + \frac{1}{n}r_c t_{oN} \quad (3.1)$$

where the term $\dot{J}_c(\omega_c - \omega_N)$ accounts for the fact that the inertia of the coiler, J_c, is a time-varying function [27-29]. The term $\omega_c - \omega_N$ is the magnitude of the angular velocity of incoming strip relative to that of the coiler. The remaining relationships for the system in Fig. 3 are

$$\tau_c = K_{1c}\phi_c\, i_{ac} \quad (3.2)$$

$$e_{ac} = R_{ac}\, i_{ac} + L_a \frac{di_a}{dt} + K_{bc}\,\omega_c \quad (3.3)$$

$$e_{fc} = R_{fc}\, i_{fc} + N_{fc}\frac{d\phi_c}{dt} \quad (3.4)$$

$$\dot{r}_c = \frac{h_{oN}}{2\pi}\,\omega_c \quad (3.5)$$

$$i_{fc} = c_1\phi_c + c_3\phi_c^3 \quad (3.6)$$

$$J_c(r_c) = J_m + \frac{1}{n^2}J_L = J_m + \frac{1}{2n^2}\pi\rho W\, r_c^4 \quad (3.7)$$

and $B_c = B_m + \frac{1}{n^2}B_L$.

All the quantities are defined in the nomenclature (Appendix 1).

A similar set of equations can be written for decoiler by interchanging subscripts c (coiler) to d (decoiler) and N to 1. For example, as an equivalent to 3.1 one obtains,

$$\Sigma\tau = \tau_d = J\dot{\omega}_d + \dot{J}_d\,(\omega_1 - \omega_d) + B_d\omega_d - \frac{1}{n}r_d t_{i1} \quad (3.8)$$

where $\tau_d = K_{1d}\phi_d i_{ad}$, etc.

Note that a nonlinear magnitization relation, (3.6) is assumed between the field current and the flux of the motors.

3.2 Main Drive

The torque, armature and field circuit equations for the jth stand are presented. The stand's drive motor must provide torque sufficient to balance frictions rolling torque, torque caused by inlet tension and the acceleration torque. The torque caused by outlet tension is helping the rolls to rotate in the prescribed directions. Thus,

$$\Sigma\tau = \tau_m + \tau_o - J_\omega - \tau_f - \tau_i - \tau_j = 0 \tag{3.9}$$

where τ_m, o, f, i, j indicate the torque of motor, outlet tension, friction, inlet tension, and jth rolls, respectively. Equation (3.9) can be written for the jth stand as

$$\tau_{mj} = J\dot{\omega}_j + B\omega_j + \frac{1}{n} Rt_{ij} = \frac{1}{n} Rt_{oj} + \tau_j \tag{3.10}$$

where,

$$\tau_{mj} = K_1 \phi_j \, i_{aj} \tag{3.11}$$

The armature and field currents are governed by,

$$e_{aj} = L_a \frac{d(i_{aj})}{dt} + R_a i_{aj} + K_b \omega_j \tag{3.12}$$

$$e_{fj} = R_f i_{fj} + N_f \dot{\phi}_j \tag{3.13}$$

The field current-flux are related by the familiar magnitization characteristic approximated by third order polynomial,

$$i_{fj} = c_1\phi_j + c_3\phi_j^3 \tag{3.14}$$

Thus three variables, ω_j, i_{aj}, and ϕ; are sufficient for a state space model of the main drive.

3.3 Screw-Down

The screw-down motors are assumed to be of the armature-controlled direct current type. The task of these motors is the adjustment of the vertical position of rolls (settings or screw pitch) [30]. The equations for the jth stand screw-down motor are,

$$\tau_{sj} = J_s \dot{\omega}_{sj} + B_s \omega_{sj} \tag{3.15}$$

$$\tau_{sj} = K_1 \phi_{sj} i_{asj} \tag{3.16}$$

$$e_{asj} = R_a i_{asj} + L_a \frac{d(i_{asj})}{dt} + K_b \omega_{sj} \tag{3.17}$$

and the screw speed is assumed to be directly proportional to the angular velocity of the motor,

$$\dot{S}_{oj} = K_s \omega_{sj} \tag{3.18}$$

where S_{oj} is the screw pitch of the jth stand rolls.

3.4 Mill Block Diagram

The mechanical and electrical subsystems for the jth stand can be shown in a general block diagram as in Fig. 4. Note that for j = 1 and N, the diagram involves the decoiler or coiler motors, respectively and are considered as special cases of this diagram.

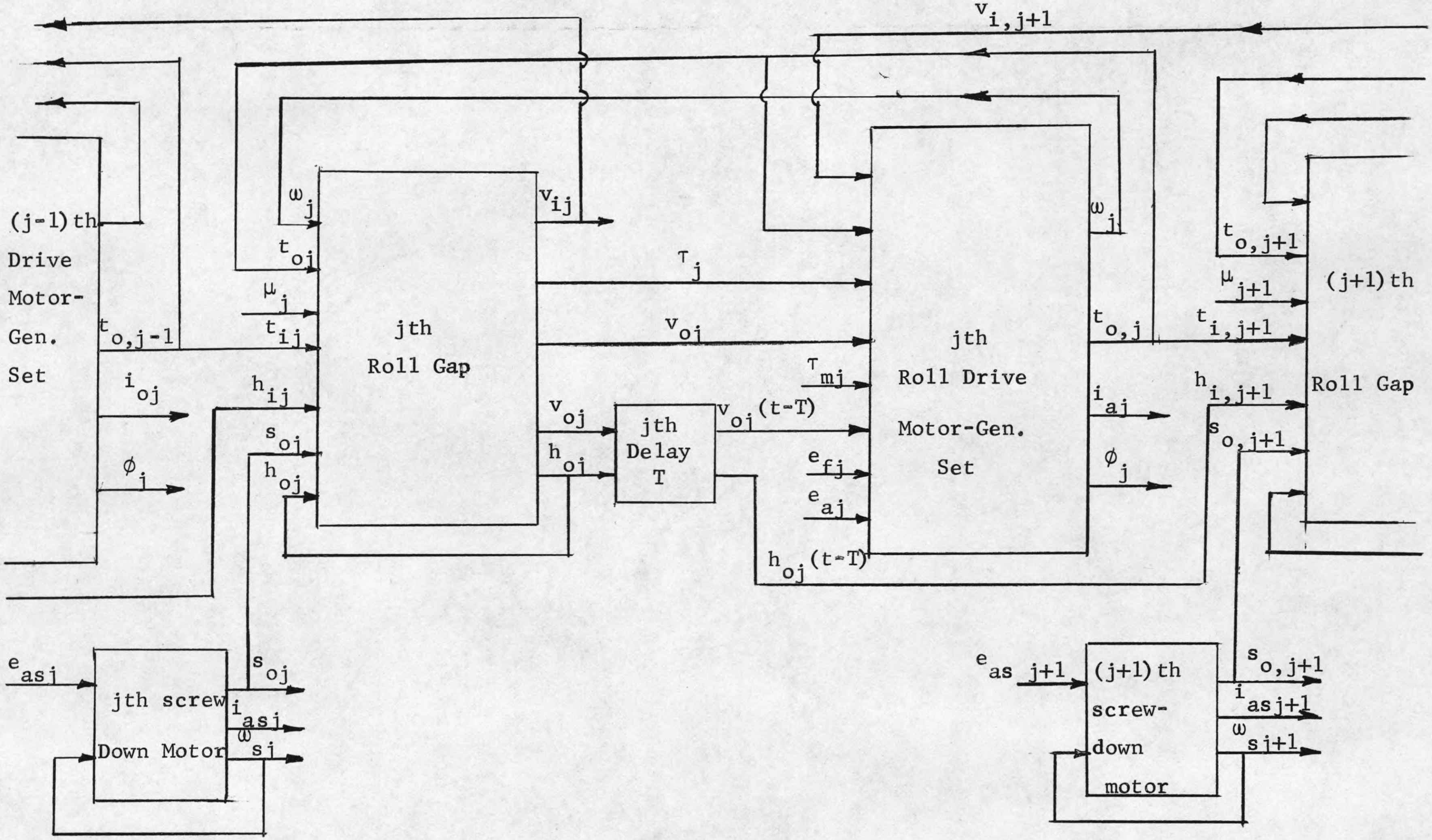

Figure 4. Mechanical-electrical subsystem block diagram.

4. State Equations

Table 1 shows a possible selection of state, control and output variables of the system of Figures 1,2. For a single stand rolling mill, i.e., $j = N = 1$ there may be as many as $7 + 8 = 15$ state variables, $3 + 4 = 7$ control variables and 2 output variables.

Let

$$\begin{aligned} x_{1j} &= \omega_j & u_{1j} &= e_{aj} \\ x_{2j} &= \tilde{t}_{0j} & u_{2j} &= e_{fj} \\ x_{3j} &= i_{aj} & u_{3j} &= e_{asj} \\ x_{4j} &= \phi_j \\ x_{5j} &= \omega_{sj} \\ x_{6j} &= S_{0j} \\ x_{7j} &= i_{asj} \end{aligned} \tag{4.1}$$

Considering (2.4) and rewrite it as:

$$\begin{aligned} \hat{F}_j &= \langle k, q_j \rangle + \langle q_j, K\, q_j \rangle \\ &= k_{22}\, \hat{h}_{0j}^2 + (k_2 + k_{12}\, \hat{h}_{ig} + k_{23}\, \tilde{t}_{ij} + k_{25}\, \mu_j)\, \hat{h}_{aj} \\ &\quad + \langle \underline{k}, q_j \rangle + \langle q_j, \underline{K}\, q_j \rangle \end{aligned} \tag{4.2}$$

where

$$\underline{k} = \underline{I}k,\ \underline{K} = \underline{I}K,\ \underline{I} = \text{diag}\,[1\ 0\ 1\ 1\ 1],$$

note that vector k and matrix K are previously computed for stand j and

Table 1

System Variables in an N-Stand Rolling Mill

Type of Variable	Stand j: Roll Gap	Stand j: Drive Motor	Stand j: Screw-Down	Decoiler	Coiler	Total no. of Variables
State	$\tilde{t}_{0j}$	ω_j, i_{aj}, ϕ_j	$\omega_{sj}, S_{0j}, i_{as_j}$	$\omega_d, i_{od}, \phi_d, r_d$	$\omega_c, i_{ac}, \phi_c, r_c$	7N + 8
Control	-	e_{aj}, e_{fj}	e_{as_j}	e_{ad}, e_{fd}	e_{ac}, e_{fc}	3N + 4
Output	$\hat{h}_{oj}$ $\tilde{t}_{oj}$	-	-	-	-	2N

the subscript has been omitted on them for simplicity.

Eliminating $\hat{F}_j$ from (4.2) and (2.13),

$$\beta_j k_{22}\hat{h}_{0j}^2 + [\beta_j(k_2 + k_{12}\hat{h}_{ih} + k_{23}\tilde{t}_{ij} + k_{24}\tilde{t}_{0j} + k_{25}\mu_j)$$

$$-1]\ \hat{h}_{0j} + (\alpha_j S_{0j} + \beta_j \langle \underline{k}, q_j \rangle + \beta_j \langle q_j, \underline{K} q_j \rangle) = 0$$

where $\alpha_j = (\frac{1}{h^*_{0j}})$, $\beta_j = (F^*_j / M h'^*_{0j})$ as defined in (2.13).

Solving for $\hat{h}_{0j}$ from the above and considering (2.9) and notations of (4.1),

$$\hat{h}_{0j} = a_{3j}\, x_{2,j-1} + a_{4j} x_{2j} \pm \eta_j(t-T) + a_{5j} \tag{4.3}$$

where, a_{ij}, i = 3,4,5 and $\eta_j(t-T)$ are defined in Appendix 3.

Similarly torque and thickness equations of (2.5) and (2.6) after the substitution of $\hat{h}_{0j}$ from (4.3) can be rewritten as,

$$\hat{t}_j = b_{3j} x_{2,j-1} + b_{4j} x_{2j} + \phi_j(t-T) + b_{5j} \tag{4.4}$$

b_{ij}, i = 3,4,5 and $\phi_j(t-T)$ are expressed in Appendix 3.

Similarly 2.6 can be reduced to,

$$\hat{h}_{nj} = C_{3j} x_{2,j-1} + C_{4j} x_{2j} + \psi_j(t-T) + C_{5j} \tag{4.5}$$

where C_{nj}, n = 3,4,5 and ψ_j have expressions identical with b_{nj}, n = 3,4,5 and ϕ_j when replacing ℓ and $\underline{L}$ by m and $\underline{M}$, respectively.

To derive the first state equation let us write the torque equation at the shaft of the j^{th} stand drive motor,

$$J\dot{\omega}_j = -B\omega_j + K_1 \, i_{aj}\phi_j - \frac{R}{n} A_{ij} \, \tilde{t}_{ij} + \frac{RA}{n}, \tilde{t}_{oj} -$$

$$\frac{1}{n}\tau^*_j \hat{\tau}_j - \frac{1}{n}(R(t^*_{ij} - t^*_{0j}) + \tau^*_j) \qquad (4.6)$$

Substituting for $\hat{\tau}_j$ from (4.7)

$$\dot{x}_{1j} = - \frac{B}{J} x_{1j} - \frac{1}{Jn}(b_{3j}\tau^*_j + RA_{ij}) \, x_{2,j-1} -$$

$$\frac{1}{Jn}(b_{4j}\tau^*_j - RA_{0j})x_{2j} - \frac{\tau^*_j}{Jn}\phi_j - d_{1j} \qquad (4.7)$$

where

$$d_{1j} = \frac{1}{Jn}(R(t^*_{ij} - t^*_{0j}) + (b_{5j}+1)t^*_j) \qquad (4.8)$$

To drive the state quantum for $\tilde{t}_{0j}$, consider equations (2.8), (2.10), and (2.11),

$$\hat{v}_{i,j+1} = -1 + \frac{v_{i,j-11}}{v^*_{i,j+1}} = -1 + \frac{1}{v^*_{i,j+1}} \, \frac{v_{n,j+1} \, h_{n,j+1}}{h_{i,j+1}}$$

$$= -1 + \frac{Rh^*_{n,j+1}}{v^*_{i,j+1}h^*_{i,j+1}} \, \frac{\omega_{j+1}(\hat{h}_{n,j+1} + 1)}{(\hat{h}_{0j}(t-T) + 1)} \qquad (4.9)$$

Similarly,

$$\hat{v}_{o,j} = \frac{v_{o,j}}{v^*_{o,j}} - 1 = -1 + \frac{Rh^*_{n,j}}{v^*_{o,j} h^*_{oj}} \cdot \frac{\omega_j(\hat{h}_{nj}+1)}{(\hat{h}_{oj}+1)} \tag{4.10}$$

then by (2.10)

$$\dot{x}_{2j} = g_{2j}(\underline{x}, \underline{x}(t-T), \underline{x}(t-2T)) \tag{4.11}$$

where g_{2j} is defined in Appendix 3. Note that by $\underline{x}$ in (4.11) it is meant a vector containing all state variables as defined by (4.1).

The remaining state equations are simple and are written according to (3.12-18).

$$\dot{x}_{3j} = -\frac{Ra}{La} x_{3j} - \frac{K_b}{L_a} x_{1j} + \frac{1}{L_a} u_{1j} \tag{4.12}$$

$$\dot{x}_{4j} = -\frac{R_f e_1}{N_f} x_{4j} - \frac{R_f e_3}{N_f} x^3_{4j} + \frac{1}{N_f} u_{2j} \tag{4.13}$$

$$\dot{x}_{5j} = -\frac{B_s}{J_s} x_{5j} + \frac{K_{is}}{J_s} x_{7j} \tag{4.14}$$

$$\dot{x}_{6j} = -K_s x_{5j} \tag{4.15}$$

$$\dot{x}_{7j} = -\frac{R_{as}}{L_{as}} x_{7j} - \frac{K_b}{L_{as}} x_{5j} + \frac{1}{L_{as}} u_{3j} \tag{4.16}$$

Thus 4.7, 4.11, 4.12 to 4.16 are severn state equations describing the dynamics of the j^{th} stand. In addition to the above, eight other equations are needed to describe coiler and decoiler;

let

$M = 7N$, N is the number of stands in the rolling mill installation, and

Decoiler:

$$x_{M+1} = \omega_d \quad x_{M+2} = L_{ad} \quad x_{M+3} = \phi_d \quad x_{M+4} = r_d \quad u_{2N+1} = e_{ad} \quad u_{3N+2} = e_{fd}$$

Coiler:

$$x_{M+5} = \omega_c \quad x_{M+6} = L_{ac} \quad x_{M+7} = \phi_c \quad x_{M+8} = r_c \quad u_{3N+3} = e_{ac} \quad u_{3N+4} = e_{fc} \tag{4.17}$$

The corresponding state equations are,

$$\dot{x}_{M+1} = - B_d \frac{x_{M+1}}{J_d(x)} - 2W\rho h^*_{ij} \frac{x_{M+4}x_{11}}{J_d(x)} + 2W\rho h^*_{i1} \frac{x^2_{M+1}x^3_{M+4}}{J_d(x)} + \frac{A_{i1}}{n} \frac{x_{M+4}x_{M+5}}{J_d(x)} + K_{1d} \frac{x_{M+2}x_{M+3}}{J_d(x)} \tag{4.18}$$

where

$$J(x) = J_m + \frac{1}{2n^2} \pi\rho W x^4_{M+4}$$

$$\dot{x}_{M+2} = -\frac{R_{ad}}{L_{ad}} x_{M+2} - \frac{K_{bd}}{L_{ad}} x_{M+1} + \frac{1}{L_{ad}} u_{3N+1} \tag{4.19}$$

$$\dot{x}_{M+3} = -\frac{R_f d_1}{N_{fd}} x_{M+3} - \frac{R_{fd} d_3}{N_{fd}} x^3_{M+3} + \frac{1}{N_{fd}} u_{3N+2} \qquad (4.20)$$

$$\dot{x}_{M+4} = \frac{h_{i1}}{2\pi} x_{M+1} \qquad (4.21)$$

$$\dot{x}_{M+5} = -B_c \frac{x_{M+5}}{J_c(x)} + 2W\rho h^*_{oN} \frac{x^3_{M+8} x_N}{J_c(x)} +$$

$$2W\rho h^*_{oN} \frac{x^2_{M+5} x^3_{M+8}}{J_c(x)} - \frac{A_{ON}}{n} \frac{x_{M+8} x_{2N}}{J_c(x)} + K_{1c} \frac{x_{M+8} x_{M+7}}{J_c(x)} \qquad (4.22)$$

when

$$J_c(x) = J_m + \frac{1}{2n^2} \pi\rho W x^4_{M+8}$$

$$\dot{x}_{M+6} = -\frac{R_{ac}}{L_{ac}} x_{M+6} - \frac{K_{bc}}{L_{ac}} x_{M+5} + \frac{1}{L_{ac}} u_{3N+3} \qquad (4.23)$$

$$\dot{x}_{M+7} = -\frac{R_{fc} c_1}{N_{fc}} x_{M+7} - \frac{R_{fc} c_3}{N_{fc}} u_{3N+3} \qquad (4.23)$$

$$\dot{x}_{M+7} = -\frac{R_{fc} c_1}{N_{fc}} x_{M+7} - \frac{R_{fc} c_3}{N_{fc}} x^3_{M+7} + \frac{1}{N_{fc}} u_{3N+4} \qquad (4.24)$$

$$\dot{x}_{M+8} = \frac{h^*_{oN}}{2\pi} x_{M+5} \qquad (4.25)$$

The above equations as well as those for the j^{th} stand can now be written in a vector form for an N- stand rolling mill as follows,

Let

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ x_{7N+8} \end{bmatrix} = \left[\begin{array}{c} x_{11} \\ x_{21} \\ \vdots \\ x_{71} \\ \hline \vdots \\ \hline x_j \\ x_{2j} \\ \vdots \\ x_{7j} \\ \hline \vdots \\ \hline x_{1N} \\ x_{2N} \\ \vdots \\ x_{7N} \\ \hline x_{7N+1} \\ \vdots \\ x_{7N+8} \end{array}\right] \begin{array}{l} \\ \text{Stand 1} \\ \\ \\ \cdots \\ \\ \text{Stand j} \\ \\ \\ \cdots \\ \\ \text{Stand N} \\ \\ \\ \text{Coiler} \\ \text{and} \\ \text{Decoiler} \end{array} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ u_{3N+4} \end{bmatrix} = \left[\begin{array}{c} u_1 \\ u_3 \\ u_3 \\ \hline \vdots \\ \hline u_j \\ u_{2j} \\ u_{3j} \\ \hline \vdots \\ \hline u_N \\ u_{2N} \\ u_{3N} \\ \hline u_{3N+1} \\ \vdots \\ u_{3N+4} \end{array}\right] \tag{4.26}$$

be 7N + 8th and 2N + 4th state and control vectors for the N- stand rolling mill then the general form of the state equations is,

$$\dot{x} = A\mathbf{x} + Bu + f(x) + g(\mathbf{x}, \mathbf{x}(t-T), \mathbf{x}(t-2T)) + d$$

where

$$
\begin{array}{lll}
A & 7N + 8 \times 3N + 8 \text{ constant matrix} & \\
B & 7N + 8 \times 2N + 4 \text{ constant matrix} & (4.27) \\
f, g, d, \text{ and } \mathbf{x} \text{ are} & 7N + 8 \text{ vector} & \\
u \text{ is} & 7N + 4 \text{ vector} &
\end{array}
$$

f and g are nonlinear vector functions.

The above matrices and vectors are defined on the following pages.

$$
A=\left[\begin{array}{ccccccc|cccc|ccc}
a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 & & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & & \\
a_{31} & 0 & a_{33} & 0 & 0 & 0 & 0 & & & & & & & \\
0 & 0 & 0 & a_{44} & 0 & 0 & 0 & & & & & & & \\
0 & 0 & 0 & 0 & a_{55} & 0 & a_{57} & & & & & & & \\
0 & 0 & 0 & 0 & a_{65} & 0 & 0 & & & & & & & \\
0 & 0 & 0 & 0 & a_{75} & 0 & a_{77} & & & & & & & \\
\hline
 & & & & & & & \ddots & & & & & & \\
\hline
 & & & & & & & 0 & a_{7N-6,7N-12} & 0 & 0 & a_{7N-6,7N-6} & a_{7N-6,7N-5} & 0 \\
 & & & & & & & & & & & 0 & 0 & 0 \\
 & & & & & & & & & & & a_{7N-7,7N-6} & 0 & a_{7N-4,7N-4} \\
 & & & & & & & & & & & 0 & 0 & 0 \\
 & & & & & & & & & & & 0 & 0 & 0 \\
 & & & & & & & & & & & 0 & 0 & 0 \\
 & & & & & & & & & & & 0 & 0 & 0 \\
\hline
 & & & & & & & & & & & & & \\
\end{array}\right]
$$

Stand 1												
...												
Stand N	0	0	0	0								
	0	0	0	0								
	0	0	0	0								
	$a_{7N-3,7N-3}$	0	0	0								
	0	$a_{7N-2,7N-2}$	0	$a_{7N-2,7N}$								
	0	$a_{7N-1,7N-2}$	0	0								
	0	$a_{7N,7N-2}$	0	$a_{7N,7N}$								
Coiler and Decoiler					0	0	0	0	0	0	0	0
					$a_{7N+2,7N+1}$	$a_{7N+2,7N+2}$	0	0	0	0	0	0
					0	0	$a_{7N+3,7N+3}$	0	0	0	0	0
					$a_{7N+4,7N+1}$	0	0	0	0	0	0	0
					0	0	0	0	0	0	0	0
					0	0	0	0	$a_{7N+6,7N+5}$	$a_{7N+6,7N+6}$	0	0
					0	0	0	0	0	0	$a_{7N+7,7N+7}$	0
					0	0	0	0	$a_{7N+8,7N+5}$	0	0	0

$$
B=\left[\begin{array}{ccc|ccc|cccc}
0 & 0 & 0 & & & & & & & \\
0 & 0 & 0 & & & & & & & \\
b_{31} & 0 & 0 & & & & & & & \\
0 & b_{42} & 0 & & & & & & & \\
0 & 0 & 0 & & & & & & & \\
0 & 0 & 0 & & & & & & & \\
0 & 0 & b_{73} & & & & & & & \\
\hline
& \cdots & & & & & & & & \\
& \cdots & & & & & & & & \\
& \cdots & & & & & & & & \\
& \cdots & & & & & & & & \\
& \cdots & & & & & & & & \\
\hline
& & & 0 & 0 & 0 & & & & \\
& & & 0 & 0 & 0 & & & & \\
& & 0 & b_{7N-4,3N-2} & 0 & 0 & & & & \\
& & & 0 & b_{7N-3,3N-1} & 0 & & & & \\
& & & 0 & 0 & 0 & & & & \\
& & & 0 & 0 & 0 & & & & \\
& & & 0 & 0 & b_{7N,3N} & & & & \\
\hline
& & & & & & 0 & 0 & 0 & 0 \\
& & & & & & b_{7N+2,3N+1} & 0 & 0 & 0 \\
& & & & & & 0 & b_{7N+3,3N+2} & 0 & 0 \\
& & & & & & 0 & 0 & 0 & 0 \\
& & & & & & 0 & 0 & 0 & 0 \\
& & & & & & 0 & 0 & b_{7N+6,3N+3} & 0 \\
& & & & & & 0 & 0 & 0 & b_{7N+7,3N+4} \\
& & & & & & 0 & 0 & 0 & 0
\end{array}\right]
\begin{array}{l}
\text{Stand 1} \\ \vdots \\ \text{Stand N} \\ \text{Coiler and Decoiler}
\end{array}
$$

$$
d = \begin{bmatrix} d_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ d_{7N-6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\qquad
f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_4 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \\ f_{7N-3} \\ 0 \\ 0 \\ 0 \\ f_{7N+1} \\ 0 \\ f_{7N+3} \\ 0 \\ f_{7N+5} \\ 0 \\ f_{7N+7} \\ 0 \end{bmatrix}
\qquad
g = \begin{bmatrix} g_1 \\ g_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ g_{7N-6} \\ g_{7N-5} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Stand 1

...

Stand N

Coiler and Decoiler

5. CONCLUSIONS

A dynamic model of an N-stand continuous cold reduction mill has been developed. A generalized state equation of (7N + 8)th order was derived taking into account the dynamics of coiler, stands, drive, screw-down motors and decoiler. The equations turned out to be highly nonlinear and in general involve time lags due to the transition of strip from one stand to the next.

This model is expected to be a more accurate description of a rolling mill than the models proposed thus far since it uses less approximations such as linearizations of force-torque as well as the conservation of the strip volume (eq. 2.8). The main reasons for avoiding these approximations is that with the existence of digital computers and efficient numerical methods more realistic models can be handled.

The state equations can be modified for rolling mills with different electrical equipment and introduction of new variables cause no difficulty in formulating the new set of equations.

REFERENCES

1. Phillips, R. A., "Analysis of Tandem Cold Reduction Mill with Automatic Gauge Control", AIEE Trans., January, 1957, pp. 355-363.

2. Smith, H. W., "Dynamic Control of a Two-Stand Cold Mill", Automatica IFAC, Vol. 5, 1969, pp. 183-190.

3. Harvie, R. A., "Electrical Drive for 56 in Five-Stand Tandem Cold Reduction Mill", ISE Journal, Vol. 27, August, 1950, p. 49.

4. Kokotović, P. V. and P. Sannuti, "Singular Perturbation Method for Reducing the Model Order in Optimal Control Design", IEE Trans. AC, Vol. AC-13, No. 4, August, 1968, pp. 377-84.

5. Sannuti, P. and P. V. Kokotović, "Near-Optimum Design of Linear Systems by a Singular Perturbation Method", IEEE Trans. AC, Vol. AC-14, No. 1, February, 1969.

6. "Singular Perturbation Method for Near Optimum Design of High-Order Nonlinear Systems", Proc. IFAC, 4th Congress, 1969.

7. Kokotović, P. V., W. R. Perkins, J. B. Cruz, Jr., and G. D'Ans, "ϵ-coupling Method for Near-Optimum Design of Large-Scale Linear Systems", Proc. IEE, Vol. 116, No. 5, May, 1969, pp. 889-92.

8. Jamshidi, M., C. Hadlock, and P. Kokotović, "Near Optimum Design of Three Time Scale Systems", to appear.

9. Gabasov, R. and S. V. Churakova, "Necessary Optimality Conditions in Time-Lag Systems," Automation and Remote Control (Trans. Russian) Consulting Bureau, New York, New York, January, 1968, pp. 37-45.

10. Lee, E. B., "Variational Problems for Systems Having Delay in the Control Action", IEEE Trans. AC, December, 1968, pp. 697-699.

11. Chayung, D. H. and E. B. Lee, "Linear Optimal Control Problem with Delays", J. SIAM on Control, Vol. 4, pp. 548-575, 1966.

12. Ragg, B. C., "Necessary Conditions for the Optimal Control of a System with Time Varying Transport Lags", IEEE Trans. AC, October, 1966.

13. Orowan, E., "The Calculation of Roll Pressure in Hot and Cold Flat Rolling", Proc. Inst. Mech. Engr., Vol. 150, 1943, pp. 140-167.

14. Finne, R., J. D. Jacob, and J. Rissanen, "A Mathematical Model of Strip Rolling Mills", IBM Nordiska Laboratorier TP18.0 94, January, 1963, Nordiska, Sweden.

15. Bland, D. P. and H. Ford, "The Calculation of Roll Force and Torque in Cold Strip Rolling With Tensions", Proc. Inst. Mech. Engrs., Vol. 159, pp. 144-63, 1948.

16. Ford, H. "Researches into the Deformation of Metals by Cold Rolling", Proc. Inst. Mech. Engrs., Vol. 159, p. 115, 1948.

17. Ford, H., F. Ellis, and D. R. Bland, "Cold Rolling with Strip Tensions", J. Iron and Steel Instr., Vol. 168, pp. 57-68, London, England.

18. Hessenberg, W. C. F., and R. B. Sims, "Principles of Continuous Gauge Control in Sheet and Strip Rolling", Inst. Mech. Engrs. J., Vol. 155, 1952, pp. 75-81, London, England.

19. Ford, H., F. Ellis, "Comparison of Calculated and Experimental Results", J. Iron and Steel Inst., Vol. 171, pp. 239-245, 1951.

20. Bland, D. R. and H. Ford, "An Approximate Treatment of the Elastic Compression of the Strip in Cold Rolling", J. Iron and Steel Inst., Vol. 171, October, 1951, pp. 245-249.

21. Jamshidi, M. and P. Kokotović, "An Approximation of Force-Torque Equation in Rolling Mills", Coordinated Science Laboratory Report R-452, University of Illinois, Urbana, Illinois.

22. Durelli, A. J., E. A. Phillips, and C. H. Tsao, Introduction to the Theoretical and Experimental Analysis of Stress and Strain, McGraw-Hill Book Co., New York, 1958.

23. Kokotovic, P. and S. Bingulac, "Model Feedback Concept in Analog Computer Design of Control Systems", Proc. Analog. Comp. Conference, pp. 291-298, Belgrade, Yoguslavia.

24. Moorev, M. V., Multivariable Control Systems, (transl. Russian), published for NASA and NSF Israel Program for Scientific Translation, Jerusalem, 1968.

25. Courcoulas, J. H. and J. M. Ham, "Incremental Control Equations for Tandem Rolling Mills", AIEE Trans., January 1957, pp. 363-371.

26. Gourishankar, V., Electromechanical Energy Conversion, International Book Co., Scranten, Pennsylvania, 1967.

27. Coughanowr and Koppel, Process Systems Analysis and Control, McGraw-Hill, New York, 1964.

28. Shinskey, M., Process Control Systems, McGraw-Hill, 1960.

29. Perkins, W. R. and J. B. Cruz, Jr., Engineering of Dynamic Systems, John Wiley and Sons Book Co., New York, 1969.

30. Johnson, F. G., J. H. Stoltz, and R. G. Chapman, Jr., "Electrical Features of Combinational Temper - Reduction Mills", ISE Journal, October, 1965, pp. 125-134.

Appendix 1. Notations and Nomenclature

Notations:

i	= "entry" or inlet plane
o	= "exit", or outlet plane
n	= neutral plane
j	= number of stand $j = 1,\ldots,N$
N	= maximum number of stands in the mill
$\wedge$	= $\hat{F} = \frac{F - F^*}{F^*}$, denotes the operating value
t_{ij}	= the inlet tension, t_i belonging to the jth stand.
s	= screw down motors
c	= coiler
d	= decoiler
m	= motor (coiler, decoiler)
L	= load

Nomenclature:

e_f	= field voltage
e_a	= armature voltage
ω	= angular velocity
S_{oj}	= screw-down setting of jth stand rolls
θ	= arc of contact (rolls and strip)
R	= radius of undeformed roll
R'	= radius of deformed roll
h	= thickness of the strip

$\tilde{t}$	=	tension per unit area
A	=	area of the strip
τ	=	roll torque per unit width
F	=	roll force per unit width
μ	=	coefficient of friction
v	=	strip velocity
E	=	modulus of elasticity (Young's modulus)
T	=	time lag (strip transit period)
M'	=	modulus of elasticity (spring constant) of "mill housing"
J	=	moment of inertia
B	=	friction loss
n	=	gear ratio
r	=	coiler or decoiler radius
K_1	=	torque constant
$\emptyset$	=	magnetic flux
i_a	=	armature current
R_a	=	armature resistance
L_a	=	armature inductance
K_b	=	back emf constant
L_f	=	field current
K_b	=	back emf constant
L_f	=	field inductance
R_f	=	field resistance
N_f	=	field winding number of turns
C_1,C_3	=	coefficients of field current polynomial

ρ	=	density of rolled material
w	=	width of the strip
K_s	=	screw-down position gears constant
d	=	disturbance vector
x	=	state vector
u	=	control vector

Appendix 2. Roll Force and Torque Equations

[Bland and Ford]

$$F = F(h_i, h_o, t_i, t_o, r) = R'\left[\int_o^{\theta_n} \frac{kh}{h_o}\left(1 - \frac{to}{k_o}\right) e^{\mu H(h_o,\theta)} d\theta + \int_{\theta_n}^{\theta_i} \frac{kh}{h_i}\left(1 - \frac{t_i}{k_i}\right) e^{\mu\left(H_i(h_o,\theta_i) - H(h_o,\theta)\right)}\right] \quad (2.1)$$

$$\tau = \tau(h_i, h_o, t_i, t_o, \mu) = RR' \int_o^{\theta_n} \frac{kh}{h_o}\left(1 - \frac{to}{k_o}\right) \theta e^{\mu + 1} d\theta + \int_{\theta_n}^{\theta_i} \frac{kh}{h_i}\left(1 - \frac{t_i}{k_i}\right) + \frac{h_i t_i - h_o t_o}{2R'} \quad (2.2)$$

where

$$H(h,\theta) = 2\sqrt{\frac{R'}{h}} \tan^{-1}\sqrt{\frac{R'}{h}}\,\theta$$

and

$$h_n = h_n(h_i, h_o, t_i, t_o\tau) = h_o\, 1 + \tfrac{1}{4} \tan^2\left[\sqrt{\frac{R'}{h_o}}.\tan^{-1} \sqrt{\frac{R'}{h_o}}\,\theta_i - \frac{1}{2\mu}\ln \frac{h_i(1 - to/ko)}{h_o\left(1 - \frac{t_i}{k_i}\right)}\ 2\right] \quad (2.3)$$

Appendix 3: Expressions Involving Equations (4.3), (4.4), and (4.11)

In Equation (4.3):

$$a_{3j} = -\frac{k_{23}}{2k_{22}}, \quad a_{4j} = -\frac{k_{24}}{2k_{22}},$$

$$a_{5j} = -\frac{k_{25}\mu_j}{2k_{22}} + \frac{\beta_j k_2 - 1}{2\beta_j k_{22}} \quad \text{, all k's are for jth stand,}$$

$$\pm \eta_j(t-T) = -\frac{k_{12}}{2k_{22}} \hat{h}_{o,j-1}(t-T) \pm \{[\beta(k_2 + k_{12} h_{o,j-1}(t-T) + k_{23} x_{2,j-1} + k_{24} x_{2,j} + k_{25}\mu_j) - 1]^2 - 4\beta k_{22}(\alpha_j x_{6j} + \beta_j \langle \underline{k}, q_j \rangle + \beta_j \langle q_j, \underline{K} q_j \rangle)\}^{\frac{1}{2}} .$$

Note that here

$$q_j = [h_{o,j-1}(t-T) \quad h_{oj}(t) \quad x_{2,j-1} \quad x_{2,j} \quad \mu_j]^T .$$

For Equation (4.4):

$$b_{3j} = (\ell_2 + \ell_{25}\mu_j) a_{3j} + 2\ell_{22} a_{3j} a_{5j} + \ell_{23} a_{5j} + \ell_3$$

$$b_{4j} = (\ell_2 + \ell_{25}\mu_j) a_{4j} + 2\ell_{22} a_{4j} a_{5j} + \ell_{24} a_{5j} + \ell_4$$

$$b_{5j} = \ell_2 a_{5j} + \ell_{22} a_{5j}^2 + \ell_{25} a_{5j}\mu_j + \ell_5 \mu_j$$

and

$$\phi_j(t-T) = (\ell_{22}a_{3j}^2 + \ell_{23}a_{3j})x_{2,j-1}^2 + (\ell_{22}a_{4j}^2 + \ell_{24}a_{4j})x_{2,j}^2$$

$$+ (\ell_{23}a_{4j} + \ell_{24}a_{3j} + 2\ell_{22}a_{3j}a_{4j})x_{2,j-1}\, x_{2j}$$

$$+ [(\ell_{23} + 2\ell_{22}a_{3j})x_{2,j-1} + (\ell_{24} + 2\ell_{22}a_{4j})x_{2j} + \ell_{12}h_{o,j-1}(t-T)$$

$$\pm (\ell_2 + 2\ell_{22}a_{5j} + \ell_{25}\mu_j)]\eta_j(t-T) + [\ell_{12}(a_{3j}x_{2,j-1} + a_{4j}x_{2,j}$$

$$+ a_{5j}) + \ell_2]h_{o,j-1}(t-T) + \langle q_j, \underline{L}, q_j\rangle + \ell_{22}\eta_j^2(t-T) \ .$$

Note that $\underline{L} = \underline{I}L$ and q_j is as in (4.2). Equation (4.11):

$$g_{2j} \triangleq \frac{ER}{T}\left[\frac{h^*_{nj}}{v^*_{oj}\, h^*_{oj}} \; \frac{x_{1j}(c_{3j}x_{2,j-1} + c_{4j}x_{2j} + \Psi_j(t-T) + c_{5j}+1)}{(a_{3j}x_{2,j-1} + a_{4j}x_{2j} + \eta_j(t-T) + a_{5j}+1)}\right.$$

$$- \frac{h^*_{n,j+1}}{v^*_{i,j+1}\, h^*_{i,j+1}} \cdot \frac{x_{1,j+1}(c_{3,j+1}x_{2,j} + c_{4,j+1}x_{2,j+1} +}{(a_{3j}x_{2,j-1}(t-T) + a_{4j}x_{2j}(t-T) +}$$

$$\left.\frac{\Psi_{j+1}(t-T) + c_{5,j+1} + 1)}{\eta_j(t-2T) + a_{5j} + 1)}\right] .$$

Distribution List as of November 1, 1969

Defense Documentation Center
Attn: DDC-TCA
Cameron Station 50 Copies
Alexandria, Virginia 22314

ESD (ESTI)
L. G. Hanscom Field
Bedford, Mass. 01731 2 Copies

Director, Electronic Programs
Attn: Code 427
Department of the Navy
Washington, D. C. 20360 3 Copies

Naval Air Systems Command
AIR 03
Washington, D.C. 20360 2 Copies

Naval Electronic Systems Command
ELEX 03, Room 2046 Munitions Building
Department of the Navy
Washington, D.C. 20360 2 Copies

Director
Naval Research Laboratory
Washington, D.C. 20390
Attn: Code 2027 6 Copies

Commander
U. S. Naval Ordnance Laboratory
Attn: Librarian
White Oak, Md. 21502 2 Copies

Commanding General
Attn: STEWS-RE-L, Technical Library
White Sands Missile Range
New Mexico 88002 2 Copies

Commander
Naval Electronics Laboratory Center
Attn: Library
San Diego, Calif 92152 2 Copies

Raytheon Company
Attn: Librarian
Bedford, Massachusetts 01730

Dr. L. M. Hollingsworth
AFCRL (CRN)
L. G. Hanscom Field
Bedford, Massachusetts 01731

Division of Engineering & Applied Physics
210 Pierce Hall
Harvard University
Cambridge, Massachusetts 02138

Director
Research Laboratory of Electronncs
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Materials Center Reading Room 13-2137
Massachusetts Institute of Technology
Cambridge, Mass. 02139

Project MAC
Document Room
Massachusetts Institute of Technology
545 Technology Square
Cambridge, Mass. 02139

Raytheon Company
Research Division Library
28 Seyon St
Waltham, Massachusetts 02154

Sylvania Electronic Systems
Applied Research Laboratory
Attn: Documents Librarian
40 Sylvan Road
Waltham, Mass. 02154

Commanding Officer
Army Materials & Mechanics Res. Center
Attn: Dr. H. Priest
Watertown Arsenal
Watertown, Mass. 02172

Lincoln Laboratory
Massachusetts Institute of Technology
Lexington, Massachusetts 02173

Commanding Officer
Office of Naval Research Branch Office
495 Summer Street
Boston, Massachusetts 02210

Commanding Officer (Code 2064)
Navy Underwater Sound Laboratory
Fort Trumbull
New London, Connecticut 06320

Yale University
Engineering Department
New Haven, Connecticut 06520

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-HL-CT-A
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-HL-CT-DD
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-HL-CT-I
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-HL-CT-L (Dr W. S. McAfee)
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-HL-CT-O
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-HL-CT-R
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-KL-D
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-KL-E
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-KL-M (Drs Schie./Hieslmair)
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-KL-S (Dr. H. Jacobs)
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-KL-T
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-NL-A
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-NL-D
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-NL-P-2 (Dr. Haratz)
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL- NL-P
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-NL-R (Mr. R. Kulnyi)
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-NL-S
Fort Monouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-RD-GF
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-RD-MT
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-SC
Fort Monmouth, New Jersey 07703

Commanding General
U.S. Army Electronics Command
Attn: AMSEL-VI-F (R. J. Niemela)
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-VL-D
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-WL-D
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-XL-C
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-XL-D (Dr. K. Schwidta)
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-XL-E
Fort Monmouth, New Jersey 07703

Commanding General
U. S. Army Electronics Command
Attn: AMSEL-XL-S (Dr. R. Buser)
Fort Mouth, New Jersey 07703

Mr. Norman J. Field, AMSEL-RD-S
Chief, Office of Science & Technology
Research and Development Directorate
U. S. Army Electronics Command
Fort Monmouth, New Jersey 07703

Mr. Robert O. Parker, AMSEL-RD-S
Executive Secretary, JSTAC
U. S. Army Electronics Command
Fort Monmouth, New Jersey 07703

Project Manager
Common Positioning & Navigation Systems
Attn: Harold H. Bahr (AMCPM-NS-TM), Bldg.439
U. S. Army Electronics Command
Fort Monmouth, New Jersey 07703

U. S. Army Munitions Command
Attn: Science & Technology Br. Bldg 59
Picatinny Arsenal, SMUPA-VA6
Dover, New Jersey 07801

N. J. A. Sloane
Bell Telephone Laboratories
Mountain Avenue
Murray Hill, New Jersey 07974

European Office of Aerospace Research
APO New York 09667

New York University
College of Engineering
New York, N. Y. 10019

Director
Columbia Radiation Laboratory
Columbia University
538 West 120th St.
New York, N. Y. 10027

Airborne Instruments Laboratory
Deer Park, New York 11729

Mr. Jerome Fox, Research Coordinator
Polytechnic Institute of Brooklyn
333 Jay St.
Brooklyn, N. Y. 11201

Syracuse University
Dept. of Electrical Engineering
Syracuse, N. Y. 13210

Rome Air Development Center
Attn: Documents Library (EMTLD)
Griffiss Air Force Base, N. Y. 13440

Mr. H. E. Webb (EMMIIS)
Rome Air Development Center
Griffiss Air Force Base, N. Y. 13440

Professor James A. Cadzow
Department of Electrical Engineering
State University of New York at Buffalo
Buffalo, N. Y. 14214

Carnegie Institute of Technology
Electrical Engineering Department
Pittsburgh, Pa. 15213

Hunt Library
Carnegie-Mellon University
Schenley Park
Pittsburgh, Pa. 15213

Lehigh University
Dept of Electrical Engineering
Bethelehem, Pennsylvania 18015

Commander (ADL)
Naval Air Development Center
Johnsville, Warminster, Pa. 18974

Technical Director (SMUFA-A2000-107-1)
Frankford Arsenal
Philadelphia, Pennsylvania 19137

Philco Ford Corporation
Communications & Electronics Div.
Union Meeting and Jolly Rods
Blue Bell, Pennsylvania 19422

Director
Walter Reed Army Institute of Research
Walter Reed Army Medical Center
Washington, D.C. 20012

Mr. M. Zane Thornton, Chief, Network Engineering, Communications & Operations Branch, Lister Hill
National Center/Biomedical Communications
8600 Rockville Pike
Bethesda, Maryland 20014

Director
Advanced Research Projects Agency
Department of Defense
Washington, D.C. 20301

Director for Materials Sciences
Advanced Research Projects Agency
Department of Defense
Washington, D.C. 20301

Distribution List, Continued

Dr. A. A. Dougal
Asst. Director (Research)
Ofc. of Defense Res. & Eng.
Department of Defense
Washington, D.C. 20301

Office of Deputy Director
(Research & Information, Rm 3D1037)
Department of Defense
The Pentagon
Washington, D. C. 20301

Headquarters
Defense Communications Agency (340)
Washington, D. C. 20305

Commanding General
U. S. Army Material Command
Attn: AMCRD-TP
Washington, D.C. 20315

Director, U. S. Army Material
Concepts Agency
Washington, D. C. 20315

AFSC (SCTSE)
Andrews Air Force Base, Maryland 20331

Hq USAF (AFRDD)
The Pentagon
Washington, D. C. 20330

Hq USAF (AFRDDG)
The Pentagon
Washington, D. C. 20330

Hq USAF (AFRDSD)
The Pentagon
Washington, D.C. 20330

Dr. I. R. Mirman
AFSC (SCT)
Andrews AFB, Maryland 20331

Naval Ship Systems Command
Ship 031
Washington, D. C. 20360

Naval Ship System Command
Ship 035
Washington, D. C. 20360

Commander
U. S. Naval Security Group Command
Attn: G43
3801 Nebraska Avenue
Washington, D. C. 20390

Director
Naval Research Laboratory
Washington, D. C. 20390
Attn: Dr. A. Brodizinsky, Sup. Elec Div

Director
Naval Research Laboratory
Washington, D. C. 20390
Attn: Dr. W. C. Hall, Code 7000

Director
Naval Research Laboratory
Attn: Library, Code 2029 (ONRL)
Washington, D. C. 20390

Dr. G. M. R. Winkler
Director, Time Service Division
U. S. Naval Observatory
Washington, D. C. 20390

U. S. Post Office Department
Library - Room 1012
12th & Pennsylvania., N. W.
Washington, D. C. 20260

Colonel E. P. Gaines, Jr.
ACDA/FO
1901 Pennsylvania Ave. N. W.
Washington, D. C. 20451

Commanding Officer
Harry Diamond Laboratories
Attn: Mr. Berthold Altman (AMXDO-TI)
Connecticut Ave. & Van Ness St., N.W.
Washington, D.C. 20438

Central Intelligence Agency
Attn: OCR/DD Publications
Washington, D. C. 20505

Dr. H. Harrison, Code RRE
Chief, Electrophysics Branch
National Aeronautics & Space Admin.
Washington, D.C. 20546

Federal Aviation Administration
Attn: Admin Stds Div (MS-110)
800 Independence Ave. S.W.
Washington, D. C. 20590

Director
Nation Security Agency
Attn: TDL
Fort George G. Meade, Md. 20755

The John Hopkins University
Applied Physics Laboratory
Attn: Document Librarian
8621 Georgia Avenue
Silver Springs, Maryland 20910

Commanding Officer
Human Engineering Laboratories
Aberdeen Proving Ground
Aberdeen, Maryland 21005

Commanding Officer (AMXRD-BAT)
U. S. Army Ballistics Research
Laboratory
Aberdeen Proving Ground
Aberdeen, Maryland 21005

Electromagnetic Compatibility
Analysis Center
(ECAC), Attn: ACLP
North Severn
Annapolis, Maryland 21402

Director
U. S. Army Engineer Geodesy
Intelligence & Mapping
Research & Development Agency
Fort Belvoir, Virginia 22060

Dr. G. M. Janney, AMSEL-HL-NVOR
Night Vision Laboratory, USAECOM
Fort Belvoir, Virginia 22060

Dr. A. D. Schnitzler, AMSEL-HL-NVII
Night Vision Laboratory, USAECOM
Fort Belvoir, Virginia 22060

U. S. Army Mobility Equipment Research
and Development Center
Attn: Technical Document Center
Bldg. 315
Fort Belvoir, Virginia 22060

Weapons Systems Evaluation Group
Attn: Colonel Blaine O. Vogt
400 Army-Navy Drive
Arlington, Virginia 22202

Head, Technical Services Division
Naval Investigative Service Headquarters
4420 North Fairfax Drive
Arlington, Virginia 22203

Physical & Engineering Sciences Division
U. S. Army Research Office
3045 Columbia Pike
Arlington, Va. 22204

Lt. Col. H. W. Jackson
Chief, Electronics Division
Directorate of Engineering Sciences
Air Force Office of Scientific Research
Arlington, Virginia 22209

Commanding General
U. S. Army Security Agency
Attn: IARD-T
Arlington Hall Station
Arlington, Virginia 22212

VELA Seismological Center
300 North Washington Street
Alexandria, Virginia 22314

U. S. Naval Weapons Laboratory
Dahlgren, Virginia 22448

Research Laboratories for the Eng.
Sciences, School of Engineering &
Applied Science
University of Virginia
Charlottesville, Va. 22903

Dr. Herman Robl
Deputy Chief Scientist
U. S. Army Research Office (Durham)
Box CM, Duke Station
Durham, North Carolina 27706

Richard O. Ulsh (CRDARD-IPO)
U. S. Army Research Office (Durham)
Box CM, Duke Station
Durham, North Carolina 27706

ADTC (ADBPS-12)
Eglin AFB, Florida 32542

Commanding Officer
Naval Training Device Center
Orlando, Florida 32813

Technical Library, AFETR
(ETV,MU-135)
Patrick AFB, Florida 32935

Commanding General
U. S. Army Missile Command
Attn: AMSMI-REX
Redstone Arsenal, Alabama 35809

Redstone Scientific Information Center
Attn: Chief, Document Section
U. S. Army Missile Command
Redstone Arsenal, Alabama 25809

AUL3T-9663
Maxwell AFB, Alabama 36112

Hq AEDC (AETS)
Attn: Library/Documents
Arnold AFS, Tennessee 37389

Case Institute of Technology
Engineering Division
University Circle
Cleveland, Ohio 44106

NASA Lewis Research Center
Attn: Library
21000 Brookpark Road
Cleveland, Ohio 44135

Professor J. J. D'Azzo
Dept. of Electrical Engineering
Air Force Institute of Technology
Wright-Patterson AFB, Ohio 54533

Director
Air Force Avionics Laboratory
Wright-Patterson AFB, Ohio 45433

AFAL (AVT) Dr H. V. Noble
Electronics Technology Division
Air Force Avionics Laboratory
Wright-Patterson AFB, 45433

AFAL (AVTA) R. D. Larson
Wright-Patterson AFB, Ohio 45433

Dr. Robert E. Fontana
Systems Research Laboratories Inc.
7001 Indian Ripple Road
Dayton, Ohio 45440

Dept. of Electrical Engineering
College of Engineering & Technology
Ohio University
Athens, Ohio 45701

Commanding Officer
Naval Avionics Facility
Indianapolis, Indiana 46241

Dr. John D. Hancock, Head
School of Electrical Engineering
Purdue University
Lafayette, Ind 47907

Professor Joseph E. Rowe
Chairman, Dept of Electrical
Engineering
The University of Michigan
Ann Arbor, Michigan 48104

Dr. G. J. Murphy
The Technological Institute
Northwestern University
Evanston, Ill. 60201

Commanding Officer
Office of Naval Research Branch Office
219 South Dearborn St.
Chicago, Illinois 60604

Illinois Institute of Technology
Dept. of Electrical Engineering
Chicago, Ill 60616

The University of Arizona
Dept. of Electrical Engineering
Tucson, Ariz. 85721

Commander Test Command (TCDT-)
Defense Atomic Support Agency
Sandia Base
Albuquerque, N. M. 87115

Los Alamos Scientific Laboratory
Attn: Report Library
P. O. Box 1663
Los Alamos, N. M. 87544

Atmospheric Sciences Office
Atmospheric Sciences Laboratory
White Sands Missile Range
New Mexico 88002

Commanding Officer
U. S. Army Electronics R & D Activity
White Sands Missile Range
New Mexico 88002

Missile Electronic Warfare
Technical Area, AMSEL-WT-MT
White Sands Missile Range
New Mexico 88002

Director
Electronic Sciences Lab.
University of Southern California
Los Angeles, Calif. 90007

Engineering & Mathematical Sciences
Library
University of California at Los Angeles
405 Hilgred Avenue
Los Angeles, Calif. 90024

Aerospace Corporation
P.O. Box 95085
Los Angeles, California 90045
Attn: Library Acquisitions Group

Det 6, Hq OAR
Air Force Unit Post Office
Los Angeles, Calif. 90045

Director, USAF PROJECT RAND
Via: Air Force Liaison Office
The RAND Corporation
Attn: Library D
1700 Main Street
Santa Monica, California 90045

Hq SAMSO (SMTTA)Lt Nelson
AF Unit Post Office
Los Angeles, Calif. 90045

Dr. Sheldon J. Wells
Electronic Properties Information Center
Mail Station E-175
Hughes Aircraft Company
Culver City, California 90230

Director
Coordinated Science Laboratory
University of Illinois
Urbana, Illinois 61801

Commandant
U. S. Army Command & General Staff College
Attn: Acquisitions, Library Division
Fort Leavenworth, Kansas 66027

Dept of Electrical Engineering
Rice University
Houston, Texas 77001

HQ AMD (AMR)
Brooks AFB, Texas 78235

USAFSAM (SMKOR)
Brooks AFB, Texas 78235

Mr B. R. Locke
Technical Advisor, Requirements
USAF Security Service
Kelly Air Force Base, Texas 78241

Director
Electronics Research Center
The University of Texas at Austin
Austin, Texas 78712

Department of Electrical Engineering
Texas Technological College
Lubbock, Texas 79409

Commandant
U. S. Army Air Defense School
Attn: Missile Sciences Div., C&S Dept.
P.O. Box 9390
Fort Bliss, Texas 79916

Director
Aerospace Mechanics Division
Frank J. Seiler Research Laboratory (OAR)
USAF Academy
Colorado Springs, Colorado 80840

Director of Faculty Research
Department of the Air Force
U. S. Air Force Academy
Colorado Springs, Colorado 80840

Academy Library (DFSLB)
U. S. Air Force Academy
Colorado Springs, Colorado 80912

Utah State University
Dept of Electrical Engineering
Logan, Utah 84321

School of Engineering Sciences
Arizona State University
Tempe, Ariz. 85281

Commanding General
U. S. Army Strategic Communications Command
Attn: SCC-CG-SAE
Fort Huachuca, Arizona 85613

Deputy Director and Chief Scientist
Office of Naval Research Branch Office
1030 East Green Street
Pasadena, California 91101

Aeronautics Library
Graduate Aeronautical Laboratories
California Institute of Technology
1201 E. California Blvd.
Pasadena, California 91109

Professor Nicholas George
California Inst. of Technology
Pasadena, California 91109

Commanding Officer
Naval Weapons Center
Corona Laboratories
Attn: Library
Corona, California 91720

Hollander Associates
P.O. Box 2276
Fullerton, California 92633

Commander, U.S. Naval Missile Center
Point Mugu, California 93041

W. A. Eberspacher, Associate Head
Systems Integration Division
Code 5340A, Box 15
U. S. Naval Missile Center
Point Mugu, California 93041

The Library
Government Publications Section
University of California
Santa Barbara, California 93106

Commander (Code 753)
Naval Weapons Center
Attn: Technical Library
China Lake, California 93555

Library (Code 2124)
Technical Report Section
Naval Postgraduate School
Monterey, California 93940

Glen A. Myers (Code 52Mv)
Assoc Professor of Elec. Engineering
Naval Postgraduate School
Monterey, California 93940

Dr. Leo Young
Stanford Research Institute
Menlo Park, California 94025

Union Carbide Corporation
Electronic Division
P.O. Box 1209
Mountain View, California 94041

Lenkurt Electric Co., Inc.
1105 County Road
San Carlos, California 94070
Attn: Mr. E. K. Peterson

Director
Microwave Physics Laboratory
Stanford University
Stanford, California 94305

Director
Stanford Electronics Laboratories
Stanford University
Stanford, California 94305

Nuclear Instrumentation Group
Bldg 29, Room 101
Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

Director, Electronics Research Laboratory
University of California
Berkeley, California 94720

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Illinois Coordinated Science Laboratory Urbana, Illinois 61801	2a. REPORT SECURITY CLASSIFICATION 2b. GROUP

3. REPORT TITLE

DYNAMIC MODEL OF A CONTINUOUS COLD ROLLING MILL

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(First name, middle initial, last name)*

JAMSHIDI, M. & KOKOTOVIC, P.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March, 1970	35	33

8a. CONTRACT OR GRANT NO. DAAB 07-67-C-0199; also in part AFOSR 68-1579A; and in part Alcoa Foundation. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) R-459 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* UILU-ENG 70-204

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Joint Services Electronics Program thru U.S. Army Electronics Command Fort Monmouth, New Jersey 07703

13. ABSTRACT

The dynamic model of a continuous cold reduction mill is developed. The model makes use of the force-torque equation expansion technique developed in an earlier report and presents the generalized (7N+8)th order state equation for an N-stand rolling mill. The equations of interstand tensions are found to be highly nonlinear with the arguments delayed due to the transition of strip between the stands. The model developed can used in the design of a controller for the entire rolling process.

DD FORM 1 NOV 65 1473

Security Classification

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
System Modelling						
Rolling Mills						
Control Systems						
State-space Approach						

Security Classification