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A DECODING METHOD FOR BOSE-CHAUDHURI BINARY CODES USING AN ERROR COUNTING TECHNIQUE

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Introduction

The first line method to solving problems in science has always been the trial and error approach. This approach is generally used until "short cuts" begin to become apparent producing end results in a more predictable manner.

As more sophisticated methods of problem solving are developed, more sophisticated tools are needed to effect these solutions. Many times, some of these tools become too unwieldy and the method must be modified to use tools more suitable to the task.

The coding, transmission, and decoding of information in the binary channel is a good example of an area beset with many problems, and many advances have been made in the development of techniques to solve these problems.

For instance, probably the simplest method of single-error detection is the simple even parity check made on an n-bit code word where the n^{th} bit is the mod 2 sum of the (n-1) bits in the rest of the code word. Hence the number of nonzero bits in the n-bit word as encoded is always even, and the occurrence of an odd number of errors will be indicated by the failure of the parity check.

A good example of a sophisticated procedure for error detection and correction is Peterson's procedure for implementation of Bose-Chaudhuri-Hocquenhem group codes[*]. These codes have very good error-correction and

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[*] W. H. Peterson, "Encoding and Error-Correction Procedures for the Bose-Chaudhuri Codes," <u>IRE Trans. on Information Theory</u>, IT-6, pp. 459-470 (1960).

detection capabilities. However, the circuits needed to encode and decode information and correct as many errors as possible are quite complex because they involve arithmetic operations in Galois fields, even though the digits of the code words are binary.

In some of these methods of information coding, direct implementations of the theory may not be practical from any one of several points of view. A system may not have time to perform very complex operations in decoding, and economic reasons may put limits on the amount of hardware that may be employed in the implementation of a decoding procedure.

The method proposed here is an error correction procedure for BCH codes in which the hardware is kept simple at the expense of operating time. It is based on the property of BCH codes that information about the number of errors in a received code word is more readily obtainable, under certain conditions, than that about the location of the digits in error. The procedure involves successive tentative corrections which are accepted or rejected depending on whether the error count in the received code word has been reduced or not.

The method and the decoding logic are illustrated for a particular BCH code, the BCH (31, 16) code with an appended parity check digit, but extensions of the principle to other BCH codes are clearly possible.

Theoretical Foundations

The proposed decoding method is essentially a trial-and-error correction procedure which, like all such procedures, is straightforward in principle. However, it is critically dependent on the existence of a method to assess the effect of each trial correction. It contrasts with decoding methods which locate errors directly, e.g., Chien[*].

In the case of BCH codes, an error number detection scheme has been proposed by Kastenholz[**]. The method proposed here employs this scheme to check whether or not a given trial correction was successful in reducing the number of errors in the received code word.

The procedure will be illustrated by means of a particular BCH code. We arbitrarily select the BCH (31, 16) code over the field $GF(2^5)$ in which the minimum distance between any of the 2^{16} code points is 7. Thus, up to 3 errors can always be corrected. (Details of the development of this code are given in Appendix II.) For reasons which will become apparent later, it is advantageous to append an odd parity check digit to this code. The augmented code, the BCH (32, 16) code, can correct up to 3 errors and detect the existence of 4 errors. The odd parity of the parity check digit ensures that no transmitted code word is all zeros.

Kastenholz's error number detection scheme involves the selection of any two distinct B_m's from the formula

$$B_m = \frac{(S_1)^{2m+1} + S_{2m+1}}{S_{2m-1}}$$

where the S_j are syndromes and $m = 1,2,\ldots$ (see Appendix I). The number of errors in a code word can then be determined from the syndromes and the two B_m's as follows:

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[*] R. T. Chien, "Cyclic Decoding Procedures for Bose-Chaudhuri-Hocquenhem Codes," IEEE Trans. on Inf. Theory, IT-10, pp. 357-363 (1964).

[**] C. E. Kastenholz, "General Purpose Computer Encoding and Decoding of Error Control Codes," NEC Proc., pp. 699-703 (1964).

for no errors, all S_j are zero

for 1 error, $B_{m1} = B_{m2} = 0$

for 2 errors, $B_{m1} = B_{m2} \neq 0$

for 3 or 4 errors, $B_{m1} \neq B_{m2}$.

In the case of the BCH (32, 16) code, more than 4 errors are of course assumed not to occur. However, the parity check digit must be used to resolve the 3 <u>or</u> 4 error indication (the parity check will fail for 3 errors, as it would for any odd number of errors, but will not fail for 4 errors), so that it is possible to distinguish the correctable cases from the case of 4 errors, which can not be corrected. This distinction is available as soon as the syndromes and the B_m functions are formed - a feature which is especially attractive in systems which retransmit uncorrectable words.

If the syndromes and the B_m functions indicate that the received code word contains 1, 2, or 3 errors, then an iterative procedure is entered which involves, at each step, (1) the tentative complementation of successive binary digits of the received code word, and (2), the application of the error number detection scheme described above to the tentatively modified code word as an acceptance criterion: If the number of errors has been reduced by the tentative complementation, an error has actually been corrected and the tentative complementation is accepted. Otherwise, it is rejected. Note that an indicated reduction of 2 or more errors can not occur for a single tentative correction.

If any step results in an error count of zero, then the remaining digits are clearly correct and can be simply copied. The procedure can similarly be terminated early when all information digits have been processed, as errors in the check digits are of no particular interest.

Implementation of the Correction Procedure for the Augmented BCH (31, 16) Code

Aside from facilities to tentatively complement and recomplement successive binary digits of the received code word, facilities must be provided to form, for each tentative correction,

a) the syndromes,

b) the two B_m functions.

In the augmented BCH (31, 16) code, three syndromes must be formed: S_1, S_3, and S_5. The two B_m functions selected are

$$B_1 = \frac{(S_1)^3 + S_3}{S_1} \quad \text{and} \quad B_2 = \frac{(S_1)^5 + S_5}{S_3} \quad .$$

Because S_1 or S_3 might become 0 during a trial correction, the B functions actually generated for comparisons with each other (and with zero) are

$$B_1 = S_3[(S_1)^3 + S_3] \text{ and } B_2 = S_1[(S_1)^5 + S_5] \ .$$

These modifications however do not affect the validity of the error number determination.

The three syndromes, derived from the received vector r(x) and the parity check matrix H^T by

$$r(x) = H^T = 0 + e(x) \cdot H^T = (S_1 \ S_3 \ S_5)$$

where e(x) is the error vector, are generated in three syndrome registers, S_1, S_3, and S_5 (see Figure 1)*, from the received code word. These syndrome registers are 5-stage cyclic shift registers arranged to perform multiplications modulo $(1 + x^3 + x^5)$ in the Galois field $GF(2^5)$.

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*For a mathematical derivation of the syndrome registers, see Appendix III.

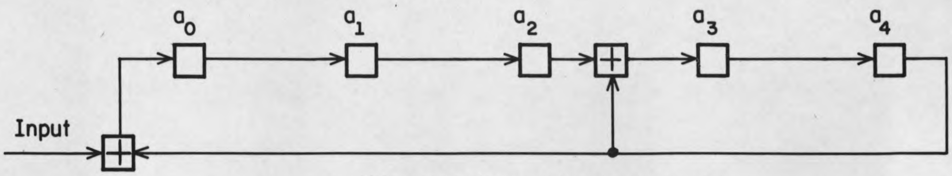

Figure 1a. Register S_1

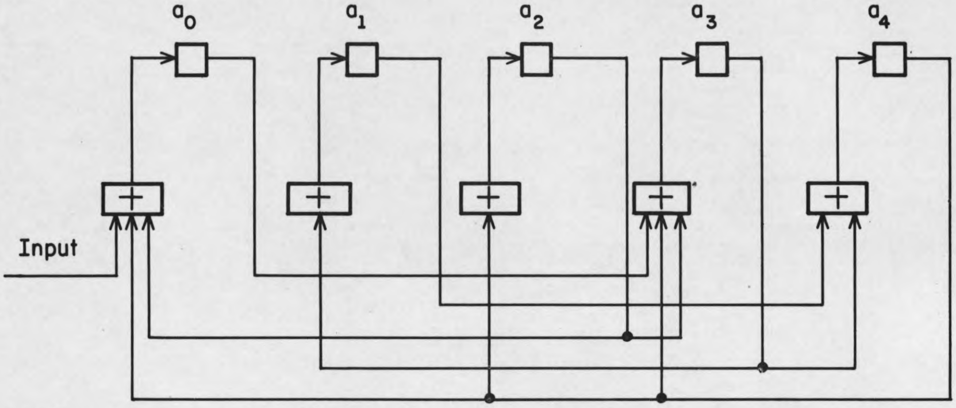

Figure 1b. Register S_3

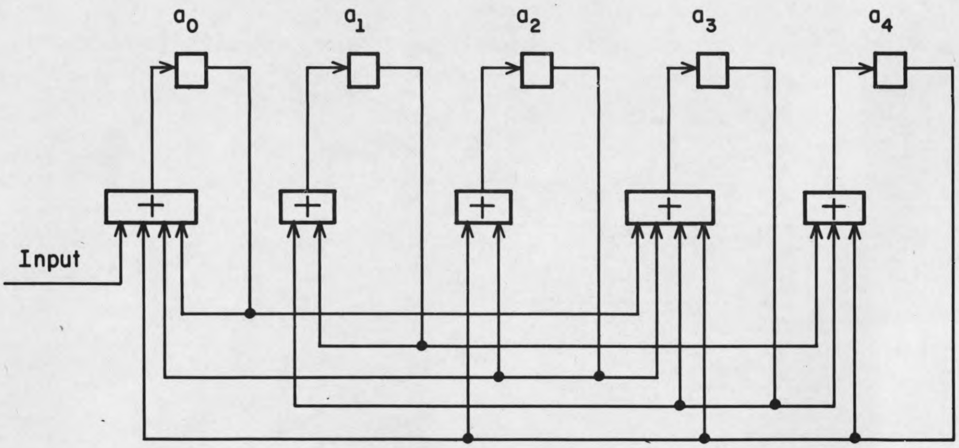

Figure 1c. Register S_5

⊞ Mod. 2 adder

☐ Storage element

Figure 1. Logical Schematics of Syndrome Registers

The two B functions are formed using the syndromes generated in the syndrome registers S_1, S_3, and S_5.

Since $B_1 = S_3[(S_1)^3 + S_3]$ and $B_2 = S_1[(S_1)^5 + S_5]$, two additional 5-stage Galois field shift registers modulo $(1 + x^3 + x^5)$ are needed to form, first, $(S_1)^3$ and $(S_1)^5$ respectively, and secondly, after the indicated polynomial additions in $GF(2^5)$ have been performed in accumulators AC_1 and AC_2, (see Figure 2) to perform the premultiplications by S_3 and S_1, respectively, with the results being placed into the accumulators AC_1 and AC_2. Countdowns and countups in $GF(2^5)$, which are necessary to control the shifts, are performed in countdown register A and countup register B (see Figure 3).

In addition to the equipment listed above, circuitry is needed to detect when all syndromes are zero, to keep track of the number of errors remaining in the code word, and to indicate when a trial correction is successful, i.e., when the error count has been reduced.

Operation of the Decoder (Brute Force Method)

The decoding procedure starts when the 32-bit code word has been received and loaded into the input buffer. We assume for purposes of discussion that bits x_0 to x_{15} are the 16 information bits, bits x_{16} to x_{30} are the BCH check bits, and x_{31} is the appended overall parity check bit. (The arrangement of the information bits within the code word may of course be varied to take advantage of certain statistical properties of error incidence.)

Initially, the overall parity check is computed, the syndromes S_1, S_3, and S_5 are generated and the B-functions are computed.

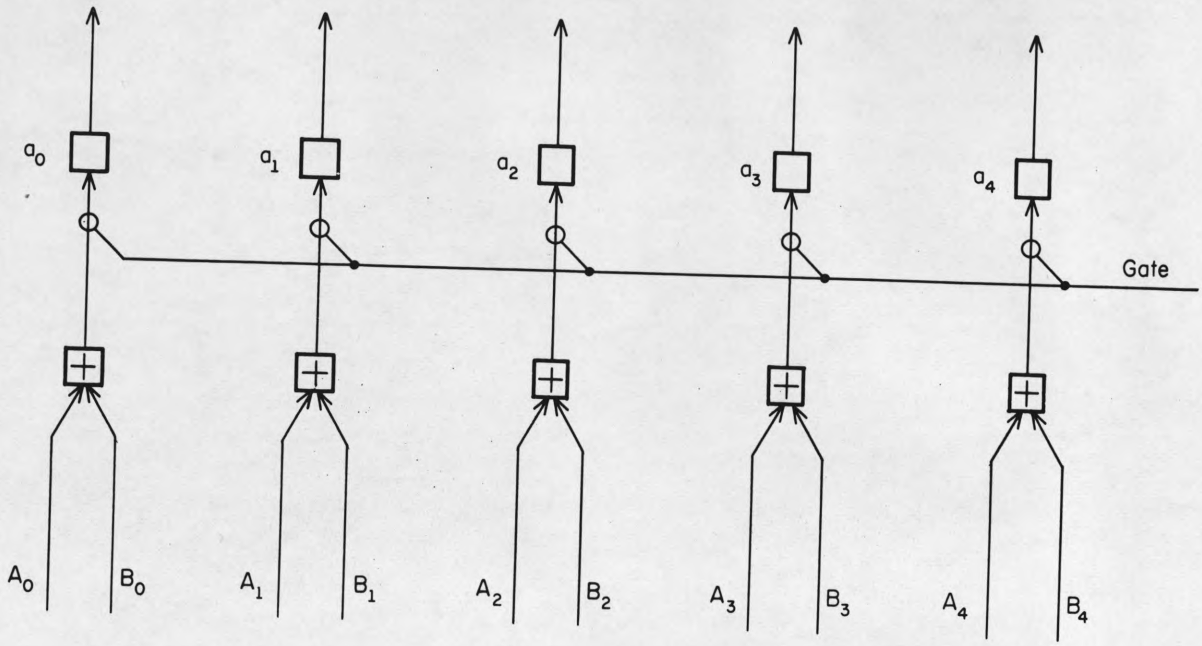

Figure 2. Galois Field (2^5) Adder Register

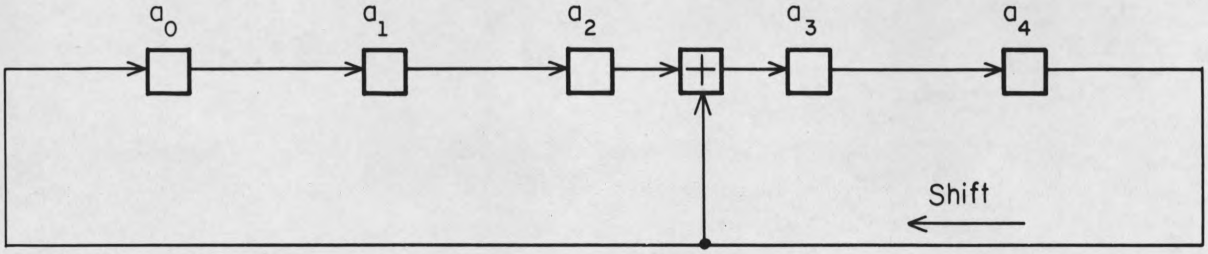

Figure 3a. Register B

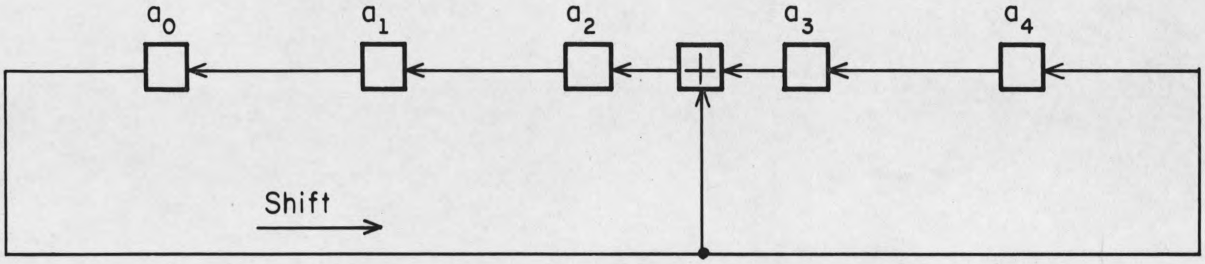

Figure 3b. Register A

Figure 3. Countup and Countdown Registers in GF(2^5)

a) If all syndromes are 0, then no error has occurred and the 16 information bits may be read out directly.

b) If $B_1 \neq B_2$ and the overall parity checks, then 4 errors have occurred during transmission. No correction is attempted under this condition, and an appropriate indication is given immediately. Note that the use of the overall parity check digit for the discrimination of 3 vs. 4 errors causes the case of 3 (correctable) errors in the code word plus a 4th error in the parity check digit to be classified as uncorrectable.

c) In all other cases, the initial error count indicated by B_1 and B_2 is remembered for later comparisons, and the sequential correction procedure is entered.

A cycle $i (i=0,1,2,...,30)$ in the iterative correction procedure consists of the following steps:

1. Perform a tentative correction at position i by complementing x_i.
2. Generate the syndromes S_1, S_3, and S_5 of the tentatively modified code word.
3. If all syndromes are now 0, all errors have been corrected. The remainder of the modified code word, i.e., digits x_i to x_{30}, may be read out directly.
4. If any syndrome is not 0, the functions B_1 and B_2 must be generated and the error count indicated by them compared with the previous error count.
5. If the number of errors was reduced by this tentative correction, then the complementation of x_i is accepted and the correct error count is updated; otherwise, x_i is recomplemented.
6. The digit x_i is now correct and may be read out.

7. The sequence of operations is repeated as cycle (i+1). The correction process terminates when all syndromes are 0 (step 3 above), which of course may not happen until the cycle for $i = 30$.

Improvements in the Method

a) Reduction to k Trial Correction Steps

Errors in the information digits are the only ones which must be corrected. Hence, regardless of the position of the information digits within the word, the correction procedure should be applied only to the k information digits, trapping whatever errors remain in the otherwise useless n-k check digits. Since $n = 2^m-1$ and $k = 2^{m-1}$, for the (31, 16), only about half the digits need be subjected to the correction phase. Of course, all errors may have been corrected in less than k cycles.

b) Elimination of Complete Regeneration of Syndromes in Each Correction Step

The generation of the syndromes, each of which is required not only initially but also in each recursive step of the correction procedure, requires 31 shifts. These shifts, during correction operations, can be eliminated in the following way:

1. An error in the r_0 position of the code word would have a syndrome of $\alpha^0 = 10000$ for S_1, S_3, S_5 if it were the only error present. Therefore, if there had been an error in the low-order position of the code word, we could add the syndrome for that error to S_1, S_3, S_5 and our detection circuit should indicate that the number of errors had decreased.

Remember that shifting right one place in S_1, S_3, S_5 is equivalent to multiplying them by α, α^3, α^5, mod $(1 + x^3 + x^5)$ in $GF(2^5)$ respectively. If we shift the code word and S_1, S_3, S_5 one place to the right, we may now add 10000 to the syndrome registers outputs and check our error count for the next bit in the sequence.

Utilizing this fact requires very little modification to our syndrome registers. We simply place mod 2 adders on the outputs adding 00000 for the initial error number count or 10000 during trial correction steps. This modification as applied to the S_1 register is shown in Figure 4 as an example. In addition, for every successful correction, 10000 is also added to S_1, S_3 and S_5 before shifting for the next cycle to correct the error in the syndrome registers.

The 31 cycle regeneration loop for calculation of S_1, S_3, S_5 in each trial correction has now been eliminated and we have gained a considerable increase in speed. Read-out is accomplished as correction is done as the code word is shifted to the right each trial.

Conclusions

We have developed and discussed a decoding procedure for BCH(n,k) codes under the constraints $m = 2^m - 1$, $d = 2t+1$, over Galois fields $GF(2^m)$ with $t \leq 3$. An error counting technique was employed as a means of judging results of a trial correction scheme rather than obtaining error locations by such methods are put forward by Chien*, Kasami**, Peterson***, et. al. The procedure proposed here seems to be independent of the redundancy of the code. We have developed a circuit for the BCH(32, 16) code which takes 16

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*R. T. Chien, "Cyclic Decoding Procedures for Bose-Chaudhuri-Hocquenhem Codes," IEEE Trans. on Inf. Theory, IT-10, pp. 357-363 (1964).

**T. Kasami, "A Decoding Procedure for Multiple-Error-Correcting Cycle Codes," IEEE Trans. on Inf. Theory, IT-10, pp. 134-138 (1964).

***W. H. Peterson, "Encoding and Error-Correction Procedures for the Bose-Chaudhuri Codes," IRE Trans. on Inf. Theory, IT-6, pp. 459-470 (1960).

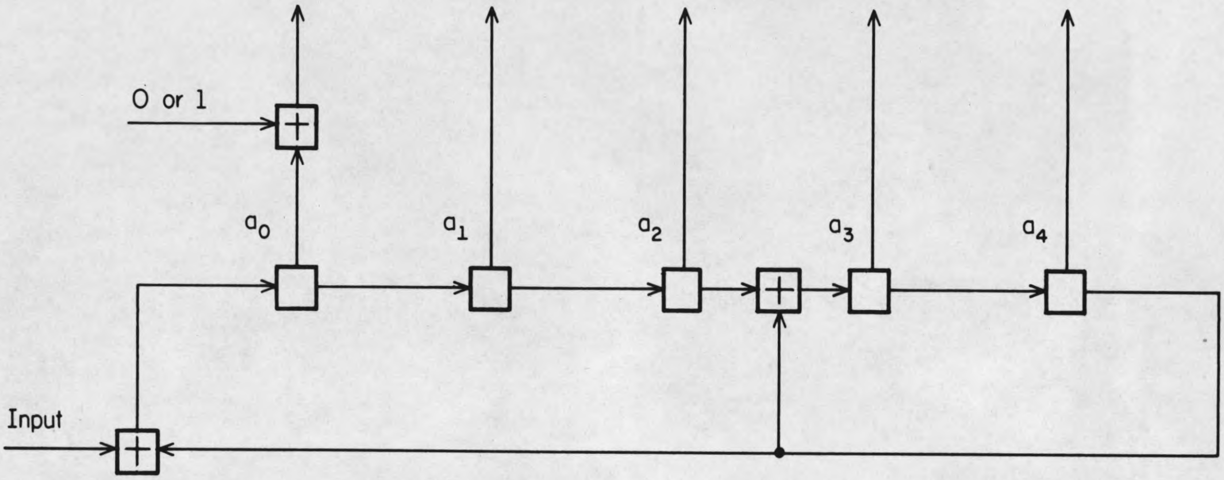

Figure 4. Tentative Correction Modification to Register S_1

correction cycles at the most to correct all acceptable errors in the information digits. As m of GF(2^m) becomes 6, 7, or 8 and the resulting codes expand, the ratio of r/n (r=n-k) decreases for the same value of t. By contrast, Kasami's procedure is dependent on high redundancy codes, the BCH (31, 11) code for example.

The big problem in implementation of the proposed decoding procedure is the error counting operation. No general theory is available at present for error counting in BCH codes. A logical layout of the error counting section of our hardware can be found in Figure 5. The "brute force" approach to the entire decoder system is laid out in Figure 6(a). The final configuration including our modifications is shown in Figure 6(b).

The Galois field arithmetic registers used in this procedure are rather simple in design. However, we paid for simplicity with time. For example, if we wished to multiply α^6 times α^{27}, we place α^{27} in register A and α^6 in register B. We then shift 28 times (until register A equals 10000). Then register B holds the result, α^3. The worst case time for a multiplication will occur when α^{30} is placed in register A to multiply any other α^i. However, as n grows large, the size of the registers involved is dependent only on the factor m of GF(2^m).

The ability to easily count the number of errors in a received code word removes the initial problem involved in Peterson's decoding procedure. In that procedure, the syndrome (power sums) matrix rank is dependent upon how many errors occurred. One way of solving this unknown factor is to assume t errors and find the determinant of M_T. If M_T is singular, try again for t-1 errors, and so on until M_T is non-singular. Massey* developed a procedure

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*Massey, private communication to R. T. Chien.

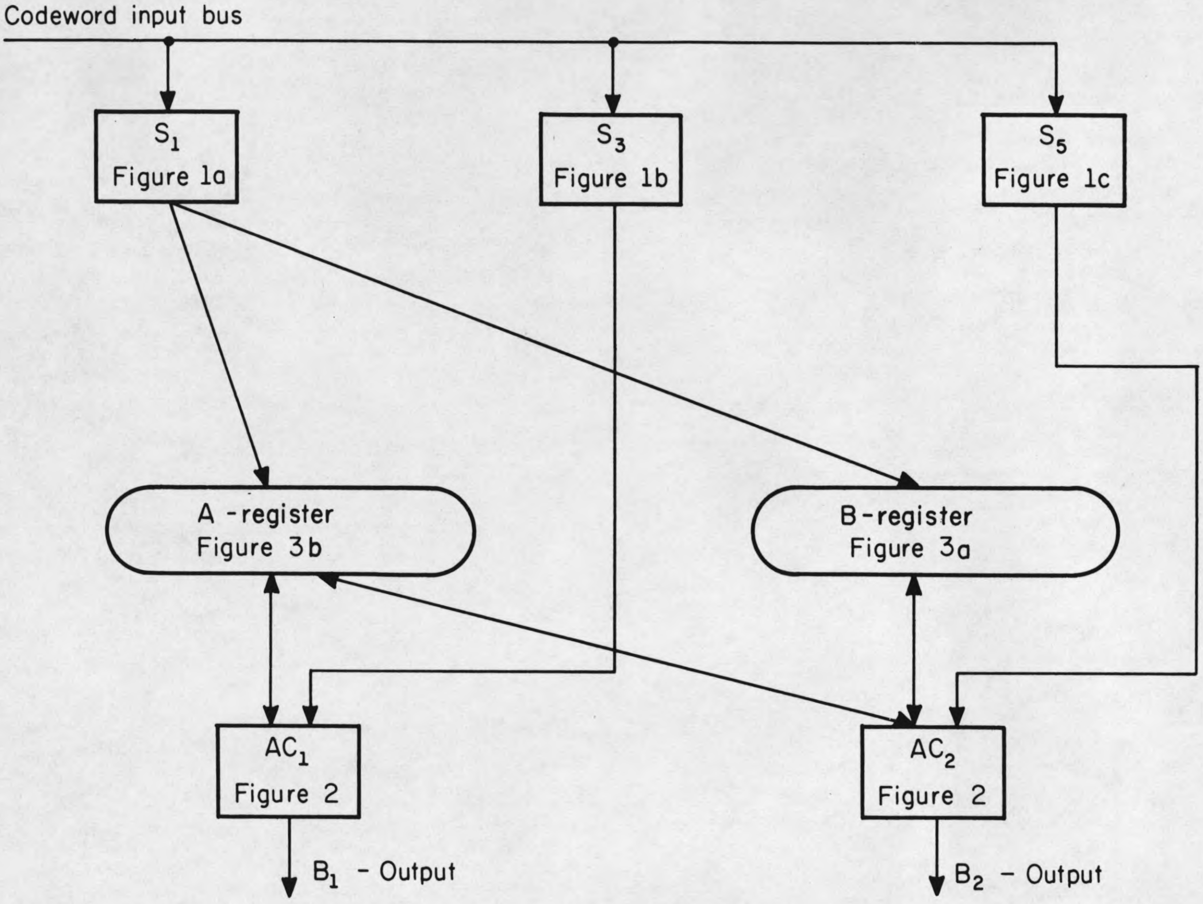

Figure 5. Error Counting Section Logical Layout

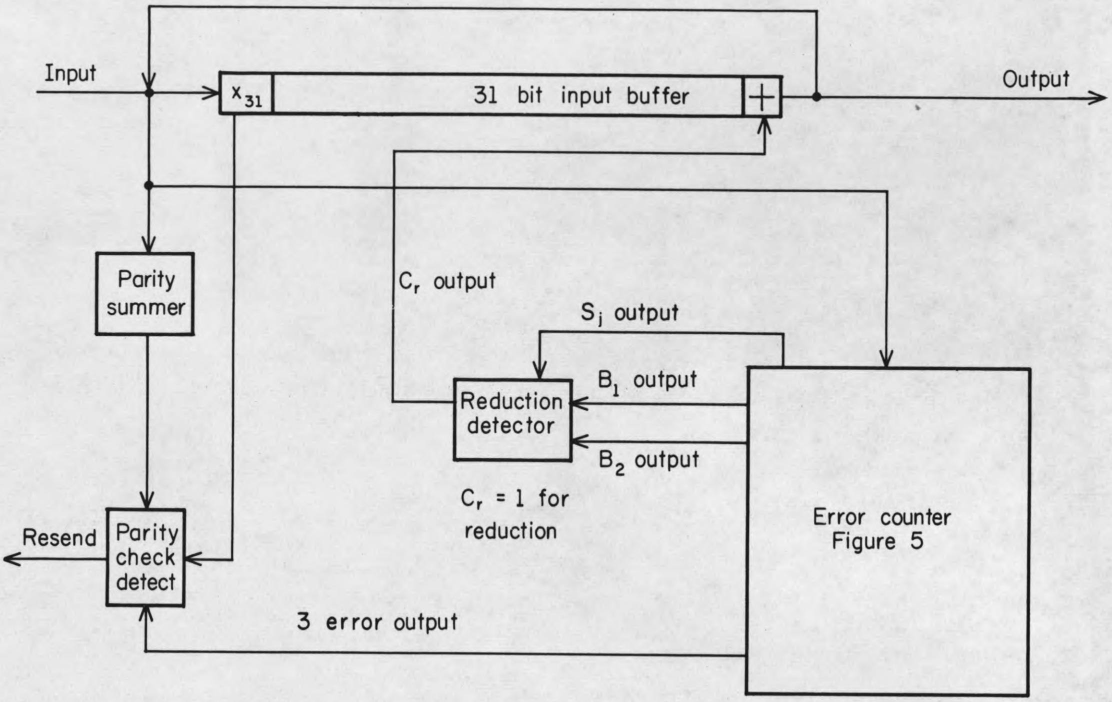

Figure 6a. Brute Force Configuration

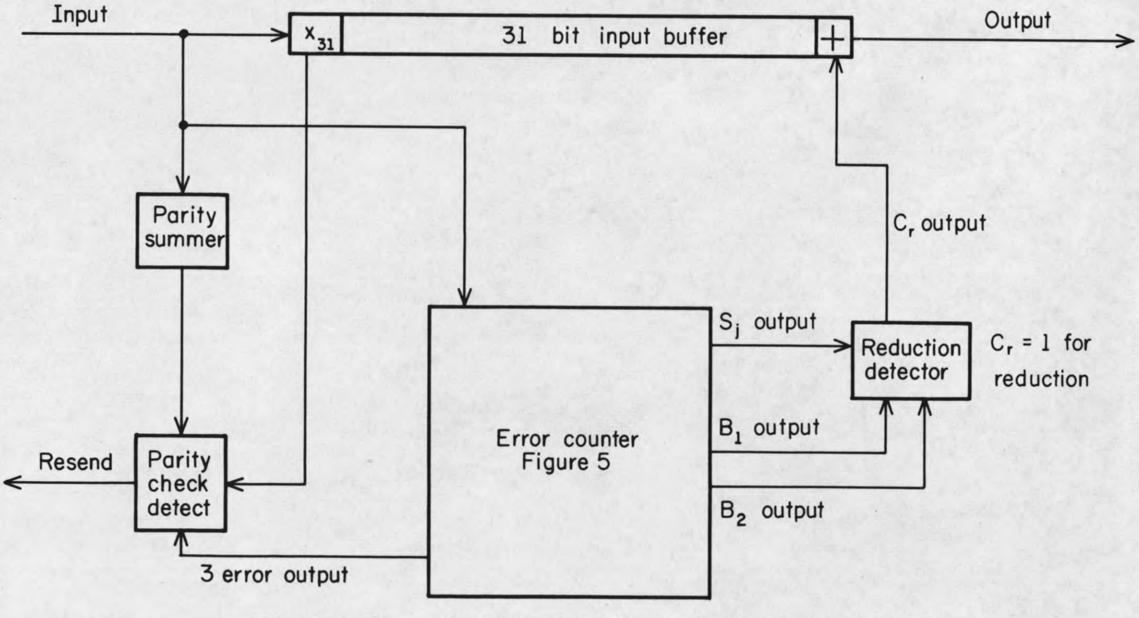

Figure 6b. Modified "Final" Configuration

Figure 6. Decoder System Logical Layouts

that is based on adding errors to the code word until M_T becomes non-singular. The proposed procedure does not rely on the non-singularity of M_T for solution and therefore is more powerful.

In our procedure, the time per correction step and hence the time for decoding one word varies according to the value of k for any BCH (n,k) code where k=n-mt. The position of the left-most error also influences the amount of decoding time by varying the number of correction cycles required.

Extension of this procedure to other BCH (n,k) codes where $t \leq 3$ is fairly straightforward. All $GF(2^5)$ field operating circuits are changed to $GF(2^m)$ and allowances for different operating times are made. A study must be made of the B_1 and B_2 functions to insure that $B_1 \neq B_2$ for 3 or 4 errors. Extending this problem to BCH codes with $t > 3$ will require the formulation of a suitable error counting scheme.

Appendix I

Kastenholz defines the functions B_1 and B_2 as:

$$B_1 = \frac{(S_1)^3 + S_3}{S_1} \qquad B_2 = \frac{(S_1)^5 + S_5}{S_3}$$

with the properties that:

for 1 error, S_1 is α^j

for 2 errors, S_1 is $\alpha^j + \alpha^{j+i}$

for 3 errors, S_1 is $\alpha^j + \alpha^{j+i} + \alpha^{j+k}$

for 4 errors, S_1 is $\alpha^j + \alpha^{j+i} + \alpha^{j+k} + \alpha^{j+h}$.

Actually, the B functions in general have the form

$$B_m = \frac{(S_1)^{2m+1} + S_{2m+1}}{S_{2m-1}}$$

Assume the existence of functions B_m and B_{m+n} $(m,n = 1,2,\ldots)$. For the one-error case, it can be easily seen that

$$B_m = B_{m+n} = 0 .$$

For the two-error cases,

$$B_m = \frac{(\alpha^j + \alpha^{j+i})\ (\alpha^{pj} + \alpha^{pj+pi}) + \alpha^{rj} + \alpha^{rj+ri}}{\alpha^{nj} + \alpha^{nj+ni}}$$

where $n = 2m-1$

$p = 2m$

$r = 2m+1$

$$B_m = \frac{\alpha^{rj+i} + \alpha^{rj+pi}}{\alpha^{nj}\ (\alpha^0 + \alpha^{ni})}$$

$$= \frac{\alpha^{rj}\ (\alpha^{i} + \alpha^{pi})}{\alpha^{nj}\ (\alpha^{0} + \alpha^{ni})}$$

$$= \alpha^{(r-n)j}\ \frac{(\alpha^{i}\ (\alpha^{0} + \alpha^{(p-1)i})}{\alpha^{0} + \alpha^{ni}}$$

Now $n = p-1$, and $r-n = 2$. Then B_m reduces to $B_m = \alpha^{2j+i}$. For the three-error case, the syndromes result from

$$\alpha^{j} + \alpha^{j+i} + \alpha^{j+k}$$

Formulating B_m and B_{m+n} and setting them equal to each other, we find (after some manipulation):

$$\begin{aligned}
&\alpha^{(2m-1)i} + \alpha^{(2m-1)k} + \alpha^{(2m-2)i} + \alpha^{(2m-2)k} \\
&\quad + \alpha^{(2m+2n-1)i + (2m-2)k} + \alpha^{(2m-2)i + (2m+2n-1)k} \\
&\quad + \alpha^{(2m-2n-2)i + (2m-2)k} + \alpha^{(2m-2)i + (2m+2n-2)k} \\
&\quad = 0 .
\end{aligned}$$

Inspection shows that the above equation will be zero only in the case where $i = k$. But in the three-error case, $i \neq 0$, $k \neq 0$ and $i \neq k$. So $B_m \neq B_{m+n}$.

For the final proof of the validity of the procedure, we need to look at the function $B_1 + B_2$ in the case of four-errors.

The formulas for B_1 and B_2 now become the following:

$$B_1 = \frac{(\alpha^{j} + \alpha^{j+i} + \alpha^{j+k} + \alpha^{j+h})\ (\alpha^{2j} + \alpha^{2(j+i)} + \alpha^{2(j+k)} + \alpha^{2(j+h)})}{D_3}$$

$$+ \frac{\alpha^{3j} + \alpha^{3(j+i)} + \alpha^{3(j+k)} + \alpha^{3(j+h)}}{D_3}$$

$$B_2 = \frac{(\alpha^{i} + \alpha^{j+i} + \alpha^{j+k} + \alpha^{j+h})(\alpha^{4j} + \alpha^{4(j+i)} + \alpha^{4(j+k)} + \alpha^{4(j+h)})}{D_1}$$

$$+ \frac{\alpha^{5j} + \alpha^{5(j+i)} + \alpha^{5(j+k)} + \alpha^{5(j+h)}}{D_1}$$

where

$$D_1 = \alpha^{j} + \alpha^{j+i} + \alpha^{j+k} + \alpha^{j+h}$$

and

$$D_3 = \alpha^{3j} + \alpha^{3(j+i)} + \alpha^{3(j+k)} + \alpha^{3(j+h)}$$

A Fortran 60 program was written on the CDC 1604 to evaluate the function $B_1 + B_2$ for all possible permissible values of i, k, and h. The constraints on i, k, and h are the following:

$$i \neq k \neq h \neq 0$$

The values of i, k, and h ran over the following ranges:

i from 1 to 30

k from i+1 to 30

h from k+1 to 30

It is obvious that these ranges satisfy the constraints on i, k, and h. In addition, the program checked only one of each type (i.e., the case $i = 1$, $k = 2$, and $h = 3$, but not also $i = 1$, $k = 3$, and $h = 2$ as the results are the same). Let us remember that this proof by perfect induction was applied only to the case of the $GF(2^5)$ field.

The number of possible combinations for $n = 30$ is

$$N_C = \frac{30!}{3!\ 27!} = 4060$$

No zeros of $B_1 + B_2$ were found in the 4060 possible cases.

In performing this test of the functions B_1 and B_2, we were mainly interested in showing that $B_1 \neq B_2$ in all permissible four-error cases. We must remind the reader that this was done only for the (31, 16) code over $GF(2^5)$. There is an interesting problem here for research by some other party in trying to find the theory behind the behavior of the B_1 and B_2 functions for (n,k) codes over $GF(2^m)$.

Appendix II

We develop here the BCH (31, 16) code used in this paper. We are interested in obtaining the h(x) polynomial for the construction of the encoder.

Recall that the BCH codes are generated by the polynomial, g(x), which has roots

$$\alpha^{m_0}, \alpha^{m_0+1}, \alpha^{m_0+2}, \ldots, \alpha^{m_0+d-2}$$

where α is the primitive element of a field $GF(2^m)$, and d is the minimum Hamming distance between any two code points in the code space.

Using Peterson's notation[*], we set $m_0 = 1$, select α as the primitive element of the field $GF(2^5)$ mod $(\alpha^5 + \alpha^3 + 1)$ and choose $d = 2t + 1 = 7 (t=3)$. Then, $\alpha, \alpha^3, \ldots, \alpha^{2t-1}$ are roots of our code vector $\{f(x)\}$. If $m_i(x)$ denotes the minimum function of α^i, then

$$g(x) = LCM(m_1(x), m_3(x), \ldots, m_{2t-1}(x))$$

For t = 3

$$g(x) = m_1(x)\ m_3(x)\ m_5(x)$$

The minimum functions are all of degree 5.

We can easily see that the roots of the three minimum functions are as follows:

$\alpha^1\ \alpha^2\ \alpha^4\ \alpha^8\ \alpha^{16}$	$m_1(x)$
$\alpha^3\ \alpha^6\ \alpha^{12}\ \alpha^{24}\ \alpha^{17}$	$m_3(x)$
$\alpha^5\ \alpha^{10}\ \alpha^{20}\ \alpha^9\ \alpha^{18}$	$m_5(x)$

We extract the minimum functions from the following equations.

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[*] Peterson, W. W., Error Correcting Codes, MIT Press, Cambridge, Mass., pp. 164-167 (1961).

$$m_1(x) = (X - \alpha)\ (X - \alpha^2)\ (X - \alpha^4)\ (X - \alpha^8)\ (X - \alpha^{16})$$

$$m_3(x) + (X - \alpha^3)\ (X - \alpha^6)\ (X - \alpha^{12})\ (X - \alpha^{24})\ (X - \alpha^{17})$$

$$m_5(x) = (X - \alpha^5)\ (X - \alpha^{10})\ (X - \alpha^{20})\ (X - \alpha^9)\ (X - \alpha^{18})$$

Table I gives the appropriate polynomial configurations for the α^i's. With some mathematical manipulation we arrive at the following minimum functions:

$$m_1(x) = 1 + X^3 + X^5$$

$$m_3(x) = 1 + X + X^2 + X^3 + X^5$$

$$m_5(x) = 1 + X + X^3 + X^4 + X^5$$

More manipulation leads to

$$g(x) = 1 + X^4 + X^5 + X^6 + X^7 + X^8 + X^{10} + X^{12} + X^{13} + X^{14} + X^{15}$$

H(x) is given by the formula $h(x) = \dfrac{X^{31} + 1}{g(x)}$

After much long division laboring, we obtain

$$h(x) = X^{16} + X^{15} + X^{12} + X^7 + X^6 + X^5 + X^4 + 1$$

The encoding scheme used in the computer simulation of this procedure is described by Peterson* as a k stage encoder with connections based upon

$$h(x) = x^n - 1/g(x)$$

Figure 7 shows the logical configuration of the encoder.

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*Op. cit., page 148.

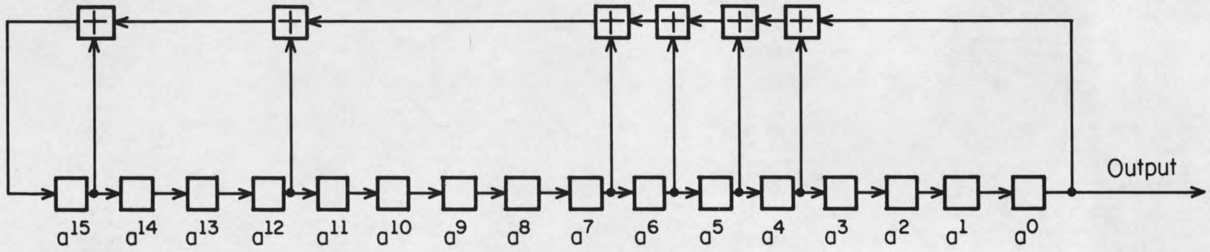


Figure 7. Encoder Register for BCH(31,16) Code

Appendix III

Since the received vector is $r(x)$, $r(x) = c(x) + e(x)$ where $e(x)$ is the error vector, $c(x)$ is divisible by $g(x)$ and $c(x)\, H^T = 0$. Therefore, we are left with

$$r(x) H^T = e(x)\, H^T = (S_1\ S_3\ S_5)$$

Since $m = 5$ and $n = 2^m - 1 = 31$, we are operating in a field $GF(2^5)$ with 32 elements.

The primitive polynomial of degree 5 is

$$K(x) = 1 + x^3 + x^5 \quad \text{or} \quad x^5 = 1 + x^3$$

There are 32 code words corresponding to

$$\alpha^j (0 \leq j \leq 30);\ \alpha^{31} = \alpha^0 = 1$$

and of course

00000

Table I lists all of the powers of α.

A Galois field shift register for $\alpha^5 = \alpha^3 + 1$ is the circuit shown in Figure 1(a).

For the S_3 syndrome, we multiply by α^3

$$1 \cdot \alpha^3 = \alpha^3$$

$$\alpha \cdot \alpha^3 = \alpha^4$$

$$\alpha^2 \cdot \alpha^3 = \alpha^5 = 1 + \alpha^3$$

$$\alpha^3 \cdot \alpha^3 = \alpha^6 = \alpha + \alpha^4$$

$$\alpha^4 \cdot \alpha^3 = \alpha^7 = 1 + \alpha^2 + \alpha^3$$

We determine our new values of a_0, a_1, a_2, a_3, and a_4 as follows:

$$\alpha^3 (a_0 + a_1\alpha + a_2\alpha^2 + a_3\alpha^3 + a_4\alpha^4)$$

$$= a_0\alpha^3 + a_1\alpha^4 + a_2(1 + \alpha^3)$$

$$+ a_3(\alpha + \alpha^4) + a_4(1 + \alpha^2 + \alpha^3)$$

$$= a_0\alpha^3 + a_1\alpha^4 + a_2 + a_2\alpha^3$$

$$+ a_3\alpha + a_3\alpha^4 + a_4 + a_4\alpha^2 + a_4\alpha^3$$

$$= (a_2 + a_4) + \alpha(a_3)$$

$$+ \alpha^2(a_4) + \alpha^3(a_0 + a_2 + a_4)$$

$$+ \alpha^4(a_1 + a_3)$$

So our old values of a_i are converted as:

$$\alpha^3$$

$$a_0 \longrightarrow a_2 + a_4$$

$$a_1 \longrightarrow a_3$$

$$a_2 \longrightarrow a_4$$

$$a_3 \longrightarrow a_0 + a_2 + a_4$$

$$a_4 \longrightarrow a_1 + a_3$$

Now we develop a shift register as shown in Figure 1(b).

For the S_5 syndrome, we must design a circuit from the following equations. Multiplying by α^5 gives:

$$1 \cdot \alpha^5 = 1 + \alpha^3$$

$$\alpha \cdot \alpha^5 = \alpha^6 = \alpha + \alpha^4$$

$$\alpha^2 \cdot \alpha^5 = \alpha^7 = 1 + \alpha^2 + \alpha^3$$

$$\alpha^3 \cdot \alpha^5 = \alpha^8 = \alpha + \alpha^3 + \alpha^4$$

$$\alpha^4 \cdot \alpha^5 = \alpha^9 = 1 + \alpha^2 + \alpha^3 + \alpha^4$$

Again, we determine new values for a_0, a_1, a_2, a_3, and a_4:

$$\alpha^5(a_0 + a_1\alpha + a_2\alpha^2 + a_3\alpha^3 + a_4\alpha^4)$$

$$= \alpha^5 a_0 + a_1\alpha^6 + a_2\alpha^7 + a_3\alpha^8 + a_4\alpha^9$$

$$= a_0(1+\alpha^3) + a_1(\alpha + \alpha^4)$$

$$+ a_2(1 + \alpha^2 + \alpha^3)$$

$$+ a_3(\alpha + \alpha^3 + \alpha^4)$$

$$+ a_4(1 + \alpha^2 + \alpha^3 + \alpha^4)$$

Now collecting terms:

$$= (a_0 + a_2 + a_4)$$

$$+ (a_1 + a_3)\alpha$$

$$+ (a_2 + a_4)\alpha^2$$

$$+ (a_0 + a_2 + a_3 + a_4)\alpha^3$$

$$+ (a_1 + a_3 + a_4)\alpha^4$$

The old values of the a_i's are converted as follows:

a^5

$a_0 \longrightarrow (a_0 + a_2 + a_4)$

$a_1 \longrightarrow (a_1 + a_3)$

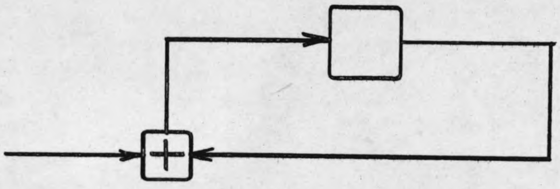
$a_2 \longrightarrow (a_2 + a_4)$

$a_3 \longrightarrow (a_0 + a_2 + a_3 + a_4)$

$a_4 \longrightarrow (a_1 + a_3 + a_4)$

The circuit follows as in Figure 1(c).

For the checking of parity for comparison with x_{31}, we use the following simple circuit.



Circuit for Summing 1's Digits in a Code Word

Figure 8

Acknowledgement

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TABLE I

Representation of GF(2^5)

$\alpha^{0} = 1 = (10000)$
$\alpha^{1} = \alpha = (01000)$
$\alpha^{2} = \alpha^{2} = (00100)$
$\alpha^{3} = \alpha^{3} = (00010)$
$\alpha^{4} = \alpha^{4} = (00001)$
$\alpha^{5} = 1 + \alpha^{3} = (10010)$
$\alpha^{6} = \alpha + \alpha^{4} = (01001)$
$\alpha^{7} = 1 + \alpha^{2} + \alpha^{3} = (10110)$
$\alpha^{8} = \alpha + \alpha^{3} + \alpha^{4} = (01011)$
$\alpha^{9} = 1 + \alpha^{2} + \alpha^{3} + \alpha^{4} = (10111)$
$\alpha^{10} = 1 + \alpha + \alpha^{4} = (11001)$
$\alpha^{11} = 1 + \alpha + \alpha^{2} + \alpha^{3} = (11110)$
$\alpha^{12} = \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} = (01111)$
$\alpha^{13} = 1 + \alpha^{2} + \alpha^{4} = (10101)$
$\alpha^{14} = 1 + \alpha = (11000)$
$\alpha^{15} = \alpha + \alpha^{2} = (01100)$
$\alpha^{16} = \alpha^{2} + \alpha^{3} = (00110)$
$\alpha^{17} = \alpha^{3} + \alpha^{4} = (00011)$
$\alpha^{18} = 1 + \alpha^{3} + \alpha^{4} = (10011)$
$\alpha^{19} = 1 + \alpha + \alpha^{3} + \alpha^{4} = (11011)$
$\alpha^{20} = 1 + \alpha + \alpha^{2} + \alpha^{2} + \alpha^{4} = (11111)$
$\alpha^{21} = 1 + \alpha + \alpha^{2} + \alpha^{4} = (11101)$
$\alpha^{22} = 1 + \alpha + \alpha^{2} = (11100)$
$\alpha^{23} = \alpha + \alpha^{2} + \alpha^{3} = (01110)$
$\alpha^{24} = \alpha^{2} + \alpha^{3} + \alpha^{4} = (00111)$
$\alpha^{25} = 1 + \alpha^{4} = (10001)$
$\alpha^{26} = 1 + \alpha + \alpha^{3} = (11010)$
$\alpha^{27} = \alpha + \alpha^{2} + \alpha^{4} = (01101)$
$\alpha^{28} = 1 + \alpha^{2} = (10100)$
$\alpha^{29} = \alpha + \alpha^{3} = (01010)$
$\alpha^{30} = \alpha^{2} + \alpha^{4} = (00101)$
$\alpha^{31} = \alpha^{0} = 1$

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13. ABSTRACT

The implementation of a decoding scheme for Bose-Chaudhuri (n,K) cyclic codes is discussed. A method of error counting is formulated and a trial-and-error correction operation is developed based on error count reduction by successful trial corrections

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