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A CLASS OF FLAT DELAY FILTER NETWORKS AND THEIR TRANSIENT RESPONSES

G. Kishi

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Summary

The transfer functions of the network studied in this report are rational functions of the frequency variable whose denominators are Bessel polynomials and whose numerators have real frequency roots. Amplitude and group delay characteristics are shown in a set of curves. Transient responses of the networks are investigated for suddenly impressed and removed sinusoidal input signals. The set of curves shows excellent transient behaviour compared with that of conventional filters.

Introduction

The group delay characteristic of a filter network is of great importance for requirements pertaining to various kinds of transmission distortion caused by the network. From this point of view, numerous contributions on flat delay networks have been made. One of the most important results along this line is a set of Hurwitz polynomials whose group delay characteristics give maximally flat approximation of constant delay [1,2].

It is well known that the amplitude characteristics determined by this set of polynomials give a set of curves which are approximately gaussian functions of the frequency variable. This means that the skirt selectivity of this function is not so high compared with conventionally used filter networks.

To make up this deficiency, insertion of a pair of transmission zeros is considered in this report[1]. Amplitude and group delay characteristics of this type of network are calculated and shown in figures. Transient responses of the networks are also investigated for suddenly impressed and removed sinusoidal input signals.

Preliminaries

Before getting into the details, we begin with a brief summary of the system of polynomials which give maximally flat group delay characteristics [1,2].

It is well known that the system of polynomials defined by the recurrence formula

$$B_n(s) = (2n-1)\ B_{n-1}(s) + s^2\ B_{n-2}(s)$$
$$B_0(s) = 1 \tag{1}$$
$$B_1(s) = 1 + 2$$

has the group delay characteristics given by

$$\frac{d}{d\omega} \arg B_n(j\omega) = 1 - \frac{\omega^{2n}}{|B_n(j\omega)|^2} \quad . \tag{2}$$

The polynomial $B_n(s)$ is a Hurwitz polynomial since its test fraction

$$\frac{\text{Even part of } B_n(s)}{\text{Odd part of } B_n(s)}$$

is a partial fraction of the continued fraction

$$\coth s = \frac{1}{s} + \cfrac{1}{\frac{3}{s} + \cfrac{1}{\frac{5}{s} + \cfrac{1}{\frac{7}{8} + \ddots}}} \tag{3}$$

having positive coefficients.

Transfer Function

Let us consider the transfer function formed by the flat delay polynomial $B_n(j\omega)$ with the insertion of transmission zeros as shown in

$$K_n(j\omega) = \frac{Q(j\omega)}{B_n(j\omega)} \tag{4}$$

where $Q(j\omega)$ is a real coefficient polynomial of $j\omega$. The group delay characteristic of this transfer function is given by

$$-\frac{d}{d\omega} \arg K_n(j\omega) = \frac{d}{d\omega} \arg B_n(j\omega) - \frac{d}{d\omega} \arg Q(j\omega) . \tag{5}$$

If we choose $Q(j\omega)$ as a polynomial satisfying

(i) $|Q(j\omega)|$ is small whenever ω is in the attenuation range

(ii) $\frac{d}{d\omega} \arg Q(j\omega)$ is small enough whenever ω is in the transmission range,

then the transfer function $K_n(j\omega)$ will have flat group delay and steep selectivity. The simplest form of $Q(j\omega)$ satisfying these two conditions is the polynomial whose conjugate pair of roots is located in the attenuation region of $j\omega$ axis. For this selection of $Q(j\omega)$, the second term in Eq. (5) is identically zero for ω within the transmission range.

Let us consider the transfer function defined by

$$T(jx) = K \cdot \frac{1 - \frac{x^2}{\rho^2}}{B_n(jkx)} \qquad (6)$$

where x is the normalized frequency variable[2]. The normalization constant k is determined as a function of ρ and n by the normalization condition

$$-20 \log_{10} \frac{|T(j)|}{|T(0)|} = 3 \text{ (db)} \qquad (7)$$

or

$$20 \log_{10} \frac{\rho^2}{1 + \rho^2} \cdot \frac{|B_n(jk)|}{|B_n(0)|} = 3 \text{ (db)} \quad . \qquad (8)$$

The curves of $k = k(n,\rho)$ are plotted in Fig. 1 for $n = 3$ to 7.

The amplitude characteristics

$$-20 \log_{10} \frac{|T_n(jx)|}{|T_n(0)|}$$

and the normalized group delay characteristics

$$\frac{d}{dx} \arg B_n(jkx)$$

are shown in Fig. 2 to 11 for $n = 3$ to 7 and $\rho = 2.2$, 2.4, 2.6, 2.8, 3, 3.2, 3.4, 3.6, 3.8, 4 and ∞.

In these figures, the curves for $\rho = \infty$ are amplitude and group delay characteristics of the all-pole functions with all zeros at infinity determined by the polynomial $B_n(jkx)$.

The amplitude characteristics for finite ρ show infinite attenuation at $x = \rho$ and larger attenuation compared with the amplitude characteristics for $\rho = \infty$ whenever $x < \rho$.

The group delay characteristics for finite ρ are smaller than those for $\rho = \infty$ by the effect of the normalization constant $k(n,\rho)$. This means that the deviation from zero frequency delay within the passband can be reduced by the insertion of transmission zeros given by the numerator in Eq. (6).

Transient Response

Let us consider the transient response of the transfer function defined by Eq. (6) for both suddenly impressed and suddenly removed sinusoidal signals with angular frequency x_c in the normalized frequency scale x. We begin with the no-detuning case. The effect of the insertion of transmission zeros upon the transient response without detuning can easily be evaluated as follows:

First of all, the transfer function can be decomposed into two parts as

$$\frac{1 + \frac{(jx)^2}{\rho^2}}{B_n(jkx)} = \frac{1}{B_n(jkx)} + (jx)^2 \cdot \frac{1}{\rho^2 \cdot B_n(jkx)} \quad . \tag{9}$$

Now let $f(\theta)$ be the unit step response of the first term of Eq. (9), where θ is the normalized time variable corresponding to the normalized frequency variable x. (If $x = \omega/\omega_b$, then $\theta = \omega_b t$.) Then the unit step response of the second term is given by

$$\frac{1}{\rho^2} \cdot f''(\theta) \tag{10}$$

where prime means differentiation with respect to θ. On the other hand, the function $f(\theta)$ can be considered as

$$f(\theta) \doteq \int_{-\infty}^{\theta/k} a\, e^{-b(\theta-\theta_0)^2}\, d\theta \qquad (11)$$

because $B_n(jkx)$ has gaussian amplitude and linear phase characteristics, approximately. Thus the effect of the insertion of transmission zeros upon the unit step response can be divided into two effects as follows:

(i) Effect of the normalization constant $k(n,\rho)$ in the integral (11),

(ii) Effect of generation of the additional term given by (10).

The first effect produces the difference of the location of the build-up in unit step responses, and quick response can be expected for small value of $k(n,\rho)$. The other effect reduces the rise time of the build-up in unit step response, but the reduction can be neglected because of the factor $1/\rho^2$.

The unit step responses for $n = 3$ to 7 are shown in Fig. 12 to 16. The curves show that the transient response of this type of filter network has practically no overshoot and ringing for the case of no detuning.

For the detuned case, in-phase and quadrature components for networks of degree $n = 3$ to 5 with $\rho = 3.4$ are shown in Fig. 17 to 22, each of which corresponds to the detuning factor $x_c = 0.2$, 0.4, 0.6, 0.8, 1 and half-gain points. Approximate values of half-gain points for $n = 3$, 4 and 5 are $x_c = 1.376$, 1.378 and 1.381, respectively.

Turning-on and turning-off envelope functions determined by these in-phase and quadrature components together with no-detuning case are shown in Fig. 23 to 29. These curves show that overshoot and ringing are practically zero whenever $x_c \leq 0.8$, and approximately 1% overshoot is observed for $x_c = 1$. Generally speaking, the transient behaviour continues from $\omega_b t = 0$ to 4 radian, and is fairly good compared with the transient response of conventionally used filter networks[3]. One of the remarkable features of these curves is the absence of the masking tail which commonly exists in turning-off envelope functions of conventional filter transient.

ACKNOWLEDGMENT

The author is grateful to Professors M. E. Van Valkenburg and J. A. Resh for many valuable suggestions and criticisms.

FOOTNOTES

1. Part of this report is publishec in Japanese [3].

2. The transfer function given here is also used in order to realize a network having a sine-squared impulse response. See [4].

3. See, for example, [5].

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[5] Kishi, G., "Transient Responses of Butterworth and Chebyshev Filter Networks", CSL Report R-194, University of Illinois, February, 1965.

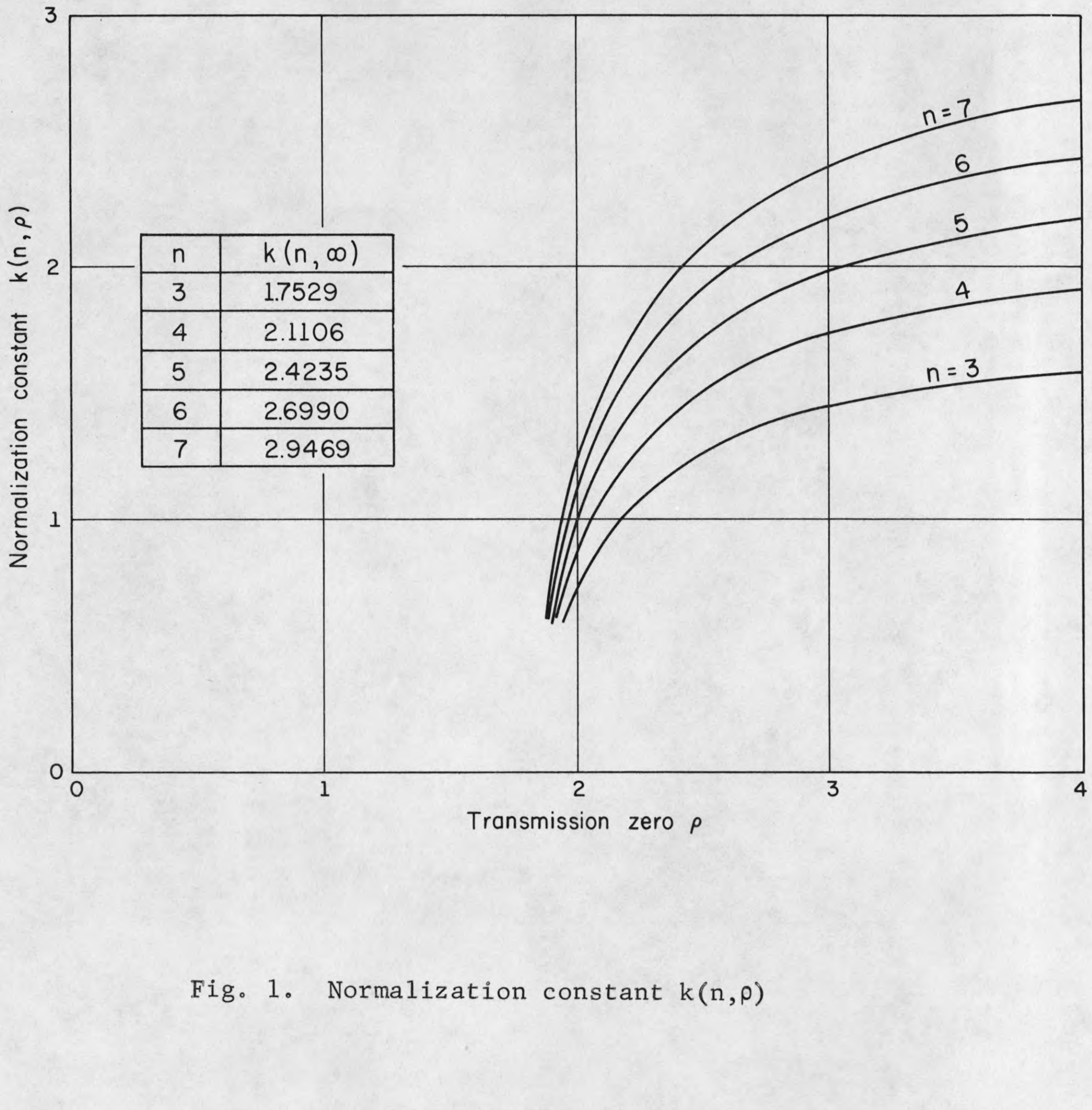

Fig. 1. Normalization constant $k(n,\rho)$

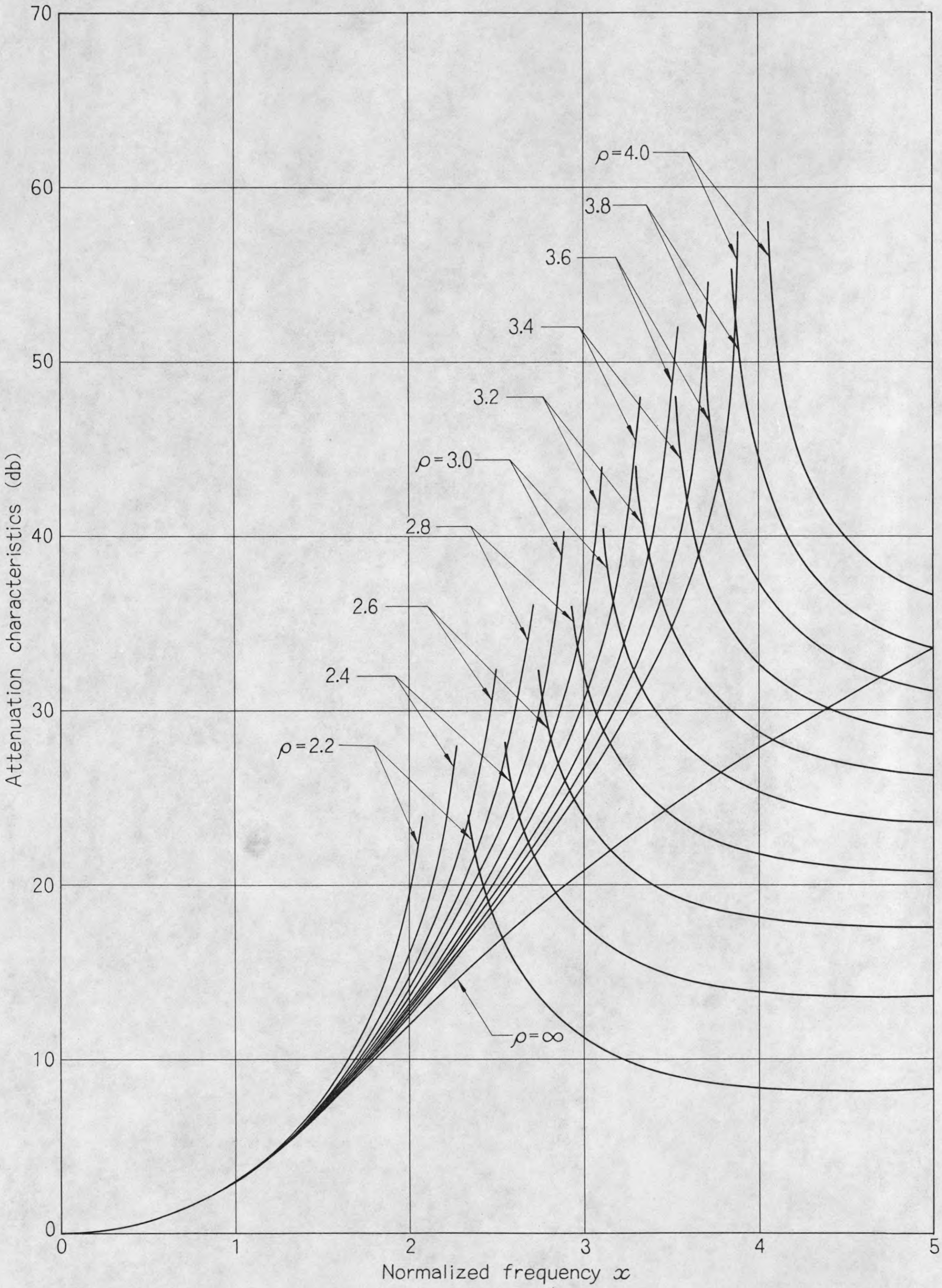

Fig. 2. Amplitude characteristic, n = 3

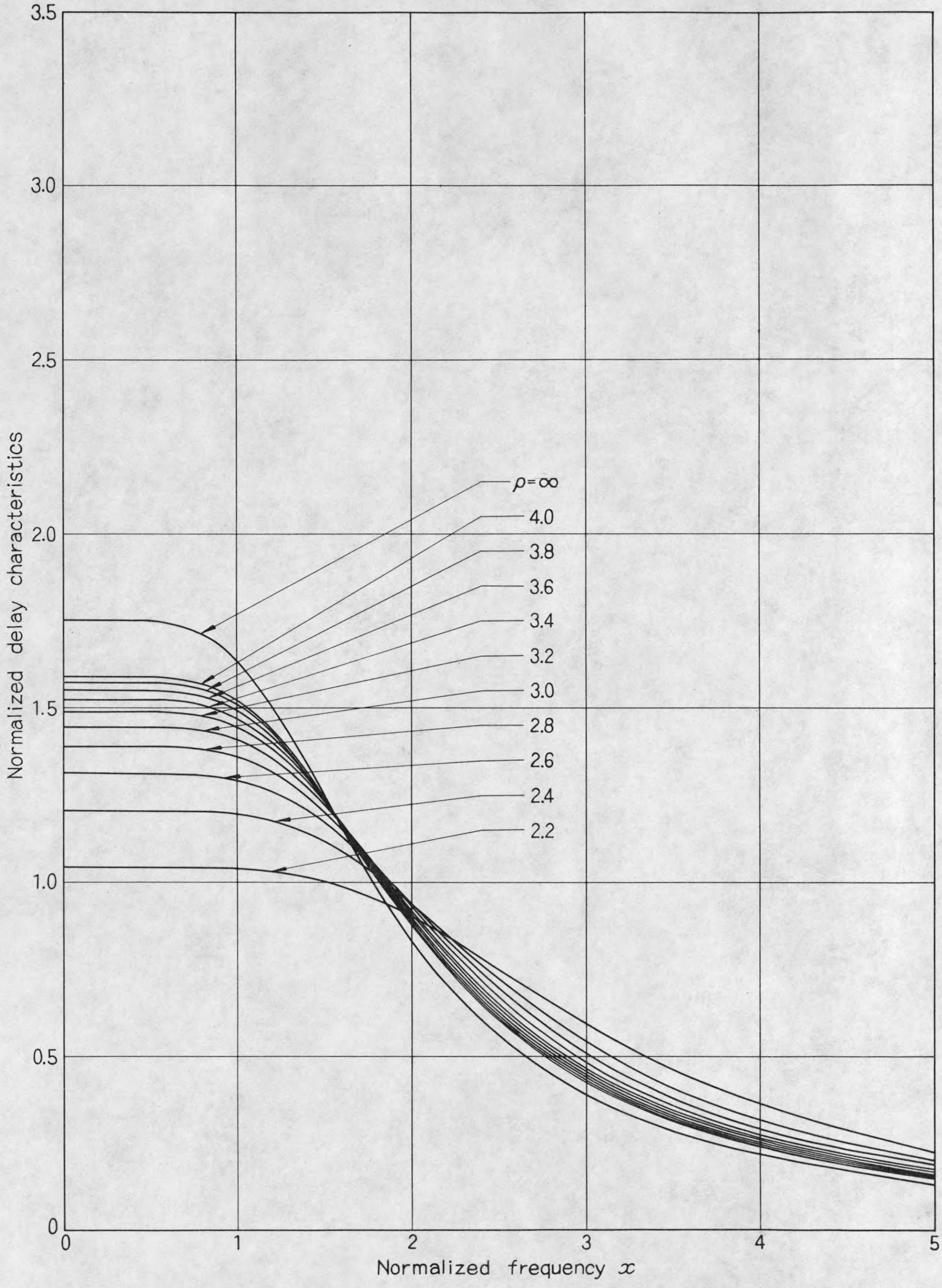

Fig. 3. Group delay characteristic, n = 3

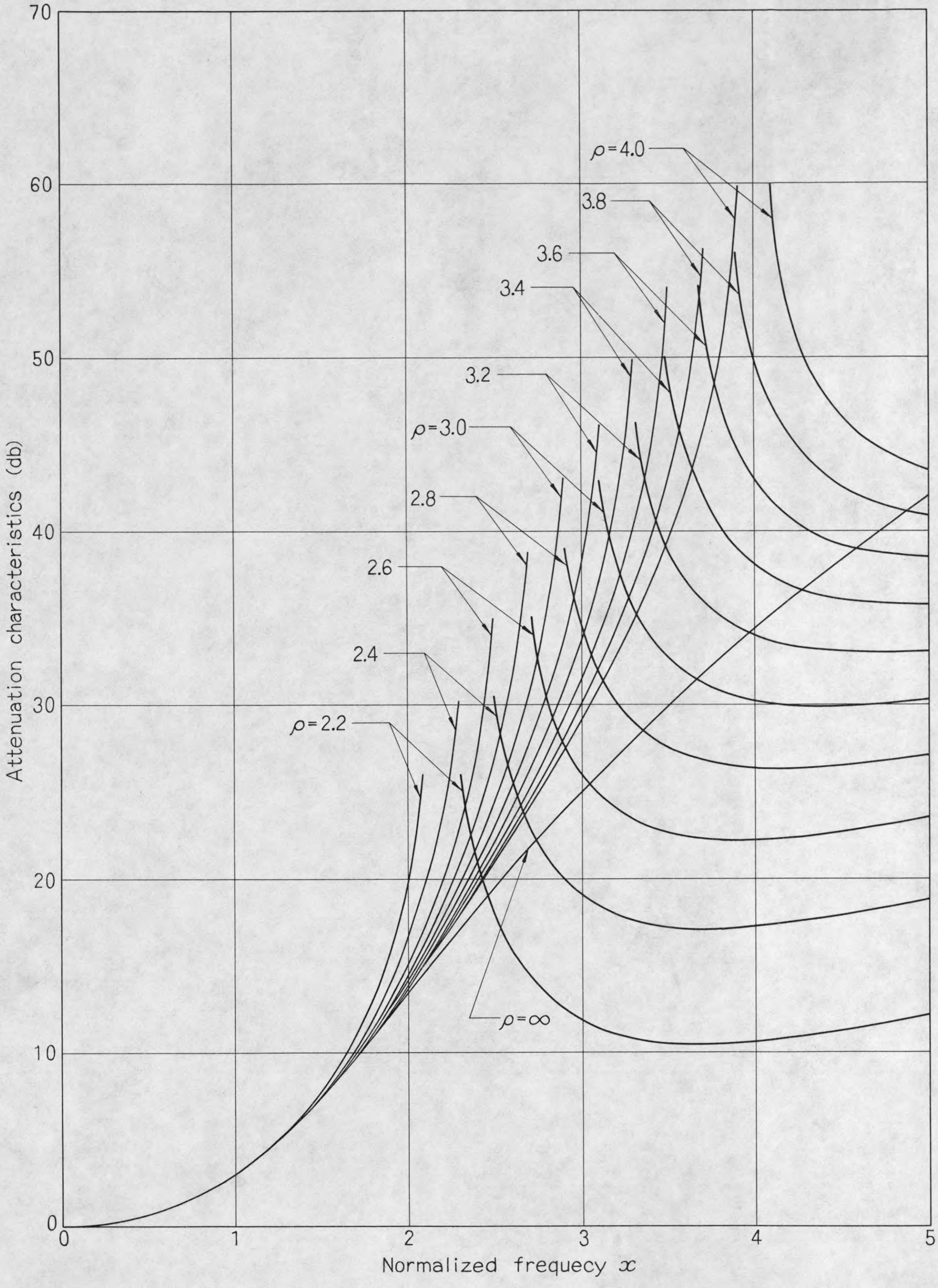

Fig. 4. Amplitude characteristic, n = 4

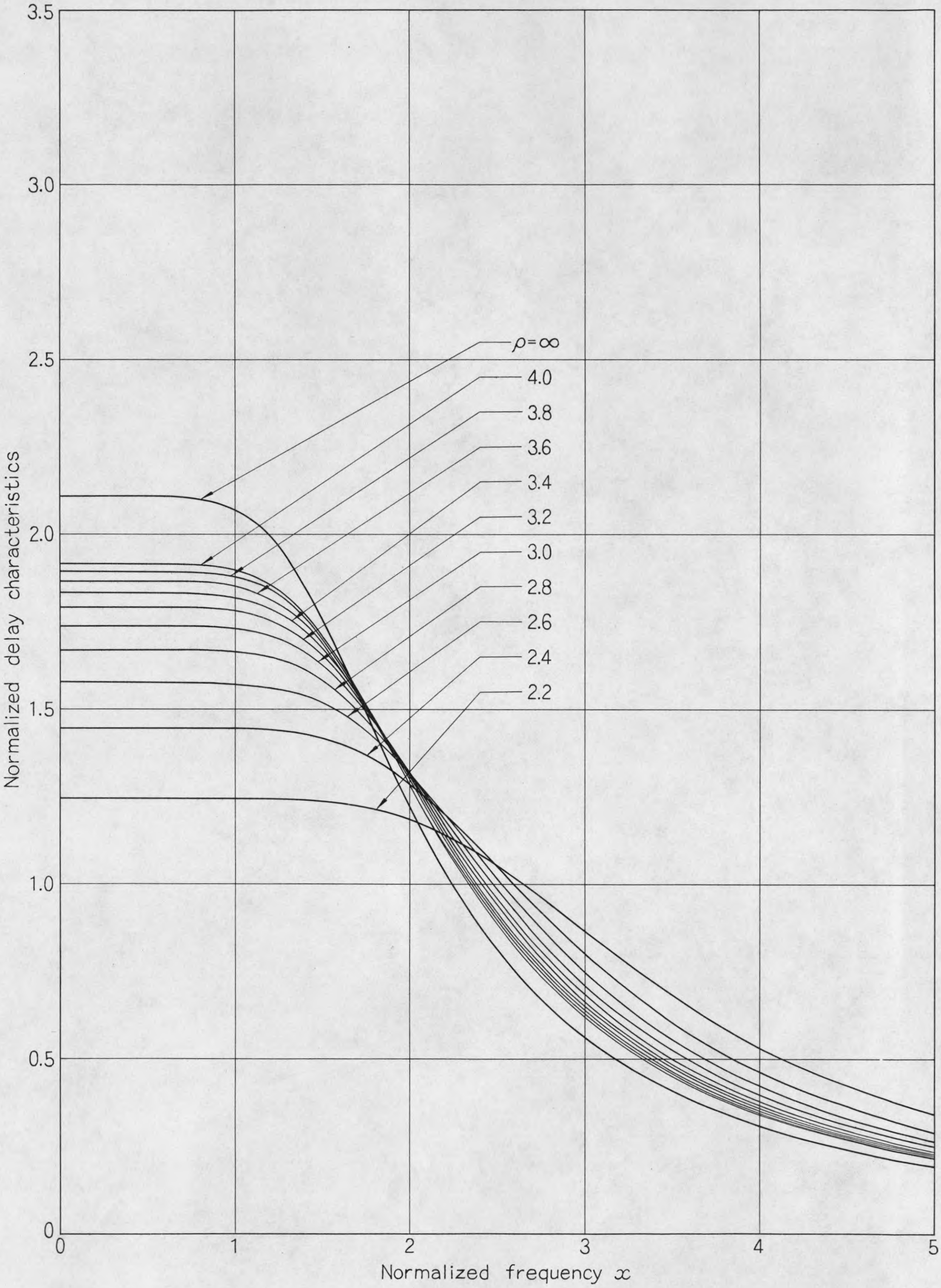

Fig. 5. Group delay characteristic, n = 4

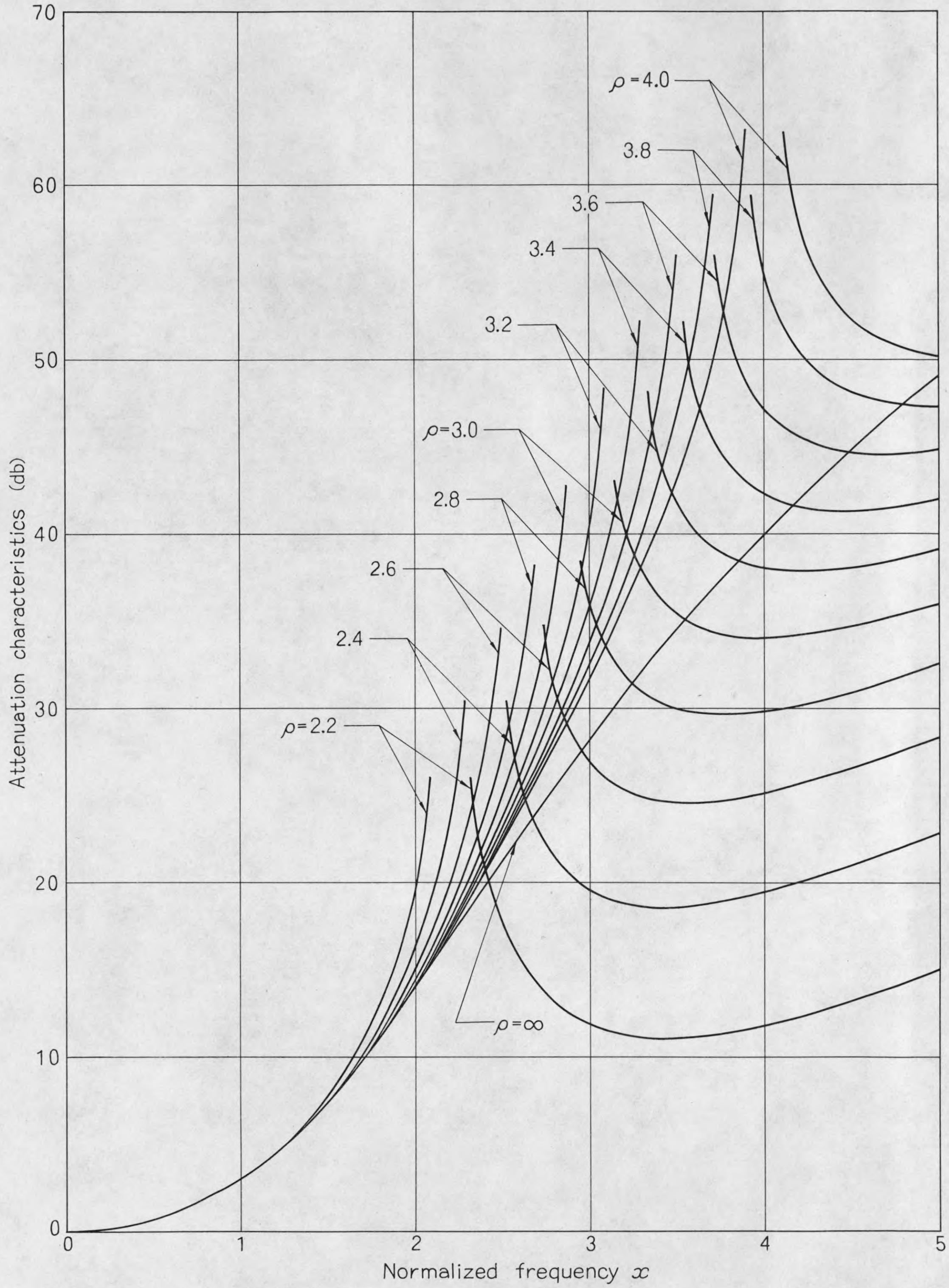

Fig. 6. Amplitude characteristic, n = 5

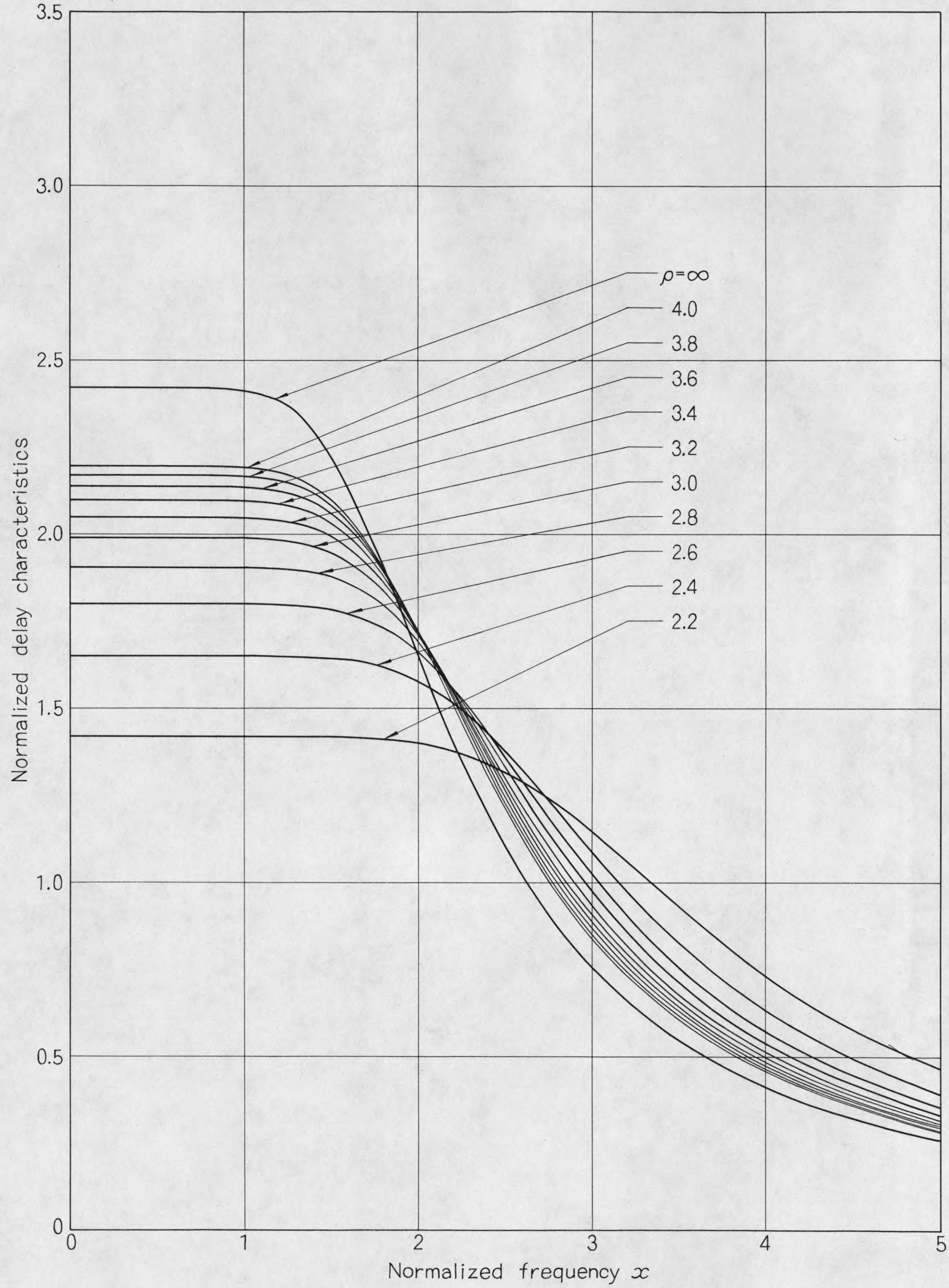

Fig. 7. Group delay characteristic, n = 5

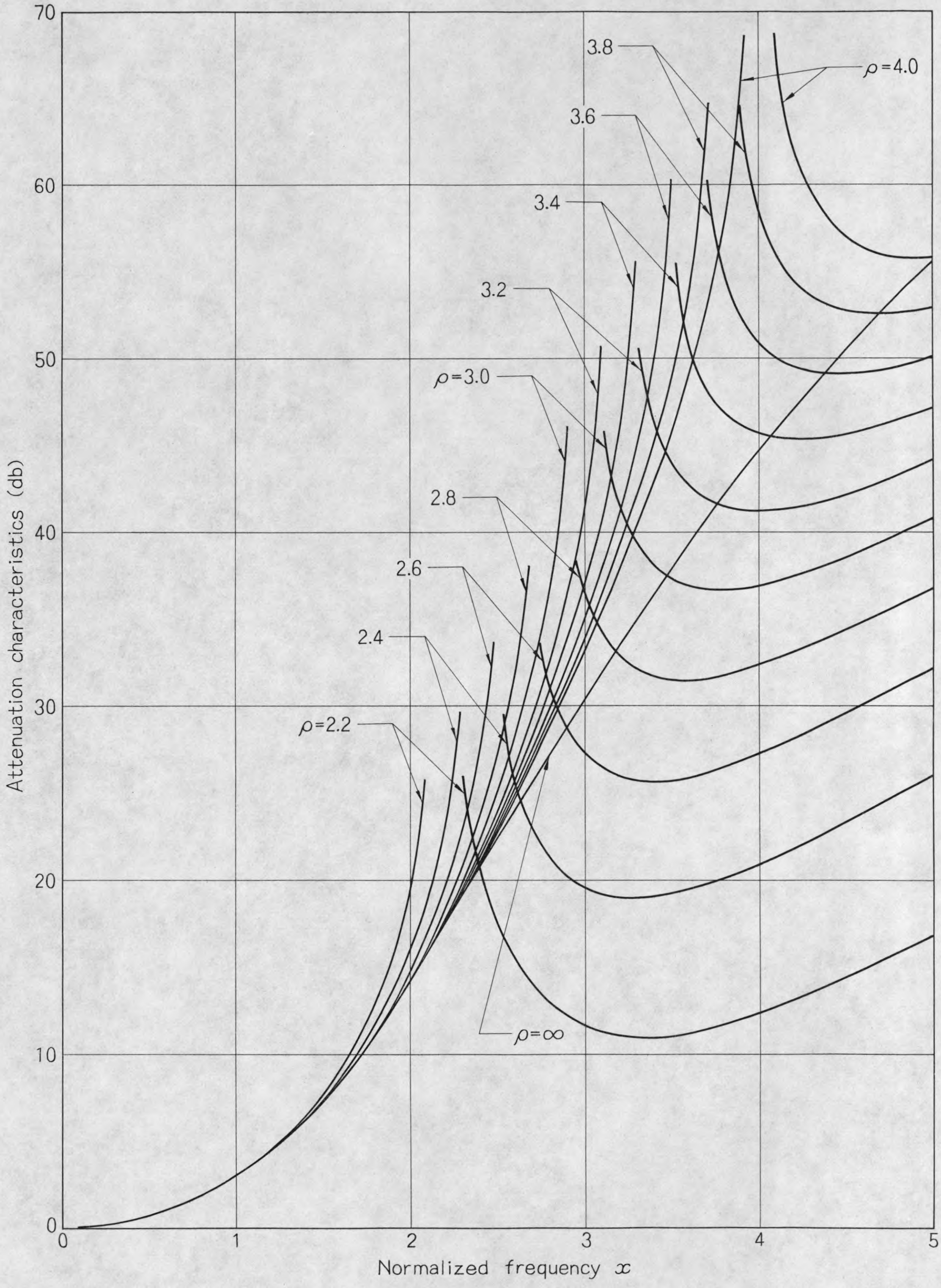

Fig. 8. Amplitude characteristic, n = 6

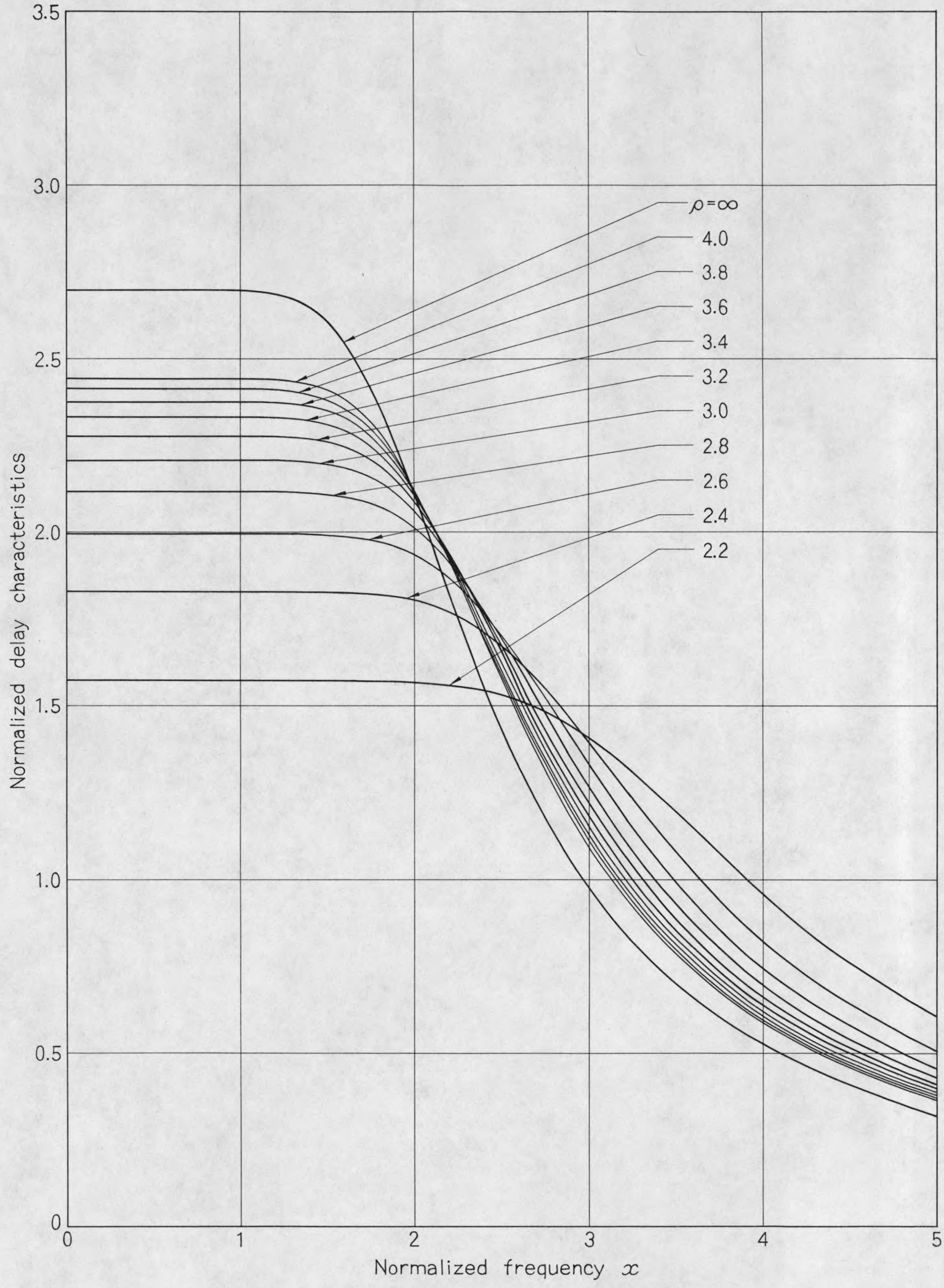

Fig. 9. Group delay characteristic, n = 6

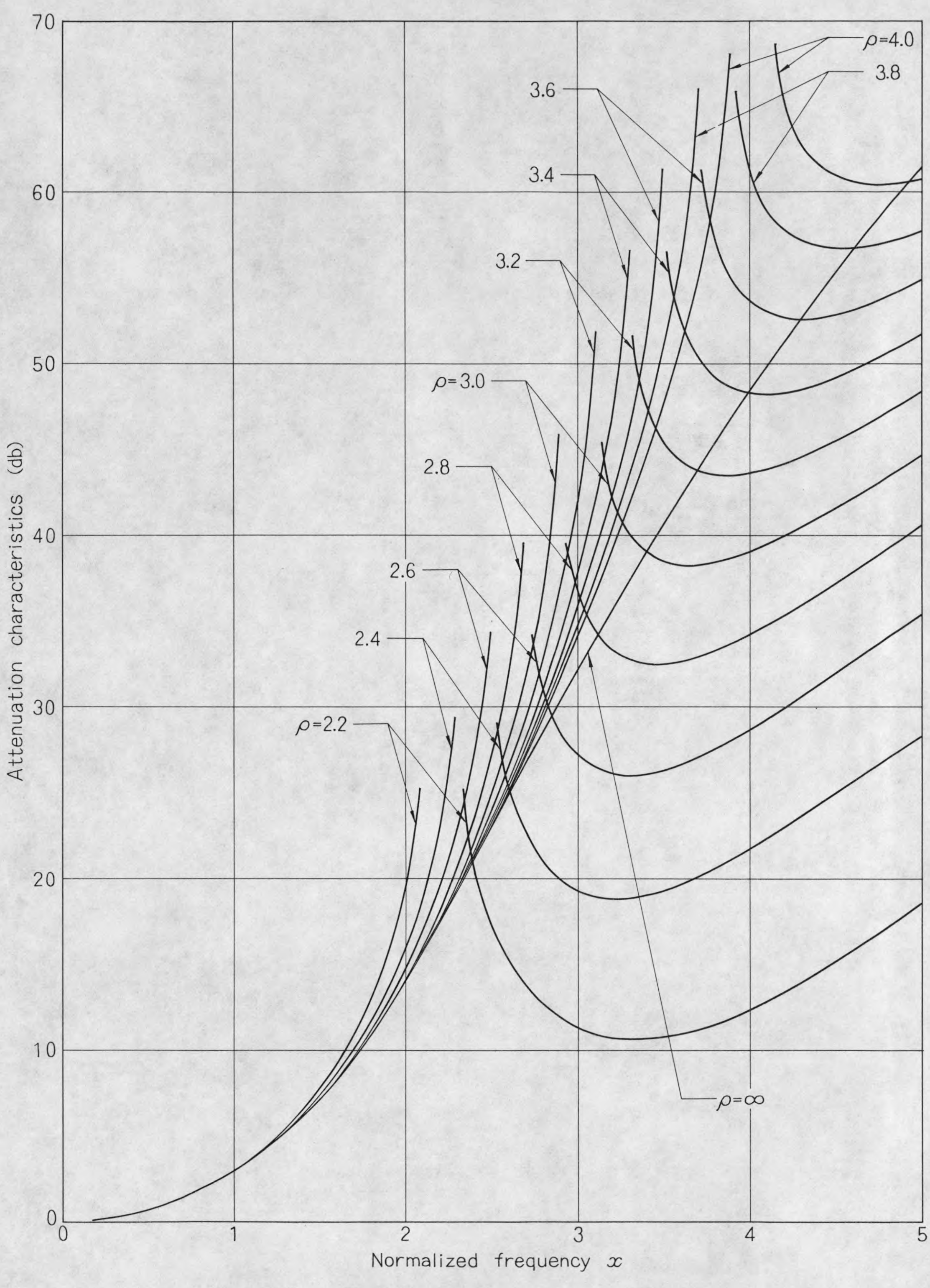

Fig. 10. Amplitude characteristic, n = 7

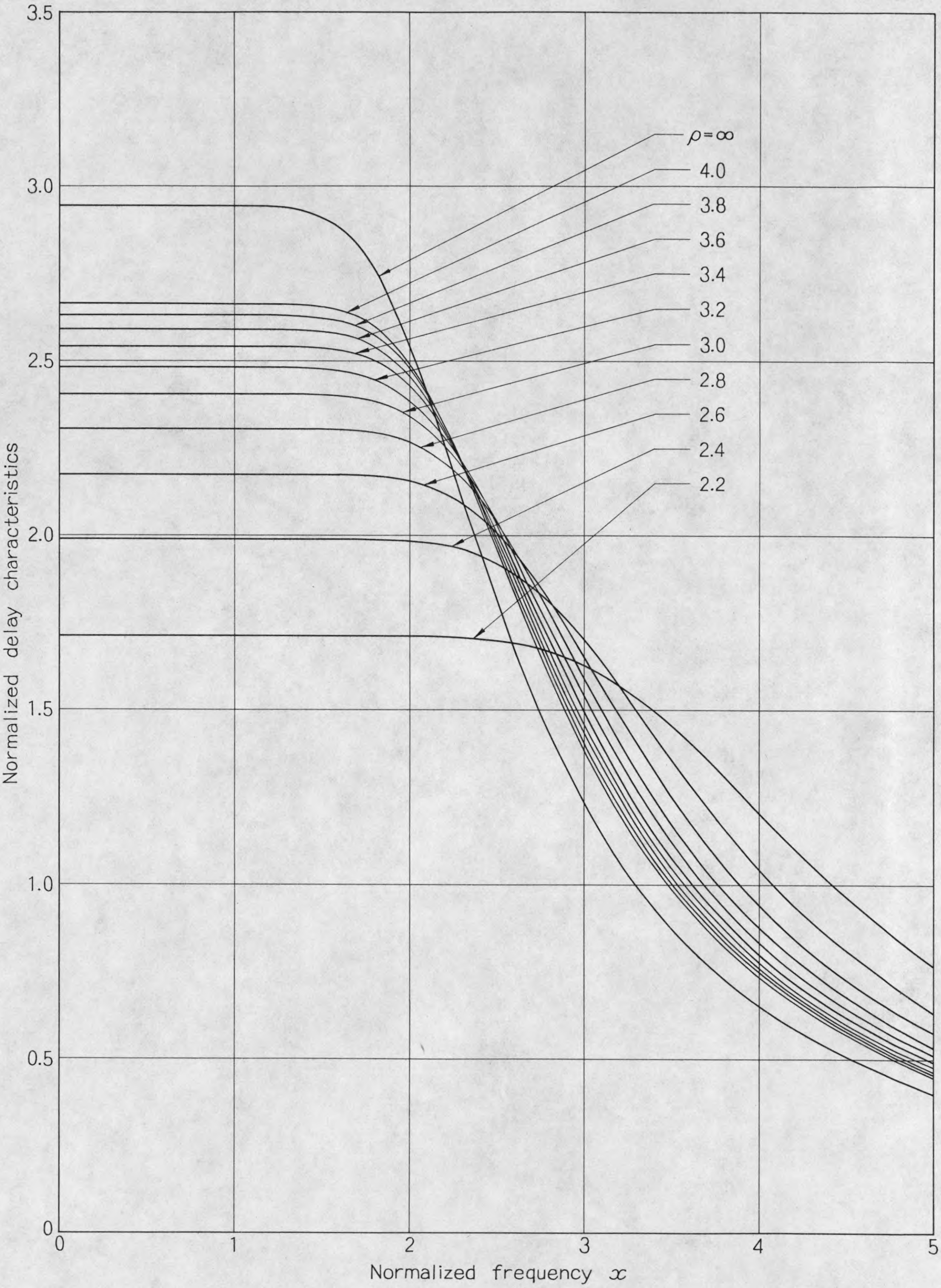

Fig. 11. Group delay characteristic, n = 7

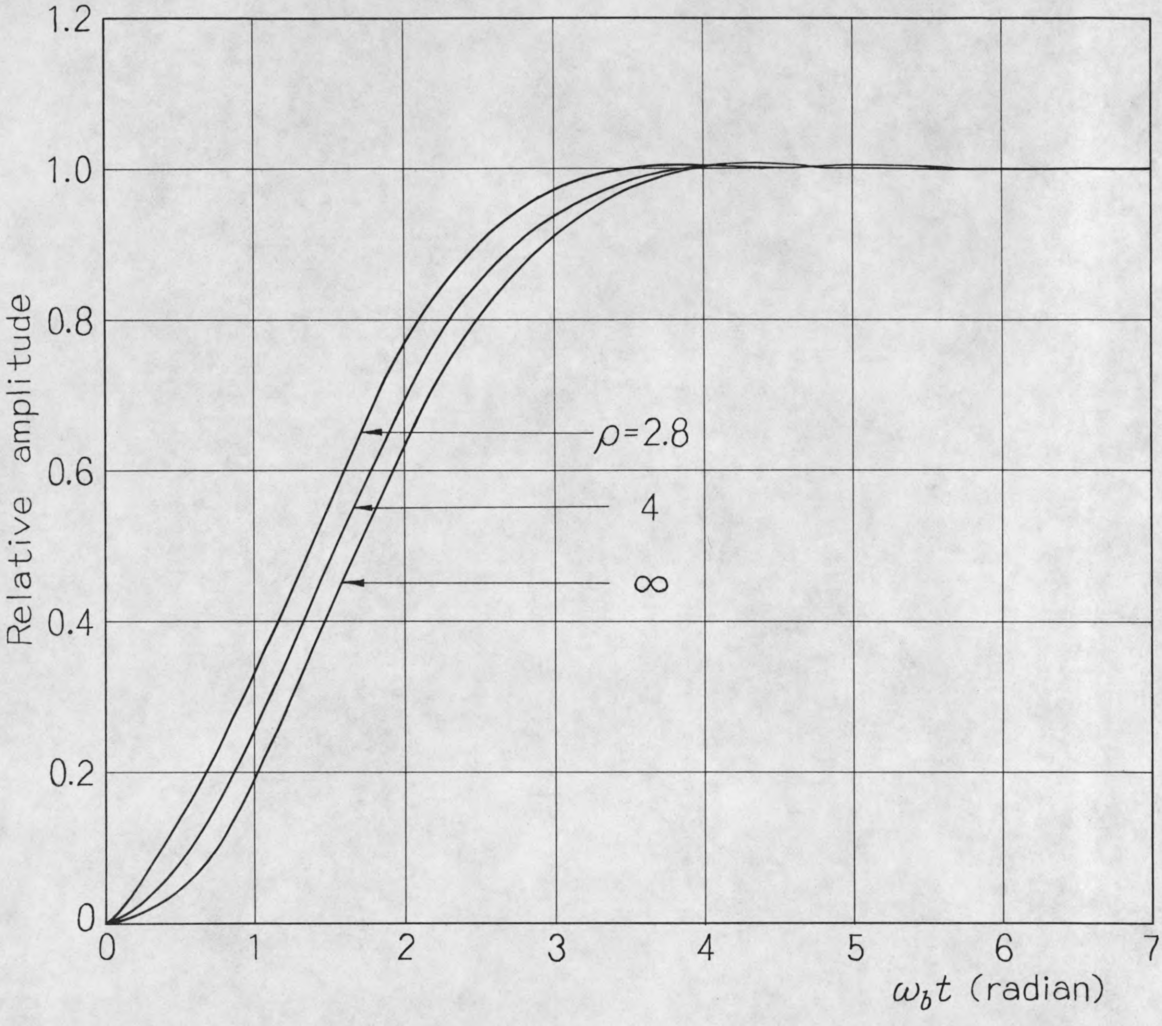

Fig. 12. Transient response, n = 3, x_c = 0

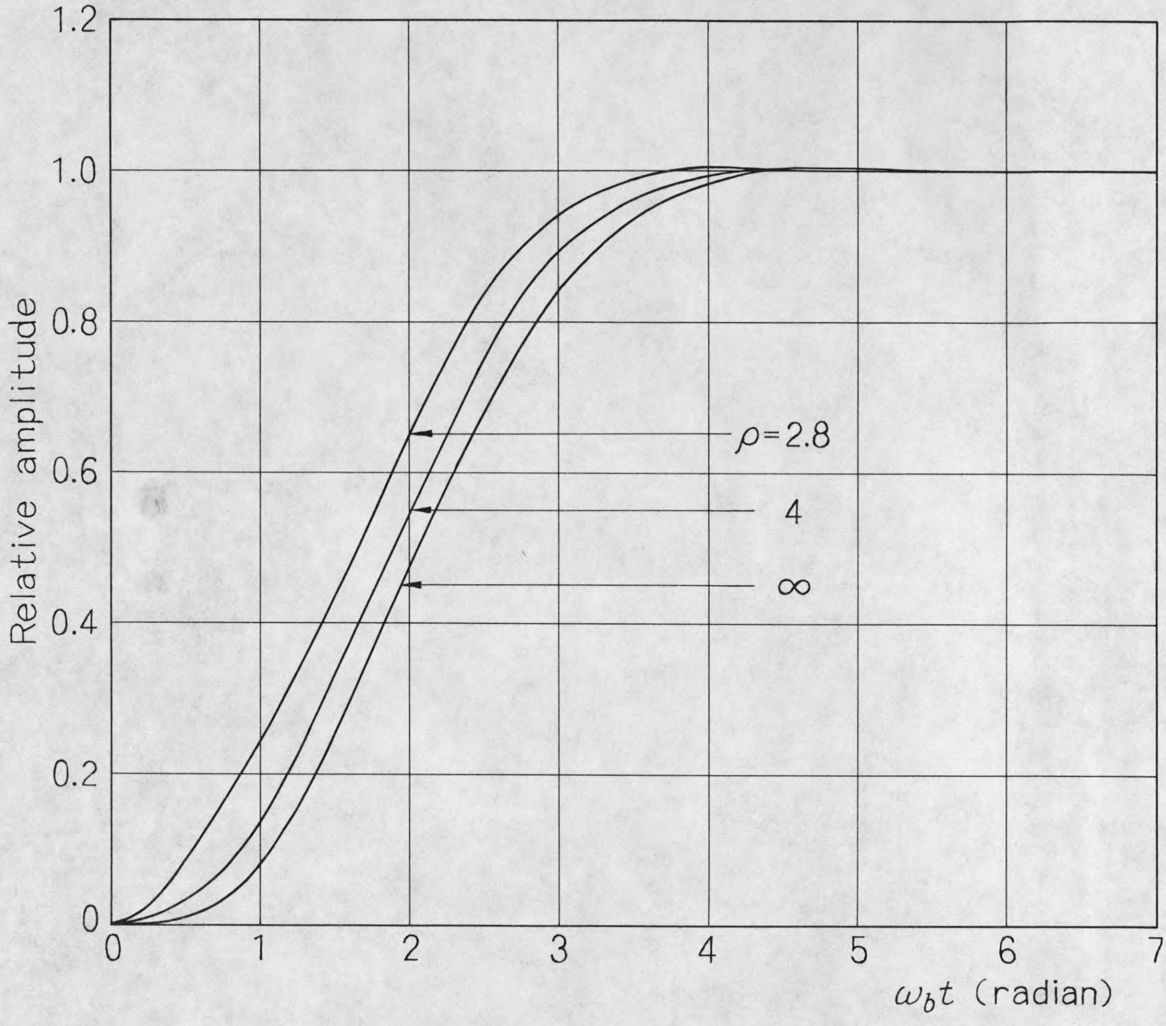

Fig. 13. Transient response, $n = 4$, $x_c = 0$

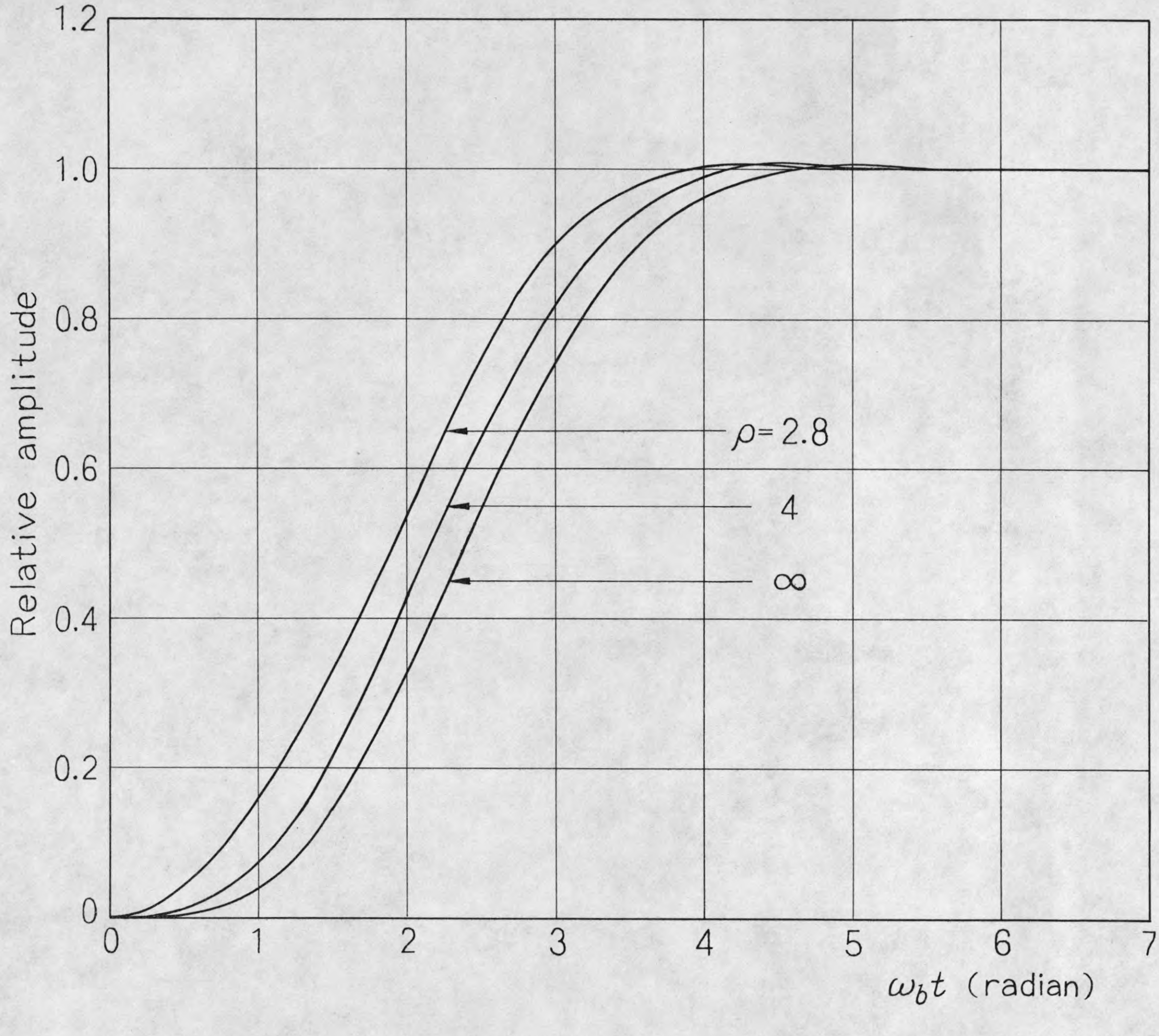

Fig. 14. Transient response, n = 5, x_c = 0

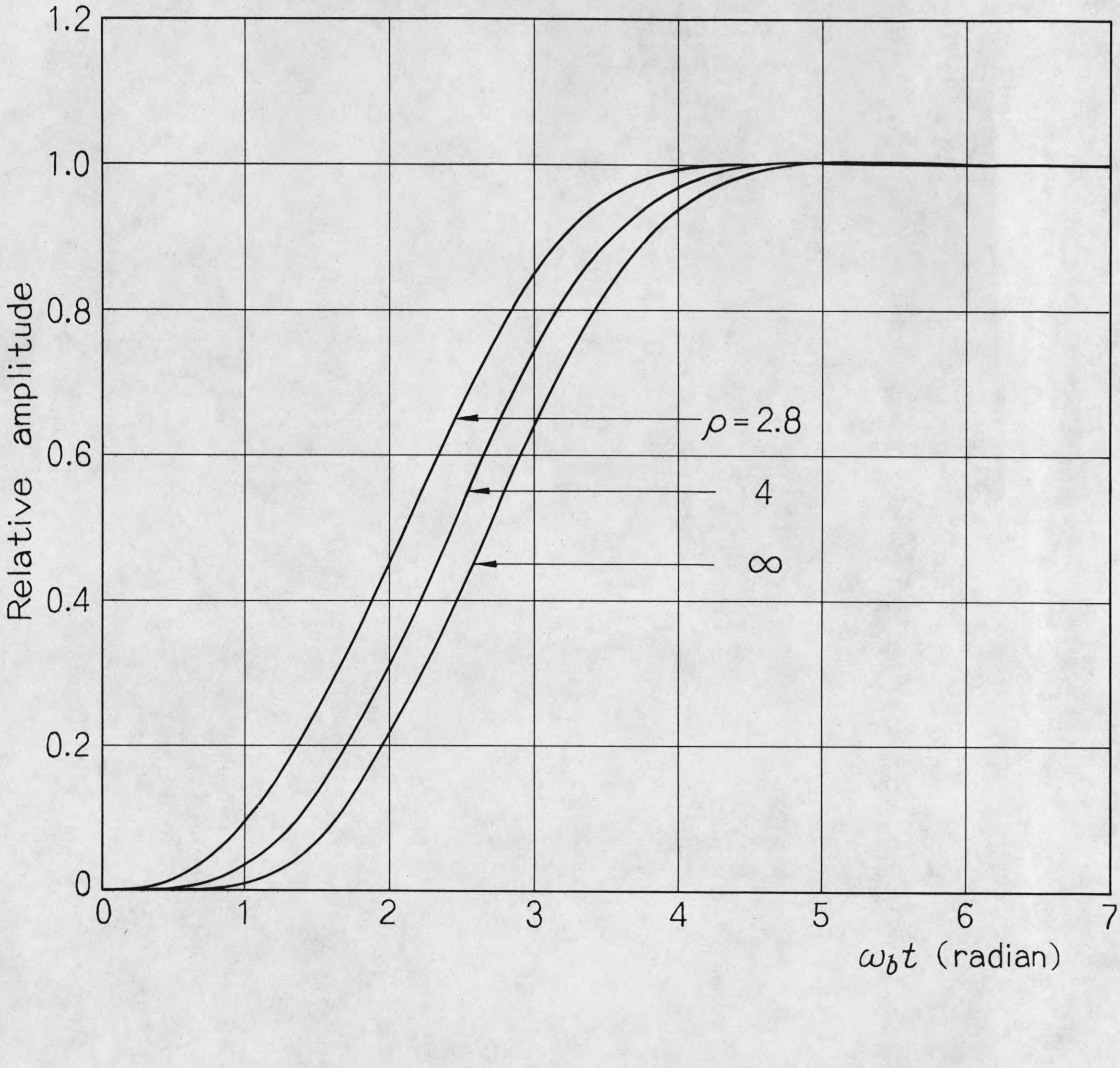

Fig. 15. Transient response, n = 6, $x_c = 0$

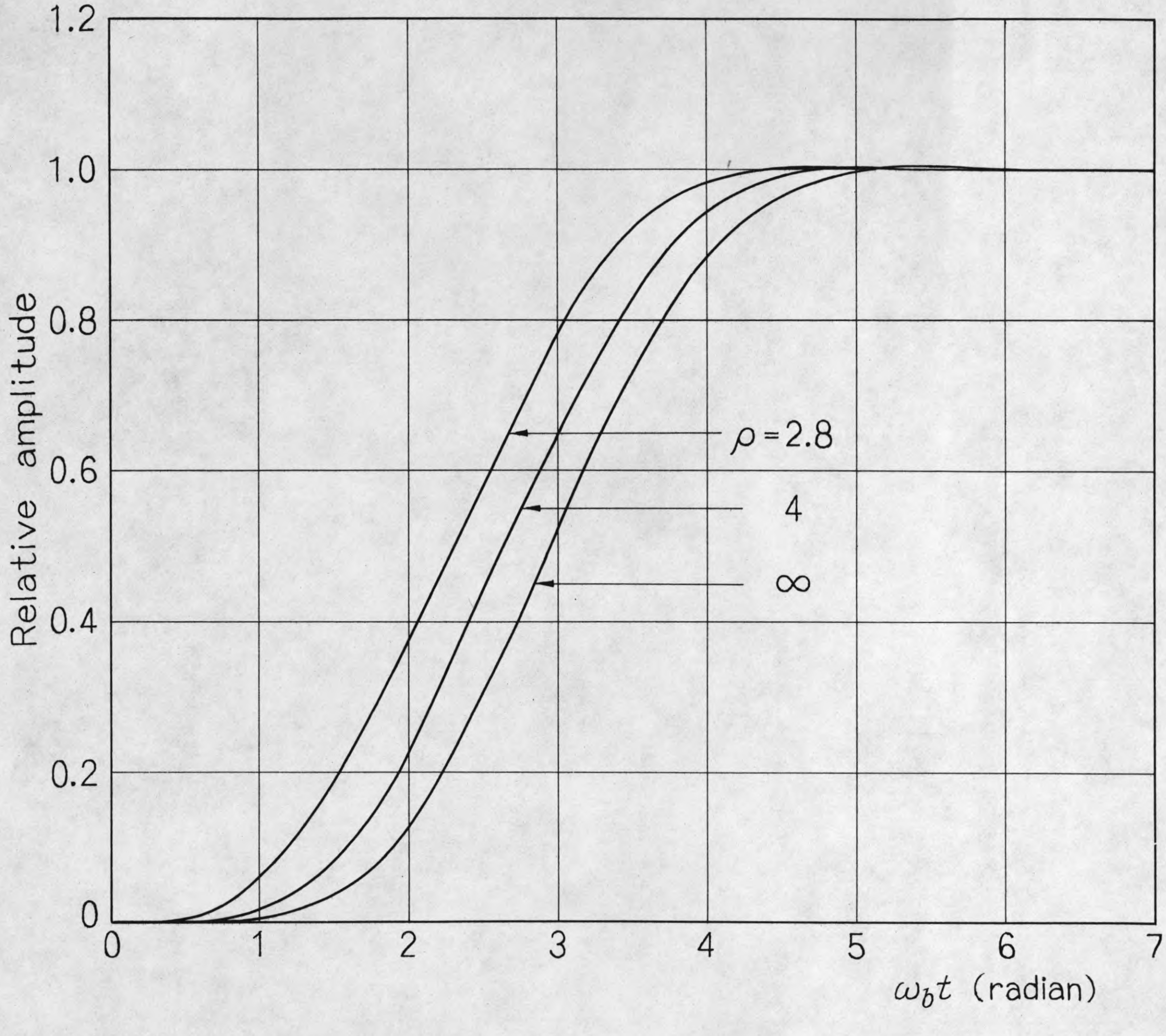

Fig. 16. Transient response, n = 7, x_c = 0

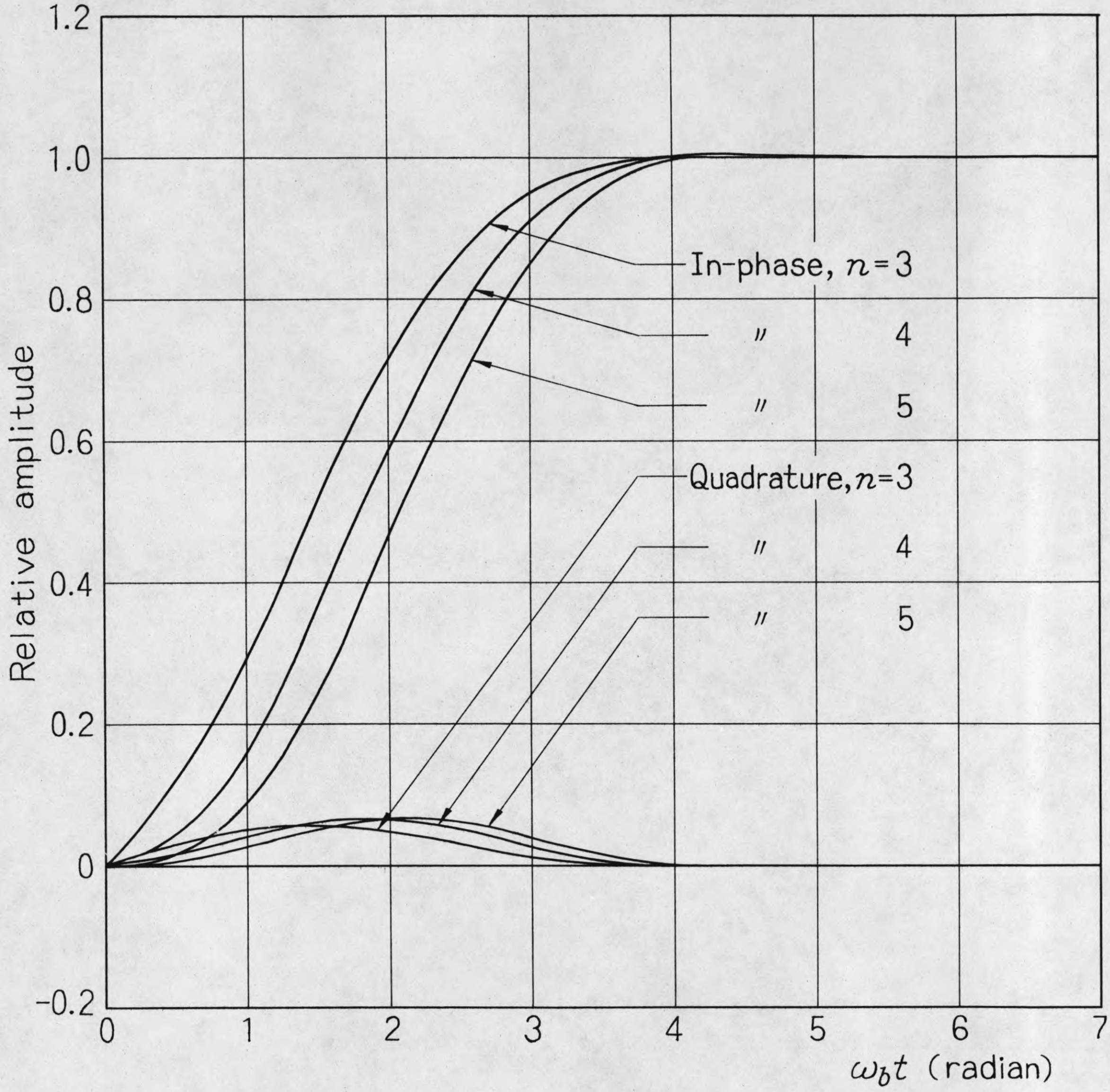

Fig. 17. In-phase and quadrature components, $\rho = 3.4$, $x_c = 0.2$

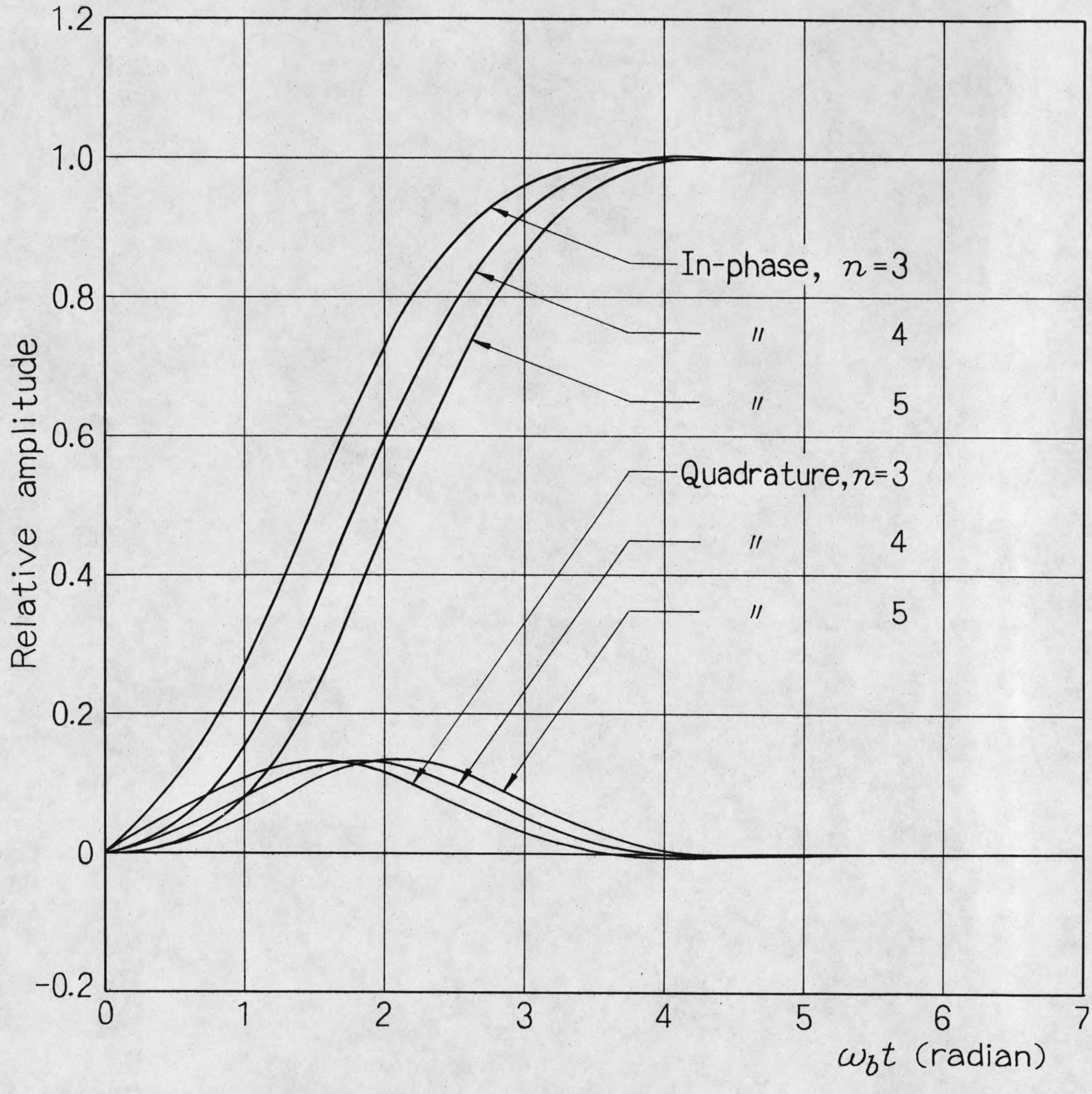

Fig. 18. In-phase and quadrature components, ρ = 3.4, x_c = 0.4

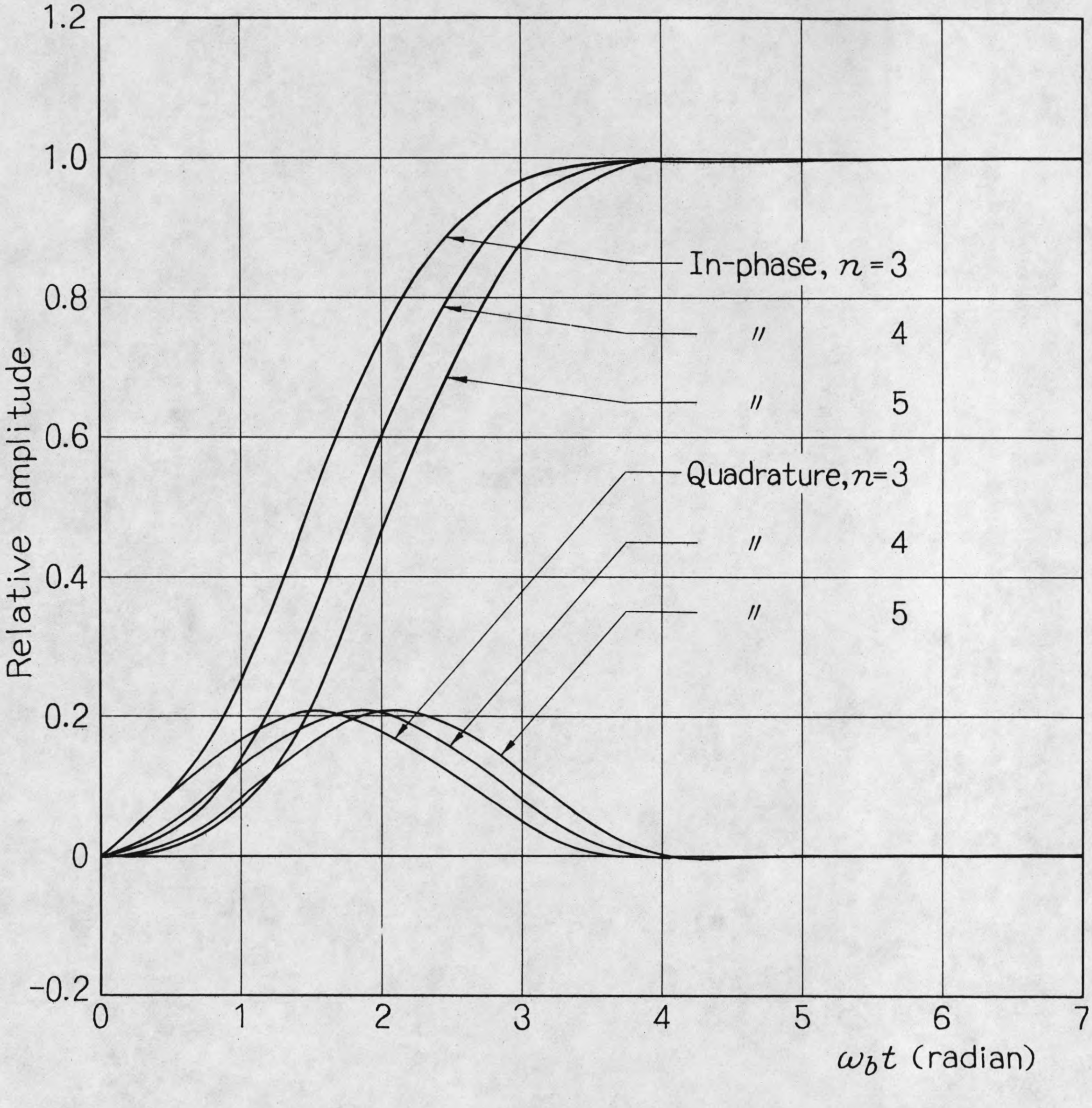

Fig. 19. In-phase and quadrature components, ρ = 3.4, x_c = 0.6

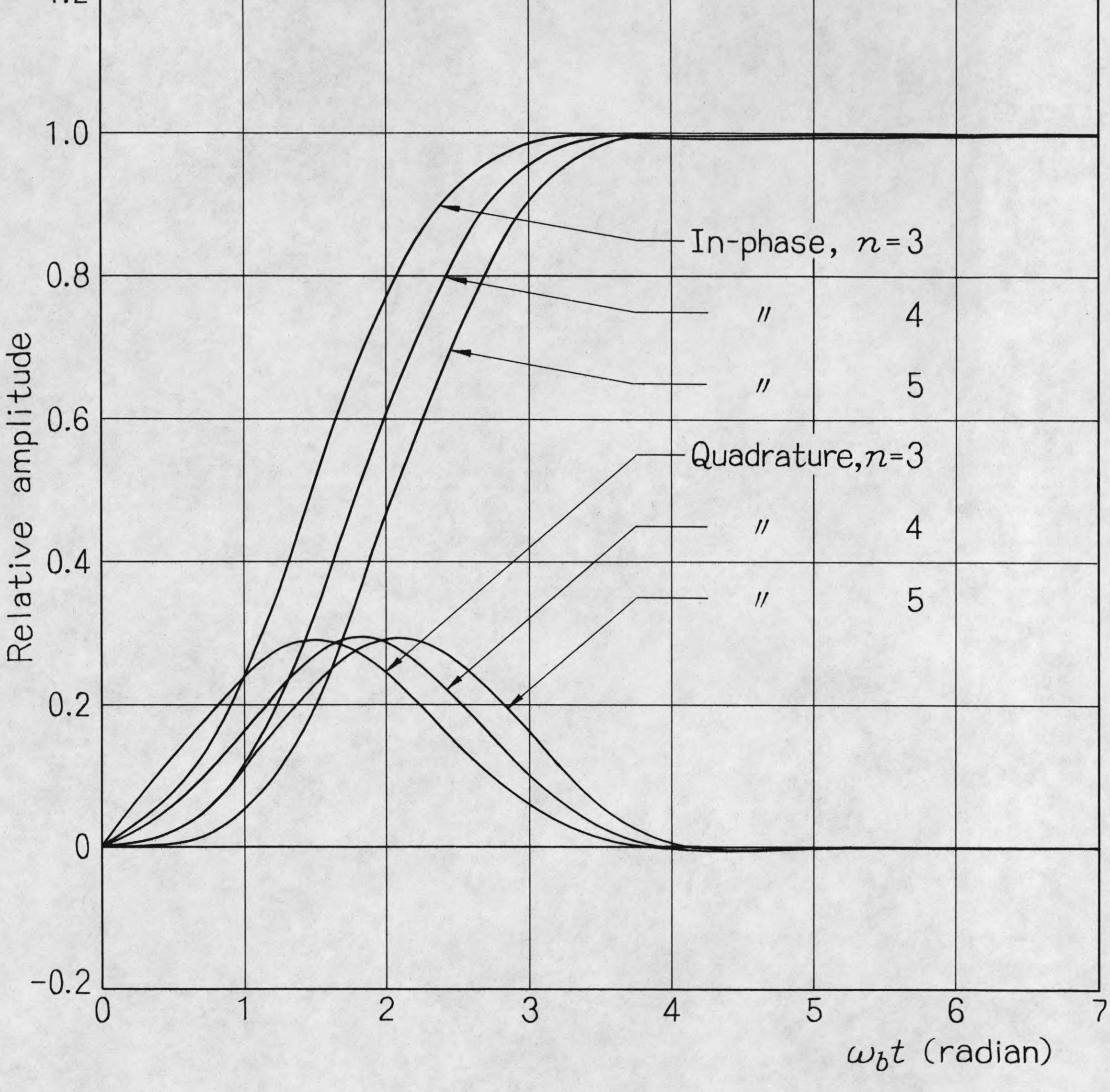

Fig. 20. In-phase and quadrature components, ρ = 3.4, x_c = 0.8

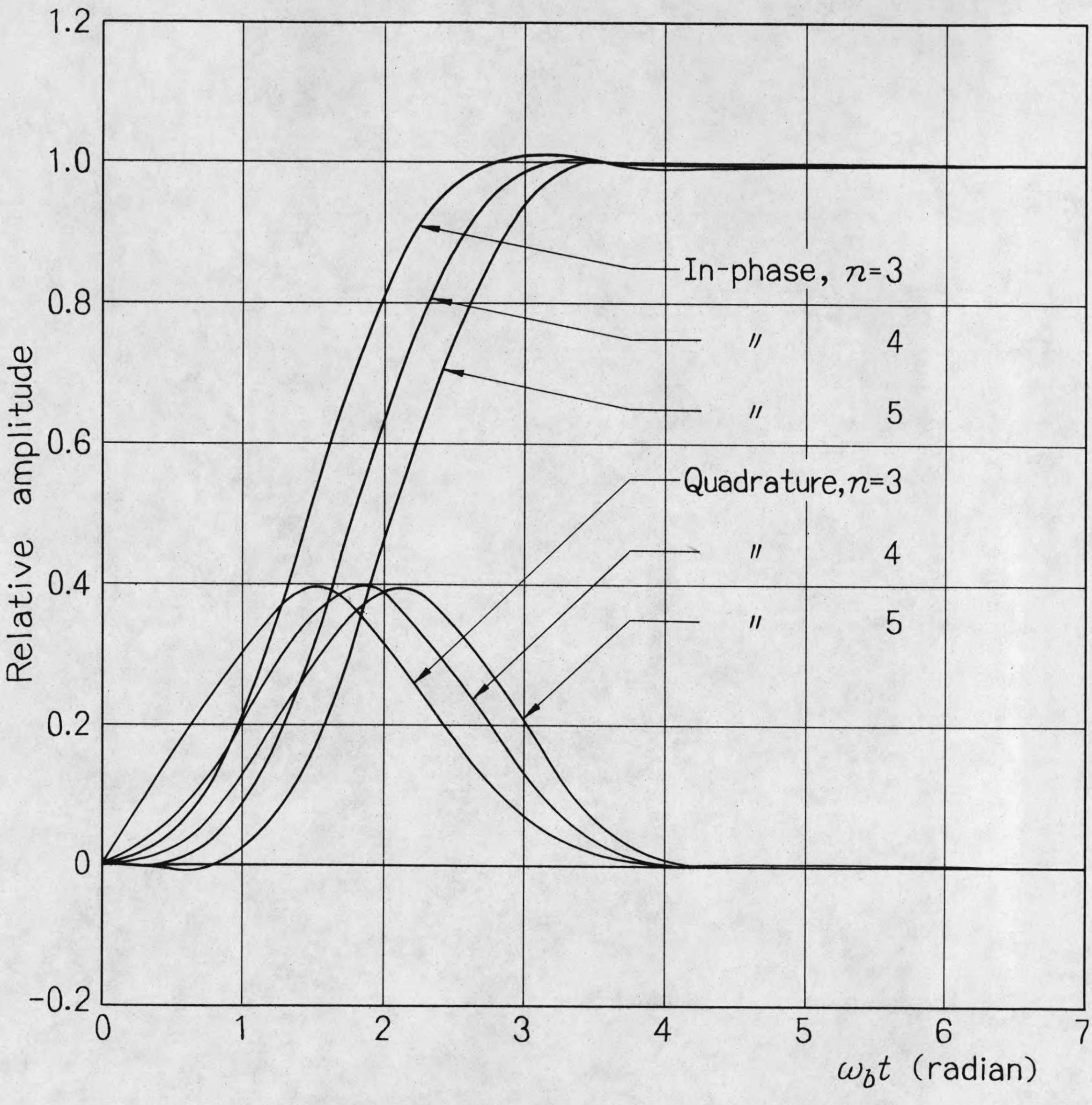

Fig. 21. In-phase and quadrature components, ρ = 3.4, x_c = 1

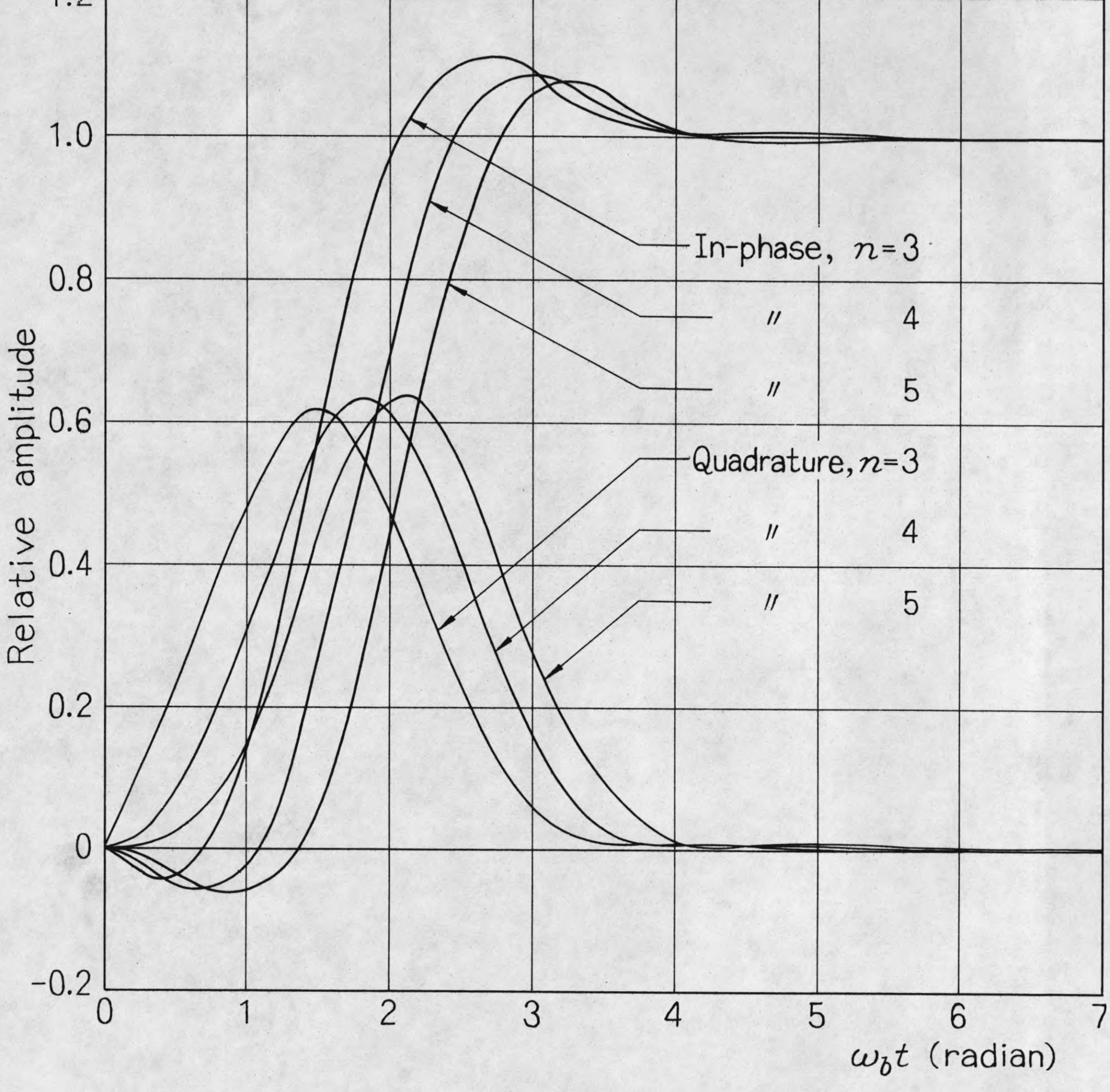

Fig. 22. In-phase and quadrature components, ρ = 3.4, x_c = bdb points

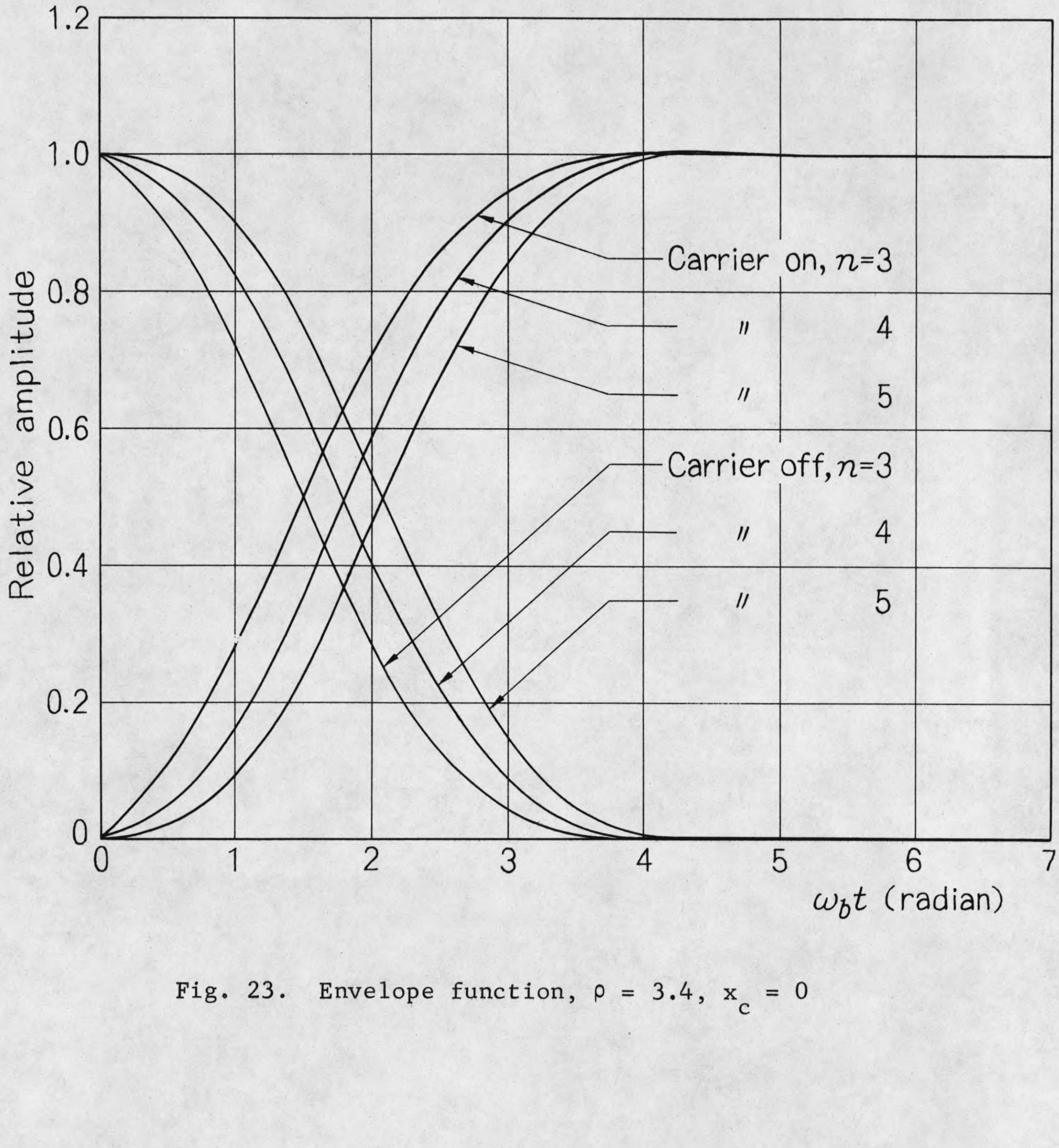

Fig. 23. Envelope function, $\rho = 3.4$, $x_c = 0$

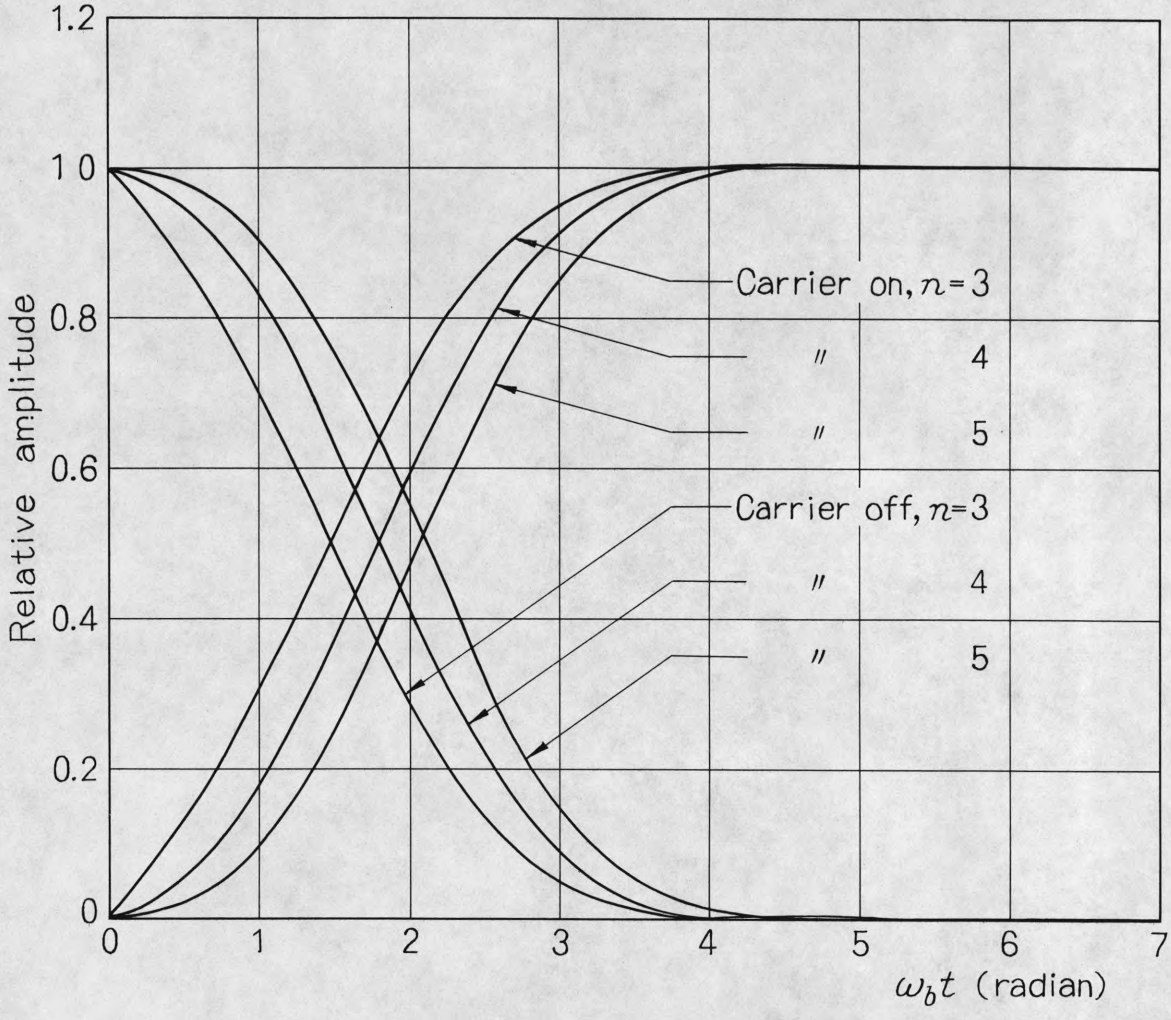

Fig. 24. Envelope function, ρ = 3.4, x_c = 0.2

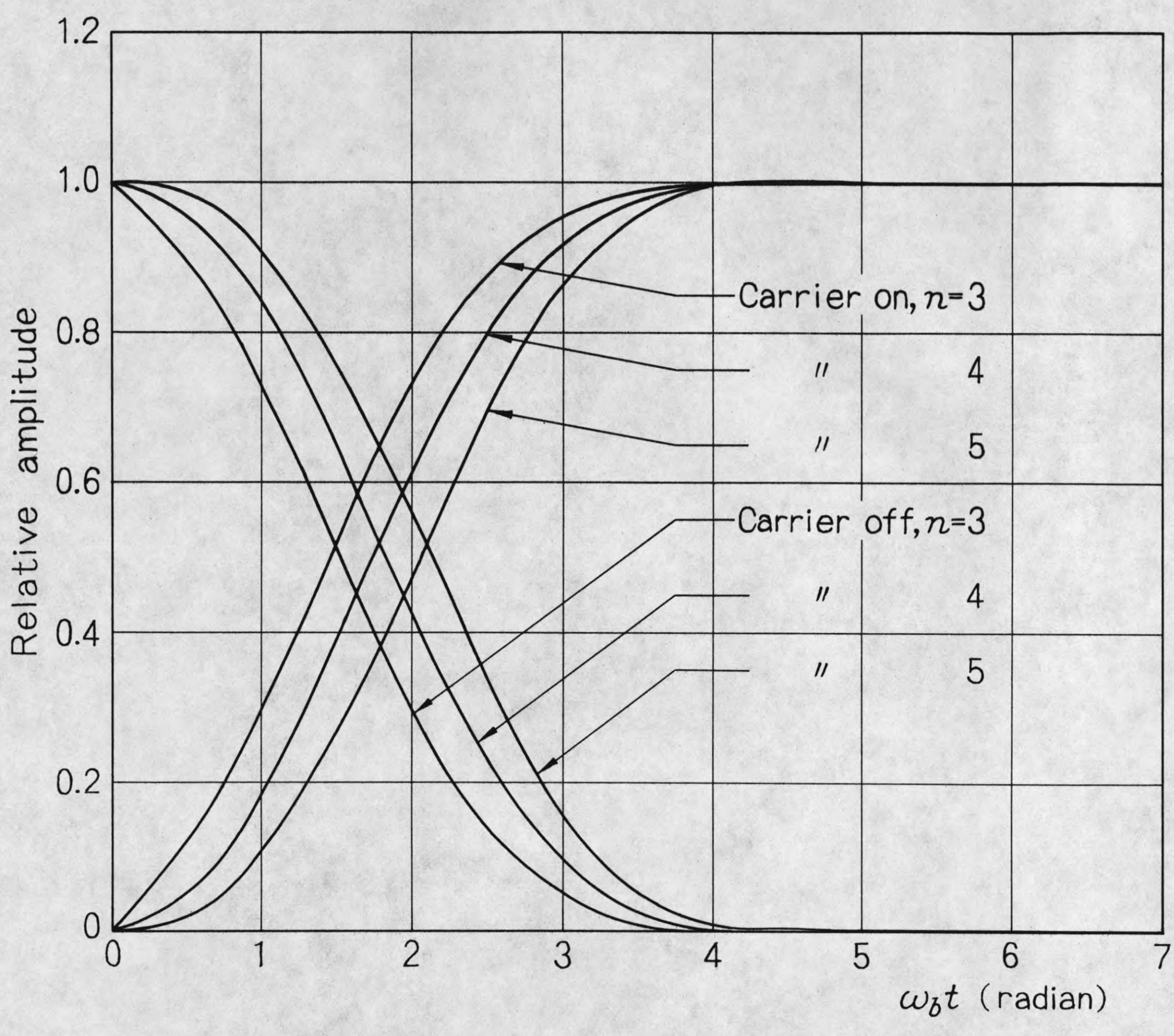

Fig. 25. Envelope function, $\rho = 3.4$, $x_c = 0.4$

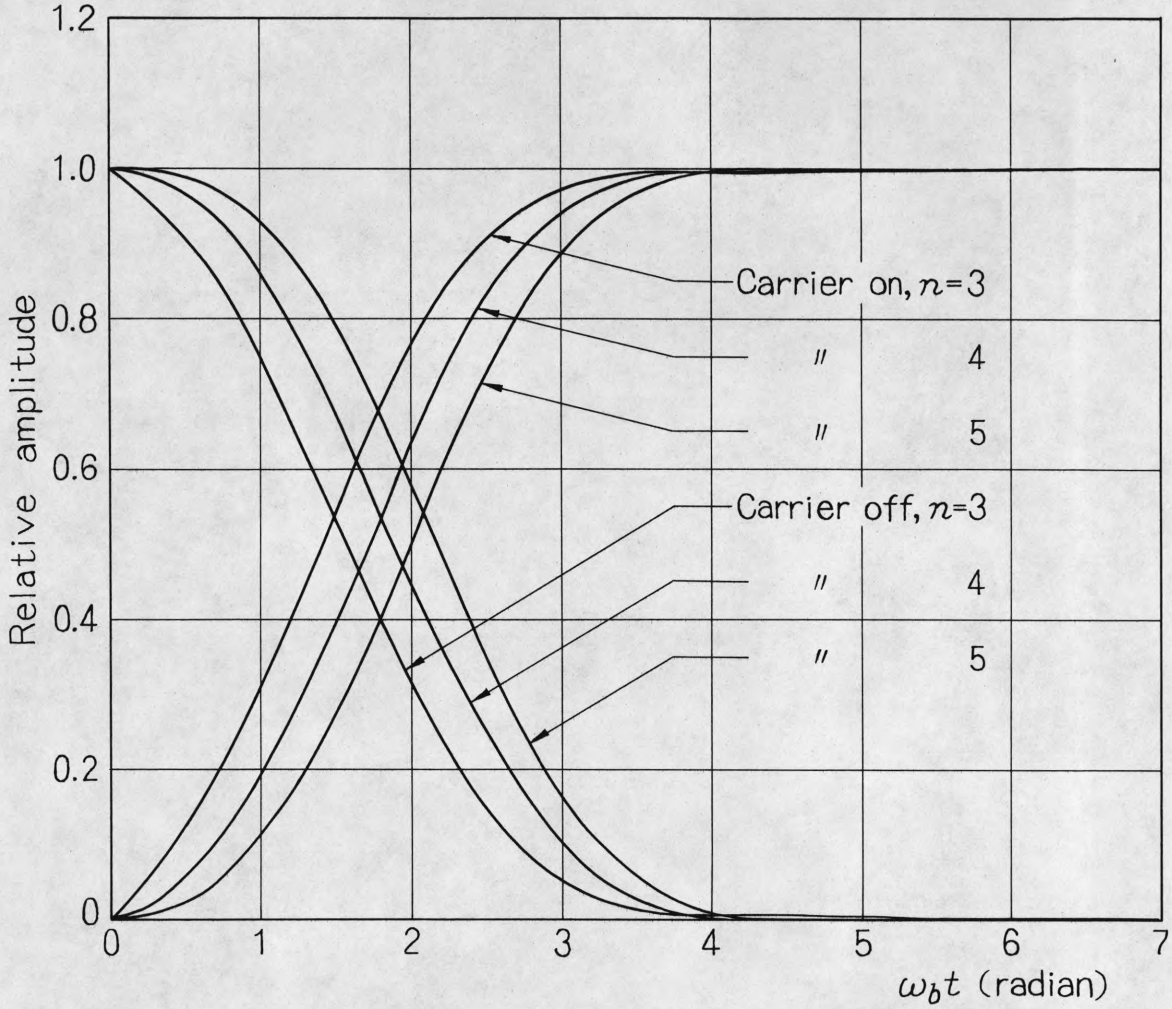

Fig. 26. Envelope function, $\rho = 3.4, x_c = 0.6$

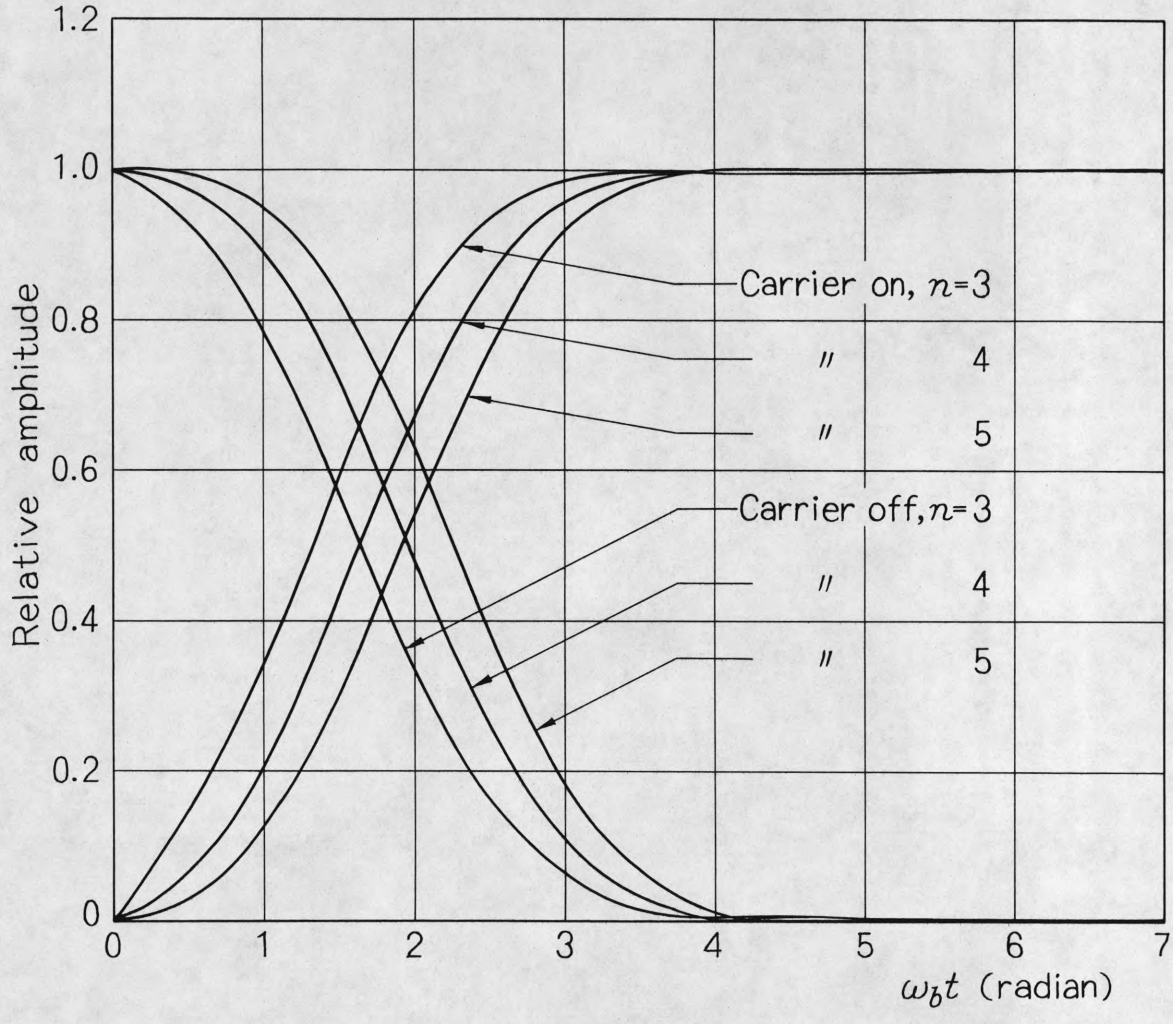

Fig. 27. Envelope function, ρ = 3.4, x_c = 0.8

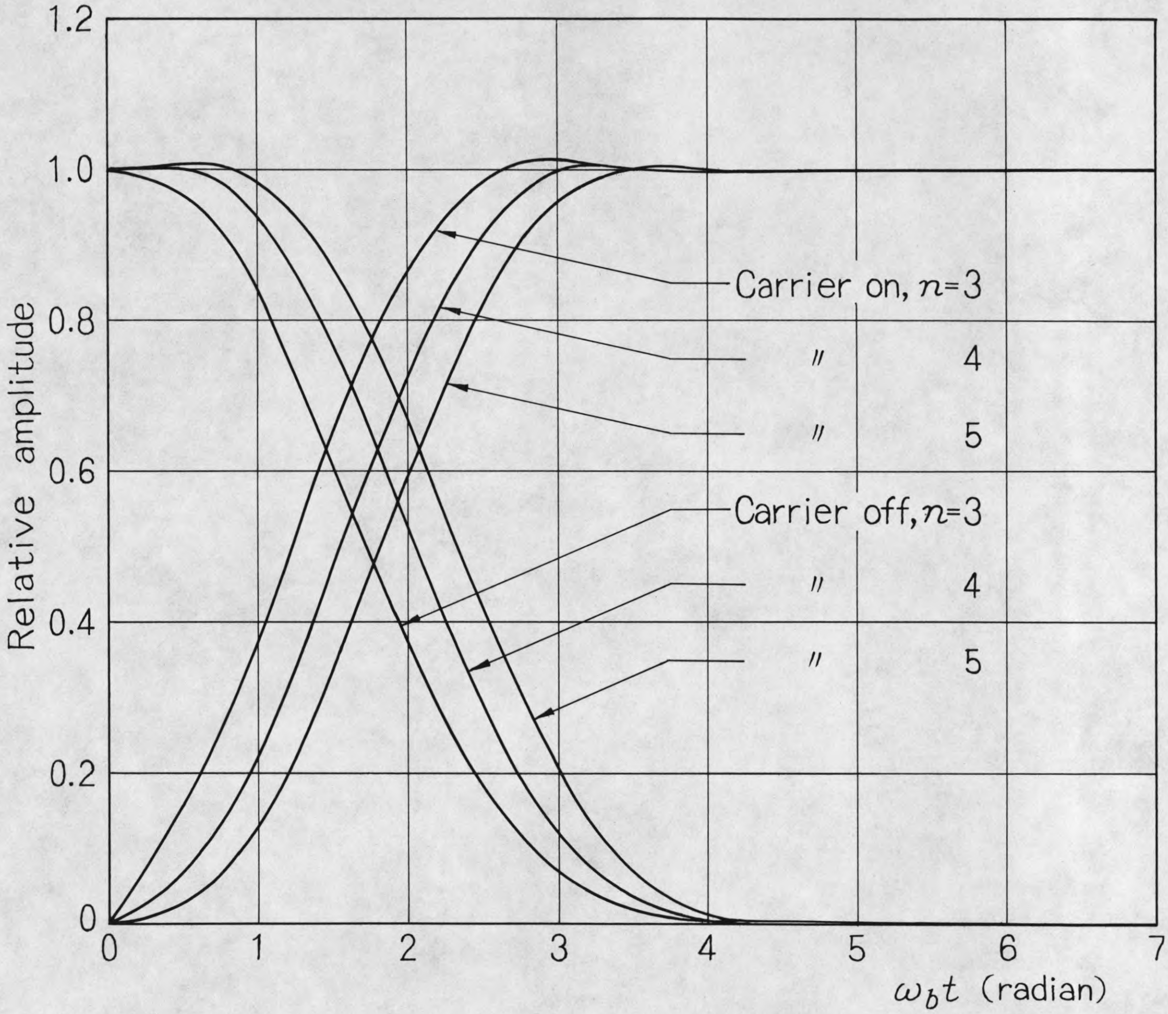

Fig. 28. Envelope function, $\rho = 3.4$, $x_c = 1$

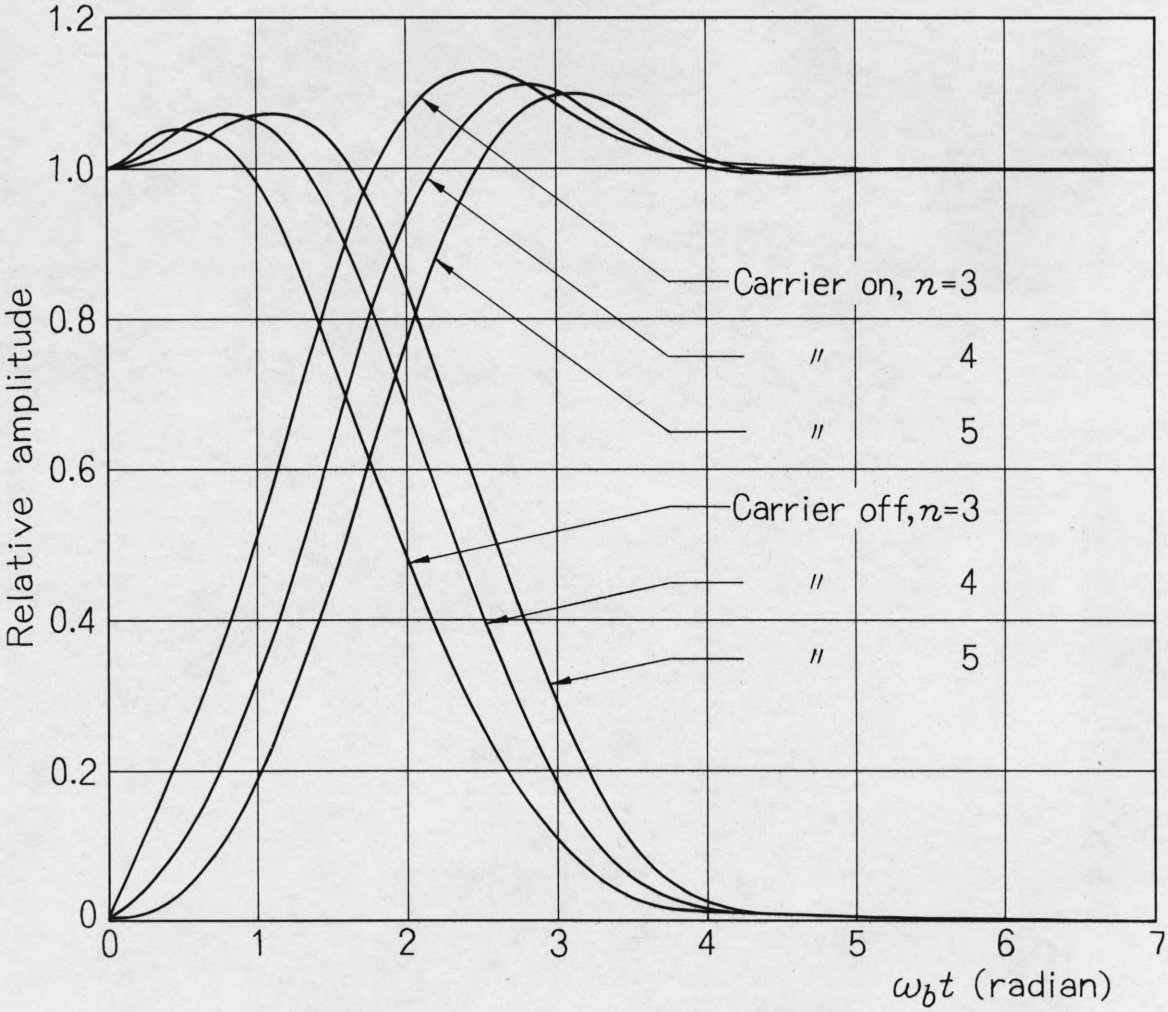

Fig. 29. Envelope function, ρ = 3.4, x_c = bdb points

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