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# PATTERN CLASSIFICATION WITH A PARTITIONED TRAINING SET

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by
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# TABLE OF CONTENTS

# 1. INTRODUCTION

Pattern classification processes are useful in many practical problems such as medical diagnosis, weather forecasting, speech and character recognition, etc. It has been under much investigation in recent years and a considerable body of literature exists.

The common objective in pattern classification is to classify patterns as members of particular categories to which they belong. That means any pattern classification scheme must have the ability to establish some sort of decision criteria for classifying memberships in different categories, and the ability to recognize members in each category. Therefore pattern classification can be considered as consisting of two parts:

1) Pattern detection - The process of learning from a set of sample patterns of known classifications and discriminating characteristics of each category; and

2) Actual classification - The process of recognizing patterns of unknown classifications as members of particular categories.

This paper is a study in the first part of the process since it is most often the more important part of any pattern classification scheme. An algorithm for establishing decision criteria of classification is described. Evaluation is made on its performance, computation time and data storage requirement.

## 2. BAYES STRATEGY

Pattern classification using Bayes decision rules involves the minimization of the "conditional average loss" (Nilsson) or the "expected risk" (Sebestyan) of classification. Consider a case of r categories, $C_1, \ldots\ldots, C_r$, and a pattern x. The "expected risk", denoted by $R(x,C_i)$, is the average loss incurred when pattern x is classified as belonging to category $C_i$. $R(x,C_i)$ is a function of loss and conditional probability and can be expressed as

$$R(x,C_i) = \sum_{i=1}^{r} \ell(C_i|C_j)p(C_j|x) \tag{1}$$

$\ell(C_i|C_j)$ is the loss incurred when a pattern belonging to category $C_j$ is classified as belonging to category $C_i$. ($\ell(C_i|C_i) = 0$ for $i = 1,\ldots,r$). $P(C_j|x)$ is the conditional probability that given a pattern x, x belongs to category $C_j$. By Bayes rule,

$$P(C_j|x) = \frac{P(x|C_j)P(C_j)}{P(x)} \tag{2}$$

$P(x|C_j)$ is the conditional probability of occurrence of pattern x given that it belongs to category $C_j$, and $P(x|C_j)$ is often referred to as the likelihood of pattern x with respect to category $C_j$. $P(C_j)$ is the _a priori_ probability of occurrence of category $C_j$ and $P(x)$ is the probability of occurrence of pattern x. Substituting (2) into (1),

$$R(x,C_i) = \sum_{j=1}^{r} \ell(C_i|C_j) \frac{P(x|C_j)P(C_j)}{P(x)}$$

$$R(x,C_i) = \frac{1}{P(x)} \sum_{j=1}^{r} \ell(C_i|C_j)P(x|C_j)P(C_j)$$

Optimal classification, in the Bayesian sense, means that each pattern is classified in such a manner that the "expected risk" of classification is minimized. In other words, pattern x is classified as belonging to category $C_i$ if

$$R(x,C_i) \leq R(x,C_j) \text{ for } j = 1,\ldots,r \text{ and } j \neq i.$$

The fact that Bayes strategy depends on the "expected risk" function presents several problems in real life and thus limits its usefulness. At best, the loss function, $\ell(C_i|C_j)$, and the a priori probability of occurrence of category $C_j$,$P(C_j)$, are estimations, and often represent subjective evaluations. (In fact, in many studies, the loss function is assumed to be uniform or symmetric for all categories.) The more serious problem is the requirement for the likelihood factor in decision making, because it depends on the probability density function within each category. It is a rare case in practical problems when probability density functions are known without preprocessing an unrealistically large number of sample patterns in each category. It is also difficult to express probability density functions analytically for easy applications. Moreover, density function based on past patterns are insensitive to sudden changes in distribution, and thus updating of decision criteria would also be difficult. For these reasons, Bayes strategy is seldom applied in practical pattern classification problems.

## 3. NONPARAMETRIC TRAINING PROCEDURE AND DECISION SURFACE

In practical classification problems, one is provided only with a set of sample patterns of known classifications, from which discriminating characteristics and decision criteria are to be determined. Therefore, nonparametric training procedures, in which information about the probability density functions within each category is not assumed, are more useful in real life situations.

Each pattern to be classified can be represented by a m-dimensional vector $x = (x_1,\ldots,x_m)$, where $x_i$'s are m measurements made on the pattern x. One basic assumption is that these m measurements provide a complete description of the characteristics of the pattern and that they are sufficient for classification pruposes.

Consider the two-category case, $C_1$ and $C_2$. A discriminant function f(x) can be defined on the representative vector x such that

$$\left.\begin{array}{l} x \text{ is classified as belonging to } C_1 \text{ if } f(x) > T \\ x \text{ is classified as belonging to } C_2 \text{ if } f(x) < T \\ \quad\quad \text{no decision is made if } f(x) = T \end{array}\right\} \quad (3)$$

T is defined as the threshold of classification.

To illustrate this geometrically, patterns are represented by points or vectors in a m-dimensional pattern space. Each dimension represents a property to be measured on the pattern. One might expect that the set of points representing patterns in one category would cluster in the pattern space, and that two different categories would be represented by two different clusters separated from one another. The discriminant function f(x) in (3) defines a decision surface in the pattern space. Patterns are classified as belonging to category $C_1$ if the points representing them lie on the positive side of the decision surface, while those represented by points lying on the negative side are classified as belonging to category $C_2$.

For a multi-category case, the rules of classification can be easily extended. When there are r categories, r different discriminant functions, $f_1(x),\ldots,f_r(x)$, can be defined. Pattern x is classified as belonging to category $C_i$ if

$$f_i(x) > f_j(x) \text{ for } j = 1,\ldots,r \text{ and } j \neq i.$$

For simplicity, only the two-category case is considered throughout this study.

## 4. LINEAR DECISION SURFACE

When the discriminant function f(x) in (3) is defined as

$$f(x) = \sum_{i=1}^{m} w_i X_i$$

where $w_i$'s are constants (weights), f(x) is called a linear discriminant function, and the decision surface defined by f(x) in the pattern space is a hyperplane.

To simplify mathematically, add 1 as the (m+1)st component of the representative vector x and call the new (m+1)-dimensional vector X.

$$X = (x_1, \ldots, x_m, 1)$$

Define a weight vector W where

$$W = (w_1, \ldots, w_m, -T)$$

then the linear discriminant function can be more easily represented by

$$F(x) = W X^T$$

where $X^T$ denotes the transpose of X. Thus the rules in (3) can be expressed as

$$\left.\begin{array}{c} x \text{ is classified as belonging to } C_1 \text{ if } F(x) > 0 \\ x \text{ is classified as belonging to } C_2 \text{ if } F(x) < 0 \\ \text{no decision is made if } F(x) = 0 \end{array}\right\} \qquad (4)$$

Categories $C_1$ and $C_2$ are said to be linearly separable if the weight vector W exists. It is well known that if the patterns are finite and linearly separable, there exists such a solution. Perception-type procedures, which process a fixed set of sample patterns of known classifications in an iterative manner, have been used to investigate such a solution. It is found that the weight vector W converges within a finite number of steps.

Although linear discriminant functions can be easily found, they are quite restricted in use since it is not always possible to have linearly separable categories. In this case, more sophisticated discriminant functions are required for practical pattern classification problems. One basic extension to the linearly decision surface is the piece-wise linear decision surface. Such a surface consists of a collection of linear decision surfaces which, when taken together, form a piece-wise linear decision surface. For example, assume category $C_1$ has a collection of p linear decision surfaces, defined by p linear discriminant functions, $F^1(x),\ldots,F^p(x)$. Then pattern x is classified as belonging to category $C_1$ if

$$F^i(x) > 0 \text{ for } i = 1,\ldots,p.$$

## 5. SUPERVISED LEARNING PROCESS

During the detection phase of the pattern classification process, a set of sample patterns of known classifications is given during training, from which a weight vector, or decision surface, will hopefully emerge. In this case, the decision surface is said to be acceptable if it is capable of classifying a large fraction of unknown patterns with a small amount of errors, but also if the process can be implemented with relative ease. This is done usually by iteration on the set of sample patterns.

Iteration starts with an initial weight vector which is modified during training according only to input sample patterns. The weight vector is tested by attempting to classify sample patterns of known classifications. When the decision is correct, the weight vector remains the same. When a decision errors occur, the weight vector is modified. The amount of weight adjustment, in case of error at step t is dependent only on the weight vector and the sample pattern tested at step t. Iteration ends when the weight vector classifies the entire set of sample patterns correctly.

A simple set of iteration rules is used and can be expressed as

$$W(t+1) = W(t) \qquad \text{when there is no error}$$

$$W(t+1) = W(t) + c(t)X \quad \text{when error occurs}$$

where $W(t)$ is the weight vector at step t and X is the vector representing pattern x tested at step t. $c(t)$ is the correction factor at step t. This correction factor can either be fixed throughout iteration, or vary with the amount of erroneous crossover from the decision surface defined by the weight vector $W(t)$ at step t. In both cases, it can be shown that iteration converges if the set of sample patterns from the two categories is finite and linearly separable. In the latter case, the correction factor c(t) at step t can be chosen in such a way that the new weight vector W(t+1) will classify the sample pattern x at step t correctly. In other words, the decision surface is moved so that the point representing the sample pattern tested at step t would be located on the correct side of the new decision surface. It can be easily shown (Appendix A) that if this is the case, the correction factor c(t) has a lower bound

$$c(t) \geq \frac{|W(t)X^T|}{XX^T} \ \text{sgn}(x)$$

where sgn(x) is either + or - depending on the correct classification of sample pattern x. In this study, this value of the lower bound is used as the correction factor at step t during iteration, although the fixed correction method would also work quite well. Convergence of the iterative process is assumed when the correction factor is very small with respect to the weight vector (<.001).*

To obtain an optimal decision surface, a representative set of sample patterns is needed. Iterative processes on this entire set present several problems that need to be considered. A large data storage for the sample patterns is required, the computation time needed for iteration to converge is quite lengthy at times, and updating of decision surface due to changes is also difficult.

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*Because of this convergence assumption, not all sample patterns are classified correctly by the weight vector obtained at the end of the iteration. This effect can be observed in later results.

## 6. SEGMENTATION

Due to the problems encountered in finding a solution for the optimal decision surface using iterative process on the entire set of sample patterns of known classifications, a new scheme is investigated. This scheme utilizes segmentation techniques, in that the set of sample patterns used in training is divided into segments. Iteration is performed on one segment at a time. The weight vector obtained after processing one segment of sample patterns is used as the initial weight vector for iteration on the next segment. This process is repeated for every segment, until at the end of the last segment, a decision surface is obtained.

Obviously, the decision surface thus obtained may not be as accurate as previously, but this process has several advantages. Data storage requirements are reduced since it needs only to be provided for sample patterns in each segment, instead of for the entire set. Also due to the small size of each segment, the computation time needed for convergence of the iterative process on each segment is much shorter, and thus the time needed for the entire training phase is reduced. Updating of decision surface can also be accomplished with relative ease by adding a new segment consisting of new sample patterns.

An inherent advantage of this scheme using segmentation technique is the fact that learning can be made more related to the stage of training. As we know, a great deal of knowledge about the decision surface is gained in the early stages of training. In subsequent stages, most of time is occupied by minor adjustments made on the decision surface. Using segmentation techniques, each segment can be thought of as a stage in the training process. If learning is concentrated more on the early stages, much time could be saved by ignoring minor adjustments on the weight vector in later stages. Computation time could then be more wisely utilized by performing only major modifications of the decision surface, such as in cases of updating.

## 7. EXPERIMENT

A series of experiments is conducted on IBM 360/75 to study the effects of segmentation on the computation of decision surfaces for pattern classification. A two-dimensional pattern space is chosen so that results can be more readily illustrated. Since each dimension measures a property of the patterns to be classified, a normalized 0-to-1 scale is used to indicate the range of probable nonexistence or existence of the property. A set of 60 sample patterns of known classifications is chosen at random for training purposes. They are divided into two categories, positive or negative, depending on the locations of their representative points in the pattern space with respect to an arbitrarily pre-selected linear decision surface.

There are two groups of experiments:

A. In the first group, the set of sample patterns used in training is evenly divided in various ways into segments, while the training algorithm remains the same in each case. The performance of the decision surface obtained in each case is evaluated and compared with that obtained without segmentation. A comparison is also made on the computation time needed and data storage required for each solution.

B. In the second group of experiments, the idea of relating learning more closely with training stages is incorporated. The set of sample patterns is divided into 5 segments of 12 patterns each. Each segment is thought of as a stage in training. Associated with each stage is a threshold level L(n), where n is the training stage. When in stage n, modification on the decision surface, in case of an error in classification, is made only when the absolute value of the product $\frac{WX^T}{\sqrt{\sum_{i=1}^{m} w_i^2}}$ is greater than the associated threshold level $L(n)$. All other minor errors are ignored. (Geometrically speaking, $\left|\frac{WX^T}{\sqrt{\sum_{i=1}^{m} w_i^2}}\right|$ measures the distance from the point representing pattern x to the decision surface defined by weight vector W, and gives an indication on the magnitude of error made by the decision surface when attempting to classify sample pattern x.) Since the assumption is that the amount of knowledge gained about the decision surface decreases with increasing training, the threshold level $L(n)$ is an increasing function of training stage n. In this study, $L(n)$ is chosen as a linear function of n

$$L(n) = S(n-1) \tag{5}$$

where S is a constant. The performances of decision surfaces obtained using the same training algorithm but with different values of S, and the time needed for computation in each case are evaluated and compared.

## 8. RESULTS AND OBSERVATIONS

Results from the first group of experiments are shown in Figures 1 through 6. As an illustrative example, Figures 1 through 4 show the training process using segmentation techniques. The set of sample patterns used in training is divided into 4 segments of 15 patterns each. Each figure shows the initial linear decision surface used and the convergent decision surface obtained by iteration on each segment. In Figure 4, the linear decision surface obtained by iteration on the entire set of sample patterns without segmentation is also shown to give a comparison with that obtained with segmentation. Figure 5 shows the different linear decision surfaces obtained when the number of sample patterns in each segment is varied. Figure 6 shows a comparison on the performances of these different linear decision surfaces obtained in each case. (The percentage of correct classifications made on the entire set of sample patterns by the decision surface is used as the performance measure of that decision surface.) A comparison is also made on the ratios of the computation time and data storage requirement needed for each decision surface with segmentation to that without segmentation.

As expected, the graphs show that the performance of decision surface improves with increasing number of sample patterns in each segment, but the amount of computation time and data storage needed for solution also increases. From iteration on 15 segments of 4 sample patterns each to iteration on the entire set, performance improves by a factor of 1.4, but data storage requirement increases by a factor of 12.8 and computation time also by a factor of 4.7. Thus, there is a trade-off between performance and requirements for computation time and data storage when segmentation technique is used in training.

Results from the second group of experiments are shown in Figures 7 through 12. Figures 7 through 10 show an example of the training process when the constant S of the threshold level function in (5) is set at 0.03. As can be observed, minor classification errors are ignored after an initial stage and only major modifications on the decision surface are performed. Figure 11 shows the different final linear decision surfaces obtained as the constant S varies from 0.0 to 0.05. In Figure 12, a comparison is made on the time needed for computation of the decision surface in each case and its performance.

As observed in the graphs, training time for the decision surface decreases while the number of classification errors increases with increasing value of the constant S in (5). Compare with S = 0, the performance of the decision surface obtained at S = 0.05 worsens by a factor of 1.25 while computation time decreases by a factor of 3.57. Therefore, a trade-off also exists.

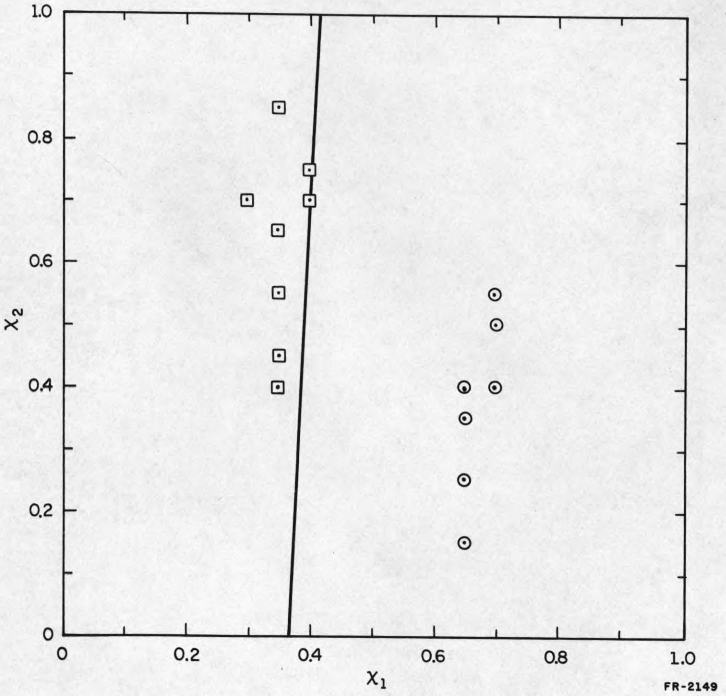


Fig. 1. Decision surface obtained after first segment.

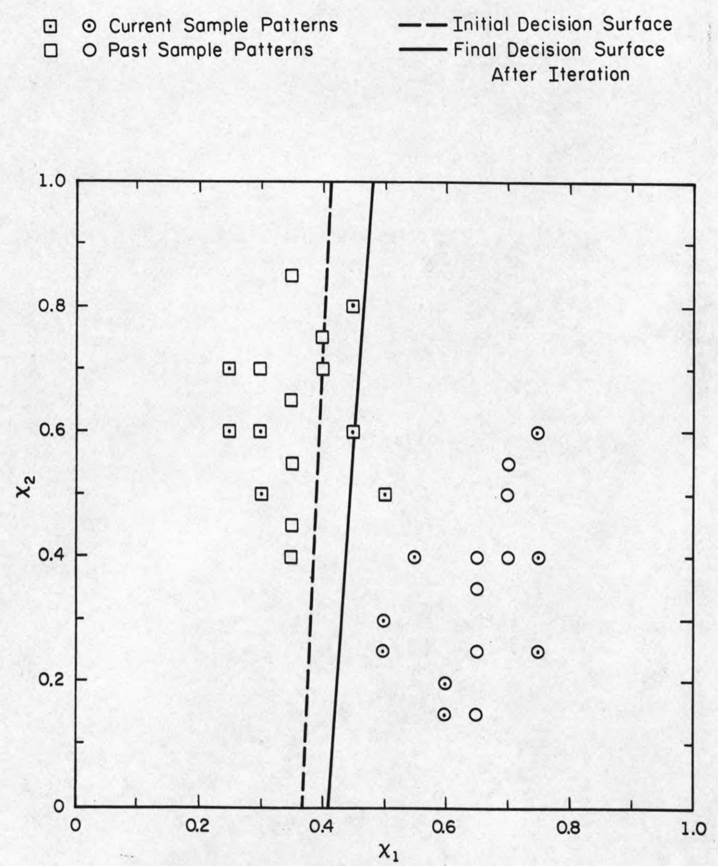


Fig. 2. Decision surface obtained after second segment.

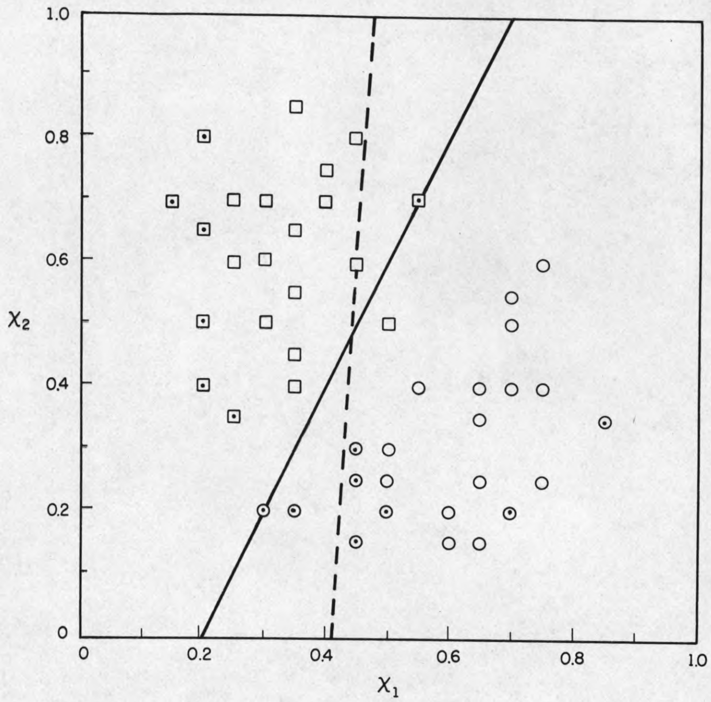


Fig. 3. Decision surface obtained after third segment.

⊡ ⊙ Current Sample Patterns
□ ○ Past Sample Patterns
– – – Initial Decision Surface
—— Final Decision Surface After Iteration
–·–·– Decision Surface Obtained Without Segmentation

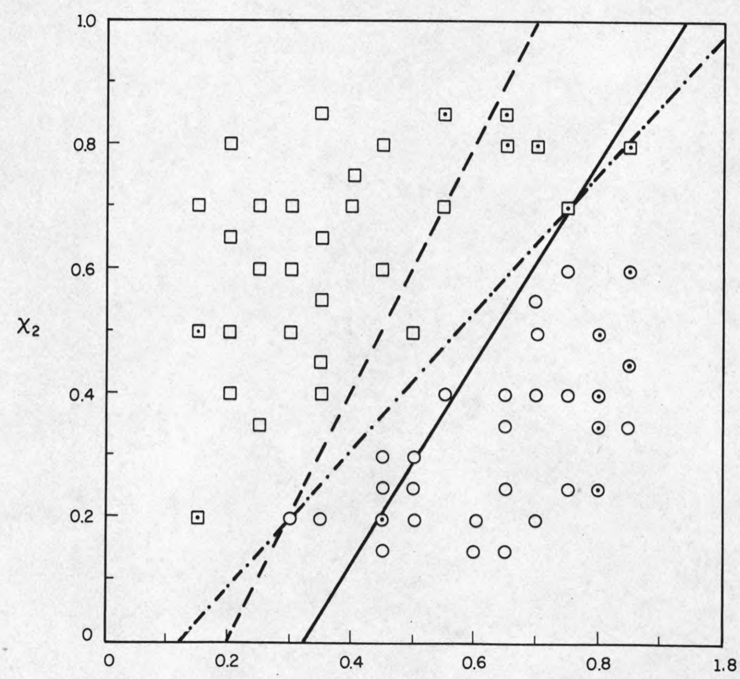


Fig. 4. Decision surface obtained after fourth segment.

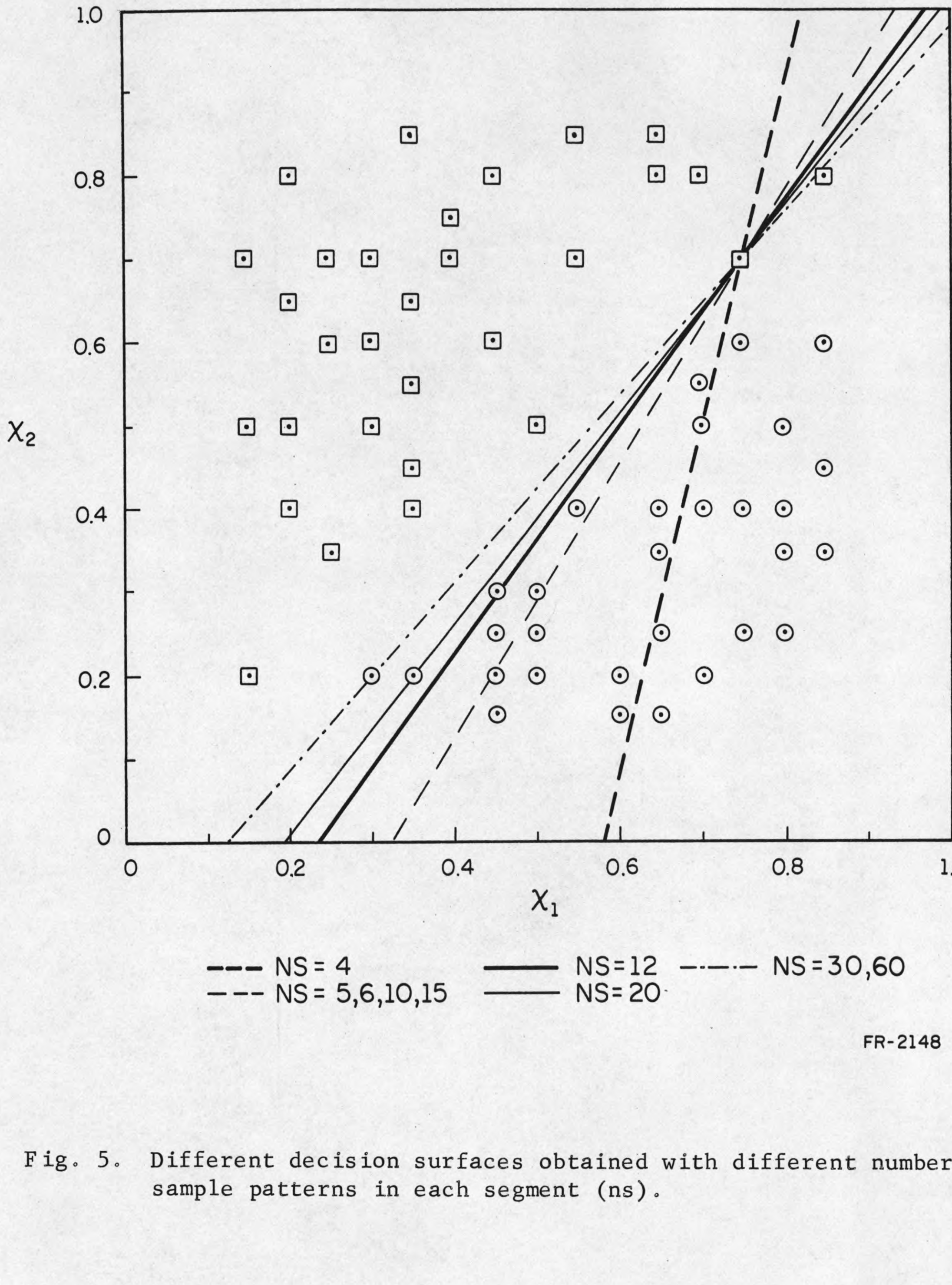


Fig. 5. Different decision surfaces obtained with different number of sample patterns in each segment (ns).

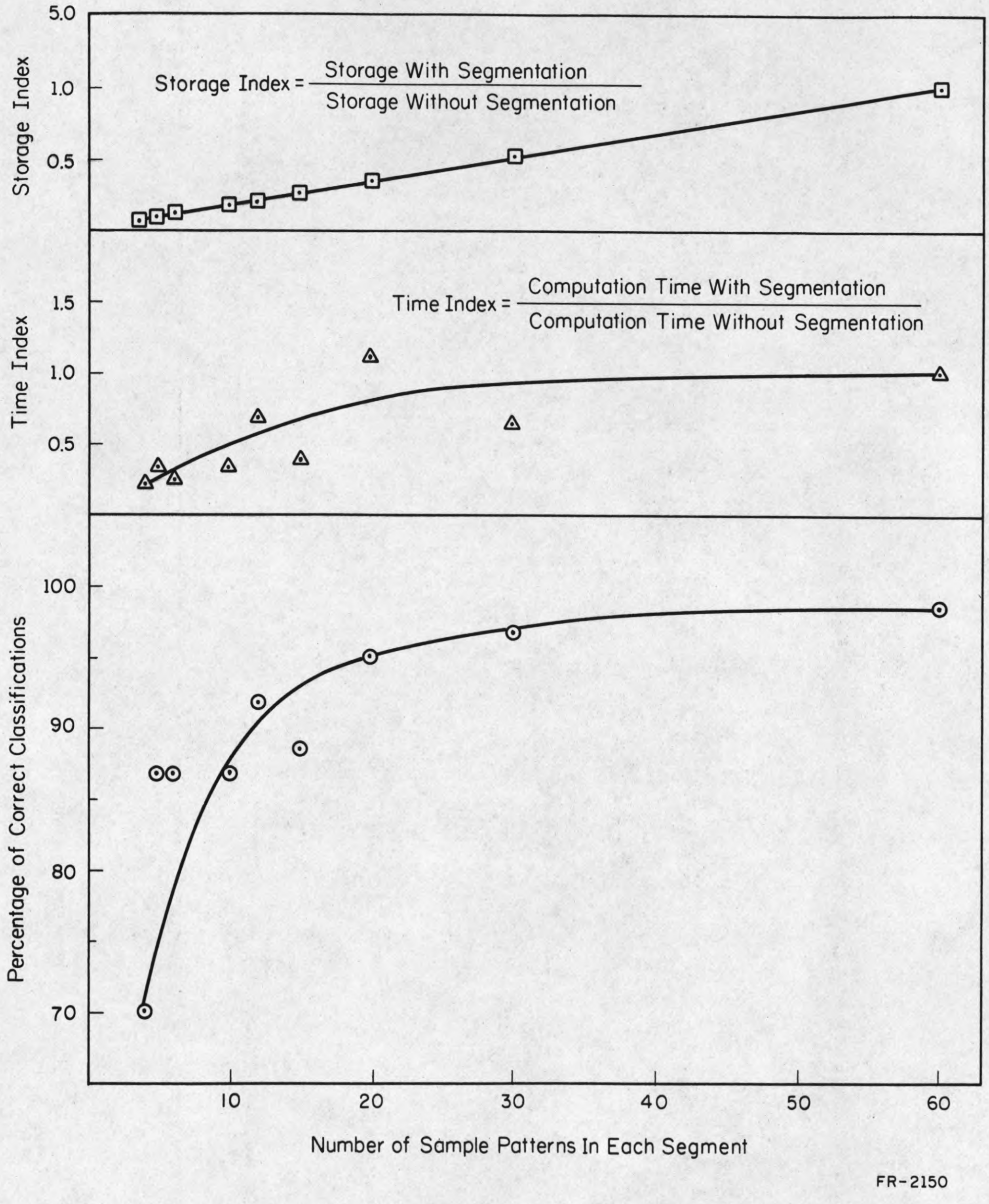


Fig. 6. Comparison on performance, computation time and data storage requirement.

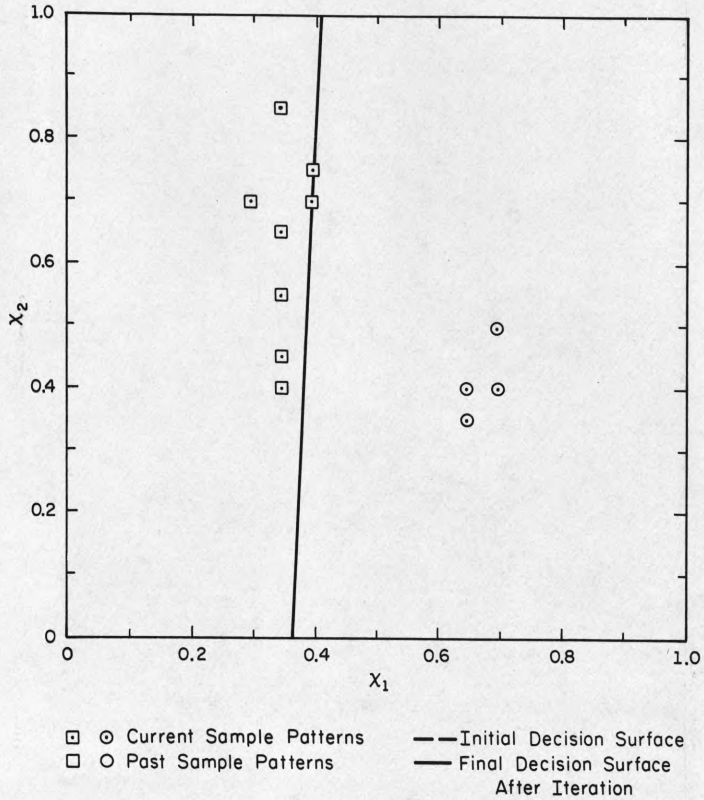


Fig. 7. Decision surface obtained after first training stage.

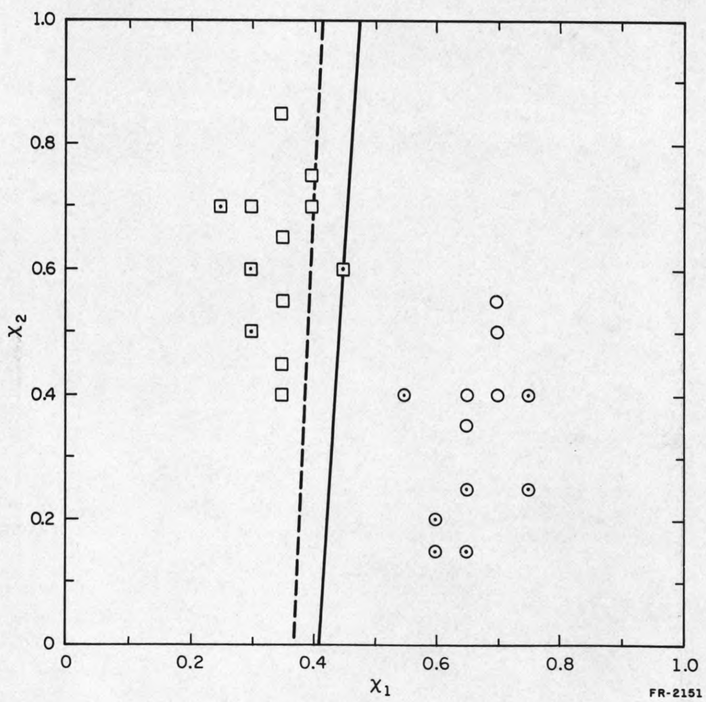


Fig. 8. Decision surface obtained after second training stage (L(2) = 0.03).

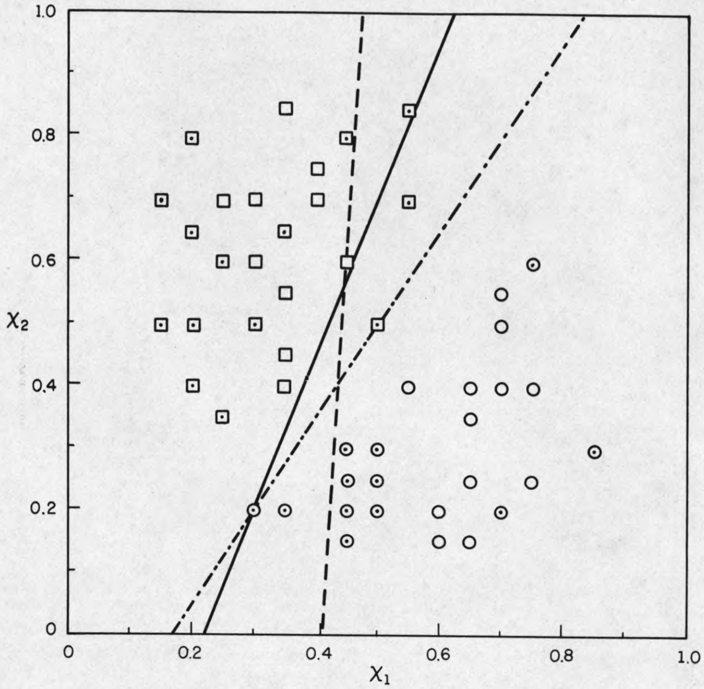


Fig. 9. Decision surface obtained after third training stage (L(3) = 0.06) and remains the same for fourth training stage (L(4) = 0.09).

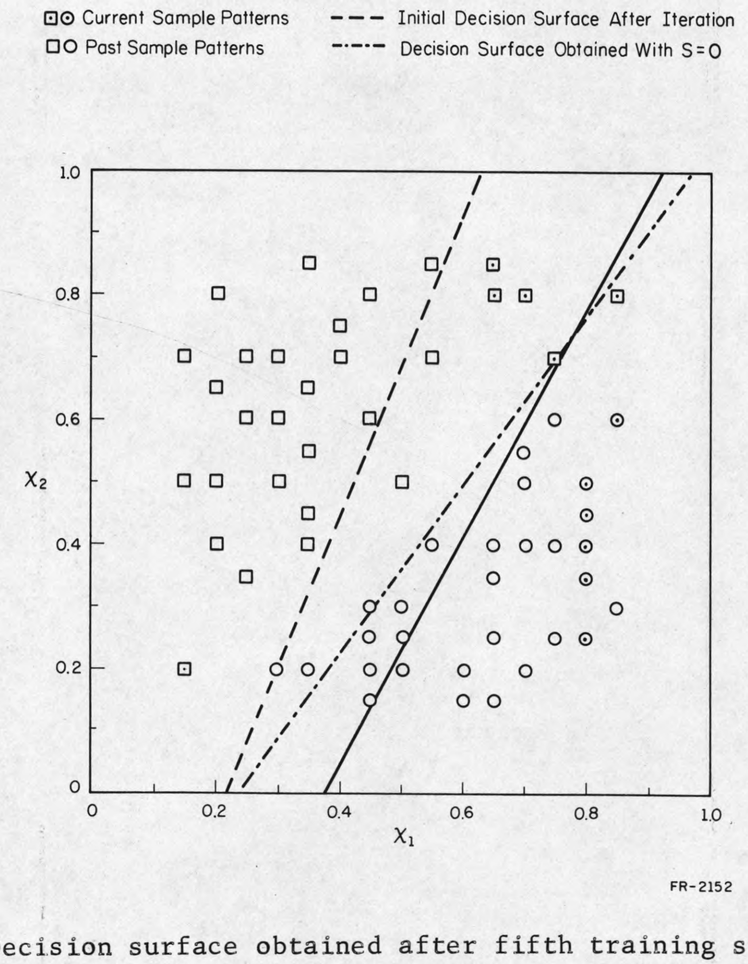


Fig. 10. Decision surface obtained after fifth training stage (L(5) = 0.12).

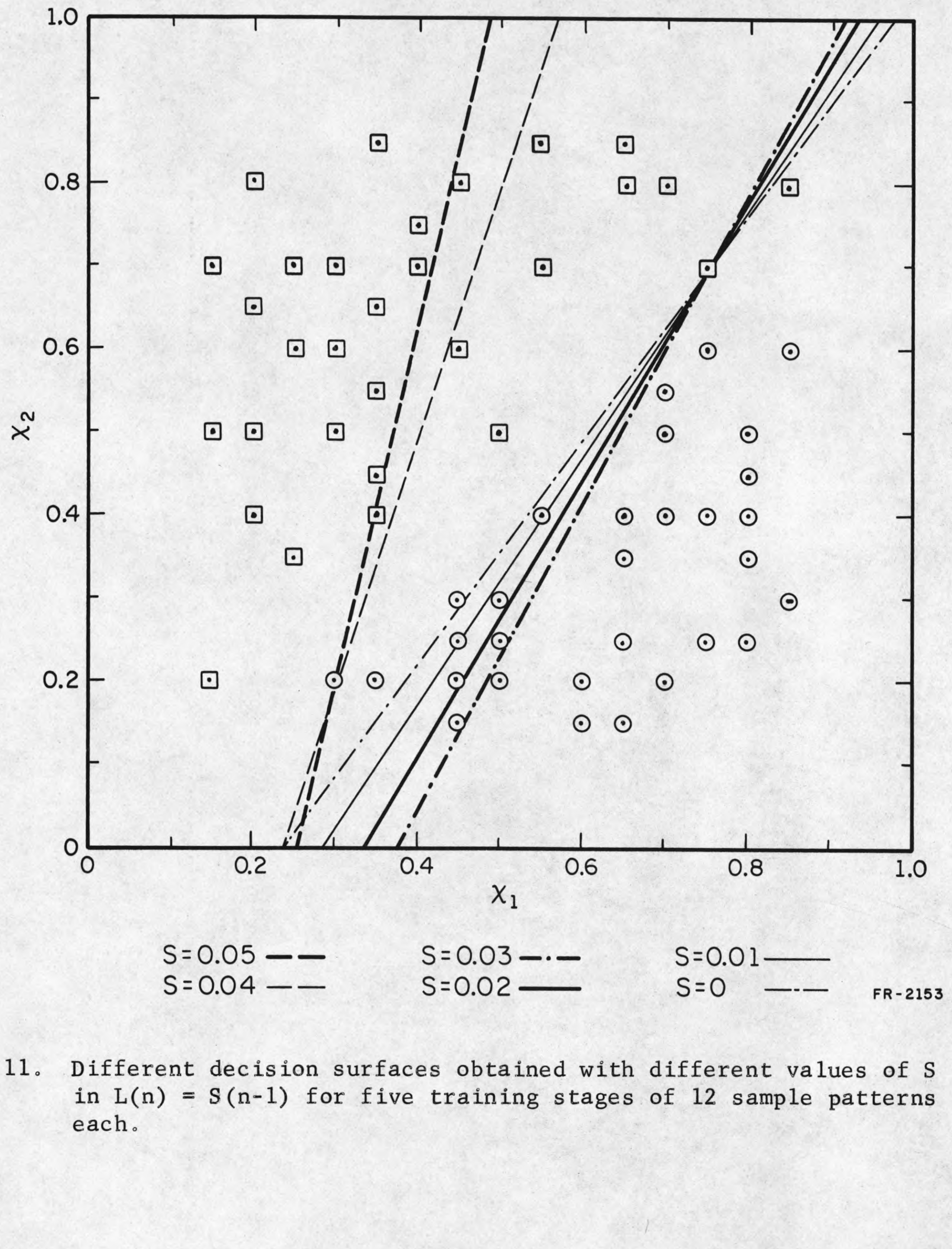


Fig. 11. Different decision surfaces obtained with different values of S in L(n) = S(n-1) for five training stages of 12 sample patterns each.

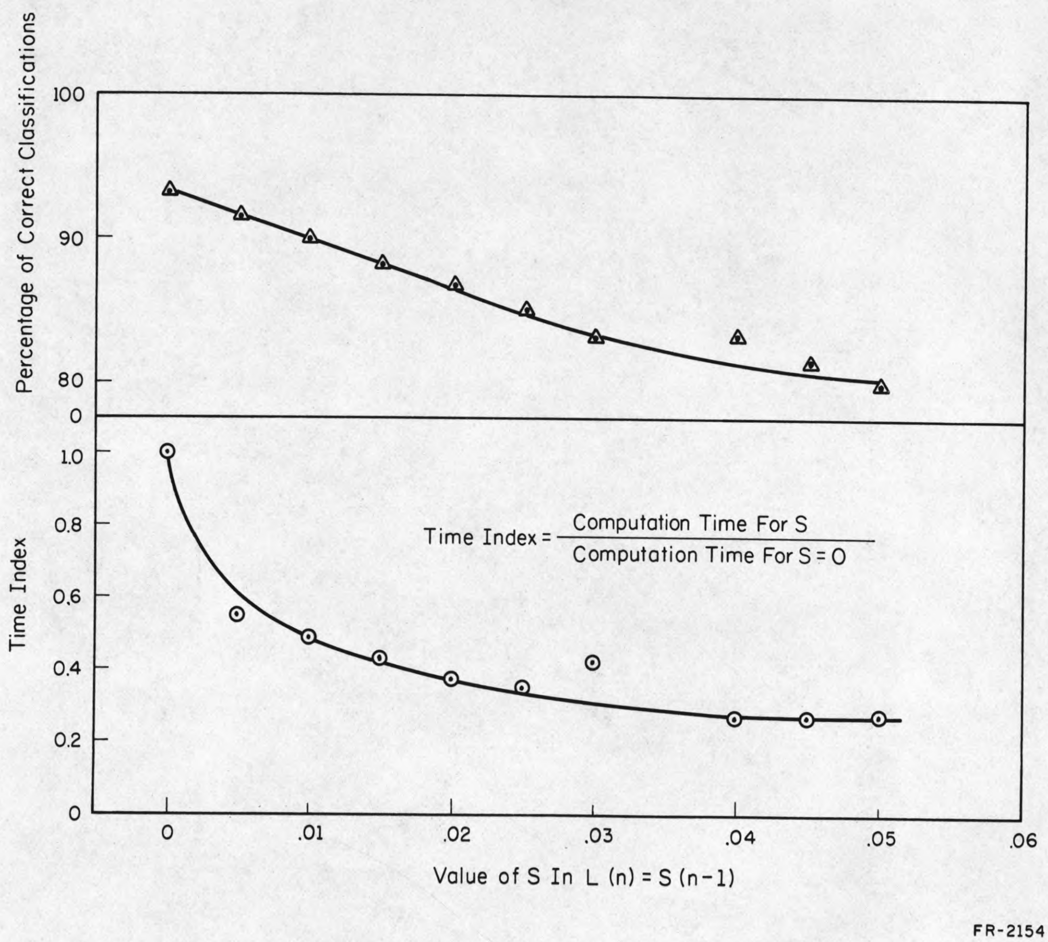


Fig. 12. Comparison on performance and computation time.

## 9. SUMMARY

An iterative training algorithm for linear decision surface in pattern classification using segmentation techniques has been described. Experimental results are observed to show the effects of segmentation on performance of the decision surface obtained, together with computation time and data storage required for solution. The idea of decreasing learning time with increasing training has been discussed, and due to the simplicity of the algorithm, updating of decision surface can easily be implemented.

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# APPENDIX A

Assume a sample pattern x belongs to category $C_1$ and the point in pattern space representing x lies on the positive side of the correct linear decision surface. Decision error at step t means

$$W(t)\, X^T < 0 \qquad (1a)$$

Correct classification by W(t+1) means

$$W(t+1)X^T > 0$$

where

$$W(t+1) = W(t) + c(t)X$$

Therefore

$$[W(t) + c(t)X]X^T > 0$$

$$W(t)X^T + c(t)XX^T > 0$$

But (1a) implies

$$c(t)XX^T > -W(t)X^T$$

Therefore

$$c(t) > \frac{-W(t)X^T}{XX^T}$$

Similarly for a sample pattern x belonging to category $C_2$, the correction factor c(t) at step t in case of error can be expressed as

$$c(t) < \frac{-W(t)X^T}{XX^T}$$

Combining the two results, the correction factor c(t) when decision error occurs at step t has a lower bound

$$c(t) > \left[\frac{|W(t)X^T|}{XX^T}\right]\operatorname{sgn}(x)$$

where sgn(x) is either + or -, depending on the correct classification of the sample pattern x.

## APPENDIX B

The set of sample patterns of known classifications is shown below on the sequence used in training.

| $x_1$ | $x_2$ | sgn(x) | $x_1$ | $x_2$ | sgn(x) | $x_1$ | $x_2$ | sgn(x) |
|---|---|---|---|---|---|---|---|---|
| 0.35 | 0.85 | - | 0.55 | 0.40 | + | 0.85 | 0.30 | + |
| 0.35 | 0.65 | - | 0.30 | 0.50 | - | 0.50 | 0.20 | + |
| 0.35 | 0.55 | - | 0.45 | 0.60 | - | 0.35 | 0.20 | + |
| 0.65 | 0.40 | + | 0.25 | 0.70 | - | 0.15 | 0.70 | - |
| 0.40 | 0.70 | - | 0.45 | 0.80 | - | 0.45 | 0.15 | + |
| 0.35 | 0.45 | - | 0.25 | 0.60 | - | 0.15 | 0.50 | - |
| 0.40 | 0.75 | - | 0.50 | 0.30 | + | 0.55 | 0.85 | - |
| 0.35 | 0.40 | - | 0.75 | 0.60 | + | 0.45 | 0.20 | + |
| 0.70 | 0.40 | + | 0.50 | 0.25 | + | 0.80 | 0.40 | + |
| 0.65 | 0.35 | + | 0.50 | 0.50 | - | 0.65 | 0.80 | - |
| 0.30 | 0.70 | - | 0.25 | 0.35 | - | 0.70 | 0.80 | - |
| 0.70 | 0.50 | + | 0.30 | 0.20 | + | 0.80 | 0.35 | + |
| 0.65 | 0.25 | + | 0.55 | 0.70 | - | 0.15 | 0.20 | - |
| 0.70 | 0.55 | + | 0.20 | 0.80 | - | 0.80 | 0.50 | + |
| 0.65 | 0.15 | + | 0.70 | 0.20 | + | 0.65 | 0.85 | + |
| 0.30 | 0.60 | - | 0.45 | 0.25 | + | 0.80 | 0.25 | - |
| 0.60 | 0.20 | + | 0.20 | 0.40 | - | 0.75 | 0.70 | + |
| 0.75 | 0.25 | + | 0.45 | 0.30 | + | 0.85 | 0.60 | + |
| 0.60 | 0.15 | + | 0.20 | 0.50 | - | 0.85 | 0.45 | + |
| 0.75 | 0.40 | + | 0.20 | 0.65 | - | 0.85 | 0.80 | - |

Results from the first group of experiments are shown below. The computation time obtained is the average execution time for 3 runs on 360/75.

| No. of Sample Patterns per Segment | Average Execution Time (Sec.) | Array Area (Bytes) | Final Weight Vector Obtained | | |
|---|---|---|---|---|---|
| | | | $w_1$ | $w_2$ | $w_3$ |
| 4 | .29 | 76 | .51 | -.12 | -.30 |
| 5 | .46 | 92 | .46 | -.28 | -.15 |
| 6 | .35 | 108 | .46 | -.28 | -.15 |
| 10 | .44 | 172 | .46 | -.28 | -.15 |
| 12 | .91 | 204 | .43 | -.32 | -.10 |
| 15 | .52 | 252 | .47 | -.29 | -.15 |
| 20 | 1.50 | 332 | .42 | -.34 | -.08 |
| 30 | .88 | 492 | .39 | -.35 | -.05 |
| 60 | 1.34 | 972 | .40 | -.36 | -.05 |

Results from the second group of experiments are shown below.

| S in L(n)=S(n-1) | Average Execution Time (Sec.) | Final Weight Vector Obtained | | |
|---|---|---|---|---|
| | | $w_1$ | $w_2$ | $w_3$ |
| 0 | 1.07 | .43 | -.32 | -.10 |
| .005 | .59 | .44 | -.31 | -.11 |
| .01 | .52 | .44 | -.29 | -.12 |
| .015 | .46 | .45 | -.28 | -.14 |
| .02 | .40 | .45 | -.27 | -.15 |
| .025 | .38 | .45 | -.26 | -.16 |
| .03 | .45 | .46 | -.25 | -.17 |
| .04 | .28 | .52 | -.17 | -.12 |
| .045 | .29 | .53 | -.14 | -.13 |
| .05 | .30 | .54 | -.12 | -.14 |

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