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DISCRETE WAREHOUSE PROBLEM

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Discrete Warehouse Problem †

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Abstract

A *discrete warehouse* is a collection of objects (robot and obstacles) which are allowed to move along the grid edges of a two-dimensional grid. In this paper, we consider motion planning problems of a unit-square robot moving on a square grid among unit-square obstacles, which are movable. In such a setup one is allowed to move some of the obstacles in order to navigate the robot between an initial and a final position of the robot in the warehouse. The final positions of the obstacles are unimportant for our problems. We consider two forms of obstacle manipulations: (a) *remote,* when the obstacles are moved by a remote mechanism, and (b) *contact,* when the obstacles are moved only by direct contact of the robot. We present necessary and sufficient conditions for the existence of a motion in both cases, and propose efficient algorithms for constructing feasible motions. Thus, we establish the first known class of tractable motion planning problems with multiple movable obstacles.

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1. Introduction

Computational geometry, or more generally, algorithmics, is a newcomer to the multifaceted area of robotics. The repertoire of robotics problems presently treated by computational geometry is admittedly narrow in view of the total scope of robotics. Most of the recent efforts have been towards studying the problems of motion planning. In this way there are basically two different options: (a) motion planning with fixed obstacles (or, obstacle-avoiding motion planning) and, (b) motion planning with movable obstacles (or, obstacle-manipulation motion planning). First of which has been called the Piano mover's problem by Schwartz and Sharir in an influential series of papers [3-6] and has been studied by many authors since (see [9] for an overview of some of these results). In its general form the Piano Mover's problem is defined as follows: Given a fixed configuration of obstacles in the the Euclidean space and an initial and a final configuration of a robot, determine whether there exists a continuous motion of the robot between its initial and final configuration during which it does not intersect any of the fixed obstacles. This problem is known to be solvable in polynomial time and space.

In contrast, the *coordinated motion planning* problem defined by Hopcroft et. al. [1] , relates to motion planning with obstacle manipulation. The problem is to determine if there exists a continuous motion of a collection of disjoint movable objects constrained to move within an enclosed region, between the initial and final configurations of the objects during which they do not penetrate either the "walls" of the enclosing box, or each other. They show that even the restricted two-dimensional version of this problem for arbitrarily many rectangles in a rectangular region is PSPACE-hard. More recently, a restricted version of the problem, involving translations of one movable polygonal obstacle, was shown to be tractable [8].

In this paper, we formulate a new class of motion planning problems similar to the "warehouseman's problem" [1]. We consider the following environment (see Figure 1): A warehouse containing a robot B and a collection of movable and fixed obstacles O. For our purposes we assume that obstacles and the robot are all square shaped with unit dimensions, and are placed on the unit square grid. In an abstract formulation we view the warehouse as a subset (subgraph) of the unit square grid (graph). Each grid point is either empty, contains a movable object, contains a fixed object, or contains the robot. Motion of robot and the obstacles is restricted to grid edges of the warehouse.

We define two cases for such a system depending on the manner in which obstacles are moved: (a) by a *contact* motion, or (b) by a *remote* motion (e.g., a mechanical arm). For a contact movement the obstacle being moved has to be adjacent to the robot. In this case, robot B is a special object which can move on its own and all other obstacles of O can move only by its consequence. For a remote movement the obstacles are moved by a remote mechanism, called the *grip*, and need not be adjacent to the robot. In this case, robot B is like just another obstacle of O. In both cases movement of an obstacle is valid if it moves in an adjacent location on the grid which is empty. This configuration of obstacles and robot resembles a warehouse where items are stored in movable shelves and have to be retrieved by a robot moving on the grid edges. We permit the robot to access an item from a shelf by being at one of the four empty grid points around it.

For both the contact and remote motions, we study two kinds of problems: (1) how to move the robot from an initial position to a final position in the warehouse; (2) how to construct a "clearing" (i.e., a free path) between two locations of the warehouse (one of these points may be a "door" on the boundary). For both problems we present necessary and sufficient conditions for the existence of a solution and propose efficient algorithms for constructing feasible motions.

In Section 2, we give the basic notation and some of the definitions required for describing our results. In Section 3, we discuss the case of remote motions. Finally, in Section 4, we outline our approach for contact motions.

2. Problem Definition

We denote the warehouse W by a rectilinear region of the unit square grid of size $m \times n$. At any instant, grid point (i, j) of the warehouse W is either *empty*, *movable*, *fixed*, or has a *robot* (e, m, f, or r, respectively). We represent the *type* of (i, j) by $T(i, j) = x$, where $x \in \{e, m, f, r\}$. For our considerations only the relative positions of obstacles is important, since all the obstacle items are indistinguishable. Configuration R of the warehouse is specified by an $m \times n$ *type* matrix T which is like a snapshot of W describing the types of items for all $m \times n$ grid points of W.

Robot B is located at the grid point of type r and its coordinates are (b_x, b_y). We call, S, a set of obstacle items *H–consecutive* if they are consecutive on a row of the array W and are all of type m. Similarly, items of S are *V–consecutive* if they are consecutive on a column of array W and all are of type m. $|S|$ denotes the number of items in S. Let $S_L(i, j)$ (resp. $S_R(i, j)$) represent the maximal H-consecutive set of items to the left (resp. right) of grid position (i, j) and let $S_U(i, j)$ (resp. $S_D(i, j)$) represent maximal V-consecutive set of items in the upward (resp. downward) direction of grid position (i, j). In the following we describe five **elementary motions (EM)** permitted (in terms of the above notation) for remote and robot motion(see Figure 2).

Elementary Contact Manipulation:

(1) *Free* motion: the robot can move to an adjacent e-type grid point along the grid edges of W.

(2) *Push* motion: the robot can push sets $S_L(b_x, b_y)$ and $S_R(b_x, b_y)$ (resp. $S_U(b_x, b_y)$ and $S_D(b_x, b_y)$) of k or less H-consecutive (resp. V-consecutive) items adjacent to it in a row (resp. column) by one position provided there is an empty space on the row (resp. column) within k spaces of it in the direction of push. Then, the robot moves one unit in the direction of push.

(3) *Pull* motion: the robot can pull sets $S_L(b_x, b_y)$ and $S_R(b_x, b_y)$ (resp. $S_U(b_x, b_y)$ and $S_D(b_x, b_y)$) of k or less H-consecutive (resp. V-consecutive) items adjacent to it in a row (resp. column) by one position provided there is an empty space right next to it on that row (resp. column) in the direction of pull.

(4) *Slide* motion: the robot can slide sets $S_L(b_x \pm 1, b_y)$ and $S_R(b_x \pm 1, b_y)$ (resp. $S_U(b_x, b_y \pm 1)$ and $S_D(b_x, b_y \pm 1)$) of k or less H-consecutive (resp. V-consecutive) items along the row (resp. column) with respect to position adjacent to it on the column (resp. row) provided there is an empty space within k spaces of $(b_x \pm 1, b_y)$ (resp. $(b_x, b_y \pm 1)$).

Elementary Remote Manipulation:

(5) *Remote* motion: Any set of k or less H-consecutive (resp. V-consecutive) items can be moved along a row (resp. column) provided there is an empty space within k spaces of its either end points.

Consider an $m \times n$ grid warehouse W, with $1 \le x \le m$ and $1 \le y \le n$. Grid point (x, y) is said to be *adjacent* to grid points $(x-1, y)$, $(x+1, y)$, $(x, y-1)$, and $(x, y+1)$ (if they exist in W). Let R_0 denote the initial configuration of W which is specified by the type-matrix T_0.

A configuration R_j is said to be *reachable* from a configuration R_i, if it can be obtained from R_i by sequence of **EM**s (i.e., elementary motions). For simplicity, we say a configuration R is reachable, to mean R is reachable from the initial configuration R_0. The *robot displacement problem (RDP)* is the problem of obtaining a reachable configuration with the robot being at a specified grid point P_f. In a configuration R there is a *clearing* from point P_a to point P_b, if there exists a sequence of distinct points $P_a = P_{\pi_1}, P_{\pi_2}, \ldots, P_{\pi_k} = P_b$ such that P_{π_i} is adjacent to $P_{\pi_{i+1}}$, for $i < k$, and $T(P_{\pi_j}) = e$, for all j. The *clearance construction problem (CCP)* involves obtaining a reachable configuration R, which has a clearing between the two specified points P_0 and P_f.

Grid points of type e and m are called *free grid points* and grid points of type f are called *rigid grid points*. We partition free grid points of W into equivalence classes $C_0, \ldots, C_t$. If a free grid point is placed in class C_i then all the adjacent free grid points are also placed in C_i (see Figure 3). Rigid points form boundaries of equivalence classes. Note that type of a rigid point remains unchanged under an elementary motion **EM**, thus:

Fact 1 : Equivalence classes are preserved under **EM**.

Consider two grid points P_a and P_b. A shortest path S_{ab} between P_a and P_b is a minimum cardinality sequence $P_a = P_{\pi_0}, P_{\pi_1}, \ldots, P_{\pi_k} = P_b$ such that P_{π_i} is adjacent to $P_{\pi_{i+1}}$, for $0 \le i \le k-1$, and P_{π_j} is free, for $1 \le j \le k-1$. If such a sequence does not exist (i.e., P_a and P_b belong to different classes) then we write $|S_{ij}| = \infty^*$. The number of e-type grid points (holes) in class C_i is denoted by h_i. By virtue of

Fact 1, h_i 's remain the same under **EM**, i.e., holes are neither destroyed nor transferred.

In the next two sections, we study RDP and CCP for the cases of remote motions and contacts motions.

3. Remote Motion

In this section we study remote motion planning. The *grip* (or, mechanical arm) is allowed to move a movable shelf into an adjacent empty spot, or equivalently, to "move" an empty spot into an adjacent movable shelf. We will adopt the later convention, for it simplifies the proofs. (Using metaphor, the e/m type movement is similar to electron/hole movement in semiconductors.) The (XY-)grip has only horizontal and vertical movements, resembling the pen movement in an XY-plotter. In compliance with the grid environment W and the XY-grip, all distances are measured in L_1 metric. The following operation, performed by the grip, changes a configuration:

OP: if $T(P_1) = e$ and $T(P_2) = m$ and P_1 and P_2 are adjacent **then** $T(P_1) := m$ and $T(P_2) := e$.

First we study robot displacement problem (RDP). Consider the initial configuration R_0 with a distinguished movable obstacle O at the grid point P_0. We aim to obtain a reachable configuration with O at a specified grid point P_f. Thus, an instance of RDP is specified by a pair (P_0, P_f) of grid points (and an initial configuration R_0). To simplify the case analysis below, we assume $P_0 \neq P_f$.

Lemma 1 : An instance (P_0, P_f) of RDP has a solution if and only if P_0 and P_f belong to C_0 and $h_0 \geq 1$, where C_0 is the class to which P_0 belongs.

Proof : *(only if)* If $h_0 = 0$ then **OP** is not possible. Therefore configuration of C_0 remains unchanged.

(if) Consider a path $(P_h = P_{\pi_1}, \ldots, P_{\pi_a} = P_0)$ from an e-type grid point (a hole) P_h to P_0. By a sequence of **OP** s (interchanging type of P_{π_i} and $P_{\pi_{i+1}}$, for $0 \leq i \leq a-2$) the hole is brought adjacent to P_0. Now, consider a path $(P_0 = P_{\psi_1}, \ldots, P_{\psi_b} = P_f)$ from P_0 to P_f. If P_{ψ_i} contains O (the distinguished obstacle) and one of the grid points adjacent to it has a hole then we can move O to $P_{\psi_{i+1}}$, as shown in Figure 4. Since $P_{\pi_{a-1}}$ is empty and it is adjacent to P_0 then (inductively) we can move O to P_f. □

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* In implementation, choose $\infty = mn + 1$.

As outlined in Lemma 1, an algorithm for RDP involves obtaining a shortest path S_{h0} between P_h and P_0 and a shortest path S_{0f} between P_0 and P_f. From the previous discussion and the algorithm in [2], we conclude:

Theorem 1 : An instance of RDP can be solved in $O(|S_{h0}|\,|S_{0f}|\log(|S_{h0}|\,|S_{0f}|))$.

Next we consider clearance construction problem (CCP). An instance of the problem involves two points P_0 and P_f (and an initial configuration R_0).

Lemma 2 : There exists a reachable configuration with a clearing from P_0 to P_f if and only if $h_0 \geq |S_{0f}|$, where C_0 is the class to which P_0 belongs.

Proof : *(only if)* Assume $h_0 < |S_{0f}|$.

Case 1)

$|S_{0f}| = \infty$: In this case, P_0 and P_f belong to different classes. Boundary of C_0 consists of a set of rigid points, and thus, cannot be changed under **OP**. Therefore, in any reachable configuration $|S_{0f}| = \infty$ (see Fact 1).

Case 2)

$|S_{0f}| \neq \infty$: In this case, P_0 and P_f belong to the same class C_0. In a clearing from P_0 to P_f there must be a path of e-type grid points from P_0 to P_f. Thus the number h_0 of e-type grid points in C_0 must be at least equal to the length of the path. Since $|S_{0f}|$ is the length of the shortest path then there is no solution.

(if) Assume $h_0 \geq |S_{0f}|$. We will show how to obtain a reachable configuration R_f with a clearing from P_0 to P_f. Consider class C_0 containing P_0 and P_f. Let $P_0 = P_{\pi_0}, P_{\pi_1}, \ldots, P_{\pi_k} = P_f$ be a shortest path from P_0 to P_f, such that P_{π_i} is adjacent to $P_{\pi_{i+1}}$, for $0 \leq i \leq k-1$. Inductively, assume there exist a reachable configuration R_i with $T(P_{\pi_0}), \ldots, T(P_{\pi_i}) = e$; this is certainly the case for $i = 0$. Next, we show that there exist a configuration R_{i+1} reachable from R_i with $T(P_{\pi_0}), \ldots, T(P_{\pi_{i+1}}) = e$. Consider an e-type point P_j ($\neq P_{\pi_0}, \ldots, P_{\pi_{i+1}}$) in C_0. Such a point must exist since $h_0 \geq |S_{0f}|$ and $i \leq k$. By virtue of

Lemma 1, there is a configuration with $T(P_l)=e$, where P_l is a grid point adjacent to P_{π_t} with $\pi_l \in \{\pi_0, \ldots, \pi_i\}$ (essentially by "switching" type of P_j and type of P_l). By sequence of $\pi_i - \pi_l$ **OP**'s we obtain a configuration with $T(P_{\pi_{t+1}}), \ldots, T(P_{\pi_{i+1}}) = e$. One more **OP** results in configuration R_{i+1} with $T(P_{\pi_{i+1}}) = e$ (by "switching" type of P_{π_t} and type of P_l, see Figure 5). When $k = i-1$, there is a clearing from P_0 to P_f. □

Lemma 2 (if part) dictates an effective procedure for constructing a clearing. Next, we will propose an efficient implementation of the outlined procedure. When the grip is in motion it is either *loaded* (moving a movable object) or *unloaded* (moving toward an object to be moved). Let τ be the total time for constructing a clearing. We can write $\tau = \tau_l + \tau_u$, where τ_l and τ_u are the total time the grip is loaded and unloaded, respectively. When the grip is loaded it moves much slower than when it is unloaded, that is, $\tau_l \gg \tau_u$. Thus, the primary objective is to minimize τ_l and the secondary objective is to minimize τ_u. Next, we present an optimal algorithm for minimizing τ_l and a 2-approximation algorithm for minimizing τ_u assuming a shortest path S_* from P_0 to P_f has been obtained.

Let λ_l be the time that it takes to move a movable shelf into an adjacent empty spot. We can write $\tau_l = \lambda_l \eta$, where η is the total number of **OP**'s. We aim to minimize η (denoted by η_*), or equivalently, τ_l.

We assume a shortest path S_*, from P_0 to P_f has been obtained. With reference to R_0, we construct a (complete) weighted bipartite graph $G = (V_e, V_p, E)$, where V_e is the set of e-type grid points in C_0 and V_p correspond to vertices of S_*. Weight $w(e)$ of an edge $e = (v_1, v_2)$ is the length of a shortest path (as before, in L_1 metric) from v_1 to v_2. Consider a matching $M = \{e_1, \cdots, e_{|S_*|}\}$ in G. A motion μ_M corresponding to M is, for each $e_i = (v_{i_1}, v_{i_2}) \in M$, a sequence of $w(e_i)$ **OP**'s resulting in a configuration with $T(v_{i_1}) = e$. The number of **OP**'s performed in μ_M is denoted by η_M.

Lemma 3: For a minimum weighted matching M in graph G, $\eta_M = \eta_*$.

Proof: Consider the path $S_* = P_{\pi_0}, \cdots, P_{\pi_k}$. In the final configuration $T(P_{\pi_i}) = e$, for all i. Let P_{σ_i} be the e-type grid point whose type was switched, by a sequence of **OP**'s, with the type of P_{π_i}. Clearly,

$\eta_* = \sum_{i=0}^{i=k} |S_{\pi_i \sigma_i}|$, where $S_{\pi_i \sigma_i}$ is a shortest path from P_{π_i} to P_{σ_i}. Since M is a minimum weighted matching in G, and for each pair $(P_{\pi_i}, P_{\sigma_i})$ there is an edge with weight $|S_{\pi_i \sigma_i}|$ in G then $w(M) \le \sum_{i=0}^{i=k} |S_{\pi_i \sigma_i}|$, or equivalently, $\eta_M \le \eta_*$. □

The motion μ_M corresponding to M can be obtained by constructing G and obtaining a minimum-weighted matching in G. Using the geometric nature of the matching problem and employing the matching algorithm of [7], we obtain the following theorem.

Theorem 2: The motion μ_M can be calculated in $O(|V_e|^{2.5}\log(|V_e|)^2)$ time.

Given a matching M, we aim to minimize the extra movement, that is, $\tau_u = \lambda_u \beta$ where λ_u is the time it takes to move the grip from one grid point to an adjacent grid point and β is the total number of grid points the grip traverses. Let β^* denote the optimal β. Consider an initial position P_s of the grip (see Figure 5). When the matching is fixed (solid edges) the grip must move from P_s to one of the holes at P_h. The hole is brought to one of the position in the path. This process is repeated until all the holes are brought to the path.

Let (P_1, P_2), ..., (P_{m-1}, P_m) be the matching constructed in the previous step. Consider the greedy unloaded motion μ_F: $(P_S = P_0, P_1)$, $\cdots$, (P_{m-2}, P_{m-1}). Let l_i denote the length (in L_1-metric) between P_i and P_{i+1}. The number of unloaded movements corresponding to μ_F is $\beta_F = l_0 + l_2 + \cdots + l_{m-2}$.

Lemma 4: $\beta_F \le 2\beta^*$.

Proof: We assume the path does not contain an empty point, i.e., $l_1, l_3, \cdots, \neq 0$ (other case is handled similarly). β^* is movement from P_0 to some hole P_i (odd i) plus some matching between the holes (odd P_i) and points on the path (even P_i).

Thus:

$$\beta^* - l_0^* \ge l_1 + l_3 + \cdots + l_{m-1} = |M| \tag{1}$$

where l_0^* is the shortest path from P_0 to some hole P_i. Also, $l_{2k+1} + 1$, due to the triangle inequality. Thus,

$$\beta_F - l_0 (= l_2 + l_4 + \cdots + l_{m-2}) \leq |M| - l_1 + \frac{m}{2}$$

$$\beta_F - l_0 + l_1 - \frac{m}{2} \leq |M| \tag{2}$$

From Equations (1) and (2):

$$\beta^* - l_0^* + l_0 + \frac{m}{2} - l_1 \geq \beta_F$$

Since $\beta^* \geq m/2$ and $l_0^* \leq l_0$, then $2\beta^* \geq \beta_F$. □

Thus, the greedy motion μ_F is provably good and is readily obtained.

Theorem 3: The motion μ_F can be calculated in $O(m)$ time.

The above theorem illustrates that the discrete warehouse problem under remote motion has a polynomial algorithmic solution, which establishes it as the first class of tractable problems with multiple moveable obstacles. In the next section, we show that the case of discrete warehouse problem with contact motions is also tractable.

4. Contact Motion

In this section, we consider motion planning in the presence of contact obstacle manipulations. As discussed before, the robot B must be adjacent to a movable obstacle in order to move it. Moreover, there must be an empty location in line with the intended direction of motion. It is easy to observe that movements of the robot in such an environment is more restrictive than in the remote environment. Here, we consider two different kinds of robots; one which can push, pull and slide (called an F-robot) and the other which can only push the adjacent obstacles (called an H-robot). We discuss necessary and sufficient conditions for RDP and CCP for these both cases.

Unlike the case of remote motions not every empty location of W is reachable by the robot. More specifically, a robot can only access empty locations which lie in a "circle" of radius k around it. The definition of the "circle' region is different for both F- and H-robots. In the case of an H-robot (one with only pushing capability) the "circle" contains locations no more than k units apart from (b_x, b_y) on the row and column containing B. In the case of an F-robot the "circle" contains locations no more than k units away from (b_x, b_y) on the rows (resp. columns) containing B, above (resp. left) B, and below (resp.

right) B (see Figure 6). If there is no empty location in the influence circle of the robot then it is stuck in that configuration. We redefine h_i as the maximum number of holes that lie in the circle of influence over all the configurations. It is easy to see that a robot does not endanger its chances of movement at any time by moving all empty locations in its circle next to itself. This can be achieved by using the push and slide EM s. It is also easy to see that the pull motions send the empty locations away from the "center" and so are not useful in this respect.

Fact 2 : Pull motions do not help in using the empty holes for motion.

First we consider *RDP*. Consider the initial configuration R_0 with B at P_0. We obtain a reachable configuration with B at p_f. Due to the freedom of manipulation, an F-robot is similar to a remote robot and it can manage to move about with the help of only one hole, whereas an H-robot needs a new hole every time it advances a step. Then we have the following lemmas.

Lemma 6 : An instance (P_0, P_f) of RDP for an F-robot has a solution if and only if P_0 and P_f belong to C_0 and $h_0 \geq 1$, where C_0 is the class to which P_0 belongs.

Lemma 5 : An instance (P_0, P_f) of RDP for an H-robot has a solution if and only if P_0 and P_f belong to C_0 and $h_0 \geq |S_{0f}|$, where C_0 is the class to which P_0 belongs.

The existence of holes in the circle of robot can be checked in time $O(k)$. Then, using a procedure similar to that for the remote motion, we have the following theorem.

Theorem 4 : An instance of RDP for contact motion can be solved in $O(|S_{h0}|\,|S_{0f}| \log(|S_{h0}|\,|S_{0f}|))$.

Next, we consider the clearance construction problem (see Figure 7). In summary, We have the following theorems:

Theorem 5 : There exists a clearing from P_0 to P_f in an instance of *CCP* for a contact motion if and only if there are at least $|S_{0f}|$ holes in the circles of influence on the path.

Again by using the Euclidean matching algorithm of [7] we can construct the minimum weight matching of the holes in the circles of influence to the points of the path. Thus,

Theorem 6 : The solution to CCP, when it exists can be found in $O(|S_{0f}|^{2.5}\log^2|S_{0f}|)$.

5. Discussion and Open Problems

In this paper we have introduced a new class of motion planning problems, called the discrete warehouse problem, for robots on a two dimensional grid in the presence of movable obstacles. Our results demonstrate that problems of this class are tractable, unlike the "warehouseman's problem" of [1].

Besides the issues about the existence of a motion of a robot and the feasibility of a clearing, as discussed in this paper, one can address other aspects of this problem. For example, in some situations it is important to know the shortest time clearing, where the goal is to reduce the total time required for a clearing and not necessarily the shortest clearing. Also, questions about motions with minimum number of elementary motions is also a natural question, which we suspect to be NP-Complete. Another interesting direction for research would be to consider this problem in a dynamic framework, where the states of the shelves of the warehouse changes dynamically by external interactions. The problem of finding lower bounds for the various problems is another open question.

This paper, establishes a framework for considering motion planning in the presence of movable obstacles which form a significant part of the robotics problems occurring in real life.

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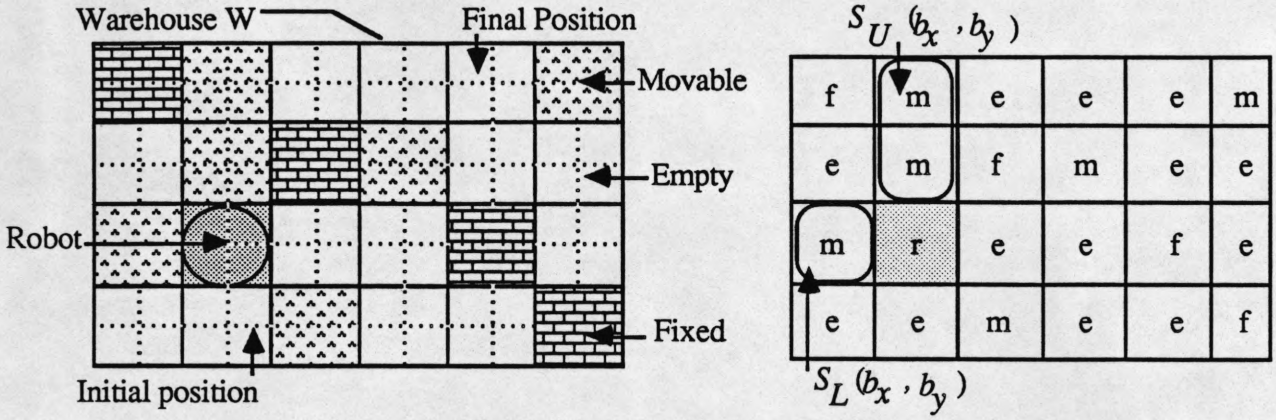

Fig. 1: Discrete Warehouse Problem

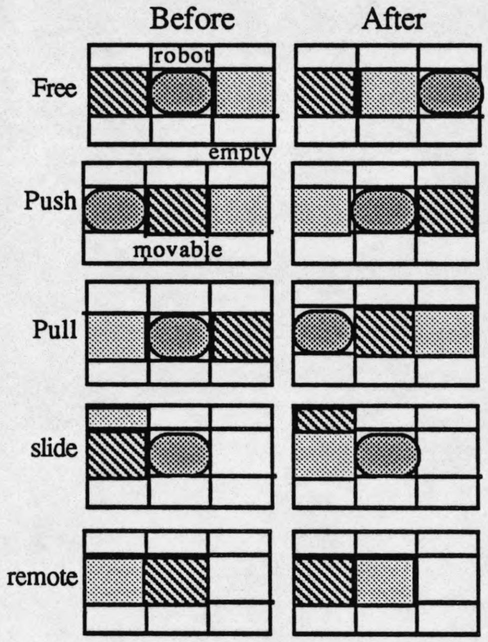

Fig. 2: Various Elementary Motions

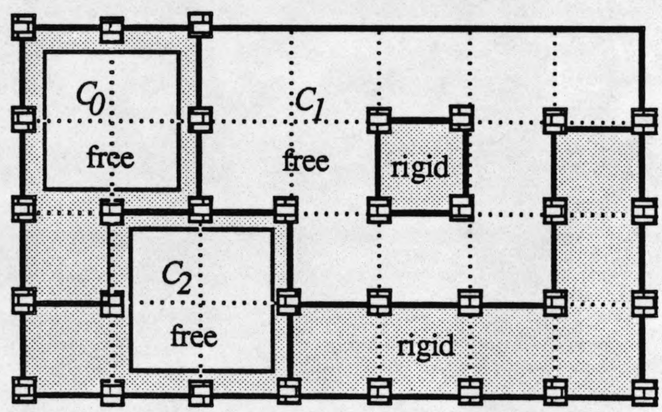

Fig. 3: Equivalence classes

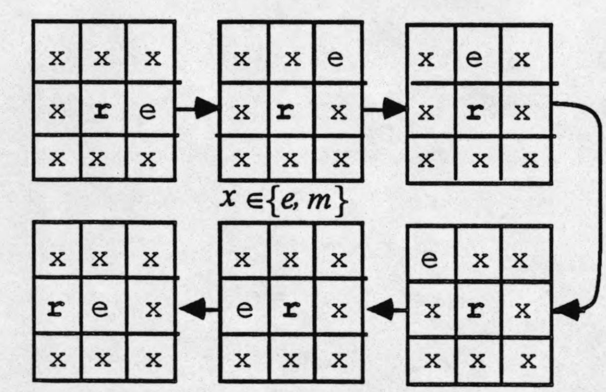

Fig. 4: Advancing the distinguished obstacle r

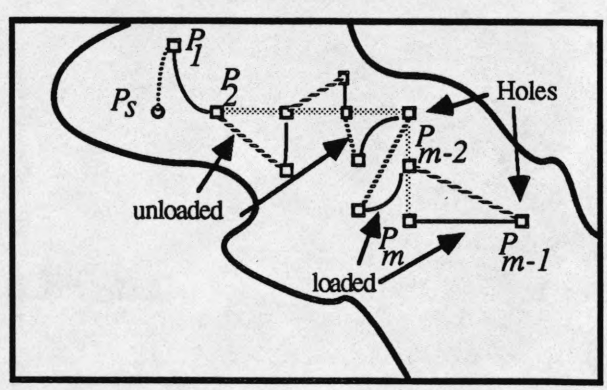

Fig. 5: Bringing holes to a path

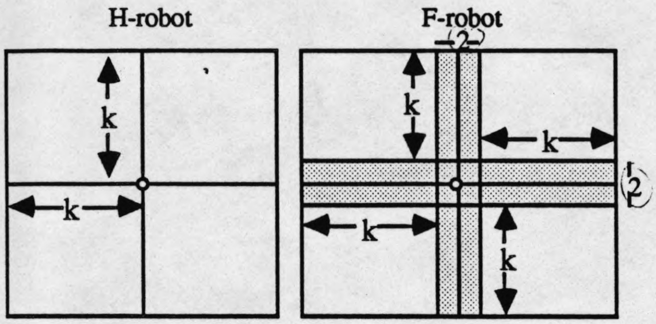

Fig. 6: "Circles" of Influence for the two contact robots

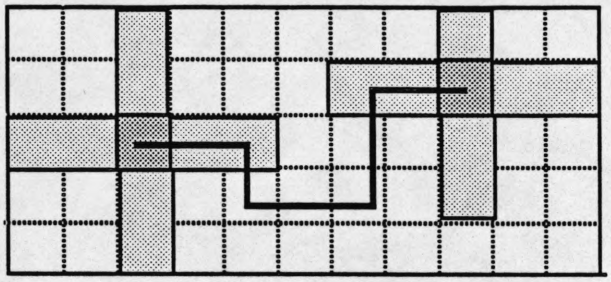
Fig. 7: Motion plan for an H-robot