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QUENCHING OF THE BEAM-PLASMA INSTABILITY BY MODE MIXING AT A DENSITY DISCONTINUITY

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by

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QUENCHING OF THE BEAM-PLASMA INSTABILITY BY MODE MIXING AT A DENSITY DISCONTINUITY

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Abstract

Experiments on the spatial instability of a beam-plasma system led to the observation that the instability is quenched when irradiated by high power microwaves. It is conjectured that this effect is due to a local decrease in plasma density in the irradiated region. A theoretical model is detailed that explains the salient features of the observed effect. Auxiliary measurements on a system with a controlled density gradient are presented that give strong support for the proposed mechanism.

I. INTRODUCTION

During simultaneous measurements of radiation and microwave scattering from unstable plasma waves in a beam-plasma system it was noticed that the radiation from the unstable waves was reduced if a beam of high power microwaves was focused onto the plasma. The effect was found to be insensitive to the frequency of the incident radiation which exceeded the plasma frequency by a factor 2 - 3. This excluded the possibility of resonant wave coupling processes as the source of the observed effect. The power density of the incident radiation, which was of the order of 1 Watt/cm^2, was far too small to cause any dynamic (parametric) effects that could lead to quenching of the instability. The only reasonable alternative was to assume that heating of the plasma was responsible for the observed effect. A direct influence of a finite plasma temperature on the growth of the unstable waves is negligible under the conditions of the experiment; the growth rate being dominated by collisions rather than temperature. An increased plasma temperature can, however, also influence the plasma density. In the above mentioned experiments the plasma was produced by a pulsed discharge and subsequently decayed by ambipolar diffusion. An increase in temperature leads to an increased diffusion coefficient and hence to a decrease in plasma density in the heated region. In this paper we want to point out that even a small variation in the background plasma density can lead to a drastic reduction in the instability level and therefore affords a reasonable explanation of the observed quenching effect.

The paper is organized as follows: in Section II we present the observations that motivated these investigations, in Section III a theoretical model is proposed that explains the salient features of the observed effect, and in Section IV subsidiary measurements on a system with a deliberately introduced density discontinuity are presented which lend support to the proposed theoretical model. Section V contains a summary of these results.

II. EXPERIMENTAL ARRANGEMENT AND OBSERVATIONS

An influence of high power microwave radiation on the instability in a beam plasma system was first noticed when radiation and scattering from the unstable waves were measured simultaneously, in order to establish the relation between the radiation and the fluctuation spectrum.[1] The experimental arrangement has been described in detail previously;[2,3] a pulsed electron beam of 50 - 500 mA current at an energy of 15-20 kV and 2 μsec duration was injected into the afterglow of a pulsed discharge in Neon. The background gas pressure was $2x10^{-2}$ Torr and most experiments were carried out at a plasma density of $10^{12}/cm^3$. The spatial development of the instability was determined by measuring the microwaves radiated by the instability as a function of distance along the axis of the system. An example of the unperturbed growth of the instability at a plasma frequency of 10.0 GHz is shown in Fig. 1 (curve A). If the system is irradiated with a focused microwave beam of frequency 28.28 GHz, 9 Watt CW and 2 cm width, the power radiated by the plasma waves is reduced by about a factor of 2 (curve B). The effect does not depend on the polarization of the incident microwaves with respect to the direction of the beam velocity, nor significantly on the ratio of incident frequency to plasma frequency. Qualitatively the same effect was observed, for example, at a plasma frequency of 16 GHz. On the other hand it was found that the influence of the incident microwave beam was greatly enhanced if the beam width along the axis of the system was increased to 10 cm (curve C). Fig. 2 shows another example of the suppression of the instability for a larger electron beam current where the instability is stronger and the unstable waves saturate at large distances.

Using an independent microwave source it was shown that the reduction of the observed microwave power was not due to a change in the receiver sensitivity by pick-up of stray radiation from the 28 GHz - source. In fact, measurements of the velocity distribution of the beam gave an independent indication that the instability was directly affected. Due to the feedback of the unstable waves on the initially monoenergetic beam, the beam becomes spread out in velocity space. Irradiation with the high power microwaves always reduces the width of the beam distribution function compared to the undisturbed case, indicating a reduction in the instability level.

A parametric quenching of the instability by radiation of frequency far in excess of the plasma frequency is possible in principle, as first proposed by Aliev and Silin[4] for the case of ion-acoustic waves. The coupling of the external field to the plasma waves is provided by the induced relative oscillatory drift of the plasma species (electrons and ions). If both particles are identical (beam and plasma electrons) the induced drift is the same for both species and no parametric coupling is expected unless, for example, the beam is relativistic or, as is the case in this experiments, the plasma electrons are subject to collisions. Following Aliev and Silin it is easy to derive the modifications introduced by a radiation field of frequency ω_o on the dispersion relation of a cold beam in a cold plasma with collisions. Within the dipole approximation the result is

$$1 - \frac{\omega_p^2}{\omega(\omega+i\nu)} - \frac{\omega_b^2}{(\omega-ku_o)^2} + (1-J_o^2(a))\,\frac{\omega_p^2\,\omega_b^2}{\omega(\omega+i\nu)(\omega-ku_o)^2} = 0$$

ω_p and ω_b are the plasma and beam plasma frequencies respectively, u_o is the beam velocity and ν the collision frequency. The argument of the Bessel function is given by

$$a = \frac{e}{m}\,\frac{kE_o\nu}{\omega_o^3}$$

For real ω and $a \ll 1$ one finds for the growth rate of the most unstable waves, except for a factor of order unity

$$\sigma \simeq \left(\frac{\omega_b}{\nu}\right)^{\frac{1}{2}} \frac{\omega_p}{u_o}\, J_o(a)$$

In a typical experiment we have $k = 6\ \mathrm{cm}^{-1}$, $E_o = 50$ V/cm, $\omega_o/2\pi = 28\times10^9$/sec and $\nu = 4\times10^9$/sec. This gives $a \sim 10^{-6}$ and the parametric influence on the instability is negligible.

Interferometric measurements of the background electron density revealed, that the density was reduced by about 5% in the region that was illuminated by the high power microwaves. This reduction is due to local heating of the plasma with a resulting increase in ambipolar diffusion. Such a local change in background plasma density can have a profound influence on the growth characteristics of unstable waves as was first remarked by Dorman.[5] His treatment, however, is not germain to the case of spatially growing waves and can therefore not be applied to the problem under consideration.

III. THEORETICAL ANALYSIS

To study the influence of the variable plasma density on the spatial growth of the beam-plasma instability, we will first consider the model of a cold beam-plasma system. The neglect of the plasma thermal motion is easily justified in the case of a homogeneous plasma, because of the large phase velocity of the electrostatic waves (of the order of the beam velocity, which is $\frac{1}{3}$ c). Since this condition also applies in the inhomogeneous case, and no other thermal effect is known to be important, we will assume that the cold plasma model is also applicable in this case. Moreover in the region upstream from the density variation $|(\omega/k)-u_o| > u_{Tb}$, where ($u_o$, v_{Tb}) is the (drift, thermal) velocity of the beam and (ω,k) refers to the most unstable mode. Hence in this region the interaction between the beam and the waves is largely nonresonant, so that the thermal spread of the beam can be neglected to first order. The influence of the beam's thermal motion is, however, quite pronounced in the downstream region. A modification of the present cold beam-plasma theory will be discussed later in Section IV.

We therefore consider a semi-infinite ($x \geq x_o$) cold beam-plasma system, in which the unperturbed plasma density, $n_o(x)$, is a function of x, and look for stationary solutions which are proportional to $e^{-i\omega t}$(ω: real). The linearized continuity equations and equations of motion for the plasma and beam (fixed ions) are

$$-i\omega\, n_p + \frac{d}{dx}\,(n_o(x)u_p) = 0 \tag{1}$$

$$-i\omega\, u_p + \frac{e}{m}\, E = -\,\nu(x)u_p \tag{2}$$

$$-i\omega\, n_b + \frac{d}{dx}\,(n_{ob}u_b + u_o n_b) = 0 \tag{3}$$

$$-i\omega\, u_b + \frac{d}{dx}\,(u_o u_b) + \frac{e}{m}\, E = 0 \tag{4}$$

Here (n_{ob},u_o) is the unperturbed beam (density, drift velocity), and all other functions are perturbed quantities. The term $-\nu\, u_p$ in Eq. (2) represents the influence of the plasma-ion collisions, which must be included to obtain a finite spatial growth rate of all ω. While we will usually consider the case where the collision frequency, ν, is constant, the following results hold for any prescribed function $\nu(x)$. The remaining equation is Poisson's equation

$$\frac{dE}{dx} = -4\pi e\ (n_p + n_b) \tag{5}$$

Eqs. (1-5) can be reduced to a single third order equation, which can be integrated to yield

$$\frac{d^2F}{dx^2} + \varkappa^2(x)F = 0 \tag{6}$$

where

$$F(x) = \varkappa^{-2}(x)E(x)e^{-i\omega x/u_o} \tag{7}$$

$$u_o^2\varkappa^2(x) = \omega_b^2\omega[\omega + i\nu(x)]/\{\omega[\omega + i\nu(x)] - \omega_p^2(x)\} \tag{8}$$

and $(\omega_b^2, \omega_p^2(x)) = (n_{ob}, n_o(x))4\pi e^2/m$. The arbitrary constant of integration is Eq. (6) is related to the situation where an electric field $E_o e^{-i\omega t}$ is applied to the system. Since this situation is not of present interest, we have set it equal to zero.

An equation similar to (6) was obtained a number of years ago by Sumi.[6] His equation however did not include the plasma-ion collisional effect, which is of basic importance for the spatial instability analysis. Moreover, the solution of this equation was only discussed in terms of the lowest order WBK approximation. To the contrary, we are concerned with the situation where the plasma density $n_o(x)$ varies both rapidly (i.e., comparable with the wavelength) and by a significant amount (5% to 50%). Under these conditions a pronounced decrease in the level of the instability occurs in the inhomogeneous region. Moreover, if there is a decrease in the density in the direction of the beam velocity, there is a significant spatial structure to the instability level in the "downstream" homogeneous region - due to the mixing ("beats") between the growing and decaying modes.

To illustrate these effects in the simplest fashion, we first consider two homogeneous plasma regions. If a density discontinuity occurs well inside the plasma (from where the beam is injected), then the only modes of significance in the region $x \lesssim 0$ (upstream) are the spatially growing modes

$$E_1(x,t) = E_1(\omega)e^{i(k_1 x-\omega t)} \qquad (\mathrm{Im}k_1<0)$$

where $k_1 = (\omega/u_o) + \varkappa_1$, and $\varkappa_1$ is given by Eq. (8) in the region. Moreover the most important mode is the one for which $|\mathrm{Im}\varkappa_1(\omega)|$ is a maximum. Note that the discontinuity at $x = 0$ has no influence on the solution in the region $x < 0$, because all modes have a positive phase velocity.[7] Thus in region 1 ($x \leq 0$) and 2 ($x \geq 0$) Eq. (6) has solutions of the form

$$F_1 = A_1 e^{i\varkappa_1 x} \quad ; \quad F_2 = A_2 e^{i\varkappa_2 x} + B_2 e^{-i\varkappa_2 x}$$

where both $\mathrm{Im}\varkappa_i < 0$. Since Eq. (6) requires that both F and dF/dx be continuous at $x = 0$, one can readily determine $E_2(x)$ in terms of $E_1(0)$, using the definition (7). One obtains

$$E_2(x) = \frac{E_1(0)\kappa_2}{2\kappa_1^2}\,[(\kappa_1+\kappa_2)e^{i\kappa_2 x} + (\kappa_2-\kappa_1)e^{-i\kappa_2 x}]e^{i\omega x/u_o} \qquad (9)$$

This yields for the power

$$|E_2(x)|^2 = \frac{|E_1(0)|^2|\kappa_2|^2}{4|\kappa_1|^4}\,[|\kappa_2+\kappa_1|^2 e^{i(\kappa_2-\kappa_2^*)x} + |\kappa_2-\kappa_1|^2 e^{-i(\kappa_2-\kappa_2^*)x}$$

$$+ 2(|\kappa_2|^2-|\kappa_1|^2)\cos(\kappa_2+\kappa_2^*)x - 2i(\kappa_2\kappa_1^*-\kappa_1\kappa_2^*)\sin(\kappa_2+\kappa_2^*)x] \qquad (10)$$

The last two terms in Eq. (10) are due to the mixing (spatial beats) between the growing and decaying modes in region 2. In order for this mixing to produce a noticeable effect on $|E_2(x)|^2$, not only must κ_2 differ from κ_1, but one must also have $|\mathrm{Re}\kappa_2| >> |\mathrm{Im}\kappa_2|$. This condition is necessary in order for the trigonometric terms in (10) to vary rapidly compared to the exponential growth term. To see what type of discontinuity may produce this effect, we note that $\kappa(\omega,\omega_p)$, Eq. (8), can be written

$$\kappa(\omega,\omega_p) = (\omega_b/u_o)(2A)^{-\frac{1}{2}}[\mathrm{sign}(\omega)\{B+(B^2+\nu^2\omega^2\omega_p^4)^{\frac{1}{2}}\}^{\frac{1}{2}}$$

$$-i\{-B+(B^2+\nu^2\omega^2\omega_p^4)^{\frac{1}{2}}\}^{\frac{1}{2}} \qquad (11)$$

where $A = (\omega^2-\omega_p^2)^2 + \omega^2\nu^2$ and $B = \omega^2(\omega^2-\omega_p^2 + \nu^2)$. Normally κ is considered as a function of ω, but for present purposes it is more important to consider it as a function of ω_p^2 (i.e., the density in the homogeneous regions), with ω held constant. In Fig. 3 κ^* is plotted vs the fractional density step $\Delta n/n = (n_2-n_1)/n_1$, and where ω has been taken to be the most unstable mode in region 1. From this figure it is clear that if $\Delta n > 0$ then $|\mathrm{Im}\kappa_2|>|\mathrm{Re}\kappa_2|$, and consequently the mixing will produce no noticeable effect on $|E_2(x)|^2$. However if there is a density drop of more than 3% from region 1 to region 2, then $|\mathrm{Re}\kappa_2|>|\mathrm{Im}\kappa_2|$ and the mixing can produce a noticeable spatial structure in the radiated power from region 2. Numerical examples illustrating this effect will be given below.

The second important fact predicted by (10) is that

$$|E_2(0)|^2 = |\kappa_2/\kappa_1|^4|E_1(0)|^2 \tag{12}$$

so that the electric field is discontinuous at $x = 0$. The ratio of the fields can be expressed quite simply if one introduces the parameter s, by $\omega_p^2 = \omega^2 + s\omega\nu$. From (11) one finds that $|\kappa|^4 = (\omega_b/u_o)^4(\omega^2+\nu^2)\nu^{-2}(1+s^2)^{-1}$, so that for constant ν, $|E_2(0)|^2/|E_1(0)|^2 = (1+s_2^2)$. Using the approximation $s_1 \simeq 3^{-\frac{1}{2}}$ (the most unstable mode in region 1), one finds

$$|E_2(0)|^2/|E_1(0)|^2 \simeq (1+3^{-\frac{1}{2}})/\{1+[(\Delta n/n)(\omega_{p1}/\nu)+3^{-\frac{1}{2}}]^2\} \tag{13}$$

Since (ω_{p1}/ν) is usually quite large ($\gtrsim 20$), this ratio is a very sensitive function of $(\Delta n/n)$. Eq. (13) furthermore predicts that for very small negative values of Δn, the power ratio can be greater than unity ($\simeq 1.5$). The reality of this conclusion is doubtful however, since, for finite gradients, Eq. (9) is not valid in the limit of vanishing Δn (see the discussion following Eq. (18)).

While this step discontinuity model is relatively simple to analyze, the result (12) is not acceptable - at least in a formal sense - because it implies that dE/dx is infinite at $x = 0$. This in turn, because of (5), implies that there is an infinite perturbation of the density and hence a breakdown in the linear theory. To determine under what conditions the results (9) et seq. are essentially correct requires the analysis of a more realistic model involving a finite gradient in $n_o(x)$. This has been done for the case where

$$\omega_p^2(x) = \omega_{p1}^2(1 + \epsilon(x/L)) \quad (L \geq x \geq 0 \; ; \; \epsilon > -1) \tag{14}$$

corresponding to a constant density gradient in the transition region. It can be shown (see Appendix 1) that the linear approximation is valid in this case provided that

$$1 \gg |\epsilon| (\omega_b/\omega_p)^2 (\omega_p/\nu)(u_o/\omega_p L) \tag{15}$$

which indeed requires that $L \neq 0$. However in the present experiments $(\omega_b/\omega_p)^2 \simeq 10^{-4}$, $\omega_p/\nu \simeq 20$, and $\lambda = u_o/\omega_p \simeq 1$ cm. Hence, if $L \gg 10^{-2} \lambda$ (where λ is the wavelength of the waves in the homogeneous region) the linear approximation is valid. This is certainly the situation in all of the present experiments. The fact that the linear approximation is satisfied does <u>not</u> prove that the expression (9) is essentially correct. To establish this requires a solution of Eq. (6) using the plasma density (14), which yields the equation

$$\frac{d^2F}{d\xi^2} + b^{-2}\xi^{-1} F = 0 \qquad (16)$$

where $\xi = a-b(\omega_b x/u_o)$, $a = [\omega(\omega+i\nu)-\omega_{pl}^2]/[\omega(\omega+i\nu)]$, and $b = (\epsilon/L)u_o\omega_{pl}^2 \cdot[\omega_b\omega(\omega+i\nu)]^{-1}$. The solution of (16) is

$$F = A\ z\ J_1(z) + B\ z\ N_1\ (z) \qquad (17)$$

where $z = 2\ \xi^{\frac{1}{2}}/b$ and (J,N) are the Bessel functions of the (first, second) kind. If one now uses (17) and matches F and dF/dx to $F(0)e^{i\kappa_1 x}$ at x =0 and to $A_2e^{i\kappa_2 x} + B_2e^{i\kappa_2 x}$ at x = L, then one finds that for $x \geq L$ (the solution for $L \geq x \geq 0$ is given in Appendix 1)

$$F(x) = \sum_{\pm} A_{\pm}\ e^{\pm i\kappa_2(x-L)} \qquad (x \geq L)$$

where

$$A_{\pm} = F(0)\ \frac{\pi}{4}\ \{[N_o(z_o) + \frac{i}{2b}\ \kappa_1 z_o N_1(z_o)][z_1J_1(z_1) \pm i\ \frac{2}{\kappa_2 b}\ J_o(z_1)]$$

$$-[J_o(z_o) + \frac{i}{2}\ b\kappa_1 z_o J_1(z_o)][z_1N_1(z_1) \pm i\ \frac{2}{\kappa_2 b}\ N_o(z_1)]\} \qquad (18)$$

and (z_o, z_1) correspond to (x = 0, x = L). These case of interest is when ϵ is not too small and (L/λ) is not very large (large rapid changes in $n_o(x)$) - which is the opposite extreme from the WKB situation. From the definitions following (16) we note that

$$\max\,(\nu/\omega_p;\epsilon) > |\xi| \text{ and } |b| \simeq \epsilon(\omega_p/\omega_b)(\lambda/L)$$

where again $\lambda = u_o/\omega_p$. Therefore if L/λ is not too large (but large enough to satisfy (15)), and ϵ is not too small, then b is large and $|\xi|$ will be fairly small. Consequently z is small in this case, and the Bessel function in (18) may be expanded for small argument. When this is done one finally obtains the approximate result

$$E_2(x) \simeq \frac{E_1(0)\varkappa_2}{2\varkappa_1^{\,2}} \sum_{\pm} [\varkappa_2 \pm \varkappa_1) \pm \frac{2i}{b} \ln\left(\frac{\varkappa_2}{\varkappa_1}\right)] e^{\pm i\varkappa_2 (x-L) + i(\omega x/u_o)} \tag{19}$$

where $x \geq L$. The result (19) may be compared with the result (9) for the simple discontinuous model. It can be seen that, provided b is large and (15) is satisfied (L not too large and not too small), the result (9) is a very good approximation.

Some numerical examples which illustrate the behavior of the $|E_2(x)|^2$, as given by Eq. (10), are shown in Figs. 4 and 5. Fig. 4 contains two examples for the variation of power with distance for the case of a density decrease. The value of ν/ω_{p1} was chosen to correspond to the experimental conditions. We notice that the period of the oscillatory behavior in the perturbed region increases with decreasing density due to the smaller value of $\mathrm{Re}\varkappa$. Fig. 5 shows the same calculation for the case of a density increase. The oscillatory component of $|E_2|^2$ is very much smaller since $|\mathrm{Re}\varkappa_2| \gg |\mathrm{Im}\varkappa_2|$.

IV. MEASUREMENTS ON A SYSTEM WITH A CONTROLLED DENSITY DISCONTINUITY

The theoretical model that was presented in the previous section provides a qualitative explanation for the observed quenching effect. It is, nevertheless, desirable to have an independent check of this model in order to ascertain that the quenching is indeed due to a density discontinuity and that the incident high power radiation has no direct dynamical effect on the instability.

To this end, measurements were performed on a system in which a discontinuity in plasma density of known location and magnitude was deliberately introduced. This was accomplished by constricting the discharge over a given length by inserting two glass tubes of different inside diameter 4.0 and 3.6 cm into the main discharge tube, which had an inside diameter of 4.3 cm. Since the pulsed discharge in the tube decays by ambipolar diffusion, the plasma density decreases more rapidly in the constricted region. At a delay time of a few 100 μsec, at which the plasma frequency is equal to the receiver frequency, the density in the constricted region of the discharge tube was 20% lower than the density in the unconstricted region for the 4 cm insert, and 50% lower for the 3.6 cm insert, as measured by a K-band microwave interferometer.

The inserted tube could be positioned a certain distance away from the entrance aperture to realize a situation in which the beam enters a region of decreased density, or it could be positioned close to the entrance aperture, in which case the beam emerged from a region of low density and entered a region of higher density. Fig. 6-9 give examples of the observed radiation in these situations. The position of the constriction is indicated in each instance. It can be seen that in each case the instability decreases sharply in the region downstream from the density discontinuity, as expected. In order to ascertain that the decrease in the observed radiation intensity was not due to a disturbance of the radiation pattern by the increased thickness of the glass part of the discharge tube, an experiment was performed in which the same glass thickness was produced by having an additional glass tube on the outside of the discharge tube. No decrease of the radiation was observed at the glass discontinuity.

The decrease in the instability level due to the discontinuity is larger than what would be expected on the basis of the cold plasma model that was developed in the previous section. It is well known, however, that the originally monoenergetic beam acquires a velocity spread as soon as the instability reaches a sizable amplitude. This velocity spread in conjunction with the influence of collisions of plasma electrons with ions (or neutral atoms) causes a marked change in the dispersion relation and leads to a drastic reduction in the growth rate. Complete quenching of the instability occurs when the velocity spread Δu exceeds a critical value given by[2,8]

$$\Delta u^2 = \frac{3}{8}^{3/2} \frac{\omega_b^2}{\omega_p \nu} u_o^2 \qquad (20)$$

It is difficult to assess the effect of a discontinuity on a smeared-out beam, short of a consistent kinetic treatment of the problem. Some insight into the influence of a velocity spread can be gained, however, by a relatively crude model, assuming that the beam stays cold up to the discontinuity and that it assumes a Lorentzian distribution behind the discontinuity. This model allows one to account for the velocity spread simply through a modification of the dispersion relation. In the equation for the electric field in the region 2, this amounts to replacing $\varkappa_2$ inside the brackets of Eq. 9 by $\varkappa_2 - i(\omega\Delta u/u_o^2)$ and multiplying the r.h.s. by $\exp(-x\omega\Delta u/u_o^2)$: An example for the influence of a relative velocity spread of 1% on the behavior of the solution in region 2 is plotted in Fig. 10 for $\Delta n/n = -0.2$. Comparing the curve for $\Delta n/n = -0.2$ in Fig. 4 with Fig. 10 it can be seen that even a small velocity spread has a profound effect on the decrease in the instability level. As a matter of fact, the 1% velocity spread is sufficient to change the originally growing mode into a strongly damped mode in the region of decreased density. For the 20% density step considered here, a 0.2% velocity spread is sufficient to cause a net damping of the mode. It should be noticed that a 1% velocity spread still leads to a growing solution in the region of unperturbed density, as can be seen from Eq. 20. The computed curve in Fig. 10 shows a certain similarity with the experimental curve in Fig. 6, which corresponds to the same density drop. In comparing these curves it should be noted that $(\omega_b x/2^{\frac{1}{2}}u_o) = 1$ corresponds to an actual distance of 24 cm. Furthermore, the perturbed region extends only over a certain distance, after which the density again assumes its unperturbed value. In this region the waves grow essentially with the unperturbed growth rate. The fact that the maxima and minima do not exactly coincide with the beginning and end of the perturbed region may have its origin in the finite resolution of the microwave horn that was used in these measurements. It had a width of 5 cm and for a steep spatial growth overemphasized the power at a given location. In view of the fact that the solution in the perturbed region is extremely sensitive to the assumed velocity spread and considering the relatively crude model that was assumed, it is unrealistic to expect a quantitative agreement with experimental results. It would appear, however, that the experimentally observed magnitude of the quenching can be explained satisfactorily by invoking the well known fact that the beam invariably acquires a small velocity spread when interacting with a plasma.

V. CONCLUSION

The experimental results of Section IV show quite clearly that a variation in the plasma density does have a profound effect on the spatial beam-plasma instability. This effect moreover appears to be the only reasonable explanation for the observed decrease in the instability level when the plasma is locally irradiated with microwaves, as noted in Section III. The theoretical analysis of this effect indicates that the resulting drop in intensity is a sensitive function of the plasma discontinuity. It also predicts that there may be a spatial oscillatory behavior downstream, due to beats between the decaying and growing modes, only if the plasma density decreases. The experimental observations presented in Section IV suggest some oscillatory behavior for a 20% density drop; the experiments are, however, not conclusive in this respect since there is no indication of oscillations for the 50% density drop. Moreover the spatial behavior in the downstream region is a very sensitive function of the beam thermal spread - due to the fact that, in this region, the waves are more nearly in resonance with the beam particles. Considering the sensitivity of the phenomena to these parameters, which are only approximately known, and the spatial resolution of the measurements, the agreement between the theory and experiment appears to be satisfactory. An improvement of the theory to account for thermal effects, particularly in the beam, therefore appears to be warranted only if the physical parameters can be determined more accurately.

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APPENDIX 1

To establish a condition for the validity of the linear approximation in the region where $\varkappa(x)$ is rapidly varying, we note first that u_p, u_b, and n_b can all be expressed in terms of $\varkappa(x)$, $F(x)$, and dF/dx. In the region of the constant density gradient (Eq. (14)), the solution of Eq. (16) is

$$F(z) = F(0)(\pi/2)\{[N_o(z_o) + \frac{1}{2}\, ib\varkappa_1 z_o N_1(z_o)]zJ_1(z)$$

$$- [J_o(z_o) + \frac{1}{2}\, ib\varkappa_1 z_o J_1(z_o)]zN_1(z)\}$$

so that, not only is dF/dx continuous, but it does not become large in the region $L \geq x \geq 0$. Thus the only perturbed quantity one must examine is n_p

$$n_p = (i\omega 4\pi e)^{-1} \frac{d}{dx}\left(\frac{\omega_p^2 \varkappa^2}{(i\omega-\nu)} F\, e^{i\omega x/u_o}\right)$$

which can become large because of the term $d(\omega_p^2\varkappa^2)/dx$. Considering only this term one has

$$\omega_{p1}^2(n_p/n_{o1}) \simeq -\frac{e}{m}\frac{\omega_b^2}{u_o^2} F\, e^{-i\omega x/u_o} \frac{d}{dx}\left(\frac{\omega_p^2\varkappa^2}{\omega(\omega+i\nu)-\omega_p^2}\right)$$

$$= -\frac{e}{m}\frac{u_o^2}{\omega_b^2}\frac{}{\omega(\omega+i\nu)}\frac{d\omega_p^2}{dx}$$

Using Eq. (14, and the fact that $\omega_b/u_o)^4(\omega^2+\nu^2)\nu^2 \gtrsim |\varkappa^4|$, we conclude that

$$\left|\frac{e}{m}\frac{\omega_b^2}{u_o^2\nu^2}\frac{\epsilon}{L}F\right| \gtrsim \left|\frac{n_p}{n_{o1}}\right|$$

From above we see that the magnitude of F can not be substantially larger than F(0). To obtain a bound on F(0) we make use of the fact that $1 \gg |n_b/n_{ob}|$ in region 1. This leads to

$$1 \gg \left|\frac{n_b}{n_{ob}}\right| = \frac{e}{m}\frac{\omega_p}{u_o^3}\frac{|E(0)|}{|\kappa_1^2|} \quad \text{or} \quad \frac{mu_o^3}{e\omega_p} \gg |F(0)|$$

Combining this with the previous expression yields

$$|\epsilon|\left(\frac{\omega_b}{\omega_p}\right)^2\left(\frac{\omega_p}{\nu}\right)\left(\frac{u_o}{\omega_p L}\right) \gg \left|\frac{n_p}{n_{o1}}\right|$$

so the linear approximation is valid provided Eq. (15) is satisfied.

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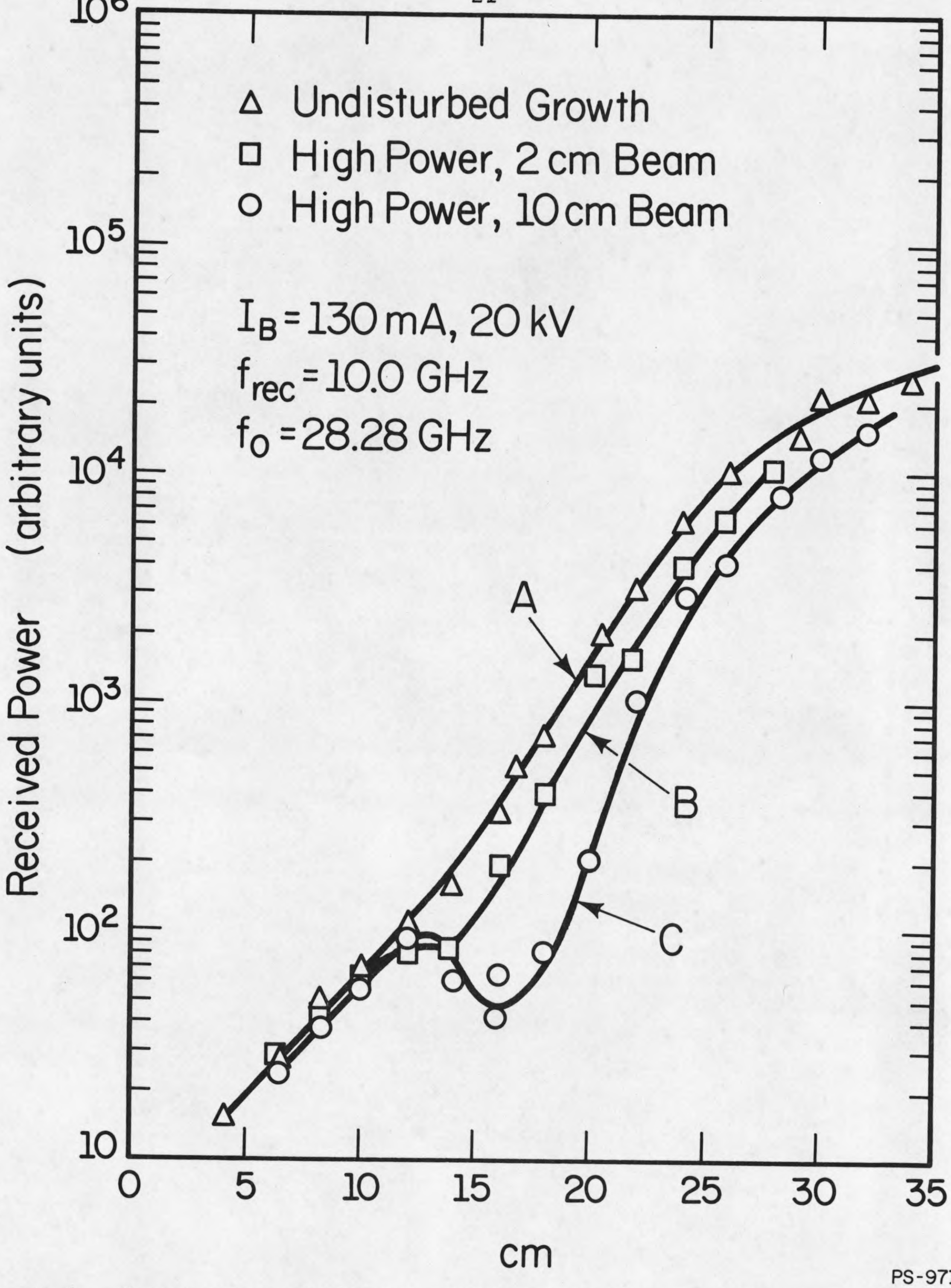

Figure 1. Suppression of the instability by high power microwave irradiation. The indicated beam width refers to the microwave beam. The E-field of the microwaves was perpendicular to the direction of the beam.

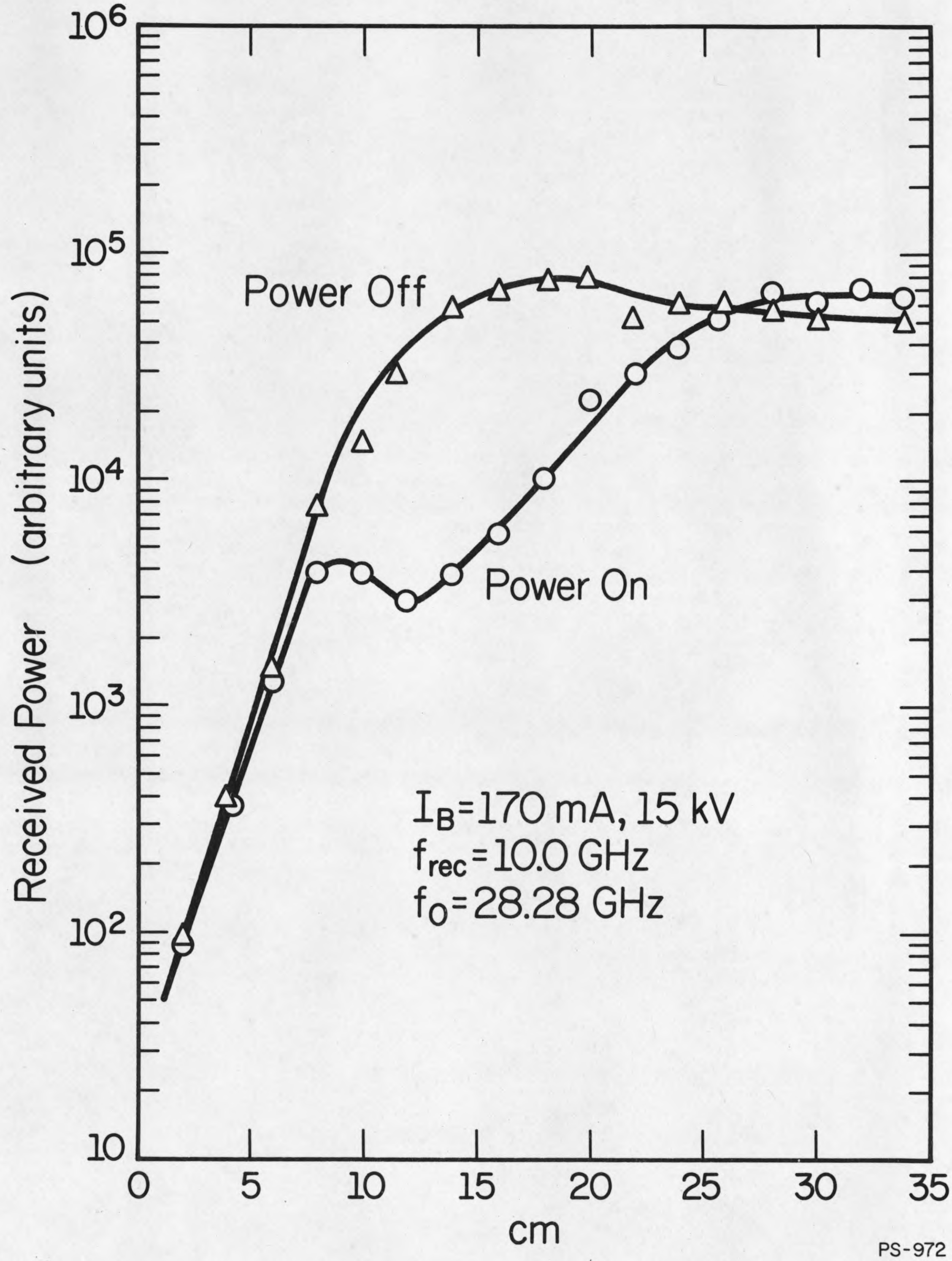

Figure 2. Example of instability suppression for strong instability. The microwave beam was 10 cm wide, and irradiated the region from 10-20 cm along the system.

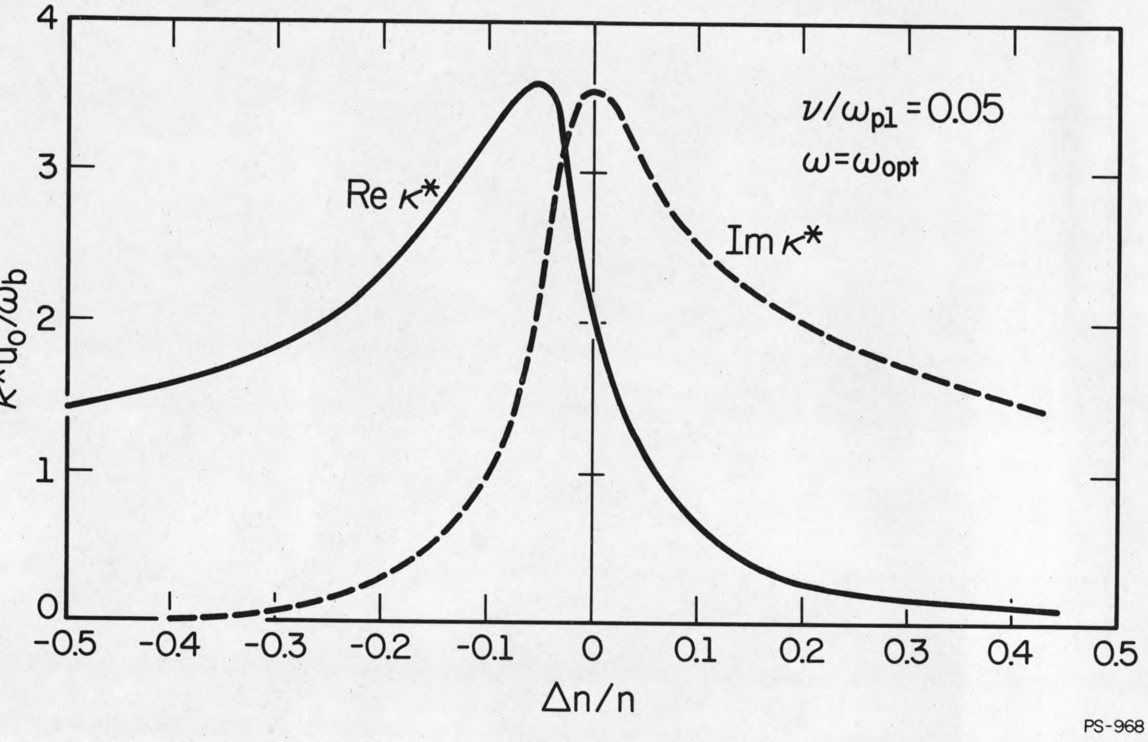

Figure 3. Plot of κ^* versus the relative density step for constant collision frequency. Allowance for a variation of collision frequency with density (coulomb collisions) introduces only negligible modifications.

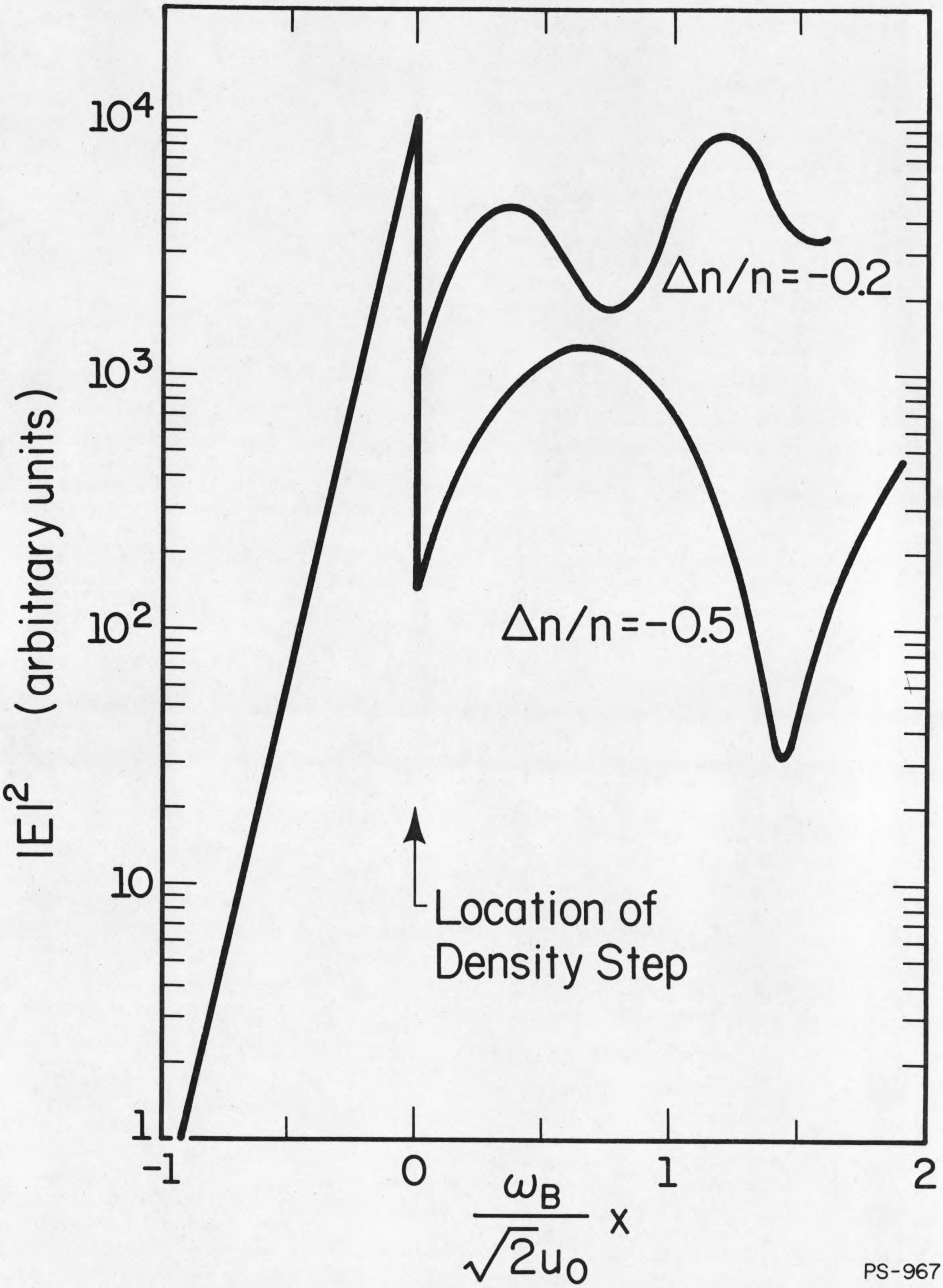

Figure 4. Plot of electric field intensity versus normalized distance for a density decrease. $\nu/\omega_{p1} = 0.05$. A purely growing solution is assumed for $x \leq 0$.

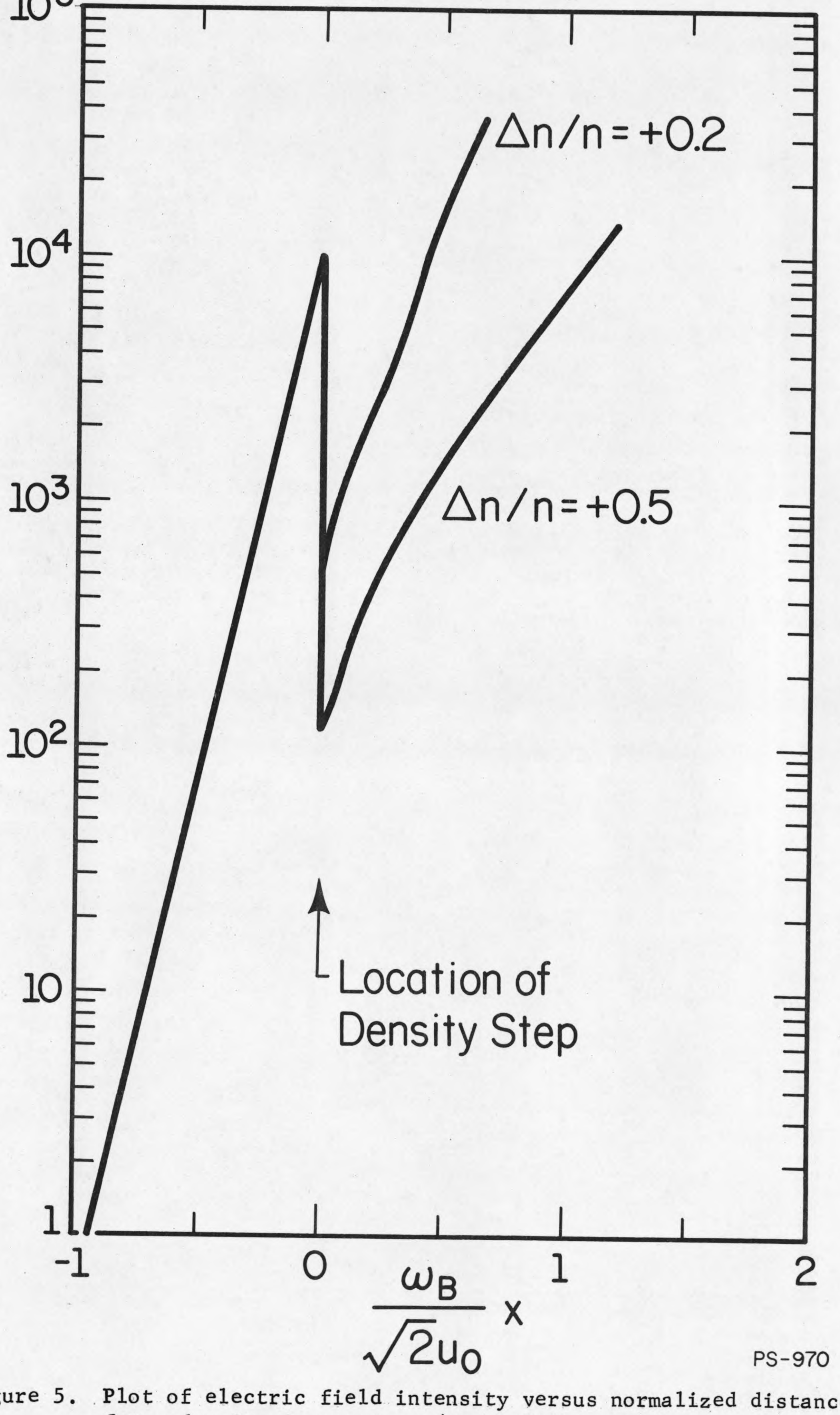

Figure 5. Plot of electric field intensity versus normalized distance for a density increase. $\nu/\omega_{p1} = 0.05$.

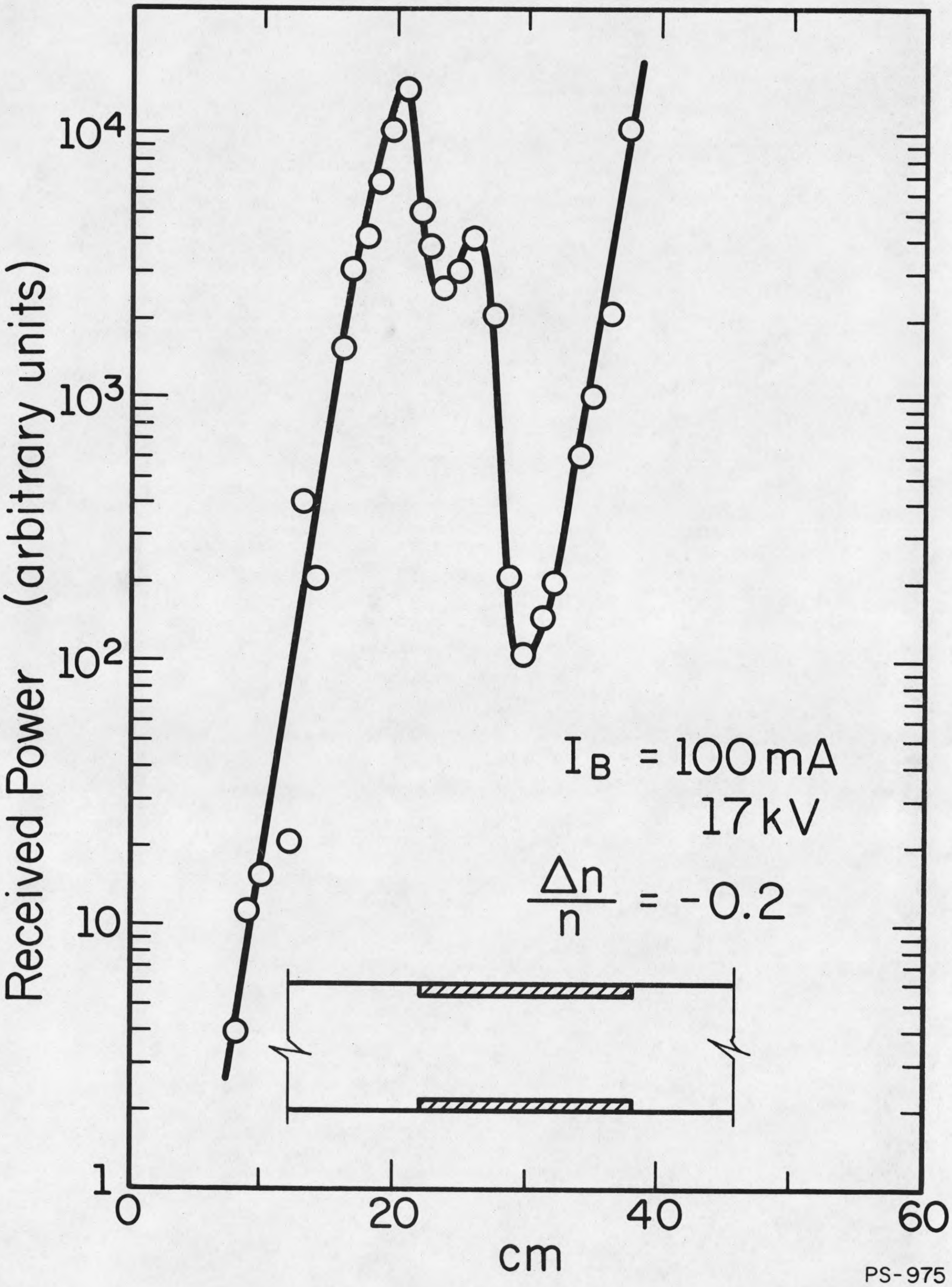

Figure 6. Measurement of radiation versus distance for a 20% density drop. The receiver frequency was 10 GHz, which corresponded to the frequency of maximum instability in the undisturbed region.

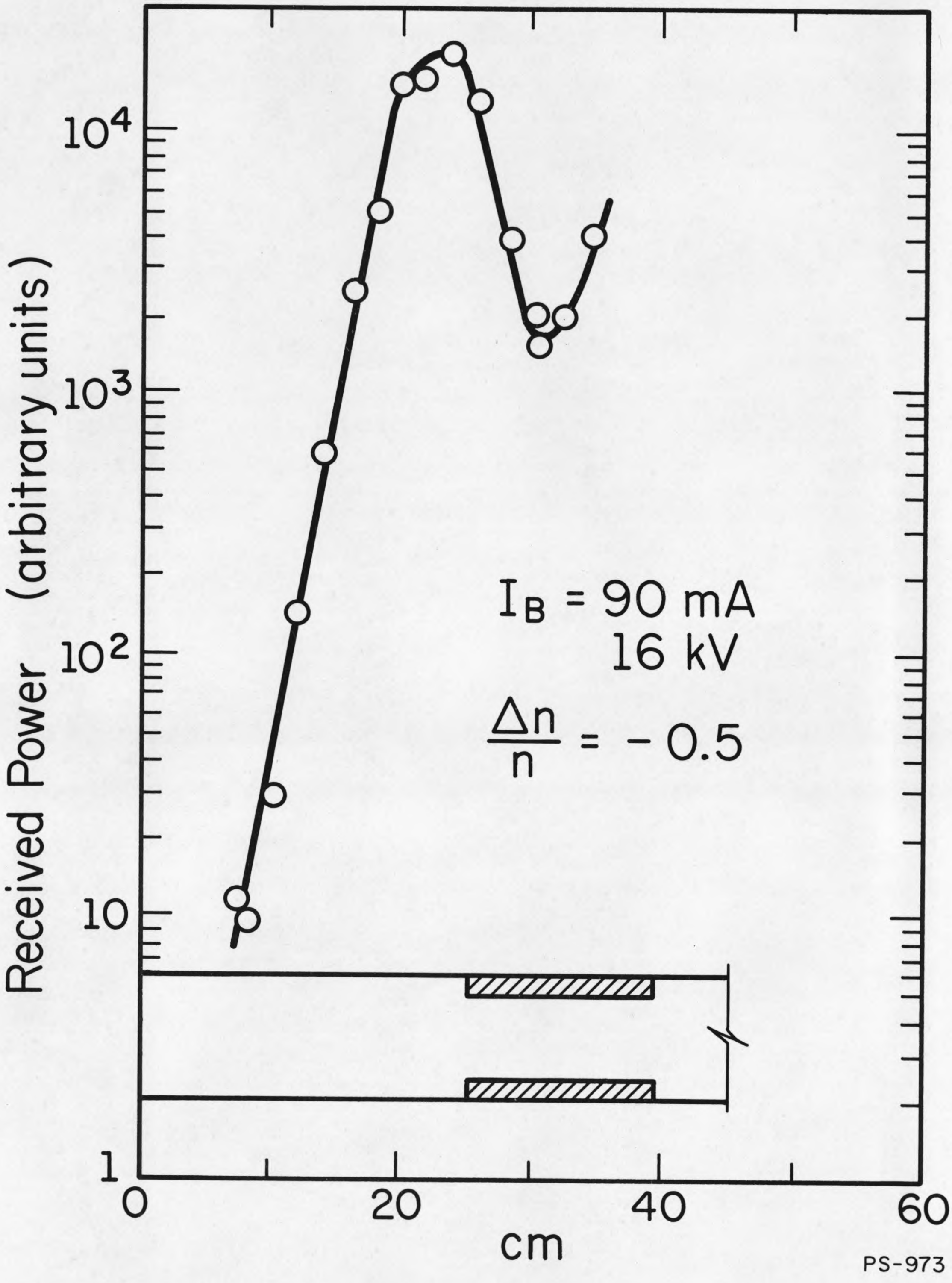

Figure 7. Measurement of radiation versus distance for a 50% density drop. Receiver frequency was 10 GHz.

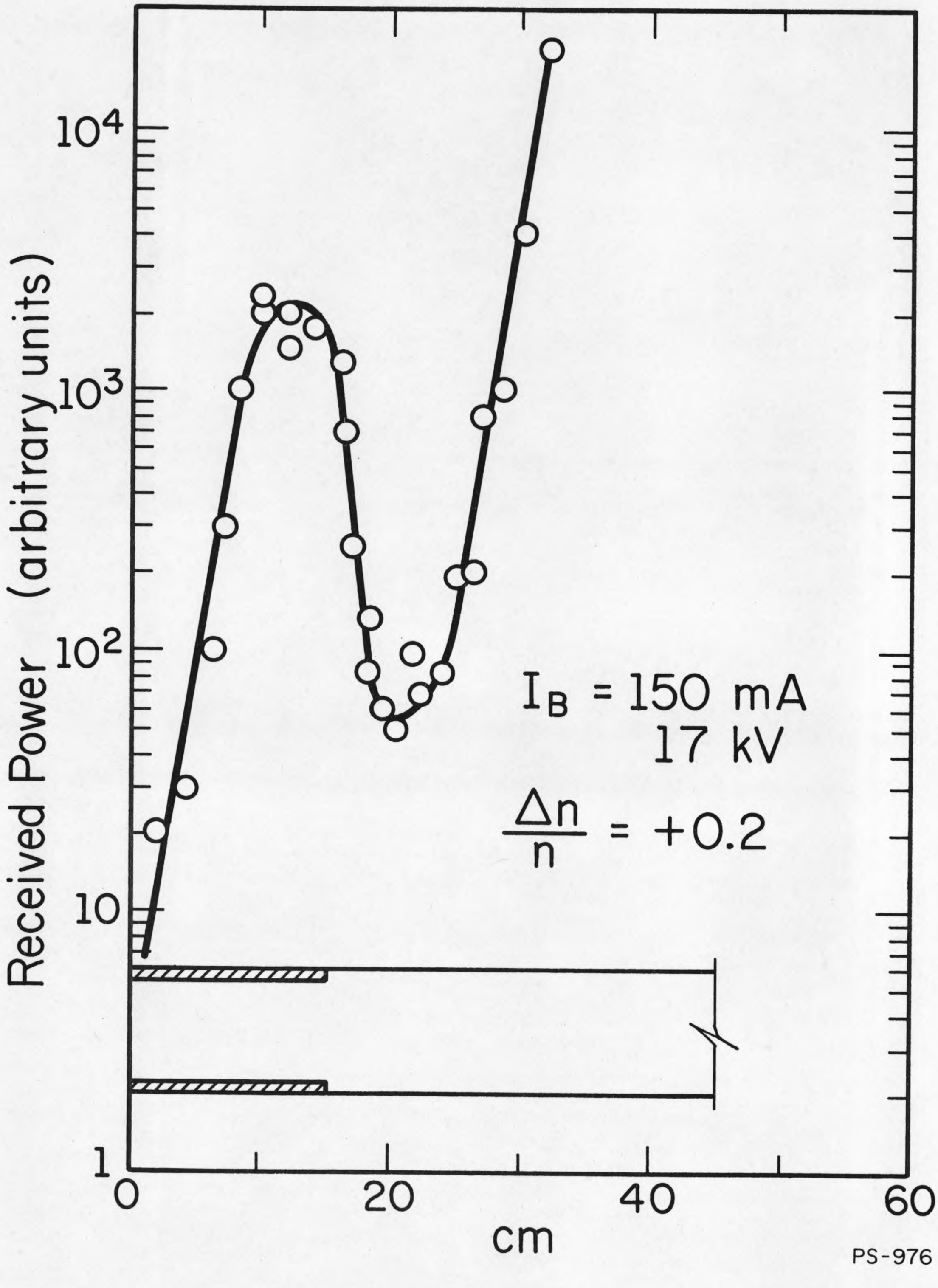

Figure 8. Measurement of radiation versus distance for a 20% density increase. Receiver frequency was 10 GHz.

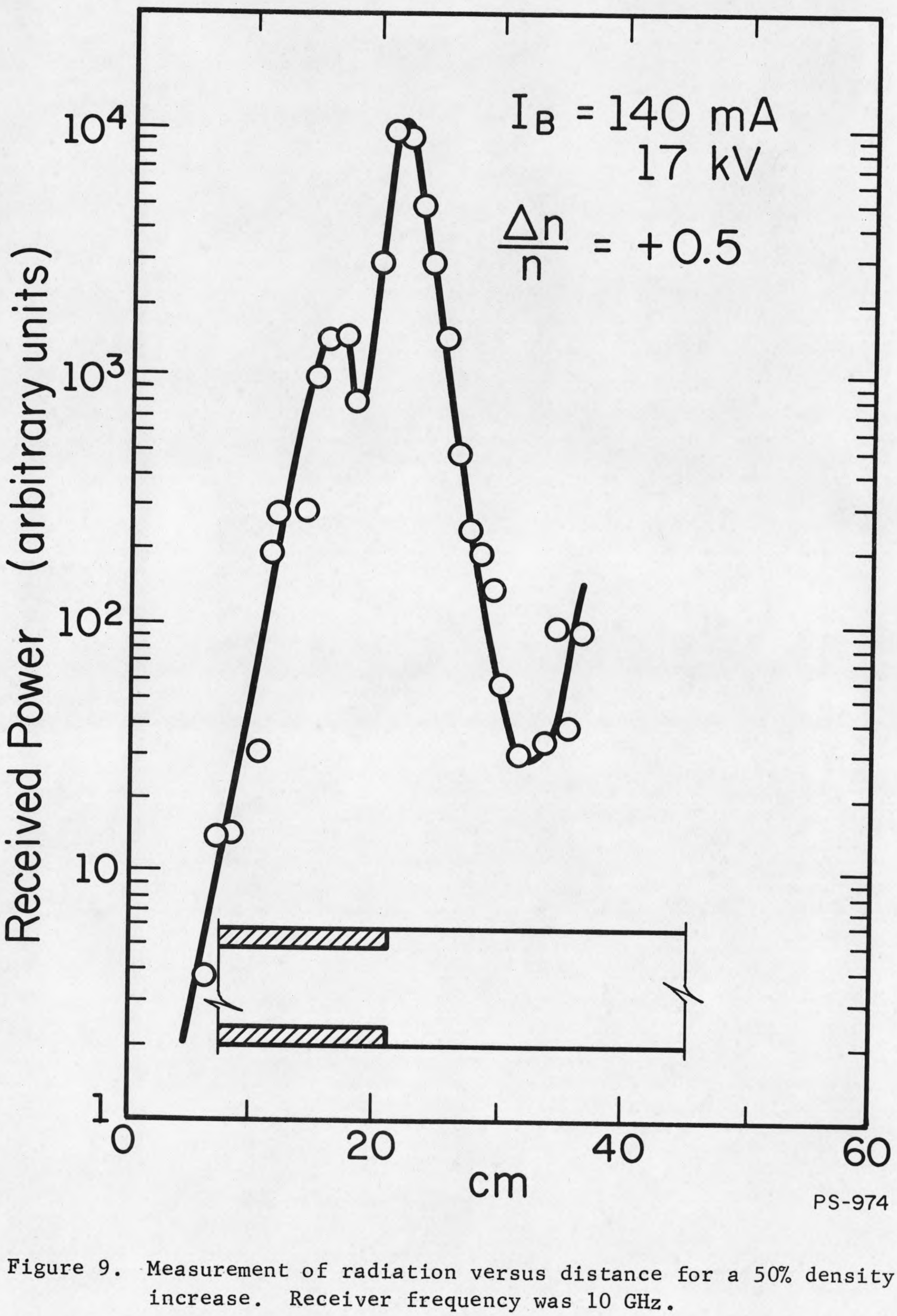

Figure 9. Measurement of radiation versus distance for a 50% density increase. Receiver frequency was 10 GHz.

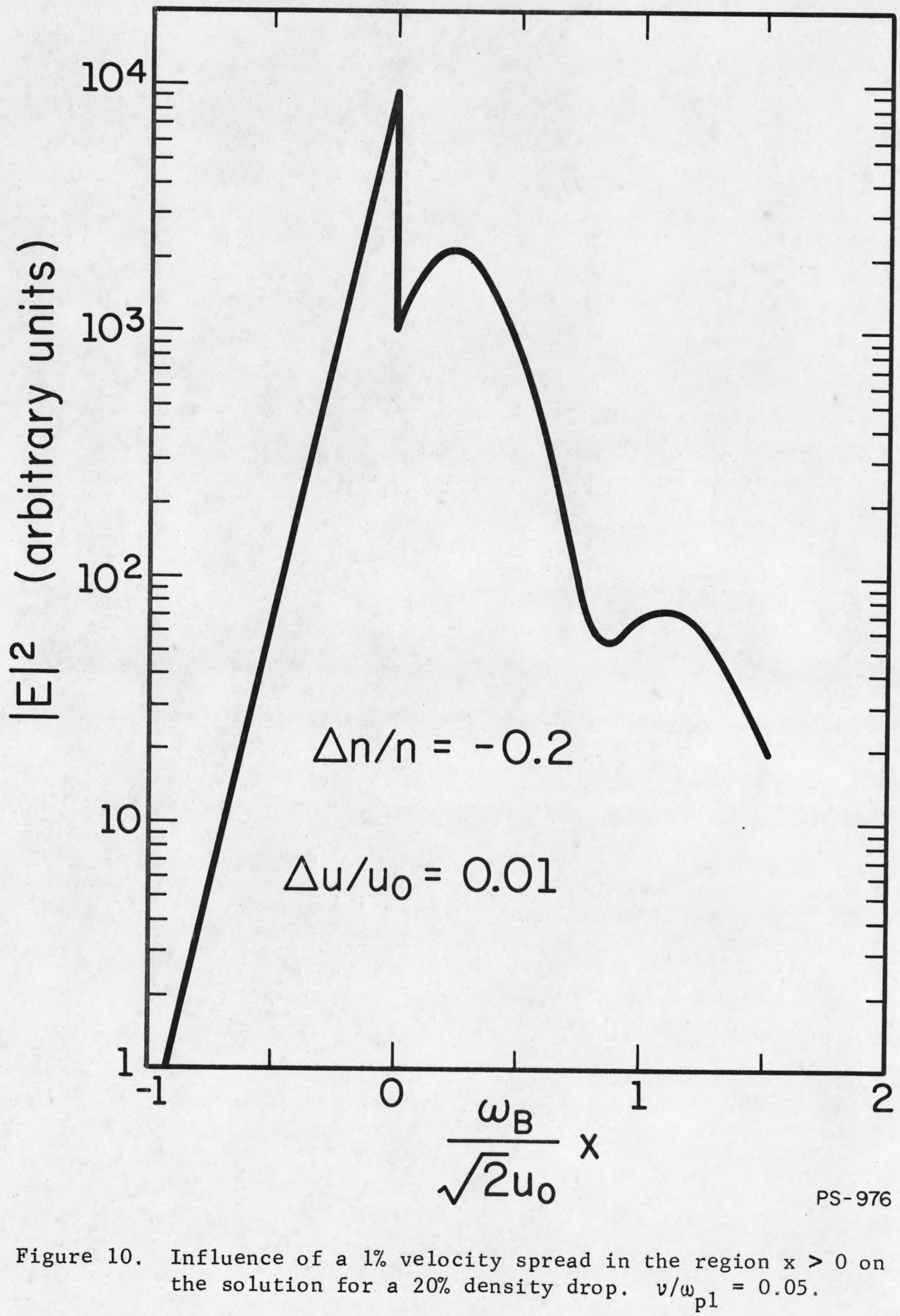

Figure 10. Influence of a 1% velocity spread in the region $x > 0$ on the solution for a 20% density drop. $\nu/\omega_{p1} = 0.05$.

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13. ABSTRACT

Experiments on the spatial instability of a beam-plasma system led to the observation that the instability is quenched when irradiated by high power microwaves. It is conjectured that this effect is due to a local decrease in plasma density in the irradiated region. A theorectical model is detailed that explains the salient features of the observed effect. Auxiliary measurements on a system with a controlled density gradient are presented that give strong support for the proposed mechanism.

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