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SET OF CUT SETS AND OPTIMUM FLOW

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SET OF CUT SETS AND OPTIMUM FLOW

by

W. Mayeda and M. E. Van Valkenburg

Abstract

An important unsolved problem in the theory of communication nets is the enumeration of the properties of a set of edge flows necessary to give a required terminal flow from one vertex to another. For example, there is no simple method for obtaining a set of edge flows to give maximum terminal flow. The relationship of these flows and the conditions necessary to obtain maximum flow are important practical problems in systems in which edge flow is limited; in the telephone system for example. Clearly an improvement would result if it were possible to reduce some edge flows and still maintain the same terminal flow.

The method to be presented stems from the work of Ford and Fulkerson which relates maximum terminal flow to the cut set separating the terminals. A new set of cut sets called a "set of M-cut sets" is introduced from which it is possible to improve edge flows while maintaining maximum terminal flow. We consider flow from vertex i to vertex j in a lossless non-oriented communication net G, and let $\psi_{ij}(e)$ be the edge flow in e such that $\psi_{ij}(e) \leq C(e)$ where C(e) is the edge capacity of e. Then one of the interesting results is that for a given set $\{\psi_{ij}(e)\}$ of edge flows which gives a maximum flow from i to j in G, there exists another set $\{\psi'_{ij}(e)\}$ of edge flows which also gives a maximum flow from i to j in G such that

$$\psi'_{ij}(e) \leq \psi_{ij}(e)$$

for every edge in G with at least one of $\psi'_{ij}(e)$ is strictly less than the corresponding edge flow $\psi_{ij}(e)$ if and only if $\{\psi_{ij}(e)\}$ can not be obtained by using a set of M cut sets with respect to vertices i and j.

Another interesting result is that for any set $\{\psi_{ij}(e)\}$ of edge flows, there exists a set $\{\psi'_{ij}(e)\}$ of edge flows which gives the same terminal flow from i to j as $\{\psi_{ij}(e)\}$ such that (1) $\psi'_{ij}(e) \leq \psi_{ij}(e)$ for all edge flows and (2) every path from i to j to which a nonzero path flow is assigned, the order to have $\{\psi'_{ij}(e)\}$, intersect every cut set in a set of M cut sets with respect to vertices i and j.

SET OF CUT SETS AND OPTIMUM FLOW

Introduction

An important unsolved problem in the theory of communication nets is the enumeration of the properties of a set of edge flows necessary to give a required terminal flow from one vertex to another. The relationship of these flows and the conditions necessary to obtain maximum flow are important practical problems in systems in which edge flow is limited; in the telephone system for example. Clearly an improvement would result if it were possible to reduce some edge flows and still maintain the same terminal flow.

The method to be presented stems from the work of Ford and Fulkerson [1] which relates maximum terminal flow to the cut set separating the terminals. A new set of cut sets is introduced from which it is possible to improve edge flows while maintaining maximum terminal flow. Further, the best assignment for edge flows may be determined in one step.

For the time being, we will consider only lossless non-oriented communication nets [2]. Let G be such a net containing edge e. We consider flow from vertex i to vertex j in G, and let $\psi_{ij}(e)$ be the edge flow in e such that $\psi_{ij}(e) \leq C(e)$ where C(e) is the edge capacity of e. Let $P_{ij} = \{e_1, e_2, \ldots, e_k\}$ be a path from i to j in G. Then $\psi(P_{ij})$ is called a path flow which satisfies

$$\psi(e_r) \leq C(e_r) - \psi_o(e_r) \tag{1}$$

for $r = 1, 2, \ldots, k$, where $\psi_o(e_r)$ is the edge flow of e_r which has been assigned to e_r initially.

A flow from i to j in G for G intact is called a terminal flow and is symbolized by $\psi_{ij}(G)$ which is determined for a particular assignment of edge flows $\{\psi_{ij}(e)\}$. The symbol $\{\psi(P^r_{ij})\}$ is used to represent a set of all path flows to obtain a terminal flow from i to j. Observe that $\{\psi(P^r_{ij})\}$ gives a set $\{\psi_{ij}(e)\}$ of edge flows where

$$\psi_{ij}(e) = \sum_r \psi^r_{ij}(e) ; \; e \in P^r_{ij} \quad . \tag{2}$$

To illustrate these notation conventions, consider the communication net of Figure 1. There we have paths $P^1_{ij} = \{abc\}$, $P^2_{ij} = \{de\}$, $P^3_{ij} = \{afe\}$, and $P^4_{ij} = \{bcdf\}$. Suppose that $\psi(P^1_{ij}) = 1$, $\psi(P^2_{ij}) = 2$, $\psi(P^3_{ij}) = 3$, and $\psi(P^4_{ij}) = 4$. Then $\psi(a) = 4$, $\psi(b) = \psi(c) = 5$, $\psi(d) = 6$, $\psi(e) = 5$, and $\psi(f) = 7$. The set of edge flows $\{\psi_{ij}(e)\}$ consists of $\psi(a)$, $\psi(b)$, $\psi(c)$, $\psi(d)$, $\psi(e)$, and $\psi(f)$. Then in order that these path flows form a set of path flows, the edge capacities in G must be $C(a) \geq 4$, $C(b) \geq 5$, $C(c) \geq 5$, $C(d) \geq 6$, $C(e) \geq 5$, and $C(f) \geq 7$. From the example, it is seen that we can obtain all possible sets of path flows from a given set of edge flows. There may be many sets of edge flows which have the same terminal flow, $\psi_{ij}(G)$, of course.

We next define the saturated state for edges and cut sets. If $\psi_{ij}(e) = C(e)$ in a given set of edge flows, $\{\psi_{ij}(e)\}$, then e is said to be a <u>saturated edge</u>. An edge is said to be a <u>basic saturated edge</u> if it is saturated and the flows through the edge which constitute $\psi_{ij}(e)$ are all in one direction. When a cut set S_{ij} consists of saturated edges only, then S_{ij} is said to be a <u>saturated cut set</u>. Similarly, when S_{ij} consists of basic saturated edges only, then S_{ij} is a <u>basic saturated cut set</u> [3].

To illustrate these saturated quantities, consider the communication net of Figure 1 with $C(a) = 4$, $C(d) = 6$, $C(f) = 7$, and $C(e) = 5$, and edge flows

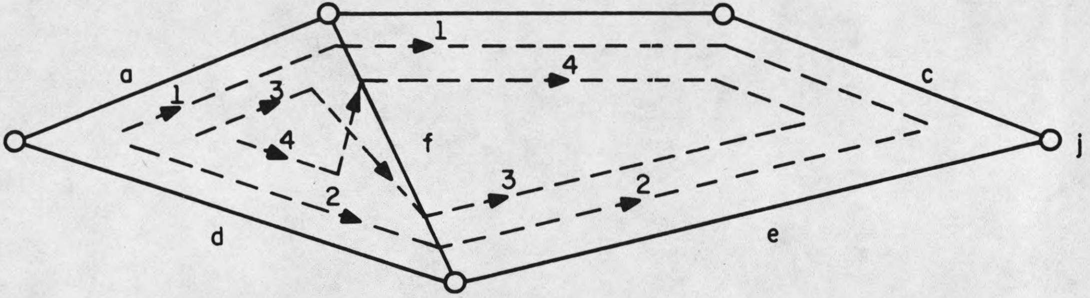


Fig. 1. A communication net and path flows.

are as given. Then cut set $S_{ij} = \{afe\}$ is a saturated cut set, but not a basic saturated cut set. However, cut set $S'_{ij} = \{ad\}$ is a basic saturated cut set.

The Set of W Cut Sets

The two notions upon which we obtain a new method for forming sets of edge flows to match a prescribed terminal flow are the following: (1) Let $\{\psi'_{ij}(e)\}$ and $\{\psi''_{ij}(e)\}$ be two sets of edge flows which can be assigned to net G to give the same terminal flow $\psi_{ij}(G)$. Then

$$\{\psi'''_{ij}(e) = \alpha\psi'_{ij}(e) + (1 - \alpha)\ \psi''_{ij}(e)\} \tag{3}$$

is a set of edge flows which also gives the same terminal flow $\psi_{ij}(G)$ for $0 \leq \alpha \leq 1$. (2) If the edge capacity C(e) of every edge in a net G is changed to $C'(e) = \alpha\, C(e)$, $0 \leq \alpha < \infty$, then the set of edge flows can be $\{\alpha\psi_{ij}(e)\}$.

Consider a non-separable [4] communication net G consisting of v vertices (terminals). Let S^1_{ij}, $S^2_{ij}, \ldots$, and S^{v-1}_{ij} be linearly independent cut sets which separate i and j. Suppose that a terminal flow from i to j, $\psi_{ij}(G)$, causes S^p_{ij}, $p = 1,2,\ldots,v-1$, to be basic saturated cut sets. Then it is clear that there exists a set $\{\psi(P^r_{ij})\}$ of path flows which gives $\psi_{ij}(G)$ and every path P^r_{ij} whose path flow $\psi(P^r_{ij})$ is nonzero passes through each cut set exactly once. Similarly, by assigning proper flows to only those paths which pass through every cut set exactly once, we can obtain a set of path flows $\{\psi(P^r_{ij})\}$ which will give the maximum terminal flow from i to j. In general, there will always be a set of linearly independent cut sets in a net such that by assigning nonzero flows only to those paths which pass through every cut set exactly once we can obtain a maximum terminal flow. The existence of such a cut set will be considered once we have defined a special set of W cut sets and given a method for its determination. A set of W cut sets is obtained as follows: The set of cut sets S_{ij}^{11}, $S_{ij}^{12}, \ldots,$ $S_{ij}^{1k_1}$ are those whose values, $\sum_{e \in S} C(e)$, are the smallest among those of all cut sets which separate i and j. Suppose cut sets S_{ij}^{11}, $S_{ij}^{12}, \ldots,$ and $S_{ij}^{1m_1}$ ($m_1 \leq k_1$) are linearly independent. Hence cut sets $S_{ij}^{1m_1+1}, \ldots,$ and $S_{ij}^{1k_1}$ can be obtained by a linear combination of these m_1 cut sets. It is clear that when a maximum terminal flow from i to j is given to G, the k_1 cut sets become all basic saturated cut sets.

We modify net G by multiplying all edge capacities by α^1 if the edge is in any of the cut sets being considered. Let the resultant net be $G(\alpha^1)$. Next choose the smallest α^1 which satisfies $1 \leq \alpha^1 < \infty$ such that we obtain a new cut set, S_{ij}^{21} to $S_{ij}^{2k_2}$ whose values are minimum of all of those which separate i and j in $G(\alpha^1)$. Let that value of α_o^1 and net with $\alpha^1 = \alpha_o^1$ be $G(\alpha_o^1)$. Notice that values of cut sets $S_{ij}^{11}, \ldots,$ and $S_{ij}^{1k_1}$ are also minimum in $G(\alpha_o^1)$. Let $S_{ij}^{21}, \ldots, S_{ij}^{2m_2}$ ($m_2 \leq k_2$) and the cut sets $S_{ij}^{11}, \ldots,$ and $S_{ij}^{1m_1}$ be linearly independent among $S_{ij}^{21}, \ldots, S_{ij}^{2k_2}$, $S_{ij}^{11}, \ldots,$ and $S_{ij}^{1k_1}$.

This procedure is repeated by multiplying edge capacities by α^2, to all edges in the cut sets being considered, and then selecting the smallest value α_o^2 which produces new cut sets whose values are the smallest among all those which separate i and j in the resulting net $G(\alpha_o^1\ \alpha_o^2)$.

The general pattern is now apparent. Let $S_{ij}^{p1}, \ldots,$ and $S_{ij}^{pk_p}$ in $G(\alpha_o^1\ \alpha_o^2\ \text{---}\ \alpha_o^{p-1})$ be the cut sets whose values are the smallest among all cut sets which separate i and j in the net when $\alpha^{p-1} = \alpha_o^{p-1}$ and none of these except

$$S_{ij}^{11}, \ldots, S_{ij}^{1m_1},\ S_{ij}^{21}, \ldots, S_{ij}^{2m_2}, \ldots,\ S_{ij}^{p-1\ 1}, \ldots, \text{ and } S_{ij}^{p-1\ m_{p-1}} \tag{4}$$

are the smallest when $\alpha^{p-1} < \alpha_o^{p-1}$. Let $S_{ij}^{p1}, \ldots, S_{ij}^{pm_p}$ and these in (4) are linearly independent among all cut sets which separate i and j in $G(\alpha_o^1 \alpha_o^2 \cdots \alpha_o^{p-1})$. Then we included $S_{ij}^{p1}, \ldots,$ and $S_{ij}^{pm_p}$ in the set of cut sets in (4).

We continue the process just described until one of the following two cases occurs:

Case 1: There is no cut set in $G(\alpha_o^1, \ldots \alpha_o^{p-1})$ which separates i and j and cannot be obtained by a linear combination of the cut sets in (4). Then a set of W cut sets with respect to i and j consists of the linear independent cut sets in (4).

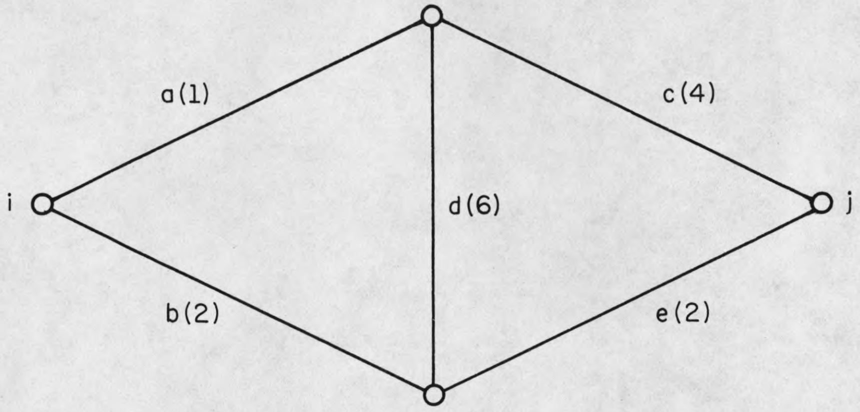
Case 2: There exists at least one cut set in $G(\alpha_o^1, \ldots \alpha_o^{p-1})$ which separates i and j but which cannot be obtained by a linear combination of the cut sets of (4). In order to be Case 2, we require that there is no α^{p-1}, $1 < \alpha^{p-1} < \infty$, which will produce at least one new independent cut set $S_{ij}^{p\,r}$ with respect to (4) in $G(\alpha_o^1 \alpha_o^2 \ldots \alpha_o^{p-1})$ whose value is the smallest of those of cut sets which separate i and j.

Suppose the values of cut sets S_{ij}^{p1}, S_{ij}^{p2}, ..., and $S_{ij}^{pk_p}$ in $G(\alpha_o^1 \alpha_o^2 \ldots \alpha_o^{p-1})$ can not be minimum for any value of α^{p-1} $(1 \leq \alpha^{p-1} < \infty$. Also suppose that

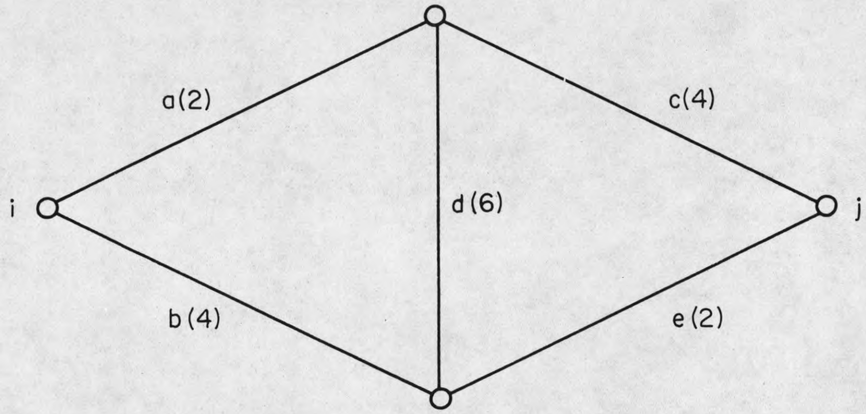
$$S_{ij}^{p1}, \ldots, S_{ij}^{pm_p} \tag{5}$$

and cut sets in (4) are linearly independent among $S_{ij}^{11}, \ldots, S_{ij}^{1k_1}, \ldots, S_{ij}^{p-11}$, $\ldots, S_{ij}^{p-1\,k_{p-1}}$, $S_{ij}^{p1}, \ldots,$ and $S_{ij}^{pk_p}$, then a set of W cut sets with respect to vertices i and j consists of the independent cut sets which are in the cut sets of (4) and (5).

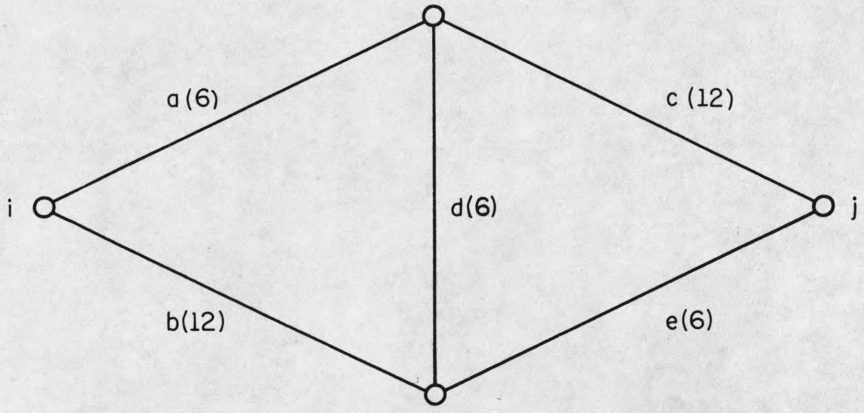
To illustrate the determination of the set of W cut sets, consider the nets of Figure 2, where the value inside the parenthesis is the capacity of



(a) a net G



(b) net G $(\alpha_0^1 = 2)$



(c) net G $(\alpha_0^1 = 2, \alpha_0^2 = 3)$

Fig. 2. Nets G, $G(\alpha_0^{1})$ and $G(\alpha_0^{1}, \alpha_0^{2})$

the edge. From (a) of the figure, $S_{ij}^{11} = \{ab\}$ is the only cut set whose value is minimum among all those of cut sets which separates i and j in the net. By multiplying edges and and b by $\alpha_p^1 = 2$, we produce a new cut set $S_{ij}^{21} = \{cd\}$ whose value is minimum in G with $\alpha_o^1 = 2$ as shown in (b) in the figure. Multiplying all edges in S_{ij}^{11} and S_{ij}^{21} by α^2 and setting $\alpha_o^2 = 3$, we have $G(\alpha_o^1 = 2, \alpha_o^2 = 3)$ as shown in (c) of the figure. Now there exists a new cut set $S_{ij}^3 = \{aed\}$ whose value is now the minimum of all applicable cut sets. Thus a set of W cut sets with respect to i and j consists of $\{ab\}$, $\{aed\}$, and $\{cd\}$.

W Cut Sets and Terminal Flow

The set of W cut sets and the terminal flow have a clear relationship which is given by the following theorem:

Theorem 1: For a given communication net G, there exists a set $\{S_{ij}\}$ of linearly independent cut sets which separate i and j and a set of $\{P_{ij}\}$ of paths between i and j of which every path intersects each cut set in $\{S_{ij}\}$ once, such that by assigning nonzero path flows to paths only in $\{P_{ij}\}$ gives any permissible terminal flow $\psi_{ij}(G)$ from i to j not exceeding the terminal capacity.

Proof: We consider the set of W cut sets given by (4) added to (5), and the modified net $G(\alpha_o^1\ \alpha_o^2\ \ldots, \alpha_o^{p-1})$ derived from the given net G. Consider two possible cases.

Case 1: For this case, the terms of (5) are absent in the set of W cut sets. In $G(\alpha_o^1 \ldots, \alpha_o^{p-1})$, all S_{ij}^{rt} in (4) are basic saturated cut sets when a maximum terminal flow exists from i to j and so there exists a set of path flows $\{\psi'(P_{ij}^r)\}$ which produce maximum terminal flow in such a way that each path with nonzero flow intersects once. Let $\{\psi'_{ij}(e)\}$ be the set of edge flows corresponding to $\{\psi'(P_{ij}^{r})\}$. By dividing each of these path flows by

$$A = \alpha_o^1 \alpha_o^2 \alpha_o^3 \ldots \alpha_o^{p-1} \tag{6}$$

is equivalent to dividing every edge flow by A. Thus edge flow in e becomes $\psi'_{ij}(e)/A$. On the other hand, the edge capacity of any edge in G is not smaller than 1/A times the edge capacity of the edge in the modified G(A). Thus the set of path flows $\{\psi'(P_{ij}^{r})/A\}$ can be assigned to G. This clearly gives a maximum terminal flow from i to j in G. Any other terminal flow can be obtained by assigning K times every path flow where $0 \leq K \leq 1$, and so the theorem is true for this case.

Case 2: When terms corresponding to (5) are in the set of W cut sets, there exists at least one edge in G(A) whose edge capacity is the same as that in the original net G. Let these edges be e_1, e_2,..., and e_q. Also let $G^-(A)$ be the net obtained by removing these edges from G(A). Then in $G^-(A)$ the cut sets corresponding to (5) become basic saturated cut sets under the same conditions that S_{ij}^{p1},..., and $S_{ij}^{pm_p}$ do because every edge in $S_{ij}^{p+1\;1}$,..., and $S_{ij}^{p+1\;m_{p+1}}$ which is not multiplied by α_o^{p-1} are removed to obtain G'(A) from G. Thus as far as G'(A) is concerned, Case 1 will apply. Since a set of path flows of G' can be assigned to G, the theorem is true for this case. Q.E.D.

The Set of M Cut Sets

We next distinguish a set of M cut sets from the set of W cut sets previously considered.

Definition 2: A set of M cut sets with respect to i and j is a set of linearly independent cut sets $\{S_{ij}\}$ which separate i and j in such a way that there exists a set of paths from i to j, $\{P^r_{ij}\}$ which intersect each cut set in $\{S_{ij}\}$ only once, and a maximum terminal flow from i to j can be obtained by assigning nonzero path flows to only these paths in $\{P^r_{ij}\}$.

Theorem 2: For any set of edge flows $\{\psi_{ij}(e)\}$ giving maximum terminal flow from i to j, there exists a set of M cut sets and a corresponding set of path flows such that the set of edge flows $\{\psi'_{ij}(e)\}$ has the property that for every edge in G

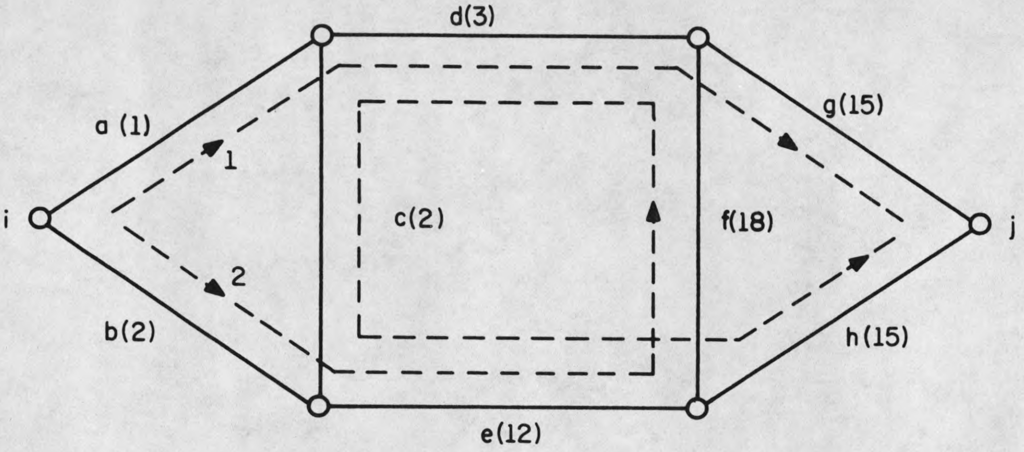
$$\psi'_{ij}(e) \leq \psi_{ij}(e) \tag{7}$$

Proof: Consider a net G' obtained from a given net G by leeting every edge capacity equal the edge flow given by $\{\psi_{ij}(e)\}$. We can obtain a set of W cut sets with respect to i and j of G' and by that we can obtain a set of edge flows $\{\psi'_{ij}(e)\}$ which gives the maximum terminal flow from i to j in G' which must be equal to the maximum terminal flow from i to j in G. Because every edge capacity in G' is equal to the edge flow in $\{\psi_{ij}(e)\}$, Eq. (7) is true for all edges. Furthermore the set of W cut sets of G' is clearly a set of M cut sets of G with respect to i and j. Thus the theorem is true. Q.E.D.

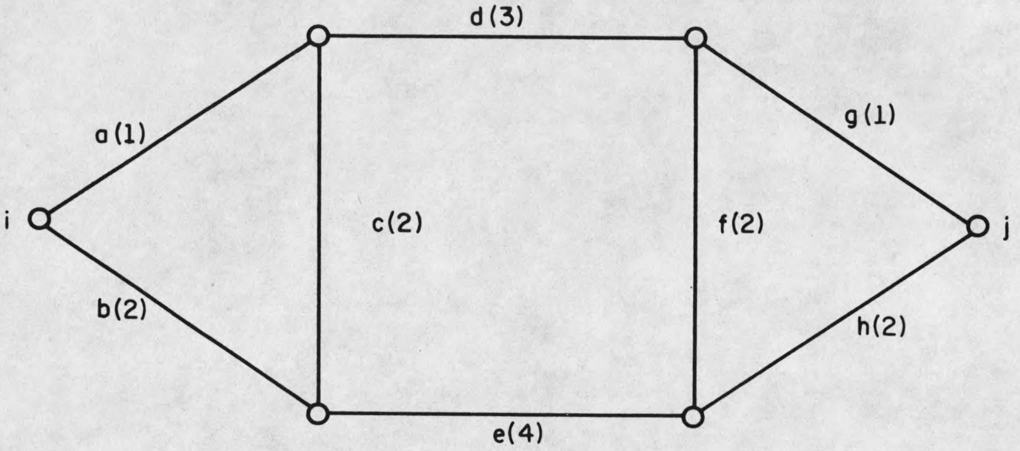
The following example will illustrate the above argument: Consider nets given in Figure 3. From net G in (a) of the figure, we can see that the set of edge flows $\{\psi_{ij}(e)\}$ is

$$\{\psi_{ij}(e)\} = \{\psi_{ij}(a) = 1,\ \psi_{ij}(b) = 2,\ \psi_{ij}(e) = 2,\ \psi_{ij}(d) = 3,$$
$$\psi_{ij}(e) = 4,\ \psi_{ij}(f) = 2,\ \psi_{ij}(g) = 1,\ \psi_{ij}(h) = 2\}$$

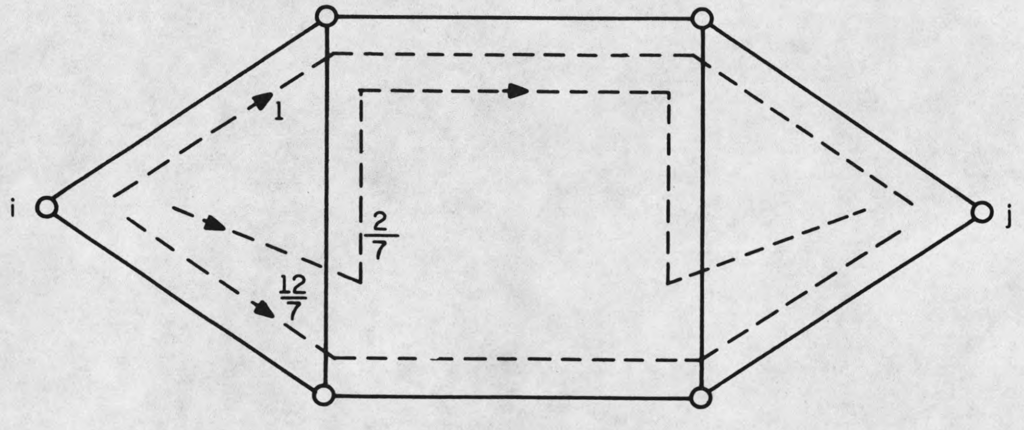
and terminal flow $\psi_{ij}(G)$ is 3.



(a) A net G and given path flows.



(b) G' obtained from G.



(c) Path flows obtained from a set of W cut sets of G'.

Fig. 3. Nets G, G' and Path Flows

Net G' given in (b) of the figure is obtained from G by setting every edge capacity equal to the edge flow in $\{\psi_{ij}(e)\}$. The set of W cut sets with respect to i and j in G' consists of

$$S_{ij}^{11} = \{ab\},\ S_{ij}^{12} = \{gh\},\ S_{ij}^{21} = \{de\},\ S_{ij}^{31} = \{ace\} \text{ and } S_{ij}^{32} = \{gfe\}.$$

The resultant net $G(\alpha_o^1 = \frac{7}{3}, \alpha_o^2 = 3)$ produced by the process of obtaining these W cut sets gives nonzero path flows and by dividing them by $\alpha_o^1 \alpha_o^2 = 7$, we have the nonzero path flows shown in (c) of the figure from which we can obtain the set of edge flows $\{\psi'_{ij}(e)\}$.

In the above example, we can see that the set of edge flows is obtained uniquely from a given set of M cut sets. For convenience, we say that such a set is the set of edge flows corresponding to a given set of M cut sets.

The following theorem shows why a set of M cut sets is important for maximum terminal flows.

Theorem 3: For a given set of edge flows $\{\psi_{ij}(e)\}$ which gives a maximum terminal flow from i to j in a net G, there exists another set of edge flows $\{\psi'_{ij}(e)\}$ which also gives a maximum terminal flow from i to j such that

$$\psi_{ij}(e) \geq \psi'_{ij}(e) \tag{8}$$

for every edge in G and there exists at least one edge in G such that inequality in Eq. (8) holds for the edge, if and only if $\{\psi_{ij}(e)\}$ is not the set of edge flows corresponding to a set of M cut sets with respect to i and j in G.

Proof: The half of the proof is directly from Theorem 2. Thus we only need to prove that if $\{\psi_{ij}(e)\}$ is the set of edge flows corresponding to a set of M cut sets, there is no edge in G such that Eq. (8) holds with inequality.

Consider a net G' obtained by letting the edge capacity of every edge e in G equal to the edge flow $\psi_{ij}(e)$ in $\{\psi_{ij}(e)\}$. Then the set of M cut sets becomes the set of W cut sets with respect to i and j in G'. Since each path flow intersects every cut set in a set of W cut sets exactly once, the value of every cut set in the set is equal to the maximum terminal flow from i to j in net G'. Thus there exist no other set of edge flows $\{\psi'_{ij}(e)\}$ which gives the maximum terminal flow from i to j which satisfies Eq. (8) with the existence of at least one edge e' in G' such that $\psi'_{ij}(e') < \psi_{ij}(e')$. Q.E.D.

By Theorem 3, it is clear that edge flow $\{\psi'_{ij}(e)\}$ corresponding to a set of M cut sets gives an optimum flow from i to j under the condition that we can not increase edge flow of any edge more than that in $\{\psi'_{ij}(e)\}$. Thus for a given set of edge flow $\{\psi_{ij}(e)$ in net G, we can obtain such an optimum flow by one step. That is, by obtaining a set of W cut sets in G' which is the modified net from G by setting every edge capacity equal to the edge flow given by $\{\psi_{ij}(e)\}$, we can obtain a set of edge flows $\{\psi'_{ij}(e)\}$ which satisfies such an optimum flow.

In conclusion, we show the importance of sets of W and M cut sets w.r.t. i-j for terminal flows in communication nets. Also we give a process of obtaining such sets. However the process given in this paper is not a simple way. In order to use the properties about terminal flows and set of W and M cut sets to obtain an economical flow, we would like to have a simple process to obtain a set of W and M cut sets. To find such a process will be an interesting and important future problem.

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An important unsolved problem in the theory of communication nets is the enumeration of the properties of a set of edge flows necessary to give a required terminal flow from one vertex to another. For example, there is no simple method for obtaining a set of edge flows to give maximum terminal flow. The relationship of these flows and the conditions necessary to obtain maximum flow are important practical problems in systems in which edge flow is limited; in the telephone system for example. Clearly an improvement would result if it were possible to reduce some edge flows and still maintain the same terminal flow.

The method to be presented stems from the work of Ford and Fulkerson which relates maximum terminal flow to the cut set separating the terminals. A new set of cut sets called a "set of M-cut sets" is introduced from which it is possible to improve edge flows while maintaining maximum terminal flow. We consider flow from vertex i to vertex j in a lossless non-oriented communication net G, and let $\psi_{ij}(e)$ be the edge flow in e such that $\psi_{ij}(e) \leq C(e)$ where C(e) is the edge capacity of e. Then one of the interesting results is that for a given set $\{\psi_{ij}(e)\}$ of edge flows which gives a maximum flow from i to j in G, there exists another set $\{\psi'_{ij}(e)\}$ of edge flows which also gives a maximum flow from i to j in G such that

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ABSTRACT (continued)

$$\psi'_{ij}(e) \leq \psi_{ij}(e)$$

for every edge in G with at least one of $\psi'_{ij}(e)$ is strictly less than the corresponding edge flow $\psi_{ij}(e)$ if and only if $\{\psi_{ij}(e)\}$ cannot be obtained by using a set of M cut sets with respect to vertices i and j.

Another interesting result is that for any set $\{\psi_{ij}(e)\}$ of edge flows, there exists a set $\{\psi'_{ij}(e)\}$ of edge flows which gives the same terminal flow from i to j as $\{\psi_{ij}(e)\}$ such that (1) $\psi'_{ij}(e) \leq \psi_{ij}(e)$ for all edge flows and (2) every path from i to j to which a nonzero path flow is assigned, the order to have $\{\psi'_{ij}(e)\}$, intersect every cut set in a set of M cut sets with respect to vertices i and j.