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A NOTE ON COMPUTING TIME FOR RECOGNITION OF LANGUAGES GENERATED BY LINEAR GRAMMARS

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Abstract

It is shown that (1) there exists a language L_o which is generated by a linear grammar and is not T(n)-recognizable by any on-line multi-tape Turing machine if $\overline{\lim_{n \to \infty}} T(n)/(n/\log n)^2 = 0$, and (2) any language generated by a linear grammar is n^2-recognizable by an on-line single-tape Turing machine in the sense of Hartmanis and Stearns.

Let V_T denote a finite set of symbols and let V_T^* denote the set of all finite sequences, including a null sequence, over V_T. The model of on-line Turing machine by Hartmanis, Stearns (1964) and Hennie (1966) will be used. An on-line Turing machine has a one-way input tape using the symbols of V_T and a one-way output tape using two symbols "1" and "0" besides working tapes. An on-line Turing machine is said to recognize a subset L of V_T^* if and only if for any input sequence on V_T, the n-th output digit is "1" if the sequence of the first n input symbols is in L and "0" otherwise. L is said to be T(n)-recognizable if and only if there is an on-line Turing machine which recognizes L and, for any input sequence, prints the n-th output digit in T(n) or fewer operations. For the details refer to Harmanis (1964) and Hennie (1966). For the definitions of context-free grammars, context-free languages and linear grammars, refer to Chomsky (1963).

F. C. Hennie (1966) showed a set of sequences (set A) for which a very good lower bound on the computation time (number of machine steps) to be recognized by an on-line (multi-tape) Turing machine can be found. A slight variation of Hennie's set A will be used here.

The set A' will be defined to be the set of sequences over the alphabet $\{0,1,s,c\}$ which satisfy the following conditions:

a) The first symbol and the last symbol is s, and the sequence consists of a number of identical-length blocks of 0's and 1's, consecutive blocks being separated by single s's or "scs".

b) The total number of blocks exceeds 2^k, where k is the number of 0's and 1's in each block. The 2^k-th block and the (2^k+1)-th block are separated by "scs", and other consecutive blocks are separated by single s's.

c) The mirror image of the last zero-one block is the same as a zero-one block in the first 2^k blocks.

Let n denote the length of input sequence. By following the same arguments as those by Hennie (1966), we have Lemma 1.

Lemma 1: If set A' is T(n)-recognizable by an on-line Turing machine, then

$$T(n) \geq C\, n^2/(\log n)^2 ,$$

where C is a constant dependent only on the number of tapes, the number of tape symbols and the number of internal states of the machine.

Let $R(\varphi)$ denote the reversal of sequence φ. For example, if $\varphi = 011$, then $R(\varphi) = 110$. Let

$$L_0 = \{\varphi_0 s\varphi_1\ s\varphi_2\ c\varphi_3 sR(\varphi_1)s \mid \varphi_0, \varphi_2, \varphi_3 \in \{0,1,s\}^* \text{ and } \varphi_1 \in \{0,1\}^*\}.$$

Then it is easy to see that L can be generated by the linear grammar $G_0 = (V_{N0}, V_{T0}, P_0, S)$, where V_{T0} (the set of terminal symbols) = $\{0,1,s,c\}$, V_{N0} (the set of nonterminal symbols) = $\{S,X,Y\}$, S is the initial symbol and P_0 (the set of production rules):

$$\begin{array}{lll} S \rightarrow 0S, & X \rightarrow 0X0, & Y \rightarrow 0Y, \\ S \rightarrow 1S, & X \rightarrow 1X1, & Y \rightarrow 1Y, \\ S \rightarrow sS, & X \rightarrow sYs, & Y \rightarrow sY, \\ S \rightarrow sXs, & Y \rightarrow c, & Y \rightarrow Y0, \\ & & Y \rightarrow Y1, \\ & & Y \rightarrow Ys, \end{array}$$

The language L_0 may be considered a variation of the language which was shown to be not n-recognizable by Hartmanis and Stearns (1964).

The next lemma follows directly from the definitions of A' and L_0.

Lemma 2: Assume that a sequence φ on alphabet $\{0,1,s,c\}$ satisfies conditions a) and b) of the definition of set A'. Then, $\varphi \in A'$ if and only if $\varphi \in L_0$.

Since it can be decided in real-time computation (Yamada, 1962), whether a given sequence satisfies conditions a) and b) of set A', it follows from Lemma 2 that if L_0 is T(n)-recognizable by an on-line Turing machine, then A' is also T(n)-recognizable by an on-line Turing machine. Consequently, the next theorem follows from Lemma 1.

Theorem 1: If L_0 is T(n)-recognizable by an on-line multi-tape Turing machine, then there exists a constant C such that

$$T(n) \geq C(n/\log n)^2 .$$

On the other hand, the following theorem holds.

Theorem 2: Any language generated by a linear grammar is Cn^2-recognizable by an on-line single-tape Turing machine, where C is a constant.

This theorem was provied in (Kasami, 1965a). A sketch of a simpler proof is shown in the Appendix.

It was shown that any context-free language is n^3-recognizable by an on-line Turing machine (Kasami, 1965b; Younger, 1966; Torii, 1966). There is a gap between $(n/\log n)^2$ and n^3.

Appendix

A SKETCH OF THE PROOF OF THEOREM 2

Let $G = (V_N, V_T, P, S)$ be a given linear grammar, where V_N is the finite set of nonterminal symbols, V_T is the finite set of terminal symbols, P is a set of rules and $S \in V_N$ is the initial symbol. For a rule $g: X \rightarrow \varphi$, let $\ell(g)$ and $r(g)$ denote X and φ respectively. By the definition of linear grammar (Chomsky, 1963), $r(g)$ contains at most one nonterminal symbol. Without loss of generality, it can be assumed that G is a normal grammar (Chomsky, 1963). Then, for any g in P, $r(g)$ is of the form:

$$aX, \ Xa \text{ or } a,$$

where $a \in V_T$, and $X \in V_N$. Let $a_i \in V_T$ be the i-th input symbol and let $\xi_i = a_1 a_2 \ldots a_i$. Let the labelled directed graphs $G_1, G_2, \ldots, G_i, \ldots$ be defined as follows:

1) Let G_0 be a graph consisting of a vertex v_0.
2) Suppose that G_{i-1} is defined. Add vertex v_i to G_{i-1} and connect vertices v_{i-1} and v_i by an edge from v_{i-1} to v_i with label a_i. Let G_{i0} denote the resulting graph.
3) Suppose that G_{ij-1} $(1 \leq j < i)$ has been defined. Then, for each rule g of P, draw an edge with label $\ell(g)$ from v_{i-j} to v_i, if and only if there is a directed path with label sequence $r(g)$ from v_{i-j} to v_i in G_{ij-1}. Let G_{ij} denote the resulting graph.
4) Let $G_i = G_{ii}$.

Then, it follows directly from the construction of G_i that ξ_i belongs to the language generated by G if and only if G_i contains an edge from v_0 to v_i with label S. The proof is essentially the same as those in (Younger, 1966; Torii, 1966). The process mentioned above can be implemented by an on-line single-tape Turing machine as follows: Divide the semi-infinite working tape into sections of the same length $T_{01}, T_{11}, T_{02}, T_{12}, \ldots, T_{0k}, T_{1k}, \ldots$, where each section $T_{\ell k}$ can store any terminal symbol, any subset of V_N and a mark * simultaneously. Assume that the finite state control unit has two working memories M_1 and M_2 which can store the last input symbol and the content of one section of the working tape respectively. Then, the operation of this machine proceeds as follows:

1) Read the first input symbol a_1 from the input tape. Store a_1 and $*a_1$ in R_1 and T_{01} respectively. For each rule g of P with $r(g) = a_1$, store $\ell(g)$ in T_{01}. If T_{01} contains the initial symbol S, write "1" on the output tape, otherwise write "0". Move the head to section T_{11}.

2) For $0 \leq \ell < 2$, let $\mu(h,\ell) \equiv h-\ell \pmod 2$, $0 \leq \mu(h,\ell) < 2$. Assume that $a_{i-k-\ell}$ ($*a_1$ if $i-k-\ell = 1$) and the set of labels (nonterminal symbols) of edges from $v_{i-1-\ell-k}$ to $v_{i-\ell-1}$ in G_{i-1} are stored in $T_{\mu(i-1,\ell)k}$ for $1 \leq k+\ell < i$ and $0 \leq \ell < 2$, and that the head of the working tape is on section $T_{\mu(i,0)1}$.

2.1) Read a_i from the input tape. Erase the content of $T_{\mu(i,0)1}$. Store a_i in $T_{\mu(i,0)1}$ and in M_1 instead of a_{i-1}. For each rule g with $r(g) = a_i$, store $\ell(g)$ in $T_{\mu(i,0)1}$. Move the head to section $T_{\mu(i,0)2}$.

2.2) Assume that $a_{i-\ell-k+1}$ and the set of labels of edges from $v_{i-\ell-k}$ to $v_{i-\ell}$ in G_{ij-1} ($j \leq i$) are stored in $T_{\mu(i,\ell)k}$ ($1 \leq k+\ell \leq i$, $0 \leq \ell < 2$ and if

$\ell = 0$, $k < j$) and that the head is on section $T_{\mu(i,0)j}$. Erase the content of $T_{\mu(i,0)j}$. Copy the contents of T_{0j-1} and T_{1j-1} into M_2. Transfer a_{i-j+1}, which is already in $T_{\mu(i,1)j-1}$, into $T_{\mu(i,0)j}$. If mark * is written in $T_{\mu(i,1)j-1}$, then also write mark * in $T_{\mu(i,0)j}$.

For each rule g of P such that either (1) $r(g) = a_{i-j+1}X$, where $X \in V_N$ is in $T_{\mu(i,0)j-1}$ or (2) $r(g) = Xa_i$, where $X \in V_N$ is in $T_{\mu(i,1)j-1}$, write $\ell(g)$ in $T_{\mu(i,0)j}$. All information required for the procedure above is currently stored in M_1 and M_2.

If mark * is not written in $T_{\mu(i,0)j}$, then move the head to section $T_{\mu(i,0)j+1}$ and repeat the same procedure for j+1. For each j, the number of required steps can be bounded above by a constant independently of i and j.

2.3) Suppose that mark * is written in $T_{\mu(i,0)j}$. Write "1" on the output tape if $T_{\mu(i,0)j}$ contains the initial symbol S, and write "0" otherwise. Return the head to section $T_{\mu(i+1,0)1}$. For each i, the number of steps required for producing the i-th output after reading the i-th input can be bounded above by Ci, where C is a constant independent of i.

Consequently, the number of steps required for producing the i-th output from the beginning can be bounded above by $Ci^2/2$.

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It is shown that (1) there exists a language L_o which is generated by a linear grammar and is not T(n)-recognizable by any on-line multi-tape Turing machine if $\overline{\lim}_{n \to \infty} T(n)/(n/\log n)^2 = 0$ and (2) any language generated by a linear grammar is n^2-recognizable by an on-line single-tape Turing machine in the sense of Hartmanis and Stearns.

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