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SUBSAMPLED MULTICHANNEL BLIND DECONVOLUTION BY SPARSE POWER FACTORIZATION

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Subsampled Multichannel Blind Deconvolution by Sparse Power Factorization

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Abstract

In this technical report, we show that sparse power factorization (SPF) is an effective solution to the subsampled multichannel blind deconvolution (SMBD) problem when the input signal follows a sparse model. SMBD is formulated as the recovery of a sparse rank-one matrix. Unlike the recovery of rank-one matrix or of sparse matrix, when there are multiple priors on the solution simultaneously, SPF outperforms convex relaxation approaches both theoretically and empirically. We confirm that SPF exhibits the same advantage in the context of SMBD.

1 Introduction

Reconstruction of an unknown input signal and/or unknown channel responses from multichannel output samples have been of interest for numerous applications in signal processing and communications. The *subsampled multichannel blind deconvolution problem* (SMBD), portrayed in Figure 1, refers to the resolution of an input signal $x \in \mathbb{C}^n$ and/or the set of M -FIR-filters of length L from the output measurements (possibly subject to noise).

Subsampling the channel outputs distinguishes SMBD from the conventional multichannel blind deconvolution problem, and the degree of subsampling is often a critical factor in designing systems for relevant applications. In parallel magnetic resonance imaging (pMRI), Fourier measurements of the input image are acquired using multiple coils. The coil sensitivity functions apply to the image as pointwise multiplication in the spatial domain, which is represented as circular convolutions in the Fourier domain. The blindness to channel information (coil sensitivity functions) is important because pre-estimates of the coil sensitivity functions are often impractical due to their time variant property. The degree of subsampling corresponds

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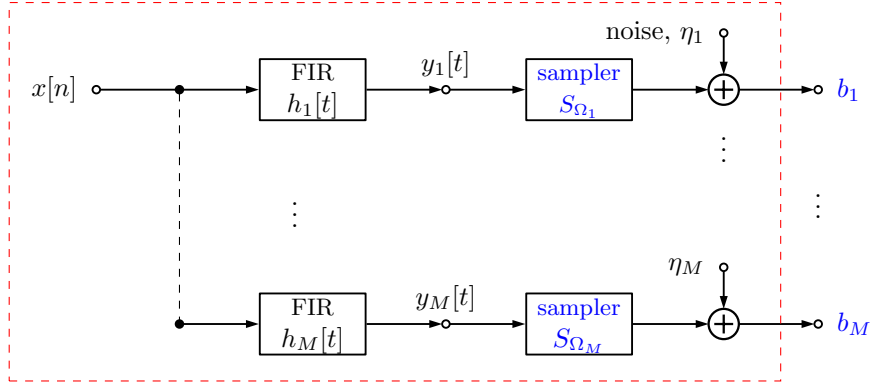


Figure 1: Subsampled Multichannel Blind Deconvolution

to the acceleration factor of image acquisition and higher acceleration is desired since MR is a slow modality. Another important application that is formulated as SMBD is the superresolution problem. It involves the reconstruction of a single higher resolution image from multiple low-resolution image frames with unknown blur. Subsampling is inherent in the problem statement. Unlike pMRI, a linear convolution model is more relevant in this application. In this case, a larger subsampling factor enables greater degree of superresolution.

Subsampling by a higher factor is desired in applications, but makes the multichannel blind deconvolution problem more difficult since fewer equations are provided to determine the unknowns. To address this, we adopt in this paper a sparse signal prior. In the past two decades, sparse signal models have been of interest since they served as good constraints or as good regularizers for solving ill-posed inverse problems whose solution is known a priori to be sparse. In particular, sparse signal models have been shown useful for accelerating imaging systems (e.g., [1, 2]), and rigorous analysis of such system was presented (e.g., [3]).

Exploiting a sparse signal model, we formulate SMBD as the recovery of a sparse rank-one matrix through the so-called “lifting” procedure. The lifted formulation in higher dimension absorbs inherent shift and scale ambiguities of the deconvolution problem within the factorization of the matrix variable; hence, solutions to the deconvolution problem up to such ambiguities are represented as a unique sparse rank-one matrix.

We propose to solve the SMBD using a particular alternating minimization algorithm called *sparse power factorization* (SPF) [4]. SPF was proposed as a solution to the recovery of a sparse rank-one matrix from its linear measurements. Empirically, SPF nearly achieves the performance of nonblind sparse blind deconvolution. That is, the recovery of the input signal with unknown channels is almost as good as with exactly known channels. Furthermore, although SPF is a parametric method, it is robust against slight error in parameters (e.g., overestimation of filter length).

Alternating minimization approaches that exploit models on signal and filters have been studied as solutions to the SMBD problem. Previous work on the SMBD problem has been in the specific context of image

superresolution (cf. [5]), or pMRI (cf. [6]). These approaches used variational formulations, employing the signal and filter models to define regularization terms expressed as penalties for deviation from the model. Furthermore, none of the previous approaches to the SMBD problem offered a theoretical performance guarantee. Instead, unlike previously known alternating minimization approaches to the SMBD problem, SPF enforces that intermediate results are within the exact parametric models, which, under certain conditions on the linear measurements, enables a recovery guarantee. For measurements obtained by inner products with i.i.d. Gaussian vectors, SPF achieves the fundamental limit on the required number of measurements in the problem up to a polylog factor of n [4]. A similar guarantee applies to recovery from structured random measurement [7]. We conjecture that a similar guarantee applies to SMBD. Let s and n denote the sparsity level and the length of the unknown input signal x , and denote by M and L the number of channels and the length of their impulse responses, respectively. The guarantee in this case would be that, up to a polylog factor of n , the number of samples per channel is proportional to the fundamental limit (lower bound) of $s/M + L$.

2 Problem Formulation

2.1 Definitions

Definition 2.1 (Bracket Evaluation). *For any vector $q \in \mathbb{C}^k$ with $k \leq n$, define*

$$q[t] := \begin{cases} q_{t+1} & 0 \leq t < k \\ 0 & \text{elsewhere} \end{cases}$$

Definition 2.2 (Circular Convolution). *For $x \in \mathbb{C}^n$ and $s \in \mathbb{C}^L$ define $x \circledast s \in \mathbb{C}^n$ with the relation*

$$(x \circledast s)[t] := \sum_{\ell=0}^{n-1} x[\ell]s[(t - \ell) \bmod n]$$

Definition 2.3 (Linear Convolution). *For $x \in \mathbb{C}^n$ and $s \in \mathbb{C}^L$ define $x * s \in \mathbb{C}^{n+L-1}$ with the relation*

$$(x * s)[t] := \sum_{\ell=0}^{n-1} x[\ell]s[(t - \ell)]$$

Note: since the bracket valuation $[\cdot]$ is defines a zero-padded sequence of length n , the circular convolution

is well defined even when x and s are not of the same length.

Definition 2.4 (Subsampling Operator S_Γ). For a subset of indices $\Gamma = \{\omega_1, \dots, \omega_k\} \subset \{0, \dots, n-1\}$, define the sampling operator $S_\Gamma : \mathbb{C}^n \rightarrow \mathbb{C}^k$ by

$$(S_\Gamma x)_\ell := x[\omega_\ell], \quad \text{for } \ell = 1, \dots, k.$$

Definition 2.5 (Circular Unit Shift Operator). For $x \in \mathbb{C}^n$ we define $\sigma x \in \mathbb{C}^n$ by $(\sigma x)[t] := x[(t+1) \bmod n]$.

Definition 2.6 (Unit Shift Operator). For $x \in \mathbb{C}^n$ we define $sx \in \mathbb{C}^n$ by $(sx)[t] := x[t+1]$.

2.2 Problem Statement

Let $h_1, h_2, \dots, h_M \in \mathbb{C}^L$ denote the filter unit pulse responses and let $x \in \mathbb{C}^n$ be the input signal.

Definition 2.7 (Sub-sampled Multi-Channel Blind Deconvolution). The reconstruction of unknown signal x and possibly the unknown unit pulse responses $(h_k)_{k=1}^M$ from noisy samples of multiple channel outputs given by

$$b_k := S_\Gamma(h_k * x), \quad k = 1, \dots, M \tag{1}$$

where $b_k \in \mathbb{C}^{|\Gamma|}$ is a vector of output samples. This problem will be referred to as *SMBD* problem. Similarly, we define the circulant variant of the problem (*SMBCD*) with the model

$$b_k := S_\Gamma(h_k \circledast x), \quad k = 1, \dots, M \tag{2}$$

The definitions almost coincide, except that S_Γ samples \mathbb{C}^{n+L-1} in the linear case, whereas it samples from \mathbb{C}^n in the circulant case.

For convenience, define the vector $b^T := \begin{bmatrix} b_1^T & \dots & b_M^T \end{bmatrix} \in \mathbb{C}^{M|\Gamma|}$, and $h \in \mathbb{C}^{ML}$ with similar definition.

Properties of the problem

- *Bilinearity*: the measurement vector b is a bilinear function of x and $(h_k)_{k=1}^M$; hence, the reconstruction is a bilinear inverse problem.
- *Scale ambiguity*: a consequence of the bi-linear nature of the equation. Any pair x and $(h_k)_{k=1}^M$ solving the problem defines infinitely many other solutions by scaling αx and $(\frac{h_k}{\alpha})_{k=1}^M$.
- *Shift ambiguity*: In case $(h_k)_{k=1}^M$ are all shorter than L , then shifted filters $(s^t h_k)_{k=1}^M$ constitute a solution of the problem with a opposite-shifted signal $s^{-t}x$. The same holds for the circulant case.

- *Algebraic Underdetermined*: due to the subsampling, the bilinear system in (1) has fewer observations than the non-sampled MBD problem; hence, the algebraic constraint count may be too small for unique reconstruction.

2.3 Formulation as Recovery of a Rank-1 and Row-Sparse coefficient matrix

Recently, Ahmed et al. [8] proposed a formulation of the single channel blind deconvolution problem as the recovery of rank-1 matrix, which has been extended to the multichannel (MBD) case by Romberg et al. [9].

We use the same suggested formulation, which is based on a linear expansion of the unknown variables, but extend it to consider downsampling. For any integer i , denote $e_i := s^i \delta$, so that $x = \sum_{\ell=0}^{n-1} x[\ell] e_\ell$ and $h_k = \sum_{j=0}^{L-1} h_k[j] e_j$, so that

$$S_\Gamma(h_k * x) = \sum_{j=0}^{L-1} \sum_{\ell=0}^{n-1} S_\Gamma(e_\ell * e_j) x[\ell] h_k[j] = \sum_{j=0}^{L-1} \sum_{\ell=0}^{n-1} S_\Gamma(e_{\ell+j}) x[\ell] h_k[j].$$

Defining $\Upsilon^{(k)} := x h_k^T \in \mathbb{C}^{n \times L}$, we have

$$b_k = S_\Gamma(h_k * x) = \sum_{j=0}^{L-1} \sum_{\ell=0}^{n-1} S_\Gamma(e_{j+\ell}) \Upsilon_{\ell,j}^{(k)}, \quad k = 1, \dots, M \quad (3)$$

This linear form can be represented by an inner product as follows. Define the matrix $E^{(i)} \in \mathbb{C}^{M \times L}$ by $E_{j,\ell}^{(i)} := (S_\Gamma(e_{j+\ell}))_i = e_{j+\ell}[\omega_i]$, and the linear operator $\mathcal{A} : \mathbb{C}^{n \times ML} \rightarrow \mathbb{C}^{M|\Gamma|}$ by the relation

$$\mathcal{A}(\Upsilon)_{k,i} := \langle E^{(i)}, \Upsilon^{(k)} \rangle, \quad k = 1, \dots, M, \text{ and } i = 1 \dots |\Gamma|$$

(here, the double subscript reads k th channel, i th entry). Then, assuming the presence of additive noise η , we can rewrite the input-output relation as

$$b = \mathcal{A}(xh^T) + \eta.$$

Therefore $\Upsilon = xh^T$ may be recovered by solving

$$\min_{\Upsilon} \{ \|b - \mathcal{A}(\Upsilon)\|_2 : \text{rank}(\Upsilon) = 1 \} \quad (4)$$

Once Υ is recovered, the vectors x and h may be determined by picking a row and a column from Υ .

Remark 2.8. *The definition for the circular convolution model is similar.*

By replacing the outer product xh^T by a general matrix of the same dimension, we increase the solution size from $p + q$ variables (in fact $p + q - 1$ modulo scaling ambiguity) to $p \times q$. This augmented formulation is also referred to as *lifting*. This seemingly redundant extension facilitates convex relaxation algorithms, as will be demonstrated later. Also note that scaling and shift ambiguities are absorbed within the product xh^T , so the solution Υ of the problem can be unique (under some circumstances), whereas its factorization $\Upsilon = xh^T$ is not necessarily unique.

2.3.1 A Sparsity Prior

It is often possible to assume that the input signal is sparse with respect to a certain dictionary $\Psi \in \mathbb{C}^{n \times r}$. That is, x can be uniquely represented as

$$x = \Psi\beta \tag{5}$$

such that

$$\|\beta\|_0 := \{\text{number of nonzero entries in } \beta\} \leq s,$$

where s is the so-called assumed sparsity level of the signal. This prior can overcome the under-determined nature of the system in (1). With the assumption of sparsity, we reformulate the problem as

$$P : \quad \min_Z \{\|b - \mathcal{A}(\Psi Z)\|_2 : \text{rank}(Z) = 1, \quad \|Z\|_{0,2} \leq s\} \tag{6}$$

where $\|Z\|_{0,2}$ counts the number of nonzero rows of Z . Note that $xh^T = \Psi Z$, where $Z = \beta h^T$ is a rank-1 matrix with s nonzero rows.

The rank-1 and the row sparsity constraints on Z define two priors.

3 Sparse Power Factorization

To solve (P), we propose to use *sparse power factorization* (SPF) [4], which is an alternating minimization algorithm that recovers a sparse rank-one matrix from its linear measurements.

Define $F : \mathbb{C}^{ML} \rightarrow \mathbb{C}^{m \times n}$ by

$$F(h) := [\mathcal{A}(e_1 h^T), \mathcal{A}(e_2 h^T), \dots, \mathcal{A}(e_n h^T)], \quad h \in \mathbb{C}^{ML},$$

where $(e_k)_{k=1}^n$ denotes the standard basis vectors in \mathbb{C}^n . Recall that $h = [h_1^T, h_2^T, \dots, h_M^T]^T$. Then, by the

linearity of the operator \mathcal{A} , $[F(h)]x = \mathcal{A}(xh^T)$, and $b = [F(h)]x + \eta$.

For given filter estimates \hat{h} , signal x is updated as a solution to a nonblind sparse deconvolution problem. To exploit the sparsity of x over dictionary Ψ , various sparse recovery algorithms can be used. In this paper, we use hardthresholding pursuit (HTP) [10] with the sensing matrix $\Phi = F(\hat{h})$, which is summarized in Alg. 2, where the *hard thresholding* (more precisely, the projection onto the ℓ_0 ball) operator $H_s : \mathbb{C}^n \rightarrow \mathbb{C}^n$ makes a given vector exactly s -sparse by zeroing all but the s elements of largest magnitude.

Define $G : \mathbb{C}^n \rightarrow \mathbb{C}^{m \times ML}$ by

$$G(x) := [\mathcal{A}(x\tilde{e}_1^T), \mathcal{A}(x\tilde{e}_2^T), \dots, \mathcal{A}(x\tilde{e}_{ML}^T)], \quad x \in \mathbb{C}^n,$$

where $(\tilde{e}_k)_{k=1}^{ML}$ denotes the standard basis vectors in \mathbb{C}^{ML} . Then, again, by the linearity of the operator \mathcal{A} , $[G(x)]h = \mathcal{A}(xh^T)$, and for given signal estimate \hat{x} , the filters h are updated as the LS solution to the linear system $b = [G(\hat{x})]\hat{h}$.

The SPF algorithm is summarized as Alg. 1. The initialization of SPF takes an estimate of the filters as the principal right singular vector of the prox matrix $\mathcal{A}^*(b) \in \mathbb{C}^n \times d$. Using a version of the restricted isometry property (RIP) of \mathcal{A} , one can show that the estimation error of this initialization is bounded from above by a certain threshold. The same RIP condition also guarantees that $[G(\hat{x})]$ has full column rank with a small condition number, and that $F(h_t)$ satisfies the sparsity-restricted isometry property; hence, the LS steps and the sparse recovery step using HTP are guaranteed to provide (near) optimal solutions to the respective subproblems. The detailed arguments establishing the above RIP of \mathcal{A} , and proving the above results will be included in a future, extended version of this report.

Algorithm 1: $\hat{x} = \text{SPF}(\mathcal{A}, b, \Psi, s)$

```

t ← 0;
h0 ← arg maxh { ||[A*(b)]h||22 : ||h||2 = 1 };
while stop condition not satisfied do
    xt+1 ← HTP(F(ht), b, Ψ, s);
    ht+1 ← arg minh ||b - [G(xt+1)]h||22;
    t ← t + 1;
end
return  $\hat{x} \leftarrow x_t$ ;

```

Algorithm 2: $\hat{x} = \text{HTP}(\Phi, b, \Psi, s)$

$t \leftarrow 0; x_0 \leftarrow 0;$
while *stop condition not satisfied* **do**
 $J \leftarrow \text{supp}(H_s(\Psi^*(x_t + \gamma\Phi^*(b - \Phi x_t))));$
 $\beta_{t+1} \leftarrow \arg \min_{\beta} \{\|b - \Phi\Psi\beta\|_2^2 : \text{supp}(w) \subset J\};$
 $x_{t+1} \leftarrow \Psi\beta_{t+1};$
 $t \leftarrow t + 1;$
end
return $\hat{x} \leftarrow x_t;$

4 Numerical Results

We compare SMBD by SPF to the following algorithms:

- Non-blind HTP: recovery of unknown sparse input signal using HTP, assuming knowledge of the filters.
- Non-blind LS: least squares recovery (without assuming sparsity) of the unknown input signal assuming knowledge of the filters.
- SMBD using a convex relaxation approach for the recovery of a sparse rank-1 matrix. Recently, it has been shown that the max function of convex surrogates for the rank and for the sparsity with certain weights performs best among all combinations of these surrogates [11]. However, the weights that provide the best performance depend on the unknown sparse rank-1 matrix, and therefore cannot be used in practice. Furthermore, more importantly, it has been shown that, when \mathcal{A} is the i.i.d. Gaussian linear operator, a necessary condition for the exact recovery using the aforementioned best convex surrogate approach is significantly more demanding than a sufficient condition for the exhaustive search. Therefore, the convex surrogate approach does not achieve the fundamental limit of the problem whereas it is achieved by SPF up to a polylog factor of n . To confirm that SPF outperforms the best convex surrogate approach in solving SMBD, we compare the two methods. The best convex surrogate approach solves the following optimization problem, where Z^* denotes the sparse rank-1 matrix that we aim to recover.

$$P_{\text{Convex}} : \quad \min_Z \frac{1}{2} \|b - A(\Psi Z)\|_2^2 + \lambda \max \left(\frac{\|Z\|_{1,2}}{\|Z^*\|_{1,2}}, \frac{\|Z\|_*}{\|Z^*\|_*} \right).$$

where $\|Z\|_*$ denotes the nuclear norm of Z , that is the sum of all singular values of Z , and $\|Z\|_{1,2}$

denotes the mixed norm defined by

$$\|Z\|_{1,2} \triangleq \sum_{k=1}^n \|z^k\|_2.$$

Here, z^k denotes the k -th row of Z . In the simulation, FISTA [12] is used to solve this convex optimization problem, and the best performing weights $\|Z^*\|_{1,2}$ and $\|Z^*\|_*$ are assumed to be known (given by an oracle). We compute an estimate \hat{x} of the unknown signal x from the best rank-1 approximation of the solution \hat{Z} to Problem P_{Convex} .

We performed simulation on the following generic benchmark: the Fourier transform of the input signal x is sparse over a random rotation dictionary. In the context of pMRI, the Fourier transform of x corresponds to the target image in the spatial domain. A random rotation dictionary is not practical but serves as a good benchmark in the following sense:

- Unlike a wavelet dictionary, a random rotation dictionary is incoherent to the Fourier basis vectors, which is desired for the success of the sparse recovery steps using HTP.
- The input signal in the Fourier domain synthesized in the aforementioned way is not sparse over the standard basis vectors. If the Fourier transform of x is sparse, then convolution of filters with x , which corresponds to pointwise multiplication in the Fourier domain, will remove information of the filters; hence, the reconstruction of filters may be impossible even without subsampling.

Filter coefficients were generated as complex i.i.d. Gaussian random variables, unless otherwise stated. The lengths of the filters are set as $L = 8$, which is relatively short compared to the length of the signal $n = 256$. We vary the number of channels, the number of samples m per channel, and the sparsity level s of the input signal, to observe their effect on the reconstruction. The subsampling pattern Γ consists of $3L - 2$ consecutive indices and $(m - 3L + 2)$ random indices according to the uniform distribution on $\{3L - 1, \dots, n\}$. This sampling pattern is motivated by sampling schemes used in MRI.

We compare the performance of the different methods in a Monte-Carlo study, averaging over 25 instances in which the input signal, noise, filters, and sampling pattern are random. Fig. 2 compares the reconstruction SNR in the noiseless case. Each square shows the reconstruction SNR as a function of the relative sparsity s/n (horizontal axis) and the subsampling rate m/n (vertical axis). As the relative sparsity decreases, high reconstruction SNR is obtained with fewer data samples (smaller m/n). Not surprisingly, we observe that as commonly known in practice, more channels provide higher reconstruction SNR for all methods. When there are many channels (the last two columns of Fig. 2), SPF nearly achieves the performance of the nonblind HTP, and outperforms the best convex surrogate approach, labelled as ‘‘convex’’. Next, Fig. 3 compares the reconstruction SNR in the presence of slight noise in the measurements at SNR of 30 dB. Again, SPF

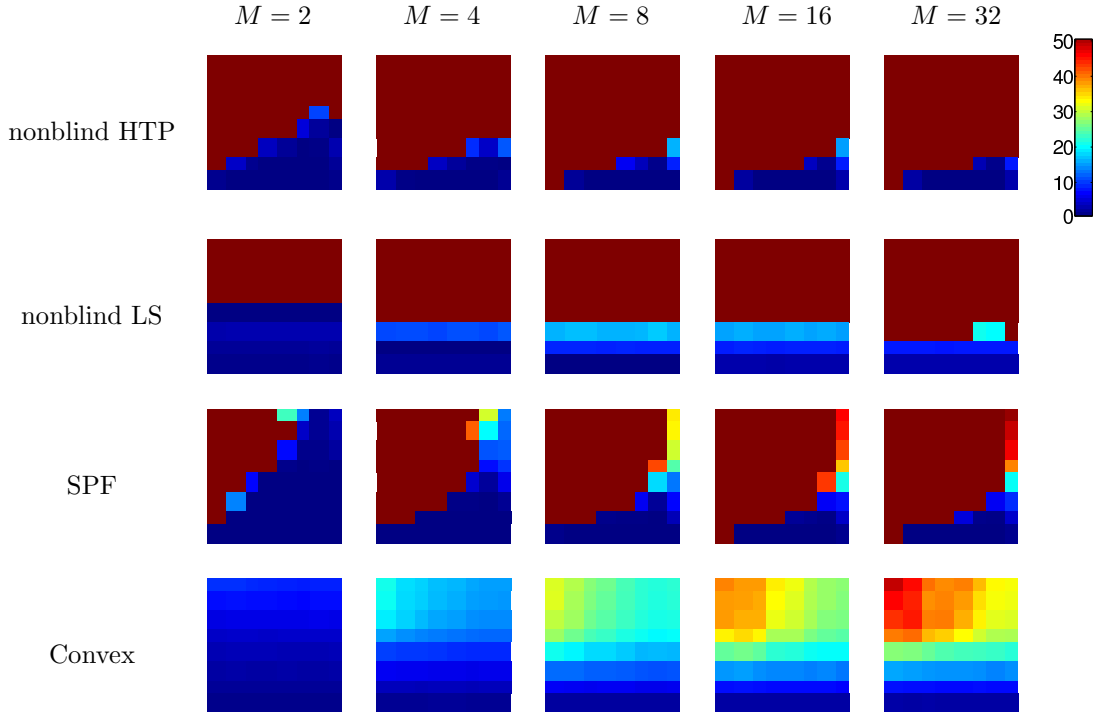


Figure 2: Reconstruction SNR (capped at 50 dB). Noiseless. Filters of length $L = 8$. M -Channels. x -axis is s/n . y -axis is m/n .

outperforms the convex surrogate approach and performs competitively to the nonblind HTP with many channels, similarly to the noiseless case. In the noisy case, we note that the use of the sparsity prior improves the performance significantly: nonblind LS with exactly known filters, only allows subsampling by factor of 2 with 32 channels, whereas nonblind HTP, and most important from a practical perspective, SPF allow higher subsampling for sparse signals.

As shown in Fig. 4, the recovery performance by SPF shows graceful degradation with an overestimate $\hat{L} = 9$ of the filter length $L = 8$, or with correlation between filters for different channels. We only compare SPF and the nonblind HTP (which always uses the known filters, of correct order) in these scenarios because they outperform the other methods. The middle column of Fig. 4 corresponds to the case where the length of the filters in SPF is overestimated as 9, while the true length is 8. To simulate the correlated filters (the last column of Fig. 4), we first generated isotropic filters with i.i.d. Gaussian coefficients (as in the first column), and then mixed them to generate correlated filters with the k th correlated filter obtained as the weighted average of all isotropic filters, where the weight on the k th isotropic filter is 1 and the weights on the other filters are 0.1.

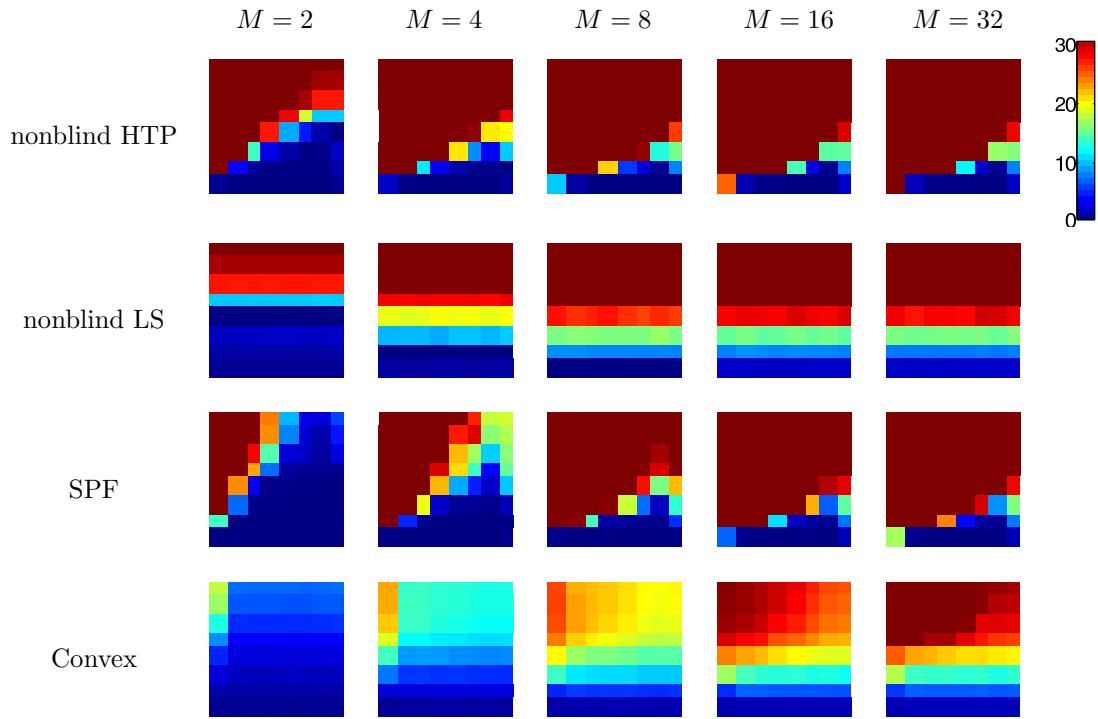


Figure 3: Reconstruction SNR (capped at 30 dB). Input SNR 30 dB. Filters of length $L = 8$. M -Channels. x -axis is s/n . y -axis is m/n .

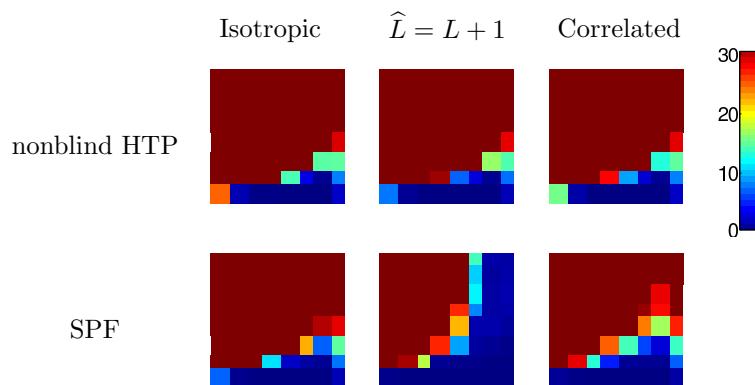


Figure 4: Reconstruction SNR (capped at 30 dB) for the overestimation of the filter length in SPF (middle column) and for the correlated filters (right column). Input SNR 30 dB. Filters of length $L = 8$. 16-Channels. x -axis is s/n . y -axis is m/n .

5 Conclusion

We proposed to use the sparse power factorization algorithm to solve the subsampled multichannel blind deconvolution (SMBD) problem by exploiting a sparse signal model. SMBD is formulated as the recovery of a sparse rank-one matrix from linear measurements produced by a known linear operator \mathcal{A} . When there are multiple priors on the solution simultaneously, SPF outperforms convex relaxation methods empirically. In solving SMBD, we verified that SPF also outperforms convex relaxation methods and provides competitive performance even compared to the nonblind sparse deconvolution with exactly known filters. A rigorous recovery guarantee of SPF for SMBD will appear in an accompanying paper [7].

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